Lab 2: Partial Derivatives

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- Sign on to an iMac with your username and password.
- When you open *Mathematica*, click **New Document** and a blank screen will appear. This is known as a notebook. If you need to open a new notebook, go to **File**, choose **New** and choose **Notebook** or simply hit `Command + N` (or `Ctrl + N` for **Windows**).
- Make the Untitled window bigger if necessary by dragging the lower right corner. Choose at least **125%** from the lower left of the Untitled window for a comfortable viewing size.
- Do not forget to **Save** the notebook periodically. Save new notebooks on the **Desktop**. The **Save** command is under the **File** menu. Give your file a name in the following format:

  `Lab2_Name1_Name2.nb`
- Follow the instructions in the paper copy of the handout.
- Write down the answers to the ? marked problems in your blue books.
- At the end of Lab session, save and quit Mathematica. **Do not delete your file or any calculation you did.** Then log off or restart the computer. Do NOT click **Shut Down**.

Exercise 1: Defining the Function and Partial Derivatives

1. Enter the following command to define the function we'll explore in this lab:

```
f[x_,y_]:=Cos[x]Sin[x+y]
```

2. Enter the following Plot command to plot the y cross-section of the graph of f at $y = 3\pi/2$ for $1.5 \leq x \leq 1.5$.

```
Plot[f[x,3*Pi/2],{x,-1.5,1.5}]
```

3. By changing the endpoints for x, zoom in around the point $x = 0.5$ until the graph looks linear. Estimate the slope of this (tangent) line.
4. Define a new function, `fx` which is the partial derivative of f with respect to x, by entering the command:

```
fx[x_,y_]:=D[f[x,y],x]
```

5. Then evaluate `fx` at the point $(0.5, 3\pi/2)$ by entering the command:

```
fx[0.5,3*Pi/2]
```

6. Repeat the above steps, but this time use an x cross-section at $x = 0.5$ and zoom in around $y = 3\pi/2$. Use `fy` to define the partial derivative with respect to y.
7. ? How are the values of `fx` and `fy` related to your estimated tangent slopes?

Exercise 2: Investigating Contour Plot of the function

8. Create a contour plot of f on $1.5 \leq x \leq 1.5, \pi \leq y \leq \pi$, and use it to find the points where the function hits its extreme values (highest and lowest). Graph the x and y cross-sections through these points and estimate the partial derivatives there. Check your estimates by evaluating `fx` and `fy` to find the exact values.
9. ? What is the value of the partial derivative function `fx` at the points where f hits its extreme values? What is the value of the partial derivative function `fy` at the points where f hits its extreme values?

10. Enter the following command to draw (and label) the **0**-level curve of the partial derivative function `fx`:

```
xpic=ContourPlot[fx[x,y]==0,{x,-1.5,1.5},{y,-Pi,Pi},ContourStyle->Red]
```

    Make sure to use a double equal-sign for `fx[x,y]==0`.

11. Alter the commands as appropriate to generate the corresponding level curve for `fy`; starting with the label `ypic`, and using the color `Blue` this time.

12. Now go back to your original contour plot of f itself and give it the name `cp` by entering `cp` equal to its generating command.

13. Combine all three plots by using the Show command:

```
Show[cp,xpic,ypic]
```

14. ? What is the significance of the points where `xpic` and `ypic` cross?

15. ? Now Show just `cp` and `ypic` together, and think about what `ypic` tells us about f. Locate points where the contours of f (from `cp`) are parallel to the y-axis. How are these points related to `ypic`? Use the meaning of the partial derivative `fy` to explain your answer.

16. ? Now Show just `cp` and `xpic`, and locate points where the contours of f are parallel to the x-axis. How are these points related to `xpic`? Use the meaning of the partial derivative `fx` to explain your answer.