Midterm 2

INSTRUCTIONS:

- Please show ALL your work! Answers without supporting justification will not be given credit.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
- Write legibly, in correct order and clearly mark the answer to each part.
- Please note that use of calculator is not allowed.
- If you write down the correct formula for an answer, you will get some partial credit regardless of whether you evaluated the exact values or not.
- Unless otherwise specified, you may use any valid method to solve a problem.

Full Name: ____

Question	Points	Score
1	25	
2	7	
3	10	
4	20	
5	8	
Total:	70	

This exam has 5 questions, for a total of 70 points.
The maximum possible point for each problem is given on the right side of the problem.
You can score a maximum of 65 **points.**

1. Consider the function

$$F(x,y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 1$$

defined on the disk D of radius $\sqrt{2}$ centered at the origin i.e.

$$D = \{(x,y) \mid x^2 + y^2 \leq 2\}.$$

Follow the steps below to find the absolute maximum and minimum of f on D.

(a) Find all the critical points of F. $\boxed{6}$
[HINT: There are 4 such points.]

(b) Find all of the second order partial derivatives of F and write down the determinant of the Hessian matrix as a function of x and y. Don't calculate its value at any specific point yet. $\boxed{3}$

(c) In the list of critical points from part (a), identify the ones lying *inside* D (excluding the boundary). $\boxed{1}$

(d) Classify each of the point(s) in part (c) as a local maximum, local minimum, or a saddle point using the Hessian. $\boxed{3}$

(e) Evaluate F at the critical point(s) from part (c). $\boxed{2}$

(f) Use Lagrange multiplier to find the maximum and minimum of $F(x, y)$ subject to the constraint $x^2 + y^2 = 2$. Note that this gives the extreme values of F on the boundary circle of D. $\boxed{8}$

(g) Compare the extreme values of F from part (e), and the extreme values of F from part (f), to find the absolute maximum and minimum of $F(x, y)$ on D. $\boxed{2}$

2. Find the following double integral. $\boxed{7}$

$$\iint\limits_{\Omega} y^2 e^{xy}\, dA$$

where Ω is the **triangle** bounded by $x = y$, $y = 4$, and $x = 0$.

[HINT: Identify the region correctly as type I vs. type II to make the integration a lot simpler.]

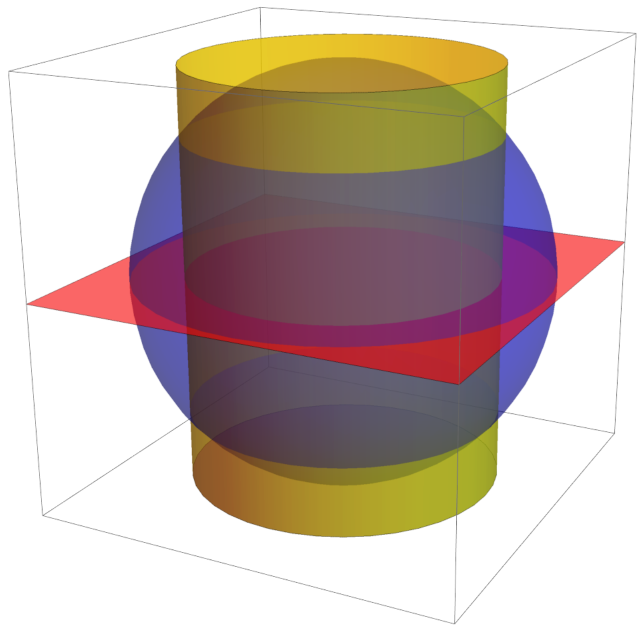

Figure 1: Three surfaces

3. Use polar coordinates to determine the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 9$, above the plane $z = 0$, and inside the cylinder $x^2 + y^2 = 5$. $\boxed{10}$

   All three surfaces are pictured above. You will get part marks for correctly setting up the volume as a double integral.

4. (a) Let $u = x - 2y$ and $v = 3x - y$. Find the transformation functions g and h such that $\boxed{3}$

$$x = g(u, v) \quad \text{and} \quad y = h(u, v)$$

[HINT: Solve the system of equation for x and y in terms of u and v.]

(b) Find the Jacobian matrix of this transformation and calculate its determinant. $\boxed{7}$

(c) Let R be the parallelogram enclosed by the four straight lines $\boxed{4}$

$$x - 2y = 0, \quad x - 2y = 4, \quad 3x - y = 1, \quad \text{and} \quad 3x - y = 8.$$

Define S to be the set of points (u, v) such that

$$R = \{(x, y) \mid x = g(u, v), y = h(u, v) \text{ for some } (u, v) \in S\}$$

i.e. S maps to R under the above transformation. What does the set S look like?

(d) Evaluate the following integral using appropriate change of variable formula: $\boxed{6}$

$$\iint\limits_R \frac{x-2y}{3x-y}\,dA$$

where R is the parallelogram defined in part (c).

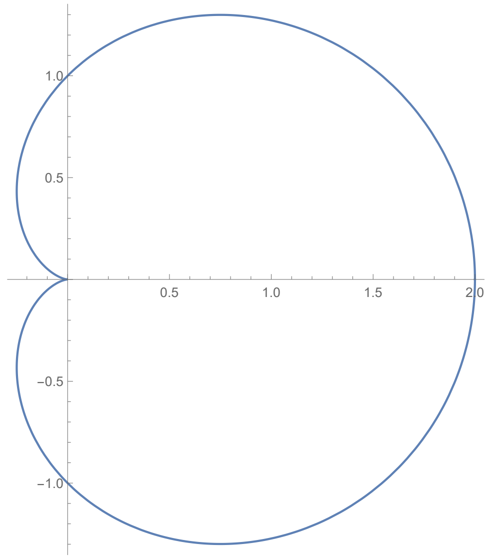

Figure 2: Polar plot of $r = 1 + \cos\theta$

5. Find the area inside the polar curve $r = 1 + \cos\theta$ as pictured above, by evaluating the following integral: $\boxed{8}$

$$\int_0^{2\pi} \int_0^{1+\cos\theta} r\,dr\,d\theta$$