Assignment 20 (7/31)

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This week we covered topics from page 52-66 of the Calculus textbook. You should read those sections. Here are some paractice $\epsilon - N, \epsilon - \delta$ proofs. **You do not have to submit them. You should expect a problem similar to** 1 **or** 2 **in the final exam.**

Exercise 1. *Give an $\epsilon - N$ proof of the following.*

(a)

$$\lim_{n\to\infty} \frac{3}{n^2} = 0$$

(b)

$$\lim_{n\to\infty} \frac{n-1}{2+n} = 1$$

Note that the N you are trying to find in above proofs, should not depend on n, it can only depend ϵ. Similarly δ in the proof below cannot depend on x. It can depend on c, l and ϵ.

Exercise 2. *Give an $\epsilon - \delta$ proof of the following.*

(a)

$$\lim_{x\to 2}(3-x) = 1$$

(b)

$$\lim_{x\to -2}(3x+5) = -1$$

Exercise 3. *Consider the sequence $\{a_n\}_{n\in\mathbb{N}}$ defined as*

$$1, -1, 1, -1, 1, -1, \ldots$$

Prove using $\epsilon - N$ that $\lim_{n\to\infty} a_n$ does not exist.

Exercise 4. *If $\lim_{x\to c} f(x) = l$, then prove that*

$$\lim_{x\to c}(2f(x)-1) = 2l - 1$$

Give an $\epsilon - \delta$ proof.