Assignment 13 (2/18)

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Problem 1

Problems 7.3.(8, 10, 27, 30, 36).

Problem 2

Problems 7.3.(47, 61, 73).

Problem 3

Recall that

$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx. \tag{*}$$

Prove that

$$\int_0^{\pi/2} \log(\tan(x)) dx = 0.$$

Problem 4

1. Prove that

$$\int_0^{2a} f(x)dx = \begin{cases} 2 \int_0^a f(x)dx & \text{if } f(2a - x) = f(x) \\ 0 & \text{if } f(2a - x) = -f(x) \end{cases}$$

2. Prove that

$$\int_0^{\pi/2} \log(\sin(2x)) dx = \int_0^{\pi/2} \log(\sin(x)) dx.$$

3. Use part (2) and (\star) to evaluate

$$\int_0^{\pi/2} \log(\sin(x)) dx.$$

Problem 5

Use the result of problem (4.3) to evaluate

$$\int_0^{\pi/2} \log(\tan\theta + \cot\theta) d\theta.$$