

STAT 320: Principles of Probability

Unit 1: A few Definitions and Notations from Set Theory

United Arab Emirates University

Department of Statistics

Outline


- 1 Definition of Sets
- 2 Relationship Between Sets
- 3 A Few Set Operations
- 4 Venn Diagrams
- 5 Disjoint Sets and Partition


Definition of a Set


Definition of a Set


Definition (Set)


*A Set is a well defined collection of **objects**.*

 **Elements:** The objects inside a set are called the **elements** of the Set

 Usually we denote sets with upper-case letters, elements with lower-case letters.

 The following notation is used to show a set membership.

 $x \in A$:means that x is a member of the set A .

 $x \notin A$:means that x is NOT a member of the set A .

Example

Example: Different Ways of Describing a Set

- 1 Listing all the elements: $A = \{1, 2, 3, 4, 5, 6\}$
- 2 Give a verbal description: A is the set of all integers from 1 to 6, inclusive.
- 3 Give a mathematical inclusion rule:

$$A = \{x \text{ is an Integer} : 1 \leq x \leq 6\}$$

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The Null Sets and the Universal Set

Definition (Null Set)

*The **Null Set** (or also called the **Empty Set**) is a set with no elements in it.*

It is often denoted by \emptyset

Definition (Universal Set)

*The **Universal Set** is the set of all elements currently under consideration.*

It is often denoted by Ω

The universal set contains all of the elements relevant to a given discussion.

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Relationship Between Sets

 **Subset:** A set A is a subset of a set B all the elements of A are also the elements of B .

If $x \in A \implies x \in B$ then $A \subseteq B$.

Subsets: If A is a subset of B then we write $A \subseteq B$.

Note: The notation for subset is very similar to the notation for "less than or equal to" and means, in terms of the sets, "included in or equal to".

 We write $A \not\subseteq B$ if A is not included in B



We say "**A is a proper subset of B**" if all the elements of A are also the elements of B, but in addition there exists at least one element c such that $c \in B$ but $c \notin A$.



Proper Subset: $A \subset B$, then we say **A is a proper subset of B**.

Comment: The notation for subset is very similar to the notation for "less than", and means, in terms of the sets, "included in but not equal to".

Subset: Example

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Few Set Operations

Set Operation: Union

A Union B:

A Union B, denoted by $A \cup B$ is the set of all elements that are either in A , or in B , or inside both the sets.

The logical operator is OR.

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Set Operation: Intersection

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The logical operator is AND.

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The logical operator is **AND**.

Set Operation: Complement

A Complement : A Complement, denoted by \bar{A} is the set of all elements that are not in A . Sometimes it is also denoted by A^c

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The logical operator is **NOT**.

Set Operation: Set Difference

A Minus B:

A Minus B, denoted by $A - B$ is the set of all elements that are only an element of A , but not an element of the set B .

$A - B$ is same as: $A \cap \bar{B}$.

Comment: $A - B$ is not same as $B - A$

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A Notation

Tuple: Let k be an integer. A k – **tuple** is the a the ordered sequence of values often written inside parenthesis while different elements are separated by comma.

Example:

- $(1.5, 2)$ is a **2-tuple**.
- $(1.5, 2, 6, 2)$ is a **4-tuple**.
- (H, T, H, H, H) is a **5-tuple**

Set Operation: Cartesian Product

Cartesian Product of A and B:

Cartesian Product of A and B , denoted by $A \times B$ is the set of all “two-tuple” objects where the first element in the tuple is from the set A and the second element is taken from the set B .

$$A \times B = \{(x, y) : x \in A, y \in B\}.$$

Comment: In general $A \times B$ is not same as $B \times A$. However, $A \times A$ is denoted by A^2 . Note that A is a set and not a number.

Cartesian Product of Multiple Sets

Let A_1, A_2, \dots, A_n are non empty sets. Then their Cartesian product is defined as following:

$$A_1 \times A_2 \times \dots \times A_n = \{(a_1, a_2, \dots, a_n) : a_i \in A_i \text{ for } i = 1, \dots, n\}.$$

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A Few Rules for Various Set Operations

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$\overline{(A \cap B)} = (\bar{A} \cup \bar{B})$$


$$\overline{(A \cup B)} = (\bar{A} \cap \bar{B})$$

Set Operation: Examples

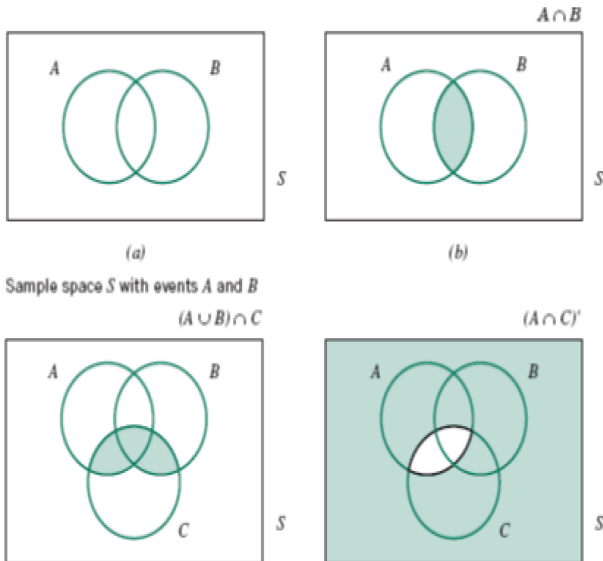
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Venn Diagrams

 **Venn Diagrams** are graphical representation of the sets that are typically used to depict the relation between various sets

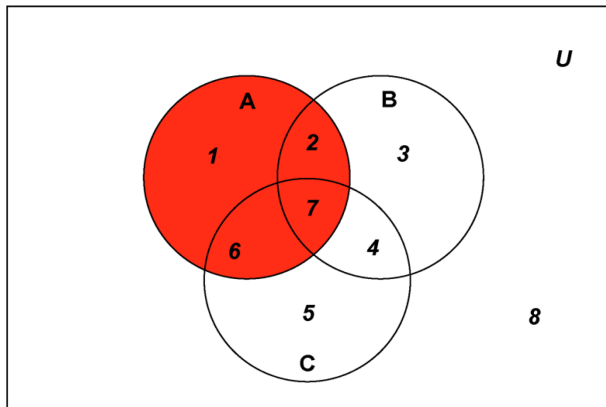
Venn Diagrams: Examples



Venn Diagrams: Examples

Let $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$,

$A = \{1, 2, 6, 7\}$, $B = \{2, 3, 4, 7\}$, and $C = \{4, 5, 6, 7\}$



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Disjoint Sets and Partition

Disjoint Sets

Disjoint Sets:

Two sets A , and B are said to be **Disjoint Sets** or **mutually exclusive sets** if A and B does not have any elements in common.

A and B are Disjoint $\Leftrightarrow A \cap B = \emptyset$.

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Exhaustive

Exhaustive:

Two sets A and B are exhaustive for the set C if $A \cup B = C$.

A and B are exhaustive for C $\Leftrightarrow A \cup B = C$.

Partition

Partition: A group of sets $\{A, B\}$ is called a **partition** for a set C if

1

$A \cap B = \emptyset$ (i.e. , A , and B are Disjoint) , and

2

$A \cup B = C$ (i.e. , A and B is exhaustive for C).

Examples

Examples

A Few Questions:

Let Ω be the universal set and \emptyset denotes the empty set.


- $A \cup \emptyset =$
- $A \cup \overline{A} =$
- $A \cap \emptyset =$
- $A - \overline{A} =$
- $A \cap \overline{A} =$
- $A \cup \Omega =$
- $A \cap \Omega =$
- If $A \subset B$ then $A \cap B =$
- If $A \subset B$ then $A \cup B =$


A Few Standard Notation


- \mathbb{Z} : Set of all Integers. i.e. $\mathbb{Z} = \{0, -1, 1, -2, 2, -3, 3, \dots\}$.
- \mathbb{Z}_+ : Set of all non-negative Integers. i.e. $\mathbb{Z}_+ = \{1, 2, 3, \dots\}$.
- \mathbb{R} : Set of all real numbers.
- \mathbb{R}_+ : Set of all positive real numbers.
- $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x_1, x_2) : x_1 \in \mathbb{R}, x_2 \in \mathbb{R}\}$
- $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x_1, x_2, x_3) : x_1 \in \mathbb{R}, x_2 \in \mathbb{R}, x_3 \in \mathbb{R}\}$
- $\mathbb{Z}_+^2 = \mathbb{Z}_+ \times \mathbb{Z}_+ = \{(z_1, z_2) : z_1 \in \mathbb{Z}_+, z_2 \in \mathbb{Z}_+\}$

Real Numbers, Intervals

For the definitions below, assume $a \in \mathbb{R}$, $b \in \mathbb{R}$, and $a < b$.

 $[a, b] := \{x \in \mathbb{R} : a \leq x \leq b\}$

 $(a, b] := \{x \in \mathbb{R} : a < x \leq b\}$

 $[a, b) := \{x \in \mathbb{R} : a \leq x < b\}$

 $(a, b) := \{x \in \mathbb{R} : a < x < b\}$

Examples of Intervals

$[0, 1] := \{x \in \mathbb{R} : 0 \leq x \leq 1\}$ = Set of all real numbers from 0 to 1, including the numbers 0 and 1.

$(0, 1) := \{x \in \mathbb{R} : 0 < x < 1\}$ = Set of all real numbers between 0 to 1. It does not include the numbers 0, and 1.

$(0, \infty) := \{x \in \mathbb{R} : 0 < x\}$ = Set of all positive real numbers. The interval **does not include 0**

$[0, \infty) := \{x \in \mathbb{R} : 0 \leq x\}$ = Set of all non-negative real numbers. The interval **does include 0**

Examples: Union & Intersection of Intervals

A Discussion on Cardinality of Various Sets

Questions?