MATH 152(45) **Midterm 2** NOVEMBER 11, 2016
SUBHADIP CHOWDHURY

Please show **all** your work! Answers without supporting work will not be given credit. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
Please note that use of calculator is not allowed.

Full Name: _____

Question	Points	Score
1	10	
2	15	
3	10	
4	10	
5	30	
6	0	
Total:	75	

This exam has 6 questions, for a total of 75 points.
The last problem is worth 4 **bonus points.**
The maximum possible point for each problem is given on the right side of the problem.

Do not open this answer script unless you are instructed to do so.

1. (a) State the second mean value theorem for integrals. 4

(b) A rod that has mass M and variable mass density extends from $x = 0$ and $x = L$ consists of two pieces. Find the mass of each piece given that the center of mass of the entire rod is at $x = \frac{2}{3}L$, the cente of mass of the first piece is at $x = \frac{1}{4}L$ and the center of mass of the second piece is at $x = \frac{7}{8}L$. 6

2. A rod of length a is placed on the $X-$axis from $x = 0$ to $x = a$. Suppose the mass density of the rod at x is given by $\lambda(x) = \dfrac{1}{16a^2 + 9x^2}$.

   (a) Find the mass of the rod. 7

   (b) Find the center of mass of the rod. You will get partial credit for correctly writing down the formula for center of mass. 8

   Answers need to be in the most simplified form possible without a calculator.

3. Consider the region Ω bounded by the two curves

$$y - x = 4, \qquad y = 4 - x^2$$

Suppose Ω is revolved around the line $x = 1$ to obtain a solid T.

(a) Express the volume of T as an integral in terms of x. 5

(b) Express the volume of T as an integral in terms of y. 5

Do NOT evaluate the integrals.

4. The base of a solid is an equilateral triangle lying in the $XY-$plane with one vertex at the origin and the opposite side perpendicular to the X-axis. 10

   Find the volume of the solid given that the cross-sections perpendicular to $X-$axis are isosceles right angled triangles with hypotenuse on the $XY-$plane.

5. Evaluate the following integrals.

(a) 10

$$\int \frac{\ln x - 1}{(\ln x)^2} dx$$

(b) 10

$$\int \frac{e^x - e^{-x}}{e^{2x} + e^{-2x} + 2} dx$$

(c) 10

$$\int_0^{\infty} \frac{1}{e^x\sqrt{4-e^{-2x}}}dx$$

6. **For each of the following problems, you have to give a formula for the function; drawing a picture is not enough.**
   (a) Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is surjective but not injective and has infinitely many discontinuities.
   (b) Give an example of a ***continuous*** function $f : \mathbb{R} \to \mathbb{R}$ which is injective but not surjective.