CAAP 2018

Quiz 3

JULY 19, 2018
SUBHADIP CHOWDHURY

Full Name:

1. Determine which of the following statements are true or false. Write T/F in the box accordingly. **If the statement is False, give a counterexample. If it's true, no explanation is necessary.**

   (a) If **P** is the statement

   There exists a natural number n such that π^n is rational,

   then **not P** is the statement

   π^n is irrational for all natural numbers n. [2]

   (b) Given natural numbers a and b, if $6 \mid ab$, then either $6 \mid a$ or $6 \mid b$. [2]

   (c) For all real numbers x, we have [2]

   $$\lfloor x+y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$$

   (d) Every composite number n must have at least two distinct positive factors other than 1 and n. [2]

   (e) Given natural numbers a, b, and c, if $a \mid b$ and $a \mid c$, then $(a,b) = (a,c)$. [2]

2. Prove by induction that the following identity holds for all natural numbers n. [10]

$$\frac{1}{2\times 5} + \frac{1}{5\times 8} + \frac{1}{8\times 11} + \cdots + \frac{1}{(3n-1)\times(3n+2)} = \frac{n}{2(3n+2)}$$

3. Prove that two consecutive odd numbers are always relatively prime to each other. [7]

   [HINT: How do you write odd numbers? Consecutive odd numbers? You might need to use the identity $(a+bc, b) = (a,b)$.]

4. Consider the following Arithmetic Progression. [8]

$$24, 21, 18, 15, \ldots$$

   If sum of the first n terms of this AP is 105, then find n.

   [Extra Credit, 1 pt.] Can you explain why there are two possible values of n?