Review and Practice Problems [Part 2]

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Problem 0 [Hints for Part 1]

1. 1. Comparison and Limit comparison test
   2. n—th term divergence test
   3. Geometric series
   4. Comparison test
   5. Root test and limit comparison test
   6. integral test
   7. Ratio test and limit comparison test
2. Diverges. Use nth term divergence test.
3. $3l = l + l^2$. Solve for l.
4. Show that first derivative of the function $\ln(x)/x^2$ is negative for $x \geq 2$.
5. Direct formula.
6. 1. f increases most rapidly in the direction of the gradient.
   2. Use chain rule.
7. Use wolfram alpha to double check your answer!

Problem 1

Suppose the three sides of an acute angled triangle $\triangle ABC$ are given by $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, and $\overrightarrow{AB} = \vec{c}$ respectively. Express the following vectors in terms of $\vec{a}, \vec{b}$, and $\vec{c}$ only.

1. $\overrightarrow{AD}$ where D is the midpoint of $\overline{BC}$.
2. $\overrightarrow{AD}$ where D is the foot of the perpendicular from A to $\overline{BC}$.
3. $\overrightarrow{AD}$ where D is the point in $\overline{BC}$ such that $\angle BAD = \angle DAC$.

   For part (3) proceed as follows:

   (a) Show that if $\angle BAD = \angle DAC$, then $\frac{BD}{DC} = \frac{AB}{AC}$.

   (b) Use above relation to find $\overrightarrow{BD}$.

   (c) Find $\overrightarrow{AD}$ from $\overrightarrow{AB}$ and $\overrightarrow{BD}$.

Problem 2

Suppose the vectors $\vec{b} = 4\hat{i} + 3\hat{j}$ and $\vec{c}$ are perpendicular to each other in the $XY-$plane.

1. Find all such $\vec{c}$.
2. Find all vectors $\vec{a}$ such that the length of the component of projection of$\vec{a}$ onto $\vec{b}$ (resp. $\vec{c}$) is 1 (resp. 2).

Problem 3

Find the directional derivative of $f(x, y, z) = xe^{y^2 - z^2}$ at $(1, 2, -2)$ toward the point $(2, 2, 1)$.

Problem 4

Problem 16.3.$(18, 28)$.

Problem 5

Problems 16.4.$(28, 31, 38(a, b))$.

Problem 6

Problems 17.3.$(13, 35)$.