Final Exam

Please show **all** your work! Answers without supporting work will not be given credit. Answer the questions in the green books provided.
Please write your name on each green book you use.
Please note that use of calculator is not allowed.
You can score a maximum of 90 points in this exam.

Full Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	15	
6	10	
7	15	
8	10	
9	5	
Total:	95	

This exam has 9 questions, for a total of 95 points, but you can score a maximum of 90 points in this exam i.e. you have 5 bonus points.
The maximum possible point for each problem is given on the right side of the problem.

1. (a) Rewrite the following word problem as a Linear Programming Problem i.e. identify the objective function and the constraints. [4]

   To beat their competition, Buzzfeed has to create at least 90 more articles and at least 60 more videos, but no more than 90 more quizzes every month. The company can hire in their new office buildings at two locations: one in Chicago and one in New York; each location can hire at most 50 people.

   The Chicago location has enough resources for one person to produce 20 articles, 10 videos, and 10 quizzes per month. The New York location has enough resources for one person to produce 10 articles, 10 videos, and 10 quizzes per month.

   The salary per person in Chicago office is \$2000 per month and in New York office is \$3000 per month.

   How many people should Buzzfeed hire at each of the new location so that they have to pay the **least** total salary?

   (b) **Plot** the polygonal solution region and identify its vertices. [4]

   (c) Find the optimal answer by checking the objective function at each of the vertices. [2]

2. Find equation of the straight line that passes through $(3, 1, 4)$ and is parallel to the line of intersection of the planes [10]

$$x + 2y + 3z = 1 \text{ and}$$
$$2x - y + z = -3$$

3. Find the **angle** at which the curve [10]

$$\vec{r}(t) = \cos(\pi t)\,\hat{i} + \sin(\pi t)\,\hat{j} + 2t\,\hat{k}, \quad t \in \mathbb{R}$$

   intersects the surface

$$2x^2 + z^2 - xy + 2xz - 1 = 0$$

   at the point $(0, 1, 1)$. **Do not simplify your final answer.**

4. Use polar coordinates to find the integral [10]

$$\iint_{\Omega} \sqrt{x^2 + y^2}\, dA$$

   where Ω is the region bounded by the circle $x^2 + y^2 = 2x$.

   [Hint: First rewrite the domain using polar coordinates to get bounds for the integral.]

5. Locate and classify all local maxima, local minima, and saddle points of the function **15**

$$f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$$

[HINT: First find the critical points, then check the Hessian at each of those points.]

6. Find the extrema of the function $F(x,y) = 2y + x$ subject to the constraint **10**

$$g(x,y) = y^2 + xy - 1 = 0.$$

7. Evaluate the integral **15**

$$\iint_R \frac{1}{x} dA$$

over the region R bounded by the four straight lines

$$\begin{aligned} x - y &= 1 \\ x - y &= 4 \\ y &= 2x \\ y &= 3x \end{aligned}$$

using the change of variable $u = x - y$ and $v = y/x$.

You get part marks for finding the domain of integration in uv-coordinates[2], for calculating determinant of the Jacobian[6], for writing the new integrand in terms of u and v variables[4] and for finishing the integration[3].

8. Find whether the following series are convergent or divergent. **Clearly mention what test(s) you are using.**

(a) **5**

$$\sum_{n=1}^{\infty} \frac{\sqrt{2n^3 - (\ln n)^2}}{\sqrt[3]{n^5 + 5}}$$

(b) **5**

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

9. Find the Taylor Series of $f(x) = \dfrac{1}{x^2}$ centered at $x = 1$. **5**