Assignment 19 (3/2)

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- *This homework is due at the beginning of class on* ***Wednesday*** 3/7. *You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material item. You are encouraged to think about the exercises marked with a* ($\star$) *or* (†) *if you have time, but you don't need to hand them in. If you correctly solve a* (†)*-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering Sequences and Series (Chapter 11) from Stewart.*

Important Points and Reading Materials

- INTEGRAL TEST:
  - Note that integral test does NOT say that $\sum_{k=1}^{\infty} f(k)$ is equal to $\int_1^{\infty} f(k)$. The integral converges iff the series converges, but they aren't equal.
  - What are the prerequisite conditions on f to apply integral test?
  - Understand p—series test.
- COMPARISON AND LIMIT COMPARISON TEST:
  - Applies only to series with positive terms.
  - If you want to show $\sum a_n$ converges and you know $\sum b_n$ converges, do you need to show $a_n \geq b_n$ or $a_n \leq b_n$. What if $\sum b_n$ diverges, and you want to show that $\sum a_n$ also does?
  - What can you do if you know what series $\sum b_n$ you want to use, but the inequality goes the wrong way. For instance, what if you want to show that $\sum \frac{1}{n+1}$ diverges by comparing it to $\sum \frac{1}{n}$.
  - Limit comparison tesy is used to compare complicated series with rational function entries to simpler series, usually a p—series. This is useful if you know what sequence you want to compare to, but its difficult to figure out which sequence is larger (i.e. you can't use comparison).
  - Also remember that the limit $\lim_{n\to\infty} a_n/b_n = L$ must be > 0 to apply limit comparison
- ALTERNATING SERIES
  - Applies only to series with alternating positive and negative terms.
  - What are the prerequisites for this test? Note that a_n must be a decreasing sequence, just $a_n \to 0$ is not enough.
  - Remember that most of our convergence tests only work for sequences with nonnegative terms. For general series, we need to try something else.
  - Absolute convergence implies convergence.
- ROOT AND RATIO TEST
  - Applies to any series. But the test is done using the absolute value of the terms.
  - Know what $\lim\limits_{n\to\infty} a^{1/n}$ and $\lim\limits_{n\to\infty} n^{1/n}$ are. More generally, if $p(n)$ is a polynomial, what are $\lim\limits_{n\to\infty} [p(n)]^{1/n}$ and $\lim\limits_{n\to\infty} \frac{p(n+1)}{p(n)}$? What does this mean for the root and ratio tests?

Problems

Exercise 1

Determine whether the following series converge or diverge. Clearly mention what test you are using. If you have to use multiple tests, mention each of the steps.

1. (11.4.5)
$$\sum \frac{n+1}{n\sqrt{n}}$$
2. (11.4.16)
$$\sum \frac{1}{n^n}$$
3. (11.4.27)
$$\sum \left(1+\frac{1}{n}\right)^2 e^{-n}$$
4. (11.4.31)
$$\sum \sin(1/n)$$
5. (11.5.11)
$$\sum (-1)^{n+1} \frac{n^2}{n^3+4}$$
6. (11.5.17)
$$\sum (-1)^n \sin(\pi/n)$$
7. (11.5.34)
$$\sum (-1)^{n-1} \frac{(\ln n)^p}{n}$$
For what values of p does it converge?
8. (11.7.10)
$$\sum \frac{(-3)^n}{(2n+1)!}$$
9. (11.7.44)
$$\sum \frac{(n!)^2}{(kn)!}$$
For what values of k does it converge?
10. (11.7.28)
$$\sum \left(\frac{-2n}{n+1}\right)^{5n}$$
11. (11.7.37)
$$\sum \frac{(-1)^n \arctan n}{n^2}$$
12. (11.7.40)
$$\sum a_n \quad \text{where} \quad a_1 = 1, \quad a_{n+1} = \frac{2+\cos n}{\sqrt{n}} a_n \text{ for } n \geq 1$$