Assignment 11 (7/16)

Subhadip Chowdhury

- *This homework is due at the beginning of class on* ***Tuesday*** 7/24. *You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Exercise* 5 *and* 8 *are Extra credit problems. You are encouraged to think about them, but only after you have finished solving the rest of the exercises. Exercise* 9 *is hard, think about it if you want a challenge.*

Practice Problems

Let a and b be positive integers with $(a,b)=d$. Then clearly $d \mid a$ and $d \mid b$. Recall that we proved in class, if $a=md$ and $b=nd$, then $(m,n)=1$. The following exercise asks you to show the converse.

Exercise 1. *Let a,b,d,m,n be positive integers such that* $a=md$, $b=nd$ *and* $(m,n)=1$. *Show that* $(a,b)=d$.
*[*HINT: *Prove by contradiction.]*

Exercise 2. *Suppose* a,b, *and* c *are positive integers such that* $(ac,bc)=d$. *Show that* $(a,b)=\frac{d}{c}$.
*[*HINT: *Use exercise* 1.*]*

Exercise 3. *Using the identity* $(a+bc,b)=(a,b)$, *show that*

$$(2^m-1, 2^{m+1}-1)=1$$

for any positive integer m.

Exercise 4. *Suppose* a,b,x *and* d *are positive integers such that* $(a,x)=d$ *and* $(b.x)=1$. *Show that* $(ab,x)=d$.

Exercise 5 (Extra Credit)**.** *Suppose* a,b *and* c *are positive integers such that* $c \mid ab$ *and* $(b,c)=1$. *Then show that* $c \mid a$.
*[*HINT: *Prove by contradiction using exercise* 4.*]*

Exercise 6. *Find whether the following statements are True or False. If it's False, give a counterexample. If it's True, give a proof.*

(a) For all integers x *and* y, *we have* $\lfloor x+y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$.

(b) For all integers x, *we have* $\lfloor 2x \rfloor = 2\lfloor x \rfloor$.

(c) For all integers x, *we have* $\left\lfloor x^2 \right\rfloor = \lfloor x \rfloor^2$.

(d) For all integers x, *we have* $\lfloor x+0.5 \rfloor = \lfloor x-0.5 \rfloor$.

(e) For all integers x, *we have* $\lfloor x \rfloor + \lfloor -x \rfloor = 0$.

Exercise 7. *Solve the following equation for* x.

$$\lfloor 5-\lfloor x \rfloor \rfloor = 15$$

Exercise 8 (Extra Credit)**.** *Assume* k *is an integer and* n *is a natural number such that* $\left\lfloor \frac{10^n}{k} \right\rfloor = 2018$. *What is the minimum possible value of* n*?*

Exercise 9 (Hard, Challenge Problem)**.** *Recall that we defined the Fractional part of* x *to be* $\{x\} = x - \lfloor x \rfloor$. *Suppose* x *is a real number such that* $\{x\}, \lfloor x \rfloor$, *and* x *are in a GP. Find* x.