Math 1800-B Handout 2: Vectors

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■ Exercise 1.

For the following problems, fill the box with either "certainly", "possibly", or "certainly not".

1. If $\vec{u} \cdot \vec{v} = \vec{w} \cdot \vec{v}$, then $\vec{u}$ is $\boxed{}$ equal to $\vec{w}$.
2. Given three vectors $\vec{u}, \vec{v}$ and $\vec{w}$, if $\vec{u} + \vec{v} = \vec{u}$, then $\vec{w} + \vec{v}$ is $\boxed{}$ equal to $\vec{w}$.
3. $||\vec{u} - \vec{v}||$ is $\boxed{}$ less than or equal to $||\vec{u} + \vec{v}||$.

■ Exercise 2.

Find a value c so that $3\hat{i} + 4\hat{j} + 5\hat{k}$ is perpendicular to $4\hat{i} + 2\hat{j} + c\hat{k}$.

■ Exercise 3.

Find the equation of the plane parallel to $2x + 4y - 3z = 1$ and passing through the point $(1, 0, -1)$.

■ Exercise 4.

In the diagram below, the force vectors $\vec{F_1}$ and $\vec{F_2}$ both have a magnitude of 10 newton. Determine the magnitude and direction of the force vector $\vec{G}$ needed to counterbalance (i.e., neutralize) the combined action of $\vec{F_1}$ and $\vec{F_2}$.

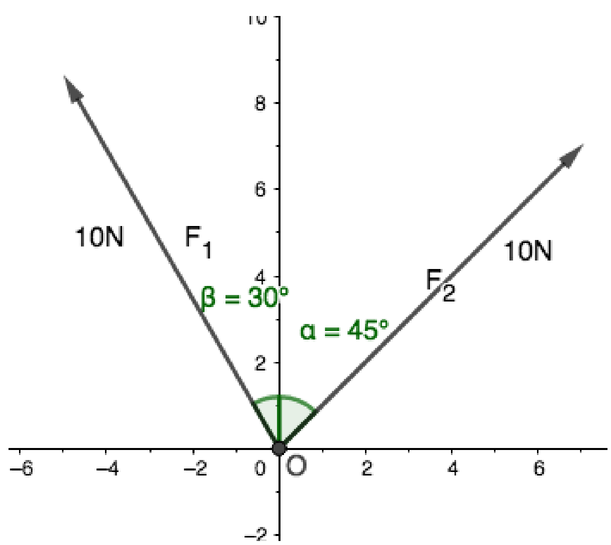

Figure 1

Solution. Let $\vec{G}$ denote the sum of $\vec{F_1}$ and $\vec{F_2}$. Then

$$\vec{G} = \langle 10\cos(45) - 10\cos(60), 10\cos(45) + 10\cos(30)\rangle = \langle 5\sqrt{2} - 5, 5\sqrt{2} + 5\sqrt{3}\rangle$$

So $\|\vec{G}\| = 15.867$ and the direction is $82.5°$ North of positive X-axis.

Now the force F needed to counterbalance G must have equal magnitude and opposite direction. So it has magnitude $15.867N$ and makes an angle $(180 + 82.5) = 262.5$ degrees with the positive $X-$axis.

■ Exercise 5.

The vertices of a triangle $\triangle ABC$ are $A = (4,3,2), B = (1,3,1)$, and $C = (-5,5,-2)$. Let D be the foot of the perpendicular from A to the side $\overline{BC}$. Find the vector $\overrightarrow{AD}$.

■ Exercise 6.

Find the distance of the point $P = (1,0,1)$ from the plane $x + y - z = 1$.

[Hint: Find a point Q on the plane. Find the normal vector $\vec{n}$ of the plane. The distance is the projection of $\overrightarrow{QP}$ in direction of $\vec{n}$.]

■ Exercise 7.

Suppose λ and μ are real numbers such that

- the three vectors

$$\begin{aligned}\vec{u} &= 2\hat{\imath} + 3\hat{\jmath} + \hat{k},\\ \vec{v} &= \hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k},\\ \vec{w} &= 7\hat{\imath} + 3\hat{\jmath} + 2\hat{k}\end{aligned}$$

  are coplanar, and

- The vector $\vec{v}$ has magnitude $\sqrt{2}$.

Find all possible values of λ and μ.

Solution. Coplanarity implies $\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$. Simplifying gives us the equation

$$1 + \lambda - 5\mu = 0$$

On the other hand, $\|\vec{v}\| = \sqrt{2}$ implies

$$\lambda^2 + \mu^2 = 1$$

Solving the two equations together we get

$$\begin{cases}\lambda &= \frac{12}{13}\\ \mu &= \frac{5}{13}\end{cases} \text{or} \begin{cases}\lambda &= -1\\ \mu &= 0\end{cases}$$

■ Exercise 8.

At each of the two points P and Q of the following topographical map draw vectors in the (instantaneous) directions you would have to walk from P and from Q to travel

1. the steepest uphill path from your starting point,
2. the steepest downhill path from your starting point, and
3. the path on which altitude remains constant.

What is the relationship between these three vectors at each point P and Q?

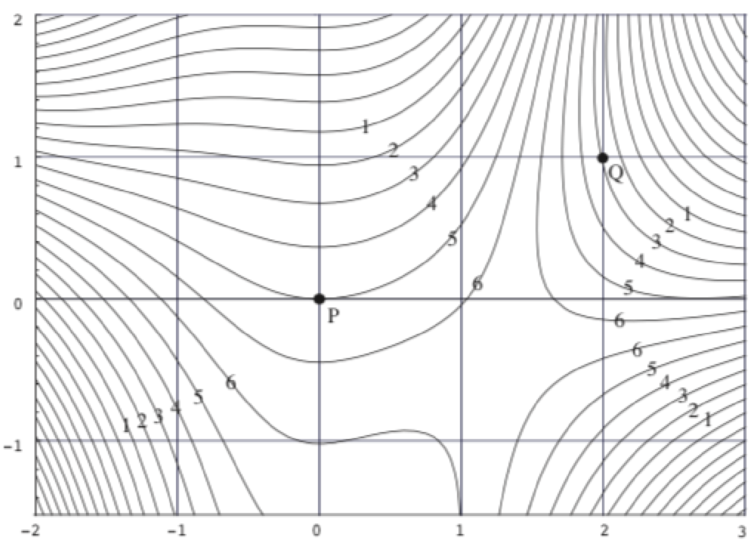

Figure 2