# Lab 3: Parametric Plotting

**Subhadip Chowdhury**

A parametric plot in the $xy$-plane can be drawn for any pair of functions $x(t)$ and $y(t)$ depending on the parameter $t$. For instance, to dynamically draw the unit circle centered at the origin, you could enter the command

```
Manipulate[ParametricPlot[{Cos[t],Sin[t]},{t,0,a},PlotRange-> 1],{a,0.001,2*Pi}]
```

and then use the slider bar to trace the circle. Note that $a$ is the last value of the parameter $t$ shown in the image.

Take a look at the curves parameterized by $(x,y) = (\mathbf{cos}[t], \mathbf{sin}[n\,t])$ with integer values of $n$. Start with $n = 2$, and experiment with the image until you understand the answers to the questions (a)--(e) below. Then fill in in the first row of the chart for future reference, and continue on to $n = 3, 4,$ and $5$.

(a) For what value of $t$ does the $x$ coordinate first return to its value at $t = 0$?

(b) For what value of $t$ does the $y$ coordinate first return to its value when $t = 0$?

(c) For what value of $t$ do both coordinates simultaneously return to their values at $t = 0$ for the first time?

(d) How many times does the **entire curve shape** cross the $x$-axis?

(e) How many times does the **entire curve shape** cross the $y$-axis?

| | Question | | | | |
|---|---|---|---|---|---|
| $n$ | (a) | (b) | (c) | (d) | (e) |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

? **Question 1:** Which of these curve shapes intersect themselves at the origin? Why?
Now fill in another table with the answers to the same questions, but use the curves parameterized by $(x,y) = (\mathbf{cos}[n\,t], \mathbf{sin}[t])$.

| | Question | | | | |
|---|---|---|---|---|---|
| $n$ | (a) | (b) | (c) | (d) | (e) |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

? **Question 2:** What is different about these curves when $n$ is even?

We can also take *rational* multiples of $t$ in our parameterizations: $(x, y) = (\cos[r\,t], \sin[t])$ with $r = m/n$.

? **Question 3:** What is the smallest range of $t$ values that will give the entire curve shapes described by $(x, y) = (\cos[r\,t], \sin[t])$ for $r = 3/4, 3/5,$ and $3/7$? (Hint: You will need to change the upper limit on the slider value $a$ at the end of the `Manipulate` command.)

? **Question 4:** For each value of $r$, find an alternate parameterization which gives the same curve while using only $0 \leq t \leq 2\pi$.

? **Question 5:** Experiment until you can say what should happen to the curves when parameterized by $x = \cos[s\,t]$ and $y = \sin[t]$ for an irrational value like $s = \sqrt{2}$. Note that you should use the `PlotPoints` option as below to keep your images smooth (and accurate):

```
Manipulate[ParametricPlot[{Cos[Sqrt[2]*t], Sin[t]},{t, 0, a}, PlotPoints->500], {a,0.001,2500}]
```