Lab 1: 3D Graphing with Mathematica

Subhadip Chowdhury

- Sign on to an iMac with your username and password.
- When you open *Mathematica*, click **New Document** and a blank screen will appear. This is known as a notebook. If you need to open a new notebook, go to **File**, choose **New** and choose **Notebook** or simply hit `Command + N` (or `Ctrl + N` for **Windows**).
- Make the Untitled window bigger if necessary by dragging the lower right corner. Choose at least 125% from the lower left of the Untitled window for a comfortable viewing size.
- Do not forget to **Save** the notebook periodically. Save new notebooks on the **Desktop**. The **Save** command is under the **File** menu.
- Follow the instructions in the paper copy of the handout. Write the answers in the blue book provided.
- At the end of Lab session, quit Mathematica, then log off or restart the computer. Do NOT click **Shut Down**.

We'll be learning to use two main commands in this lab. The first one (`Plot3D`) produces 3D surfaces, and the second (`ContourPlot`) produces contour plots for functions of two variables.

Exercise 1: Defining custom functions

1. Recall that to define a function $f(x,y)$ we write

   `f[x_,y_]:=`*definition*

   Note the 'underscore' after `x` and `y` in the left hand side. Type

   ```
   f[x_,y_]:=x^2+y^2
   ```

   to define $f(x,y)=x^2+y^2$.

Exercise 1: 3D Graphing

2. Type

   ```
   Plot3D[f[x,y],{x,-1,1},{y,-1,1}]
   ```

   to plot the graph of $f(x,y)$ over the domain $-1 \le x \le 1, -1 \le y \le 1$.

3. Type

   ```
   ContourPlot[f[x,y],{x,-1,1},{y,-1,1}]
   ```

   to produce the contour plot of the same function.

Recall the "monkey saddle" function from the last lab.

$$M(x,y) = 3xy^2 - x^3$$

4. Draw the `ContourPlot` of the monkey saddle function.
5. Explain what the cross sections parallel to x, y, and z axis look like.

Exercise 3: More on Contour Plots

Use the following format to get a better picture for the contour diagram. Explre other customization options as well.

```
ContourPlot[f[x,y], {x, -10, 10}, {y, -10, 10},
ContourLabels -> True, ContourStyle -> Thickness[.005]]
```

Draw the graph and the contour plots of the following functions.

6. $2x + y + 4$
7. $\frac{x}{2} + \frac{y}{4} + 1$
8. Why do you get similar contour plots both times?

There are rare occasions when you may wish to use a set of non-evenly-spaced z values to compute contour lines. For the last example, we are going to start with evenly-spaced z values $1, 2, ..., 9, 10$, but then switch to $10, 20, ..., 50, 60$. The reason is when z gets large, the corresponding contour lines become too close together, hence it makes sense to "change the scale" by upping the Δz value from 1 to 10.

9. First plot the graph and the contour plot of $x^2 + 2y^2$ as before. Observe why we need to display contours selectively.
10. Next, add the option

    ```
    Contours -> {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60},
    ```

    to selectively display contours in the last picture.

Exercise 4: Level Surface

The `ContourPlot3D` command is designed to plot the surfaces consisting of all the input triples (x, y, z) generating the same output from a function of three variables $g(x, y, z)$ (these are called "level surfaces").

11. Type the following and compare your output to problem 1.

    ```
    ContourPlot3D[x^2 + y^2 - z == 0,{x,-1,1},{y,-1,1},{z,0,1}]
    ```

12. Recall that the graph of f is defined by the assignment $z = f(x, y)$. Use a simple algebra manipulation to show that, for the three-variable function defined by $g(x, y, z) = f(x, y) - z$, the set of triples (x, y, z) satisfying $g(x, y, z) = 0$ is the same as the set of points on the graph of f. Thus the 'level surface' of g at level 0 becomes the graph of f.

Exercise 5: Exploratory Plotting

Use the graphing commands to plot some surfaces. For instance, try plotting

13. a plane which intersects x-axis at $(3, 0, 0)$, y-axis at $(0, 4, 0)$, and z-axis at $(0, 0, 5)$.
14. a hemispherical dome above the XY-plane, that has radius 2 and center at $(1, 1, 0)$.
15. a cone with vertex at the origin and the laeral surface making an angle of 45° with the XY-plane.