

# Assignment 12 (5/1)

Subhadip Chowdhury

## Problem 1

Problems 11.3.(55, 58, 62, 64).

## Problem 2

Below are some sequences defined recursively. Assume that the sequences are convergent. Find the limit.

(a)  $a_1 = 1, a_n = \frac{1}{2} \left( a_{n-1} + \frac{R}{a_{n-1}} \right) \quad \forall n \geq 2$

(b)  $a_1 = 1, a_n = \sqrt{6 + a_{n-1}} \quad \forall n \geq 2$

(c)  $a_1 = 1, a_{n+1} = a_n + \cos(a_n) \quad \forall n \geq 1$

## Problem 3

The Fibonacci sequence is defined as

$$F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$$

(a) Evaluate  $F_3, \dots, F_6$ .

(b) Define  $r_n = \frac{F_{n+1}}{F_n}$ . Assuming that the sequence  $\{r_n\}$  converges, find its limit.

(c) Find  $\lim_{n \rightarrow \infty} \frac{F_{n+3}}{F_n}$ . [Hint: For any convergent sequence  $\{a_n\}$ , we have  $a_n \rightarrow l \Rightarrow a_{n+1} \rightarrow l$ ]

## Problem 4

Suppose  $\{a_n\}$  is a sequence defined as

$$a_n = \sum_{i=1}^n \frac{i}{n^2}$$

Does the sequence converge? If so, to what?

### Problem 5

Suppose  $\{a_n\}$  is defined by  $a_1 = 1$  and

$$a_n = \frac{5a_{n-1}^2 + a_{n-1}}{2a_{n-1} + 2}$$

Define another sequence  $\{b_n\}$  by

$$b_n = \frac{a_n}{a_{n-1}}$$

- (a) Prove that  $\{a_n\}$  is an increasing sequence.
- (b) Is the sequence  $\{a_n\}$  convergent?
- (c) Prove that  $\{b_n\}$  is an increasing sequence.
- (d) Prove that  $\{b_n\}$  is bounded above.
- (e) Find  $\lim_{n \rightarrow \infty} b_n$ .