MATH 1800-B HANDOUT 1: FUNCTIONS OF SEVERAL VARIABLES

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Exercise 1

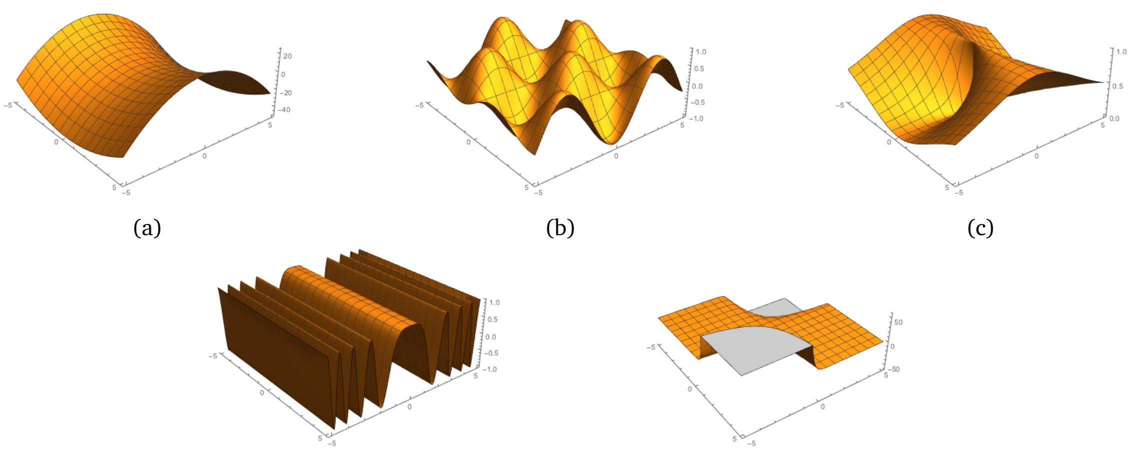
For the following problems, fill the box with either “certainly”, “possibly”, or “certainly not”.

1. The point $(a,-1,3)$ is $\boxed{\qquad}$ on the sphere $(x-2)^2+(y+1)^2+(z-3)^2=1$.
2. If all the y cross-sections (i.e. the cross sections parallel to XZ-plane) of the graph of $f(x,y)$ are straight lines, then the graph is $\boxed{\qquad}$ a plane.
3. If $f(x,y)$ is a linear function, then the graph of f is $\boxed{\qquad}$ parallel to XZ-plane.
4. The graph of $f(x,y)=x^2+y^2-1$ is $\boxed{\qquad}$ the same set of points as the 1-level surface of $g(x,y,z)=x^2+y^2-z$.

Exercise 2

Math the following graphs with the functions.

$$\text{(I): } xe^{-xy}, \quad \text{(II): } \frac{x^2}{x^2+y^2}, \quad \text{(III): } \sin(x)\cos(y), \quad \text{(IV): } x^2-2y^2, \quad \text{(V): } \cos(y^2)$$



(a) (b) (c) (d) (e)

Figure 1

Exercise 3

Consider the function $z=f(x,y)=\frac{x}{2}-2y+1$.

1. Sketch the contour plot for the graph with z-increment value of 1.
2. (a) Starting at any point (x,y), what is the slope of the surface in the $x-$direction?
   (b) What is the slope in $y-$direction?
   (c) What is the slope along the line $x=y$?
3. What kind of surface is the graph? Sketch a picture.

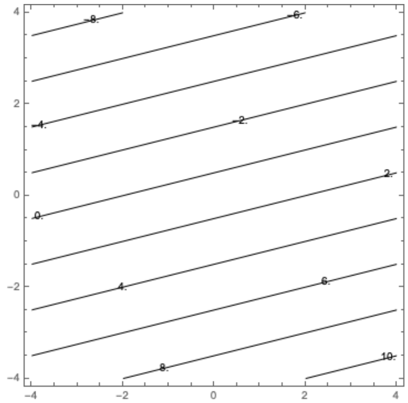


Figure 2: 3.1

Solution.

1.

2. (a) $\frac{\Delta z}{\Delta x} = 1/2$

   (b) $\frac{\Delta z}{\Delta x} = -2$

   (c) $f(0,0) = 1$, $f(1,1) = -1/2$. So slope in the direction of $x = y$ is $\frac{-1/2-1}{\text{distance from (1,1) to (0,0)}} = -\frac{3}{2\sqrt{2}}$.

3. The surface is a plane. It passes through $(-2,0,0),(0,1/2,0),(0,0,1)$. The picture looks as follows:

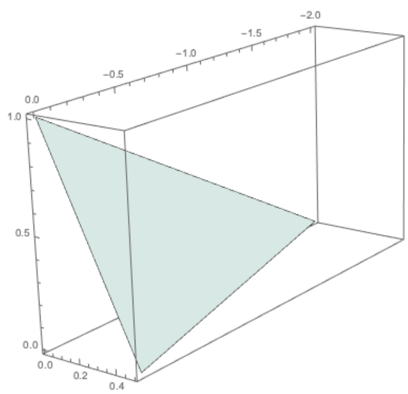


Figure 3: 3.3

Exercise 4

Find the linear functions whose contour plots are shown in figure 4.

Exercise 5

Find an equation for the plane that contains the line in the XY-pane where $y = 1$, and the line in the XZ-pane where $z = 2$.

Solution. Observe that the plane is parallel to the X-axis. So its equation looks like $z = ny + c$. Plugging in two points (e.g. $(0,1,0)$ and $(0,0,2)$) into the equation we get that $c = 2, n = -2$. So the equation is $z = -2y + 2$.

Exercise 6

Consider the contour plot for the function $f(x,y) = x^2 + y$.

1. Sketch the cross-section of the graph with the plane $x = 4$.

2. Compute the rate of change of z with respect to y as (x,y) moves towards increasing y-value, along the line $x = 4$.

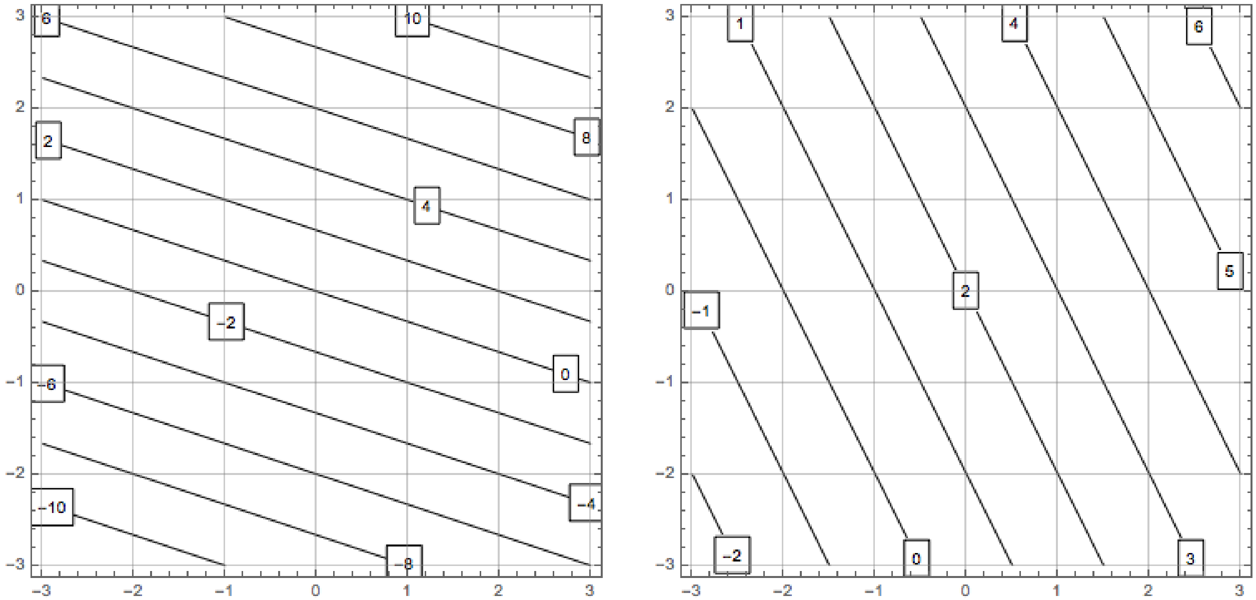


Figure 4: Plots for Exercise 4

3. What happens to the rate of change of z with respect to x as you move from $(4,5)$ towards increasing x-value along the line $y = 5$.
4. Starting at the point $(1,1)$, what direction would yield the maximal rate of change of z with respect to the distance in XY-plane.

Understanding how to answer these questions will be critical for graphically estimating partial derivatives and gradient vectors that we will learn about next week.

Solution.

1. When $x = 4$, the cross-section looks like $z = 16 + y$, a straight line.
2. $\frac{\Delta z}{\Delta y} = 1$, slope of the straight line $z = 16 + y$.
3. $f(4,5) = 21, f(5,5) = 30, f(6,5) = 41$. So $\frac{\Delta z}{\Delta y}$ is increasing.
4. The maximal rate of change would be in a direction perpendicular to the level curve passing through $(1,1)$. [Exact vector direction or coordinates not required.]