

Assignment 7 (1/20)

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Problem 1

Problems 9.2.(5, 9, 15, 18, 23, 25(a, b)).

Problem 2

Problems 11.5.(14, 22, 23, 41, 49, 52).

Problem 3

Problem 11.5.47. We will find

$$L = \lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x}$$

Follow these steps:

(a) Show that

$$\lim_{x \rightarrow 0} (1+x)^{1/x} = e$$

Hence L'Hôpital's Rule can be applied!

(b) Let $f(x) = (1+x)^{1/x}$. What is $f'(x)$?

(c) Apply L'Hôpital's rule. Prove that

$$L = \lim_{x \rightarrow 0} f'(x) = \lim_{x \rightarrow 0} f(x) \left[\frac{1}{x(x+1)} - \frac{\ln(1+x)}{x^2} \right] = \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} \left[\frac{x - (x+1)\ln(1+x)}{x^2(x+1)} \right] = e \cdot L' \text{ (let)}$$

(d) We need to find L' now. **Justify that L'Hôpital's rule can be applied again.** Do it.

$$L' = \lim_{x \rightarrow 0} \frac{(-\ln(1+x))}{3x^2 + 2x}$$

(e) Same thing for one last time. **Justify that L'Hôpital's rule can be applied again.**

$$L' = \lim_{x \rightarrow 0} \frac{-1/(1+x)}{6x + 2}$$

(f) Hence

$$L = e \cdot \lim_{x \rightarrow 0} \frac{-1}{2(1+x)} = -\frac{e}{2}$$

by part (a).

When a problem is asking you to justify whether L'Hôpital can be applied or not, you need to check that your limit is indeed $0/0$ (or ∞/∞) form. Note that

$$f(x) \rightarrow e \text{ and } f'(x) \rightarrow L$$

i.e. both of the limits are finite!

Let me know if you find an easier method of solving this limit!