STAT 320: Principles of Probability
Unit 1: A few Definitions and Notations from Set Theory

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Department of Statistics

Outline

Definition of a Set

Definition of a Set

Definition (Set)

A Set is a well defined collection of **objects**.

- **Elements:** The objects inside a set are called the **elements** of the Set

- Usually we denote sets with upper-case letters, elements with lower-case letters.

- The following notation is used to show a set membership.
  - $x \in A$:means that x is a member of the set A.
  - $x \notin A$:means that x is NOT a member of the set A.

Example

Example: Differnt Ways of Describing a Set

1. Listing all the elements: $A = \{1, 2, 3, 4, 5, 6\}$
2. Give a verbal description: A is the set of all integers from 1 to 6, inclusive.
3. Give a mathematical inclusion rule:

$$A = \{x \text{ is an Integer} \mid 1 \leq x \leq 6\}$$

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The Nulll Sets and the Universal Set

Definition (Null Set)

The **Null Set** *(or also called the* **Empty Set***) is a set with no elements in it.*

It is often denoted by $\emptyset$

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The **Universal Set** *is the set of all elements currently under consideration.*

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Relationship Between Sets

Subset: A set A is a subset of a set B all the elements of A are also the elements of B.

If $x \in A \implies x \in B$ then $A \subseteq B$.

Subsets: If A is a subset of B then we write $A \subseteq B$.

Note: The notation for subset is very similar to the notation for "less than or equal to" and means, in terms of the sets, "included in or equal to".

We write $A \not\subseteq B$ if A is not included in B

- We say **"A is a proper subset of B"** if all the elements of A are also the elements of B, but in addition there exists at least one element c such that $c \in B$ but $c \notin A$.
- **Proper Subset:** $A \subset B$, then we say **A is a proper subset of B.**

Comment: The notation for subset is very similar to the notation for "less than", and means, in terms of the sets, "included in but not equal to".

Subset: Example

Outline

Few Set Operations

Set Operation: Union

A Union B: A Union B, denoted by $A \cup B$ is the set of all elements that are either in A, or in B, or inside both the sets.

The logical operator is OR.

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Set Operation: Set Difference

A Minus B: A Minus B, denoted by $A - B$ is the set of all elements that are only an element of A, but not an element of the set B.

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Comment: $A - B$ is not same as $B - A$

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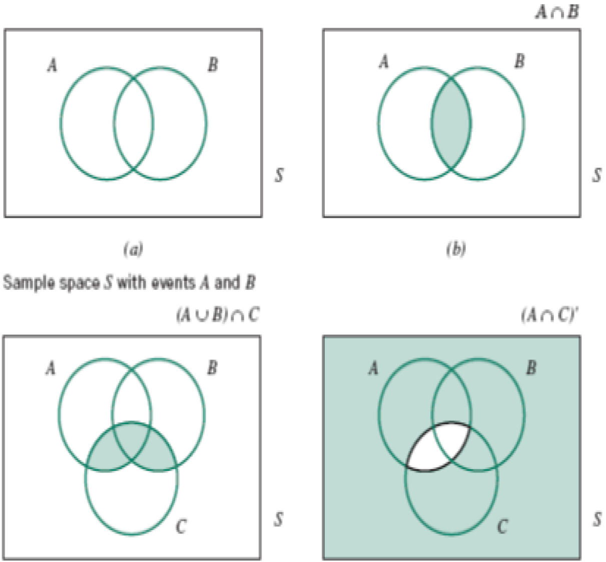
Set Operation: Examples

Outline

Venn Diagrams

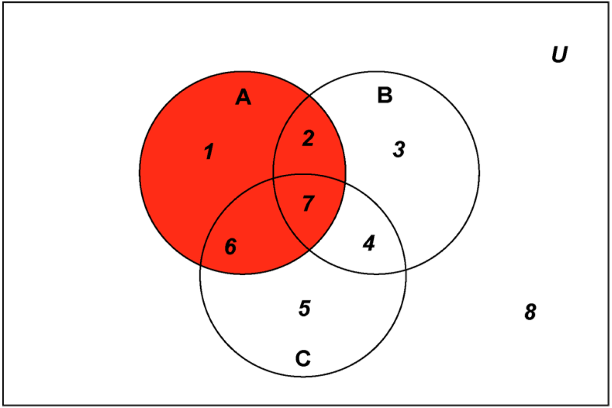
Venn Diagrams are graphical representation of the sets that are typically used to depict the relation between various sets

Venn Diagrams: Examples



Venn Diagrams: Examples

Let $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$,
$A = \{1, 2, 6, 7\}$, $B = \{2, 3, 4, 7\}$, and $C = \{4, 5, 6, 7\}$



Outline

Disjoint Sets and Partition

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A and B are Disjoint $\Leftrightarrow A \cap B = \emptyset$.

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Exhaustive

Exhaustive: Two sets A and B are exhaustive for the set C if $A \cup B = C$.

A and B are exhaustive for C $\Leftrightarrow$ $A \cup B = C$.

Partition

Partition: A group of sets $\{A, B\}$ is called a **partition for a set** C if

1. $A \cap B = \emptyset$ (i.e. A, and B are Disjoint) , and
2. $A \cup B = C$ (i.e. A and B is exhaustive for C).

Examples

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