MATH 1600 **Quiz 10 (Solution)** OCT 25, 2018
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1. Find the equation of the tangent line at $x = 0$ to the graph of the function $f(x)$ where [8]

$$f(x) = \frac{3x^2 + 1}{e^x + 7x}$$

HINT: Slope of the required tangent line is $f'(0)$.

Solution: Note that

$$f'(x) = \frac{(e^x + 7x)(6x) - (3x^2 + 1)(e^x + 7x)}{(e^x + 7x)^2}$$

So $f'(0) = -8$. Also $f(0) = 1$. The equation of the tangent line to the graph of $f(x)$ at $x = x_0$ is $y = f'(x_0)(x - x_0) + f(x_0)$. So the answer to our question is

$$y = -8x + 1$$

2. For what values of x, is the function $f(x) = e^x(x^2 - 4x + 2)$ concave down? [7]

HINT: A function is concave down when its second derivative is negative.

Solution:

$$f'(x) = e^x(2x - 4) + e^x(x^2 - 4x + 2) = e^x(x^2 - 2x - 2)$$
$$f''(x) = e^x(2x - 2) + e^x(x^2 - 2x - 2) = e^x(x^2 - 4)$$

Since e^x is never zero, $f''(x) \leq 0$ when $x^2 - 4 \leq 0$ i.e. when $-2 \leq x \leq 2$.