Math 1800-B Handout 3: Lines and Planes in 3D

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■ Exercise 1.

Below is a list of vectors and a list of properties. Match the two sets in such a way that each entry in left column matches a different entry in right column.

A. $\langle 3, -2, 8\rangle$	I. is parallel to the straight line $\frac{x-1}{2} = y - 3 = z$
B. $\langle 4, 2, 2\rangle$	II. is perpendicular to the plane $z - 2y - x = 3$
C. $\langle 3, 1, -1\rangle$	III. is perpendicular to both $\langle 2, 3, 0\rangle$ and $\langle -2, 5, 2\rangle$
D. $\langle 1, 2, -1\rangle$	IV. lies in the plane $x - y + 2z = 3$

■ Exercise 2.

Find the point(s) on the surface $xy + yz + zx + 4 = 0$ where the tangent plane is parallel to the XY-plane.

■ Exercise 3.

(a) Find parametric equations for the line through the points $(6, 1, 1)$ and $(9, 1, 4)$. Call this line L_1.

(b) Find parametric equations for the line through the points $(-4, 4, 0)$ and $(-6, 5, 1)$. Call this line L_2.

(c) Find parametric equations for the line through the points $(6, -1, -5)$ and $(2, 1, -3)$. Call this line L_3.

(d) Verify that L_2 and L_3 are parallel. (Their direction vectors should be parallel.) Are they the same line? How could you tell?

(e) Do lines L_1 and L_2 intersect? If so, where?

(f) Find the intersection of L_1 with the plane given by the equation $2x + y + 3z = 7$.

(g) Find the point on the plane $2x + y + 3z = 7$ which is closest to the origin.

(h) Find the point on L_2 closest to the origin.

■ Exercise 4.

Suppose the curve given by $\vec{r}(t) = \langle \cos(\pi t), \sin(\pi t), t\rangle$ intersects the paraboloid $z = x^2 + y^2$ at a point $P = (x_0, y_0, z_0)$.

(a) Find the coordinates of P.

(b) Find equation of the tangent plane to the paraboloid at P.

(c) What is the equation of the tangent line to the curve $\vec{r}(t)$ at P?

(d) What is the angle of intersection between the curve and the paraboloid? This is the angle between the tangent line in part (3) and the plane in part (2).