

Please show **all** your work! Answers without supporting work will not be given credit. Answer the questions in the spaces provided on the question sheets.

**If you run out of room for an answer, continue on the back of the page.**

**Please note that use of calculator is not allowed.**

Full Name: \_\_\_\_\_

Question	Points	Score
1	5	
2	5	
3	15	
4	15	
5	15	
6	10	
7	10	
Total:	75	

This exam has 7 questions, for a total of 75 points.  
The maximum possible point for each problem is given on the right side of the problem.

1. Consider the straight line in 3 dimension that passes through the origin and the point  $(3, 4, 12)$ . What is the angle between the straight line and the positive  $X$ -axis?

2. Find  $\lambda$  and  $\mu$  if the vector  $\vec{a} = 3\hat{i} + \lambda\hat{j} - \hat{k}$  is perpendicular to the vector  $\vec{b} = 2\hat{i} + \hat{j} + \mu\hat{k}$  and  $\|\vec{a}\| = \|\vec{b}\|$ .

3. The vertices of a triangle  $\triangle ABC$  are  $A = (4, 3, 2)$ ,  $B = (1, 3, 1)$ , and  $C = (-5, 5, -2)$ . Let  $D$  be the foot of the perpendicular from  $A$  to the side  $\overline{BC}$ .

(a) Find the vectors  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ .

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(b) Find the vector  $\overrightarrow{BD}$ .

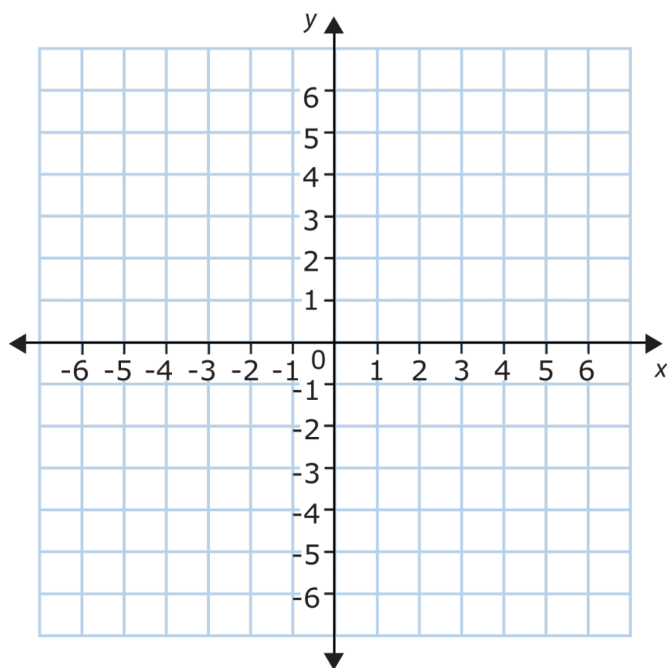
6

[HINT:  $\overrightarrow{BD}$  is the projection of  $\overrightarrow{BA}$  on to  $\overrightarrow{BC}$ ]

(c) Find the length of  $\overline{BD}$  and  $\overline{AD}$ .

5

4. (a) Find the equation of the plane that passes through the point  $P(4, 0, 1)$  and is perpendicular to  $\vec{n} = \hat{i} - \hat{j} + 4\hat{k}$ . 5
- (b) Suppose this plane intersects the  $X$ ,  $Y$ , and  $Z$  axes at  $A$ ,  $B$ , and  $C$  respectively. Find the coordinates of  $A$ ,  $B$ , and  $C$ . 3
- (c) Find the vector  $\overrightarrow{BC}$ . 1
- (d) Find the equation of the straight line that passes through  $A$  and is parallel to  $\overrightarrow{BC}$ . 6

Figure 1: Level curve of  $f(x, y)$  passing through  $(4, 1)$ 

5. (a) Identify the level curve of  $f(x, y) = \ln(x - y^2)$  that passes through  $(2, 1)$ . Describe and draw a picture of it **in the given coordinate grid**. 5
- (b) Find the gradient of  $f$  at  $(2, 1)$ . 5
- (c) Find  $\frac{\partial^2 f}{\partial x \partial y}$  and  $\frac{\partial^2 f}{\partial y \partial x}$  at  $(2, 1)$ . 5

6. Let

$$f(x, y) = \frac{x^3 + xy}{y\sqrt{4y - x^2}}$$

(a) Find and describe the domain of the function  $f(x, y)$ .

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(b) Show that

$$\lim_{(x,y) \rightarrow (0,0)} f(x, y)$$

6
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does not exist by considering one path to the origin along the  $Y$  axis and another path along the parabola  $y = x^2$ .

7. Find the directional derivative of  $f(x, y, z) = e^x \cos(xyz)$  in the direction parallel to the line

10
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$$\frac{x-1}{4} = \frac{y-4}{5} = \frac{z-2}{-3}$$