Assignment 1

STAT 230
UAEU

There are a total of 7 problems. You may review Unit1 and Unit2 slides while answering the questions. Show your steps to get entire credit for your solutions.

1. Let A, B, C be three events. Use the basic properties of probabilities to show that
   (a) (2.5 points) $P(A \cup B) \leq P(A) + P(B)$
   (b) (2.5 points) $P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$
   (c) (2.5 points) $P(A \cap B) \geq P(A) + P(B) - 1$
   (d) (2.5 points) $P(A \cap B \cap C) \geq P(A) + P(B) + P(C) - 2$

2. (10 points) A car repair can be performed either on time or late and either satisfactorily or unsatisfactorily. The probability of a repair being on time and satisfactory is 0.26. The probability of a repair being on time is 0.74. The probability of a repair being satisfactory is 0.41. What is the probability of a repair being late and unsatisfactory?

3. (5 points)Three cards are selected randomly **with replacement** from the deck of 52 cards. What is the probability that all the cards are 'Spades' (♠)?

4. (5 points)Three cards are selected randomly **without replacement** from the deck of 52 cards. What is the probability that none of them is an 'Ace'?

5. A pair of fair dice is rolled.
   (a) (5 points) What is the probability that the sum of the numbers displayed by the upturned faces is 5?
   (b) (5 points) What is the probability that the second die lands on a higher value than does the first?

6. A typical ATM pin consists of 4 digits. Assume that all the integers between 0 to 9 are equally likely for selecting each of the digits. Find the probability of the following events.
   (a) (3 points) What is the total number of all possible different ATM PINs that a user can choose?
   (b) (4+3 points) The bank has selected a group of 150 ATM users randomly. What is the probability that **atleast two of the users** have exact same ATM pin? *Assume that all the possible four digit numbers are equally likely to be considered as a PIN of a randomly selected user. Also, compute the probability upto 3 decimal places.*

   Comment: 4 points to compute the probability in terms of the notations, 3 points to compute the exact probability value.

7. (5 points) A forest contains 50 tigers, of which 10 are captured, tagged, and then released. A certain time later, 6 of the 20 tigers are captured. What is the probability that 3 of these 6 have been tagged?

8. (5 points) If there are 7 strangers in a room, what is the probability that no two of them celebrate their birthday in the same month?

9. Suppose that there is a practice session for a UAE football (soccer) team. In that session all the 11 players randomly picked up a jersey from a basket where exactly 11 jerseys were kept. Note that, all the jersey's have the players name written on it, therefore in the basket there were exactly one jersey that would match with a specific player.
   (a) (5 points) What is the probability that **none** of the players have picked up the jersey that would match their name?
   (b) (5 points) What is the probability that **exactly three** of the players have picked up the jersey that would match their name?