STAT 320: Principles of Probability
Unit 3: Introduction to Probability

United Arab Emirates University

Department of Statistics

Outline

Sample Space & Events

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Definition (Random Experiment)

A process of observation whose outcome is not known in advance with certainty.

Experiment : Single throw of a 6-sided die.

Definition (Outcome)

An outcome is defined as any possible result of a random experiment.

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The set, S, of all possible outcomes of a particular experiment is called the sample space for the experiment.

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Example : Experiment: Determination of and recording of the sex of a newborn child.

Sample Space: $S = \{B, G\}$ where the outcome G means that the child is a girl and B that it is a boy.

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Example : Consider a context of horse race where 7 horses have participated the race. They are marked as $1, 2, \ldots, 7$.

Experiment: Recording the order of the horse numbers of 7 horses according to their completion time. The positions for the horses can be 1, 2, 3, 4, 5, 6, and 7.

Sample Space: $S =$ All 7! permutations of (1, 2, 3, 4, 5,6,7)

An outcome (2,3, 1, 6, 5, 4, 7) means, for instance, that the number 2 horse comes in first, then the number 3 horse, then the number 1 horse, and so on.

Question : Let A be the event that horse 3 wins the race. Write down the explicit description of A.

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Example : Consider the **single flip** of a coin.

Experiment: Recording the outcome after flipping a coin

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Experiment: Recording the outcome after flipping **two coins.**

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$$S = \{H, T\} \times \{H, T\} = \{(H, H), (H, T), (T, H), (T, T)\}$$

As there is no ambiguity in this case, for brevity of notation we often/will use the following notation:

Sample Space: $S = \{HH, HT, TH, TT\}$

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Example : Consider the rolling of a dice **two times**

Experiment: Recording the outcome after rolling a dice **two times.**

Sample Space: The sample space consists of the 36 points

S ={1,2,3,4,5,6}×{1,2,3,4,5,6}

$S = \{(i,j) : i,j = 1,2,3,4,5,6\}$ where the outcome (i,j) is said to occur if i appears on the first through and j on the second. other die.

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Experiment: (in hours) the lifetime of a transistor.

Sample Space: The sample space consists of all non-negative real numbers; that is, $S = \{x \in \mathbb{R} : x > 0\} = \mathbb{R}_+$.

Question : Let A be the event that the transistor does not last longer than 5 hours. Write down the event A in the notation of set theory.

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Reminder: Disjoint Events and Partition

Disjoint Events & Partition

Definition (Pairwise Disjoint Events)

Two events A and B are disjoint (or mutually exclusive) if $A \cap B = \emptyset$. A collection of events $\{A_i\}_{i=1}^n$ are pairwise disjoint (or mutually exclusive) if $A_i \cap A_j = \emptyset$ for all $i \neq j$.

Definition (Partition)

$A_1, A_2, \ldots, A_n$ are called partition of the sample space S if $\{A_i\}_{i=1}^n$ is pairwise disjoint and $\cup_{i=1}^n A_i = S$.

Comment : Any set A and it's complement, $\overline{A}$, creates a partition of S.

Comment : In the above definition, we may replace n by ∞ and the definition naturally extends.

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Axioms of Probability

Combinations

Binomial and Multinomial Coefficient

Questions?