Lab 3: Derivatives

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- Sign on to an iMac with your username and password.
- When you open *Mathematica*, click **New Document** and a blank screen will appear. This is known as a notebook. If you need to open a new notebook, go to **File**, choose **New** and choose **Notebook** or simply hit `Command + N` (or `Ctrl + N` for **Windows**).
- Make the Untitled window bigger if necessary by dragging the lower right corner. Choose at least **125%** from the lower left of the Untitled window for a comfortable viewing size.
- Do not forget to **Save** the notebook periodically. Save new notebooks on the **Desktop**. The **Save** command is under the **File** menu. Give your file a name in the following format:

  `Lab2_Name1_Name2.nb`
- Follow the instructions in the paper copy of the handout.
- Write down the answers to the ? marked problems in the blue book provided.
- At the end of Lab session, save and quit Mathematica. **Do not delete your file or any calculation you did.** Then log off or restart the computer. Do NOT click **Shut Down**.

Exercise I : Numerical Calculation

Let $f(x) = x^2 - 5x$.

1. Define the function $f(x)$ in Mathematica by typing

   ```
   f[x_]:=x^2-5*x
   ```

   Remember to use the the ':=' in your command.

2. Next Plot the function $f(x)$ from $-10 \leq x \leq 10$ using the command `Plot`. Remember to use square brackets to denote the function inside the `Plot` command.

3. ? On paper, sketch the graphs of $f(x)$ and sketch the graph of $f'(x)$, the derivative directly below it.

4. Now let's sketch the derivative function $f'(x)$ numerically as follows. We know

   $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

   So that means we can numerically approximate $f'(x)$ by setting h to be a small quantity. Let $h = 0.001$.

   In Mathematica, define a function $df(x)$ by typing

   ```
   df[x_] := (f[x + 0.001] - f[x])/0.001
   ```

   Then $df(x)$ is an approximation for $f'(x)$.

5. Plot $df(x)$ from $-10 \leq x \leq 10$.

6. Do your hand sketch from **(3)** and the Mathematica sketch of $df(x)$ agree? What are the similarities and differences? Did your sketch cross the x-axis in the right place? Did it have the correct sign everywhere? Was it the right shape? It can be hard to get exactly the right shape when you sketch the derivative "by eye". But it should be possible to get the correct signs, the correct x-axis crossings, and roughly the right shape.

Exercise II : Algebraic Calculation

Next, we will calculate the derivative algebraically, using $f(x) = x^2 - 5x$ again. Write this part as carefully as possible, as we will come round and carefully critique your work, to help you get good at this.

7. First calculate, and simplify as far as possible

$$\frac{f(x+h) - f(x)}{h}$$

   Notice there are no limits in this part.

8. ? Now use part (a) to find a simple formula for

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

9. Plot the graph of your formula for $f'(x)$ on Mathematica. How does it compare with the numerical approximation to $f'(x)$ that we graphed in problem **5**?

Exercise III : Other Examples

10. ? Repeat exercise I and II with $g(x) = x^2 - 5x + 10$. What do you notice about the derivatives of $f(x)$ and $g(x)$. Write a careful sentence or two to explain what's going on.
11. Now repeat exercise I with some crazy functions of your choice. Remember to try exponentials, trig functions, logs, rationals of polynomials, etc. Make sure you sketch the derivative BY HAND before you plot it on Mathematica to check your work.
12. What happens when you try to repeat exercise II with your crazy functions? My prediction is that sometimes the algebra works out, but sometimes it doesn't simplify nicely...
13. Repeat exercise I with $f(x) = e^x$. Can you guess the equation for $f'(x)$ from the picture?
14. Repeat exercise I with $g(x) = \ln x$. Can you guess the equation for $g'(x)$ from the picture?