Review and Practice Problems [Part 1]

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Problem 1

Determine whether the following series converge or diverge. In each case, clearly mention what test(s) or theorem you are using.

1. $$\sum \frac{\sqrt{(ln(n))^3 + \ln(n^3)}}{n^3+1}$$

2. $$\sum \left(2+\frac{1}{n}\right)^n$$

3. $$\frac{1}{2}+\frac{2}{3^2}+\frac{4}{4^3}+\frac{8}{5^4}+\frac{16}{6^5}+\cdots$$

4. $$\sum \sin\left(\frac{\pi}{4n^2}\right)$$

5. $$\sum \frac{(2k+1)^{2k}}{(5k^2+1)^k}$$

6. $$\sum \frac{1}{k}\left(\frac{1}{\ln k}\right)^{3/2}$$

7. $$\sum \frac{(k!)^2}{(pk)!}, p \geq 2, p \in \mathbb{Z}$$

Problem 2

Suppose $\{a_k\}_{k\in\mathbb{N}}$ is a sequence such that $a_{k+10} = a_k$ for all $k \geq 1$. Does the series $\sum_{i=1}^{\infty} a_i$ converge or diverge?

Problem 3

Let $\{a_n\}_{n\geq 1}$ be a sequence defined as

$$a_n = \frac{3a_{n-1}}{1+a_{n-2}},\ a_1 = a_2 = 1$$

Assume that the sequence $\{a_n\}_{n\geq 1}$ converges to l. Find l.

Problem 4

Show that the sequence

$$a_n = \frac{\ln n}{n^2}$$

is a decreasing sequence for $n \geq 2$.

Problem 5

Let S be a surface given by the equation

$$\frac{x^2}{4} + \frac{y^2}{9} - z^2 = 1$$

Find the equation of the tangent plane to the surface S at the point $(2,3,1)$.

Problem 6

Let

$$f(x,y) = \frac{x}{y}$$

1. In what direction is $f(x,y)$ increasing most rapidly at $(1,1)$?
2. Suppose $x = \ln(3u)$ and $y = e^{4v}$. Find $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ at $u = 2, v = 0$.

Problem 7

Find

$$\int_0^4 \int_{-\sqrt{x}}^{\sqrt{x}} \int_x^y 2\, dz\, dy\, dx$$