Lab 3: Parametric Plotting with Lissajous curves

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A parametric plot in the xy-plane can be drawn for any pair of functions $x(t)$ and $y(t)$ depending on the parameter t. For instance, to dynamically draw the unit circle centered at the origin, you could enter the command

```
Manipulate[ParametricPlot[{Cos[t],Sin[t]},{t,0,a},PlotRange-> 1],{a,0.001,2*Pi}]
```

and then use the slider bar to trace the circle. Note that a is the last value of the parameter t shown in the image.

■ Exercise 1.

Take a look at the curves parameterized by $(x, y) = (\cos[t], \sin[n\,t])$ with integer values of n. Start with $n = 2$, and experiment with the image until you understand the answers to the questions (a)--(e) below. Then fill in the first row of the chart for future reference, and continue on to $n = 3, 4,$ and 5.

1. For what value of t does the x coordinate first return to its value at $t = 0$?
2. For what value of t does the y coordinate first return to its value at $t = 0$?
3. For what value of t do both coordinates simultaneously return to their values at $t = 0$ for the first time?
4. How many times does the **entire curve shape** cross the x-axis?
5. How many times does the **entire curve shape** cross the y-axis?

	Question				
n	(a)	(b)	(c)	(d)	(e)
2					
3					
4					
5					

Which of these curve shapes intersect themselves at the origin? Why?

Now fill in another table with the answers to the same questions, but use the curves parameterized by $(x, y) = (\cos[n\, t], \sin[t])$.

	Question				
n	(a)	(b)	(c)	(d)	(e)
2					
3					
4					
5					

■ Exercise 2.

What is different about the curves in exercise **2** when n is even?

We can also take *rational* multiples of t in our parameterizations: $(x, y) = (\cos[r\, t], \sin[t])$ with $r = m/n$.

■ Exercise 3.

What is the smallest range of t values that will give the entire curve shapes described by $(x, y) = (\cos[r\, t], \sin[t])$ for $r = 3/4, 3/5,$ and $3/7$? (Hint: You will need to change the upper limit on the slider value a at the end of the `Manipulate` command.)

■ Exercise 4.

For each value of r, find an alternate parameterization which gives the same curve while using only $0 \leq t \leq 2\pi$.

■ Exercise 5.

Experiment until you can say what should happen to the curves when parameterized by $x = \cos[s\, t]$ and $y = \sin[t]$ for an irrational value like $s = \sqrt{2}$. Note that you should use the `PlotPoints` option as below to keep your images smooth (and accurate):

```
Manipulate[ParametricPlot[{Cos[Sqrt[2]*t],Sin[t]},{t,0,a},PlotPoints->500],{a,0.001,2500}]
```