MATH 1800

Midterm 2

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INSTRUCTIONS:

- Please show ALL your work! Answers without supporting justification will not be given credit.
- Answer the questions in the white space provided. If you run out of room, use the back page.
- Write legibly and clearly mark the answer.
- Please note that use of any books or notes is not allowed. You are allowed to use the one page of handwritten letter-sized note that you brought. Use of calculators are allowed.
- If you write down the correct formula for an answer, you will get some partial credit regardless of whether you evaluated the exact values or not.
- Unless otherwise specified, you may use any valid method to solve a problem.

Full Name: _____

Question	Points	Score
1	12	
2	15	
3	8	
4	10	
5	10	
6	10	
Total:	65	

This exam has 6 questions, for a total of 65 points.
The maximum possible point for each problem is given on the right side of the problem.

Do not turn the page unless you are instructed to do so.

1. For the following problems, determine whether each statement is true or false and then fill the box with either "True" or "False". If it is true, explain your reasoning briefly. If it is false, explain why or give a counterexample that disproves the statement.

(a) There exists a function f with continuous second-order partial derivatives such that 3

$$f_x(x,y) = x + y^2 \text{ and } f_y(x,y) = x - y^2.$$

(b) If $z = f(x-y)$, then $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$. 3

(c) If $f(x,y)$ has two local maxima, then $f(x,y)$ must have a local minimum. 3

(d) Different parametrizations of the same curve result in identical tangent vectors at a given point on the curve. 3

2. If a sound with frequency f_s is produced by a source traveling along a line with speed v_s and an observer is traveling with speed v_o along the same line from the opposite direction toward the source, then the frequency of the sound heard by the observer is

$$f_o = \left(\frac{c + v_o}{c - v_s}\right) f_s$$

where c is the speed of sound, approximately **332** m/s. (This is called the *Doppler effect*.) Suppose that, at a particular moment, you are in a train traveling at **34** m/s and accelerating at **1.2** m/s^2 . A train is approaching you from the opposite direction on the other track at **40** m/s , accelerating at **1.4** m/s^2, and sounds its whistle, which has a frequency of **460** Hz.

(a) At that instant, what is the perceived frequency that you hear? 3

(b) How fast is the perceived frequency changing? 12

3. Let $\boldsymbol{C}$ be the curve with equation

$$\boldsymbol{x = 2 - t^3, \quad y = 2t - 1, \quad z = \ln t}$$

(a) Find where $\boldsymbol{C}$ intersects the $\boldsymbol{XZ}$-plane. 2

(b) Find the parametric equation of the tangent line to $\boldsymbol{C}$ at $\boldsymbol{(1, 1, 0)}$. 6

4. Consider the surface

$$\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{c}.$$

(a) Find the equation of the tangent plane to this surface at a point (p, q, r). 7

(b) Show that the sum of X-intercept, Y-intercept and Z-intercept of the above tangent plane does not depend on $p, q,$ and r. 3

5. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the point on this ellipse that is nearest to the origin. [10]

   HINT: Use Lagrange multipliers.

6. Find all the stationary critical points for the following function and classify them as local minima, local maxima, or saddle points. [10]

$$f(x,y) = (x^2 + y^2)\, e^{y^2 - x^2}$$