

Assignment 4+5 (1/10 + 1/12)

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- This homework is due at the beginning of class on **Friday** 1/19. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.
- Hand in the exercises only, not the reading material part. You are encouraged to think about the exercises marked with a (*) or (†) if you have time, but you don't need to hand them in. If you correctly solve a (†)-marked problem, you will get a candy!
- Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.
- We are currently covering Chapter 14 from Stewart.

Important Points and Reading Materials

- Functions of Multiple Variables
 - Definition - domain - range
 - How many dimension do you need to draw a graph of a multivariable function?
 - Know how to sketch the domain of a function of two variable - including regions bounded by parabola, circle, ellipse, curves etc.
 - What is a level curve and a contour map (for a function of 2 variables)? What is a level surface?
 - How to identify contour maps from pictures of graphs and vice versa.
- Limit and Continuity
 - Understand that we need to consider all the directional limits for a function of more than one variable.
 - If limits along two different curve of approach do not match, then limit does not exist.
 - Define continuity using limit.
 - Note that rational functions i.e. functions that are ratios of integer polynomials, are continuous on their domain. As such if you are asked to calculate limit of a rational function at a point, and the functional value exists, then limit is equal to the functional value.

Problems

Exercise 1

Find and sketch the domain of the following functions:

$$f(x, y) = \arcsin(x^2 + y^2 - 2)$$

$$f(x, y) = \ln(9 - x^2 - 9y^2)$$

Exercise 2

Problems 14.1.(61 – 66).

Exercise 3

(14.2.(20,9,24))

Show that the following limits do not exist:

1.

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2}$$

2.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$$

3.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6}$$

[HINT: In each case, find two direction/curves to approach towards (0,0) such that limits aren't equal.]

Exercise 4★

Find the distance between the planes $3x + y - 4z = 2$ and $3x + y - 4z = 24$.

Exercise 5

(14.1.49) Sketch a contour map of the function $f(x, y) = ye^x$ showing several level curves.

Exercise 6

Identify the level curve of $f(x, y) = \ln(x - y^2)$ that passes through (2, 1).