

Assignment 19 (3/2)

Subhadip Chowdhury

- This homework is due at the beginning of class on **Wednesday** 3/7. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.
- Hand in the exercises only, not the reading material item. You are encouraged to think about the exercises marked with a (*) or (†) if you have time, but you don't need to hand them in. If you correctly solve a (†)-marked problem, you will get a candy!
- Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.
- We are currently covering Sequences and Series (Chapter 11) from Stewart.

Important Points and Reading Materials

- INTEGRAL TEST:
 - Note that integral test does NOT say that $\sum_{k=1}^{\infty} f(k)$ is equal to $\int_1^{\infty} f(k)$. The integral converges iff the series converges, but they aren't equal.
 - What are the prerequisite conditions on f to apply integral test?
 - Understand p -series test.
- COMPARISON AND LIMIT COMPARISON TEST:
 - Applies only to series with positive terms.
 - If you want to show $\sum a_n$ converges and you know $\sum b_n$ converges, do you need to show $a_n \geq b_n$ or $a_n \leq b_n$. What if $\sum b_n$ diverges, and you want to show that $\sum a_n$ also does?
 - What can you do if you know what series $\sum b_n$ you want to use, but the inequality goes the wrong way. For instance, what if you want to show that $\sum \frac{1}{n+1}$ diverges by comparing it to $\sum \frac{1}{n}$.
 - Limit comparison test is used to compare complicated series with rational function entries to simpler series, usually a p -series. This is useful if you know what sequence you want to compare to, but it's difficult to figure out which sequence is larger (i.e. you can't use comparison).
 - Also remember that the limit $\lim_{n \rightarrow \infty} a_n/b_n = L$ must be > 0 to apply limit comparison
- ALTERNATING SERIES
 - Applies only to series with alternating positive and negative terms.
 - What are the prerequisites for this test? Note that a_n must be a decreasing sequence, just $a_n \rightarrow 0$ is not enough.
 - Remember that most of our convergence tests only work for sequences with nonnegative terms. For general series, we need to try something else.
 - Absolute convergence implies convergence.
- ROOT AND RATIO TEST
 - Applies to any series. But the test is done using the absolute value of the terms.
 - Know what $\lim_{n \rightarrow \infty} a^{1/n}$ and $\lim_{n \rightarrow \infty} n^{1/n}$ are. More generally, if $p(n)$ is a polynomial, what are $\lim_{n \rightarrow \infty} [p(n)]^{1/n}$ and $\lim_{n \rightarrow \infty} \frac{p(n+1)}{p(n)}$? What does this mean for the root and ratio tests?

Problems

Exercise 1

Determine whether the following series converge or diverge. Clearly mention what test you are using. If you have to use multiple tests, mention each of the steps.

1. (11.4.5)

$$\sum \frac{n+1}{n\sqrt{n}}$$

2. (11.4.16)

$$\sum \frac{1}{n^n}$$

3. (11.4.27)

$$\sum \left(1 + \frac{1}{n}\right)^2 e^{-n}$$

4. (11.4.31)

$$\sum \sin(1/n)$$

5. (11.5.11)

$$\sum (-1)^{n+1} \frac{n^2}{n^3 + 4}$$

6. (11.5.17)

$$\sum (-1)^n \sin(\pi/n)$$

7. (11.5.34)

$$\sum (-1)^{n-1} \frac{(\ln n)^n}{n}$$

For what values of p does it converge?

8. (11.7.10)

$$\sum \frac{(-3)^n}{(2n+1)!}$$

9. (11.7.44)

$$\sum \frac{(n!)^2}{(kn)!}$$

For what values of k does it converge?

10. (11.7.28)

$$\sum \left(\frac{-2n}{n+1}\right)^{5n}$$

11. (11.7.37)

$$\sum \frac{(-1)^n \arctan n}{n^2}$$

12. (11.7.40)

$$\sum a_n \quad \text{where} \quad a_1 = 1, \quad a_{n+1} = \frac{2 + \cos n}{\sqrt{n}} a_n \quad \text{for } n \geq 1$$