Assignment 13 (2/7)

Subhadip Chowdhury

- *This homework is due at the beginning of class on* ***Friday*** 2/16. *You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material item. You are encouraged to think about the exercises marked with a* ($\star$) *or* (†) *if you have time, but you don't need to hand them in. If you correctly solve a* (†)*-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering Chapter* 15 *from Stewart.*

Important Points and Reading Materials

- Change of Variable for double integrals
  - Understand how transformations work in 2 variables. How do you figure out what a given region in the uv-plane maps to in the xy-plane? What region in the uv-plane maps to a given region in the xy-plane?
  - Know how to compute Jacobians, and understand where this formulas comes from.
  - Know how to compute a double integral using change of variables. If a specific variable substitution is not given to you, know how to possibly find one (either by making the function f simpler, or by making the region of integration simpler). Keep in mind that there may be more than one variable substitution which may work. Try to find one that will make your life as easy as possible.
  - Know how to sometimes find a variable substitution that will turn a complicated region into a rectangle, or other simple shape. It may help to write the boundaries of the region in the form $f(x, y) = a$, and then let $u = f(x, y)$, or to find parametric equations for the boundaries.

Problems

Exercise 1

Evaluate the following integrals using appropriate change of variable formula:

(a)

$$\iint_R \frac{x-2y}{3x-y}\,dA$$

where R is the parallelogram enclosed by the four straight lines

$$x-2y=0, \quad x-2y=4, \quad 3x-y=1, \quad \text{and} \quad 3x-y=8.$$

(b) ($\star$)

$$\iint_R \frac{1}{x}\,dA$$

over the region R bounded by the four straight lines

$$x-y=1, \quad x-y=4, \quad y=2x, \quad y=3x$$

(c)

$$\iint\limits_R y\,dA$$

where R is the region bounded by

$$x^2y = 4, \quad x^2y = 9, \quad y = x, \quad y = 2x$$

(d)

$$\iint\limits_R x^2\,dA$$

where R is the region bounded by the ellipse $9x^2 + 4y^2 = 36$.

[HINT: Use $x = 2u, y = 3v$ and then polar coordinates.]

(e)

$$\iint\limits_R \cos\left(\frac{y-x}{y+x}\right)dA$$

where R is the trapezoidal region with vertices $(1,0), (2,0), (0,2)$, and $(0,1)$.

Exercise 2

Let R be the region bounded by the circle $x^2 + y^2 = 2x$. Find the region S in $r\theta$-plane that maps to R under the change of variables $x = r\cos\theta$ and $y = r\sin\theta$.

Exercise 3

Let R be the region bounded by the hyperbolas $y = 1/x, y = 4/x$ and the lines $y = x, y = 4x$. What change of variables would you use so that we can find a rectangular region in uv-plane that maps to R. Find the Jacobian of the transformation.