Assignment 16 (5/15)

Subhadip Chowdhury

Problem 1

Problems 12.2.(30, 32, 42).

Problem 2

Consider a real number a and consider the sequence:

$$a, \cos(a), \cos(\cos(a)), \cos(\cos(\cos(a))), \dots$$

Does this sequence converge? If it does, find the limit.

Problem 3

Suppose a sequence a_n converges to l. Define another sequence $\{m_n\}$ by

$$m_n = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

Prove that $m_n \to l$.

Problem 4

Find

$$\lim_{x \to \infty} x e^{-x^2} \int_0^x e^{x^2} dx$$

Problem 5

Suppose S is a nonempty set which is bounded below. let B be the set of lower bounds of S. Prove that $\mathsf{lub}(B) = \mathsf{glb}(S)$.

Problem 6

Find whether following integrals converge, and if so, evaluate them:

(a)
$$\int_0^9 \frac{1}{(x-1)^{2/3}} dx$$

(b)
$$\int_0^1 \frac{1}{1-x^2} dx$$

(c)
$$\int_{1}^{\infty} \frac{\sin(\pi/x)}{x^2} dx$$

(d)
$$\int_0^\infty e^{-rx}\cos(2x)dx$$
, $r \in \mathbb{R}$