Assignment 7 (1/19)

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- *This homework is due at the beginning of class on **Friday** 1/26. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material part. You are encouraged to think about the exercises marked with a ($\star$) or (†) if you have time, but you don't need to hand them in. If you correctly solve a (†)-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering Chapter 14 from Stewart.*

Important Points and Reading Materials

- Tangent plane and Linear approximation
  - The linear approximation $L(x,y)$ of $f(x,y)$ at a point (x_0,y_0) is
  $$L(x,y)=f(x_0,y_0)+f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)$$
  - Know how to tell if a function is differentiable. It is not enough to know that f_x and f_y exist; but it is sufficient if f_x, f_y are continous functions. Consequently, if f_x and f_y are continuous, all the directional derivatives exist.
- The Chain Rule
  - Calculation using the rule.
  - If x and y are functions of two variables s and t and z is a function of x and y, the chain rule becomes
  $$\frac{\partial z}{\partial s}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$
  and similarly for $\frac{\partial z}{\partial t}$.

Problems

Exercise 1

1. Show that $f(x,y)=xe^{xy}$ is differentiable at $(1.1,0.1)$ and find its linear approximation there.
2. Do the same for $f(x,y)=1+x\ln(xy-5)$ at $(2,3)$.

Exercise 2

Find equation of the tangent plane for the given surface at the specified point:

$$z=3y^2-2x^2+x \text{ at } (2,-1,-3)$$

Exercise 3

Use chain rule to find dw/dt.

$$w=xe^{y/z},\quad x=t^2,\quad y==1-t,\quad z=1+2t$$

Exercise 4

Find $\partial z/\partial s$ and $\partial z/\partial t$.

$$z = \tan(u/v),\ u = 2s + 3t,\ v = 3s - 2t$$

Exercise 5

If $u = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

Exercise 6

If $z = f(x - y)$, show that

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

Exercise 7

Show that a function of the form

$$z = f(x + at) + g(x - at)$$

is a solution to the wave equation

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$$