

Assignment 20 (11/16)

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Problem 1

Problems 7.2.(8, 22, 23).

Problem 2

$\log_a(b)$ is defined as $\frac{\ln(b)}{\ln(a)}$. Thus in particular, $\log_a b$ is not defined for $a = 1$. Prove that the equation

$$x^{\log_{\sqrt{x}} 2x} = 4$$

has no solution.

Problem 3

Suppose for $a, b, c > 1$, we have

$$\frac{\ln a}{b - c} = \frac{\ln b}{c - a} = \frac{\ln c}{a - b}.$$

Prove that

$$a^a \cdot b^b \cdot c^c = 1.$$

Problem 4

Problems 7.3.(8, 10, 21, 29, 33, 60, 61).

Problem 5

Prove that

$$\int_a^b f(x)dx = \int_a^b f(a + b - x)dx. \quad (\star)$$

Using this result show that

$$\int_0^{\pi/2} \log(\tan(x))dx = 0.$$

Problem 6

1. Prove that

$$\int_0^{2a} f(x)dx = \begin{cases} 2 \int_0^a f(x)dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

2. Prove that

$$\int_0^{\pi/2} \log(\sin(2x))dx = \int_0^{\pi/2} \log(\sin(x))dx.$$

3. Use part (2) and (\star) to evaluate

$$\int_0^{\pi/2} \log(\sin(x))dx.$$