

Assignment 13 (7/19)

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- This homework is due at the beginning of class on **Thursday** 7/26. You are encouraged to work together on these problems, but you must write up your solutions independently.

Problems

Problem 1-3 in this assignment can be proved using induction.

Exercise 1. Show that the following statements are true for all natural numbers n by inducting on n .

(a) $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(4n^2-1)}{3}$

(b) $\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

(c) $2^n > n$

(d) $9^n + 7$ is divisible by 8

Exercise 2. If $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$ for all $n > 1$, show that

$$a_n = 2 \cos \frac{\pi}{2^{n+1}}$$

[HINT: Use induction on n and the trigonometric identity $\cos(2\theta) = 2\cos^2 \theta - 1$]

Exercise 3. If n is an odd natural number, prove that $n(n^2 - 1)$ is divisible by 24.

Exercise 4. If $\sin \theta + \cos \theta = \lambda$ for some angle θ , prove that $\sin \theta - \cos \theta = \pm \sqrt{2 - \lambda^2}$.

Exercise 5. Prove that $2\sin^2 \theta + 3\cos^2 \theta \geq 2$ for all θ .

Exercise 6 (Extra Credit). Prove that $\sin^4 \theta + \cos^4 \theta \geq \frac{1}{2}$ for all θ .