Assignment 12 (7/17)

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- *This homework is due at the beginning of class on **Tuesday** 7/24. You are encouraged to work together on these problems, but you must write up your solutions independently.*

Problems

Exercise 1. *Find and sketch the solution set of the following equations/inequalities. If the equation has one variable, the answer is an interval. If it has two variables, then it's a region on the plane. You must explain how you found your answer.*

(a) $|x| + |y| < 1$

(b) $\dfrac{x + 3y}{3x + y} < 1$

(c) $|x - 1| + |x - 2| < 5$

(d) $x^2 - |x + 2| + x > 0$

All of the following problems use the Discriminant.

Exercise 2. *If $x^2 - ax + 3 > 0$ for all real numbers x, then find all possible values of a.*

Exercise 3. *Suppose λ is a real number such that the two roots of the equation*

$$(\lambda - 1)(x^2 + x + 1) - (\lambda + 1)(x^2 - x + 1) = 0$$

are real and distinct. Prove that

$$\lambda < -2 \text{ or } \lambda > 2.$$

Exercise 4. *If the two roots of of the equation*

$$(b - c)x^2 + (c - a)x + (a - b) = 0$$

are equal to each other, then show that $a, b,$ and c are in an AP.

Exercise 5. *Suppose $a, b,$ and c are three real numbers in a GP and x is a real number such that $a + b + c = bx$. Find all possible values of x.*