STAT 320: Principles of Probability Unit 3: Introduction to Probability

United Arab Emirates University

Department of Statistics

Outline



Sample Space & Events



Sample Space & Events

Definition (Random Experiment)

A process of observation whose outcome is not known in advance with certainty.

Experiment: Single throw of a 6-sided die.

Definition (Outcome)

An outcome is defined as any possible result of a random experiment

Experiment: Single throw of a 6-sided die.

An Outcome: The number 5 appear in the die-throw example.

Sample Space & Events

Definition (Random Experiment)

A process of observation whose outcome is not known in advance with certainty.

Experiment : Single throw of a 6-sided die.

Definition (Outcome)

An outcome is defined as any possible result of a random experiment.

Experiment: Single throw of a 6-sided die.

An Outcome: The number 5 appear in the die-throw example.

Sample Space

Definition (Sample Space)

The set, S, of all possible outcomes of a particular experiment is called the sample space for the experiment.

Experiment : Single throw of a 6-sided die.

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Sample Space

Definition (Sample Space)

The set, S, of all possible outcomes of a particular experiment is called the sample space for the experiment.

Experiment: Single throw of a 6-sided die.

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Events

Definition (Events)

An event is any collection of possible outcomes of a particular experiment, that is, any subset of S (including \emptyset and S itself).

Experiment : Single throw of a 6-sided die.

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Example of Events: $A = \{2, 4, 6\}, B = \{3\}$

All Possible Events?

Events

Definition (Events)

An event is any collection of possible outcomes of a particular experiment, that is, any subset of S (including \emptyset and S itself).

Experiment: Single throw of a 6-sided die.

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Example of Events: $A = \{2, 4, 6\}, B = \{3\}$

All Possible Events?

Example: Experiment: Determination of and recording of the sex of a newborn child.

Sample Space: $S = \{B, G\}$ where the outcome G means that the child is a girl and B that it is a boy.

Example: Experiment: Determination of and recording of the sex of a newborn child.

Sample Space: $S = \{B, G\}$ where the outcome G means that the child is a girl and B that it is a boy.

Example: Consider a context of horse race where 7 horses have participated the race. They are marked as 1, 2, ..., 7.

Experiment: Recording the order of the horse numbers of 7 horses according to their completion time. The positions for the horses can be 1, 2, 3, 4, 5, 6, and 7.

Sample Space: S = All 7! permutations of (1, 2, 3, 4, 5,6,7)

An outcome (2,3, 1, 6, 5, 4, 7) means, for instance, that the number 2 horse comes in first, then the number 3 horse, then the number 1 horse, and so on.

Example: Consider a context of horse race where 7 horses have participated the race. They are marked as 1, 2, ..., 7.

Experiment: Recording the order of the horse numbers of 7 horses according to their completion time. The positions for the horses can be 1, 2, 3, 4, 5, 6, and 7.

Sample Space: S = All 7! permutations of (1, 2, 3, 4, 5,6,7)

An outcome (2,3, 1, 6, 5, 4, 7) means, for instance, that the number 2 horse comes in first, then the number 3 horse, then the number 1 horse, and so on.

Example:

Consider the single flip of a coin.

Experiment: Recording the outcome after flipping a coin

Sample Space: $S = \{H, T\}$

Example:

Consider the single flip of a coin.

Experiment: Recording the outcome after flipping a coin

Sample Space: $S = \{H, T\}$

Example:

Consider the flipping of two coins.

Experiment: Recording the outcome after flipping two coins.

Sample Space:

$$S = \{H, T\} \times \{H, T\} = \{(H, H), (H, T), (T, H), (T, T)\}$$

As there is no ambiguity in this case, for brevity of notation we often/will use the following notation:

Sample Space: $S = \{HH, HT, TH, TT\}$

Example: Consider the flipping of **two coins**.

Experiment: Recording the outcome after flipping two coins.

Sample Space:

$$S = \{H, T\} \times \{H, T\} = \frac{\{(H, H), (H, T), (T, H), (T, T)\}}{\{H, H, T\}}$$

As there is no ambiguity in this case, for brevity of notation we often/will use the following notation:

Sample Space: $S = \{HH, HT, TH, TT\}$

Example: Consider the flipping of **two coins**.

Experiment: Recording the outcome after flipping two coins.

Sample Space:

$$S = \{H, T\} \times \{H, T\} = \frac{\{(H, H), (H, T), (T, H), (T, T)\}}{\{H, H, T\}}$$

As there is no ambiguity in this case, for brevity of notation we often/will use the following notation:

Sample Space: $S = \{HH, HT, TH, TT\}$

Example: Consider the rolling of a dice **two times**

Experiment: Recording the outcome after rolling a dice two times.

Sample Space: The sample space consists of the 36 points

 $S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

$$S = \{(i,j): i,j = 1,2,3,4,5,6\}$$
 where the outcomes

where the outcome (i, j) is said to occur if i appears on the first

through and j on the second. other die

Example: Consider the rolling of a dice two times

Experiment: Recording the outcome after rolling a dice two times.

Sample Space: The sample space consists of the 36 points

 $S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

 $S = \{(i,j): i,j=1,2,3,4,5,6\}$ where the outcome (i,j) is said to occur if i appears on the first

through and j on the second. other die.

Reminder: Disjoint Events and Partition

Definition (Pairwise Disjoint Events)

Two events A and B are disjoint (or mutually exclusive) if $A \cap B = \Phi$. A collection of events $\{A_i\}_{i=1}^n$ are pairwise disjoint (or mutually exclusive) if $A_i \cap A_j = \Phi$ for all $i \neq j$.

Definition (Partition)

 A_1, A_2, \ldots, A_n are called partition of the sample space S if $\{A_i\}_{i=1}^n$ is pairwise disjoint and $\bigcup_{i=1}^n A_i = S$.

Comment: Any set A and it's complement, \overline{A} , creates a partition of S.

Definition (Pairwise Disjoint Events)

Two events A and B are disjoint (or mutually exclusive) if $A \cap B = \Phi$. A collection of events $\{A_i\}_{i=1}^n$ are pairwise disjoint (or mutually exclusive) if $A_i \cap A_j = \Phi$ for all $i \neq j$.

Definition (Partition)

 A_1, A_2, \ldots, A_n are called partition of the sample space S if $\{A_i\}_{i=1}^n$ is pairwise disjoint and $\bigcup_{i=1}^n A_i = S$.

Comment: Any set A and it's complement, \overline{A} , creates a partition of S.

Definition (Pairwise Disjoint Events)

Two events A and B are disjoint (or mutually exclusive) if $A \cap B = \Phi$. A collection of events $\{A_i\}_{i=1}^n$ are pairwise disjoint (or mutually exclusive) if $A_i \cap A_j = \Phi$ for all $i \neq j$.

Definition (Partition)

 A_1, A_2, \ldots, A_n are called partition of the sample space S if $\{A_i\}_{i=1}^n$ is pairwise disjoint and $\bigcup_{i=1}^n A_i = S$.

Comment: Any set A and it's complement, \overline{A} , creates a partition of S.

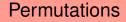
Definition (Pairwise Disjoint Events)

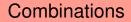
Two events A and B are disjoint (or mutually exclusive) if $A \cap B = \Phi$. A collection of events $\{A_i\}_{i=1}^n$ are pairwise disjoint (or mutually exclusive) if $A_i \cap A_j = \Phi$ for all $i \neq j$.

Definition (Partition)

 A_1, A_2, \ldots, A_n are called partition of the sample space S if $\{A_i\}_{i=1}^n$ is pairwise disjoint and $\bigcup_{i=1}^n A_i = S$.

Comment: Any set A and it's complement, \overline{A} , creates a partition of S.





Binomial and Multinomial Coefficient

