Math 1600 Handout : Practice Problems for Second Derivative, Power Rule, Exponential Function, and Product Rule

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Power Rule and Derivative of Exponential Functions

The *Power Rule* states that

$$(x^a)' = ax^{a-1}$$

for any real number a. Note that this rule applies only when the base is x and the exponent doesn't depend on x.

The derivative of a^x is given by

$$(a^x)' = a^x \ln(a)$$

As a specific case, derivative of e^x is e^x since $\ln e = 1$.

Exercise 1

Let $f(x) = x^{100}$. Which of the following statements are true?

(I) The 100^{th} derivative of f is zero everywhere.

(II) The 101^{st} derivative of f is zero everywhere.

(III) The 102^{nd} derivative of f is zero everywhere.

Ans: (II) and (III) but not (I)

Exercise 2

Let $f(x) = 2x^{12} + 5x^{11} - 6x^7 + 1$. We want to find the seventh derivative of f. Without actually evaluating the derivative, can you choose the correct answer from following choices?

(A) $7983360x^6 + 8316000x^4 + 30240$

(B) $7983360x^6 + 8316000x^4 - 30240$

(C) $7983360x^5 + 8316000x^3 - 30240$

(D) $7983360x^5 + 8316000x^4 + 30240$

(E) $7983360x^5 + 8316000x^4 - 30240$

Ans: E

Exercise 3

If $f(x) = x^2 + x$ and $g(x) = x^3 + \lambda$, for what value of λ do we have $f(\lambda) = g(\lambda)$ and $f'(\lambda) = g'(\lambda)$?
Ans: 1

Exercise 4

Let $P(x) = ax^3 + bx^2 + cx + d$. If $P(0) = P(1) = -2$, $P'(0) = -1$, and $P''(0) = 10$, what is $P'''(0)$?

Ans: -24

Exercise 5

Consider a function $f(x)$ defined as follows:

$$f(x) = \begin{cases} b + ax - x^2 & \text{for } x < 2 \\ ax^2 + bx + 2 & \text{for } x \geq 2 \end{cases}$$

If both $f(x)$ and $f'(x)$ are continuous at $x = 2$, then find a and b.

Ans: $a = 2, b = -10$

Exercise 6

Suppose $f(x) = e^x$ and $g(x) = ax^2 + bx + c$. If $f(0) = g(0)$, $f'(0) = g'(0)$, and $f''(0) = g''(0)$, then find $a, b,$ and c.

Ans: $1/2, 1, 1$

Exercise 7

(a) Find the slope of the graph of $f(x) = 1 - e^x$ at the point where it crosses the x-axis.

(b) Find the equation of the tangent line to the curve at this point.

(c) Find the equation of the line perpendicular to the tangent line at this point.

Ans: (a)-1, (b)$y = -x$, (c)$y = x$

Exercise 8

For what value(s) of a are the graphs of $y = a^x$ and $y = 1 + x$ tangent to each other at $x = 0$? Explain.

Ans: e

Product and Quotient Rule

The *product rule* for derivatives states that

$$(fg)' = f'g + g'f$$

The *quotient rule* for derivatives states that

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

Exercise 9

Suppose f and g are functions with $g(3) = 2$, $f'(3) = -1$, and $g'(3) = 0$. What is the derivative of $h(x) = \frac{f(x)}{g(x)}$ at $x = 3$?

Ans: $-\frac{1}{2}$

Exercise 10

Suppose f, g, and h are nonzero differentiable functions with $h(x) = f(x)g(x)$ for all real x. Suppose also that

$$h'(1) = 12h(1), \quad f'(1) = 4f(1), \quad g'(1) = \lambda g(1)$$

Then find the value of λ.

Ans: 8

Exercise 11

Let f be a function with $f(5) = 2$ and $f'(5) = -1$. Let $g(x) = x^2 f(x)$. Find $g'(5)$.

Ans: -5

Exercise 12

Suppose $h(x) = f(x)g(x)$ and $g'(x) = xg(x)$. If $g(2) = 1$, $f'(2) = 3$, and $f(2) = 4$, then find $h'(2)$.

Ans: 11

Curve Sketching

Exercise on Increasing and Decreasing Function

1. Let $f(x) = \frac{1}{4}x^4 - x^3$. On what intervals is f increasing and on what intervals is it decreasing?

   (a) Increasing on $(3, \infty)$; decreasing on $(-\infty, 3)$
   (b) Increasing on $(4, \infty)$; decreasing on $(-\infty, 4)$
   (c) Increasing on $(-\infty, 0)$ and $(2, \infty)$; decreasing on $(0, 2)$
   (d) Increasing on $(-\infty, 0)$ and $(3, \infty)$; decreasing on $(0, 3)$
   (e) Increasing on $(-\infty, 0)$ and $(4, \infty)$; decreasing on $(0, 4)$

   Ans: a

2. If f is a differentiable function with $f'(x) < 0$ everywhere and $f(0) = 0$, then which of the following are possibilities for $f(-1)$?

   (I) $f(-1) > 0$
   (II) $f(-1) = 0$
   (III) $f(-1) < 0$

   (a) (I) only
   (b) (III) only
   (c) (I) and (II) only
   (d) (II) and (III) only
   (e) There is not enough information to say anything about $f(-1)$.

   Ans: a

Exercise on Concavity

1. Let $f(x) = \dfrac{1}{x+3}$. On what intervals is f concave up and on what intervals is it concave down?

   (a) The function f is concave up on $(-\infty, -3) \cup (-3, \infty)$, and concave down nowhere

   (b) The function f is concave up nowhere, and concave down on $(-\infty, -3) \cup (-3, \infty)$

   (c) The function f is concave up on $(-\infty, -3)$, and concave down on $(-3, \infty)$

   (d) The function f is concave up on $(-3, \infty)$, and concave down on $(-\infty, -3)$

   (e) The function f is neither concave up nor concave down since $x + 3$ is linear.

Ans: d

2. Suppose f is concave down everywhere and $y = ax + b$ is the equation of the line tangent to the graph of $f(x)$ at $x = 0$. Which of the following inequalities is true?

   (a) $ax + b > f(x)$ for all x

   (b) $ax + b \geq f(x)$ for all x

   (c) $ax + b \leq f(x)$ for all x

   (d) $ax + b < f(x)$ for all x

   (e) None of these inequalities must be true.

Ans: b