Assignment 2 (7/26)

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This homework is due in class on Friday, 8/4. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.

You are encouraged to think about the problems marked with a ($\star$), but they are not to be handed in.

Problem 0$\star$

Read section 1.3, 2.1.

Problem 1

A vector $\vec{v}$ is said to be a *linear combination* of vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3, \ldots, \vec{u}_n$ if there exists real numbers $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ such that

$$\vec{v} = \lambda_1 \vec{u}_1 + \lambda_2 \vec{u}_2 + \ldots + \lambda_n \vec{u}_n$$

1. Show that any vector $\vec{v} \in \mathbb{R}^n$ is a linear combination of the elementary vectors $e_1, e_2, \ldots, e_n$.
2. Show that for a matrix A and vectors $\vec{u}, \vec{v}$, we have $A(\vec{u} + \vec{v}) = A\vec{u} + A\vec{v}$.
3. Let T be the linear transformation corresponding to A. Show that the vector $T(\vec{v})$ is a linear combination of $T(e_1), T(e_2), \ldots, T(e_n)$.

Problem 2$\star$

Prove that two nonzero vectors $\vec{u}$ and $\vec{v}$ are perpendicular to each other iff $\vec{u} \cdot \vec{v} = 0$.

Problem 3

Find a 3×3 matrix A such that $A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$, $A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}$.

Problem 4

Problems 1.3.(1, 2, 3, 4, 19, 22, 24, 28, 29, 31, 54$\star$, 55, 57).

$\boxed{\textbf{Problem 5}}$

Problems 2.1.(1, 2, 3, 6, 24, 26, 30, 33, 44,)