Assignment 12 (10/26)

Subhadip Chowdhury

Problem 1

Assume f is continuous and

$$\int_0^x f(t)dt = \frac{2x}{4+x^2}$$

Determine $f(0)$. Find the roots of f, if any.

Problem 2

Evaluate

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \sin\left(\frac{i\pi}{n}\right)$$

Problem 3

Let

$$F(x) = \int_0^x x^2 \sin t \, dt$$

Find $F'(\pi/4)$.

Problem 4

Assume that f is a continuous function such that

$$\int_0^x f(t)\cos^2\left(t^2\right) dt = 6x^2 + 8\cos\left(x^2\right) - \sin\left(2x^2\right).$$

1. What is $f(\sqrt{\pi})$?
2. What is the area bounded by the curve $y = f(x)$ and the x-axis with $x \in [0, \sqrt{\pi}]$?

Problem 5

Evaluate

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} \sum_{i=0}^{n-1} \frac{1}{\sqrt{i}}$$

Problem 6

Evaluate

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n^2+k^2}$$

You might need to use the fact that $\int \frac{du}{u} = \ln|u|$.

Problem 7⋆

THIS IS NOT AN EASY PROBLEM. YOU DO NOT HAVE TO SUBMIT THIS. IF YOU DO AND HAVE THE CORRECT ANSWER, YOU WILL GET A CHOCOLATE!

Let $a \in \mathbb{R}$ be a constant such that $a \neq -1$. Evaluate

$$\lim_{n\to\infty} \frac{(1^a + 2^a + 3^a + \ldots + n^a)}{n^{a-1}\left[(na+1) + (na+2) + \ldots + (na+n)\right]}$$