Lab 2: Partial Derivatives

Subhadip Chowdhury

Exercise 1: Defining the Function and Partial Derivatives

1. Enter the following command to define the function we'll explore in this lab:

   ```
   f[x_,y_]:=Cos[x]Sin[x+y]
   ```

2. Enter the following Plot command to plot the cross-section parallel to $XZ-$plane of the graph of f at $y = 3\pi/2$ for $-1.5 \leq x \leq 1.5$.

   ```
   Plot[f[x,3*Pi/2],{x,-1.5,1.5}]
   ```

3. By changing the endpoints for the range of x in the last command, zoom in around the point $x = 0.5$ until the graph looks linear. Estimate the slope of this (tangent) line by hand from the picture.

4. Find the partial derivative of f with respect to x, by entering the command:

   ```
   D[f[x,y],x]
   ```

   Then define a new function `fx` by copying the above answer as follows:

   `fx[x_,y_]:=`*the output of the last command*

5. Then evaluate `fx` at the point $(0.5, 3\pi/2)$ by entering the command:

   ```
   fx[0.5,3*Pi/2]
   ```

6. How is the value of `fx[0.5,3*Pi/2]` related to your estimated slope of the straight line?

7. Repeat the above steps, but this time use an x cross-section at $x = 0.5$ and zoom in around $y = 3\pi/2$. Use `fy` to define the partial derivative with respect to y. Compare the value of `fy[0.5,3*Pi/2]` with the slope you find.

Exercise 2: Investigating Contour Plot of the function

8. Create a contour plot of f on $-1.5 \leq x \leq 1.5, -\pi \leq y \leq \pi$, and use it to estimate the points where the function hits its extreme values (highest and lowest).

9. Graph the x and y cross-sections through these points and estimate the partial derivatives there. Check your estimates by evaluating `fx` and `fy` to find the exact values.

10. What is the exact value of the partial derivative function `fx` and `fy` at the points where f hits its extreme values? Explain your reasoning.

11. Enter the following command to draw (and label) the $\mathbf{0}$-level curve of the partial derivative function `fx`:

```
xpic=ContourPlot[fx[x,y]==0,{x,-1.5,1.5},{y,-Pi,Pi},ContourStyle->Red]
```

    Make sure to use a double equal-sign for `fx[x,y]==0`.

12. Alter the commands as appropriate to generate the corresponding level curve for `fy`; starting with the label `ypic`, and using the color `Blue` this time.

13. Now go back to your original contour plot of f itself and give it the name `cp` by entering

    `cp =` *the code for the contour plot from 8*

14. Combine all three plots by using the Show command:

```
Show[cp,xpic,ypic]
```

15. What is the significance of the points where `xpic` and `ypic` cross?

16. Now Show just `cp` and `ypic` together, and think about what `ypic` tells us about f. Locate points where the contours of f (from `cp`) are parallel to the y-axis. How are these points related to `ypic`? Use the meaning of the partial derivative `fy` to explain your answer.

17. Now Show just `cp` and `xpic`, and locate points where the contours of f are parallel to the x-axis. How are these points related to `xpic`? Use the meaning of the partial derivative `fx` to explain your answer.

Exercise 3 : Using LaTeX

Open Overleaf and write the code that produces the following equation as output.

$$\alpha \leq \frac{e^{b_1}}{b_2 + \sqrt{b_3}}$$