Math 1800-C Handout 10
Practice Problems for the Final Exam

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§1. Change of Variables (Optional)

■ Exercise 1.

Use change of variables to set up the following integrals in terms of (u, v)-coordinates so that the domain of integration becomes a rectangle with sides parallel to the axes.

(a) $\iint_R (4x + 8y)\,dA$ where R is the parallelogram with vertices $(-1, 3)$, $(1, -3)$, $(3, -1)$, and $(1, 5)$.

(b) $\iint_R e^{x+y}\,dA$ where R is the region $|x| + |y| \leq 1$.

■ Exercise 2.

Evaluate

$$\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$$

where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$, and $(0, 1)$.

■ Exercise 3.

Let f be continuous on $[0, 1]$ and let R be the triangular region with vertices $(0, 0)$, $(0, 1)$, and $(1, 0)$. Show that

$$\iint_R f(x+y)\,dA = \int_0^1 u f(u)\,du$$

§2. Line Integrals

■ Exercise 1.

Evaluate the following line integrals.

(a) $\oint_C y\,dx + \left(x + y^2\right) dy$ where C is the ellipse $4x^2 + 9y^2 = 36$ with counterclockwise orientation.

(b) $\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x, y) = xy\,\vec{\mathbf{i}} + x^2\,\vec{\mathbf{j}}$ and C is given by $\vec{\mathbf{r}}(t) = \sin t\,\vec{\mathbf{i}} + (1+t)\,\vec{\mathbf{j}}$, $0 \leq t \leq \pi$

(c) $\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x, y) = \left(4x^3y^2 - 2xy^3\right)\vec{\mathbf{i}} + \left(2x^4y - 3x^2y^2 + 4y^3\right)\vec{\mathbf{j}}$ and C is given by $\vec{\mathbf{r}}(t) = (t + \sin \pi t)\,\vec{\mathbf{i}} + (2t + \cos \pi t)\,\vec{\mathbf{j}}$, $0 \leq t \leq 1$

(d) $\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x, y, z) = \sin y\,\vec{\mathbf{i}} + x \cos y\,\vec{\mathbf{j}} - \sin z\,\vec{\mathbf{k}}$, and C is the helix $x = 3\cos t, y = t, z = 3\sin t$ from $(3, 0, 0)$ to $(0, \pi/2, 3)$

(e) $\oint_C \sqrt{1 + x^3}\,dx + 2xy\,dy$ where C is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 3)$

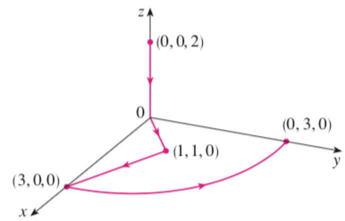

Figure 1

(f) $\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x, y, z) = \left(3x^2yz - 3y\right)\, \vec{\mathbf{i}} + \left(x^3z - 3x\right)\, \vec{\mathbf{j}} + \left(x^3y + 2z\right)\, \vec{\mathbf{k}}$ and C is the curve shown in figure 1.

(g) $\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x, y) = \frac{\left(2x^3+2xy^2-2y\right)\vec{\mathbf{i}}+\left(2y^3+2x^2y+2x\right)\vec{\mathbf{j}}}{x^2+y^2}$ and C is the curve shown in figure 2.

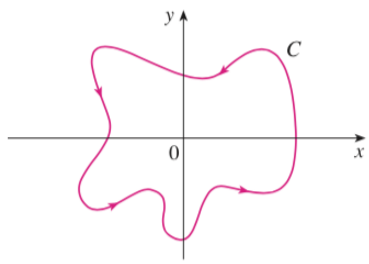

Figure 2

(h) $\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x, y, z) = \langle y \cos x - xy \sin x, xy + x \cos x \rangle$, and C is the triangle from $(0, 0)$ to $(0, 4)$ to $(2, 0)$ to $(0, 0)$.

(i) $\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x, y) = xy^2\, \vec{\mathbf{i}} + x^2y\, \vec{\mathbf{j}}$, and C is $\vec{\mathbf{r}}(t) = \cos t\, \vec{\mathbf{i}} + 2 \sin t\, \vec{\mathbf{j}}, \quad 0 \leq t \leq \pi/2$

■ Exercise 2.

(a) A 160 lb man carries a 25 lb can of paint up a helical staircase that encircles a silo with a radius of 20 ft. If the silo is 90 ft high and the man makes exactly three complete revolutions climbing to the top, how much work is done by the man against gravity?

(b) Suppose there is a hole in the can of paint and 9 lb of paint leaks steadily out of the can during the mans ascent. How much work is done?