Math 1800-C Handout 6: Vector Fields and recap on Integrals

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Exercise 1

Fill the boxes with 'certainly', 'possibly', or 'certainly not'.

(a) The plot of the vector field $\vec{G}(x,y) = \vec{F}(2x,2y)$ is $\boxed{}$ drawn by doubling the length of all the arrows in the plot of $\vec{F}(x,y)$.

(b) If the flow lines for the vector field $\vec{F}(x,y)$ are all concentric circles centered at the origin, then the dot-product $\vec{F}(x,y)\cdot(x\hat{\imath}+y\hat{\jmath})$is $\boxed{}$ equal to zero.

(c) If the flow lines for the vector field $\vec{F}(x,y)$ are all straight lines parallel to the constant vector $\vec{v} = 3\hat{\imath}+5\hat{\jmath}$, then $\vec{F}(x,y)$ is $\boxed{}$ equal to $\vec{v}$.

(d) The flow lines of the vector field $\vec{F}(x,y) = e^x\hat{\imath}+y\hat{\jmath}$ $\boxed{}$ cross the X-axis.

Exercise 2

Find a vector field whose flow lines are of the form $\vec{r}(t) = t\hat{\imath}+t^2\hat{\jmath}$.

Exercise 3

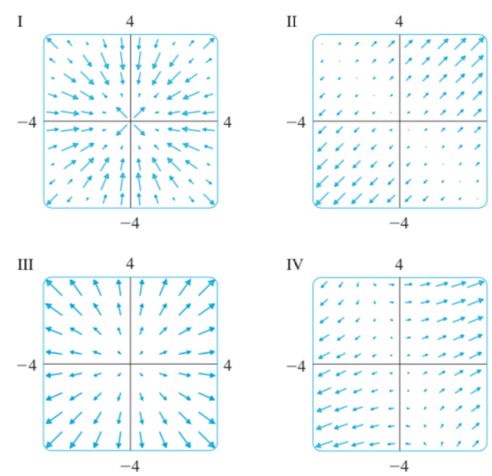
Match the following functions with their gradient vector fields.

(a) x^2+y^2

(b) $x(x+y)$

(c) $(x+y)^2$

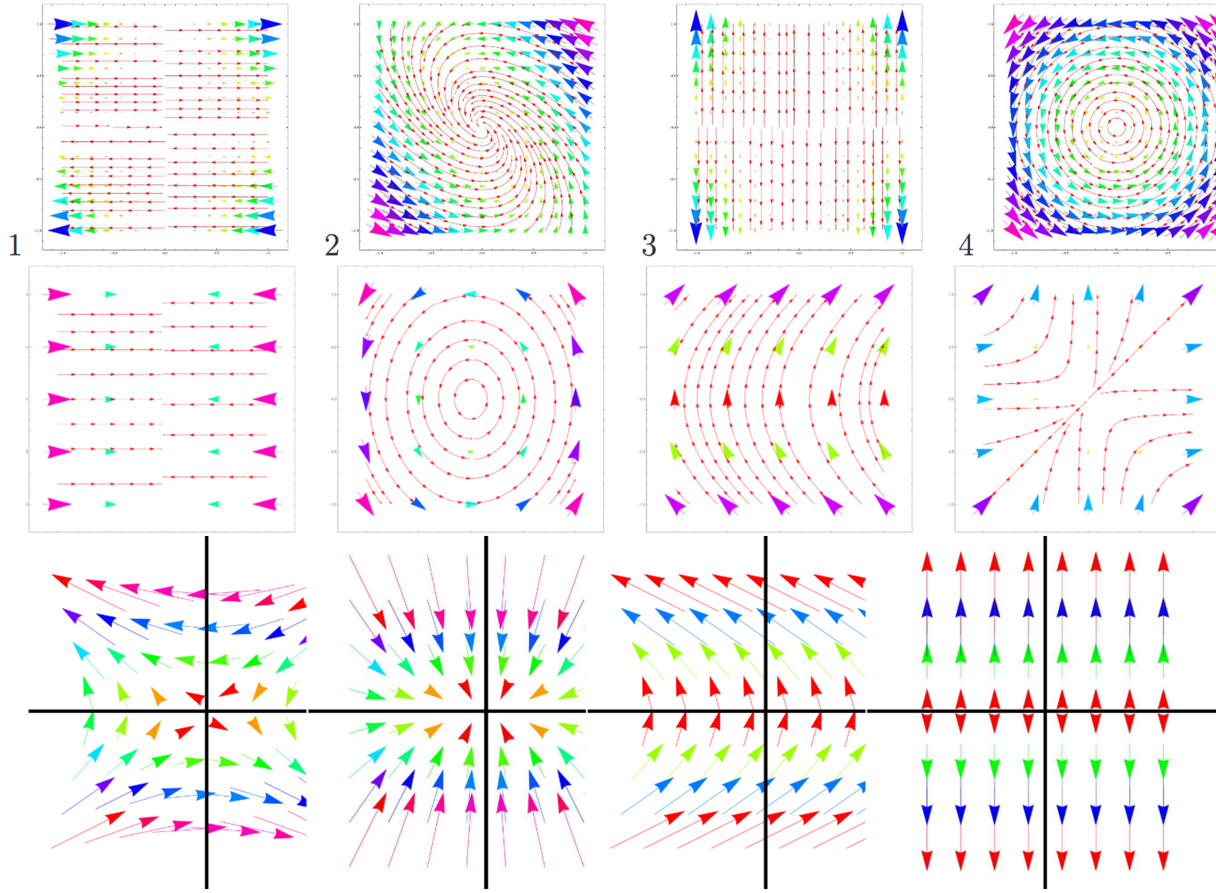
(d) $\sin\sqrt{x^2+y^2}$



Exercise 4

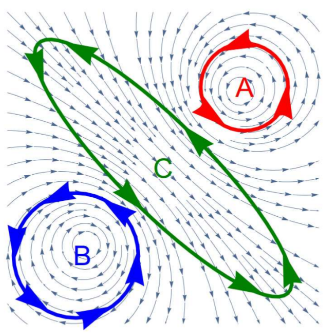
Match the vector fields.

a) $\langle y,1\rangle$	b) $\langle 0,2y\rangle$	c) $\langle -x,-2y\rangle$	d) $\langle -2y,3x\rangle$
e) $\langle 0,x^2y\rangle$	f) $\langle -2y,-x\rangle$	g) $\langle x^2y,0\rangle$	h) $\langle -x,0\rangle$
i) $\langle -2y,1\rangle$	j) $\langle -y-x,x\rangle$	k) $\langle -y,x\rangle$	l) $\langle x^2,y^2\rangle$



Exercise 5

Is the line integral around the following curves A, B, C positive, negative or zero?



Recap on Integrals

Exercise 1

Sketch the region of integration of the following iterated integral, switch the order to $dydzdx$ and then evaluate the integral.

$$\int_0^{\pi}\left(\int_{\sqrt{z}}^{\sqrt{\pi}}\left(\int_0^{x}\sin(xy)dy\right)dx\right)dz$$

Exercise 2

Consider the surface given by the graph of the function

$$z = f(x,y) = \frac{100}{1+(x^2+y^2)^2}\arctan\left(\frac{\pi}{8}\left(x^2+y^2\right)\right)$$

Find the volume under the surface and above the region $x^2+y^2 \leq 16$.

Exercise 3

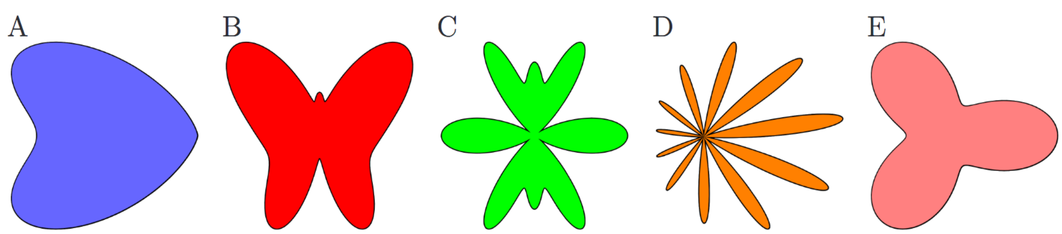
Find the double integrals

a) $\int_0^3\int_y^3 \frac{\sin(2x)}{x}dxdy$

b) $\int_0^8\int_{y^{1/3}}^2 \frac{y^2e^{x^2}}{x^8}dxdy$

Exercise 4

Match the given regions.



a) $r(t) \leq |2+\cos(3t)|$

b) $r(t) \leq |\cos(5t) - 5\cos(t)|$

c) $r(t) \leq |1+\cos(t)\cos(7t)|$

d) $r(t) \leq |\sin(11t)+\cos(t)/2|$

e) $r(t) \leq |8-\sin(t)+2\sin(3t)+2\sin(5t) - \sin(7t)+3\cos(2t)-2\cos(4t)|$

Exercise 5

Find the mas of the tetrahedron $x+y+z \leq 1; x \geq 0, y \geq 0, z \geq 0$ with density function given by $f(x,y,z) = 24x$.