Notes from Class and Homework

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Point of Inflection

An **inflection point** is a point on a curve at which the sign of the concavity changes. More rigorously,

Definition 0.1. A point $(c, f(c))$ is called a point of inflection if $\exists \delta > 0$ such that $f(x)$ is concave in one direction on $(c - \delta, c)$ and concave in the other direction on $(c, c + \delta)$.

0.1 How to determine points of inflection?

The best way to find out if a point is a point of inflection or not is to check the first derivative on both sides. The following characterisation is useful in determining which points to check:

Theorem 0.2. *If $(c, f(c))$ is a point of inflection then either $f''(c) = 0$ or $f''(c)$ does not exist.*

Note that the implication in the theorem is **one way**. $f''(c) = 0$ does **not** necessarily imply that c is a point of inflection.

Exercise 0.3. Give example of a function $f(x)$ such that f is twice differentiable, has $f''(c) = 0$ at a point c, but c is not a point of inflection.

Exercise 0.4. Give example of a function $f(x)$ such that $f''(c)$ does not exist but c is not a point of inflection.

Exercise 0.5. Consider the function $f(x) = x^{\frac{1}{3}}$. Give rough plots of $f(x)$ and $f'(x)$. Is $(0, 0)$ a point of inflection? What can you say about $f''(0)$?

Exercise 0.6. Consider the function $f(x) = x^{\frac{9}{5}} - x$. Give rough plots of $f(x)$ and $f'(x)$. What can you say about $f''(0)$? Is $(0, 0)$ a point of inflection?

Exercise 0.7. Describe the concavity of the graph of $f(x) = 2\sin^2(x) - x^2$ for $x \in [0, \pi]$.

Exercise 0.8. Describe the concavity of the graph of $f(x) = 2\sin^2(x) - x^2$ for $x \in [0, \pi]$.

Exercise 0.9. Problems 4.6.$(3, 38, 42, 47)$. Problem 4.8.55.