A Few Problems Aiming the Midterm Exam

1. An airline has four flights (4 flight numbers) from New-York to California and five flights (5 different flight numbers) from California to Hawaii per day. If the flights are to be made on separate days, how many different flight arrangements can the airline offer from New-York to Hawaii?

2. A car manufacturer provides cars with the following different variations:
   - Manual or automatic transmission
   - Three different stereo systems
   - Four possible exterior colors

   How many different types of car the manufacturer sells?

3. How many different numbers can be constructed using a 64 bits binary digits?

4. A typical lisence plate number in Abu-Dhabi consists of 5 digits. How many different lisence plate numbers (excluding '00000') are possible that esnds with 2 ?

Example : How many different "words" (sequence of three letters that may or may not be dictionary words) can you make with the letters "MISSISSIPPI" ?

5. A class in probability theory consists of 10 men and 8 women. An examination is given, and the students are ranked according to their performance. Assume that no two students obtain the same score.
   a). How many different rankings are possible?
   b). Assuming equally likeli setup, what is the probability a Female student appears to be first rank.

6. An investor has 20 thousand dollars to invest among 4 possible investments. Each investment must be in units of a thousand dollars. If the total 20 thousand is to be invested,
   (a) how many different investment strategies are possible?
   (b) how many different investment strategies are possible if not all the money need be invested?

7. 12 identical balls are distributed among 4 kids. Assuming all possible of number of distributions of the balls are equally likely, idensity the following probabilities. What is probability that each of the kids obtains one ball or more?

8. A smoke detector system uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is 0.95; by device B, 0.90; and assume that both the devices work independently.
   (a) If smoke is present, find the probability that the smoke will be detected by either device A or B, or both devices.
   (b) Find the probability that the smoke will be undetected.

9. A smoke detector system uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is 0.95; by device B, 0.90; and by both devices, 0.88.
   (a) If smoke is present, find the probability that the smoke will be detected by either device A or B, or both devices.
   (b) Find the probability that the smoke will be undetected.

Example : Suppose that A and B are mutually exclusive events for which $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that
   (a) either A or B occurs?
   (b) A occurs but B does not?
   (c) both A and B occur?

10. A car repair can be performed either on time or late and either satisfactorily or unsatisfactorily. The probability of a repair being on time and satisfactory is 0.26. The probability of a repair being on time is 0.74. The probability of a repair being satisfactory is 0.41. What is the probability of a repair being late and unsatisfactory?

Let A, B be two events such that

$$P(A) = \frac{2}{3} \text{ and } P(\overline{B}) = \frac{1}{4}.$$

Can A and B be disjoint? Explain.

11. A fair-dice is thrown 8 times and all the the 8 numbers that appear are recorded.
   (a) What is the probability that there is no Six?
   (b) What is the probability that there is at least one Six?
   (c) What is the probability that there will be exactly 3 $Four$'s and 2 $Three$'s among the eight throws?

12. A typical ATM pin consists of 4 digits. Assume that all the integers between 0 to 9 are equally likely for selecting each of the digits. Find the probability of the following events.
   (a) What is the probability that all the four digits of a randomly selected ATM pin is different?
   (b) *The bank has selected a group of 120 ATM users randomly. What is the probability that atleast two of the users have exact same ATM pin? Assume that all the possible four digit numbers are equally likely to be considered as a PIN of a randomly selected user.
   (c) *What would be corresponding probability if 175 ATM users are randomly chosen instead?

13. A card is selected randomly from the deck of 52 cards.
   - Let A be the event that the card is an 'Ace'.
   - Let B be the event that the card is either 'King' or a 'Queen'.
   - Let C be the event that the card is 'Diamonds'.

   (a) Is the events A and B statistically independent? (Justify your answer)
   (b) Is the events B and c statistically independent? (Justify your answer)

14. If two events, A and B, are such that $P(A) = 0.6$, $P(B) = 0.2$, and $P(A \cap B) = 0.1$, find the following
   (a) $P(A \mid B)$
   (b) $P(B \mid A)$
   (c) $P(A \mid A \cup B)$

15. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The companys statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability is 0.2 for a person who is not accident prone. If we assume that 30% of the population is accident prone, what is the probability that a new policyholder will have an accident within a year of purchasing a policy?

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Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

16. Suppose that we toss 2 fair dice. Let E_1 denote the event that the sum of the dice is 6 and F denote the event that the first die equals 4. Is E_1 statistically independent of F?

17. If two events A, and B are statistically independent, then
   (a) A and $\overline{B}$ are also statistically independent,
   (b) $\overline{A}$ and B are also statistically independent.
   (c) $\overline{A}$ and $\overline{B}$ are also statistically independent.

18. If A and B are independent events with $P(A) = 0.5$ and $P(B) = 0.2$, find the following:
    (a) $P(A \cap B)$ (b) $P(A \cup B)$ (c) $P(\bar{A} \cap \bar{B})$ (d) $P(\bar{A} \cup \bar{B})$

19. A diagnostic test for a disease is such that it (correctly) detects the disease in 92% of the individuals who actually have the disease. Also, if a person does not have the disease, the test will report that he or she does not have it with probability 0.9. Only 2% of the population has the disease in question. If a person is chosen at random from the population and the diagnostic test indicates that she has the disease, what is the conditional probability that she does, in fact, have the disease? Are you surprised by the answer? Would you call this diagnostic test reliable?

20. After production, an electrical circuit is given a quality score of A, B, C, or D. Over a certain period of time, 77% of the circuits were given a quality score A, 11% were given a quality score B, 7% were given a quality score C, and 5% were given a quality score D. Furthermore, it was found that 2% of the circuits given a quality score A eventually failed, and the failure rate was 10% for circuits given a quality score B, 14% for circuits given a quality score C, and 25% for circuits given a quality score D.
    (a) If a circuit did not fail, what is the probability that it had received a quality score A?
    (b) If a circuit failed, what is the probability that it had received a quality score either C or D?

21. We know the following about a colormetric method used to test lake water for nitrates. If water specimens contain nitrates, a solution dropped into the water will cause the specimen to turn red 95% of the time. When used on water specimens without nitrates, the solution causes the water to turn red 10% of the time (because chemicals other than nitrates are sometimes present and they also react to the agent). Past experience in a lab indicates that nitrates are contained in 30%of the water specimens that are sent to the lab for testing. If a water specimen is randomly selected
    (a) from among those sent to the lab, what is the probability that it will turn red when tested?
    (b) It it turns red when tested, what is the probability that it actually contains nitrates?

22. Four radar sets, operating independently, are set to detect any aircraft flying through a certain area. Each set has a probability of .95 to correctly detect a plane in its area. If an aircraft enters the area, what is the probability that it
    (a) goes undetected?
    (b) is detected by at least one radar set?
    (c) is detected by exactly 2 radars?
    (d) is detected by at least 2 radars?

23. Consider 6 tosses of a fair coin. For example, two typical sequences of outcomes that are considered different is $`HTTTTT'$, and $`THTTTT'$.
    (a) What is the probability that there are be exactly 2 Heads ?
    (b) What is the probability that there are be exactly 4 Heads ?
    (c) What is the probability that there are no Heads?
    (d) What is the probability that there is at least one Tail?

24. If A and B are two events such that $P(A) > 0$ and $P(B > 0)$ then,
   (a) If the events A and B are Statistically Independent then they can not be disjoint./mutually exclushive.
   (b) If the events A and B are disjoint then they can not be statistically independent.

25. Suppose a random variable X has the following support $\mathbb{S}_X = \{1, 2, 3, 4, 5\}$.

x	1	2	3	4	5
$p_X(x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	c	$\frac{1}{10}$

   (a) What is the value of c so that $p_X(x)$ become a valid probability mass function on the support $\mathbb{S}_X$?
   (b) What is the probability that $X \in A$, where the event $A = \{2, 4\}$.
   (c) What is the expected value of the random variable?

26. If X be a discrete random variable on the support $\mathbb{S}_X = \{1, 2, 3, 4\}$ with the corresponding pmf specified as $p_X(1) = \frac{1}{4}, p_X(2) = \frac{1}{2}, p_X(3) = \frac{1}{8}$, and $p_X(4) = \frac{1}{8}$. Calculate the Expected value of X.

27. The probability distribution of X, the number of daily network blackouts is given by

$x \in \mathbb{S}_X$	0	10	40
$p_X(x)$	0.7	0.2	0.1

   (a) Find the Expected value, variance of the random variable **X**.
   (b) Find the MGF of the random variable **X**.
   (c) Find $E\left(\frac{1}{2+|X|}\right)$
   (d) Find $E\left(\sqrt{X+1}\right)$
   (e) Find the CDF of the distribution.

28. Derive the MGF of the Poisson(λ) distribution where $\lambda > 0$.

29. Derive the MGF of a Geometric(π) distribution where $1 > \pi > 0$.

30. Derive the MGF of a Poisson(λ) distribution where $\lambda > 0$.

31. If $X \sim \text{Binomial}(n = 50, \pi = 0.1)$ then
   (a) Obtain the value of $E(X)$, $\text{Var}(X)$, and $E(X^2)$.
   (b) What is the MGF of X?

32. If $X \sim \text{Geometric}(\pi = 0.2)$ then
   (a) Obtain the value of $E(X)$, $\text{Var}(X)$, and $E(X^2)$.
   (b) What is $E(4X + 10)$?
   (c) What is $\text{Var}(4X + 10)$?
   (d) What is the MGF of X?

33. If $X \sim \text{Poisson}(\lambda = 5)$, then
   (a) Obtain the value of $E(X)$, $\text{Var}(X)$, and $E(X^2)$
   (b) What is $E(3X + 50)$?
   (c) What is $\text{Var}(3X + 50)$?
   (d) What is the MGF of X?

34. Of a population of consumers, 60% are reputed to prefer a particular brand, A, of toothpaste. If a group of randomly selected consumers is interviewed,
   (a) what is the probability that **exactly 3** people have to be interviewed to encounter the first consumer who prefers brand A?
   (b) what is the probability that **at least** 3 people have to be interviewed to encounter the first consumer who prefers brand A?

35. Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour.
   (a) During a given hour, what are the probabilities that no more than two customers arrive?
   (b) During a given hour, what are the probabilities that exactly five customers arrive?
   (c) During a **given two hours period**, what are the probabilities that exactly 6 customers arrive?

36. On a specific project, an oil exploration firm has decided to allocate the required finance for 10 explorations. The probability of a particular exploration being successful is 0.10 . Assume the explorations are statisticallly independent to each other.
   (a) Find the mean and variance of the number of successful explorations.
   (b) Suppose the firm has a fixed cost of $20,000 in preparing equipment prior to doing its first exploration. If each successful exploration costs $30,000 and each unsuccessful exploration costs $15,000, find the expected total cost to the firm for its ten explorations.

37. An oil prospector will drill a succession of holes in a given area to find a productive well. The probability that he is successful on a given trial is 0.1.
   (a) What is the probability that the third hole drilled is the first to yield a productive well?
   (b) What is the expected number of drills that are required to get the first productive well?

38. An oil prospector will drill a succession of holes in a given area to find a productive well. The probability that he is successful on a given trial is 0.40. The company wants to fund as much drills that are required to obtain 3 productive wells.
   (a) What is the probability that the company need 9 drills to obtain the 3 productive wells.
   (b) What is the expected number of drills that are required to get the first productive well?
   (c) Suppose the firm require a fixed cost of \$20,000 in preparing equipment prior to doing its first exploration. If each successful exploration costs \$0,000 and each unsuccessful exploration costs \$10,000. Compute the expected budget that is required to be allocated for the project.

39. An airline company have sold 50 tickets of a specific route of flight. From the past experiences, the company knows that the probability of a randomly selected customer would cancel ticket is 2%. Assume that the customers (if cancels) cancels tickets independently.
   (a) what is the probability that exactly two customers will cancel their ticket.
   (b) What is the expected number of coustomers who may cancel their tickets.

40. A particular sale involves four items randomly selected from a large lot that is known to contain 10% defectives. Let Y denote the number of defectives among the 20 sold. The purchaser of the items will return the defectives for repair, and the repair cost is given by $U = 3Y^2 + Y + 2$. Find the expected repair cost, $E(U)$.