Math 1800-C Handout 4

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Exercise 1

Show that every plane that is tangent to the cone $x^2 + y^2 = z^2$ passes through the origin.

Exercise 2

Where does the normal line to the paraboloid $z = x^2 + y^2$ at the point $(1, 1, 2)$ intersect the paraboloid a second time?

Exercise 3

The plane $y + z = 3$ intersects the cylinder $x^2 + y^2 = 5$ in an ellipse. Find parametric equations for the tangent line to this ellipse at the point $(1, 2, 1)$.

Exercise 4

Find the angle of intersection between the curve given by its parametric equation $\vec{r}(t) = \langle t, 2t^2 \rangle$, and the parabola $y = x^2 + 4$.

Exercise 5

The length of a side of a triangle is increasing at a rate of 3 in/s, the length of another side is decreasing at a rate of 2 in/s, and the contained angle θ is increasing at a rate of 0.05 radian/s. How fast is the area of the triangle changing when $x = 40$ in, $y = 50$ in, and $\theta = \pi/6$?

Exercise 6

Let $p = g(u, v)$ be a differentiable function of two variables. Let $u = \frac{x}{y}$ and $v = \frac{y}{z}$. Show that

$$x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} + z\frac{\partial p}{\partial z} = 0$$

Exercise 7

Consider the function

$$F(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 1$$

defined on the disk D of radius $\sqrt{2}$ centered at the origin i.e.

$$D = \{(x, y) \mid x^2 + y^2 \leq 2\}.$$

Follow the steps below to find the absolute maximum and minimum of f on D.

(a) Find all the stationary critical points of F. [Hint: There are 4 such points.]

(b) Find all of the second order partial derivatives of F and write down the determinant of the Hessian matrix as a function of x and y. Don't calculate its value at any specific point yet.

(c) In the list of critical points from part (a), identify the ones lying *inside* D (excluding the boundary).

(d) Classify each of the point(s) in part (c) as a local maximum, local minimum, or a saddle point using the Hessian.

(e) Evaluate F at the critical point(s) from part (c).

(f) Use Lagrange multiplier to find the maximum and minimum of $F(x, y)$ subject to the constraint $x^2 + y^2 = 2$. Note that this gives the extreme values of F on the boundary circle of D.

(g) Compare the extreme values of F from part (e), and the extreme values of F from part (f), to find the absolute maximum and minimum of $F(x, y)$ on D.

Exercise 8

Do the same steps to find all local and global extrema of the function

$$f(x, y) = 2x^3 + 2y^3 - 3x^2 - 3y^2 + 6$$

on the disc $D = \{(x, y) \mid x^2 + y^2 \leq 4\}$.

Exercise 9

Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint.

(a) $f(x, y, z) = yz + xy; \quad xy = 1, \quad y^2 + z^2 = 1$

(b) $f(x, y, z) = x^2 + y^2 + z^2; \quad x - y = 1, \quad y^2 - z^2 = 1$

Exercise 10

Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.