STAT 320: Principles of Probability
Unit 1: Practice

United Arab Emirates University

Department of Statistics

Outline

A Few Practice Problems

What is the cartesian product between the sets $A = \{1, 2, 3\}$, and $B = \{H, T\}$,

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What is the cardinality of A and B? What is the cardinality of the corresponding cartesian product?

What is the cardinality of $\{H, T\}^5$, and $\{1, 2, 3, 4, 5, 6\}^3$

Let $\mathscr{S} = \{1, 2, 3, 4, 5, 6, 7, 8\}$,
$A = \{1, 2, 6, 7\}$, $B = \{2, 3, 4, 7\}$, and $C = \{4, 5, 6, 7\}$

Consider three tosses of a coin. What is the corresponding universal that provides the list of all possible results of the three tosses.

1. Let A be the set containig all the possibilities that corresponds to at least one Head. Write down the explicit description of A.
2. Let B be the set containig all the possibilities that corresponds to at least one Tail. Write down the explicit description of B.
3. Let D be the set containig all the possibilities that corresponds to none of them is Head.
4. What is the relation netween A and D

Consider two roll of a dice. What is the universal set in this context?

1. Let A : The set that at least one of the roll appears to be 6?
2. Let B : The set of outcomes when none of the roll is a 6?
3. What is $A \cap B =$?
4. What is $A \cup B =$?
5. What is $\overline{A}$?
6. What sbout $\overline{B} =$?
7. Is it true that $B = \overline{A}$
8. Can we say that A, B creates a partition of the universal set.

Let $\lambda > 0, a > 0$ be two constants, then $\sum_{x=0}^{\infty} \frac{a^{3x}\lambda^x}{x!} = ?$.

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Let $q > 0$ be such that $|q| < 1$, then $\sum_{x=0}^{\infty} x\, q^x = ?$

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Let $t \in \mathbb{R}0$ and, $q \in \mathbb{R}$ be such that $|q| < 1$, then $\sum_{x=0}^{\infty} e^{tx} \, q^x = ?$.

Binomial Series

Let n be a positive integer, then

$$1 + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-1} + \binom{n}{n} =$$

Let $a > 0$, and n be a positive integer, then

$$\sum_{x=0}^{n} x \binom{n}{x} a^x =$$

Let $t \in \mathbb{R}$, $a > 0$, and n be a positive integer, then

$$\sum_{x=0}^{n} e^{tx} \binom{n}{x} a^x =$$

$\Gamma(6) =$

$$\frac{\Gamma(8.7)}{\Gamma(6.7)} =$$

Gamma Function: Example

$$\int_0^\infty x^{\frac{5}{2}} e^{-x} dx =$$

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Let $\lambda > 0$, then $\int_0^\infty x^5 e^{-\lambda x} dx =$

$\mathscr{B}(3,2) =$

$$\int_0^1 x^2(1-x)^3 dx =$$

$$\int_0^1 x^{\frac{5}{2}}(1-x)^{\frac{1}{2}}dx =$$

$\Phi(x)$ Function, $x \in \mathbb{R}$

The Standard Normal CDF

The Standard Normal CDF (PHI function) $\Phi(x)$, $x \in \mathbb{R}$.

$$\Phi(x) := \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \quad \text{for all } x \in \mathbb{R}.$$

- $\Phi(0) = \frac{1}{2}$
- $\Phi(-x) + \Phi(x) = 1 \implies \Phi(-x) = 1 - \Phi(x)$ for all $x \in \mathbb{R}$
- $0 \leq \Phi(x) \leq 1$
- $\lim_{x \to -\infty} \Phi(x) = 0$, and $\lim_{x \to \infty} \Phi(x) = 1$

Discussion on Various Concepts

Log (function) Equation of Line, Circles

Log (function) Gamma, Beta, Phi function Equation of Line and Regions Circles and Regions

Questions?