CAAP 2018

Quiz 1

JULY 6, 2018
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Full Name:

1. Determine which of the following statements are true or false. Write T/F in the blank accordingly. No explanation is necessary.

   (a) ____**T**____ $\{2,4,7\} \subset \{1,2,4,6,7,9\}$ $\boxed{1}$

   (b) ____**T**____ $\frac{\sqrt{12}}{\sqrt{3}} \in \mathbb{Q}$ $\boxed{1}$

   (c) ____**T**____ $\{-15, \frac{3}{4}, \pi\} \subset \mathbb{R}$ $\boxed{1}$

2. Write the inverse and contrapositive of the following statement: $\boxed{3}$

   "If $1+1=3$, then I am the Pope."

> **Solution:**
>
> - Inverse: If $1+1 \neq 3$, then I am not the Pope.
> - Contrapositive: If I am not the pope, then $1+1 \neq 3$.

3. Suppose x is a real number. Prove that if x^2 is irrational, then x is irrational. $\boxed{6}$

> **Solution:** Assume, for the sake of contradiction, that x^2 is irrational but x is rational. Then we can find integers p and q such that $x = \frac{p}{q}$. Then, $x^2 = \frac{p^2}{q^2}$. But p^2 and q^2 are integers, which implies that x^2 is a rational number. This is clearly a contradiction. $\square$

4. Prove by induction that the following identity holds for all natural numbers n. $\boxed{8}$

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

> **Solution:** We will prove the identity by inducting on n.
>
> **Base Case:** When $n=1$, the LHS$=1$ and the RHS$=\frac{1^2(1+1)^2}{4} = 4/4 = 1$. Hence the identity is true for $n=1$.
>
> **Induction Hypothesis:** Assume that the identity is true for some natural number k.

Induction Step: By our induction assumption we have,

$$1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{k^2(k+1)^2}{4}$$

Adding $(k+1)^3$ to both sides we get,

$$\begin{aligned}
1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 &= \frac{k^2(k+1)^2}{4} + (k+1)^3 \\
&= (k+1)^2\left(\frac{k^2}{4} + (k+1)\right) \\
&= (k+1)^2\frac{k^2+4k+4}{4} \\
&= (k+1)^2\frac{(k+2)^2}{4} \\
&= \frac{(k+1)^2((k+1)+1)^2}{4}
\end{aligned}$$

Thus the identity holds when $n = k + 1$.

Hence by the Induction Principle, the identity is true for all natural number n.