Math 1800-B Handout 6: Vector Fields and Flow Lines

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■ Exercise 1.

Fill the boxes with 'certainly', 'possibly', or 'certainly not'.

(a) The plot of the vector field $\vec{\mathbf{G}}(x, y) = \vec{\mathbf{F}}(2x, 2y)$ is $\boxed{}$ drawn by doubling the length of all the arrows in the plot of $\vec{\mathbf{F}}(x, y)$.

(b) If the flow lines for the vector field $\vec{\mathbf{F}}(x, y)$ are all concentric circles centered at the origin, then the dot-product $\vec{\mathbf{F}}(x, y) \cdot (x\hat{\imath} + y\hat{\jmath})$is $\boxed{}$ equal to zero.

(c) If the flow lines for the vector field $\vec{\mathbf{F}}(x, y)$ are all straight lines parallel to the constant vector $\vec{\mathbf{v}} = 3\hat{\imath} + 5\hat{\jmath}$, then $\vec{\mathbf{F}}(x, y)$ is $\boxed{}$ equal to $\vec{\mathbf{v}}$.

(d) The flow lines of the vector field $\vec{\mathbf{F}}(x, y) = e^x\hat{\imath} + y\hat{\jmath}$ $\boxed{}$ cross the X-axis.

■ Exercise 2.

Show that the flow lines of the vector field $\vec{\mathbf{F}}(x, y) = \langle y, x\rangle$ lies on level curves of the function $f(x, y) = x^2 - y^2$.

■ Exercise 3.

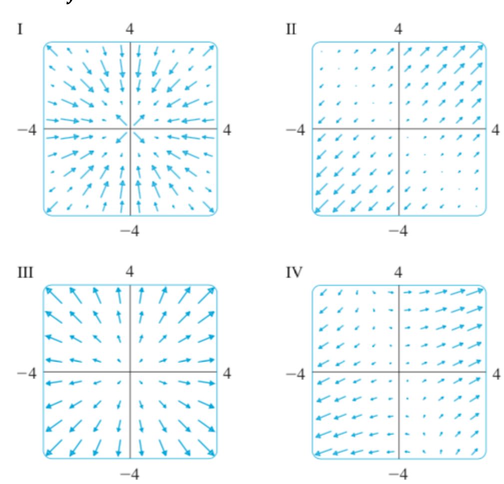
Show that $\vec{\mathbf{r}}(t) = \langle c_1e^t, c_2e^{-t}\rangle$ is a flow line of the vector field $\vec{\mathbf{F}}(x, y) = \langle x, -y\rangle$.

■ Exercise 4.

Find a vector field whose flow lines are of the form $\vec{\mathbf{r}}(t) = t\hat{\imath} + t^2\hat{\jmath}$.

■ Exercise 5.

Match the following functions with their gradient vector fields.



(a) $x^2 + y^2$

(b) $x(x + y)$

(c) $(x + y)^2$

(d) $\sin\sqrt{x^2 + y^2}$

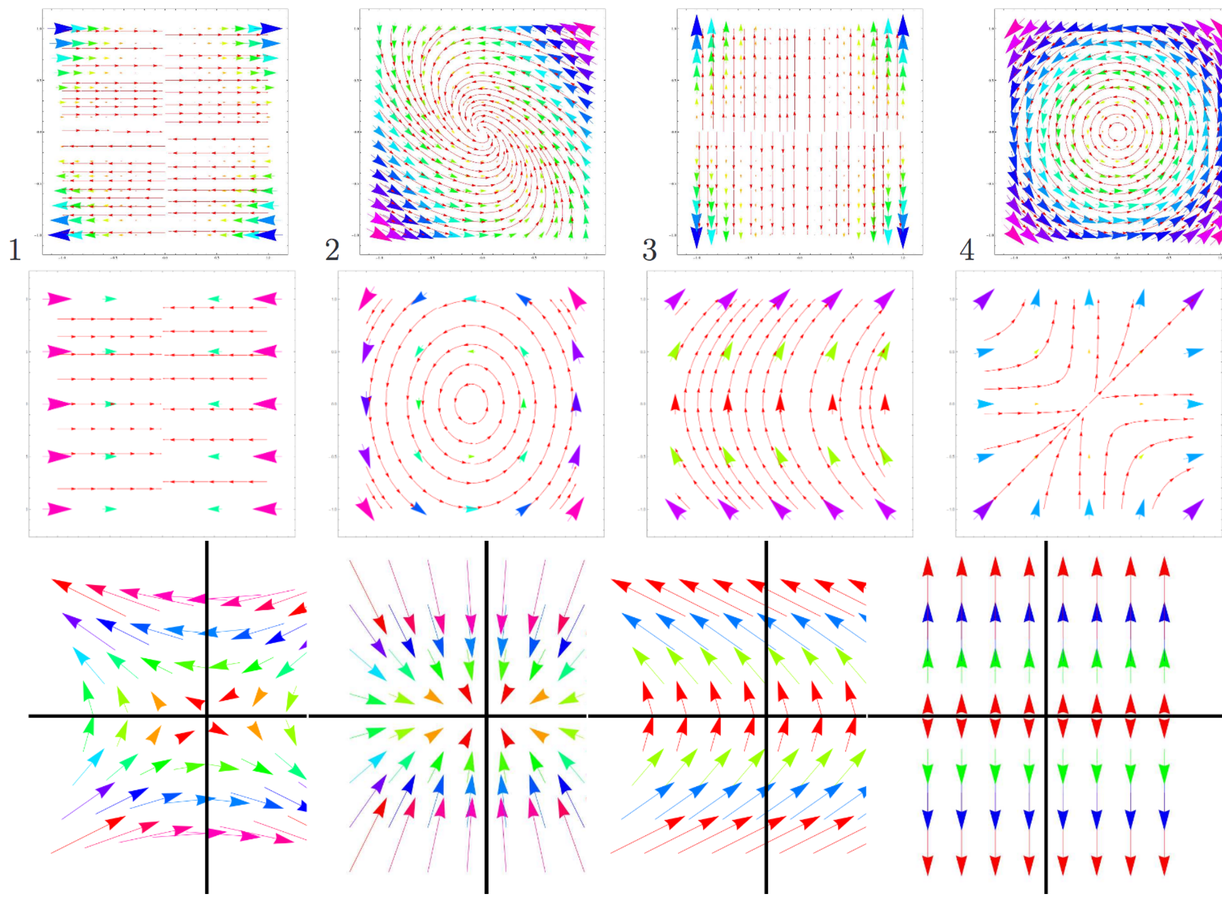
■ Exercise 6.

Match the vector fields.

a) $\langle y, 1\rangle$ b) $\langle 0, 2y\rangle$ c) $\langle -x, -2y\rangle$ d) $\langle -2y, 3x\rangle$

e) $\langle 0, x^2y\rangle$ f) $\langle -2y, -x\rangle$ g) $\langle x^2y, 0\rangle$ h) $\langle -x, 0\rangle$

i) $\langle -2y, 1\rangle$ j) $\langle -y - x, x\rangle$ k) $\langle -y, x\rangle$ l) $\langle x^2, y^2\rangle$



§1. Recap on Integrals

■ Exercise 1.

Sketch the region of integration of the following iterated integral, switch the order to $dydzdx$ and then evaluate the integral.

$$\int_0^{\pi} \left(\int_{\sqrt{z}}^{\sqrt{\pi}} \left(\int_0^{x} \sin(xy)\,dy \right) dx \right) dz$$

■ Exercise 2.

Consider the surface given by the graph of the function

$$z = f(x, y) = \frac{\arctan\left(x^2 + y^2\right)}{1 + (x^2 + y^2)^2}$$

Find the volume under the surface and above the region $x^2 + y^2 \leq 16$.

■ Exercise 3.

Find the double integrals

a) $\displaystyle\int_0^3 \int_y^3 \frac{\sin(2x)}{x} dxdy$

b) $\displaystyle\int_0^8 \int_{y^{1/3}}^2 \frac{y^2 e^{x^2}}{x^8} dxdy$