Assignment 1 (9/29)

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Problem 1

Problem 1.2.49 from Text.

Problem 2

Problem 1.2.76 from Text.

Problem 3

Statement of the Least Upper Bound Property of Real numbers:
Let S be a non-empty set of real numbers.
- A real number x is called an *upper bound* for S if $x \geq s$ for all $s \in S$.
- A real number x is the *least upper bound* (or supremum) for S if x is an upper bound for S and $x \leq y$ for every upper bound y of S.

The *least-upper-bound property* states that any non-empty set of real numbers that has an upper bound must have a least upper bound in real numbers.

Using above definition or otherwise, prove that the set

$$A = \{x \in \mathbb{Q} \mid x^2 < 5\}$$

does not have a least upper bound in *rational numbers* (i.e. $\mathbb{Q}$).

Problem 4

Problems $1.3.6, 1.3.9$ and $1.3.25, 1.3.30$ from Text.

Problem 5 (Test about Factorials and factorizations)

This problem will not be graded. Solving this will earn you a candy!

Is $4^{(4!)!} + 1$ a prime number? Justify your answer.

Here $n!$ denotes n factorial, defined as

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Thus in particular, $1! = 1, 2! = 2, 3! = 6, 4! = 24$ and so on.