Please show **all** your work! Answers without supporting work will not be given credit.
Clearly mention what theorem(s), if any, you are using.
Write answers in spaces provided.
You have 30 minutes to complete this Quiz.
You can get MAXIMUM $(2+2+2+2)+(6+1+2)+(5+3)+5=30$ marks.

Name:

1. Find whether the following statements are TRUE or FALSE. *Justify your answers as briefly as possible.*
   (a) If A is an invertible matrix, then the kernel of A and A^{-1} are isomorphic.
   (b) Every two dimensional subspace of $\mathbb{R}^{2\times 2}$ contains at least one invertible matrix.
   (c) The space $\mathbb{R}^{3\times 3}$ is isomorphic to P_9.
   (d) The kernel of the linear transformation $T(f(x))=f(x^2)$ from P_2 to P_2 is $\{0\}$.

2. Consider the linear transformation $T: P_2 \to P_2$ defined as

   $$T(f(x))=f(1)+f'(1)(x-1)$$

   (a) Consider the basis $\mathfrak{B}$ of P_2 given by $\mathfrak{B}=\{1,(x-1),(x-1)^2\}$. Find the $\mathfrak{B}$-matrix T.
   (b) Is T an isomorphism?
   (c) Denote the matrix obtained in part (a) by A. What are the rank and nullity of A?

3. (a) Show that the matrix $A=\begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$ is similar to $\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$.
   (b) Consider the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(v)=Av$. Find a basis $\mathfrak{B}$ of $\mathbb{R}^2$ for which the $\mathfrak{B}$-matrix of T is diagonal.

4. If B is a diagonal 3×3 matrix, what are the possible dimensions of the space V of all 3×3 matrices A that commute with B (i.e. $AB=BA$)?

   [HINT: If $B=\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$, consider the following 3 cases: (1) when $x=y=z$, (2) any 2 of x,y,z are equal but the third is not, (3) none of them are equal to each other.]