Assignment 13 (7/19)

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- *This homework is due at the beginning of class on* ***Thursday*** 7/26. *You are encouraged to work together on these problems, but you must write up your solutions independently.*

Problems

Problem 1-3 in this assignment can be proved using induction.

Exercise 1. *Show that the following statements are true for all natural numbers n by inducting on n.*

(a) $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \dfrac{n(4n^2-1)}{3}$

(b) $\dfrac{1}{1\times 2\times 3} + \dfrac{1}{2\times 3\times 4} + \ldots + \dfrac{1}{n(n+1)(n+2)} = \dfrac{n(n+3)}{4(n+1)(n+2)}$

(c) $2^n > n$

(d) $9^n + 7$ *is divisible by* 8

Exercise 2. *If* $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$ *for all* $n > 1$, *show that*

$$a_n = 2\cos\frac{\pi}{2^{n+1}}$$

*[*HINT: *Use induction on n and the trigonometric identity* $\cos(2\theta) = 2\cos^2\theta - 1$*]*

Exercise 3. *If n is an odd natural number, prove that* $n(n^2-1)$ *is divisible by* 8.

Exercise 4 (Extra Credit)**.** *If n is an odd natural number, prove that* $n(n^2-1)$ *is divisible by* 24.

Exercise 5. *If* $\sin\theta + \cos\theta = \lambda$ *for some angle* θ, *prove that* $\sin\theta - \cos\theta = \pm\sqrt{2-\lambda^2}$.

Exercise 6. *Prove that* $2\sin^2\theta + 3\cos^2\theta \geq 2$ *for all* θ.

Exercise 7 (Extra Credit)**.** *Prove that* $\sin^4\theta + \cos^4\theta \geq \frac{1}{2}$ *for all* θ.