Midterm 2

INSTRUCTIONS:

- Please show ALL your work! Answers without supporting justification will not be given credit.

- Answer the questions in the white space provided. If you run out of room, use the back page.

- Write legibly and clearly mark the answer.

- Please note that use of any books or notes is not allowed. You are allowed to use the one page of handwritten letter-sized note that you brought. Use of calculators are allowed.

- If you write down the correct formula for an answer, you will get some partial credit regardless of whether you evaluated the exact values or not.

- Unless otherwise specified, you may use any valid method to solve a problem.

Full Name: _____

Question	Points	Score
1	15	
2	5	
3	5	
4	14	
5	14	
6	5	
7	7	
Total:	65	

This exam has 7 questions, for a total of 65 points.
The maximum possible point for each problem is given on the right side of the problem.

1. Find the derivative of the following functions. Remember to show your work. Be careful when naming your variables. You do not need to simplify your answers.

   (a) $f(x) = 2^{\cos x}$

   $\boxed{3}$

   (b) $f(z) = \dfrac{\ln z + \tan z}{\pi - e^z}$

   $\boxed{3}$

   (c) $f(\theta) = 5 \arcsin \left(e^{-2\theta} + 3\theta^3 \right)$

   $\boxed{4}$

   (d) $f(t) = 7 \left(\sin \left(t^2 \arctan t \right) \right)^3$

   $\boxed{5}$

2. (a) Consider the curve $x^3 + 2xy + y^2 = 4$. Find $\dfrac{dy}{dx}$ at the point $(1,1)$. $\boxed{5}$

   (b) **(Bonus)** Find the equation of the tangent line to the curve in part (a) at $(1,1)$. $\boxed{3 \text{ (bonus)}}$

3. For the following statements, circle the correct choice of the words out of the three options. No explanation is necessary. $\boxed{5}$

   (a) If $h'(a) < 0$, then $h(x)$ must be $\boxed{\text{concave down } | \text{ negative } | \text{ decreasing}}$ at $x = a$.

   (b) If $h''(b) > 0$, then $h'(x)$ must be $\boxed{\text{concave up } | \text{ positive } | \text{ increasing}}$ at $x = b$.

   (c) If $f''(x)$ and $g''(x)$ exist, and the functions $f(x)$ and $g(x)$ are concave *up* everywhere, then $f(x) - g(x)$ $\boxed{\text{must } | \text{ might } | \text{ cannot}}$ be concave *down* everywhere.

   (d) If $\lim\limits_{x \to \infty} h(x) = 3$, then $\lim\limits_{x \to \infty} h'(x)$ $\boxed{\text{must } | \text{ might } | \text{ cannot}}$ be equal to 3.

   (e) If $u(t)$ is the quantity in grams of a chemical produced after t minutes and $v(t)$ is the quantity of the same chemical in kilograms produced after t hours, then
$\boxed{60u'(t) = 1000v'(t) \ | \ 1000u'(t) = 60v'(t) \ | \ 60000u'(t) = v'(t)}$

4. Suppose $f(x)$ and $g(x)$ and their derivatives have the values given in the table. Then evaluate the following.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	3	5	-1
3	5	-2	1	7
4	2	10	-2	-4

Find the following.

(a) $p'(3)$ if $p(x) = 5f(x) - 8g(x)$.

$\boxed{2}$

(b) $q'(2)$ if $q(x) = xf(2x)$.

$\boxed{4}$

(c) $r'(4)$ if $r(x) = g(f(x))$.

$\boxed{3}$

(d) $s'(1)$ if $s(x)$ is the inverse function of $g(x)$.

$\boxed{5}$

5. Consider a Physics experiment with a water tank where water is coming out of a tap near the bottom of the tank, and we are measuring the water level in the tank over time. Assume that at time t (in minutes), the water level is given by the function $h(t)$ (in meters) where

$$h(t) = 1 + 24e^{-0.5t}$$

Give proper units for all of the answers below.

(a) What is the water level in the tank when the experiment is starting?   `2`

(b) At what rate is the water level changing **8** minutes after the experiment started?   `3`

(c) **8** minutes after the experiment started, the tank develops a leak and the water level starts to decrease at a constant rate thereafter. In other words, for $t \geq 8$ minutes, the water level decreases at a rate equal to the instantaneous rate of change of the level at $t = 8$ minutes. What would be the water level at $t = 10$ minutes?   `6`

(d) With the leak, when does the tank become empty (i.e. the water level reaches **0** meters)?   `3`

(e) **(Bonus)** Does the tank ever become empty in the original scenario (i.e if there hadn't been any leak)? If yes, at what time? If no, explain why.   `2 (bonus)`

6. Let $R(p)$ denote the revenue, in thousands of dollars, earned by a gas station when the price of gas is p dollars per gallon.

   (a) What is the unit of $R'(3)$. ☐1

   (b) What is the unit of $R^{-1}(5)$. ☐1

   (c) Explain the meaning of the following statement: $(R^{-1})'(5) = 3$. ☐3

7. On the axes given, carefully sketch a possible graph of a continuous function $y = f(x)$ with all of the following properties. Use the back page for scratch work, if needed. ☐7

   - $\lim\limits_{x \to -\infty} f(x) = \infty$,
   - $\lim\limits_{x \to \infty} f(x) = 3$
   - $f'(3) = 1$
   - $f''(x) > 0$ for $x < 3$
   - $f''(x) < 0$ for $x > 3$