Please show **all** your work! Answers without supporting work will not be given credit.
Clearly mention what theorem(s), if any, you are using.
Write answers in space provided. Use the backside if needed.
You have 15 minutes to complete this Quiz.
You can get MAXIMUM $(6+9) = 15$ marks.

Name:

A furniture manufacturer makes two types of furniture - chairs and sofas. The production of the sofas and chairs requires three operations - carpentry, finishing, and upholstery. Manufacturing a chair requires 3 hours of carpentry, 9 hours of finishing, and 2 hours of upholstery. Manufacturing a sofa requires 2 hours of carpentry, 4 hours of finishing, and 10 hours of upholstery. The factory has allocated at most 66 labor hours for carpentry, 180 labor hours for finishing, and 200 labor hours for upholstery. The profit per chair is $90 and the profit per sofa is $75. How many chairs and how many sofas should be produced each day to maximize the profit?

Formulate the problem as a LPP [6 points] and solve using Simplex Method [9 points].

SOLUTION: Let $x =$ number of chairs and $y =$ number of sofas to be produced each day. Then the Linear Programming Problem is as follows:

$$\begin{aligned} \text{Maximize } & 90x + 75y = P \\ \text{Subject to } & 3x + 2y \leq 66 \\ & 9x + 4y \leq 180 \\ & 2x + 10y \leq 200 \\ & x, y \geq 0 \end{aligned}$$

Solution using Simplex method:

x_1	x_2	s_1	s_2	s_3	P	b
3	2	1	0	0	0	66
$\boxed{9}$	4	0	1	0	0	180
2	10	0	0	1	0	200
−90	−75	0	0	0	1	0

$R_2 \mapsto R_2/9$

3	2	1	0	0	0	66
1	4/9	0	1/9	0	0	20
2	10	0	0	1	0	200
−90	−75	0	0	0	1	0

$R_1 \mapsto R_1 - 3R_2$

$R_3 \mapsto R_3 - 2R_2$

$R_4 \mapsto R_4 + 90R_2$

0	$\boxed{2/3}$	1	−1/3	0	0	6
1	4/9	0	1/9	0	0	20
0	82/9	0	−2/9	1	0	160
0	−35	0	10	0	1	1800

$R_1 \mapsto 3R_1/2$

0	1	3/2	−1/2	0	0	9
1	4/9	0	1/9	0	0	20
0	82/9	0	−2/9	1	0	160
0	−35	0	10	0	1	1800

x_1	x_2	s_1	s_2	s_3	P	b

$R_2 \mapsto R_2 - 4/9R_1$

$R_3 \mapsto R_3 - 82/9R_1$

$R_4 \mapsto R_4 + 35R_1$

0	1	3/2	−1/2	0	0	9
1	0	−2/3	1/3	0	0	16
0	0	−41/3	$\boxed{13/3}$	1	0	78
0	0	105/2	−15/2	0	1	2115

$R_3 \mapsto 3R_3/13$

$R_4 \mapsto R_4 + 15/2R_3$

$R_1 \mapsto R_1 + 1/2R_3$

$R_2 \mapsto R_2 - 1/3R_3$

0	1	−1/13	0	3/25	0	18
1	0	5/13	0	−1/13	0	10
0	0	−41/3	1	3/13	0	18
0	0	375/13	0	45/16	1	2250

Hence the solution is $y = 18, x = 10$ for a maximum of $2250.

The End.