STAT 320: Principles of Probability
Unit 1: A few Definitions and Notations from Set Theory

United Arab Emirates University

Department of Statistics

Outline

Definition of a Set

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Definition (Set)

A Set is a well defined collection of **objects**.

- **Elements:** The objects inside a set are called the **elements** of the Set

- Usually we denote sets with upper-case letters, elements with lower-case letters.

- The following notation is used to show a set membership.
  - $x \in A$:means that x is a member of the set A.
  - $x \notin A$:means that x is NOT a member of the set A.

Example

Example: Differnt Ways of Describing a Set

1. Listing all the elements: $A = \{1, 2, 3, 4, 5, 6\}$
2. Give a verbal description: A is the set of all integers from 1 to 6, inclusive.
3. Give a mathematical inclusion rule:

$$A = \{x \text{ is an Integer} : 1 \leq x \leq 6\}$$

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The Nulll Sets and the Universal Set

Definition (Null Set)

*The **Null Set** (or also called the **Empty Set**) is a set with no elements in it.*

It is often denoted by $\emptyset$

Definition (Universal Set)

*The **Universal Set** is the set of all elements currently under consideration.*

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Relationship Between Sets

Subset: A set A is a subset of a set B all the elements of A are also the elements of B.

If $x \in A \implies x \in B$ then $A \subseteq B$.

Subsets: If A is a subset of B then we write $A \subseteq B$.

Note: The notation for subset is very similar to the notation for "less than or equal to" and means, in terms of the sets, "included in or equal to".

We write $A \not\subseteq B$ if A is not included in B

- We say **"A is a proper subset of B"** if all the elements of A are also the elements of B, but in addition there exists at least one element c such that $c \in B$ but $c \notin A$.
- **Proper Subset:** $A \subset B$, then we say **A is a proper subset of B.**

Comment: The notation for subset is very similar to the notation for "less than", and means, in terms of the sets, "included in but not equal to".

Subset: Example

Outline

Few Set Operations

Set Operation: Union

A Union B: A Union B, denoted by $A \cup B$ is the set of all elements that are either in A, or in B, or inside both the sets.

The logical operator is OR.

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Set Operation: Set Difference

A Minus B: A Minus B, denoted by $A - B$ is the set of all elements that are only an element of A, but not an element of the set B.

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A Notation

Tuple: Let k be an integer. A $k-$ **tuple** is the a the ordered sequence of values often written inside parenthesis while different elements are separated by comma.

Example:

- $(1.5, 2)$ is a **2-tuple**.
- $(1.5, 2, 6, 2)$ is a **4-tuple**.
- (H, T, H, H, H) is a **5-tuple**

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Set Operation: Cartesian Product

Cartesian Product of A and B: Cartesian Product of A and B, denoted by $A \times B$ is the set of all "two-tuple" objects where the first element in the tuple is from the set A and the second element is taken from the set B.

$$A \times B = \{(x, y) : x \in A, y \in B\}.$$

Comment: In general $A \times B$ is not same as $B \times A$. However, $A \times A$ is denoted by A^2. Note that A is a set and not a number.

Cartesian Product of Multiple Sets Let A_1, A_2, ..., A_n are non empty sets. Then their Cartesian product is defined as following:

$$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) : a_i \in A_i \text{ for } i = 1, \ldots n\}.$$

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A Few Rules for Various Set Operations

$$A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C = A \cap B \cap C$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$\overline{(A \cap B)} = (\overline{A} \cup \overline{B})$$

$$\overline{(A \cup B)} = (\overline{A} \cap \overline{B})$$

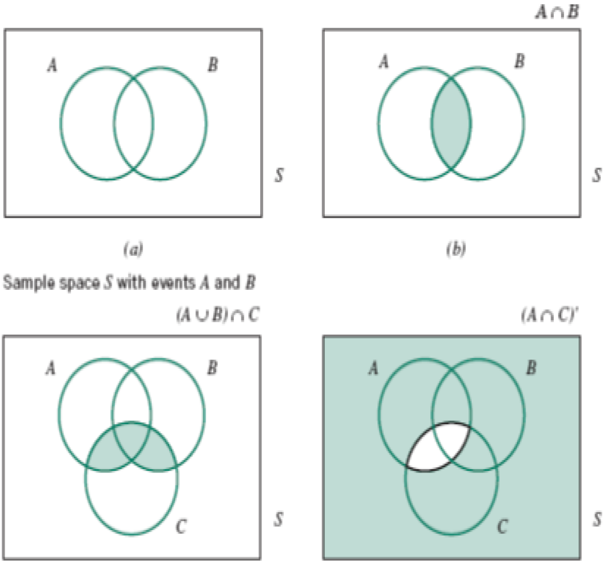
Set Operation: Examples

Outline

Venn Diagrams

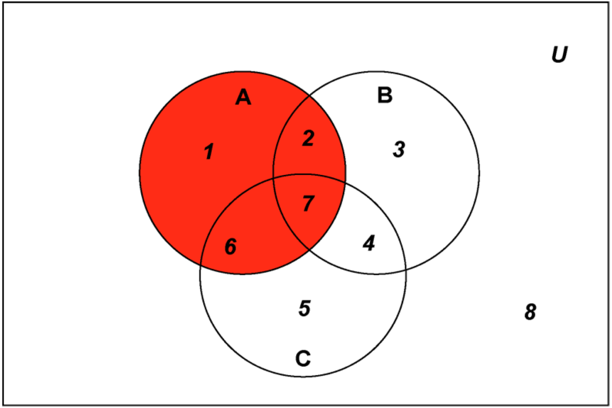
Venn Diagrams are graphical representation of the sets that are typically used to depict the relation between various sets

Venn Diagrams: Examples



Venn Diagrams: Examples

Let $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$,
$A = \{1, 2, 6, 7\}$, $B = \{2, 3, 4, 7\}$, and $C = \{4, 5, 6, 7\}$



Outline

Disjoint Sets and Partition

Disjoint Sets

Disjoint Sets: Two sets A, and B are said to be **Disjoint Sets** or **mutually exclusive sets** if A and B does not have any elements in common.

A and B are Disjoint $\Leftrightarrow$ $A \cap B = \emptyset$.

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Exhaustive

Exhaustive: Two sets A and B are exhaustive for the set C if $A \cup B = C$.

A and B are exhaustive for C $\Leftrightarrow$ $A \cup B = C$.

Partition

Partition: A group of sets $\{A, B\}$ is called a **partition for a set** C if

1. $A \cap B = \emptyset$ (i.e. ,A, and B are Disjoint) , and
2. $A \cup B = C$ (i.e. , A and B is exhaustive for C).

Examples

Examples

A Few Questions:

Let Ω be the universal set and $\emptyset$ denotes the emptyset.

- $A \cup \emptyset =$
- $A \cup \overline{A} =$
- $A \cap \emptyset =$
- $A - \overline{A} =$
- $A \cap \overline{A} =$
- $A \cup \Omega =$
- $A \cap \Omega =$
- If $A \subset B$ then $A \cap B =$
- If $A \subset B$ then $A \cup B =$

A Few Standard Notation

- $\mathbb{Z}$: Set of all Integers. i.e. $\mathbb{Z} = \{0, -1, 1, -2, 2, -3, 3, \ldots\}$.
- $\mathbb{Z}_+$: Set of all non-negative Integers. i.e. $\mathbb{Z}_+ = \{0, 1, 2, 3, \ldots\}$.
- $\mathbb{R}$: Set of all real numbers.
- $\mathbb{R}_+$: Set of all positive real numbers.
- $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x_1, x_2) : x_1 \in \mathbb{R}, x_2 \in \mathbb{R}\}$
- $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x_1, x_2, x_3) : x_1 \in \mathbb{R}, x_2 \in \mathbb{R}, x_3 \in \mathbb{R}\}$

Real Numbers, Intervals

For the definitions below, assume $a \in \mathbb{R}$, $b \in \mathbb{R}$, and $a < b$.

- $[a, b] := \{x \in \mathbb{R} : a \leq x \leq b\}$
- $(a, b] := \{x \in \mathbb{R} : a < x \leq b\}$
- $[a, b) := \{x \in \mathbb{R} : a \leq x < b\}$
- $(a, b) := \{x \in \mathbb{R} : a < x < b\}$

Examples of Intervals

$[0, 1] := \{x \in \mathbb{R} : 0 \leq x \leq 1\}$= Set of all real numbers from 0 to 1, including the numbers 0 and 1.

$(0, 1) := \{x \in \mathbb{R} : 0 < x < 1\}$=Set of all real numbers between 0 to 1. It does not include the numbers 0, and 1.

$(0, \infty) := \{x \in \mathbb{R} : 0 < x\}$=Set of all positive real numbers. The interval **does not include 0**

$[0, \infty) := \{x \in \mathbb{R} : 0 < x\}$=Set of all non-negative real numbers. The interval **does include 0**

Examples: Union & Intersection of Intervals

A Discussion on Cardinality of Various Sets

Questions?