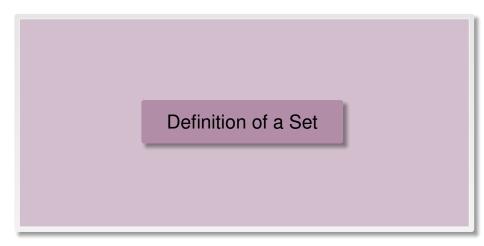
# STAT 320: Principles of Probability Unit 1: A Few Definitions and Notations From Set Theory

United Arab Emirates University

Department of Statistics

- **Definition of Sets**



#### Definition of a Set

#### Definition (Set)

A Set is a well defined collection of objects.

- **Elements:** The objects inside a set are called the **elements** of the Set
- Usually we denote sets with upper-case letters, elements with lower-case letters.

The Following Notation is Used to Show a Set Membership.
$x \in A$ : means that x is a member of the set A.
$x \notin A$ :means that x is NOT a member of the set A.

# Venn Diagrams

Venn Diagrams are graphical representation of the sets that are often used to depict the relation between various sets

For the rest of the slides, we shall include 'Venn Diagrams' for visualization of different set-theoritic concepts.

# Example

## Example: Differnt Ways of Describing a Set

• Listing all the elements:  $A = \{1, 2, 3, 4, 5, 6\}$ 

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# Example: Differnt Ways of Describing a Set

- **①** Listing all the elements:  $A = \{1, 2, 3, 4, 5, 6\}$
- Give a verbal description: A is the set of all integers from 1 to 6, inclusive.
- Give a mathematical inclusion rule:

$$A = \{x \text{ is an Integer} : 1 \le x \le 6\}$$

#### The Null Set

#### Definition (Null Set)

The **Null Set** (or also called the **Empty Set**) is a set with no elements in it.

It is often denoted by  $\emptyset$ 

#### The Universal Set

#### Definition (Universal Set)

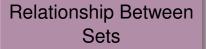
The **Universal Set** is the set of all elements currently under consideration.

In the context of probability theory, it is often denoted by  $\mathscr{S}$  (or  $\Omega$ )

The universal set contains all of the elements relevant to a given discussion.

#### Outline

- Relationship Between Sets



Subset: A set A is called a subset of a set B if all the elements of A are also the elements of B.

If  $x \in A \implies x \in B$  then  $A \subseteq B$ .

**Subsets:** If A is a subset of B then we write  $A \subseteq B$ .

Note: The notation for subset is very similar to the notation for "less than or equal to" and means, "included in or equal to" in terms of the sets.

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Note: The notation for subset is very similar to the notation for "less than or equal to" and means, "included in or equal to" in terms of the sets.

- We say "A is a proper subset of B" if all the elements of A are also the elements of B, but in addition there exists at least one element c such that  $c \in B$  but  $c \notin A$ .
- Proper Subset:  $A \subset B$ , then we say A is a proper subset of B.

Comment: The notation for subset is very similar to the notation for "less than", and means, in terms of the sets, "included in but not equal to".

#### Outline

- A Few Set Operations



#### Set Operation: Union

A Union B, denoted by  $A \cup B$  is the set of all elements that are either in A, or in B, or inside both the sets.

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The logical operator is OR.

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The logical operator is AND.

#### Set Operation: Complement

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The logical operator is **NOT**.

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A - B is same as:  $A \cap \overline{B}$ .

**Comment:** A - B is not same as B - A

## A Few Rules for Various Set Operations

$$A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C = A \cap B \cap C$$

#### Distributive laws of Union & Intersection

$$A\cap (B\cup C)=(A\cap B)\cup (A\cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

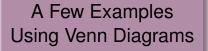
#### DeMorgan's laws

$$\overline{(A \cap B)} = (\overline{A} \cup \overline{B})$$

$$\overline{(A \cup B)} = (\overline{A} \cap \overline{B})$$



## Set Operation: Examples



Venn Diagrams are graphical representation of the sets that are typically used to depict the relation between various sets

## Venn Diagrams: Examples

# Venn Diagrams: Examples

Let 
$$\mathscr{S} = \{1, 2, 3, 4, 5, 6, 7, 8\}$$
,  $A = \{1, 2, 6, 7\}$ ,  $B = \{2, 3, 4, 7\}$ , and  $C = \{4, 5, 6, 7\}$ 

#### Outline

- Definition of Sets
- Relationship Between Sets
- A Few Set Operations
- Disjoint Sets & Partition
- Real Numbers & Intervals
- 6 The Notion of Cartesian Product
- A Discussion on Cardinality of Sets

Disjoint Sets & Partition

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A and B are Disjoint  $\Leftrightarrow A \cap B = \emptyset$ .

#### **Exhaustive:**

A collection of sets  $A_1, A_2, \dots, A_k$  are exhaustive for the set C if  $A_1 \cup A_2 \cup \cdots \cup A_k = C$ .

A and B are exhaustive for  $C \Leftrightarrow \bigcup A_i = C$ .

## Exhaustive: Special Case

#### **Exhaustive:**

Two sets A and B are exhaustive for the set C

if 
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.

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Partition:

A collection of sets  $\{A_1, A_2, \dots, A_k\}$  is called a

partition for a set C if

 $\mathcal{A}_i\cap\mathcal{A}_j=\emptyset$  for  $1\leq i
eq j\leq k$  (i.e. , $\mathcal{A}_i$ , and  $\mathcal{A}_j$  are Disjoint if i
eq j) , and

 $A_1 \cup A_2 \cup \cdots \cup A_k = C$  (i.e. , A and B is exhaustive for C).

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## Example

## Examples

## A Few Questions:

Let  $\Omega$  be the universal set and  $\emptyset$  denotes the emptyset.

- $\bullet$   $A \cup \emptyset =$
- $\bullet$   $A \cup \overline{A} =$
- $\bullet$   $A \cap \emptyset =$
- $\bullet$   $A \overline{A} =$
- $\bullet$   $A \cap \overline{A} =$
- $\bullet$   $A \cup \Omega =$
- $\bullet$   $A \cap \Omega =$
- If  $A \subset B$  then  $A \cap B =$
- If  $A \subset B$  then  $A \cup B =$

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# Real Numbers $(\mathbb{R})$ & Intervals

## Real Numbers, Intervals

For the definitions below, assume  $a \in \mathbb{R}$ ,  $b \in \mathbb{R}$ , and a < b.

## Examples of Intervals

$$[0,1] := \{x \in \mathbb{R} : 0 \leq x \leq 1\} = \text{Set of all real numbers from 0 to 1, including the numbers 0 and 1.}$$

 $(0,1):=\{x\in\mathbb{R}:0< x<1\}$ =Set of all real numbers between 0 to 1. It does not include the numbers 0, and 1.

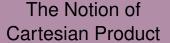
$$(0,\infty):=\{ extit{x}\in\mathbb{R}:0< extit{x}\}$$
 =Set of all positive real numbers. The interval **does not include 0**

$$[0,\infty):=\{ extit{x}\in\mathbb{R}:0< extit{X}\}$$
 =Set of all non-negative real numbers. The interval **does include 0**

## Examples: Union & Intersection of Intervals

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## The Notion of 'Tuple'

**Tuple:** A k-tuple is the a the ordered sequence of k values that are written inside a parenthesis while different elements are separated by comma.

#### Example:

- (1.5, 2) is a **2-tuple**.
- (1.5, 2, 6, 2) is a 4-tuple.
- (H, T, H, H, H) is a **5-tuple**

## Set Operation: Cartesian Product

Cartesian Product of A and B: Cartesian Product of A and B, denoted by  $A \times B$  is the set of all "two-tuple" objects where the first element in the tuple is from the set A and the second element is taken from the set B.

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$$A \times B = \{(x,y) : x \in A, y \in B\}.$$

**Comment:** In general  $A \times B$  is not same as  $B \times A$ . However,  $A \times A$  is denoted by  $A^2$ . Note that A is a set and not a number

## Set Operation: Cartesian Product

Cartesian Product of Multiple Sets

Let  $A_1$ ,  $A_2$ , ...,  $A_n$  are non

empty sets. Then their Cartesian product is defined as following:

$$A_1 \times A_2 \times ... \times A_n = \{(a_1, a_2, ..., a_n) : a_i \in A_i \text{ for } i = 1, ... n\}.$$

#### A Few Standard Notation

 $\mathbb{Z}$ : Set of all Integers. i.e.  $\mathbb{Z} = \{0, -1, 1, -2, 2, -3, 3, \ldots\}$ .

 $\mathbb{Z}_+$  : Set of all non-negative Integers. i.e.  $\mathbb{Z}_+ = \ \{1,2,3,\ldots\}$  .

 $\mathbb{R}$ : Set of all real numbers.

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Set of all real numbers.

 $\mathbb{R}_+$ : Set of all positive real numbers.

- $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{ (x_1, x_2) : x_1 \in \mathbb{R}, x_2 \in \mathbb{R} \}$

- $\longrightarrow$  is not a real number. It is a concept to denote the supremum in the real number system. We may think it to be larger than any given/prespecified number. Below are a few of its convensions/properties.
  - $\bullet \infty + x = \infty$  for any  $x \in \mathbb{R}$
  - $\bullet$   $-\infty + x = -\infty$  for any  $x \in \mathbb{R}$
  - $\bullet$   $\infty + \infty = \infty$
  - $\bullet$   $-\infty \infty = -\infty$
  - $|\infty \infty| =$ Undefined

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Discussion on Cardinality of Sets

## A Discussion on Cardinality of Various Sets

#### Finite Set

### Examples:

```
\{1,2,3,4,5,6\},\\ \{HH,HT,TH,TT\}
```

## Infinite Set: Countably Infinite Set

Examples: 
$$\mathbb{Z}_+ := \{1,2,3,\ldots,\}$$
 ,  $\mathbb{Z}_+^2$ 

#### Infinite Set: Uncountable Set

Examples:  $\mathbb{R}$  ,  $\mathbb{R}^2$  ,  $\mathbb{R}_+$ 

