Assignment 18 (2/28)

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- *This homework is due at the beginning of class on **Wednesday** 3/7. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material item. You are encouraged to think about the exercises marked with a ($\star$) or (†) if you have time, but you don't need to hand them in. If you correctly solve a (†)-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering Sequences and Series (Chapter 11) from Stewart.*

Important Points and Reading Materials

- SEQUENCES AND SERIES Review your notes from previous courses on Sequence/Series. Next lecture, I will quickly recap section 11.1 and 11.2 main results before moving on to 11.3,4,6. Section 11.7 is a recap of the first 6 sections. Halfway through Friday I plan to start 11.8, discuss 11.9 on Monday and parts of 11.10 on Wednesday. There will probably be a last quiz on next week Wednesday.

  Review the following concepts before this Friday:

  - Definition of Convergence of Sequence - Squeeze Theorem (p738), Monotonic Sequence Theorem (p742)
  - Definition of Convergence of Series - Examples (Geometric and Harmonic series)
  - Tests for determining convergence:
    * n^{th} term divergence test - p753, thm 6,7
    * integral test - p761; specific case - p—series
    * comparison test - p767
    * limit comparison test - p769
    * alternating series test - p772
    * ratio test - p779
    * rrot test - p781

    For each of these tests, you should know three things:

    * When the test is applicable
    * How to perform the test, i.e. what to check
    * What are the conclusions

    In particular you should be able to at least recall the statement and one example for each test.

  Below are some warm-up problems from the first two sections.

- GEOMETRIC SERIES (p749-750, Example 2 from Stewart)
  - An important example of an infinite series is the Geometric Series, which is one of the few cases where we can explicitly find the value of the sum when the series converges.
  - A geometric series is defined as
    $$a_n = ar^{n-1} \text{ for } n \geq 1$$
    where a, r are constant real numbers.
  - If $|r| < 1$, we can show that (read the book)
    $$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$
    Note that for calculating the exact value of the sum, it is important to specify the starting value of the index n.
  - If $|r| \geq 1$, the series $\sum ar^{n-1}$ diverges.

Problems

Exercise 1

Determine whether the following series is convergent or divergent. Specifically mention what test you are using.

(a) (11.2.29)

$$\sum \frac{2+n}{1-2n}$$

(b)

$$\sum_{n=1}^{\infty} \frac{2^{3n-1}}{3^{2n+1}}$$

(c) (11.2.43)

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1}$$

[Hint: Can you write a formula for the partial sums s_n?]

(d) (11.3.24)

$$\sum ne^{-n^2}$$

(e) (11.3.30)

$$\sum \frac{1}{n(\ln n)[\ln(\ln n)]^p}$$

where p is a Natural number. Find the values of p for which the series is convergent.

(f) (11.3.45)

$$\sum_{n=1}^{\infty} b^{\ln n}$$

where b is a positive real number. find the values of p and b for which the series is convergent.

Exercise 2

Find the values of x for which the following series converges.

(a)

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$$

[Hint: Treat it as a geometric series. When does a geometric series converge?]

(b)

$$\sum e^{nx}$$

Exercise 3⋆

If you like Biology, do problem 11.2.70 from Stewart.
If you like Economics, do problem 11.2.73 from Stewart.
If you like Physics, do problem 11.2.74 from Stewart.
If you like Fractals, do problem 11.2.89 from Stewart.

Exercise 4

(11.3.33) The Riemann zeta-function (from the infamous Riemann Hypothesis) for $x \in \mathbb{R}$ is defined by

$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$$

(a) What is the domain of $\zeta(x)$? Note that the domain consists of the values of x for which $\zeta(x)$ is defined i.e. when the series is convergent.

(b) Euler proved that

$$\zeta(2) = \frac{\pi^2}{6} \text{ and } \zeta(4) = \frac{\pi^4}{90}$$

Using this information, evaluate

(i)

$$\sum_{n=2}^{\infty} \frac{1}{(n+3)^2}$$

(ii)

$$\sum_{n=4}^{\infty} \frac{1}{(n-2)^4}$$