Assignment 3 (4/1)

Subhadip Chowdhury

Problem 1

Let Ω be the region bounded by the hyperbola $x^2 - y^2 = 1$, the X-axis and the line $y = \tanh(t)x$. Note that Ω has parts in both first and third (or second and fourth) quadrant.

(a) Show that the area of Ω is given by

$$A(t) = \cosh(t)\sinh(t) - 2\int_1^{\cosh(t)} \sqrt{x^2 - 1}dx$$

(b) Show that $A'(t) = 1$.

(c) Prove that $A(t) = t$.

Problem 2

Find the following integrals

$$\int \sinh^3(x)\cosh^5(x)dx,$$

$$\int \tanh(x)dx$$

Problem 3

(a) Apply the mean-value theorem to the function $f(x) = \ln(1 + x)$to show that for all $x > -1$

$$\frac{x}{1+x} < \ln(1+x) < x$$

(b) Use the result in part(a) to show that

$$\lim_{x\to 0} \frac{ln(1+x)}{x} = 1$$

Problem 4

Given $|a| < 1$, find the value of b for which

$$\int_0^1 \frac{b}{\sqrt{1-b^2x^2}}dx = \int_0^a \frac{1}{\sqrt{1-x^2}}dx$$