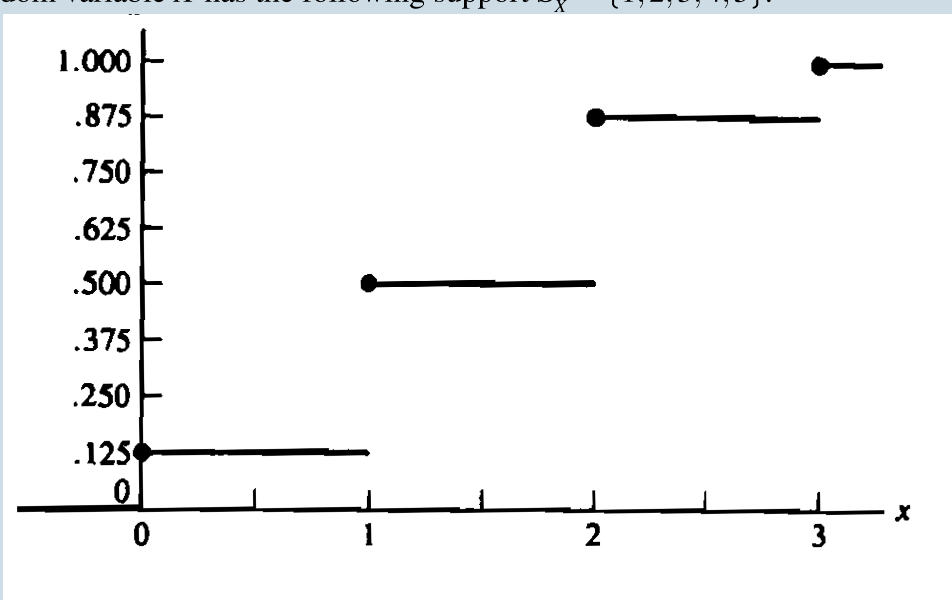
A Few Problems Aiming the Final Exam

1. If $X \sim \text{Binomial}(n = 50, \pi = 0.1)$ then
   (a) Obtain the value of $E(X)$, $\text{Var}(X)$, and $E(X^2)$.
   (b) What is the MGF of X?

2. If $X \sim \text{Geometric}(\pi = 0.2)$ then
   (a) Obtain the value of $E(X)$, $\text{Var}(X)$, and $E(X^2)$.
   (b) What is $E(4X + 10)$?
   (c) What is $\text{Var}(4X + 10)$?
   (d) What is the MGF of X?

3. If $X \sim \text{Poisson}(\lambda = 5)$, then
   (a) Obtain the value of $E(X)$, $\text{Var}(X)$, and $E(X^2)$
   (b) What is $E(3X + 50)$?
   (c) What is $\text{Var}(3X + 50)$?
   (d) What is the MGF of X?

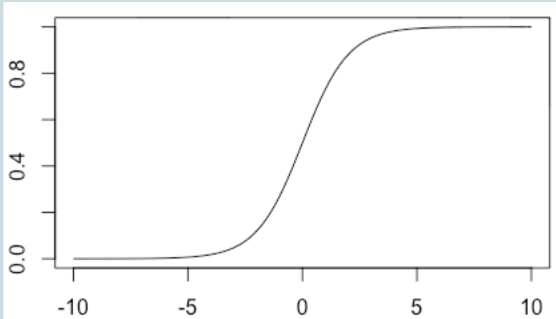
4. Suppose a random variable X has the following support $\mathbb{S}_X = \{1, 2, 3, 4, 5\}$.



   (a) What is the probability that $X = 2$?
   (b) What is the probability that $X = 1.5$
   (c) Obtain $P(1 < X \leq 3)$
   (d) Obtain $P(1 \geq X < 3)$

5. Consider the following CDF of the random variable

$$F_X(x) := \frac{1}{1+e^{-x}} \text{ for all } x \in \mathbb{R}.$$



(a) What is the probability that $X = 0$?
(b) What is the probability that $X = 2$
(c) Obtain $P(0 < X \leq 1)$
(d) Obtain $P(0 \geq X < 2)$
(e) Identify the nature of the random variable (Discrete/Continuous/ Mixture of Discrete and Continuous.)

6. Example : Suppose that X is a continuous random variable whose probability density function is given by

$$f(x) := \begin{cases} C(4x - 2x^2) & \text{if } 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) For what value of C the provided function is a valid probability density function?
(b) Find $P(X > 1)$.
(c) Find $P(X \leq 1)$.
(d) Obtain *mean* (Expected value) of X.

7. Example : For a given IT technician in a support center, let X denote the percentage of time, out of a 40-hour work week, that he is directly serving customers. Suppose that X has a probability density function given by

$$f(x) := \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the probability that the technician will spend 20% to 70% of hisworkweek serving customers.
(b) Obtain, $F(x)$, the CDF of X.
(c) Use $F(x)$ to compute $P(0.5 < X \leq 0.8)$.
(d) Obtain *mean* (Expected value) of X.
(e) find the *median* and First Quartile (Q_1) of the distribution

8. **Example :** The lifetime in hours of a certain kind of radio tube is a random variable having a probability density function given by

$$f(x) := \begin{cases} \frac{100}{x^2} & \text{if } x > 100 \\ 0 & \text{if } x \leq 100. \end{cases}$$

(a) What is the probability a randomly selected tube in a radio set will have to be replaced within the fist 150 hours of operation?
(b) Obtain the CDF function of the distribution
(c) Obtain *median* lifetime of a randomly selected radio tube.
(d) Does the *mean* / Expected value of the distribution exist?

9. **Example :** Find $E(e^X)$ and the Moment Generating Function for the continuous random variable with probability density function

$$f(x) := \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

10. **Example :** Let X denote the resistance of a randomly chosen resistor, and suppose that its pdf is given by

$$f(x) := \begin{cases} \frac{x}{18} & \text{if } 8 \leq x \leq 10 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $P(8.6 < X \leq 9.8)$.
(b) Find the median of the resistance of such resistors.
(c) Find the mean and variance of X.

11. **Example :** The length of time required by students to complete a one-hour exam is a random variable with a density function given by

$$f(x) := \begin{cases} cy^2 + y & \text{for } 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find c that makes this function a valid probability density function.
(b) Find the F(y)
(c) Find the probability that a randomly selected student will finish in less than half an hour.
(d) Find the time that 95% of the students finish before it.
(e) Given that a particular student needs at least 15 minutes to complete the exam, find the probability that she will require at least 30 minutes to finish.