MATH 151(41) **Midterm 1** OCTOBER 24, 2014
SUBHADIP CHOWDHURY

Please show **all** your work! Answers without supporting work will not be given credit. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Full Name: _____

Question	Points	Score
1	10	
2	10	
3	8	
4	18	
5	11	
6	6	
7	12	
Total:	75	

This exam has 7 questions, for a total of 75 points. The maximum possible point for each problem is given on the right side of the problem.

Do not open this answer script unless you are instructed to do so.

1. (a) Solve the inequality and express the solution set *as an interval or as a union of intervals*: 5

$$\frac{5x^2(x-3)}{x-2} \leq 0.$$

(b) Mark the solution set on a Real number line. 2

(c) Give a sketch of the graph of the function $f(x) = \frac{5x^2(x-3)}{x-2}$. You only need to point out the X-intercepts, the signs and the behaviour of the function near any asymptote in the picture. 3

2. Let $f(x) = \cos(2x + \pi)$.

   (a) Find the period of $f(x)$. Justify your answer. [3]

   (b) Find all the numbers x in the interval $[-\pi, 0]$ such that $f(x) = -\frac{1}{2}$. [7]

3. Prove by mathematical induction that [8]

$$\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\ldots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$$

for all positive integers $n \geq 2$.

4. (a) Let $f(x)$ be a function defined on the Real line. Give the $\epsilon - \delta$ definition of what it means to write:

   i. $\lim_{x \to c} f(x) = L$. [5]

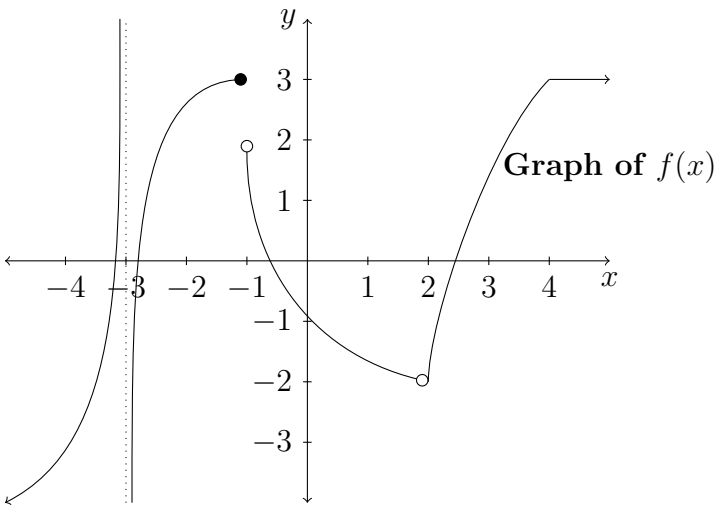
   ii. $\lim_{x \to c^-} f(x) = L$. [3]

   (b) Give an $\epsilon - \delta$ proof of the following: [10]

$$\lim_{x \to 1} (2x^2 - 3) = -1.$$

5. (a) Suppose $f(x)$ is a function defined on the real line. Define what it means to say $f(x)$ is continuous at a point $x = c$. [3]

(b) i. Determine the following from the graph below. **If a value is not defined or it does not exist, write '×' in the box.** [6]



c	-3	-1	2
$\lim_{x\to c^-} f(x)$			
$\lim_{x\to c^+} f(x)$			
$\lim_{x\to c} f(x)$			
$f(c)$			

ii. Determine whether the function in part (i) is left continuous, right continuous, both or neither at $c = -1$ and 2. [2]

6. Suppose $f(x)$ is a function defined on the real line as follows: [6]

$$f(x) = \begin{cases} \dfrac{1}{Ax-2}, & x \leq 1 \\ 3 - 2Ax + x^2, & x > 1 \end{cases}$$

Find the value of A that makes $f(x)$ continuous at $x = 1$ or prove that such a value does not exist.

7. (a) Let 6

$$f(x) = \frac{x^2 - 4}{x^2 - x - 2}.$$

Find the values of x where $f(x)$ is discontinuous. Justify your answer by evaluating appropriate limits (or showing that the limits do not exist).

(b) Classify each of the discontinuities of f as either a removable or an essential discontinuity. For any removable discontinuities, give a value for the function that would make it continuous at those points. 6