Assignment 2 (1/6)

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Problem 1

Problems 11.2.$(5, 6, 15, 21, 42, 61, 63)$.

Problem 2

Guess the nth term for the sequences 11.2.$(46, 52, 55)$.

Problem 3

The Floor function $f(x) = \lfloor x \rfloor$, aka the greatest integer function is defined to be the greatest integer $\leq x$. Similarly $g(x) = \lceil x \rceil$, the Ceiling function is defined to be the least integer $\geq x$. Read more about Floor and Ceiling functions here: WIKIPEDIA LINK.

(a) What is $\lfloor 1.5 \rfloor, \lfloor -2.3 \rfloor, \lceil 1.5 \rceil, \lceil -2.3 \rceil$?

(b) How does the graph of $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \lfloor x \rfloor$ look like?

(c) We define the fractional part of x as $\{x\} = x - \lfloor x \rfloor$. How does the graph of fractional part of x look like?

(d) Suppose a sequence $\{a_1, a_2, \ldots\}$ is defined as

$$a_i = f(i) = \begin{cases} 0 & \text{if } i \text{ is even} \\ 1 & \text{if } i \text{ is odd} \end{cases}$$

Can you give a closed form formula for $f(i)$ using floor and ceiling functions? By a closed form, I mean that the definition should not be casewise, there should be one single formula.