# **Assignment 17** (11/6)

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*This homework is due at the beginning of class on Friday 11/10. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.*

*You are encouraged to think about the problems marked with a* ($\star$) *if you have time, but you don't need to hand them in.*

Remember that you can always use the result of the previous assignment problems without proof to solve the new assignment problems.

## Problem 0$\star$

Over this week we will be covering chapter 15. Try to read the corresponding sections from book everyday after class. Once you have solved a homework problem, look up the nearby exercises to understand how else a similar problem could be formulated. Also look through all the examples in the section.

Midterm 2 is next week Wednesday.

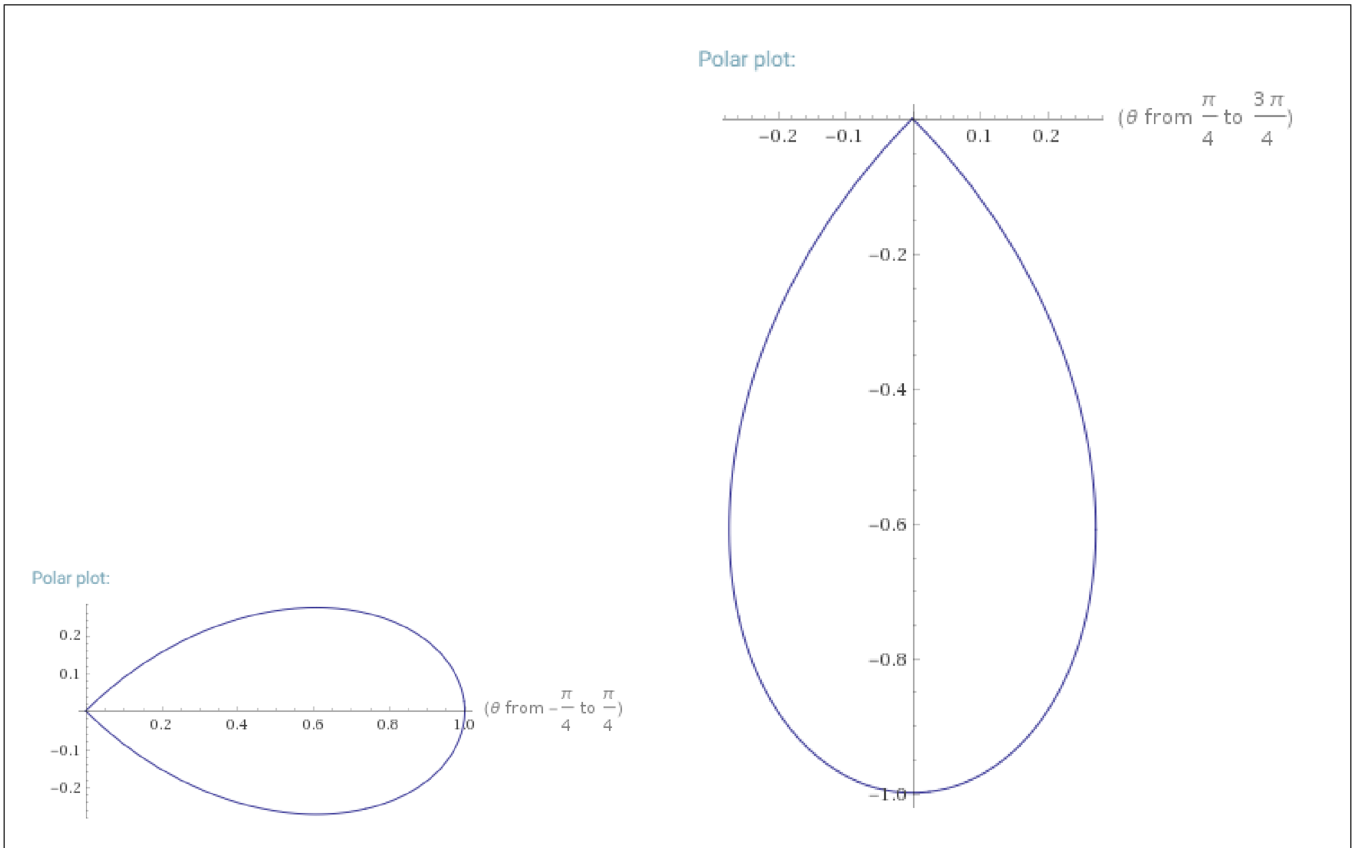
## Problem 1$\star$

### An explanation of the picture for curve $r = \cos(2\theta)$

When plotting any curve of the form $r = f(\theta)$, we regard $r$ as a 'signed' radius. That means if for some value of $\theta$, the value of $r$ is negative, we go across origin and take the diametrically opposite point.

To see this in the example $r = \cos(2\theta)$, consider the case when $\theta$ is in $[-\pi/4, \pi/4]$. This gives the right petal (of the 4 petals that we drew in class) in the figure below as expected.

On the other hand, when $\theta \in [\pi/4, 3\pi/4]$, we get the *bottom* petal! So for example, when $\theta = \pi/3$, $\cos(2\theta) = \cos(2\pi/3) = -1/2$, so we plot the diametrically opposite point $(r, \theta) = (1/2, \pi/2 + 2\pi/3) = (1/2, 7\pi/6)$.




(a) (b)

Figure 1

Also compare the plot when $\theta \in [0, \pi/2]$.

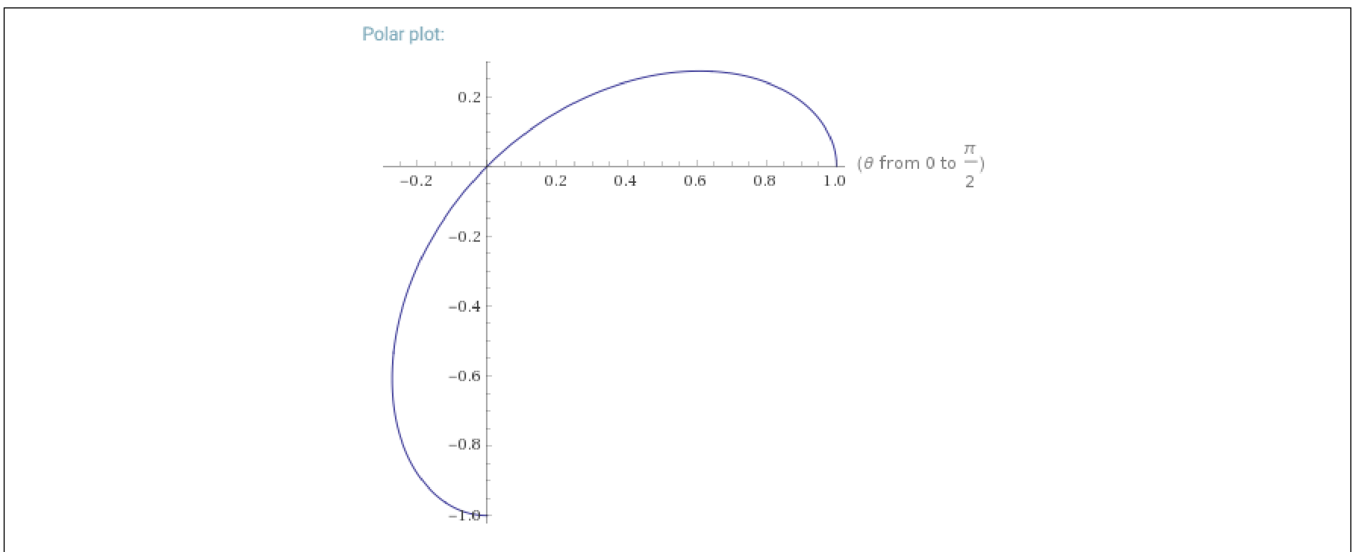



Figure 2

In other words we treat a point like $(f(\theta), \theta)$ as $(-f(\theta), \theta + \pi/2)$ when $f(\theta) \leq 0$. However, this is for plotting purpose only. While doing the integral, this is automatically taken care of. So for example, to calculate the total area of the two petals above, we only need to integrate over $\theta \in [-\pi/4, 3\pi/4$.

$$\int_{-\pi/4}^{3\pi/4} \int_{0}^{\cos(2\theta)} r \, dr \, d\theta = \pi/4$$

Similarly,

$$\int_0^{2\pi}\int_0^{\cos(2\theta)} r\,dr\,d\theta = \pi/2$$

## Problem 2

Problems 15.3.(6, 15.16, 18). Use a computer to find out what a cardioid looks like.

For more examples look HERE.

## Problem 3

Problems 15.3.(10, 13, 25).