

Practice Problems and review notes

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Problem 1

Decide whether or not the following functions are injective. If no domain is specified, find the domain.

(a) $x + \frac{1}{x}$

(b) $\frac{x}{|x|}$

(c) $\frac{1}{(x+1)^{2/3}}$

Problem 2

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{(1-x)(x-2)(x-6)}{x^2(x-5)}$$

Is f surjective onto \mathbb{R} ?

Problem 3

Suppose f has an inverse function. Recall that

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Suppose $f(3) = -6$, and $f'(3) = 2/3$. If $g = 1/(f^{-1})$, what is $g'(-6)$?

[Hint; Use Chain rule and quotient rule to find out g' .]

Problem 4

Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is a surjective function. Define

$$g(x) = e^{f(x)}$$

Is the function g injective? Is the function g surjective onto \mathbb{R} ?

Problem 5

Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous and injective function. Define

$$g(x) = e^{f(x)}$$

Is the function g injective?

Problem 6

- Note that the function $f : [0, \infty) \rightarrow [0, \infty)$ defined as $f(x) = x^2$ is surjective.
- Prove that the function $g : \mathbb{N} \rightarrow \mathbb{N}$ defined as $g(n) = n^2$ is NOT surjective.

Problem 7

Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 1 - e^{-x}$. What is the domain of f^{-1} ?