# Assignment 7 (4/13)

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## Problem 1

Find a formula $f(n) = \ldots\ldots$, (i.e. not a recursive definition) for each of the following sequences:

(a) $-1, \frac{2}{3}, -\frac{3}{5}, \frac{4}{7}, -\frac{5}{9}, \cdots$

(b) $-\frac{1}{3}, \frac{4}{9}, -\frac{9}{27}, \frac{16}{81}, \cdots$

(c) $f(n) = f(n-1) + 3n^2 + 3n + 1$, $f(1) = 8$

(d) $f(n+1) = 3f(n)$, $f(1) = 5$

(e) $2, 0, \frac{1}{2}, 0, 2, 0, \frac{1}{2}, 0, \cdots$

(f) $f(n) = -\frac{1}{f(n-1)}$, $f(1) = 1$

## Problem 2

Prove that the sequence defined by $a_n = \frac{n^3}{3^n}$ is a decreasing sequence.

## Problem 3

In problem 9.1.53, find $A_n$ and $B_n$ as a function of $n$.

## Problem 4

Give an $\epsilon - N$ proof of the fact that $\lim\limits_{n\to\infty} \frac{1}{n^p} = 0$ for any positive integer $p$.

## Problem 5

Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $\lim\limits_{x\to\infty} f(x)$ exists i.e. the graph of $f$ has an asymptote. Define a sequence $\{a_n\}_{n\geq 1}$ by $a_n = f(n)$. Does this sequence converge?

Conversely, let $f : \mathbb{R} \to \mathbb{R}$ be a function such that the sequence $f(1), f(2), f(3), f(4), \cdots$ converges. Does $\lim\limits_{x\to\infty} f(x)$ necessarily exist?