Please show **all** your work! Answers without supporting work will not be given credit.
Clearly mention what theorem(s), if any, you are using.
Write answers in spaces provided.
You have 15 minutes to complete this Quiz.
You can get MAXIMUM $(1+5+3+6=)15$ marks.

Name:

Suppose $\vec{a}$ and $\vec{b}$ are two **unit** vectors, and let $\vec{u} = \vec{a} + 2\vec{b}$ and $\vec{v} = 5\vec{a} - 4\vec{b}$.

1. What are $\|\vec{a}\|$ and $\|\vec{b}\|$?
   $\|\vec{a}\| = \|\vec{b}\| = 1$ since both $\vec{a}$ and $\vec{b}$ are unit vectors.

2. Suppose $\vec{u}$ and $\vec{v}$ are perpendicular to each other. Find $\vec{a} \cdot \vec{b}$.
   Note that $\vec{u} \perp \vec{v} \implies \vec{u} \cdot \vec{v} = 0$. Since dot product distributes over sum, we have

   $$\vec{u}\cdot\vec{v} = (\vec{a}+2\vec{b})\cdot(5\vec{a}-4\vec{b}) = 5\vec{a}\cdot\vec{a} - 4\vec{a}\cdot\vec{b} + 10\vec{b}\cdot\vec{a} - 8\vec{b}\cdot\vec{b} = 5\|\vec{a}\|^2 - 8\|b\|^2 + 6\vec{a}\cdot\vec{b} = 5 - 8 + 6\vec{a}\cdot\vec{b} = -3 + 6\vec{a}\cdot\vec{b}$$

   Hence,

   $$-3 + 6\vec{a}\cdot\vec{b} = 0 \implies \vec{a}\cdot\vec{b} = 1/2$$

   For the last two parts, keep assuming that $\vec{u}$ and $\vec{v}$ are perpendicular.

3. Suppose the angle between $\vec{a}$ and $\vec{b}$ is θ. Find the value of θ.
   We have,

   $$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|a\|\|b\|} = 1/2 \implies \theta = \pi/3$$

4. Let $\vec{w} = \vec{u} + \vec{v}$. Find the projection of $\vec{w}$ on to $\vec{a}$; i.e. $\text{Proj}_{\vec{a}}\vec{w}$.

   $$\text{Proj}_{\vec{a}}\vec{w} = \text{Proj}_{\vec{a}}(6\vec{a} - 2\vec{b}) = \frac{(6\vec{a} - 2\vec{b})\cdot\vec{a}}{\|a\|^2}\vec{a} = \frac{6\|\vec{a}\|^2 - 2\vec{b}\cdot\vec{a}}{1}\vec{a} = \left(6 - \frac{2}{2}\right)\vec{a} = 5\vec{a}$$