STAT 320: Principles of Probability
Unit 5: Discrete Random Variables

United Arab Emirates University

Department of Statistics

Outline

Random Variables

Random Variables

1. Frequently, when an experiment is performed, we are interested mainly in some function of the outcome as opposed to the actual outcome itself.

2. For instance, in tossing dice, we are often interested in the sum of the two dice and are not really concerned about the separate values of each die.

3. These quantities of interest, or, more formally, these real-valued functions defined on the sample space, are known as random variables.

Random Variables

Events of major interest to the scientist, engineer, or businessperson are those identified by numbers, called numerical events. The research physician is interested in the event that ten of ten treated patients survive an illness; the businessperson is interested in the event that sales next year will reach $3 million. Let Y denote a variable to be measured in an experiment. The realized value of Y will vary depending on the outcome of the experiment. Thus it is called a random variable.

Definition (Random Variable)

A random variable is a function from a sample space $\mathscr{S}$ into the real numbers.

Example: Random Variable

Experiment	Random Variable
Toss two dice	X= sum of the numbers
Toss a coin 25 times	X number of heads in 25 tosses
Apply different amounts of fertilizer to corn plants	X = yield/acre

Notation: Random variables will always be denoted with uppercase letters and the realized values of the variable (or its range) will be denoted by the corresponding lowercase letters. Thus, the random variable X can take the value x.

Support/Range of a Random Variable

Definition (Support/Range of a Random Variable)

The set containing the all possible values of a random variable is called its **support** or **range**.

Notation: We will use the notation $\mathbb{S}_x$ (or simply $\mathbb{S}$ if there is no ambiguity) to denote the support of a random variable X.

Example: Consider the experiment of tossing a fair coin 3 times from. Define the random variable X to be the number of heads obtained in the 3 tosses. The Support of the random variable is $\mathbb{S}_x = \{0, 1, 2, 3\}$

Example

Example : Suppose that our experiment consists of tossing 3 fair coins. If we let Y denote the number of heads that appear, then Y is a random variable taking on one of the values $0, 1, 2$, and 3 with respective probabilities.

$$p_Y(0) = P(Y = 0) =$$
$$p_Y(1) = P(Y = 1) =$$
$$p_Y(2) = P(Y = 2) =$$
$$p_Y(3) = P(Y = 3) =$$

Example

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Outline

Discrete Random Variables & its Probability Mass Function (pmf)

Definition (Discrete Random Variables)

A random variable that can take on at most a countable number of possible values is said to be discrete. *That is, if* $\mathbb{S}$*, the support of a random variable is finite or countable infinite then the corresponding random variable is discrete.*

Probability Mass Function (pmf)

For a discrete random variable X, we define the probability mass function (pmf) $p_x(x)$ of X by

$$p_x(x) = P(X = x) \text{ for all } x \in \mathbb{S}_x$$

Let X be a discrete random variable with probability mass function $p(x)$ defined on the support $\mathbb{S}$. Let $A \subset \mathbb{S}$ be an event, then

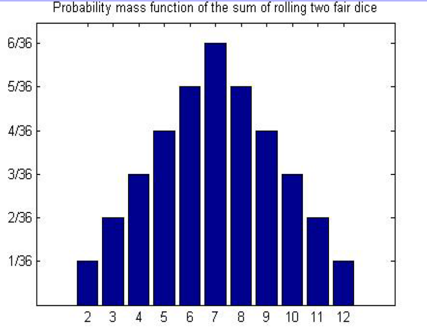
$$P\left(A\right) := P(X \in A) = \sum_{\left\{x \in A\right\}} p(x).$$

The pmf of the random variable representing the sum when two dice are rolled can be represented in multiple ways.

As a Tabular Format:

x	2	3	4	5	6	7	8	9	10	11	12
$p_X(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

As a Plot/Graph:



As a Function

$$p_X(x) = \begin{cases} \frac{x-1}{36} & \text{if } 2 \le x \le 7, \\ \frac{13-x}{36} & \text{if } 8 \le x \le 12 \end{cases}$$

Characterization of a pmf

Let $p(x)$ is **probability mass function** of a discrete random variable on the support $\mathbb{S}$, **if and only if** it satisfies the following conditions:

1. *Positivity:* $p(x) > 0$ for all $x \in \mathbb{S}$
2. *Total Probability:* $\sum_{\{x \in \mathbb{S}\}} p(x) = 1$.

Example

Example : A system consists of 2 components connected in parallel, then at least one must work correctly for the system to work correctly. Each component operates correctly with probability 0.8 and independent of the other. Let X be the number of components that work correctly. Find the probability distribution of X.

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Example : A system consists of 2 components connected in parallel, then at least one must work correctly for the system to work correctly. Each component operates correctly with probability 0.8 and independent of the other. Let X be the number of components that work correctly. Find the probability distribution of X.

Solution: X can take on only three possible values; 0, 1, or 2. Let E_i denote the event that component i works correctly. Then $P(E_i) = 0.8$. Thus, we have

- $p_X(0) = P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1})P(\overline{E_2}) = (0.2)(0.2) = 0.04.$
- $p_X(1) = P(\overline{E_1} \cap E_2) + P(E_1 \cap \overline{E_2}) = (0.2)(0.8) + (0.8)(0.2) = 0.32.$
- $p_X(2) = P(E_1 \cap E_2) = P(E_1)P(E_2) = (0.8)(0.8) = 0.64.$

x	0	1	2
$p_X(x)$	0.04	0.32	0.64

Cumulative Distribution Function (CDF) of a discrete Random Variable

cumulative distribution function

Definition (cumulative distribution function)

Let X be a discrete random variable on the support $\mathbb{S}_x$ with the corresponding probability mass function

$$P(X = x) = p_x(x) \text{ for } x \in \mathbb{S}_x.$$

Then for any $a \in \mathbb{R}$, the cumulative distribution function (cdf), denoted by $F_x(\cdot)$ is the following quantity

$$F_x(a) = P(X \leq a) = \sum_{\{x \leq a \,:\, x \in \mathbb{S}_x\}} p_x(x)$$

A **pmf** of a discrete random variable is only positive/ revelant on the support of the random variable $\mathbb{S}$, However the **CDF** is defined for any real number.

Example

Example : If X be a discrete random variable on the support $\mathbb{S}_x = \{1, 2, 3, 4\}$ with the corresponding pmf specified as $p_x(1) = \frac{1}{4}, p_x(2) = \frac{1}{2}, p_x(3) = \frac{1}{8}$, and $p_x(4) = \frac{1}{8}$. Calculate the CDF function of X.

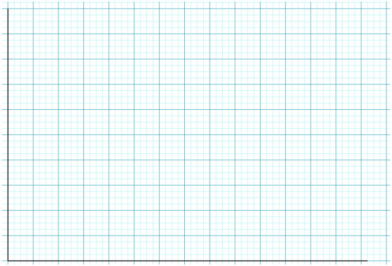
Example

Example : If X be a discrete random variable on the support $\mathbb{S}_x = \{1, 2, 3, 4\}$ with the corresponding pmf specified as $p_x(1) = \frac{1}{4}, p_x(2) = \frac{1}{2}, p_x(3) = \frac{1}{8}$, and $p_x(4) = \frac{1}{8}$. Calculate the CDF function of X.

Solution:

$$F_X\left(a\right) = \begin{cases} 0 & \text{if } a < 1 \\ \frac{1}{4} & \text{if } 1 \leq a < 2 \\ \frac{3}{4} & \text{if } 2 \leq a < 3 \\ \frac{7}{8} & \text{if } 3 \leq a < 4 \\ \frac{7}{8} & \text{if } 4 \leq a \end{cases}$$

Graph of CDF of X



Example

Let the pmf of a discrete random varibale X is given as

x	0	1	2
$p_X(x)$	0.04	0.32	0.64

Find the corresponding CDF.

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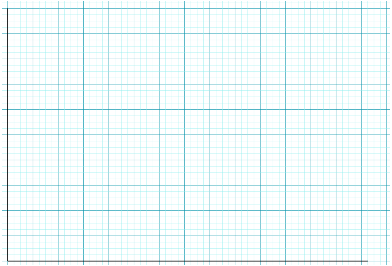
Find the corresponding CDF.

Solution:

$$F_X\left(\boxed{a}\right) = \begin{cases} 0 & \text{if } \boxed{a} < 0 \\ 0.04 & \text{if } 0 \leq \boxed{a} < 1 \\ 0.36 & \text{if } 1 \leq \boxed{a} < 2 \\ 1 & \text{if } 2 \leq \boxed{a} \end{cases}$$

Graph of CDF of X



Outline

Expected Value & Variance of a Discrete Random Variable

The "**Expected Value**" or "**Mean**" of a Discrete Random Variable

Definition (The "**Expected Value**" or "**Mean**" of a Discrete Random Variable)

If X is a random variable with pmf $p_X(x)$ on the support $\mathbb{S}_X$, then the expected value (the mean) of X denoted by $E(X)$ (or μ_X) is given by

$$\boxed{E\left(X\right) = \sum_{\left\{x \in \mathbb{S}_X\right\}} x \; p_X(x),}$$

assuming the above summation/series exists /well-defined.

Definition (The Expected Value of a Function of a Discrete Random Variable)

Let the random variable X has the probability mass funciton $p_X(x)$ for all $x \in \mathbb{S}_X$, the support of X. Let $h(x)$ be any* function, then the expected value of $h(X)$ is defined as

$$\boxed{E\Big(h(X)\Big) = \sum_{\{x \in \mathbb{S}_X\}} h(x)\, p_X(x),}$$

assuming the above summation/series exists /well-defined.

Variance

Variance The variance of X, denoted by Var(X) is deifined as

$$\boxed{\mathrm{Var}(X) := E\left(X - \mu_X\right)^2},$$

where $\mu_X = E(X)$, the mean of the random variable.

Definition (Variance)

The variance of X, denoted by Var(X) is deifined as

$$\boxed{\mathrm{Var}(X) := E(X^2) - \left(E(X)\right)^2}$$

Standard Deviation

Definition (Variance)

The variance of X, denoted by Var(X) is deifined as

$$\boxed{\sigma_X = \mathrm{SD}(X) := \sqrt{\mathrm{Var}(X)}}$$

Var(X) is often denoted by σ^2.

Properties of Expected Value and Variance

1. $E(a + bX) = a + bE(X)$
2. $\mathrm{Var}(a + bX) = b^2\mathrm{Var}(X)$

Moment Generating Function (mgf)

Moment Generating Function (mgf)

Definition (Moment Generating Function)

The Moment Generating Function (mgf) of X, denoted by $\mathrm{M}_x(t)$ is deifined as

$$\boxed{\mathrm{M}_x(t) := E\left(e^{tX}\right)},$$

whenever it exists.

If the random variable X has the probability mass funciton $p_x(x)$ for all $x \in \mathbb{S}_x$, the support of X, then assuming it exists

$$M_x(t) := E\left(e^{tX}\right) = \sum_{\{x \in \mathbb{S}_x\}} e^{tx}\, p_x(x).$$

Properties of a Moment Generating Function

- If it exists, the moment generating function is unique for a random variable. It means, no two random variable/distribution can have same moment geneting function. Therefore, a distribution can be identified by the form of its moment generating function.

- If it exists, the mgf can be used to obtain the moments of a random variable in the following way:

$$\frac{d}{dt}\{M_X(t)\}\mid_{t=0}= E(X)$$

$$\frac{d^k}{dt^k}\{M_X(t)\} \mid t = 0 = E(X^k) \text{ for } k = 1, 2, \ldots$$

Example

The probability distribution of X, the number of daily network black-outs is given by

x	0	1	2
$p_X(x)$	0.7	0.2	0.1

Find the Expected value and variance of the random variable **X.**

Example

The probability distribution of X, the number of daily network black-outs is given by

x	0	1	2
$p_X(x)$	0.7	0.2	0.1

Find the Expected value and variance of the random variable **X.**

Solution:

$$\begin{aligned}\mu_X = E(X) &= \sum_{x\in\{0,1,2\}} x p_X(x)\\ &= 0\times p_X(0) + 1\times p_X(1) + 2\times p_X(2)\\ &= 0\times 0.7 + 1\times 0.2 + 2\times 0.1\\ &= 0.4\end{aligned}$$

$$\begin{aligned}E(X^2) &= \sum_{x\in\{0,1,2\}} x^2 p_X(x)\\ &= 0^2\times p_X(0) + 1^2\times p_X(1) + 2^2\times p_X(2)\\ &= 0\times 0.7 + 1\times 0.2 + 4\times 0.1\\ &= 0.6\end{aligned}$$

Hence $\mathrm{Var}(X) := E(X^2) - (E(X))^2 = 0.6 - (0.4)^2 = 0.6 - 0.16 = 0.44$

The Standard Deviation $\sigma = \sqrt{\sigma^2} = \sqrt{\mathrm{Var}(X)} = \sqrt{0.44} = 0.6633$

Example

The probability distribution of X, the number of daily network blackouts is given by

x	0	1	2
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A small internet trading company estimates that each network blackout results in a $500 loss. Compute expectation and variance of this company's daily loss due to blackouts.

Example

The probability distribution of X, the number of daily network blackouts is given by

x	0	1	2
$p_X(x)$	0.7	0.2	0.1

A small internet trading company estimates that each network blackout results in a $500 loss. Compute expectation and variance of this company's daily loss due to blackouts.

The daily loss due to blackouts is given by $h(X) = 500X$. We need to find $E(h(X))$ and Varinace of $Var(h(X))$.

Solution:

$$\begin{aligned}
\mu_X = E(X) &= \sum_{x\in\{0,1,2\}} x p_X(x) \\
&= 0 \times p_X(0) + 1 \times p_X(1) + 2 \times p_X(2) \\
&= 0 \times 0.7 + 1 \times 0.2 + 2 \times 0.1 \\
&= 0.4
\end{aligned}$$

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&= 0^2 \times p_X(0) + 1^2 \times p_X(1) + 2^2 \times p_X(2) \\
&= 0 \times 0.7 + 1 \times 0.2 + 4 \times 0.1 \\
&= 0.6
\end{aligned}$$

Let the pmf of a discrete is given as

$$p_x(x) := \frac{1}{2^x} \text{ for } x = 1, 2, 3, \ldots$$

Exercises on Computing $E(X)$, $Var(X)$, MGF

Find $E(X)$ and $Var(X)$, where X is the outcome when we roll a fair die.

Exercises on Computing $E(X)$, $Var(X)$, MGF

We say that $\mathbb{I}_A(x)$ is an indicator function for the event A if

$$\mathbb{I}_A(x) := \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

Obtain $E(\mathbb{I}_A(X))$ and $\text{Var}\left(\mathbb{I}_A(x)\right)$.

Outline

Binomial Distribution

Binomial Distribution

- A **Bernoulli experiment** is a random experiment, the outcome of which can be classified in one of two mutually exclusive and exhaustive ways, say, “1=success" or “0=failure." Let Y be the number of success on a Bernoulli trial, then Y is called the Bernoulli random variable.

- If a sequence of n independent Bernoulli trials is performed under the same condition, we call it a set of n Bernoulli trials a Binomial experiment.

Binomial Distribution

Definition (Binomial Experiment)

An experiment is called a Binomial experiment if it satisfies the following 4 conditions:

- The experiment consists of n Bernoulli trials.
- Each trial results in a success (S) or a failure (F).
- The trials are independent.
- The probability of a success, p, is fixed throughout n trials.

Binomial Distribution Binomial(n, p)

1. Given a Binomial experiment consisting of n Bernoulli trials with success probability p, the Binomial random variable X associated with this experiment is defined as the number of successes among the n trials.
2. The random variable X has the Binomial Distribution with parameters n and p; denoted by$X \sim Binomial(n, p)$.
3. The behavior of Binomial Distribution with different n and p.

Binomial Distribution Binomial(n, p)

Definition (Binomial Distribution)

Let $p \in (0, 1)$, then the probability mass function of Binomial(n, p) is given by

$$\boxed{p(x) := \binom{n}{x} p^x (1-p)^{n-x}}, \text{ for } x \in \mathbb{S}_x, \text{ where } \mathbb{S}_x = \{0, 1, \ldots, n\}$$

Mean

$E(X) = np$

Variance

$\text{VAR}(X) = np(1-p)$

Expected Value of Binomial Distribution

$$
\begin{aligned}
E(X) &:= \sum_{y \in \mathbb{S}_X} y \, p_x(y) \\
&= \sum_{y=0}^{n} y \binom{n}{y} p^y (1-p)^{n-y} \\
&= (1-p)^n \sum_{y=0}^{n} y \binom{n}{y} \left(\frac{p}{1-p} \right)^y \\
&= (1-p)^n \frac{np}{(1-p)^n} \\
&= np
\end{aligned}
\tag{1}
$$

Expected Value of Binomial Distribution

$$
\begin{aligned}
E(X^2) &:= \sum_{y \in \mathbb{S}_X} y^2\, p_X(y) \\
&= \sum_{y=0}^{n} y^2 \binom{n}{y} p^y (1-p)^{n-y} \\
&= (1-p)^n \sum_{y=0}^{n} y^2 \binom{n}{y} \left(\frac{p}{1-p}\right)^y \\
&= (1-p)^n \frac{mp + n(n-1)p^2}{(1-p)^n} \\
&= np + n(n-1)p^2 \qquad (2)
\end{aligned}
$$

$Var(X) = E(x^2) - (E(X))^2 = np + n(n-1)p^2 - n^2p^2 = np - np^2 = np(1-p)$.

Expected Value of Binomial Distribution

$$
\begin{aligned}
M_X(t) &:= \sum_{y \in \mathbb{S}_X} e^{ty}\, p_x(y) \\
&= (1-p)^n \sum_{y=0}^{n} e^{ty} \binom{n}{y} \left(\frac{p}{1-p}\right)^y \\
&= (1-p)^n \sum_{y=0}^{n} \binom{n}{y} \left(\frac{pe^t}{1-p}\right)^y \\
&= (1-p)^n \sum_{y=0}^{n} e^{ty} \binom{n}{y} \left(\frac{p}{1-p}\right)^y \\
&= (1-p)^n \left(1 + \frac{pe^t}{1-p}\right)^n = \left(1 - p + pe^t\right)^n
\end{aligned} \tag{3}
$$

Example

Example : Five fair coins are flipped. If the outcomes are assumed independent.

1. Find the probability mass function of the number of heads obtained.
2. Find the probability that at least 3 heads are obtained.
3. Find the probability that at most 2 heads are obtained.

Example

Example : Five fair coins are flipped. If the outcomes are assumed independent.

1. Find the probability mass function of the number of heads obtained.
2. Find the probability that at least 3 heads are obtained.
3. Find the probability that at most 2 heads are obtained.

Solution: Let X = The number of heads in 5 tossed coins. $X \sim Binomial(n = 5, p = 0.5)$.

1. $P(X = 0) = 0.5^5 = 0.0313$
2. $P(X = 1) = \binom{5}{1} 0.5^5 = 0.1563$
3. $P(X = 2) = \binom{5}{2} 0.5^5 = 0.3125$
4. $P(X = 3) = \binom{5}{3} 0.5^5 = 0.3125$
5. $P(X = 4) = \binom{5}{4} 0.5^5 = 0.1563$
6. $P(X = 0) = \binom{5}{5} 0.5^5 = 0.0313$

Example

Example : It is known that screws produced by a certain company will be defective with probability .01, independently of each other. The company sells the screws in packages of 10 and offers a money-back guarantee that at most 1 of the 10 screws is defective. What proportion of packages sold must the company replace? Use the Binomial Calculator or Statistical Tables.

Example

Example : The following gambling game, known as the wheel of fortune (or chuck-a-luck), is quite popular at many carnivals and gambling casinos: A player bets on one of the numbers 1 through 6. Three dice are then rolled, and if the number bet by the player appears i times, i = 1; 2; 3, then the player wins i units; if the number bet by the player does not appear on any of the dice, then the player loses 1 unit. Is this game fair to the player?

Outline

Poisson Distribution

Poisson Distribution

The Poisson distribution models the number of occurrences of an event when there is a known average rate per unit time or space λ.

Definition (Poisson Distribution)

The requirements for a Poisson distribution are that:

1. no two events can occur simultaneously,
2. events occur independently in different intervals, and
3. the expected number of events in each time interval remain constant.

Poisson Distribution: pmf, Expected Value

The Poisson distribution models the number of occurrences of an event when there is a known average rate per unit time or space λ.

Definition (Poisson Distribution: pmf, Expected Value)

The requirements for a Poisson distribution are that:

1. The probability mass function of Poisson(λ) is given by

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ for } x = 0, 1, 2, 3, \ldots$$

2. If $X \sim Poisson(\lambda)$, then $E(X) = \lambda$, and $\mathrm{Var}(X) = \lambda$.

Expected Value of Binomial Distribution

$$
\begin{aligned}
M_X(t) &:= \sum_{y \in \mathbb{S}_X} e^{ty}\, p_x(y) \\
&= \sum_{y=0}^{\infty} e^{ty} \frac{e^{-\lambda} \lambda^y}{y!} \\
&= e^{-\lambda} \sum_{y=0}^{\infty} \frac{\left(\lambda e^t\right)^y}{y!} \\
&= e^{\lambda e^t - \lambda}
\end{aligned}
\tag{4}
$$

A few Examples of Poisson Distribution

Example : The number of customers arriving at a service counter within one-hour period.

Example : The number of typographical errors in a book counted per page.

Example : The number of email messages received at the technical support center daily.

Example : The number of traffic accidents that occur on a specific road during a month.

A Few Examples of Poisson Distribution

Example : Messages arrive at an electronic message center at random times, with an average of 9 messages per hour.

1. What is the probability of receiving exactly five messages during the next hour?
2. What is the probability that more than 10 messages will be received within the next two hours?

1. The number of messages received in an hour, X is modeled by Poisson distribution with $\lambda = 9$, i.e. $X \sim \text{Poisson}(9)$. $P(X = 5) = \frac{9^5 \exp(-9)}{5!}$
2. The number of messages received within a 2-hour period, Y is another Poisson distribution with $Y = (2)(9) = 18$, i.e. $Y \sim \text{Poisson}(18)$. $P(Y > 10) = 1 - P(Y \leq 10) = \ldots = 0.9696$

Group Work

1. Develop a real life example in which you can easily apply:
    1. Group 1: Poisson distribution.
    2. Group 2: Binomial distribution.
    3. Group 3: Poisson distribution
2. In each case, propose two problems which can be solved using the Statistical Calculator.
3. Can you propose an idea in which you can mix both distributions? (extra)

Outline

Geometric Distribution

Geometric Distribution

1. Suppose that independent trials, each having a probability p, $0 < p < 1$, of being a success, are performed until a success occurs.
2. Example: The first head in tossing coin several times.
3. Then, Geometric distribution models the number of trials performed until a success occurs.

Definition (Geometric Distribution)

The probability mass function of *Geometric*(p) is given by

$$p(x) = (1-p)^{x-1}p \text{ for } x = 1, 2, 3, \ldots,$$

- If $X \sim \text{Geometric}(p)$ then $E(X) = \frac{1}{p}$, and $\text{Var}(X) = \frac{1-p}{p^2}$

Expected Value of Binomial Distribution

$$\begin{aligned}
M_X(t) &:= \sum_{y \in \mathbb{S}_X} e^{ty}\, p_x(y) \\
&= \sum_{y=1}^{\infty} e^{ty}(1-p)^{y-1}p \\
&= p\sum_{z=0}^{\infty} e^{tz+t}(1-p)^z \\
&= pe^t \sum_{z=0}^{\infty} \left((1-p)e^t\right)^z \\
&= \frac{pe^t}{1-(1-p)e^t} \qquad (5)
\end{aligned}$$

Geometric Distribution: Example

Example : Suppose that the probability of engine malfunction during any one-hour period is $p = 0.02$. Find the probability that a given engine will survive two hours.

Geometric Distribution: Example

Example : Suppose that the probability of engine malfunction during any one-hour period is p = 0.02. Find the probability that a given engine will survive two hours.

Solution:
Letting Y denote the number of one-hour intervals until the first malfunction, we have

$$
\begin{aligned}
& P(\text{Survival for Next Two Hours}) \\
= \quad & P(Y \geq 3) \\
= \quad & 1 - P(Y \leq 2) \\
= \quad & 1 - \sum_{y=1}^{2} p(y) \\
= \quad & 1 - \{p(1) + p(2)\} \\
= \quad & 1 - 0.02 - 0.98 \times 0.02 \\
= \quad & 0.9604
\end{aligned}
$$

Exercise Find the mean and standard deviation of Y.

Outline

Negative Binomial Distribution

1. Suppose that independent trials, each having probability p, $0 < p < 1$, of being a success are performed until a total of r successes is accumulated.
2. Example: The third head in tossing coin several times.
3. Then, Negative Binomial distribution models the number of trials performed until a the rth success occurs.

Definition (Negative Binomial Distribution)

The probability mass function of Negative Binomial RV, denoted by Negative-Binomial(r, p) is given by

$$p(x) = \binom{x-1}{r-1} p^{r-1}(1-p)^{x-r} \text{ for } x = r+1, r+2, r+3, \ldots,$$

- If $X \sim$ Negative-Binomial(r, p) then $E(X) = \frac{r}{p}$, and $\text{Var}(X) = \frac{r(1-p)}{p^2}$

Geometric Distribution: Example

Example : A machine produces 1% defective parts. Using the statistical calculator, calculate the probability that

1. 10 parts have to be selected until to get 2 defective parts.
2. Between 20 to 25 parts have to be selected to get 2 defective parts.

Geometric Distribution: Example

Example : A machine produces 1% defective parts. Using the statistical calculator, calculate the probability that

1. 10 parts have to be selected until to get 2 defective parts.
2. Between 20 to 25 parts have to be selected to get 2 defective parts.

Solution:
Letting Y denote the number of

$$AA \tag{6}$$

Exercise Find the mean and standard deviation of Y.

Questions?