Assignment 1 (1/3)

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- *This homework is due at the beginning of class on **Friday** 1/12. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material part. You are encouraged to think about the exercises marked with a ($\star$) or (†) if you have time, but you don't need to hand them in. If you correctly solve a (†)-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering Chapter 12 from Stewart.*

Important Points and Reading Materials

- Review of material from 150s/130s.
  - Limit of rational functions.
  - Derivative of functions of one variables - Chain Rule.
  - Graph function of the form $y = f(x)$ where f is a polynomial, rational function, trigonometric, inverse trigonometric, exponential, or logarithmic function.
  - Trigonometry - Addition identities, values of $\sin, \cos, \tan$ at $0, \pi/6, \pi/4, \pi/3, \pi/2$ and corresponding angles in other quadrants - identities of the form $\cos(3\pi/2 - \theta) = -\sin(\theta)$.
- 3-dimensional geometry
  - In two dimensions, giving a single equation will (usually) describe a curve. What would a single equation describe in three dimensions? Would it be a curve or a surface? How many equations would you need to typically describe a curve?
  - In two dimensions, an equation of the form $y = f(x)$ describes a curve. If you take the same equation in three dimensions, it will now give you a surface. How does that surface relate to the original curve?
  - What do we mean by equation of an object? E.g. if a surface S has equation $z = x^3 + y$, that means the set of points (x, y, z) in 3 dimensions that are solution to the equation $z = x^3 + y$, collectively make up the surface S.
  - We mentioned in class that equation of a sphere with radius r and center at the origin is
  $$x^2 + y^2 + z^2 = r^2.$$
  What is the equation of a sphere with radius r and center at (a, b, c)?
  - If $f(x, y, z) = c$ denotes equation of a surface, what kind of objects are denoted by $f(x, y, z) > c$ and $f(x, y, z) < c$? E.g. What does the set of points $\{(x, y, z) \mid z < x^2 + y^2\}$ look like?

Problems

Exercise 1

Show that the equation
$$x^2 + y^2 + z^2 + 6x - 8y - 2z + 25 = 0$$
denotes a sphere. Find its center and radius. Do the same for
$$3x^2 + 3y^2 + 3z^2 = 10 + 6y + 12z$$

Exercise 2

(12.1.10) Let $P = (2, -1, 0), Q = (4, 1, 1), R = (4, -5, 4)$. Show that the triangle PQR is a right angled triangle.
[HINT: Use Pythagoras identity.]

Exercise 3

(12.1.15) Find equation of the sphere that passes through the point $(4, 3, 1)$ and has center $(3, 8, 1)$.

Exercise 4

(12.1.23c) Find equation of the sphere with center $(2, -3, 6)$ that touches the XZ-plane.

Exercise 5

(12.1.40) Write inequalities to describe the solid cylinder that lies on or below the plane $z = 8$ and on or above the disk in the XY-plane with center the origin and radius 2.

Exercise 6†

Find equation of the sphere that passes through the points $(1, 0, 0), (0, 1, 0)$, and $(0, 0, 1)$ and has the smallest possible radius.

Exercise 7⋆

Let $P = (p_1, p_2, p_3)$ and $Q = (q_1, q_2, q_3)$. Let R be a point on the straight line segment $\overline{PQ}$ such that $PR/RQ = \lambda$. Find the coordinates of R.

Exercise 8†

Find the ratio in which the YZ-plane divides the line joining $(2, 4, 5)$ and $(3, 5, 7)$.