Assignment 3

STAT 230
UAEU

There are a total of 6 problems. You may review **Unit6 slides** while answering the questions. **Show your steps to get entire credit for your solutions.**

1. The time in days between breakdowns of a machine is exponentially distributed with rate parameter $\lambda = 0.2$
   (a) (5 points) Write down the probability density function of the random variable X that records time in days between breakdowns of the machine.
   (b) (5 points) What is the expected time (in days) between machine breakdowns?
   (c) (8 points) Let a, b be two positive constants such that $a > b$. Derive an expression for
   $$P(X > a \mid X > b).$$
   (d) (7 points) Use the part(c) to obtain the probability that the machine lasts at least two more days before breaking down **given** that the machine has already performed satisfactorily for last 15 days?

2. A new battery supposedly with a charge of 1.5 volts actually has a voltage with a Uniform distribution between 1.43 and 1.60 volts.
   (a) (5 points) What is the expected value of the voltage?
   (b) (5 points) What is the standard deviation of the voltage?
   (c) (10 points) What is the probability that a battery has a voltage less than 1.48 volts?

3. (10 points) Let X follows a Uniform(0,1) distribution. Let Y be a random variable such that
   $$Y = X^{0.5}(1-X)^{1.5}.$$
   Derive the value of $E(Y)$.

4. A Wall Street analyst estimates that the annual return from the stock of company A can be considered to be an observation from a normal distribution with mean $\mu = 8.0$ percent and standard deviation $\sigma = 1.5$ percent. The analyst's investment choices are based upon the considerations that any return greater than 5 percent is "satisfactory" and a return greater than 10 percents "excellent."

(a) (5 points) What is the probability that company A's stock will prove to be "unsatisfactory" ?

(b) (5 points) What is the probability that company A's stock will prove to be "Excellent" ?

5. Suppose a certain mechanical component produced by a company has a width that is normally distributed with a mean $\mu = 2600$ and a standard deviation $\sigma = 0.6$.

(a) (8 points) What proportion of the components have a width outside the range 2599 to 2601?

(b) (7 points) If the company needs to be able to guarantee to its purchaser that no more than 1 in 1000 of the components have a width outside the range 2599 to 2601, then what should be the specification of (value of) σ?

6. The velocities of gas particles, denoted here by V, can be modeled by the Maxwell distribution. For simplicity of notations, consider the random variable X, that refers to the square of the velocity, i.e. , $X = V^2$. It can be shown that the probability density functionof X is given as

$$f_X(x) := \begin{cases} 4\pi\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}} \sqrt{x}\, e^{-\left(\frac{m}{2KT}\right)x} & \text{for } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

where m is the mass of the particle, K is Boltzmann's constant, and T is the absolute temperature. From the perpespective of the probability distribution, assume m, K, T to ve constants.

(a) (5 points) Identify the probability distribution of the random variable X.

(b) (8 points) The kinetic energy of a particle is given by $\frac{1}{2}mV^2$. Find the mean (expected value of the) kinetic energy for a particle.

(c) (7 points) Derive the mean velocity (Expected Value of V) of these particles.