CAAP 2018

Quiz 4

JULY 27, 2018
SUBHADIP CHOWDHURY

Full Name:

1. **CIRCLE** the correct option (only one) in each of the following. In problems (a)-(c), 'lub' stands for least upper bound and 'glb' stands for greatest lower bound.

   (a) Let $M = lub$ of a nonempty bounded subset S of $\mathbb{R}$. Then M is also equal to [2]
   - A. the glb of the set of all lower bounds (in $\mathbb{R}$) of S.
   - B. the lub of the set of all lower bounds (in $\mathbb{R}$) of S.
   - C. the glb of the set of all upper bounds (in $\mathbb{R}$) of S.
   - D. the lub of the set of all upper bounds (in $\mathbb{R}$) of S.

   (b) Consider the following four intervals where $a < b$ are both fixed real numbers: [2]

   $$(a, b),\ [a, b],\ [a, b),\ \text{and}\ (a, b]$$

   Which of the following is true about the upper and lower bounds of these intervals?
   - A. The glb for the intervals $[a, b)$ and $[a, b]$, while same to each other, differs from the glb of the intervals $(a, b]$ and (a, b), which in turn are the same. Similarly, The lub for the intervals $(a, b]$ and $[a, b]$, while same to each other, differs from the lub of the intervals $[a, b)$ and (a, b), which in turn are same.
   - B. The intervals $[a, b]$ and $[a, b)$ have a glb and the intervals $(a, b]$ and (a, b) do not. Further, the intervals $[a, b]$ and $(a, b]$ have a lub, and the intervals $[a, b)$ and (a, b) do not.
   - C. $[a, b]$ is the only interval among the four intervals that has a glb and a lub.
   - D. All of them have the same lub and glb.

   (c) Suppose S is a nonempty bounded subset of $\mathbb{R}$. Denote by $-S$ the set $\{-s \mid s \in S\}$. Which of the following is true about S? [2]
   - A. $lub(-S) = lub(S)$ and $glb(-S) = glb(S)$
   - B. $lub(-S) = glb(S)$ and $glb(-S) = lub(S)$
   - C. $lub(-S) = -lub(S)$ and $glb(-S) = -glb(S)$
   - D. $lub(-S) = -glb(S)$ and $glb(-S) = -lub(S)$

   (d) If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ can be [2]
   - A. $-4/5$ but not $4/5$
   - B. $4/5$ but not $-4/5$
   - C. either $-4/5$ or $4/5$

   (e) $\tan\left(\frac{30\pi}{4} + \theta\right) =$ [2]
   - A. $\tan\theta$
   - B. $-\tan\theta$
   - C. $\cot\theta$
   - D. $-\cot\theta$

2. Suppose x is a real number such that $\cos x + \sin x = \sqrt{2}\cos x$. Prove that $\cos x - \sin x = \sqrt{2}\sin x$. [10]

3. Suppose the equation $(5x^2 - 4x + 2) + m(4x^2 - 2x - 1) = 0$ has no solution. Find all possible values of m. [10]

4. In a Geometric Progression, the $(m+n)^{th}$ term is p and the $(m-n)^{th}$ term is q. [5 (bonus)]

   Show that its m^{th} term is $\sqrt{pq}$.