STAT 320: Principles of Probability

Unit 5(PART:B)

A Few Discrete Random Variables

United Arab Emirates University

Department of Statistics

Outline

- A quality controlling team in TV manufacturer evaluates a products before they are send to market. They are interested in idensifying the number of defective products and the resources that should be optimally allocated to mitigate the defectives items.
- A broker in a stock exchange speculates how many shares out of 10 newly introduced shares will go up next day.
- A airline company is interested in identifying the number of last minute cancellations that may take place.
- A car insurance company have sold 2000 insurance policies on a specific new cars. They need to estimate the funts that should be made available/reserved to compensate the losses that may occur to the insured cars during the next one year.

Bernoulli Distribution

A Bernoulli Trial/ Experiment

- The random experiment has only two outcomes. Namely SUCCESS, and FAILURE
- Events corresponding to the successive trials/experiemnts are statistically independent.
- All such trials/experiements have the **same chance/probability of success**.

Bernoulli Distribution Binomial(n, π)

Definition (Bernoulli Distribution (Bernoulli(π)))

Let $\pi \in (0,1)$. A discrete random variable on the support, $\mathbb{S} = \{0,1\}$ is called a **Bernoulli(π)** distribution. The corresponding probability mass function can be represented as $p(x) = \begin{cases} \pi & \text{if } x = 1, \\ (1-\pi) & \text{if } x = 0. \end{cases}$

The above pmf can also be reprrsented as the following:

$$\boxed{p(x) := \pi^x (1-\pi)^{1-x}}, \text{ for } x \in \mathbb{S}, \text{where } \mathbb{S} = \{0,1\}$$

Let $X \sim \text{Bernoulli}(\pi)$

Mean

$E(X) = \pi$

Variance

$\text{VAR}(X) = \pi(1 - \pi)$

MGF

$M_X(t) = \left(1 - \pi + \pi e^t\right)$

Distribution	Support $\mathbb{S}_X$	pmf $p_X(x)$	Mean $E(X)$	Variance $\text{Var}(X)$	mgf $M_X(t)$
Bernoulli(π)	$\{0, 1\}$	$\binom{n}{x}\pi^x(1-\pi)^{1-x}$	π	$\pi(1-\pi)$	$\left(1 - \pi + \pi e^t\right)$

Binomial Distribution

A Bernoulli Trial/ Experiment

- The random experiment has only two outcomes. Namely SUCCESS, and FAILURE
- Events corresponding to the successive trials/experiemnts are statistically independent.
- All such trials/experiements have the **same chance/probability of success**.

If a sequence of n independent Bernoulli trials are performed under the same condition, the random variable that records the **total number of** SUCCESS is called the **Binomial Random variable.**

Binomial Distribution

Definition (Binomial Experiment)

An experiment is called a Binomial experiment if it satisfies the following 4 conditions:

- The experiment consists of n Bernoulli trials.
- Each trial results in a success (S) or a failure (F).
- The trials are independent.
- The probability of a success, π, is fixed throughout n trials.

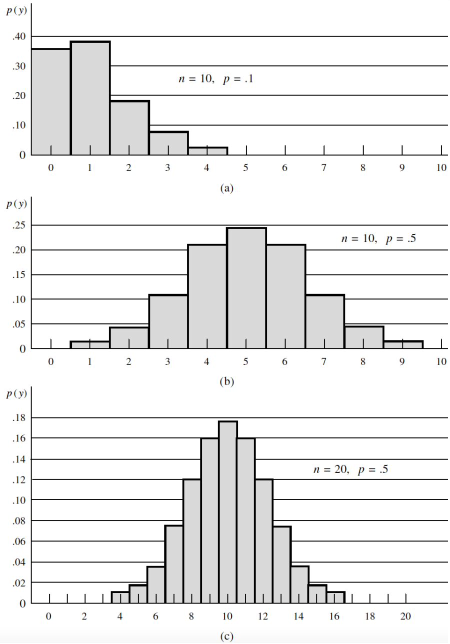
$$\text{Let } X \sim \text{Binomial}(n, \pi)$$

Binomial Distribution Binomial(n, π)

Definition (Binomial Distribution (Binomial(n, π)))

Let $\pi \in (0, 1)$, and n be a positive integer. A discrete random variable on the support, $\mathbb{S} = \{0, 1, 2, \ldots, n\}$ is called a **Binomial(n, π)** if the corresponding probability mass function can be represented as Binomial(n, π) is given by

$$\boxed{p(x) := \binom{n}{x} \pi^x (1 - \pi)^{n-x}}, \text{ for } x \in \mathbb{S}, \text{where } \mathbb{S} = \{0, 1, \ldots, n\}$$



p(y)
.40
.30
.20
.10
0
n = 10, p = .1
0 1 2 3 4 5 6 7 8 9 10
(a)
p(y)
.25
.20
.15
.10
.05
0
n = 10, p = .5
0 1 2 3 4 5 6 7 8 9 10
(b)
p(y)
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
n = 20, p = .5
0 2 4 6 8 10 12 14 16 18 20
(c)

Let $X \sim \text{Binomial}(n = 2, \pi = 0.2)$

Let $X \sim \text{Binomial}(n = 100, \pi = 0.2)$

Suppose: $n = 100, \pi = 0.2$

$$\mathbf{p}(x) = \binom{n}{x} \pi^x (1-\pi)^x$$

For $x \in \{0, 1, \ldots, 100\}$

$$E(X) = n\pi = 100 \times 0.2 = 20.$$

$\pi = 0.2$

x	p(x)
0	2.03704E-10
1	5.09259E-09
2	6.30208E-08
3	5.1467E-07
4	3.12019E-06
5	1.49769E-05
6	5.92835E-05
7	0.000199023
8	0.000578411
9	0.001478163
10	0.00336282
11	0.006878495
12	0.012753877
13	0.021583484
14	0.033531484
15	0.048061794
16	0.06383207
17	0.07885138
18	0.090898119
19	0.098074286
20	0.099300215

x	p(x)
21	0.094571633
22	0.084899534
23	0.07198004
24	0.05773399
25	0.043877833
26	0.031642668
27	0.021681087
28	0.014131423
29	0.008771228
30	0.005189643
31	0.002929637
32	0.001579258
33	0.000813557
34	0.000400796
35	0.000188947
36	8.52885E-05
37	3.68815E-05
38	1.52864E-05
39	6.07537E-06
40	2.31624E-06

x	p(x)
41	8.474E-07
42	2.976E-07
43	1.0035E-07
44	3.2501E-08
45	1.0111E-08
46	3.0224E-09
47	8.6814E-10
48	2.3964E-10
49	6.3579E-11
50	1.6213E-11
51	3.9737E-12
52	9.3611E-13
53	2.1195E-13
54	4.6118E-14
55	9.643E-15
56	1.9372E-15
57	3.7385E-16
58	6.929E-17
59	1.2331E-17
60	2.1066E-18

x	p(x)
61	3.4534E-19
62	5.4308E-20
63	8.1893E-21
64	1.1836E-21
65	1.6389E-22
66	2.1727E-23
67	2.7564E-24
68	3.3442E-25
69	3.8773E-26
70	4.2928E-27
71	4.5346E-28
72	4.5661E-29
73	4.3785E-30
74	3.9939E-31
75	3.4614E-32
76	2.8465E-33
77	2.2181E-34
78	1.6351E-35
79	1.1384E-36
80	7.4705E-38

x	p(x)
81	4.611E-39
82	2.671E-40
83	1.448E-41
84	7.328E-43
85	3.448E-44
86	1.504E-45
87	6.049E-47
88	2.234E-48
89	7.53E-50
90	2.301E-51
91	6.321E-53
92	1.546E-54
93	3.325E-56
94	6.189E-58
95	9.773E-60
96	1.273E-61
97	1.312E-63
98	1.004E-65
99	5.071E-68
100	1.268E-70

$$\mathrm{SD}(X) = \sqrt{n\pi(1-\pi)} = \sqrt{100 \times 0.2(1-0.2)} = \sqrt{16} = 4.0$$

Let $X \sim \text{Binomial}(n, \pi)$

Mean

$E(X) = n\pi$

Variance

$\text{VAR}(X) = n\pi(1-\pi)$

MGF

$\text{M}_X(t) = \left(1 - \pi + \pi e^t\right)^n$

Distribution	Support $\mathbb{S}_X$	pmf $p_X(x)$	Mean $E(X)$	Variance $\text{Var}(X)$	mgf $M_X(t)$
$\text{Binomial}(n, \pi)$	$\{0, 1, \ldots, n\}$	$\binom{n}{x}\pi^x(1-\pi)^{n-x}$	$n\pi$	$n\pi(1-\pi)$	$\left(1-\pi+\pi e^t\right)^n$

Reminder from Unit1: Binomial Series

Let $a \in \mathbb{R}$ be any real number, and $n \in \mathbb{Z}_+$ be any positive integer, then

$$(1+a)^n = \sum_{y=0}^{n} \binom{n}{y} a^y$$

$$(1+a)^n = 1 + \binom{n}{1} a + \binom{n}{2} a^2 + \cdots + \binom{n}{n-1} a^{n-1} + \binom{n}{n} a^n$$

Let $a \in \mathbb{R}$ be any real number, and $n \in \mathbb{Z}_+$ be any positive integer, then

$$(a+b)^n = \sum_{y=0}^{n} \binom{n}{i} a^y b^{n-y}$$

$$(a+b)^n = b^n + \binom{n}{1} ab^{n-1} + \binom{n}{2} a^2 b^{n-2} + \cdots + \binom{n}{n-1} a^{n-1} b + \binom{n}{n} a^n$$

Reminder from Unit1: Binomial Series

Let $a \in \mathbb{R}$ be any real number, and $n \in \mathbb{Z}_+$ be any positive integer, then

$$\sum_{y=0}^{n} y \binom{n}{y} a^y = n \times a(1+a)^{n-1}$$

Let $a \in \mathbb{R}$ be any real number, and $n \in \mathbb{Z}_+$ be any positive integer, then

$$\sum_{y=0}^{n} y^2 \binom{n}{y} a^y = n\, a(1+a)^{n-1} + n(n-1)\, a^2(1+a)^{n-2}$$

Expected Value of Binomial Distribution

$$
\begin{aligned}
E(X) &:= \sum_{y \in \mathbb{S}_X} y \, p_x(y) \\
&= \sum_{y=0}^{n} y \binom{n}{y} \pi^y (1-\pi)^{n-y} \\
&= (1-\pi)^n \sum_{y=0}^{n} y \binom{n}{y} \left(\boxed{\frac{\pi}{1-\pi}} \right)^y \\
&= (1-\pi)^n \left[n \boxed{\frac{\pi}{(1-\pi)}} \times \boxed{\left(1 + \frac{\pi}{1-\pi}\right)^{n-1}} \right] \\
&= (1-\pi)^n \frac{n\pi}{(1-\pi)^n} \\
&= n\pi
\end{aligned}
$$

Expected Value of Binomial Distribution

$$
\begin{aligned}
& E(X^2) \\
:= \quad & \sum_{y \in \mathbb{S}_X} y^2 \, p_x(y) \\
= \quad & \sum_{y=0}^{n} y^2 \binom{n}{y} \pi^y (1-\pi)^{n-y} \\
= \quad & (1-\pi)^n \left[\sum_{y=0}^{n} y^2 \binom{n}{y} \left(\frac{\pi}{1-\pi} \right)^y \right] \\
= \quad & (1-\pi)^n \left[n \, \frac{\pi}{1-\pi} \left(1 + \frac{\pi}{1-\pi}\right)^{n-1} + n(n-1) {\frac{\pi}{1-\pi}}^{2} \left(1 + \frac{\pi}{1-\pi}\right)^{n-2} \right] \\
= \quad & (1-\pi)^n \frac{n\pi + n(n-1)\pi^2}{(1-\pi)^n} \\
= \quad & n\pi + n(n-1)\pi^2
\end{aligned}
$$

$$
\begin{aligned}
Var(X) &= E(X^2) - (E(X))^2 \\
&= n\pi + n(n-1)\pi^2 - n^2\pi^2 \\
&= n\pi + n^2\pi^2 - n\pi^2 - n^2\pi^2 \\
&= n\pi - n\pi^2 \\
&= \boxed{n\pi(1-\pi)}.
\end{aligned}
$$

Expected Value of Binomial Distribution

$$
\begin{aligned}
M_X(t) := \sum_{y \in \mathbb{S}_X} e^{ty}\, p_x(y) &= (1-\pi)^n \sum_{y=0}^{n} e^{ty} \binom{n}{y} \left(\frac{\pi}{1-\pi}\right)^y \\
&= (1-\pi)^n \sum_{y=0}^{n} e^{ty} \binom{n}{y} \left(\frac{\pi}{1-\pi}\right)^y \\
&= (1-\pi)^n \left[\sum_{y=0}^{n} \binom{n}{y} \left(\frac{\pi e^t}{1-\pi}\right)^y\right] \\
&= (1-\pi)^n \left[\left(1 + \frac{\pi e^t}{1-\pi}\right)^n\right] \\
&= \boxed{\left(1-\pi+\pi e^t\right)^n}
\end{aligned}
$$

Example

Example : Five fair coins are flipped. If the outcomes are assumed independent.

1. Find the probability mass function of the number of heads obtained.
2. Find the probability that at least 3 heads are obtained.
3. Find the probability that at most 2 heads are obtained.

Example

Example : Five fair coins are flipped. If the outcomes are assumed independent.

1. Find the probability mass function of the number of heads obtained.
2. Find the probability that at least 3 heads are obtained.
3. Find the probability that at most 2 heads are obtained.

Solution: Let X = The number of heads in 5 tossed coins. $X \sim Binomial(n = 5, \pi = 0.5)$.

1. $P(X = 0) = 0.5^5 = 0.0313$
2. $P(X = 1) = \binom{5}{1}0.5^5 = 0.1563$
3. $P(X = 2) = \binom{5}{2}0.5^5 = 0.3125$
4. $P(X = 3) = \binom{5}{3}0.5^5 = 0.3125$
5. $P(X = 4) = \binom{5}{4}0.5^5 = 0.1563$
6. $P(X = 0) = \binom{5}{5}0.5^5 = 0.0313$

Example

Example : It is known that screws produced by a certain company will be defective with probability .01, independently of each other. The company sells the screws in packages of 10 and offers a money-back guarantee that at most 1 of the 10 screws is defective. What proportion of packages sold must the company replace? Use the Binomial Calculator or Statistical Tables.

Outline

Poisson Distribution

- Number of calls received by a customer desk in an hour.
- Number of imperfections in every square-meter of a glass panel used for making LCD TV.
- Number of robot malfunctions per day in an assembly line.
- Number of car accidents in a segment of roads that occurs during a year.

Poisson Distribution

The Poisson distribution models the number of occurrences of an event when there is a known average rate per unit time or space λ.

Definition (Poisson Distribution)

The requirements for a Poisson distribution are that:

1. no two events can occur simultaneously,
2. events occur independently in different intervals, and
3. the expected number of events in each time interval remain constant.

Poisson Distribution: pmf, Expected Value

The Poisson distribution models the number of occurrences of an event when there is a known average rate per unit time or space λ.

Definition (Poisson Distribution)

A discrete random variable on the support, $\mathbb{S} = \{0, 1, 2, 3, \ldots\}$, is called a **Poisson distribution with mean parameter** λ if the corresponding probability mass function is specified as

$$\boxed{p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}}$$

for $x = 0, 1, 2, 3, \ldots$, where $\lambda > 0$.

Let $X \sim \text{Poisson}(\lambda)$

Mean

$E(X) = \lambda$

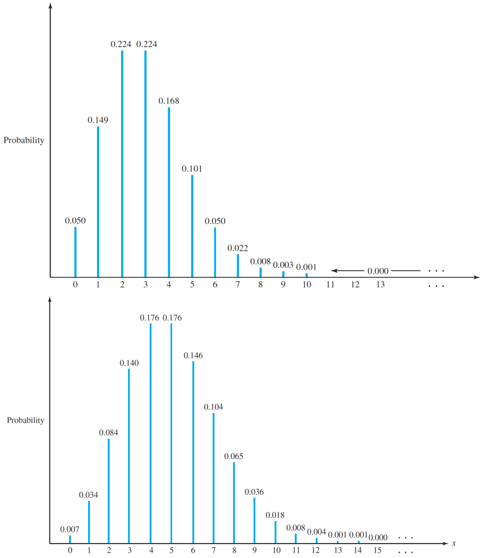
Variance

$\text{VAR}(X) = \lambda$

MGF

$\text{M}_x(t) = e^{\lambda e^t - \lambda}$

Distribution	Support $\mathbb{S}_x$	pmf $p_x(x)$	Mean $E(X)$	Variance $Var(X)$	mgf $M_x(t)$
Poisson(λ)	$\{0, 1, 2, \ldots\}$	$\frac{e^{-\lambda}\lambda^x}{x!}$	λ	λ	$e^{\lambda e^t - \lambda}$


Probability
0.050
0.149
0.224 0.224
0.168
0.101
0.050
0.022
0.008 0.003 0.001
0.000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 . . .
Probability
0.007
0.034
0.084
0.140
0.176 0.176
0.146
0.104
0.065
0.036
0.018
0.008 0.004 0.001 0.001 0.000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 . . .
x

Reminder from Unit1: Exponential Series e^a or $(\exp(a))$

Definition (Exponential Series)

For any real number $a \in \mathbb{R}$, the exponential series e^a (or sometimes denoted as $\exp(a)$) is defined as,

$$e^{a} = \sum_{n=0}^{\infty} \frac{a^n}{n!} = 1 + \frac{a}{1!} + \frac{a^2}{2!} + \frac{a^3}{3!} + \cdots,$$

Expected Value of Binomial Distribution

$$
\begin{aligned}
M_X(t) &:= \sum_{y \in \mathbb{S}_X} e^{ty}\, p_x(y) \\
&= \sum_{y=0}^{\infty} e^{ty} \frac{e^{-\lambda}\lambda^y}{y!} \\
&= e^{-\lambda}\left[\sum_{y=0}^{\infty} \frac{\left(\boxed{\lambda e^t}\right)^y}{y!}\right] \\
&= e^{-\lambda}\left[e^{\lambda e^t}\right] \\
&= \exp\left[\lambda e^t - \lambda\right]
\end{aligned}
$$

- The number of customers arriving at a service counter within one-hour period.
- The number of typographical errors in a book counted per page.
- The number of email messages received at the technical support center daily.
- The number of traffic accidents that occur on a specific road during a month.

Poisson Process: Most Simple Version

1. The Number of Events Between the interval (can be time-interval or space-interval) $(s, t]$ follows

$$\text{Poisson}(\lambda \times (t - s))$$

   where $\lambda > 0$ denotes of rate of events per unit length of the interval.
2. Events pertaiing to the two distinct intervals are Statistically Independent
3. Rate of occurance of the events remain same for each of the subintervals with same length.

A Few Examples of Poisson Distribution

Example : Messages arrive at an electronic message center at random times, with an average of 9 messages per hour.

1. What is the probability of receiving exactly five messages during the next hour?
2. What is the probability that more than 10 messages will be received within the next two hours?

1. The number of messages received in an hour, X is modeled by Poisson distribution with $\lambda = 9$, i.e. $X \sim \text{Poisson}(9)$. $P(X = 5) = \frac{9^5 \exp(-9)}{5!}$
2. The number of messages received within a 2-hour period, Y is another Poisson distribution with $Y = (2)(9) = 18$, i.e. $Y \sim \text{Poisson}(18)$. $P(Y > 10) = 1 - P(Y \leq 10) = \ldots = 0.9696$

Outline

Geometric Distribution

Geometric Distribution

1. Suppose that independent trials, each having a probability π, $0 < \pi < 1$, of being a success, are performed until a success occurs.
2. Example: The first head in tossing coin several times.
3. Then, Geometric distribution models the number of trials performed until a success occurs.

Definition (Geometric Distribution)

A discrete random variable on the support $\mathbb{S} = \{1, 2, 3, \ldots\}$ is defined to be the **Geometric(π)** random variable if the corresponding probability mass function can be represented as the following

$$p(x) = \pi(1-\pi)^{x-1} \text{ for } x = 1, 2, 3, \ldots,$$

where $0 < \pi < 1$.

Let $X \sim \text{Geometric}(\pi)$

Mean

$$E(X) = \frac{1}{\pi}$$

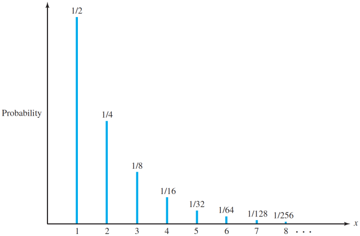
Variance

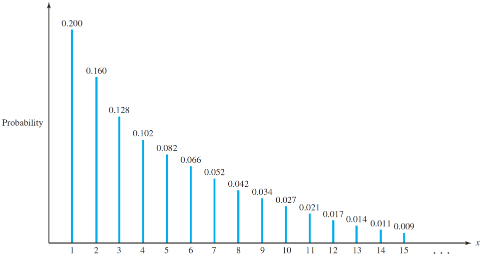
$$\text{VAR}(X) = \frac{1-\pi}{\pi^2}$$

MGF

$$M_X(t) = \frac{\pi e^t}{1-(1-\pi)e^t}$$

Distribution	Support $\mathbb{S}_X$	pmf $p_X(x)$	Mean $E(X)$	Variance $\text{Var}(X)$	mgf $M_X(t)$
Geometric(π)	$\{1, 2, \ldots\}$	$\pi(1-\pi)^{x-1}$	$\frac{1}{\pi}$	$\frac{1-\pi}{\pi^2}$	$\frac{\pi e^t}{1-(1-\pi)e^t}$


Probability
1/2
1/4
1/8
1/16
1/32
1/64
1/128
1/256
1 2 3 4 5 6 7 8 . . .
x


Probability
0.200
0.160
0.128
0.102
0.082
0.066
0.052
0.042
0.034
0.027
0.021
0.017
0.014
0.011
0.009
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 . . .
x

Reminder from Unit1: Geometric Series

Let $p \in \mathbb{R}$ be such that $|p| < 1$, then

$$\sum_{y=0}^{\infty} p^y = 1 + p + p^2 + p^3 + \cdots = \frac{1}{1-p}.$$

1. What is the value of $1 + 0.7 + (0.7)^2 + (0.7)^3 + \cdots =$
2. What is the value of $1 - 0.7 + (0.7)^2 - (0.7)^3 + \cdots =$

Reminder from Unit1: Geometric Series

Let $p \in \mathbb{R}$ be such that $|p| < 1$, then

$$\sum_{y=0}^{\infty} y\, p^y = \frac{p}{(1-p)^2}$$

Let $p \in \mathbb{R}$ be such that $|p| < 1$, then

$$\sum_{y=0}^{\infty} y^2 p^y = \frac{p}{(1-p)^2} + \frac{2p^2}{(1-p)^3}$$

Expected Value of Geometric Distribution

$$
\begin{aligned}
M_X(t) = \sum_{x \in \mathbb{S}_X} e^{tx}\, p_x(x) &= \sum_{x=1}^{\infty} e^{tx}(1-\pi)^{x-1}\pi \\
&= \pi \sum_{y=0}^{\infty} e^{ty+t}(1-\pi)^y \\
&= \pi e^t \left[\sum_{y=0}^{\infty} \left((1-\pi)e^t \right)^y \right] \\
&= \pi e^t \left[\frac{1}{1-(1-\pi)e^t} \right] \\
&= \boxed{\frac{\pi e^t}{1-(1-\pi)e^t}}
\end{aligned}
$$

Geometric Distribution: Example

Example : Of a population of consumers, 60% are reputed to prefer a particular brand, A, of toothpaste. If a group of randomly selected consumers is interviewed,

1. what is the probability that **exactly five** people have to be interviewed to encounter the first consumer who prefers brand A?
2. what is the probability that **at least** five people have to be interviewed to encounter the first consumer who prefers brand A?

Outline

Negative Binomial Distribution

1. Suppose that independent trials, each having probability π, $0 < \pi < 1$, of being a success are performed until a total of r successes is accumulated.
2. Example: The third head in tossing coin several times.
3. Then, Negative Binomial distribution models the number of trials performed until a the rth success occurs.

Definition (Negative Binomial Distribution)

Let $\pi \in (0, 1)$ and r be a positive integer. A discrete random variable on the support $\mathbb{S} = \{r + 1, r + 2, r + 3, \ldots\}$ is defined to be the **Negative-Binomial(r, π)** random variable if the corresponding probability mass function can be represented as

$$p(x) = \binom{x-1}{r-1}\pi^r(1-\pi)^{x-r} \quad \text{for } x = r+1, r+2, r+3, \ldots,$$

Let $X \sim$ Negative-Binomial(r, π)

Mean

$$E(X) = \frac{r}{\pi}$$

Variance

$$\text{VAR}(X) = \frac{r(1-\pi)}{\pi^2}$$

MGF

$$\text{M}_X(t) = \left(\frac{\pi e^t}{1-(1-\pi)e^t}\right)^r$$

Distribution	Support $\mathbb{S}_X$	pmf $p_X(x)$	Mean $E(X)$	Variance Var(X)	mgf $M_X(t)$
Negative-Binomial(r, π)	$\{r+1, r+2, \ldots\}$	$\binom{x-1}{r-1}\pi^r(1-\pi)^{x-r}$	$\frac{r}{\pi}$	$\frac{r(1-\pi)}{\pi^2}$	$\left(\frac{\pi e^t}{1-(1-\pi)e^t}\right)^r$

Geometric Distribution: Example

Example : It is known that a machine produces 1% defective parts. What is the probability that

1. 10 parts have to be selected until to get 2 defective parts.
2. Between 20 to 25 parts have to be selected to get 2 defective parts.

Solution:

Exercise Find the mean and standard deviation of Y.

Distribution	Support $\mathbb{S}_X$	pmf $p_X(x)$	Mean $E(X)$	Variance $\text{Var}(X)$	mgf $M_X(t)$
Bernoulli(π)	$\{0, 1\}$	$\pi^x(1-\pi)^{1-x}$	π	$\pi(1-\pi)$	$\left(1-\pi+\pi e^t\right)$
Binomial(n, π)	$\{0, 1, \ldots, n\}$	$\binom{n}{x}\pi^x(1-\pi)^{n-x}$	$n\pi$	$n\pi(1-\pi)$	$\left(1-\pi+\pi e^t\right)^n$
Poisson(λ)	$\{0, 1, 2, \ldots\}$	$\frac{e^{-\lambda}\lambda^x}{x!}$	λ	λ	$e^{\lambda e^t-\lambda}$
Geometric(π)	$\{1, 2, \ldots\}$	$(1-\pi)^{x-1}\pi$	$\frac{1}{\pi}$	$\frac{1-\pi}{\pi^2}$	$\frac{\pi e^t}{1-(1-\pi)e^t}$
Negative-Binomial(r, π)	$\{r+1, r+2, \ldots\}$	$\binom{x-1}{r-1}(1-\pi)^{x-r}\pi^r$	$\frac{r}{\pi}$	$\frac{r(1-\pi)}{\pi^2}$	$\left(\frac{\pi e^t}{1-(1-\pi)e^t}\right)^r$

Outline

Miscellaneous Problems

If $X \sim \text{Binomial}(n = 50, \pi = 0.1)$ then

1. Obtain the value of $E(X)$, $\text{Var}(X)$, and $E(X^2)$.
2. What is the MGF of X?

If $X \sim$ Geometric($\pi = 0.2$) then

1. Obtain the value of $E(X)$, Var(X), and $E(X^2)$.
2. What is the MGF of X?

If $X \sim \text{Poisson}(\lambda = 5)$, then

1. Obtain the value of $E(X)$, $\text{Var}(X)$, and $E(X^2)$
2. What is the MGF of X?

Identify the distribution from the MGF (Using the uniqueness property of MGF)

$M_X(t) = (0.7 + 0.3e^t)^4$, What is the distribution of X?

$M_X(t) = \frac{0.9e^t}{1-0.1e^t}$, What is the distribution of X?

$M_X(t) = \exp\left(7e^t - 7\right)$, What is the distribution of X?

$M_X(t) = \frac{1}{8}(1 + e^t)^3$, What is the distribution of X?

$M_X(t) = \frac{8e^t}{10-2e^t}$, What is the distribution of X?

Example : Approximately 10% of the glass bottles coming off a production line have serious flaws in the glass. 20 bottles are randomly selected from the product liine.

1. What is the probability that none of the selected bottles are flawed?
2. What is the probability that exactly 10 of the selected bottles are flawed?
3. Find the expected number (mean) of flawed bottles and the corresponding standard deviation.

Example : A company's toll-free complaints line receives an average of about 40 calls per hour. Use the Poisson distribution to obtain the following:

1. what is the probability that there is no calls during a 1-minute the period?
2. what is the probability that at least 1 calls arrives during the 2 minute period?
3. what is the probability two or more calls arrives in a 2- minute perior?
4. What is the expected number of calls in a 20 minutes period.

Geometric Distribution: Example

Example : From a database of a tele-marketing company, It is know that only 20% responds positively and agree to participate in their telephonic survey about a specific type of product. A tele-marketing agent was assigned to a task where it is required for positive participation from 10 individuals.

1. What is the probability that the agent needs to call 50 individuals to get a all these 10 positive responses?
2. What is the expected number of individuals the agent needs to make calls to obtain 10 positive responses?
3. What would be the required expected number of individuals the agent needs to make calls to obtain 15 positive responses instead?

Example : Suppose that a lot of electrical fuses contains 5% defectives. If a random sample of 10 fuses are tested,

1. find the probability of observing at least one to be defective.
2. What is the expected number of defective fuses?

Example : An oil exploration firm is formed with enough capital to finance ten explorations. The probability of a particular exploration being successful is 0.10 . Assume the explorations are independent.

1. Find the mean and variance of the number of successful explorations.
2. Suppose the firm has a fixed cost of \$20,000 in preparing equipment prior to doing its first exploration. If each successful exploration costs \$30,000 and each unsuccessful exploration costs \$15,000, find the expected total cost to the firm for its ten explorations.

Example : A food manufacturer uses an extruder (a machine that produces bite-size cookies and snack food) that yields revenue for the firm at a rate of \$200 per hour when in operation. However, the extruder breaks down an average of two times every day it operates. If Y denotes the number of breakdowns per day, the daily revenue generated by the machine is $R = 1600 - 50Y^2$. Find the expected daily revenue for the extruder.

Example : A particular sale involves four items randomly selected from a large lot that is known to contain 10% defectives. Let Y denote the number of defectives among the 20 sold. The purchaser of the items will return the defectives for repair, and the repair cost is given by $\text{Cost} = 3Y^2 + Y + 2$. Find the expected repair cost.

Example : You throw darts at a board until you hit the center area. Your probability of hitting the center area is 0.17. You want to find the probability that it takes eight throws until you hit the center. What values does X take on?

Example : In a certain population, it is known that 80% of the individuals have the Rhesus (Rh) factor present in their blood.

1. If 5 volunteers are randomly selected from the population, what is the probability that at least one does not have the Rh factor?
2. If five volunteers are randomly selected, what is the probability that at most four have the Rh factor?

Example : Consider rolling a fair dice multiple times untill the first 6 appears.

1. Find the expected number of throws required to get the first 6.
2. What is the probability that more then 8 throws are required to obtain the first 6?

Example : A tollbooth operator has observed that cars arrive randomly at a rate of 360 cars per hour

1. what is the probability that exactly two cars will come during a specific one-minute period?
2. Find the probability that 40 cars arrive between 10 am to 10:10 am
3. Find the expected number of cars between 10 am to 10:10 am

Example : A certain type of tree has seedlings randomly dispersed in a large area, with the mean density of seedlings being approximately five per square yard. If a forester randomly locates ten 1-square-yard sampling regions in the area. Find the probability that none of the regions will contain seedlings.

Example : Accident records collected by an automobile insurance company give the following information. The probability that an insured driver has an automobile accident is .15. If an accident has occurred, the damage to the vehicle amounts to 20% of its market value with a probability of 0.80, to 60% of its market value with a probability of 0.12, and to a total loss with a probability of .08. What premium should the company charge on a 120,000 AED car so that the expected gain by the company is zero?

Example : An integrated circuit manufacturer produces wafers that contain 18 chips. Each chip has a probability of 0.085 of not being placed quite correctly on the wafer.

1. Find the probability that a wafer contains at least three incorrectly placed chips.
2. What is the probability that a wafer contains no more than one incorrectly placed chip?
3. What is the expected number of incorrectly placed chips on a wafer?

Questions?