Lab 2: Interpolation

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- Sign on to an iMac with your username and password.
- When you open *Mathematica*, click **New Document** and a blank screen will appear. This is known as a notebook. If you need to open a new notebook, go to **File**, choose **New** and choose **Notebook** or simply hit `Command + N` (or `Ctrl + N` for **Windows**).
- Make the Untitled window bigger if necessary by dragging the lower right corner. Choose at least **125%** from the lower left of the Untitled window for a comfortable viewing size.
- Do not forget to **Save** the notebook periodically. Save new notebooks on the **Desktop**. The **Save** command is under the **File** menu. Give your file a name in the following format: `Lab2_Name1_Name2.nb`, where Name1 & Name2 are your last names.
- Follow the instructions in the paper copy of the handout.
- Write down the answers to the ? marked problems in the blue book provided.
- At the end of Lab session, save and quit Mathematica. **Do not delete your file or any calculation you did.** Then log off or restart the computer. Do NOT click **Shut Down**.

Exercise 1: Interpolation - Sinusoidal curve

Given a single-variable function $f(x)$, *interpolation* is the process of using known values $f(x_0), f(x_1), f(x_2), \ldots, f(x_n)$ to find (approximate) values for $f(x)$ at points x other than $x_i, i = 0, 1, 2, \ldots, n$.

In a Physics lab, we are trying to determine the amplitude and period of a radio wave. We know that the wave function $F(x)$ is of the form

$$F(x) = A\sin(Bx + C\pi/6).$$

We have done some experimental observations and obtained the following data points as a a result.

Our goal in the following problems is to estimate the constants A, B, and C.

1. Go to **Blackboard -> Lab handouts -> Data Files for Lab 2** and copy the contents of the `data1.rtf` file. Paste that as input in Mathematica. Your input should look like

   ```
   data1={{-5,-1.36394},{-4.5,-2.98536},...,{5,-2.996}}
   ```

   This makes a list named `data1` out of above data points.

2. Use the `ListPlot` command to plot `data1`. Give this plot a name, e.g. `plot1`.

   [HINT: Type `plot1=ListPlot[data1]`]

3. Use the command

   ```
   F:=Interpolation[data1]
   ```

   to create an interpolating function F that fits the data points.

4. ? Find $F(4\pi/3)$. [HINT: Type `F[4*Pi/3]`]

5. Next plot the function $F(x)$ for $-5 \le x \le 5$. Give this plot a name, e.g. `plot2`.

   [HINT: Type `plot2=Plot[F[x],{x,-5,5}]`]

6. Show `plot1` and `plot2` together in the same picture. Make sure they overlap.

   [HINT: Use the `Show` command]

7. Next we are going to change the labels on the X-axis in `plot2` to multiples of $\pi/6$. We are hoping that the period is a rational multiple of π that can be approximated from the picture.

   Add the option

   ```
   Ticks -> {Range[-2 Pi, 2 Pi, Pi/6], Automatic}
   ```

to the Plot command in plot2. To add an option you insert above line after a comma ',' before the ending].

[HINT: Type `plot2=Plot[F[x],{x,-5,5}, Ticks -> {Range[-2 Pi, 2 Pi, Pi/6], Automatic}]]`

Here we are telling Mathematica that the ticks in X-axis will happen on a range of -2π to 2π with regular interval of $\pi/6$. The ticks in Y-axis will be automatic, i.e. at regular interval of 1 unit.

[HINT: Your graph should look like figure 1. It ranges from -3 to 3, crosses X-axis at $\pi/3$ and $5\pi/6$ etc.]

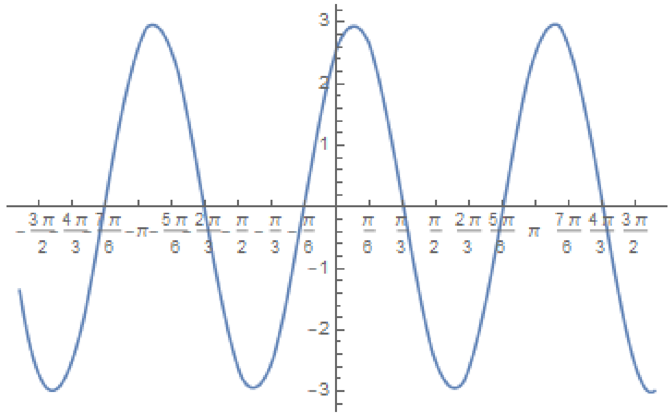

Figure 1

8. (?) Recall that the function is known to be of the form

$$F(x) = A\sin\left(Bx + \frac{C\pi}{6}\right)$$

What is the period and amplitude of the graph you found above?

9. (?) Assuming $A, B,$ and C are integers, estimate their values from the graph.

Exercise 2: Interpolation - Polynomial

We are going to repeat the above exercise with the data2.rtf file from Blackboard. This time you are told that the values come from a polynomial of degree 4.

10. Look at the list of values more closely to find out the possible values of the roots. Confirm your guess by plotting the interpolated function.

11. (?) Find an approximate possible formula for the polynomial.

[HINT: It has three roots, one negative and two positive. One of the positive roots has even degree.]