Quiz 1 Solution (10/3)

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Note: The marks for Quiz 1 are up on Chalk. It will NOT be counted towards your final grade! The decision is due to the following fact. After reviewing the answer, I realized that even if most of you can find the correct final answer, you are unaware of how to formally "write" a mathematical proof. Also, since the idea of surjective and injective functions are not discussed in the book, it might have been unfair to put that in a Quiz. In any case, I have included a model solution below. There are couple of things to note:

- Words like 'and', 'or', 'hence' etc. or symbols like ' $\implies$ ', '$\therefore$', '$\because$' etc. are not to be used irresponsibly/incorrectly. On the other hand, you must use them when they are needed so that the proof itself is readable.
- While drawing a graph, the axes and the plotted function itself must be marked.

$\boxed{\text{Problem 1}}$

Problem: Solve the inequality and express the solution set *as an interval or as a union of intervals*: $4 \leq |x-3| < 6$.

Rule of thumb: 'Or' turns into '$\cup$' , 'And' turns into '$\cap$'.

Approach 1:

$$\begin{aligned} & 4 \leq |x-3| < 6 \\ \implies & 4 \leq |x-3| \text{ and } |x-3| < 6 \end{aligned}$$

Now,

$$\begin{aligned} & 4 \leq |x-3| \\ \implies & x-3 \geq 4 \text{ or } x-3 \leq -4 \\ \implies & x \geq 7 \text{ or } x \leq -1 \\ \implies & x \in (-\infty, -1] \cup [7, \infty) \end{aligned}$$

Similarly,

$$\begin{aligned} & |x-3| < 6 \\ \implies & x-3 < 6 \text{ or } x-3 > -6 \\ \implies & x < 9 \text{ or } x > -3 \\ \implies & x \in (-3, 9) \end{aligned}$$

Hence x satisfies given inequality when $x \in (-3, 9) \cap ((-\infty, -1] \cup [7, \infty))$. Hence the solution set is given by

$$x \in (-3, -1] \cup (7, 9]$$

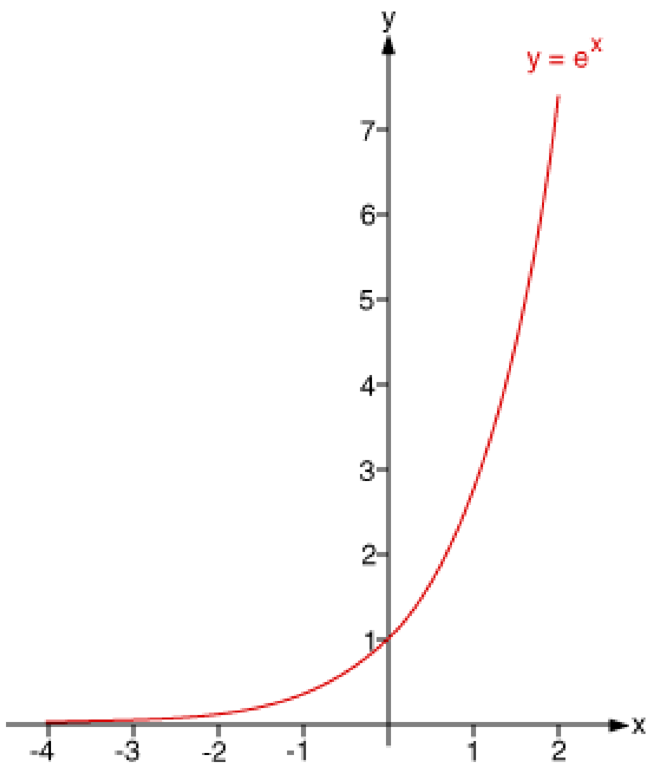
Approach 2:

$$\begin{aligned}
&4 \leq |x-3| < 6 \\
\implies &\begin{cases} 4 \leq x-3 < 6 & \text{when } x > 3 \\ \text{or} \\ 4 \leq -x+3 < 6 & \text{when } x \leq 3 \end{cases} \\
\implies &\begin{cases} 7 \leq x < 9 & \text{when } x > 3 \\ \text{or} \\ -1 \geq x > -3 & \text{when } x \leq 3 \end{cases} \\
\implies &x \in [7, 9) \cup (-3, -1]
\end{aligned}$$

Problem 2

Problem: Give an example of a function from $\mathbb{R}$ to $\mathbb{R}$ which is injective (one-one) but not surjective (onto). Drawing a graph of the function without giving an explicit formula will be accepted as a correct answer.

An example of such a function would be $f(x) = e^x$. The graph is as follows:



Notes: there are of course other solutions. Any such function must satisfy the following properties:

1. The function must be defined on all of $\mathbb{R}$. E.g. $f(x) = \sqrt{x}$ are not allowed.
2. To be an **injective** function, the graph can cross any horizontal line **at most once**. E.g. $f(x) = x^2$ is not allowed.
3. For the function to be **non-surjective**, there must be some horizontal line which the graph does **not** cross. E.g. $f(x) = x + 1$ or x^3 etc. are not allowed.