MATH 1800-C HANDOUT 1: FUNCTIONS OF SEVERAL VARIABLES

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Exercise 1

For the following problems, fill the box with either “certainly”, “possibly”, or “certainly not”.

1. The point $(a, -1, 3)$ is $\boxed{}$ on the sphere $(x-2)^2 + (y+1)^2 + (z-3)^2 = 1$.
2. If all the y cross-sections (i.e. the cross sections parallel to XZ-plane) of the graph of $f(x, y)$ are straight lines, then the graph is $\boxed{}$ a plane.
3. If $f(x, y)$ is a linear function, then the graph of f is $\boxed{}$ parallel to XZ-plane.
4. The graph of $f(x, y) = x^2 + y^2 - 1$ is $\boxed{}$ the same set of points as the 1-level surface of $g(x, y, z) = x^2 + y^2 - z$.

Exercise 2

An equilateral triangle is standing vertically in 3-space with a vertex above the XY-plane and its two other vertices at $(7, 0, 0)$ and $(9, 0, 0)$. What are the coordinates of the third vertex?

Exercise 3

Consider the function $z = f(x, y) = \frac{x}{2} - 2y + 1$.

1. Sketch the contour plot for the graph with z-increment value of 1.
2. (a) Starting at any point (x, y), what is the slope of the surface in the x—direction?
   (b) What is the slope in y—direction?
   (c) What is the slope along the line $x = y$?
3. What kind of surface is the graph? Sketch a picture.

Exercise 4

Find the linear functions whose contour plots are shown in next page.

Exercise 5

Find an equation for the plane that contains the line in the XY-pane where $y = 1$, and the line in the XZ-pane where $z = 2$,

Exercise 6

Consider the contour plot for the function $f(x, y) = x^2 + y$.

1. Sketch the cross-section of the graph with the plane $x = 4$.
2. Compute the rate of change of z with respect to y as (x, y) moves towards increasing y-value, along the line $x = 4$.
3. What happens to the rate of change of z with respect to x as you move from $(4, 5)$ towards increasing x-value along the line $y = 5$.
4. Starting at the point $(1, 1)$, what direction would yield the maximal rate of change of z with respect to the distance in XY-plane.

Understanding how to answer these questions will be critical for graphically estimating partial derivatives and gradient vectors that we will learn about next week.

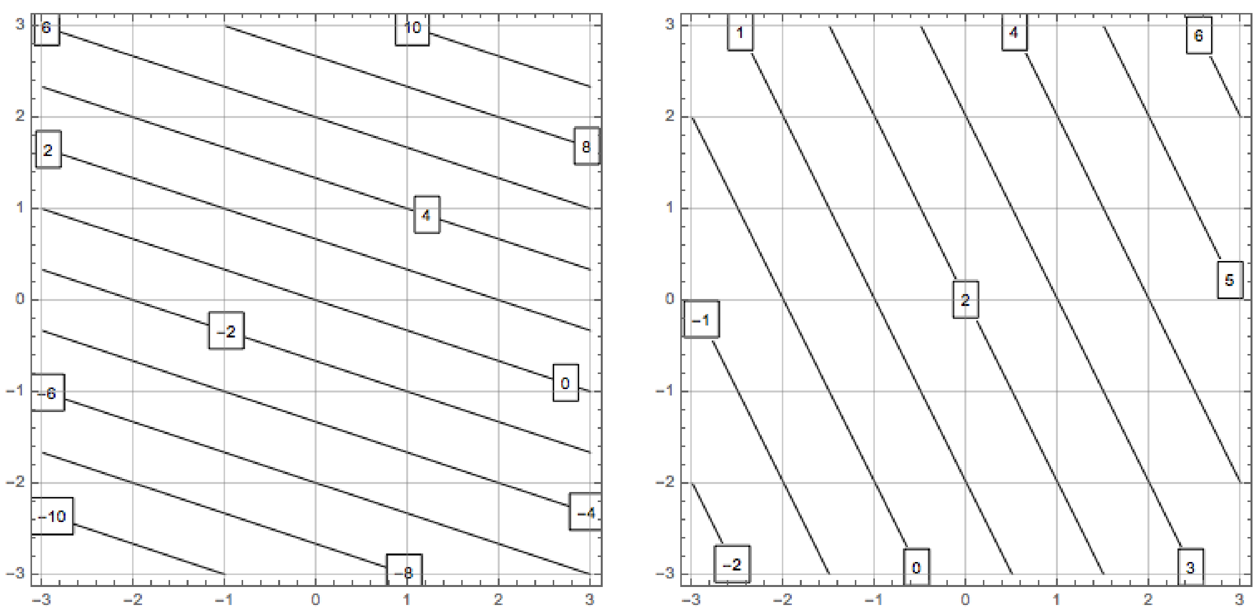

Figure 1: Plots for Exercise 4