Assignment 12 (5/4)

The One With endless L'Hôpital's chains...

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Problem 1

Problems $11.5.(14, 22, 23, 41, 49, 52)$.

Problem 2

Problem $11.5.47$. We will find

$$L = \lim_{x\to 0} \frac{(1+x)^{1/x} - e}{x}$$

Follow these steps:

(a) Show that

$$\lim_{x\to 0}(1+x)^{1/x} = e$$

Hence L'Hôpital's Rule can be applied!

(b) Let $f(x) = (1+x)^{1/x}$. What is $f'(x)$?

(c) Apply L'Hôpital's rule. Prove that

$$L = \lim_{x\to 0} f'(x) = \lim_{x\to 0} f(x)\left[\frac{1}{x(x+1)} - \frac{\ln(1+x)}{x^2}\right] = \lim_{x\to 0} f(x).\lim_{x\to 0}\left[\frac{x-(x+1)\ln(1+x)}{x^2(x+1)}\right] = e.L'\,(,let)$$

(d) We need to find L' now.**Justify that L'Hôpital's rule can be applied again.** Do it.

$$L' = \lim_{x\to 0} \frac{(-\ln(1+x))}{3x^2+2x}$$

(e) Same thing for one last time. **Justify that L'Hôpital's rule can be applied again.**

$$L' = \lim_{x\to 0} \frac{-1/(1+x)}{6x+2}$$

(f) Hence

$$L = e.\lim_{x\to 0}\frac{-1}{2(1+x)} = -\frac{e}{2}$$

by part (a).

When a problem is asking you to justify whether L'Hôpital can be applied or not, you need to check that your limit is indeed $0/0$ (or ∞/∞) form. Note that

$$f(x) \to e \text{ and } f'(x) \to L$$

i.e. both of the limits are finite!

Let me know if you find an easier method of solving this limit!