Assignment 2 (1/5)

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- *This homework is due at the beginning of class on* ***Friday*** 1/12*. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material part. You are encouraged to think about the exercises marked with a* $(\star)$ *or* $(\dagger)$ *if you have time, but you don't need to hand them in. If you correctly solve a* $(\dagger)$*-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering Chapter* 12 *from Stewart.*

Important Points and Reading Materials

- Vectors:
  - What is a vector - an object with a magnitude and a direction.
  - Two vectors are parallel iff they are scalar multiple of each other.
  - What is the unit vector in the direction of a vector $\vec{u}$?
- Dot Products:
  - Two ways of defining.
  - Use dot products to find angle between vectors.
  - Understand the scalar projection, $comp_{\vec{v}}\vec{u}$ and vector projection $Proj_{\vec{v}}\vec{u}$ of $\vec{u}$ onto $\vec{v}$. How do you compute these? What do they mean geometrically?
  - How do the direction cosines relate to projections on to $\hat{i}, \hat{j}, \hat{k}$?

Problems

Exercise 1

(12.2.8) If three vectors $\vec{u}, \vec{v}$ and $\vec{w}$ satisfy $\|\vec{u}\| = \|\vec{v}\| = 1$, $\vec{u} \perp \vec{v}$, and $\vec{u} + \vec{v} + \vec{w} = 0$, then what is $\|\vec{w}\|$?

Exercise 2

Find a unit vector that has the same direction as $3\hat{i} - 4\hat{j} + 12\hat{k}$. Find another vector in the same direction that has length 4.

Exercise 3

(12.3.4,10) Find $\vec{a} \cdot \vec{b}$ where

1. $\vec{a} = \langle 6, -2, 3\rangle, \vec{b} = \langle 2, 5, -1\rangle$
2. $\|\vec{a}\| = 80, \|\vec{b}\| = 50$, the angle between $\vec{a}$ and $\vec{b}$ is $3\pi/4$.

Exercise 4

(12.3.27) Find a unit vector that is orthogonal to both $\hat{i} + \hat{j}$ and $\hat{i} + \hat{k}$.

Exercise 5

(12.3.32) The angle between two curves is the angle between their tangent lines at the point of intersection. Find the acute angle between the curves $y = \sin x$ and $y = \cos x$ at the point of their intersection in $[0, \pi/2]$.

[HINT: derivative is same as slope of the tangent.]

Exercise 6

Suppose $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{u} = 3\vec{a} + 2\vec{b}$ and $\vec{v} = \vec{a} - 4\vec{b}$ are perpendicular. Find $\vec{a} \cdot \vec{b}$ and the angle between $\vec{a}$ and $\vec{b}$.

Exercise 7

Consider the plane $\mathscr{P}$ that passes through the point $A = (3, 4, 5)$ and is perpendicular to the vector $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$. Let B be the point $(6, 6, 6)$ and let D be the foot of the perpendicular from B to $\mathscr{P}$.

1. Find the vector $\overrightarrow{AB}$.
2. Find the projection of $\overrightarrow{AB}$ onto $\vec{n}$. [HINT: Use the projection formula. This is the vector AD.]
3. What is the length of $\overline{BD}$? [HINT: $\triangle ABD$ is a right angled triangle. Use Pythagoras.]

Note that you do not need to use equation of lines/planes for this problem.

Exercise 8

Suppose the three sides of an acute angled triangle $\triangle ABC$ are given by $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, and $\overrightarrow{AB} = \vec{c}$ respectively. Express the following vectors in terms of $\vec{a}, \vec{b}$, and $\vec{c}$ only.

1. $\overrightarrow{AD}$ where D is the midpoint of $\overline{BC}$.
2. $\overrightarrow{AD}$ where D is the foot of the perpendicular from A to $\overline{BC}$.
3. (†) $\overrightarrow{AD}$ where D is the point in $\overline{BC}$ such that $\angle BAD = \angle DAC$.