Assignment 3 (1/8)

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- *This homework is due at the beginning of class on **Friday** 1/12. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material part. You are encouraged to think about the exercises marked with a ($\star$) or (†) if you have time, but you don't need to hand them in. If you correctly solve a (†)-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering Chapter 12 from Stewart.*

Important Points and Reading Materials

- Cross Products:
  - Definition - either using angle or using components
  - Observe that unlike any of the other vector concepts we've talked about, the cross product makes sense only in three dimensions. You can't take the cross product of n-dimensional vectors.
  - If you know about determinants, try to learn the 3×3 determinant formula for the cross product (see the textbook). If you don't know about determinants, don't worry about this.
  - What is $\vec{u} \times \vec{u}$? What is $\hat{i} \times \hat{j}, \hat{j} \times \hat{k}, \hat{k} \times \hat{i}$? What is $\hat{j} \times \hat{i}$?
  - Know how to use the right hand rule to find the direction of cross product.
  - How can we use cross products to determine if two vectors are parallel?
  - How do we use the scalar triple product to find the volume of a parallelepiped?
- Equation of Lines and Planes:
  - Know how to find the Cartesian or parametric equations for a line through a point $\mathbf{p}$, parallel to a vector $\vec{v}$. What about the straight line through two points A and B?
  - Given two straight lines in three dimension, it is not always possible to find a plane containing both lines. If the lines intersect each other, we can find such a plane. If they do not, then they are either parallel (in which case, there is a plane containing them) or they are not (in which case there is no plane containing them). Two non-parallel, non-intersecting lines are said to be 'skew'. Can you give examples of two such lines?
  - A vector perpendicular to a plane is called the 'normal' vector to the plane. Know how to find equation of a plane containing a point $\mathbf{p}$ and having normal vector $\vec{v}$.
  - Know how to find equation of a plane containing three given points by using cross product to produce the normal vector. Look at example 12.5.5.
  - Read all the examples of chapter 12.5.

Problems

Please note that specifically for this assignment, I expect that you should be able to do the star marked problems (except exercise 3), if given in quiz or exam, even if you are not submitting their solution.

Exercise 1

(12.4.6) Find $\langle t, \cos t, \sin t\rangle \times \langle 1, -\sin t, cos t\rangle$.

Exercise 2

(12.4.19) Find two unit vectors orthogonal to both $\langle 3,2,1\rangle$ and $\langle -1,1,0\rangle$.

Exercise 3$\star$

(12.4.50) Show that for three vectors $\vec{u}, \vec{v}$, and $\vec{w}$,

$$\vec{u}\times(\vec{v}\times\vec{w}) = (\vec{u}\cdot\vec{w})\vec{v} - (\vec{u}\cdot\vec{v})\vec{w}$$

(12.4.51) Consequently show that

$$\vec{u}\times(\vec{v}\times\vec{w}) + \vec{v}\times(\vec{w}\times\vec{u}) + \vec{w}\times(\vec{u}\times\vec{v}) = 0$$

Exercise 4

(12.4.31) Let $P=(0,-2,0), Q=(4,1,-2), R=(5,3,1)$. Find area of the triangle $\triangle PQR$.

Exercise 5

(12.4.36) Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS where $P=(3,0,1), Q=(-1,2,5), R=(5,1,-1)$, and $S=(0,4,2)$.

Exercise 6$\star$

(12.4.37) Verify that the vectors $\vec{u}=\langle 1,5,-2\rangle, \vec{v}=\langle 3,-1,0\rangle$, and $\vec{w}=\langle 5,9,-4\rangle$ are coplanar.

Exercise 7$\star$

(12.5.1) Do all the T/F questions in exercise 12.5.1.

Exercise 8

(12.5.4) Find equation of the straight line through the point $(0,14,-10)$ and parallel to the line $x=-1+2\lambda, y=6-3\lambda, z=3+9\lambda$.

Exercise 9$\star$

Know how to do problems 12.5.(5, 9, 10, 11, 24, 27, 31, 50).

Exercise 10

(12.5.12) Find equation of the line of intersection of the planes $x+2y+3z=1$ and $x-y+z=1$.

[Hint: First determine the normal vectors to the two planes. Observe that the line of intersection is perpendicular to both of those normal vectors.]

Exercise 11

(12.5.30) Find equation of the plane that contains the line $x=1+t, y=2-t, z=4-3t$ and is parallel to the plane $5x+2y+z=1$.

Exercise 12

(12.5.63) Find equation of the plane with $x-$intercept a, $y-$intercept b, and $z-$intercept c.