Assignment 1

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Problem 1

(Oblique Asymptote) Let $r(x) = p(x)/q(x)$ be a rational function. If $\deg(p) = \deg(q) + 1$, then r can be written in the form

$$r(x) = ax + b + \frac{Q(x)}{q(x)}$$

with $\deg(Q) < \deg(q)$. Show that $[r(x) - (ax + b)] \to 0$ both as $x \to \infty$ and as $x \to -\infty$. Thus the graph of f "approaches the line $y = ax + b$" both as $x \to \infty$ ans as $x \to -\infty$. The line $y = ax + b$ is called an *oblique asymptote.*

Problem 2

Sketch the graph of the functions showing all **vertical and oblique asymptotes**. You do **not** need to point out critical points, increasing-decreasing etc.

(a) $\frac{x^3-1}{(x+1)^2}$,

(b) $(2x^2 + 3x - 2)/(x + 1)$.

Problem 3

Suppose f is a real valued continuous funtion on $[2, 7]$. Prove that given positive real numbers a and b, there exists a value $c \in [2, 7]$ such that

$$f(c) = \frac{(a + b)f(2)f(7)}{af(2) + bf(7)}.$$

Problem 4

Does $\lim\limits_{x\to\frac{\pi}{2}} \frac{3\tan(x)-4}{\sqrt{4\tan^2(x)-3}}$ exist? What about $\lim\limits_{x\to\frac{\pi}{2}} \frac{3\tan(x)-4}{2\tan(x)-3}$? Justify your claims.

Problem 5

Suppose f is continuous at $x = c$. We say that the graph of f has a *vertical cusp* at the point $(c, f(c))$ if as x tends to c from one side, $f'(x) \to \infty$ and from the other side $f'(x) \to -\infty$.

The graph of f is said to have a *vertical tangent* at the point $(c, f(c))$ if $\lim_{x \to c} f'(x)$ is either ∞ or $-\infty$.

Let p and q be positive integers, q odd and let $p < q$. Let $f(x) = x^{p/q}$. Specify conditions on p and q such that

1. the graph of f has a vertical tangent at $(0, 0)$.
2. the graph of f has a vertical cusp at $(0, 0)$.

Problem 6 [Pratice problems for Quiz 1]

You do not need to submit answers to these.

Sketch the graphs of

(a) $\frac{1}{4}x^4 - 2x^2 + \frac{7}{4}$

(b) $\frac{x^2-3}{x^3}$

(c) $\frac{3}{5}x^{5/3} - 3x^{2/3}$

(d) $x(x-1)^{1/5}$

(e) $\frac{2x^2}{x+1}$