

Practice Problems and review notes

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- The following are a set of practice/review problems for Final. Over the next couple of days, I will be going over these problems in office hour, class etc. Some of these problems might be on the final exam itself.
- Make sure you can solve all of the problems listed below and ask me via email or in person if you have questions.
- Apart from these problems, you should also go through the extra problems (i.e. those outside the book) that I had assigned in homeworks over the quarter. Ask me if you need clarification with any of those.

Problem 0

Problems regarding Wednesday's class.

Problem 7.6.(7, 20, 21, 38).

Problem 1

Consider the region Ω bounded by the two curves

$$y - x = 4, \quad y = 4 - x^2$$

Suppose Ω is revolved around the X -axis to obtained a solid S .

1. Express the volume of S as an integral in terms of x .
2. Express the volume of S as an integral in terms of y .

Suppose Ω is revolved around the Y -axis to obtained a solid T .

1. Express the volume of S as an integral in terms of x .
2. Express the volume of S as an integral in terms of y .

Try to calculate the volume of the solid in each case.

Problem 2

Consider the function $f : [1, 2] \rightarrow \mathbb{R}$ defined as

$$f(x) = e^x - \ln(x).$$

Is f an injective function? Justify your answer.

Problem 3

Find the following integral

$$\int \frac{\sin x - \cos x}{1 + \sin(2x)} dx.$$

Problem 4

Find

$$\int \frac{e^x - 1}{e^x + 1} dx.$$

Problem 5

Find

$$\int \frac{e^x(1+x)}{\sin^2(xe^x)} dx.$$

Problem 6

Find

$$\int \sin^2 x \cos^5 x \, dx$$

Problem 7

$$\int \frac{e^{3x} - e^x}{e^{4x} + 1 + 3e^{2x}} dx$$

Problem 8

Suppose f and g are two continuous function. Prove that

$$\int_{-1}^1 [f(x) + f(-x)][g(x) - g(-x)] dx = 0$$

Problem 9

Differentiate 2^{2^x} and 2^{2x} and 2^{x^2} .

Problem 10

Find

$$\int \frac{dx}{\sqrt{4 - 6x - 9x^2}}$$

Problem 11

Find

$$\int \frac{dx}{x\sqrt{9 - 4x^2}}$$

Problem 12

Suppose f and g are two continuous and differentiable functions such that $f' = g$ and $g' = -f$. Prove that between every two consecutive roots of f , there is a root of g and between every two consecutive roots of g , there is a root of f .

Please check back in case I add more problems. Also don't forget to go over all the homework problems and past practice problems.