Math 1800-C Handout 9: Summary of Chapter 8 - How to calculate line integrals

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Summary of Chapter 8

We learned the following theorems in chapter 8.

Parametrized Curves: If the curve C can be parametrized as $\vec{\mathbf{r}}(t)$, $a \leq t \leq b$, then

$$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_a^b \vec{\mathbf{F}}(\vec{\mathbf{r}}(t))\vec{\mathbf{r}}'(t)dt$$

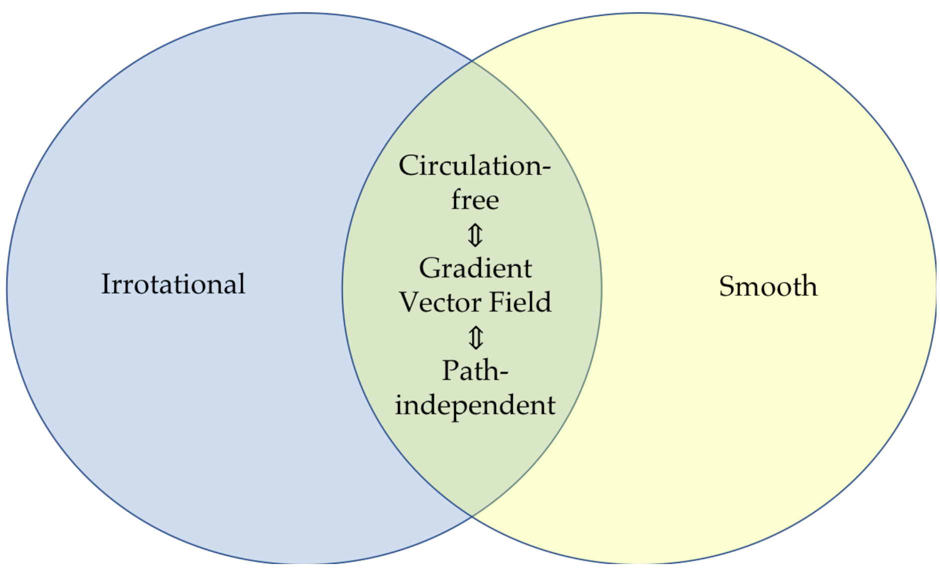
Fundamental Theorem of Line Integrals: If the vector field $\vec{\mathbf{F}}$ is a gradient vector field i.e. $\vec{\mathbf{F}} = \nabla f$, and the curve C starts at P and ends at Q, then

$$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_C \nabla f \cdot d\vec{\mathbf{r}} = f(Q) - f(P)$$

Green's Theorem: If C is a *simple, closed, oriented* curve and the vector field $\vec{\mathbf{F}}$ is *smooth* over the region R enclosed by C (oriented so that R is always to the left of C), then

$$\oint_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \iint_R \mathbf{curl}\, \vec{\mathbf{F}}\, dA$$

Vector Fields Venn Diagram



Calculating Line Integral - a flowchart