MATH 195

Midterm 1

OCTOBER 20, 2017
SUBHADIP CHOWDHURY

INSTRUCTIONS:

- Please show ALL your work! Answers without supporting justification will not be given credit.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
- Write legibly and in correct order.
- Please note that use of calculator is not allowed.
- If you write down the correct formula for an answer, you will get some partial credit regardless of whether you evaluated the exact values or not.
- Unless otherwise specified, you may use any valid method to solve a problem.

Full Name: _____

Question	Points	Score
1	15	
2	10	
3	15	
4	10	
Total:	50	

This exam has 4 questions, for a total of 50 points.
The maximum possible point for each problem is given on the right side of the problem.

Do not open this answer script unless you are instructed to do so.

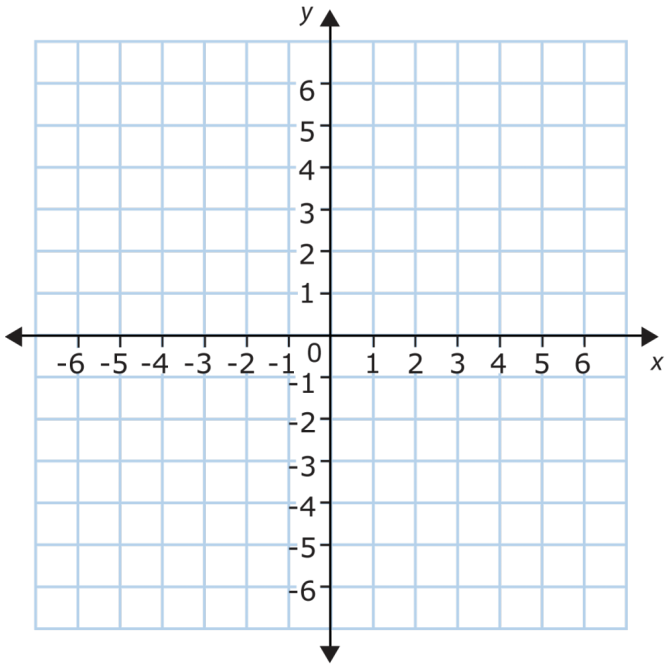

Figure 1: Level curve of $f(x, y)$ passing through $(2, 1)$

1. (a) Identify the level curve of $f(x, y) = \ln(x - y^2)$ that passes through $(2, 1)$. Describe and draw a picture of it **in the coordinate grid above**. [5]

   (b) In what direction does f increase most rapidly at $(2, 1)$? [5]

   (c) What is the rate of change of f at $(2, 1)$ in the direction of the point $(5, 5)$? [5]

2. Suppose λ and μ are real numbers such that [10]

   - the three vectors

$$\vec{u} = 2\hat{i} + 3\hat{j} + \hat{k},$$
$$\vec{v} = \hat{i} + \lambda\hat{j} + \mu\hat{k},$$
$$\vec{w} = 7\hat{i} + 3\hat{j} + 2\hat{k}$$

     are coplanar, and

   - The vector $\vec{v}$ has magnitude $\sqrt{2}$.

   Find all possible values of λ and μ.

   [HINT: If three vectors are coplanar, then the volume of the parallelepiped determined by those three vectors is zero i.e. the scalar triple product is zero.]

3. Suppose the curve given by $\vec{r}(t) = \langle \cos(\pi t), \sin(\pi t), t \rangle$ intersects the paraboloid $z = x^2 + y^2$ at a point $P = (x_0, y_0, z_0)$.

   (a) Find the coordinates of P. 3

   (b) Find equation of the tangent plane to the paraboloid at P. 4

   (c) What is the equation of the tangent line to the curve $\vec{r}(t)$ at P? 3

   (d) What is the angle of intersection between the curve and the paraboloid? This is the angle between the tangent line in part (c) and the plane in part (b). 5

4. If $u = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that 10

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$