

# STAT 320: Principles of Probability

## Unit 6 Part:A

### Continuous Random Variables

United Arab Emirates University

Department of Statistics

# Outline

- 1 Characterization of any CDF function
- 2 Continuous Random Variables
- 3 Percentiles, Quantiles, and Median
- 4 Expected Value, Variance & MGF of a Continuous Random Variable
- 5 A Few Examples

## Reminder: The Cumulative Distribution Functions

# Distribution Functions

## Definition (Cumulative Distribution Function (cdf))

The **cumulative distribution function** or **cdf** of a *any* variable  $X$ , denoted by  $F_X(x)$ , is defined by

$$F_X(x) = P(X \leq x) \text{ for all } x \in \mathbb{R}.$$

# CDF: Example

Consider the experiment of tossing three fair coins, and let  $X$  = number of heads observed. We have already seen that

$x$	0	1	2	3
$p_X(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

The cdf of  $X$  is:

$$F_X(x) = \begin{cases} 0 & \text{if } -\infty < x < 0 \\ \frac{1}{8} & \text{if } 0 \leq x < 1 \\ \frac{4}{8} & \text{if } 1 \leq x < 2 \\ \frac{7}{8} & \text{if } 2 \leq x < 3 \\ 1 & \text{if } 3 \leq x < \infty \end{cases}$$

$x$	0	1	2	3
$P_X(X = x)$	$\frac{1}{8} = 0.125$	$\frac{3}{8} = 0.375$	$\frac{3}{8} = 0.375$	$\frac{1}{8} = 0.125$

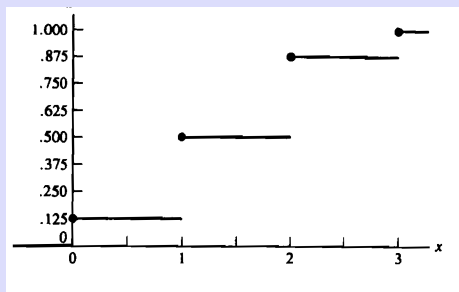


Figure: The plot of  $F_X(x)$ : CDF of the random variable  $X$

Note that  $F_X(\cdot)$  is defined for all values of  $x \in \mathbb{R}$ , not just for  $x \in \mathbb{S}_X := \{0, 1, 2, 3\}$ . For example,  $2.5 \notin \mathbb{S}_X$ , however

$$F_X(2.5) = P_X(x \leq 2.5) = P_X(X = 0) + P_X(X = 1) + P_X(X = 2) = \frac{7}{8}.$$

## Characterization of *any* CDF Function

# Characterization of a CDF

## Theorem

*The function  $F(x)$  is a cdf **if and only if** the following three conditions hold:*

- 1  $\lim_{x \rightarrow -\infty} F(x) = 0$  and  $\lim_{x \rightarrow \infty} F(x) = 1$ .
- 2  $F(x)$  is a nondecreasing function of  $x$
- 3  $F(x)$  is right-continuous; that is, for every real number  $x_0$ ,  
 $\lim_{x \downarrow x_0} F(x) = F(x_0)$ .



**Comment:** Let  $X$  be a random variable with the corresponding cdf  $F_X(x)$  for  $x \in \mathbb{R}$ . Let  $x_0 \in \mathbb{R}$  is arbitrary. Then

$$P(X = x_0) := P(X \in \{x_0\}) = \lim_{x \downarrow x_0} F_X(x) - \lim_{x \uparrow x_0} F_X(x).$$

# Example: CDF continuous

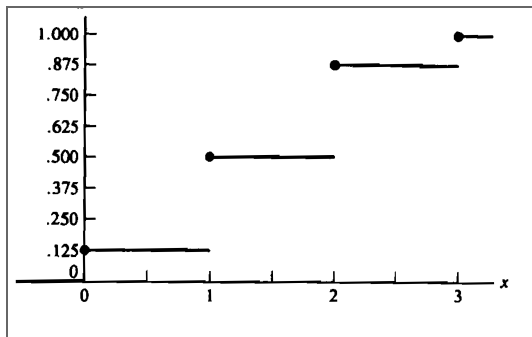


Figure: The plot of  $F_X(x)$ : CDF of the random variable  $X$

# Example: CDF continuous

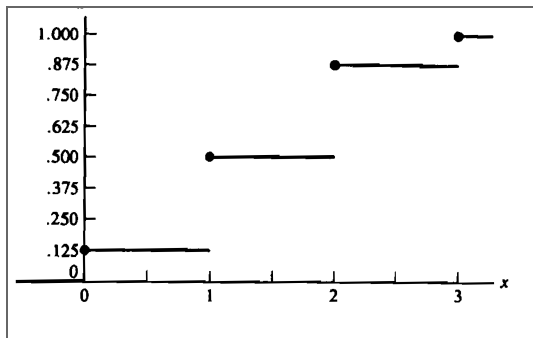


Figure: The plot of  $F_X(x)$ : CDF of the random variable  $X$

Let  $F_X(x)$  denotes the cdf function included in the above image. Therefore,

$$P(X = 1) = \lim_{x \downarrow 1} F_X(x) - \lim_{x \uparrow 1} F_X(x) = 0.5 - 0.125 = 0.375.$$

$$P(X = 1.5) = \lim_{x \downarrow 1.5} F_X(x) - \lim_{x \uparrow 1.5} F_X(x) = 0.5 - 0.5 = 0.$$

# Example: CDF continuous

**Example:** An example of a continuous cdf is the function

$$F_X(x) := \frac{1}{1 + e^{-x}} \text{ for all } x \in \mathbb{R}.$$

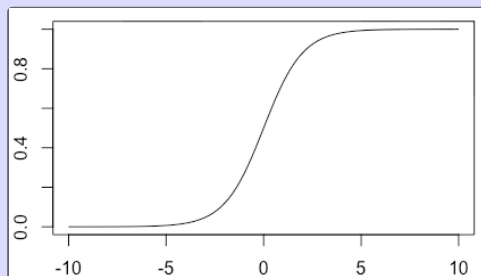


Figure: The plot of  $F_X(x)$ : CDF of the random variable  $X$

Verify: The above function satisfies the three conditions required to be a CDF.

# Reminder: Discrete Random Variable

## Definition (Discrete Random Variable)

A random variable  $X$  is discrete if its support  $\mathbb{S}_X$  is finite or countable infinite.

**Alternative Characterization of Discrete Distributions:** A random variable  $X$  is discrete if the corresponding cdf  $F_X(x)$  is a step function of  $x$ . i.e.  $F_X(x)$  increases only via jumps.

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# Continuous Random Variables

# Continuous and Discrete Random variable

## Definition (Continuous Random Variable)

A random variable  $X$  is continuous if the corresponding cumulative distribution function (cdf)  $F_X(x)$  is a continuous and continuously differentiable (at all points except for a finitely/countably many points) function of  $x$ .

## Definition (Probability Density Function)

Let  $F_X(x)$  be a cumulative distribution function (CDF) of a continuous random variable, then the corresponding **probability density function** or **pdf**, denoted as  $f_X(x)$  is a function that satisfies the following criteria.


$$F_X(x) = \int_{-\infty}^x f_X(y) dy$$



## Definition (Support of a Continuous Random Variable)

Let  $X$  be a continuous random variable with probability density function  $f_x(x)$ . The support of the random variable is defined to be

$$\mathbb{S}_x = \{x : f_x(x) > 0\}$$

 Support of a random variable refers to the all possible value or outcome of the random variable. Technically, if a point is a possible outcome, the corresponding value of the pdf is positive.

# Relation between pdf and CDF of a continuous random variable

**pdf** of a continuous random variable can be obtained by differentiating the corresponding **CDF**

$$f_x(x) = \frac{d}{dx} F_x(x)$$

**CDF** of a continuous random variable can be obtained by integrating the corresponding **pdf**

$$F_x(x) = \int_{-\infty}^x f_x(y) dy$$

# Probability Density Function (pdf): For continuous RV

**Comment:** Using the Fundamental Theorem of Calculus, if  $f_x(x)$  is continuous, we have the further relationship  $f_x(x) = \frac{d}{dx} F_x(x)$ .

If  $X$  is a continuous random variable, then probabilities can be obtained by integrating its pdf over suitable region. Specifically, for  $a, b \in \mathbb{R}$ ,  $a < b$ ,

$$P(a < X \leq b) = F_x(b) - F_x(a) = \int_a^b f_x(x) dx.$$

**Question:** Is it true that a random variable must be continuous if its support is an interval ?

**Question:** Is it true that a random variable must be continuous if its support is  $\mathbb{R}$ ?

**Question:** Is it possible for a continuous random variable to have a support that has finitely many points?

# Example: CDF continuous

**Example:** An example of a continuous cdf is the function

$$F_X(x) := \frac{1}{1 + e^{-x}} \text{ for all } x \in \mathbb{R}.$$

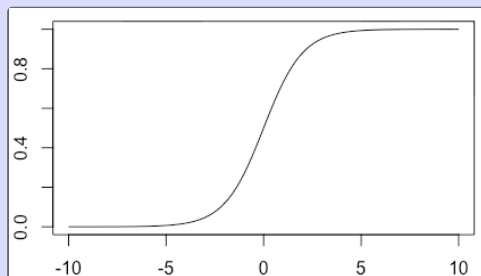


Figure: The plot of  $F_X(x)$ : CDF of the random variable  $X$

**Verify:** The above function satisfies the three conditions required to be a CDF.

# Example: CDF continuous

## Example of CDF of a Continuous Random Variable:

$$F_X(x) := \begin{cases} 0 & \text{if } x \leq 0 \\ 1 - e^{-x} & \text{if } x > 0 \end{cases}$$

**Verify:** The above function satisfies the three conditions required to be a CDF.

# Characterization of Probability Density Function (pdf)

## Definition (Characterization of Probability Density Function (pdf))

A function  $f(x)$  is a probability density function (pdf) for some continuous random variable if and only if it satisfies the following two conditions:

1 **Non-negativity:**  $f(x) \geq 0$  for all  $x$ .

2 **Total Probability:**  $\int_{-\infty}^{\infty} f(x) dx = 1$

**Support of a Continuous Random Variable** Let  $X$  be a continuous random variable with probability density function  $f(x)$ . The support of the random variable is defined to be

$$\mathbb{S}_X = \{x : f_X(x) > 0\}$$

# Result:

If  $X$  is a continuous random variable then,

- $P(X = c) = 0$  for any  $c \in \mathbb{R}$
- $P(a \leq X \leq b) = P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)$ .



## A Few Common Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} \text{ for any integer } n, n \neq -1.$$

$$\int \frac{1}{x} dx = \log(x)$$

$$\int e^{-x} dx = -e^{-x}.$$

$$\int e^{mx} dx = \frac{e^{mx}}{m} \text{ for any nonzero real number } m \in \mathbb{R}, m \neq 0.$$

\* Note: We have not included the constant term that appears as a constant while writing an indefinite integral. For the majority, if not all, of the integrals in this course will be definite integrals with a lower and upper limit.

Assume  $f'(x) := \frac{d}{dx} f(x)$  and  $g'(x) := \frac{d}{dx} g(x)$  for the following formula

Integral By Parts:  $\int f(x)g'(x)dx = f(x) \left( \int g'(x)dx \right) - \int \left\{ f'(x) \left( \int g(x)dx \right) \right\} dx$

Addition Rule:  $\int \left\{ c_1 f(x) + c_2 g(x) \right\} dx = c_1 \int f(x)dx + c_2 \int g(x)dx$  for any constant  $c_1, c_2 \in \mathbb{R}$ .

# Examples

# Example

Example :

Suppose that  $X$  is a continuous random variable whose probability density function is given by

$$f(x) := \begin{cases} C(4x - 2x^2) & \text{if } 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

- a). What is the value of  $C$ ?
- b). Find  $P(X > 1)$ .

# Example

Example :

Suppose that  $X$  is a continuous random variable whose probability density function is given by

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- What is the value of  $C$ ?
- Find  $P(X > 1)$ .

According to the property of the pdf

$$\begin{aligned} \int_{\mathbb{S}_X} f(x) dx &= 1 \\ \Rightarrow \int_0^2 C(4x - 2x^2) dx &= 1 \\ \Rightarrow C \left( 2x^2 - \frac{2x^3}{3} \right) \Big|_0^2 &= 1 \\ \Rightarrow C \left( 8 - \frac{16}{3} \right) &= 1 \\ \Rightarrow C &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} P(X > 1) &= \int_1^2 f(x) dx = \int_1^2 C(4x - 2x^2) dx \\ &= C \left( 2x^2 - \frac{2x^3}{3} \right) \Big|_1^2 = 1 \\ &= C \left\{ \left( 8 - \frac{16}{3} \right) - \left( 2 - \frac{2}{3} \right) \right\} \\ &= C \left\{ \frac{8}{3} - \frac{4}{3} \right\} \\ &= \frac{3}{8} \times \frac{4}{3} \\ &= \frac{1}{2}. \end{aligned}$$

# Example

**Example :**

For a given IT technician in a support center, let  $X$  denote the percentage of time, out of a 40-hour work week, that he is directly serving customers. Suppose that  $X$  has a probability density function given by

$$f(x) := \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a). Make a Graph of the above pdf.
- b). Find the probability that the technician will spend less than 30% of his workweek serving customers.
- c). Find the probability that the technician will spend 20% to 70% of his workweek serving customers.

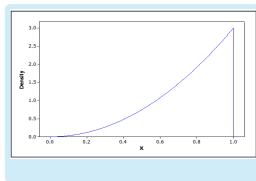
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- Make a Graph of the above pdf.
- Find the probability that the technician will spend less than 30% of his workweek serving customers.
- Find the probability that the technician will spend 20% to 70% of his workweek serving customers.



$$\begin{aligned} P(X < 0.3) &= \int_0^{0.3} f(x) dx \\ &= \int_0^{0.3} 3x^2 dx \\ &= (x^3) \Big|_0^{0.3} \\ &= (0.3)^3 - (0)^3 \\ &= 0.027 \end{aligned}$$

$$\begin{aligned} P(0.2 < X < 0.7) &= \int_{0.2}^{0.7} f(x) dx \\ &= \int_{0.2}^{0.7} 3x^2 dx \\ &= (x^3) \Big|_{0.2}^{0.7} \\ &= (0.7)^3 - (0.2)^3 \\ &= 0.335 \end{aligned}$$

# Exercise

**Example :**

The amount of time in hours that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) := \begin{cases} 100 e^{-\frac{x}{100}} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

- a). What is the probability that a computer will function between 50 and 150 hours before breaking down?
- b). What is the probability that it will function for fewer than 100 hours?



# Exercise

**Example :**

The lifetime in hours of a certain kind of radio tube is a random variable having a probability density function given by

$$f(x) := \begin{cases} \frac{100}{x^2} & \text{if } x > 100 \\ 0 & \text{if } x \leq 100. \end{cases}$$

What is the probability that exactly 2 of 5 such tubes in a radio set will have to be replaced within the first 150 hours of operation? Assume

that the events  $E_i, i = 1, 2, 3, 4, 5$ , that the  $i$ th such tube will have to be replaced within this time are independent.

# Example

Example :

For a given IT technician in a support center, let  $X$  denote the percentage of time, out of a 40-hour work week, that he is directly serving customers. Suppose that  $X$  has a probability density function given by

$$f(x) := \begin{cases} 3x^2 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a). Obtain,  $F(x)$ , the CDF of  $X$ .
- b). Use  $F(x)$  to compute  $P(0.5 < X \leq 0.8)$ .

# Example

Example :

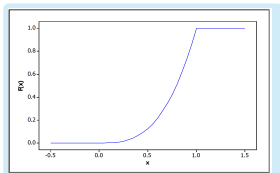
For a given IT technician in a support center, let  $X$  denote the percentage of time, out of a 40-hour work week, that he is directly serving customers. Suppose that  $X$  has a probability density function given by

$$f(x) := \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

- Obtain,  $F(x)$ , the CDF of  $X$  and Graph it.
- Use  $F(x)$  to compute  $P(0.5 < X \leq 0.8)$ .

$$\begin{aligned} F(x) = P(X \leq x) &= \int_0^x f(y) dy \\ &= \int_0^x 3y^2 dy \\ &= (y^3) \Big|_0^x \\ &= x^3 \end{aligned}$$

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^3 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1. \end{cases}$$



$$\begin{aligned} &P(0.5 < X < 0.8) \\ &= F(0.8) - F(0.5) \\ &= (0.8)^3 - (0.5)^3 \\ &= 0.387 \end{aligned}$$

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## Percentiles, Quantiles, and Median

# Percentiles, Quantiles, and Median


## Definition (Percentiles)

Let  $p$  be a number between 0 and 1. The  $(100)^{\text{th}}$  percentile of the distribution of a continuous random variable  $X$ , we shall denote by  $c$ , is that value for which

$$F(c) = p$$

i.e.  $c = F^{-1}(p)$  . where  $F^{-1}(\cdot)$  is the inverse cumulative distribution function.


# Three Special Percentiles

-  **Median**    The median of a continuous distribution, denoted by  $m$ , is the 50th percentile. So  $m$  satisfies

$$m = F^{-1}(0.5)$$

-  **First Quartile**    The first quartile is defined to be

$$Q_1 = F^{-1}(0.25)$$

-  **Third Quartile**    The third quartile is defined to be

$$Q_3 = F^{-1}(0.75)$$

# Example

**Example :** For a given IT technician in a support center, let  $X$  denote the percentage of time, out of a 40-hour work week, that he is directly serving customers. Suppose that  $X$  has a probability density function given by

$$f(x) := \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a). find the median, and
- b). the interquartile range of the distribution.



# Example

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For a given IT technician in a support center, let  $X$  denote the percentage of time, out of a 40-hour work week, that he is directly serving customers. Suppose that  $X$  has a probability density function given by

$$f(x) := \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a). find the median, and
- b). the interquartile range of the distribution.

We have already Shown

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^3 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1. \end{cases}$$

Note that if  $F(x) = y \implies x^3 = y \implies x = y^{\frac{1}{3}} \implies F^{-1}(y) = y^{\frac{1}{3}}$ .

$$m = F^{-1}(0.5) = (0.5)^{\frac{1}{3}} = 0.794$$

IQR

$$\begin{aligned} &= Q_3 - Q_1 \\ &= F^{-1}(0.75) - F^{-1}(0.25) \\ &= (0.75)^{\frac{1}{3}} - (0.25)^{\frac{1}{3}} \\ &= 0.909 - 0.630 \\ &= 0.279 \end{aligned}$$

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## Expected Value, Variance , & MGF of a Continuous Random Variable

Expected Value, or **mean** of a Continuous Random VariableDefinition (Expected Value or **mean** of a Continuous Random Variable)

If  $X$  is a continuous random variable with pdf  $f(x)$  on the support  $\mathbb{S}_X$ , then the expected value (the mean) of  $X$  denoted by  $E(X)$  is given by

$$E(X) = \int_{\mathbb{S}_X} x f(x) dx,$$

assuming the above integral exists.

$E(X)$  is sometimes also denoted by  $\mu_X$

## Definition (Expected Value of a function of a Continuous Random Variable)

Let  $h(x)$  be any\* function. If  $X$  is a continuous random variable with pdf  $f(x)$  on the support  $\mathbb{S}_X$ , then the expected value  $h(X)$  denoted by  $E(h(X))$  is given by

$$E(h(X)) = \int_{\mathbb{S}_X} h(x)f(x)dx,$$

assuming the above integral exists.

# Variance of a Random Variable

## Definition (Variance a Random Variable)

Variance of a random variable  $X$  is defined to be

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

# A Few Properties of Expected Value and Variance of a Random Variable

Let  $a$  and  $b$  be constants, then

1  $E(aX + b) = aE(X) + b$

2  $\text{Var}(aX + b) = a^2\text{Var}(X)$

3  $\text{SD}(aX + b) = |a|\text{SD}(X)$

# Moment Generating Function (mgf)

## Definition (Moment Generating Function)

The Moment Generating Function (mgf) of  $X$ , denoted by  $M_X(t)$  is defined as

$$M_X(t) := E \left( e^{tX} \right),$$

whenever it exists.

If a continuous random variable  $X$  has the probability density function  $f_X(x)$  for all  $x \in \mathbb{S}_X$ , the support of  $X$ , then assuming it exists

$$M_X(t) := E \left( e^{tX} \right) = \int_{\{x \in \mathbb{S}_X\}} e^{tx} f_X(x) dx.$$



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## A Few Examples

# Example

Example :

For a given IT technician in a support center, let  $X$  denote the percentage of time, out of a 40-hour work week, that he is directly serving customers. Suppose that  $X$  has a probability density function given by

$$f(x) := \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a). Find the expected value of percentage of time the technician spends serving customers.
- b). variance of percentage of time the technician spends serving customers.

# Example

Example :

For a given IT technician in a support center, let  $X$  denote the percentage of time, out of a 40-hour work week, that he is directly serving customers. Suppose that  $X$  has a probability density function given by

$$f(x) := \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a). Find the expected value of percentage of time the technician spends serving customers.
- b). variance of percentage of time the technician spends serving customers.

$$\begin{aligned} E(X) &= \int_{\mathbb{S}_X} xf(x)dx \\ &= \int_0^1 x(3x^2)dx \\ &= \int_0^1 (3x^3)dx \\ &= \left. \frac{3x^4}{4} \right|_0^1 \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} E(X^2) &= \int_{\mathbb{S}_X} x^2 f(x)dx \\ &= \int_0^1 x^2 (3x^2)dx \\ &= \int_0^1 (3x^4)dx \\ &= \left. \frac{3x^5}{5} \right|_0^1 \\ &= \frac{3}{5} \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= \frac{3}{5} - \left(\frac{3}{4}\right)^2 \\ &= 0.6 - (0.75)^2 \\ &= 0.0375 \end{aligned}$$

# Example

**Example :** Find  $E(X)$  and  $\text{Var}(X)$  when the density function of  $X$  is

$$f(x) := \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

# Example

Example :

Find  $E(X)$  and  $\text{Var}(X)$  when the density function of  $X$  is

$$f(x) := \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\begin{aligned} E(X) &= \int_{\mathbb{S}_X} xf(x)dx \\ &= \int_0^1 x(2x)dx \\ &= \int_0^1 (2x^2)dx \\ &= \left. \frac{2x^3}{3} \right|_0^1 \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} E(X^2) &= \int_{\mathbb{S}_X} x^2 f(x)dx \\ &= \int_0^1 x^2 (2x)dx \\ &= \int_0^1 (2x^3)dx \\ &= \left. \frac{2x^4}{4} \right|_0^1 \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= \frac{1}{2} - \left(\frac{2}{3}\right)^2 \\ &= \frac{1}{2} - \frac{4}{9} \\ &= \frac{10}{9} \end{aligned}$$

# Example

**Example :** Find  $E(e^X)$  and the Moment Generating Function for the continuous random variable with probability density function

$$f(x) := \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

# Example

**Example :** Find  $E(e^X)$  and the Moment Generating Function for the continuous random variable with probability density function

$$f(x) := \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\begin{aligned} E(e^X) &= \int_{\mathbb{S}_X} e^x f(x) dx \\ &= \int_0^1 e^x (1) dx \\ &= e^x \Big|_0^1 \\ &= e^1 - e^0 \\ &= e - 1 \end{aligned}$$

$$\begin{aligned} M_X(t) := E(e^{tX}) &= \int_{\mathbb{S}_X} e^{tx} f(x) dx \\ &= \int_0^1 e^{tx} (1) dx \\ &= \frac{1}{t} e^{tx} \Big|_0^1 \\ &= \frac{1}{t} e^t - \frac{1}{t} e^0 \\ &= \frac{e^t - 1}{t} \end{aligned}$$



# Exercise

Example :

Let  $X$  denote the resistance of a randomly chosen resistor, and suppose that its pdf is given by

$$f(x) := \begin{cases} \frac{x}{18} & \text{if } 8 \leq x \leq 10 \\ 0 & \text{otherwise.} \end{cases}$$

- 1 Find and graph the cdf of  $X$ .
- 2 Find  $P(8.6 < X \leq 9.8)$ .
- 3 Find the median of the resistance of such resistors.
- 4 Find the mean and variance of  $X$ .

# Exercise

Example :

The length of time to failure (in hundreds of hours) for a transistor is a random variable  $X$  with cumulative distribution function given by

$$F(x) := \begin{cases} 1 - e^{-x^2} & \text{for } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

- 1 Find a pdf of  $X$   $f(x)$ .
- 2 Find the probability that the transistor operates for at least 200 hours.
- 3 Find the 30<sup>th</sup> percentile of  $X$ .

# Exercise

Example :

Weekly CPU time used by an accounting firm has probability density function (measured in hours) given by

$$f(x) := \begin{cases} \frac{3}{64}x^2(4 - x) & \text{for } 0 \leq x \leq 4 \\ 0 & \text{otherwise.} \end{cases}$$

- 1 Find the  $F(x)$  for weekly CPU time.
- 2 Find the probability that the of weekly CPU time will exceed two hours for a selected week.
- 3 Find the expected value and variance of weekly CPU time.
- 4 Find the probability that the of weekly CPU time will be within half an hour of the expected weekly CPU time.
- 5 The CPU time costs the firm \$200 per hour. Find the expected value and variance of the weekly cost for CPU time.

# Exercise

Example :

The length of time required by students to complete a one-hour exam is a random variable with a density function given by

$$f(x) := \begin{cases} cy^2 + y & \text{for } 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- 1 Find c that makes this function a valid probability density function.
- 2 Find the F(y)
- 3 Find the probability that a randomly selected student will finish in less than half an hour.
- 4 Find the time that 95% of the students finish before it.
- 5 Given that a particular student needs at least 15 minutes to complete the exam, find the probability that she will require at least 30 minutes to finish.

Questions?