Assignment 5 (7/5)

Subhadip Chowdhury

- *This homework is due at the beginning of class on **Tuesday** 7/10. You are encouraged to work together on these problems, but you must write up your solutions independently.*

Explaining the Grading Rubric for first set of Assignments

Assignment 1 and 2 scores are not being added to the overall score. I read over the proofs of problem 1 and 2 and John graded the rest using his rubric. Here's what the symbols mean:

- $\circ$ means the proof (both the math and the structure) is perfect.
- $\triangle$ means the proof is almost perfect, except for some one or two wrong sentences that do not affect the Math.
- $\square$ means that Math is almost correct but there's problem with the format (e.g. statements out of order, grammatically incorrect, repetitive arguments etc.)
- $\otimes$ means that the structure makes it hard to understand whether or not the proof is mathematically correct (e.g. too verbose, extraneous statements, incorrect usage of terminology etc.)
- $\star$ means there are problems with the Math itself and student's understanding of logical inferences may be unclear.

John's grading rubric for letters are as follows. The $+$ and $-$ are somewhere in-between as appropriate.

- **A**: Exhibits thorough and precise reasoning, in complete ideas. A few sentences may be extraneous, or perhaps slight details are overlooked, but the proof clearly and precisely accomplishes what it intended to prove.
- **B**: Some important details may have been skipped over, and there may be issues with clarity and organization. While its ideas may be lacking in precision, the proof shows an effectiveness at reasoning through the broad strokes of the problem.
- **C**: Some major pieces of reasoning are missing from the proof, or perhaps the understanding the problem is flawed. One or more core ideas of the problem are overlooked. These proofs may also have repetitive, irrelevant, or disorganized ideas that distract from the proof.

Please come talk to me in office hour (or set up an one-on-one appointment) if you got low grades and/or having difficulty with the course.

Divisibility

Definition 2.1. Given two integers a and b, we say a divides b if there exists an integer c such that $b = ac$. If a divides b, we say a is a factor or divisor of b.

Example 2.2. What integers divide 0? What integer(s) are divisible by 0?

Example 2.3. What are the factors of 24?

Properties of Divisibility Relation

The following hold for all integers a, b, c.

- If $a \mid b$ and $b \mid c$, then $a \mid c$.
- If $a \mid b$ and $a \mid c$, then $a \mid b + c$.
- If $a \mid b$ and $a \mid c$, then $a \mid b - c$.
- If $a \mid b$, then $a \mid nb$ for any integer n.
- If $a \mid b$ and $a \mid c$, then $a \mid (nb + mc)$ for any integer n and m.
- If $a \mid b$ and $b \geq 0$, then $a \leq b$.

Exercise 1. *For each of the following statements, either prove it is true or provide an example that shows it is not true. Be brief about the proof, don't worry about the format too much; make sure that the mathematical reasoning is correct.*

(a) If $a \mid c$ and $b \mid c$, then $ab \mid c$.

(b) If $2a \mid 4b$, then $a \mid b$.

(c) If $a \mid b$, then $a^2 \mid b^3$.

(d) If $a \mid b$, then $a + 2 \mid b + 2$.

(e) If $a \mid b$ and $c \mid d$, then $ac \mid bd$.

(f) If $ab \mid c$, then $a \mid \frac{c}{b}$.

Greatest Common Divisor

Definition 3.1. Given two integers a and b, we say d is a common divisor of a and b if both $ad \mid a$ and $d \mid b$. The largest among the common divisors of a and b is called the gcd of a and b, denoted (a, b).

Definition 3.2. a and b are called relatively prime to each other or 'coprime' if $(a, b) = 1$.

Properties

The following hold for all integers a, b.

- If $a \mid b$, then $(a, b) = a$.
- Let $(a, b) = d$. Then if $a = md$ and $b = nd$, then $(m, n) = 1$.

  Make sure you understand the proof of the last property above. Write it out yourself for practice.

Example 3.3. $(24, 27) = 3$. $(35, 49) = 7$. $(1111, 1112) = 1$.

Exercise 2. *For each of the following statements, either prove it is true or provide an example that shows it is not true. Be brief about the proof, don't worry about the format too much; make sure that the mathematical reasoning is correct.*

(a) If $(a, b) = 1$ and $(b, c) = 1$, then $(a, c) = 1$.

(b) If $n \mid (a, b)$, then $n \mid a$ and $n \mid b$.

The following exercise is in fact theorem 3.7.2 from the book. Please give it a try yourself before looking in the book.

Exercise 3. *For any integers a, b, and c, show that $(a+bc, b) = (a, b)$.*

Exercise 4. *Using exercise 3, show that $(n, n+1) = 1$ for any integer n.*

Exercise 5. *Show that for all positive integers a, b, and c we have*

$$(ac, bc) = c(a, b)$$

*[*HINT: *Start by assuming $d = (a, b)$ and prove that $dc = (ac, bc)$]*