

# PROOF-BASED METHODS IN CALCULUS

CAAP SUMMER ACADEMY 2018

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<b>Time:</b>	MTRF 11–12:30	<b>Classroom:</b>	RY 277

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## Course Webpages

1. For announcement and grades, check Canvas (<http://canvas.uchicago.edu/>)
2. For assignment and course policies, check  
<https://subhadipchowdhury.github.io/teaching/Summer2018.CAAP>

## Office Hours

TR 7-9PM or by appointment, or email me your questions.

## Textbooks:

We will be using materials mainly from the following two textbooks. I will also provide a summary of topics covered in each class, with appropriate references, at the beginning of assignments.

- *Number, Shape, & Symmetry*, 1<sup>st</sup> edition, by Herrmann and Sally, Jr.
- *Calculus - One Variable*, 3<sup>rd</sup> edition, by Salas, Hille, Etgen

## Course Objectives

This course intends to acclimate incoming students to the sophistication of UChicago math courses, and to give them a head start on the harder topics in first-year calculus.

## Homework Policy

- Homework is to be turned in on the given date at the **BEGINNING** of the class. Usually an assignment will be set in every class. Homework sets will be due **TWICE** a week.
- **Thursday and Friday's assignments are due on Tuesday next week. Monday and Tuesday's assignments are due on Friday (same week).**
- The point of the homework is for you to work out what you do and don't understand. You should help each other to understand things and come and ask me if all of you get stuck together. When your graded homework has been handed back to you, you should go through it and see if you understand what has been written on it by the TA. If you don't, you should come to office hours and ask.
- I encourage you to work together on homework, but you should make sure you understand what you have written down. If you write up what was done in a group without understanding it, that counts as cheating.

## Participation

Student participation is an integral part of this class and is highly valued. Everyone is expected to make thoughtful contributions in the form of questions, statements, and reasoned arguments. You might be also occasionally invited to present something on the board. Please express yourself within the bounds of courtesy and respect. Please share your thoughts and be willing to listen attentively to perspectives that may differ from your own.

## Class Project

In the second half of this course, you will be working a class project. This project is open-ended, and you can choose any topic that interests you. I will post a list of project ideas sometime around the middle of July. Possible projects include parsing the solution strategy of an interesting puzzle, applying mathematical ideas to model a real life example, or understanding the multistep proof of a complicated result etc. You are also encouraged to come up with ideas on your own. I think it is a very good idea to work in groups of two, but you may choose not to do so. The project has two components - an in-class Presentation and a written Final Paper. More guidelines and details on these will be posted soon.

## Grading Policy

Apart from homework, and the class project, there will be three in-class quizzes and a final exam. The individual weights are as follows:

Participation .....	5%
Homework assignments .....	15%
Quizzes .....	15%
Class Project .....	25%
Final exam .....	40%

## Important Dates

Quiz # 1 .....	Friday, July 6, 2018
Quiz # 2 .....	Friday, July 13, 2018
Quiz # 3 .....	Thursday, July 19, 2018
Project Sign-up .....	Friday, July 20, 2018
Project Outline .....	Sunday, July 22, 2018
Progress Meetings .....	Thursday & Friday, July 26-27, 2018
In class Presentations .....	Thursday & Friday, August 2-3, 2018
Project Report Due .....	Sunday, August 5, 2018
Final Exam .....	Tuesday, August 7, 2015

## Class Policy

- Regular attendance is essential and expected.
- Be courteous when using mobile devices. Make sure your cell phone is turned fully off, or silent. If you must make or receive a call, please go outside the classroom.
- Use of computers is permitted for note-taking (with prior permission). Please turn off your Wi-fi and sound.
- The final exam is based on all material covered in class. If you have to miss a lecture, then I strongly recommend you study the material you missed before you return to class. I recommend doing the following steps:
  - Look at the course schedule written below.
  - Find someone who was in class and make a copy of their notes,
  - Read the relevant sections from the textbooks, class note, Wikipedia, etc.

Once you have done these steps, and you need more clarification on lectures you missed, email me to schedule an appointment to review the materials.

- The class on Friday July 20 will be spent discussing possible projects for presentation at the end of the course.

- For any communication regarding this course, please email me from your uchicago email address. This is mainly for identity verification purposes.

### Miscellaneous Items of Interest

- This course is open to all students who meet the academic requirements for participation. Any student who has documented a need for accommodation should contact Student Disability Services and the faculty member or instructor privately to discuss the specific situation as soon as possible. Student Disability Services can be reached at (773) 702-6000 or [disabilities@uchicago.edu](mailto:disabilities@uchicago.edu). SDS staff will coordinate accommodations for students.
- At times, you may experience some academic challenges, and you may find it helpful to utilize some of the services provided by the Student Services of The College. Various professional staff and advanced students are committed to helping you address your academic challenges in a variety of ways. Visit their [webpage](#) to learn about tutoring and mentoring options (low-cost, individual, free group, and drop-in), guidance on study skills and time management, and one-on-one assistance to ensure that you are using the best possible strategies for success in your course work.

### Tentative Course Outline

We will discuss the ‘Main Topics’ in class using the ‘Illustrative Examples’. The ‘Workout’ problems will be covered if time permits and will be assigned as homework or reading topics otherwise.

Time	Description
June 28,29 [RF]	<p>WARM-UP PROBLEMS</p> <p><b>Illustrative Examples:</b></p> <ul style="list-style-type: none"> <li>• The Pigeonhole Principle - Birthday Problem</li> <li>• The Invariance Principle - Königsberg Bridge Problem</li> </ul> <p><b>Weekend Workout:</b> Friends and Strangers: <math>R(3, 3) = 6</math></p>
July 2,3 [MT]	<p>SET THEORY AND LOGIC</p> <p><b>Main Topics:</b></p> <ul style="list-style-type: none"> <li>• Number Sets - <math>\mathbb{N}</math>, <math>\mathbb{Z}</math>, <math>\mathbb{Q}</math>, and <math>\mathbb{R}</math>.</li> <li>• Logical Comparisons using Examples - Implication, Inverse, Converse, Contrapositive, and Negation. Necessary vs. Sufficient.</li> <li>• Proof techniques and strategies</li> </ul>

**Illustrative Examples:**

- Proof by Contradiction: The Extremal Principle - set of midpoints (biggest),  $n\sqrt{2}$  is not an integer (smallest).
- The Induction Principle -  $1 + 2 + \cdots + n, \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$

**Wednesday Workout:** True/False logic quiz, Knights and knaves puzzle, NYT confirmation bias problem, The  $0 = 1$  fallacy

July 5,6,9 [RFM]

DIVISIBILITY AND PRIMES

**Main Topics:**

- Definition and fundamental properties of the Divisibility Relation
- Greatest Common Divisor
- Prime and Composite numbers
- Fundamental Theorem of Arithmetic (statement only)

**Illustrative Examples:**

- The infinitude of primes
- Set of triple primes
- The greatest integer function  $\lfloor \cdot \rfloor$  - multiplicity of a prime divisor
- $\sqrt{2}$  is irrational.

**Weekend Workout:** Sophie Germain Identity, Divisibility tests

July 10,12 [TR]

SEQUENCES

**Main Topics:**

- Definition - Explicit vs. Recursive
- Arithmetic and Geometric Progression
- Solving Linear Recurrence

**Illustrative Examples:**

- Binet's Formula

**Wednesday Workout:** Periodic Sequences

July 12,13 [RF]

BASIC ENUMERATIVE COMBINATORICS

**Main Topics:**

- Counting!
- Binomial Theorem

**Illustrative Examples:**

- Pascal's Triangle and Binomial coefficients

**Weekend Workout:** Erdős-Szekeres theorem

July 16-19 [MTR] PRECALCULUS REVIEW

**Main Topics:**

- Fractions and Decimals
- Absolute value and Inequalities
- Factorization of quadratic (and higher degree) polynomials
- Trigonometry

**Illustrative Examples:**

- Decimal Expansion
- Triangle Inequality
- Sketching Region of Solution
- Triangle Problems

**Wednesday Workout:** Different Bases, Coordinate Geometry

July 20 [F] DISCUSSING CLASS PROJECT IDEAS

July 23,24 [MT] FUNCTIONS AND GRAPHS

**Main Topics:**

- Functions as Mappings
- Bijective Functions and Inverses
- Graphs of Functions

**Illustrative Examples:**

- Linear, Quadratic, Rational, Trigonometric, Exponential, and Logarithmic Functions.
- Dirichlet function and Thomae's function

**Wednesday Workout:** Properties of Transcendental functions

July 26,27 [RF] INTRODUCTION TO LIMITS

**Main Topics:**

- $\epsilon - \delta$  definition
- Arithmetic of Limits
- One-sided Limit

**Illustrative Examples:**

- Finding Limit from Graphs
- Linear and Quadratic functions

July 30,31 [MT] MORE ON LIMITS AND CONTINUITY

**Main Topics:**

- Definition of Continuity
- Types of Discontinuity

August 2,3 [RF]      PROJECT PRESENTATIONS

August 6 [M]      REVIEW

August 7 [T]      FINAL EXAM