

Assignment 3

STAT 230
UAEU

There are a total of 6 problems. You may review **Unit6 slides** while answering the questions. **Show your steps to get entire credit for your solutions.**

The time in days between breakdowns of a machine is exponentially distributed with rate parameter $\lambda = 0.2$

1.
 - (a) (5 points) Write down the probability density function of the random variable X that records time in days between breakdowns of the machine.
 - (b) (5 points) What is the expected time (in days) between machine breakdowns?
 - (c) (8 points) Let a, b be two positive constants such that $a > b$. Derive an expression for
$$P(X > a \mid X > b).$$
 - (d) (7 points) Use the part(c) to obtain the probability that the machine lasts at least two more days before breaking down **given** that the machine has already performed satisfactorily for last 15 days?

A new battery supposedly with a charge of 1.5 volts actually has a voltage with a Uniform distribution between 1.43 and 1.60 volts.

2.
 - (a) (5 points) What is the expected value of the voltage?
 - (b) (5 points) What is the standard deviation of the voltage?
 - (c) (10 points) What is the probability that a battery has a voltage less than 1.48 volts?

(10 points) Let X follows a Uniform(0,1) distribution. Let Y be a random variable such that

3.
$$Y = X^{0.5}(1 - X)^{1.5}.$$

Derive the value of $E(Y)$.

A Wall Street analyst estimates that the annual return from the stock of company A can be considered to be an observation from a normal distribution with mean $\mu = 8.0$ percent and standard deviation $\sigma = 1.5$ percent. The analyst's investment choices are based upon the considerations that any return greater than 5 percent is "satisfactory" and a return greater than 10 percent is "excellent."

4. (a) (5 points) What is the probability that company A's stock will prove to be "unsatisfactory" ?
- (b) (5 points) What is the probability that company A's stock will prove to be "Excellent" ?

Suppose a certain mechanical component produced by a company has a width that is normally distributed with a mean $\mu = 2600$ and a standard deviation $\sigma = 0.6$.

5. (a) (8 points) What proportion of the components have a width outside the range 2599 to 2601?
- (b) (7 points) If the company needs to be able to guarantee to its purchaser that no more than 1 in 1000 of the components have a width outside the range 2599 to 2601, then what should be the specification of (value of) σ ?

The velocities of gas particles, denoted here by V , can be modeled by the Maxwell distribution. For simplicity of notations, consider the random variable X , that refers to the square of the velocity, i.e. , $X = V^2$. It can be shown that the probability density function of X is given as

$$f_X(x) := \begin{cases} 4\pi \left(\frac{m}{\pi K T} \right)^{\frac{3}{2}} \sqrt{x} e^{-\left(\frac{m}{2 K T} \right) x} & \text{for } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

6. where m is the mass of the particle, K is Boltzmann's constant, and T is the absolute temperature. From the perspective of the probability distribution, assume m, K, T to be constants.

- (a) (5 points) Identify the probability distribution of the random variable X .
- (b) (8 points) The kinetic energy of a particle is given by $\frac{1}{2}mV^2$. Find the mean (expected value of the) kinetic energy for a particle.
- (c) (7 points) Derive the mean velocity (Expected Value of V) of these particles.