Assignment 13 (7/19)

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- *This homework is due at the beginning of class on* ***Thursday*** *7/26. You are encouraged to work together on these problems, but you must write up your solutions independently.*

Problems

Problem 1-3 in this assignment can be proved using induction.

Exercise 1. *Show that the following statements are true for all natural numbers n by inducting on n.*

(a) $1^2+3^2+5^2+\ldots+(2n-1)^2=\dfrac{n(4n^2-1)}{3}$

(b) $\dfrac{1}{1\times2\times3}+\dfrac{1}{2\times3\times4}+\ldots+\dfrac{1}{n(n+1)(n+2)}=\dfrac{n(n+3)}{4(n+1)(n+2)}$

(c) $2^n>n$

(d) 9^n+7 *is divisible by* 8

Exercise 2. *If* $a_1=\sqrt{2}$, $a_{n+1}=\sqrt{2+a_n}$ *for all* $n>1$, *show that*

$$a_n=2\cos\frac{\pi}{2^{n+1}}$$

[HINT: *Use induction on n and the trigonometric identity* $\cos(2\theta)=2\cos^2\theta-1$]

Exercise 3. *If n is an odd natural number, prove that* $n(n^2-1)$ *is divisible by* 8.

Exercise 4. *If* $\sin\theta+\cos\theta=\lambda$ *for some angle* θ, *prove that* $\sin\theta-\cos\theta=\pm\sqrt{2-\lambda^2}$.

Exercise 5. *Prove that* $2\sin^2\theta+3\cos^2\theta\geq2$ *for all* θ.

Exercise 6 (Extra Credit)**.** *Prove that* $\sin^4\theta+\cos^4\theta\geq\frac{1}{2}$ *for all* θ.