CAAP 2018 | **Quiz 2** | JULY 13, 2018 SUBHADIP CHOWDHURY

Quiz 2

Full Name:

1. Suppose x and y are positive integers. Prove that if 3 divides $x+y$, then 3 divides $10x+y$. $\boxed{5}$

Solution: If $3 \mid x+y$, then we can write $x+y=3k$ for some integer k. Then $10x+y = 9x+3k = 3(3x+k)$. Thus we have shown $10x+y$ is equal to $3l$ for some integer l. Hence $3 \mid 10x+y$.

2. Recall that you proved $(a+bc, b) = (a, b)$ for all integers a, b, and c, in your assignment. $\boxed{10}$

   Now choosing suitable values of a, b, and c in above identity, prove that

$$(x-4y, y) = (4x+y, x) \text{ for all integers } x \text{ and } y.$$

[HINT: You have to use the identity twice.]

Solution: Taking $a = x, b = y, c = -4$ in the identity above, we get

$$(x-4y, y) = (x, y)$$

Taking $a = y, b = x, c = 4$ in the identity above, we get

$$(y+4x, x) = (y, x)$$

We can rewrite the last equality as $(4x+y, x) = (x, y)$. Combining the results above, we get

$$(x-4y, y) = (y+4x, x)$$

3. Observe that by definition of the greatest integer function, the real numbers t that satisfy $\lfloor t \rfloor = n$ for some $n \in \mathbb{N}$ are given by $t \in [n, n+1)$. $\boxed{8}$

   Using this fact, solve the following equation for x.

$$\lfloor \lfloor x+0.3 \rfloor + 0.7 \rfloor = 1$$

Solution: Let $y = \lfloor x+0.3 \rfloor$. Observe that y is (by definition of box function) an integer. The only integer y that satisfies

$$\lfloor y+0.7 \rfloor = 1$$

is $y = 1$. Hence

$$\lfloor x+0.3 \rfloor = y = 1 \implies 1 \le x+0.3 < 2 \implies 0.7 \le x < 1.7$$

4. Consider an Arithmetic Progression whose first term is 4 and the common difference is 3. If the nth term in this AP is 181, find the value of n. $\boxed{7}$

Solution: The nth term of this arithmetic progression is given by $4+3(n-1)$. Hence

$$4+3(n-1)=181 \implies 3(n-1)=177 \implies n-1=59 \implies n=60$$