

# Assignment 13 (7/19)

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- This homework is due at the beginning of class on **Thursday** 7/26. You are encouraged to work together on these problems, but you must write up your solutions independently.

## Problems

Problem 1-3 in this assignment can be proved using induction.

**Exercise 1.** Show that the following statements are true for all natural numbers  $n$  by inducting on  $n$ .

(a)  $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{4n^2-1}{3n}$

(b)  $\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

(c)  $2^n > n$

(d)  $9^n + 7$  is divisible by 8

**Exercise 2.** If  $a_1 = \sqrt{2}$ ,  $a_{n+1} = \sqrt{2 + a_n}$  for all  $n > 1$ , show that

$$a_n = 2 \cos \frac{\pi}{2^{n+1}}$$

[HINT: Use induction on  $n$  and the trigonometric identity  $\cos(2\theta) = 2\cos^2 \theta - 1$ ]

**Exercise 3.** If  $n$  is an odd natural number, prove that  $n(n^2 - 1)$  is divisible by 24.

**Exercise 4.** If  $\sin \theta + \cos \theta = \lambda$  for some angle  $\theta$ , prove that  $\sin \theta - \cos \theta = \pm \sqrt{2 - \lambda^2}$ .

**Exercise 5.** Prove that  $2\sin^2 \theta + 3\cos^2 \theta \geq 2$  for all  $\theta$ .

**Exercise 6** (Extra Credit). Prove that  $\sin^4 \theta + \cos^4 \theta \geq \frac{1}{2}$  for all  $\theta$ .