Please show **all** your work! Answers without supporting work will not be given credit.
Clearly mention what theorem(s), if any, you are using.
Write answers in spaces provided.
You have 45 minutes to complete this Quiz.
You can get MAXIMUM $(2+2+2+2+2)+(2+6+4+3)+(5+(2+5+3)) = 40$ marks.

Name:

1. $(2+2+2+2+2)$

   Find whether the following statements are TRUE or FALSE. *Justify your answers as briefly as possible.*

   (a) If $\vec{v}$ is a vector in $\mathbb{R}^n$ and A is a subspace of $\mathbb{R}^n$ then $\vec{v} \cdot \mathrm{Proj}_A \vec{v} \geq 0$.

   (b) There exists a subspace V of $\mathbb{R}^5$ such that V and $V^\perp$ are isomorphic. Here $V^\perp$ denotes the orthogonal complement of V.

   (c) If $A\vec{x} = \vec{b}$ is a consistent system, then it always has a solution in $(\ker A)^\perp$.

   (d) There is no orthogonal transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that

   $$T \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} \text{ and } T \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$$

   (e) There exist real invertible 3×3 matrices A and S such that $S^T A S = -A$.

2. $(2+6+4+3)$

   Consider the 2-dimensional subspace V in $\mathbb{R}^3$ defined by the equation $x_1 + 3x_2 - 2x_3 = 0$.

   (a) Construct a basis $\mathscr{B} = \{v_1, v_2\}$ of V such that neither v_1 nor v_2 has any negative components.

   (b) Find the $\mathscr{B}$-matrix of orthogonal projection onto V. Recall that there is a direct formula for this matrix.

   (c) Find an orthonormal basis $\mathscr{A}$ of V from $\mathscr{B}$ using the Gram-Schimdt process.

   (d) Find the $\mathscr{A}$-matrix of orthogonal projection onto V.

3. $(5+(2+5+3))$

   Let V be the subspace of $\mathbb{R}^{n\times n}$ consisting of matrices A such that $A^T = A$. These are called the symmetric matrices.

   (a) What is the dimension of V? Note that your answer will depend on n.

   (b) Consider the case when $n = 2$ i.e. let V be the subspace of symmetric 2×2 matrices. Let

   $$T(M) = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} M + M \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

   be a linear transformation from V to V.

   i. Find a basis of V. Check that the number of basis vectors matches the answer from part (a) in the case of $n = 2$.

   ii. What is the matrix of T with respect to this basis?

   iii. Find the determinant of the matrix obtained in part (ii).