Assignment 6 (10/10)

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Note: I understand that the quiz was a bit long. Depending on average class performance the second quiz might be weighed differently.

Problem 1

Problem 2.2.(21,22).

Problem 2

Give an $\epsilon - \delta$ proof of the following:

(a) $\lim\limits_{x\to 3}(x-2) = 1$.

(b) $\lim\limits_{x\to 1} \frac{3x+2}{5} = 1$.

(c) $\lim\limits_{x\to 2} 1 = 1$.

Follow this proforma to write the proofs: For example, let's show that $\lim\limits_{x\to 0}(2x+1) = 1$.

Proof: Let us take an arbitrary $\epsilon > 0$. We want to prove that there exists $\delta > 0$ such that

$$0 < |x - 0| < \delta \implies |2x + 1 - 1| < \epsilon.$$

So, we want to find a δ such that $0 < |x| < \delta$ implies $|2x| < \epsilon$. Clearly it suffices to take $\delta = \frac{\epsilon}{2}$, since that would imply

$$|x| < \frac{\epsilon}{2} \implies |2x| < \epsilon.$$

[Proved]

Couple of notes:

- In these proofs, your main goal is to find a δ that works.
- δ depends on ϵ. But ϵ does not depend on anything. We only know that $\epsilon > 0$.
- We are finding a δ such that the '*implication*', i.e. the following claim:

$$0 < |x - c| < \delta \implies |f(x) - l| < \epsilon$$

  is true. It is not enough for only one of the above two statements to be true, we need to show that one *implies* the other.

- δ is a function of ϵ, but not necessarily bigger or smaller than ϵ.
- For an ϵ, there might be multiple values of δ which work. For example, in the above proof, if we had chosen $\delta = \epsilon/4$, the proof would still work. We will then write

$$|x| < \frac{\epsilon}{4} \implies |2x| < \epsilon/2 \implies |2x| < \epsilon$$

since $\epsilon > 0$.