Assignment 10 (1/31)

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- *This homework is due at the beginning of class on* ***Friday*** *2/9. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material item. You are encouraged to think about the exercises marked with a ($\star$) or (†) if you have time, but you don't need to hand them in. If you correctly solve a (†)-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering Chapter 14 from Stewart.*

Important Points and Reading Materials

- Finding Local max/min/saddle point
  - First find all the points where f_x and f_y are **simultaneously** zero.
  - Define the Hessian to be $H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$
  - Let $D = \det H$.
  - $\begin{cases} \text{If } D > 0 \text{ and } f_{xx} > 0 & \text{then local min} \\ \text{If } D > 0 \text{ and } f_{xx} < 0 & \text{then local max} \\ \text{If } D < 0 & \text{then saddle point} \end{cases}$
- Finding Absolute max/min over a closed region
  - First find all the local max/min in the inside of the region using second derivative test.
  - Next use Lagrange multipliers to find the max/min over the boundary of the region.
  - Evaluate the function over all of these points to find global max/min.

Problems

Exercise 1

Find the critical points of the following functions and determine their type.

1. $f(x, y) = 12x^2 + y^3 - 12xy$
2. $f(x, y) = e^x \cos y$ [From assignment 9]
3. $f(x, y) = 2 - x^4 + 2x^2 - y^2$

Exercise 2

Consider the function
$$F(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 1$$
defined on the disk D of radius $\sqrt{2}$ centered at the origin i.e.
$$D = \{(x, y) \mid x^2 + y^2 \leq 2\}.$$
Follow the steps below to find the absolute maximum and minimum of f on D.

(a) Find all the critical points of F. This is from assignment 9.

(b) Find all of the second order partial derivatives of F and write down the determinant of the Hessian matrix as a function of x and y. Don't calculate its value at any specific point yet.

(c) In the list of critical points from item (a), identify the ones lying *inside* D (excluding the boundary).

(d) Classify each of the point(s) in item (c) as a local maximum, local minimum, or a saddle point using the Hessian.

(e) Evaluate F at the critical point(s) from item (c).

(f) Use Lagrange multiplier to find the maximum and minimum of $F(x, y)$ subject to the constraint $x^2 + y^2 = 2$. Note that this gives the extreme values of F on the boundary circle of D.

(g) Compare the extreme values of F from item (e), and the extreme values of F from item (f), to find the absolute maximum and minimum of $F(x, y)$ on D.

Exercise 3

Follow the same steps as in exercise 2 to find the absolute maximum and minimum value of $f(x, y) = x^2 + y^2 + x^2 y + 4$ defined on the square $D = \{(x, y) \mid |x| \leq 1, |y| \leq 1\}$.

Exercise 4

Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$.