Assignment 1 (9/26)

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Note: This is longer than a usual homework, mostly because we are covering 2/3 chapters at a time as review. Homeworks will usually have $10-15$ problems starting next week. Also because of its length, this is **due next Monday, October** 3 instead of this Friday.

Although I don't expect all of you to start writing proofs using full sentences and lots of words (at least not immediately); the following is the bare minimum that I expect:

- The lines should be connected by either words (proper conjunctions) or symbols (e.g. " $\implies$ ", "$\because$" etc.)

Problem 1

Problem 1.2.76.

Problem 2

Problems 1.3.(43, 45).

Problem 3

Problems 1.5.(30, 73, 74).

Problem 4

Problems 1.6.(33, 58, 67).

Problem 5

Let $P(x) = x^2(x+1)^3(x-1)^2(x-3)^5$. Sketch the graph of $P(x)$.

Problem 6

Solve the inequality and express the solution set as an interval or as a union of intervals:

$$4 \leq |x-3| < 6$$

Problem 7 (Don't have to submit, review yourself)

Give an $\epsilon - \delta$ proof of the following: $\lim_{x \to 1} \frac{3x+2}{5} = 1$.

Follow this proforma to write the proof: For example, let's show that $\lim_{x \to 0}(2x + 1) = 1$.

Proof: Let us take an arbitrary $\epsilon > 0$. We want to prove that there exists $\delta > 0$ such that

$$0 < |x - 0| < \delta \implies |2x + 1 - 1| < \epsilon.$$

So, we want to find a δ such that $0 < |x| < \delta$ implies $|2x| < \epsilon$. Clearly it suffices to take $\delta = \frac{\epsilon}{2}$, since that would imply

$$|x| < \frac{\epsilon}{2} \implies |2x| < \epsilon.$$

[Proved]

Couple of notes:

- In these proofs, your main goal is to find a δ that works.
- δ depends on ϵ. But ϵ does not depend on anything. We only know that $\epsilon > 0$.
- We are finding a δ such that the '*implication*', i.e. the following claim:

  $$0 < |x - c| < \delta \implies |f(x) - l| < \epsilon$$

  is true. It is not enough for only one of the above two statements to be true, we need to show that one *implies* the other.
- δ is a function of ϵ, but not necessarily bigger or smaller than ϵ.
- For an ϵ, there might be multiple values of δ which work. For example, in the above proof, if we had chosen $\delta = \epsilon/4$, the proof would still work. We will then write

  $$|x| < \frac{\epsilon}{4} \implies |2x| < \epsilon/2 \implies |2x| < \epsilon$$

  since $\epsilon > 0$.

Problem 8 (Don't have to submit, review yourself)

Give $\epsilon - \delta$ proofs of the following: $2.2.(49, 53, 58)$.

Problem 9

Let $\lim_{x \to c} f(x) = L$. Prove that $\lim_{x \to c} |f(x)| = |L|$. [HINT: Triangle inequality]

Problem 10

Problems $2.3.(45, 46)$.

$\boxed{\textbf{Problem 11 (Extra Credit)}}$

Let $h(x) = \max\{f(x), g(x)\}$. If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then find $\lim_{x\to c} h(x)$. You may assume the result of problem 2.3.52.

$\boxed{\textbf{Problem 12}}$

Problems 2.4.$(12, 29, 34, 37)$.

$\boxed{\textbf{Problem 13}}$

Let

$$f(x) = \frac{x^2 - 4}{x^2 - x - 2}.$$

Find the values of x where $f(x)$ is discontinuous. Justify your answer by evaluating appropriate limits (or showing that the limits do not exist).

Classify each of the discontinuities of f as either a removable or an essential discontinuity. For any removable discontinuities, give a value for the function that would make it continuous at those points.