Assignment 15 (7/23)

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- *This homework is due at the beginning of class on* ***Tuesday*** 7/31. *You are encouraged to work together on these problems, but you must write up your solutions independently.*

Least Upper Bound Axiom

Look up the first two pages of section 11.1 from the Calculus textbook. We will cover the proof of theorem 11.1.2 tomorrow. You will be encountering these topics later in your math courses again.

Definition 1.1. A real number M is called an *upper bound* of a set S if $M \geq x$ for all $x \in S$. The smallest upper bound of set S is called the least upper bound of S, denoted $lub(S)$. Similarly we can define the greatest lower bound, $glb(S)$.

A set S is said to be *bounded above* (resp. below) if it has an upper (resp. lower) bound. So for example, $S = (1, \infty)$ is not bounded above since it does not have any upper bound, but it is bounded below. In particular, $lub(S)$ does not exist, and $glb(S) = 1$.

Sequences and mathematical Induction

Although section 11.2 covers sequences, you will need ideas of function, differentiation etc. to properly understand the material. We haven't covered those yet, and probably won't have time to do so. But you can still do some of the exercise problems from this section with the ideas we have learned so far, specifically using mathematical induction. We will cover some parts of 11.3 tomorrow as well.

Problems

Exercise 1 (11.1.n)**.** *What are the lub and glb of the following sets? Write DNE if it doesn't exist. No proof is necessary.*

(a) $(-\infty, 1)$

(b) $\{x \mid x^3 \leq 8\}$

(c) $\left\{2\frac{1}{2}, 2\frac{1}{3}, 2\frac{1}{4}, \ldots\right\}$

(d) $\{0.9, 0.99, 0.999, 0.9999, \ldots\}$

(e) $\{x \mid x^2 + x + 2 \geq 0\}$

(f) $\{x \mid |x-1| > 2\}$

Exercise 2 (11.1.30)**.** *Find an example to show that the lub of a set of rational numbers may not be rational.*

Exercise 3. *Consider a sequence defined as*

$$a_1 = 2, \quad a_n = 1 - \frac{1}{a_{n-1}}$$

(a) *Prove by induction that* $a_n \leq 3$ *for all* n.

(b) *Show that* $\{a_n\}_{n\in\mathbb{N}}$ *is a periodic sequence.*

(c) *Find the least upper bound of the sequence.*

Exercise 4 (Do not submit these). *The following are practice problems from the book on a variety of topics we have already covered in class. You don't have to submit these, but make sure you know how to solve them.*

Problems 11.2.(46, 53, 58, 60, 63, 68(a, b)).