Assignment 16 (2/23)

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- *This homework is due at the beginning of class on* ***Friday*** 3/2. *You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*
- *Hand in the exercises only, not the reading material item. You are encouraged to think about the exercises marked with a* ($\star$) *or* (†) *if you have time, but you don't need to hand them in. If you correctly solve a* (†)*-marked problem, you will get a candy!*
- *Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.*
- *We are currently covering the Simplex Method for solving LPP.*

Important Points and Reading Materials

- THE SIMPLEX METHOD

  First Read up on the algorithm from the following link:

  https://people.richland.edu/james/ictcm/2006/simplex.html

  Ignore the stuff about Table Method in there. Make sure you can answer the following conceptual questions.

  - Understand why "find all the corners and plug them into the function" isn't a very good way to solve linear programming problems, except for very simple ones.
  - Understand the intuition behind the simplex method. Namely, "start at one corner, and keep moving along edges to increase the value of the function." Why does this actually give you an absolute maximum, and not just a local maximum?
  - Understand how to turn inequalities into equalities by introducing slack variables. Why is this useful?

As long as the inequality is satisfied by the point $(0, 0, \ldots, 0)$ we can introduce slack variables to change them to equalities. Thus, for example,

$$2x + 3y \leq 8 \text{ becomes } 2x + 3y + s = 8$$

$$4x - 5y \geq -3 \text{ becomes } 4x - 5y - s = -3$$

But if $(0, 0, \ldots, 0)$ does not satisfy the inequality, we can't start simplex method. We need to "initialize", i.e. find an appropriate starting point by considering the Auxiliary LPP, to be discussed next time.

WARNING: Even when $(0, 0, \ldots, 0)$ is not feasible, you might be able to perform the steps of simplex method, but it will not give you the correct answer!

  - Understand how to preform the simplex method mathematically. How do the steps you are taking correspond to the intuitive description of the simplex method. When in the process are you moving along an edge?

At each step of the Simplex method, the current value of the objective function is obtained by taking all non-basic variables to be zero. In particular, we start with $(x_1, x_2, \ldots, x_n) = \mathbf{0}$. So we need to make sure that $(0, 0, \ldots, 0)$ is in fact a feasible solution. Once that is done, we move along an edge until we reach the next vertex to get the next feasible solution. The direction of the edge is chosen by Pivot column, whereas the choice of Pivot row tells us when to stop moving along the edge. Pivoting corresponds to a change of variables (axes).

– How do you decide on the Pivot Column? There's not always going to be one right choice, but the thing that matters most is that you're always increasing the value of the function. How can you tell which variables you're allowed to increase, just by looking at the table?

> Choose one containing the least negative number in the objective function row.

– Once you've picked a variable to increase, how do you tell how far you can increase it? How do you figure out the Pivot Row?

> The 'ratio' b_j/c_{ij} indicates the maximum distance we can go along the chosen line yet stay within the feasible region. Choose the least Positive one. If there is no positive ratio, the LPP is *unbounded* i.e. the maximum is $+\infty$.

– How do you perform a pivot using row-reductions?

> Change pivot to 1 by dividing the pivot row by the pivot. Change every other entry in pivot column to 0 using Row-operations.

– How do you know when you've finished the simplex method, i.e. how do you tell when you can't move along another edge to increase the function further?

> If there are no negative entry in the obejective function row, the algorithm is finished. You can find the optimal value of the objective function by taking all non-basic variables equal to zero.

Then read Vanderbei Chapter 2, section 1 and 2 for a more theoretical explanation. Note that the notations in Vanderbei differ a little bit, namely in signs; but they are the same method in theory. I would prefer if you worked with the table as we discussed in class today (matches the notes from the link above) rather than those in Vanderbei. However, I will give marks to any convention as long as it is mathematically correct.

Problems

Exercise 1

Solve problem 2.$(1, 8, 9)$ from Vanderbei using Simplex Method.

Exercise 2†

Write a computer program in your favourite language that implements the Simplex Method for $n = 2$ variables. Note that you will need to define a subroutine for row-reductions.