Midterm 1

OCTOBER 2, 2018
SUBHADIP CHOWDHURY

INSTRUCTIONS:

- Please show ALL your work! Answers without supporting justification will not be given credit.
- Answer the questions in the white space provided. If you run out of room, use the back page.
- Write legibly and clearly mark the answer.
- Please note that use of any books or notes is not allowed. You are allowed to use the one page of handwritten letter-sized note that you brought. Use of calculators are allowed.
- If you write down the correct formula for an answer, you will get some partial credit regardless of whether you evaluated the exact values or not.
- Unless otherwise specified, you may use any valid method to solve a problem.

Full Name: ____________________________

Question	Points	Score
1	10	
2	10	
3	15	
4	20	
5	10	
Total:	65	

This exam has 5 questions, for a total of 65 points.
The maximum possible point for each problem is given on the right side of the problem.

1. For the following problems, fill the box with either "**always**", "**sometimes**", or "**never**". *No explanation is necessary.*

   (a) The level surface of a three variable function $g(x, y, z)$ can be $\boxed{}$ represented as the graph of a two-variable function $f(x, y)$. $\boxed{2}$

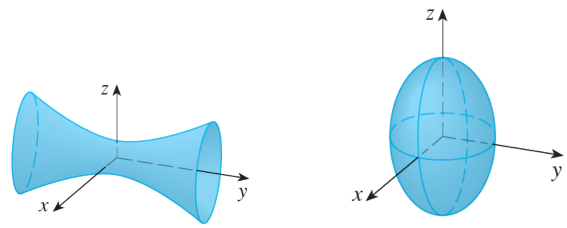
   (b) Assuming a and b are positive real numbers, the graphs of $f(x, y) = ax^2 + ay^2$ and $g(x, y) = 1 - bx^2 - by^2$ $\boxed{}$ intersect in a circle. $\boxed{2}$

   (c) Given three vectors $\vec{u}, \vec{v}$, and $\vec{w}$ in the space, $\vec{u} \cdot (\vec{v} \times \vec{w})$ is $\boxed{}$ equal to $(\vec{u} \times \vec{v}) \cdot \vec{w}$. $\boxed{2}$

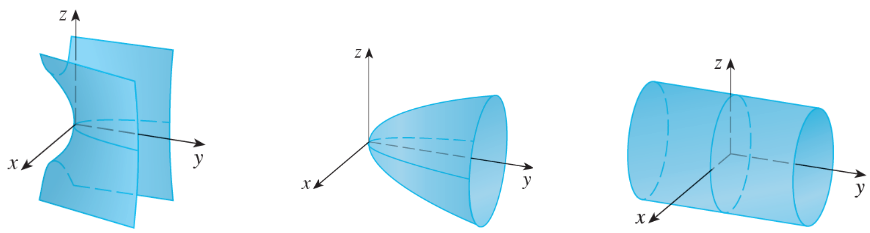
   (d) If $\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\|$, then $\|\vec{u} + \vec{v}\|$ is $\boxed{}$ equal to $\|\vec{u}\| + \|\vec{v}\|$. $\boxed{2}$

   (e) If you zoom in close enough near a point (a, b) on the contour diagram of a differentiable function, the contours are $\boxed{}$ parallel and equally spaced. $\boxed{2}$

2. (a) Match the following surfaces (labelled I-V) with its equation. *No explanation is necessary.* 5



(I) (II)



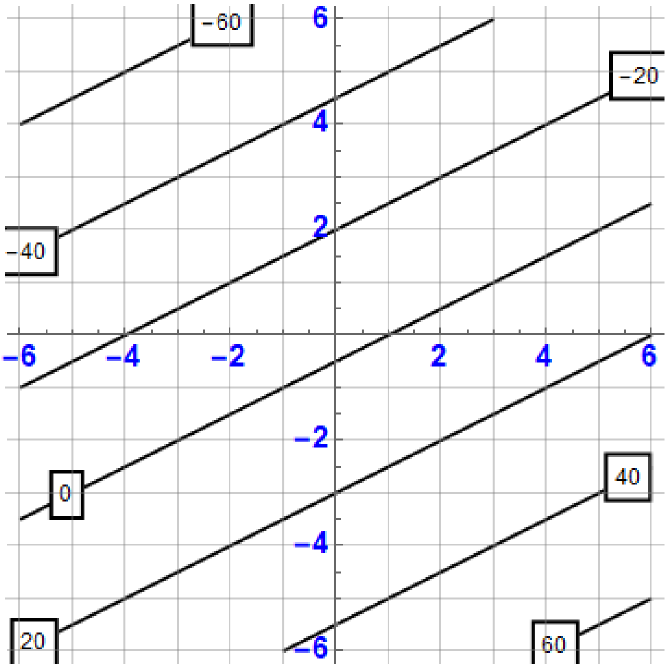
(III) (IV) (V)

$(A)\ 9x^2+4y^2+z^2=1$ ☐ $(B)\ y=2x^2+z^2$ ☐ $(C)\ y=x^2-z^2$ ☐

$(D)\ x^2+2z^2=1$ ☐ $(E)\ x^2-y^2+z^2=1$ ☐

(b) For each of the surface above, give a rough sketch of the level **0** and level **1** curve in its contour diagram. The curves do not have to be up to scale and the coordinates are not important. You only need to show the relative position of the two level curves. No explanation is necessary. 5

3. (a) Find the linear function $z = f(x, y)$ whose contour plot is shown below. Explain your reasoning. $\boxed{5}$



(b) Note that the graph of above linear function is a plane. Find an **unit vector** perpendicular to this plane. $\boxed{5}$

(c) Find the angle between this unit vector and the positive $\boldsymbol{Y}$-axis. $\boxed{5}$

4. (a) Find the equation of the plane that passes through the three points $\boldsymbol{P \equiv (2,1,0)}$, $\boldsymbol{Q \equiv (5,0,0)}$ and $\boldsymbol{R \equiv (-1,-1,-2)}$. $\boxed{10}$

(b) Find the perpendicular distance of the point $\boldsymbol{S \equiv (6,1,1)}$ from this plane. $\boxed{10}$

5. (a) Find the linear approximation of the function $\boxed{8}$

$$(x, y) = \frac{e^x \cos(xy)}{x^2 + y^2}$$

near the point $(0, 1)$.

(b) Use it to approximate $f(0.01, 0.99)$. $\boxed{2}$