Assignment 12 (2/5)

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- *This homework is due at the beginning of class on* ***Friday*** 2/9. *You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material item. You are encouraged to think about the exercises marked with a* ($\star$) *or* (†) *if you have time, but you don't need to hand them in. If you correctly solve a* (†)*-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering Chapter* 15 *from Stewart.*

Important Points and Reading Materials

- Double integration over domains of type I and II
  - A region of the form $\Omega_1 = \{(x,y) \mid a \le x \le b, f(x) \le y \le g(x)\}$ is called a region of type I and a region of the form $\Omega_2 = \{(x,y) \mid c \le y \le d, f(y) \le x \le g(y)\}$ is called a region of type II.
  - Integrals over regions of type I and II as above can be transformed to iterated integrlas as follows:

$$\iint_{\Omega_1} P(x,y)dA = \int_a^b \int_{f(x)}^{g(x)} P(x,y)\,dy\,dx$$

$$\iint_{\Omega_2} P(x,y)dA = \int_c^d \int_{f(y)}^{g(y)} P(x,y)\,dx\,dy$$

  - The order of integration is sometime forced by what the domain looks like. If there is a choice as to whether we treat the domain as type I or type II, then choose the one that makes the integration easier.
  - Keep in mind that the dependent variable gets integrated first. In particular, the final answer of a definite (double) integral has to be a number (i.e. can't have x, y in answer).
  - Sometimes we may need to break a region into pieces which look like type I or II; and evaluate each part separately.
- Triple Integral
  - Analogous to double integrals; only one dimension higher, has one more variable.

Problems

Exercise 0$\star$

Draw an example of a region that is

1. type I but not type II
2. type II but not type I
3. both type I and type II
4. Neither type I nor type II

Exercise 1

Set up the following integral for both orders of integration. Then evaluate using the easier order and explain why it's easier.

(a)

$$\iint_D y^2 e^{xy}\, dA$$

where D is the triangle bounded by $y = x$, $y = 4$, and $x = 0$.

(b)

$$\iint_D e^{x^2}\, dA$$

where $D = \{(x, y) \mid 0 \le y \le 1,\ 3y \le x \le 3\}$

Exercise 2

Evaluate

$$\iint_D (x^2 + 2y)\, dA$$

where D is the region bounded by $y = x$ and $y = x^2$.

Exercise 3

Find the volume of the solid enclosed by the cylinders $z = x^2$, $y = x^2$, and the planes $z = 0$, $y = 4$.

Exercise 4

Sketch the domain of integration and rewrite the iterated integral after changing the order of integration.

1. $$\int_1^2 \int_0^{\ln x} f(x, y)\, dy\, dx$$

2. $$\int_0^1 \int_{\arctan x}^{\pi/4} f(x, y)\, dy\, dx$$