

Please show **all** your work! Answers without supporting work will not be given credit. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Full Name: _____

Question	Points	Score
1	20	
2	10	
3	10	
4	7	
5	8	
Total:	55	

This exam has 5 questions, for a total of 55 points. The maximum possible point for each problem is given on the right side of the problem.

1. (a) Write down the precise statement of the Fundamental theorem of Integral Calculus.

4

- (b) Evaluate the following integrals.

i. $\int_1^{\sqrt{7}} \frac{x^3}{\sqrt[3]{x^2 + 1}} dx$

5

ii. $\int_{-\pi/2}^{\pi/2} (\sin x - x \cos x) dx$

3

- (c) Find the area bounded by the curves $y = |\cos x|$ and $y = 1$ between $x = 0$ and $x = 2\pi$.

2. Evaluate the following limit.

10

$$\lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \frac{1}{n} \cos \left(\frac{(i+1)\pi}{2n} \right)$$

3. Let

$$F(x) = \int_3^x \frac{2t - 3F'(t)}{x} dt$$

Evaluate $F'(3)$.

10

4. Show that the equation $x^3 - x^2 + x - 1 = 0$ has exactly one real root.

5. The thin lens equation in Physics is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, where u is the object distance from the lens, v is the image distance from the lens, and f is the focal length of the lens. Suppose that a certain lens has a focal length of 9 cm and that an object is moving towards the lens at the rate of 3 cm/sec. How fast is the image distance changing at the instant the object is 12 cm from the lens?