Assignment 18 (11/9)

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- **Definition(Injective function):** (Definition 7.1.1)
  A function $f : A \to B$ is said to be injective if $f(a) = f(b) \implies a = b$ for all $a, b \in A$.
- **Definition(Surjective function):** A function $f : A \to B$ is called surjective if for every $b \in B$, there exists some $a \in A$ such that $f(a) = b$.

Problem 1

Find the largest subset of $\mathbb{R}$ that can be the domain of the following functions. Then decide whether or not they are injective. Determine the range of each function as well.

(a) $x + \frac{1}{x}$

(b) $\frac{x}{|x|}$

(c) $\frac{1}{(x+1)^{2/3}}$

Problem 2

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{(1-x)(x-2)(x-6)}{x^2(x-5)}$$

Is f surjective onto $\mathbb{R}$?

Problem 3

In each of the following case, give examples of a function f and two sets A, B, both subsets of $\mathbb{R}$; such that $f : A \to B$ satisfies:

(a) $f'(x) > 0$ for all $x \in A$ and f is surjective onto B, but f is not injective.

(b) $f : A \to B$ is not injective but $f : A' \to B$ is injective for some subset $A' \subseteq A$.

(c) $B = \mathbb{R}$, $f : A \to \mathbb{R}$ is bijective and f has infinitely many discontinuities.

(d) f is not a constant function and $f(A') = B$ for infinitely many subsets $A' \subseteq A$.

$\boxed{\textbf{Problem 4}}$

Prove that a continuous function f is injective if and only if it is strictly monotone (i.e. either strictly increasing or strictly decreasing).

$\boxed{\textbf{Problem 5}}$

- Prove that the function $f : [0, \infty) \to [0, \infty)$ defined as $f(x) = x^2$ is surjective.
- Prove that the function $g : \mathbb{N} \to \mathbb{N}$ defined as $g(n) = n^2$ is NOT surjective.