

Assignment 3 (1/8)

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- This homework is due at the beginning of class on **Friday** 1/12. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.
- Hand in the exercises only, not the reading material part. You are encouraged to think about the exercises marked with a (*) or (†) if you have time, but you don't need to hand them in. If you correctly solve a (†)-marked problem, you will get a candy!
- Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.
- We are currently covering Chapter 12 from Stewart.

Important Points and Reading Materials

- Cross Products:
 - Definition - either using angle or using components
 - Observe that unlike any of the other vector concepts we've talked about, the cross product makes sense only in three dimensions. You can't take the cross product of n -dimensional vectors.
 - If you know about determinants, try to learn the 3×3 determinant formula for the cross product (see the textbook). If you don't know about determinants, don't worry about this.
 - What is $\vec{u} \times \vec{u}$? What is $\hat{i} \times \hat{j}$, $\hat{j} \times \hat{k}$, $\hat{k} \times \hat{i}$? What is $\hat{j} \times \hat{i}$?
 - Know how to use the right hand rule to find the direction of cross product.
 - How can we use cross products to determine if two vectors are parallel?
 - How do we use the scalar triple product to find the volume of a parallelepiped?
- Equation of Lines and Planes:
 - Know how to find the Cartesian or parametric equations for a line through a point \mathbf{p} , parallel to a vector \vec{v} . What about the straight line through two points A and B ?
 - Given two straight lines in three dimension, it is not always possible to find a plane containing both lines. If the lines intersect each other, we can find such a plane. If they do not, then they are either parallel (in which case, there is a plane containing them) or they are not (in which case there is no plane containing them). Two non-parallel, non-intersecting lines are said to be 'skew'. Can you give examples of two such lines?
 - A vector perpendicular to a plane is called the 'normal' vector to the plane. Know how to find equation of a plane containing a point \mathbf{p} and having normal vector \vec{v} .
 - Know how to find equation of a plane containing three given points by using cross product to produce the normal vector. Look at example 12.5.5.
 - Read all the examples of chapter 12.5.

Problems

Please note that specifically for this assignment, I expect that you should be able to do the star marked problems (except exercise 3), if given in quiz or exam, even if you are not submitting their solution.

Exercise 1

(12.4.6) Find $\langle t, \cos t, \sin t \rangle \times \langle 1, -\sin t, \cos t \rangle$.

Exercise 2

(12.4.19) Find two unit vectors orthogonal to both $\langle 3, 2, 1 \rangle$ and $\langle -1, 1, 0 \rangle$.

Exercise 3★

(12.4.50) Show that for three vectors \vec{u} , \vec{v} , and \vec{w} ,

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$$

(12.4.51) Consequently show that

$$\vec{u} \times (\vec{v} \times \vec{w}) + \vec{v} \times (\vec{w} \times \vec{u}) + \vec{w} \times (\vec{u} \times \vec{v}) = 0$$

Exercise 4

(12.4.31) Let $P = (0, -2, 0)$, $Q = (4, 1, -2)$, $R = (5, 3, 1)$. Find area of the triangle $\triangle PQR$.

Exercise 5

(12.4.36) Find the volume of the parallelepiped with adjacent edges PQ , PR , and PS where $P = (3, 0, 1)$, $Q = (-1, 2, 5)$, $R = (5, 1, -1)$, and $S = (0, 4, 2)$.

Exercise 6★

(12.4.37) Verify that the vectors $\vec{u} = \langle 1, 5, -2 \rangle$, $\vec{v} = \langle 3, -1, 0 \rangle$, and $\vec{w} = \langle 5, 9, -4 \rangle$ are coplanar.

Exercise 7★

(12.5.1) Do all the T/F questions in exercise 12.5.1.

Exercise 8

(12.5.4) Find equation of the straight line through the point $(0, 14, -10)$ and parallel to the line $x = -1 + 2\lambda$, $y = 6 - 3\lambda$, $z = 3 + 9\lambda$.

Exercise 9★

Know how to do problems 12.5.(5, 9, 10, 11, 24, 27, 31, 50).

Exercise 10

(12.5.12) Find equation of the line of intersection of the planes $x + 2y + 3z = 1$ and $x - y + z = 1$.

[HINT: First determine the normal vectors to the two planes. Observe that the line of intersection is perpendicular to both of those normal vectors.]

Exercise 11

(12.5.30) Find equation of the plane that contains the line $x = 1 + t$, $y = 2 - t$, $z = 4 - 3t$ and is parallel to the plane $5x + 2y + z = 1$.

Exercise 12

(12.5.63) Find equation of the plane with x -intercept a , y -intercept b , and z -intercept c .