Exam Assistance Note

Properties of Probability

Let $(\mathscr{S}, P)$ be a sample space along with the Probability measure. Let A, B be two events. Then,

- $P(\varnothing) = 0$ where $\varnothing$ denotes the Empty set (Null set).
- $P(A) \leq 1$.
- If $A \subseteq B$ then $P(A) \leq P(B)$.
- $P\left(\overline{A}\right) = 1 - P(A)$, where $\overline{A}$ denotes the complementary event to A.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Let E, and F are two events such that $P(F) > 0$, then the conditional probability of E **given** F is defined to be,

$$P(E \mid F) := \frac{P(E \cap F)}{P(F)}.$$

Let E and F are two events, then $P(E \cap F) := P(E \mid F) \times P(F)$.

Bayes' Theorem

Let A, B are two events such that $P(A) > 0$, and $P(B) > 0$, then

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A \mid B)P(B) + P\left(A \mid \overline{B}\right)P\left(\overline{B}\right)}$$

Statistical Independence

Two events E and F are said to be statistically independent if $P(E \cap F) = P(E) \times P(F)$

Characterization of a pmf

Let $p(x)$ is **probability mass function** of a discrete random variable on the support $\mathbb{S}$, **if and only if** it satisfies the following conditions:

1. *Positivity:* $p(x) > 0$ for all $x \in \mathbb{S}$
2. *Total Probability:* $\sum_{\{x \in \mathbb{S}\}} p(x) = 1$.

"CDF" of a Discrete Random Variable

Let X be a discrete random variable on the support $\mathbb{S}_X$ with the corresponding probability mass function

$$P(X = x) = p_X(x) \text{ for } x \in \mathbb{S}_X.$$

Then for any $a \in \mathbb{R}$, the cumulative distribution function (cdf), denoted by $F_X(\cdot)$ is the following quantity

$$F_X(a) = P(X \leq a) = \sum_{\{x \leq a \,:\, x \in \mathbb{S}_X\}} p_X(x)$$

"Expected Value" or "Mean" of a Discrete Random Variable

If X is a random variable with pmf $p_X(x)$ on the support $\mathbb{S}_X$, then the expected value (the mean) of X denoted by $E(X)$ (or μ_X) is given by

$$E(X) = \sum_{\{x \in \mathbb{S}_X\}} x \, p_X(x),$$

assuming the above summation/series exists /well-defined. Additionally, assuming it exists, for any* funciton $h(x)$,

$$E\left(h(X)\right) = \sum_{\{x \in \mathbb{S}_X\}} h(x) \, p_X(x),$$

"Variance & Standard Deviation (SD)" of a Random Variable

The variance of X, denoted by $\mathrm{Var}(X)$ is deifined as

$$\mathrm{Var}(X) := E(X^2) - \left(E(X)\right)^2$$

$$\mathrm{E}(X^2) := \mathrm{Var}(X) + \left(E(X)\right)^2$$

$$\sigma_X = \mathrm{SD}(X) := \sqrt{\mathrm{Var}(X)}$$

"Moment Generating Function (MGF)"

The Moment Generating Function (mgf) of X, denoted by $\mathrm{M}_X(t)$ is deifined as

$$\mathrm{M}_X(t) := E\left(e^{tX}\right)$$ whenever it exists.

If X is a discrete random variable with p.m.f $p_X(x)$ then

$$M_X(t) := E\left(e^{tX}\right) = \sum_{\{x \in \mathbb{S}_X\}} e^{tx}\, p_X(x) .$$

If X is a continuous random variable with a probability density function $f_X(x)$ then

$$M_X(t) := E\left(e^{tX}\right) = \int_{\{x \in \mathbb{S}_X\}} e^{tx}\, f_X(x)\, dx .$$

Let X be a r.v. with the moment generating function $M_X(t)$, then, assuming existence, the r^{th} raw moments (non-centered), for the random variable can be obtained as

$$E(X^r) = \left.\frac{d^r M_X(t)}{dt^r}\right|_{t=0}$$ for any positive integer r.

Specifically,

$$E(X) = \left.\frac{dM_X(t)}{dt}\right|_{t=0}$$

"Probability Density Function"

Let $F_X(x)$ be a cumulative distribution function (CDF) of a continuous random variable, then the corresponding **probability density function** or **pdf**, denoted as $f_X(x)$ is a function that satisfies the following creteria.

$$F_X(x) = \int_{-\infty}^{x} f_X(y)\,dy$$

pdf of a continuous random variable can be obtained by differentiating the corresponding **CDF**

$$f_X(x) = \frac{d}{dx}F_X(x)$$

If X is a continuous random variable, then probabilities can be obtained by integrating its pdf over suitable region. Specifically, for $a, b \in \mathbb{R}$, $a < b$,

$$P\left(a < X \leq b\right) = F_X\left(b\right) - F_X\left(a\right) = \int_a^b f_X(x)dx.$$

"Characterization of Probability Density Function (pdf)"

A function $f(x)$ is is a probability density function (pdf) for some continuous random variable if and only if it satisfies the following two conditions:

1. **Non-negativity:** $f(x) \geq 0$ for all x.
2. **Total Probability:** $\int_{-\infty}^{\infty} f(x)dx = 1$

"Quantiles, and Percentiles"

Let p be a number between 0 and 1. The $(100)^{\text{th}}$ percentile of the distribution of a continuous random variable X, we shall denote by c, is that value for which

$$F\left(c\right) = p$$

i.e. $c = F^{-1}\left(p\right)$. where $F^{-1}(\cdot)$ is the inverse cumulative distribution function.

Median The median of a continuous distribution, denoted by m, is the 50th percentile. So m satisfies

$$m = F^{-1}(0.5)$$

First Quartile The first quartile is defined to be

$$Q_1 = F^{-1}(0.25)$$

Third Quartile The third quartile is defined to be

$$Q_3 = F^{-1}(0.75)$$

"Expected Value" or "Mean" of a Continuous Random Variable

If X is a continuous random variable with pdf $f(x)$ on the support $\mathbb{S}_X$, then the expected value (the mean) of X denoted by $E(X)$ is given by

$$E(X) = \int_{\{x \in \mathbb{S}_X\}} x \, f_X(x) \, dx,$$

assuming the above summation/series exists /well-defined. Additionally, assuming it exists, for any* funciton $h(x)$,

$$E\left(h(X)\right) = \int_{\{x \in \mathbb{S}_X\}} h(x) \, f_X(x) \, dx,$$

Z-Transformation

If $X \sim \mathrm{Normal}(\mu, \sigma^2)$ for some $\mu \in \mathbb{R}$ and $\sigma^2 > 0$ then ,

$$\boxed{Z \sim \mathrm{Normal}(0,1) \text{ where } Z = \frac{X-\mu}{\sigma}}$$

Probability Mass Function (pmf)

For a discrete random vector (X,Y), we define the probability mass function (pmf) $p_{X,Y}(x,y)$ of X by $p_{X,Y}(x,y) = P(X = x, Y = y)$ for all $(x,y) \in \mathbb{S}_{X,Y}$

Bivariate CDF

Let X, Y be two discrete random variables. The joint cumulative distribution function is given by $F_{X,Y}(x, y) := P\left(X \le x, Y \le y\right)$.

Marginal Distribution of a Discrete Random Variable

The marginal probability mass function of X is given by

$$p_X(x) = \sum_{\{t : (x,t) \in \mathbb{S}_{XY}\}} p_{X,Y}(x, t)$$

The marginal probability mass function of X is given by

$$p_Y(y) = \sum_{\{s : (s,y) \in \mathbb{S}_{XY}\}} p_{X,Y}(s, y)$$

Joint Density

We say that X and Y are jointly continuous if there exists a function $f_{X,Y}(x,y)$, defined for all real x and y, having the property that, for every set C of pairs of real numbers (that is, C is a set in the two-dimensional plane),

$$P\Big((X,Y) \in C \Big) = \iint_{\{(x,y)\in C\}} f_{X,Y}(s,t)\,ds\,dt$$

$f_{X,Y}(x,y)$ is called the joint probability density function of the random vector (X,Y).

Joint CDF from Joint p.d.f.

If the joint probability density function of X and Y is $f_{X,Y}(x,y)$. then

$$F_{X,Y}(x,y) = \int\int_{\left\{\begin{array}{l} s \le x,\ t \le y \\ \text{where } (s,t) \in \mathbb{S}_{X,Y} \end{array}\right\}} f_{X,Y}(s,t)\,ds\,dt$$

If CDF of a bivariate continuous random variable is provided, then the corresponding p.d.f is obtained by following:

$$f_{X,Y}(x,y) = \frac{d^2 F(x,y)}{dx\,dy}$$

Marginal Distribution of a Continuous Random Variable

The marginal probability mass function of X is given by

$$f_X(x) = \int_{\{y:(x,y)\in \mathbb{S}_{XY}\}} f_{X,Y}(x,y)\,dy$$

The marginal probability mass function of Y is given by

$$f_Y(y) = \int_{\{x:(x,y)\in \mathbb{S}_{XY}\}} f_{X,Y}(x,y)\,dx$$

Conditional p.m.f. of a Discrete Random Variable

If $p_{XY}(x, y)$ denotes the joint probability mass function (pmf) of two discrete random variables X and Y and if $p_X(x)$ and $p_Y(y)$ denote the marginal probability function of X, (Y respectively), then

The conditional probability of X given $Y = y$ is given by

$$\boxed{p_{X|Y}(x \mid y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}}$$

The conditional probability of Y given X = x is given by

$$\boxed{p_{Y|X}(y \mid x) = \frac{p_{X,Y}(x,y)}{p_X(x)}.}$$

Conditional p.d.f. of a Continuous Random Variable

If $f_{XY}(x, y)$ denotes the joint probability density function of two continuous random variables X and Y and if $f_X(x)$ and $f_Y(y)$ denote the marginal probability density function of X, (Y respectively), then

The conditional probability density of X given $Y = y$ is given by

$$\boxed{f_{X|Y}(x \mid y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}}$$

The conditional probability density of Y given X = x is given by

$$\boxed{f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)}.}$$

Statistically Independent Random Variable

The random variables X and Y are said to be **statistically independent** random variables if, for any two events A and B,

$$P(X \in A, Y \in B) = P(X \in A) \times P(Y \in B)$$

Statistically Independent Continuous Random Variables

- Let (X, Y) be bivariate continuous random vector with a probability density function $f_{X,Y}(x, y)$ on the support $(x, y) \in \mathbb{S}_{X,Y}$.
- Let $f_X(x)$ be the marginal density of the random variable X on the support $\mathbb{S}_X$
- Let $f_Y(y)$ be the marginal density of the random variable X on the support $\mathbb{S}_Y$.

The continuous random variables X and Y are **statistically independent** if the corresponding joint probability density function

$$f_{X,Y}(x, y) = f_X(x) \times f_Y(y)$$

for all x and y, and $\mathbb{S}_{X,Y} = \mathbb{S}_X \times \mathbb{S}_Y$.

Statistically Independent Discrete Random Variables

- Let (X,Y) be bivariate continuous random vector with a probability density function $f_{X,Y}(x,y)$ on the support $(x,y) \in \mathbb{S}_{X,Y}$.
- Let $f_X(x)$ be the marginal density of the random variable X on the support $\mathbb{S}_X$
- Let $f_Y(y)$ be the marginal density of the random variable X on the support $\mathbb{S}_Y$.

The continuous random variables X and Y are **statistically independent** if the corresponding joint probability density function

$$f_{X,Y}(x,y) = f_X(x) \times f_Y(y)$$

for all x and y, and $\mathbb{S}_{X,Y} = \mathbb{S}_X \times \mathbb{S}_Y$.

A Few Properties of Statistically Indipendent Random Variables

Let X,Y be any two statistically independent random variables then then the following facts are true:

- For any two events A,B $P(X \in A, Y \in B) = P(X \in A) \times P(Y \in B)$
- For any two functions* $h(x)$ and $g(y)$ $E(g(X)h(Y)) = E(g(X))E(h(Y))$
- If the X, Y has the marginal CDFs $F_X(x)$ and $F_Y(y)$ respectively, then the joint CDF $F_{X,Y}(x,y) = F_X(x) \times F_Y(y)$ for all x, y.

"Expected Value" of a function of a Bivariate Random Vectors

Let X, Y be two discrete random variables with joint probability function $p_{X,Y}(x,y)$. Then the expected value of $g(X,Y)$ is given by

$$E(g(X,Y)) = \sum\sum_{(x,y)\in\mathbb{S}_{XY}} g(x,y)p_{X,Y}(x,y)$$

Let X, Y be two continuous random variables with joint probability density function $f_{X,Y}(x,y)$. Then the expected value of $g(X,Y)$ is given by

$$E(g(X,Y)) = \int\int_{(x,y)\in\mathbb{S}_{XY}} g(x,y)f_{X,Y}(x,y)dx\,dy$$

"Covariance" Between Two Random Variables

Let X, and Y be two random variables with a joint distribution. Then

$$\text{Cov}(X,Y) = E\left((X-\mu_X)(Y-\mu_Y)\right),$$

where μ_X and μ_Y denotes the mean of the random variables X, and Y respectively.

An Alternative Formulation for the covariance is the following:

$$\text{Cov}(X,Y) = E(XY) - E(X)E(Y)$$

Theorem: If X, and Y are two **statistically independent** random variables, then $\text{Cov}(X,Y) = 0$.

However, the converse of the result is not true in general. i.e. $\text{Cov}(X,Y) = 0$ does not imply that X, Y must be Statistically Independent.

Mean, Variance, and Covariance" of Linear Combinations of Random Variables

Let $X_1, X_2, \ldots X_n$ are random variables and $Y = a_0 + \sum_{i=1}^{n} a_i X_i$, where $a_i^{\prime}s$ are constants then

- $E(Y) = a_0 + \sum_{i=1}^{n} a_i E(X_i)$
- $\text{Var}(Y) = \sum_{i=1}^{n} a_i^2 \text{Var}(X_i) + 2 \sum\sum_{1 \le i < j \le n} a_i a_j \text{Cov}(X_i, X_j)$

Let $X_1, X_2, \ldots X_n$ are random variables $Y_1 = a_0 + \sum_{i=1}^{n} a_i X_i$, and $Y_2 = b_0 + \sum_{i=1}^{n} b_i X_i$, where $a_i^{\prime}s$ abd $b_i^{\prime}s$ are constants then

$$\text{Cov}(Y_1, Y_2) = \sum_{i=1}^{n} a_i b_i \text{Var}(X_i) + 2 \sum\sum_{1 \le i < j \le n} a_i b_j \text{Cov}(X_i, X_j)$$

- If $X_1, X_2, \ldots X_n$are mutually pairwise **statistically independent** then,

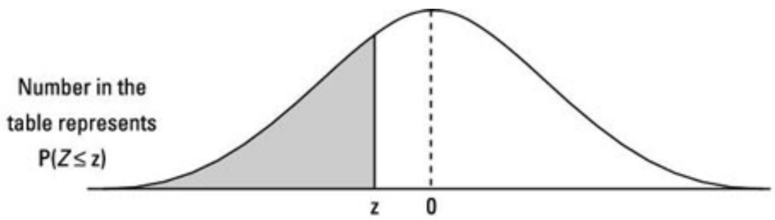
$$\text{Var}(Y) = \textstyle\sum_{i=1}^{n} a_i^2 \text{Var}(X_i), \quad \text{Cov}(Y) = \displaystyle\sum_{i=1}^{n} a_i b_j \text{Var}(X_i)$$

Standard Properties of a few Discrete Distributions

Distribution	Support $\mathbb{S}_X$	pmf $p_X(x)$	Mean $E(X)$	Variance $\mathrm{Var}(X)$	mgf $M_X(t)$
Binomial(n,π)	$\{0,1,\ldots,n\}$	$\binom{n}{x}\pi^x(1-\pi)^{n-x}$	$n\pi$	$n\pi(1-\pi)$	$(1-\pi+\pi e^t)^n$
Poisson(λ)	$\{0,1,2,\ldots\}$	$\frac{e^{-\lambda}\lambda^x}{x!}$	λ	λ	$e^{\lambda e^t-\lambda}$
Geometric(π)	$\{1,2,\ldots\}$	$(1-\pi)^{x-1}\pi$	$\frac{1}{\pi}$	$\frac{1-\pi}{\pi^2}$	$\frac{\pi e^t}{1-(1-\pi)e^t}$
Negative-Binomial(r,π)	$\{r,r+1,r+2,\ldots\}$	$\binom{x-1}{r-1}(1-\pi)^{x-r}\pi^r$	$\frac{r}{\pi}$	$\frac{r(1-\pi)}{\pi^2}$	$\left(\frac{\pi e^t}{1-(1-\pi)e^t}\right)^r$

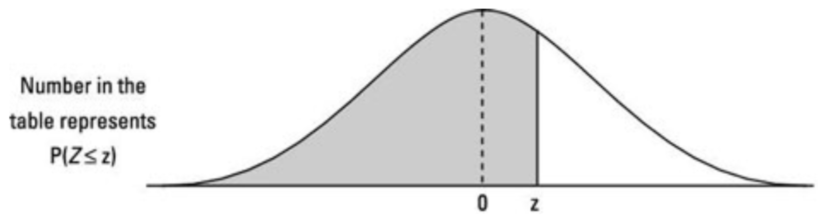
Standard Properties of a few Continuous Distributions

Distribution	Support $\mathbb{S}_X$	pdf $f_X(x)$	Mean $E(X)$	Variance $\mathrm{Var}(X)$	mgf $M_X(t)$
Uniform(a,b)	$[a,b]$	$\begin{cases}\frac{1}{b-a} & \text{if } x\in[a,b]\\ 0 & \text{otherwise .}\end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$M_X(t)=\begin{cases}\frac{e^{tb}-e^{ta}}{t(b-a)} & \text{if } t\neq 0\\ 1 & \text{if } t=0\end{cases}$
Exponential(λ)	$(0,\infty)$	$\begin{cases}\lambda e^{-\lambda x} & \text{if } x>0\\ 0 & \text{otherwise .}\end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}$ if $0\le t<\lambda$
Gamma(α,λ) shape $=\alpha$, rate $=\lambda$	$(0,\infty)$	$\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$ if $x>0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\frac{1}{(1-\frac{t}{\lambda})^\alpha}$ if $0\le t<\lambda$
Beta(α,β)	$(0,1)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}x^{\beta-1}$ if $0<x<1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$---$
Normal(μ,σ^2) mean $=\mu$, Var $=\sigma^2$	$(-\infty,\infty)$	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ for $x\in\mathbb{R}$	μ	σ^2	$e^{\mu t+\frac{t^2\sigma^2}{2}}$



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997