Assignment 2

STAT 230
UAEU

There are a total of 9 problems. You may review **Unit3 slides** while answering the questions. **Show your steps to get entire credit for your solutions.**

1. Let A, B, C be three events. Use the basic properties of probabilities to show that
   (a) (2.5 points) $P(A \cup B) \leq P(A) + P(B)$
   (b) (2.5 points) $P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$
   (c) (2.5 points) $P(A \cap B) \geq P(A) + P(B) - 1$
   (d) (2.5 points) $P(A \cap B \cap C) \geq P(A) + P(B) + P(C) - 2$

2. A car repair can be performed either on time or late and either satisfactorily or unsatisfactorily. The probability of a repair being on time and satisfactory is 0.30. The probability of a repair being on time is 0.75. The probability of a repair being satisfactory however it is performed late is 0.20.
   (a) (5 points) What is the probability of a repair being late and unsatisfactory?
   (b) (5 points) What is the probability that the car repair is unsatisfactory?

3. (5 points) **Three** cards are selected randomly **with replacement** from the deck of 52 cards. What is the probability that all the cards are 'Spades' (♠)?

4. (5 points) **Two** cards are selected randomly **without replacement** from the deck of 52 cards. What is the probability that none of them is an 'Ace'?

5. A pair of fair dice is rolled.
   (a) (5 points) What is the probability that the sum of the numbers displayed by the upturned faces is 5?
   (b) (5 points) What is the probability that the second die lands on a higher value than does the first?

6. A typical ATM pin consists of 4 digits. Assume that all the integers between 0 to 9 are equally likely for selecting each of the digits. Find the probability of the following events.

   (a) (3 points) What is the total number of all possible different ATM PINs that a user can choose?

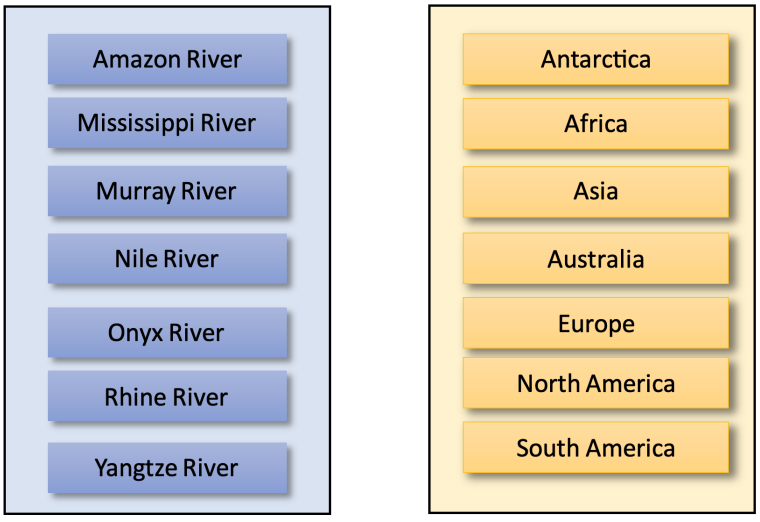
   (b) (4+3 points) The bank has selected a group of 150 ATM users randomly. What is the probability that **atleast two of the users** have exact same ATM pin? *Assume that all the possible four digit numbers are equally likely to be considered as a PIN of a randomly selected user. Also, compute the probability upto 3 decimal places.*

   Comment: 4 points to compute the probability in terms of the notations, 3 points to compute the exact probability value.

7. (5 points) A forest contains 50 tigers, of which 10 are captured, tagged, and then released. A certain time later, 6 of the 20 tigers are captured. What is the probability that 3 of these 6 have been tagged?

8. (5 points) If there are 7 strangers in a room, what is the probability that no two of them celebrate their birthday in the same month?

9. Consider the following problem of matching a River's name listed in left-side column with its corresponding continents's name listed in the right side column. Note that, there is exactly one match for each of the rivers name.



A kid has to draw lines to match the river's names to the corresponding (correct) continent's name. If a kid have randomly chosen the continent's names to match a corresponding river's name (also, the kid made sure one river's name in the left column is matched with only continent's name in the right column) , what is the probability of the following events.

(a) (5 points) What is the probability that **none** of the answers are correct?

(b) (5 points) What is the probability that **exactly three** of the answers are correct?