Math 1800-C Handout 3: Parametrized Curves and Chain Rule

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Exercise 1

1. Find parametric equations for the line through the points $(6, 1, 1)$ and $(9, 1, 4)$. Call this line L_1.
2. Find parametric equations for the line through the points $(-4, 4, 0)$ and $(-6, 5, 1)$. Call this line L_2.
3. Find parametric equations for the line through the points $(6, -1, -5)$ and $(2, 1, -3)$. Call this line L_3.
4. Verify that L_2 and L_3 are parallel. (Their direction vectors should be parallel.) Are they the same line? How could you tell?
5. Do lines L_1 and L_2 intersect? If so, where?
6. Find the intersection of L_1 with the plane given by the equation $2x + y + 3z = 7$.
7. **Challenge:** Find the point on the plane $2x + y + 3z = 7$ which is closest to the origin.
8. **Challenge:** Find the point on L_2 closest to the origin.

Exercise 2

Parametrize the line segment connecting $(3, 7)$ to $(5, -2)$, so that $t = 0$ corresponds to $(3, 7)$ and $t = 1$ corresponds to $(5, -2)$.
Hint: Find the vector from the first point to the second, and parametrize by scaling the vector!

Exercise 3

Suppose the curve given by $\vec{r}(t) = \langle \cos(\pi t), \sin(\pi t), t \rangle$ intersects the paraboloid $z = x^2 + y^2$ at a point $P = (x_0, y_0, z_0)$.

1. Find the coordinates of P.
2. Find equation of the tangent plane to the paraboloid at P.
3. What is the equation of the tangent line to the curve $\vec{r}(t)$ at P?
4. What is the angle of intersection between the curve and the paraboloid? This is the angle between the tangent line in part (3) and the plane in part (2).

Exercise 4

Let $z = f(x, y) = x^2 + y^3$, and $x = x(s, t)$ and $y = y(s, t)$; i.e., x and y are functions of s and t. Suppose that when $(s, t) = (0, 1)$, we have:

$$x(0,1) = -1, \quad x_s(0,1) = -4, \quad x_t(0,1) = -7, \quad y(0,1) = 2, \quad y_s(0,1) = 10, \quad y_t(0,1) = 5.$$

Compute $\dfrac{\partial z}{\partial t}$ at $(s, t) = (0, 1)$.

Exercise 5

If $u = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$