

## INSTRUCTIONS:

- Please show ALL your work! Answers without supporting justification will not be given credit.
- Answer the questions in the white space provided. If you run out of room, use the back page.
- Write legibly and clearly mark the answer.
- Please note that use of any books or notes is not allowed. You are allowed to use the one page of handwritten letter-sized note that you brought. Use of calculators are allowed.
- If you write down the correct formula for an answer, you will get some partial credit regardless of whether you evaluated the exact values or not.
- Unless otherwise specified, you may use any valid method to solve a problem.

Full Name: \_\_\_\_\_

Question	Points	Score
1	10	
2	4	
3	5	
4	10	
5	5	
6	4	
7	5	
8	12	
Total:	55	

This exam has 8 questions, for a total of 55 points.  
The maximum possible point for each problem is given on the right side of the problem.

1. The exponential growth model

$$P = 300e^{0.01854t}$$

describes the population of a town  $t$  years after **1900**. Use this model to solve the following problems. Remember to show your work.

- (a) What was the population in the year **1900**?

2
---

- (b) What was the population in the year **1930**?

2
---

- (c) How long does it take for the population to be doubled?

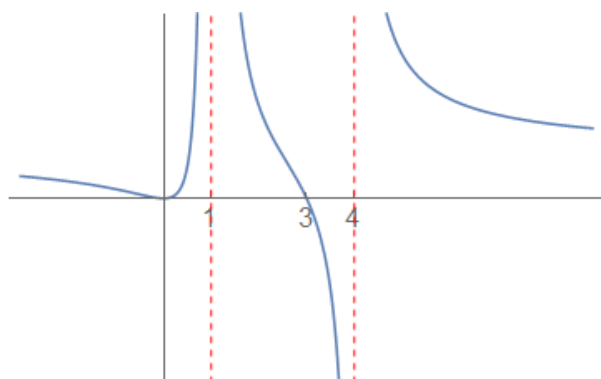
3
---

- (d) When will the town's population become **2400**?

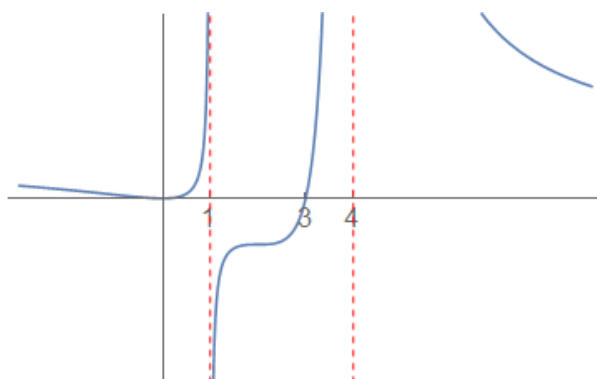
3
---

4

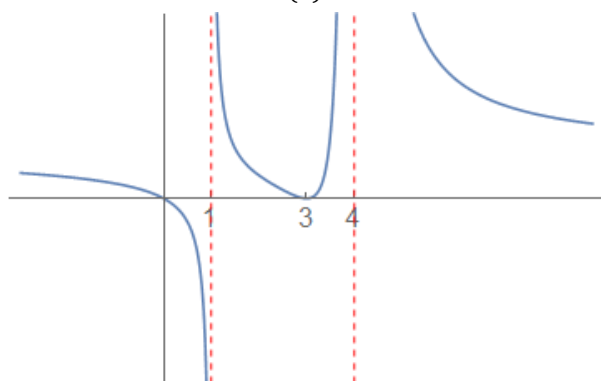
2. Match the following graphs with the rational functions.



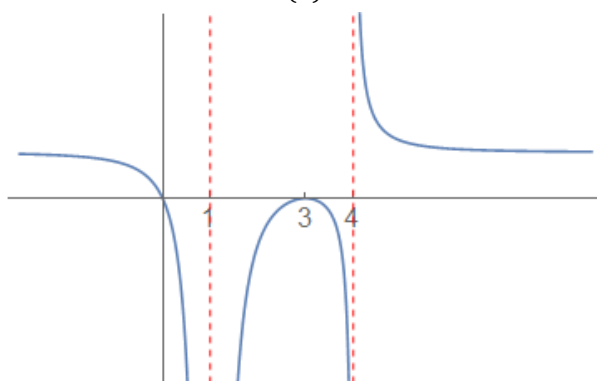
(a)



(b)



(c)



(d)

$$(I) \frac{x(x-3)^2}{(x-1)(x-4)^2}$$

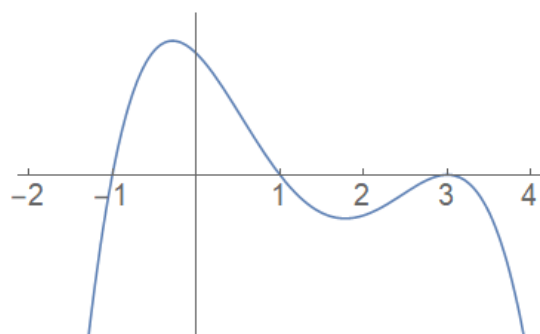
$$(II) \frac{x(x-3)^2}{(x-1)^2(x-4)}$$

$$(III) \frac{x^2(x-3)}{(x-1)(x-4)^2}$$

$$(IV) \frac{x^2(x-3)}{(x-1)^2(x-4)}$$

5

3. Find a possible polynomial function whose graph looks as follows.



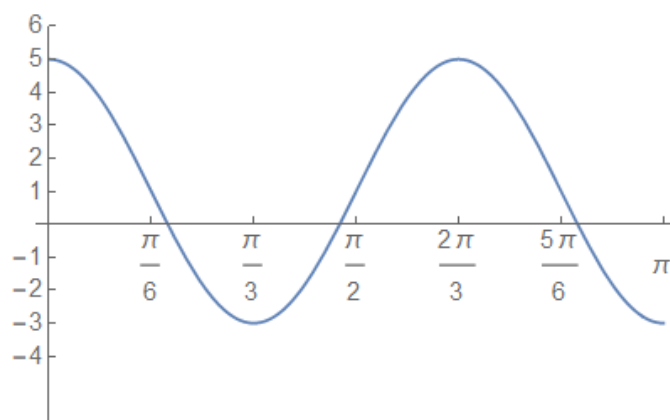


Figure 2: Periodic curve

4. Consider a periodic trigonometric function  $f(x)$  whose graph looks as figure 2.

(a) Find the Amplitude of  $f(x)$ .

2

(b) Find the Period of  $f(x)$ .

3

(c) Find the formula of  $f(x)$ .

5

5. The graph of  $\arcsin(x)$  looks as follows.

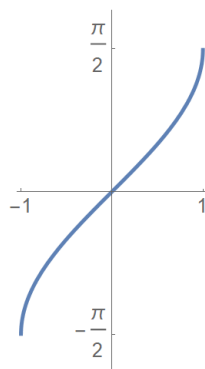


Figure 3: Graph of  $\arcsin(x)$

Consider the function  $g(x) = 2 \arcsin(x + 1)$ .

(a) Draw the graph of  $g(x)$ .

3

(b) Find the domain and range of  $g(x)$ .

2

6. Suppose a function  $h(x)$  has a vertical asymptote at  $x = 4$  and a horizontal asymptote at  $y = 3$ . Find the vertical and horizontal asymptotes of  $3h(2x)$ .

4

7. Determine which of the following statements are **True** or **False** and write **T/F** in the blank accordingly. Note that **T** means the statement is always true and **F** means the statement is not always true. *No explanation is necessary.*

(a) \_\_\_\_\_ If  $\lim_{x \rightarrow 3+} f(x) = 5$  and  $f$  is continuous, then  $\lim_{x \rightarrow 3-} f(x) = 5$ .

1

(b) \_\_\_\_\_ If  $\lim_{x \rightarrow 3-} f(x) = 5$  and  $\lim_{x \rightarrow 3+} f(x) = 5$ , then  $f(x)$  is continuous at  $x = 5$ .

1

(c) \_\_\_\_\_ If  $\lim_{x \rightarrow 3-} f(x) = 2$  and  $\lim_{x \rightarrow 3+} f(x) = 7$ , then  $f(3)$  does not exist.

1

(d) \_\_\_\_\_ If  $f'(x) = g'(x)$  for all  $x$ , then  $f(x) = g(x)$ .

1

(e) \_\_\_\_\_ The derivative of  $f(x) = (x + 1)^4$  at  $x = 0$  is given by  $\lim_{h \rightarrow 0} \frac{(1+h)^4 - 1}{h}$ .

1

8. In this question we start with the graph of a function, and consider its properties to help us sketch the derivative. For the graph of  $y = k(x)$  shown in figure 4,

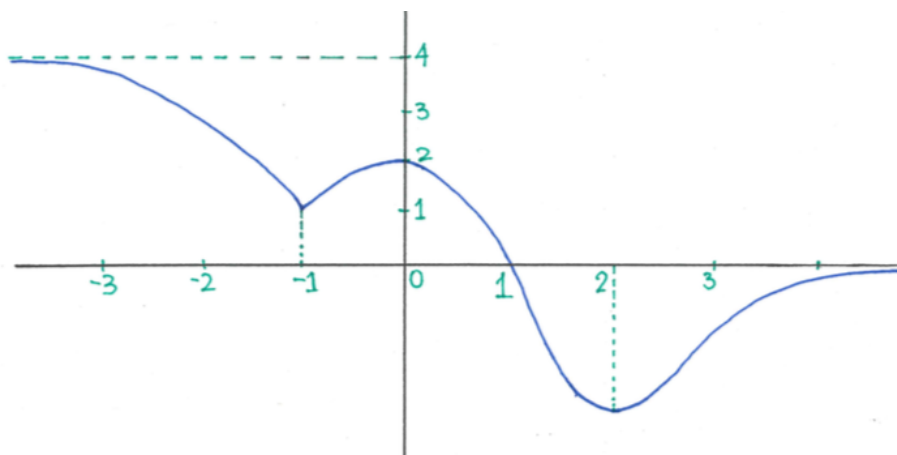


Figure 4: Graph of  $f(x)$

- (a) For what values of  $x$  is  $k'(x)$  positive? 2
- (b) For what values of  $x$  is  $k(x)$  decreasing? 2
- (c) For what values of  $x$  is  $k'(x) = 0$ ? 2
- (d) For what values of  $x$  is  $k'(x)$  not defined? 2
- (e) What does the end behaviour of  $k'(x)$  appear to be as  $x \rightarrow \infty$  and as  $x \rightarrow -\infty$ ? 2
- (f) Carefully sketch the graph of  $k'(x)$  on the axes given below. 2

