Assignment 3 (1/9)

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Note that the pinching theorem (Thm. 11.3.9) also works for sequences. You may need that for some of these problems.

Problem 1

Problems 11.3.(52, 55, 58, 62).

Problem 2

Below are some sequences defined recursively. Assume that the sequences are convergent. Find the limit.

(a) $a_1 = 1, a_n = \frac{1}{2}\left(a_{n-1} + \frac{R}{a_{n-1}}\right) \quad \forall n \geq 2$

(b) $a_1 = 1, a_n = \sqrt{6 + a_{n-1}} \quad \forall n \geq 2$

(c) $a_1 = 1, a_{n+1} = a_n + \cos(a_n) \quad \forall n \geq 1$

Problem 3

The Fibonacci sequence is defined as

$$F_1 = 1,\ F_2 = 1,\ F_n = F_{n-1} + F_{n-2}$$

(a) Evaluate $F_3, \ldots, F_6$.

(b) Define $r_n = \frac{F_{n+1}}{F_n}$. Assuming that the sequence $\{r_n\}$ converges, find its limit.

(c) Find $\lim\limits_{n\to\infty} \dfrac{F_{n+3}}{F_n}$. [Hint: For any convergent sequence $\{a_n\}$,we have $a_n \to l \Rightarrow a_{n+1} \to l$]

Problem 5

Suppose $\{a_n\}$ is defined by $a_1 = 1$ and

$$a_n = \frac{5a_{n-1}^2 + a_{n-1}}{2a_{n-1} + 2}$$

Define another sequence $\{b_n\}$ by

$$b_n = \frac{a_n}{a_{n-1}}$$

(a) Prove that $\{a_n\}$ is an increasing sequence.

(b) Is the sequence $\{a_n\}$ convergent?

(c) Prove that $\{b_n\}$ is an increasing sequence.

(d) Prove that $\{b_n\}$ is bounded above.

(e) Find $\lim_{n\to\infty} b_n$.