Midterm 1

JANUARY 26, 2018
SUBHADIP CHOWDHURY

INSTRUCTIONS:

- Please show ALL your work! Answers without supporting justification will not be given credit.
- Answer the questions in the green books provided.
- Write legibly and start each question on a new page. You can answer the problems out of order. In fact, I suggest working out the easier ones first.
- Please note that use of calculator, books, or notes is not allowed.
- If you write down the correct formula for an answer, you will get some partial credit regardless of whether you evaluated the exact values or not.
- Unless otherwise specified, you may use any valid method to solve a problem.

Full Name: ______________________________

Question	Points	Score
1	5	
2	10	
3	5	
4	7	
5	5	
6	8	
7	10	
Total:	50	

This exam has 7 questions, for a total of 50 points.
The maximum possible point for each problem is given on the right side of the problem.

1. Suppose α and β are real numbers such that $\boxed{5}$

   - the vector $\vec{u} = \hat{\imath} + \hat{\jmath} + \hat{k}$, the vector $\vec{v} = 4\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, and the vector $\vec{w} = \hat{\imath} + \alpha\hat{\jmath} + \beta\hat{k}$ are coplanar; and
   - the vector $\vec{w}$ has magnitude $\sqrt{3}$.

   Find all possible values of α and β.

   [HINT: If three vectors are coplanar, then the volume of the parallelepiped determined by those three vectors is zero.]

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2. Let

$$f(x,y) = \frac{x^2+y^2}{2x}$$

   (a) Show that the level curves of f are circles passing through the origin. $\boxed{5}$

   (b) Draw a sketch showing the two level curves that pass through the two points $(2,0)$ and $(-4,0)$ respectively. $\boxed{2}$

   (c) Determine whether $\boxed{3}$

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

   exists or not, and give a reason for your answer.

   [HINT: Use the two level curves as directions of approach.]

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3. Suppose for some differentiable function $f(x,y,z)$, we know that the maximum value of the directional derivative $D_{\vec{u}}$ at the point $(1,1,1)$ is 2, and this maximum occurs in the direction of the point $(2,3,3)$. Find ∇f at $(1,1,1)$. $\boxed{5}$

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4. Find the point(s) on the surface $xy + yz + zx + 4 = 0$ where the tangent plane is parallel to the XY-plane. $\boxed{7}$

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5. Find the angle of intersection between the curve given by its parametric equation $\vec{r}(t) = (t, 2t^2)$, and the parabola $y = x^2 + 4$. Do not simplify the $\arccos(\theta)$ part of the answer. $\boxed{5}$

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6. Let $p = g(u,v)$ be a differentiable function of two variables. Let $u = \frac{x}{y}$ and $v = \frac{y}{z}$. Show that $\boxed{8}$

$$x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} + z\frac{\partial p}{\partial z} = 0$$

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7. Use Lagrange multipliers to find the global maximum and minimum values of $\boxed{10}$

$$f(x,y) = x^2 + 2y^2 - 4y$$

   subject to the constraint $x^2 + y^2 = 9$.