

# STAT 320: Principles of Probability

## Unit 1 (Part-B): Review of a Few Mathematical Functions

United Arab Emirates University

Department of Statistics

# Outline

- 1 A Few Mathematical Function and Notation
- 2 Exponential Series  $e^x$  or  $(\exp(x))$
- 3 Geometric Series
- 4 Binomial Series  $(1+x)^n$ ,  $(a+b)^n$
- 5 A Few Common Derivatives
- 6 A Few Common Integrals

# A Few Mathematical Function and Notation

# Absolute Value

Absolute value of a real number is the magnitude of the real number ignoring its sign. Formally, we have the following definition.

## Definition (Absolute Value)

Let  $x \in \mathbb{R}$  be any real number, then the **absolute value of  $x$**  (denotes as  $|x|$ ) is defined to be

$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0, \end{cases}$$

- $|5| =$
- $|-7.6| =$
- $|0| =$
- $|1005.7| =$
- $|-200| =$

## Definition (Indicator Function)

Let  $A$  be a set. The **indicator function for the set  $A$** , denoted by  $\mathbb{I}_A(x)$ , is defined to be

$$\mathbb{I}_A(x) := \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

- $\mathbb{I}_{[0,5]}(1.33) =$
- $\mathbb{I}_{[0,5]}(-9.12) =$
- $\mathbb{I}_{\{HH, TT, HT\}}(HH) =$
- $\mathbb{I}_{\{HH, TT, HT\}}(HHHH) =$
- $\mathbb{I}_{\mathbb{Z}}(1.87) =$
- $\mathbb{I}_{\mathbb{R}_+}(-4.87) =$
- $\mathbb{I}_{\mathbb{R}_+}(14) =$

# A Few Mathematical Function and Notation

## Definition (Factorial)

Let  $n$  be a **non-negative integer**, then the **factorial of  $n$** , denoted as  $n!$  is defined to be

- 1  $0! = 1$ ,
- 2  $1! = 1$ ,
- 3  $n! = n \times (n - 1) \times \dots \times 1$  for  $n \geq 2$

**Result:**  $n! = n \times \{(n - 1)!\}$  for  $n \geq 2$ .

- $3! =$
- $5! =$
- $6! =$

# $n$ choose $r$ , $\binom{n}{r}$

## Definition

Let  $n, r$  be two **non-negative integers**, such that  $r \leq n$ , then the  **$n$  choose  $r$** , denoted by  $\binom{n}{r}$ , is defined to be

$$\binom{n}{r} := \frac{n!}{(r!) \times ((n-r)!)}$$

## Result:

If  $n, r$  be two **non-negative integers**, such that  $r \leq n$ , then  $\binom{n}{r} = \binom{n}{n-r}$

- $\binom{0}{0} = 1$ ,  $\binom{1}{0} = 1$ ,  $\binom{5}{3} = \frac{5!}{3! \times ((5-3)!)} = \frac{5!}{3! \times (2!)} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$ .
- $\binom{10}{2} =$
- $\binom{100}{95} =$

# Outline

- 1 A Few Mathematical Function and Notation
- 2 Exponential Series  $e^x$  or  $(\exp(x))$
- 3 Geometric Series
- 4 Binomial Series  $(1+x)^n$ ,  $(a+b)^n$
- 5 A Few Common Derivatives
- 6 A Few Common Integrals



# The mathematical constant $e$

## Definition ("e")

The mathematical constant  $e$  is an transcendental real number given by,

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots,$$

Approximately  $e \approx 2.7183$

# Exponential Series $e^x$ or $(\exp(x))$

## Definition (Exponential Series)

For any real number  $x \in \mathbb{R}$ , the exponential series  $e^x$  (or sometimes denoted as  $\exp(x)$ ) is defined as,

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$$

$$\square \quad e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots,$$

$$\bullet \quad e^{x^2} =$$

# Outline

- 1 A Few Mathematical Function and Notation
- 2 Exponential Series  $e^x$  or  $(\exp(x))$
- 3 Geometric Series
- 4 Binomial Series  $(1+x)^n$ ,  $(a+b)^n$
- 5 A Few Common Derivatives
- 6 A Few Common Integrals

# Geometric Series

# Geometric Series

Let  $x \in \mathbb{R}$  be such that  $|x| < 1$ , then

$$\sum_{i=0}^{\infty} x^i = 1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}.$$

1 What is  $1 + 0.7 + (0.7)^2 + (0.7)^3 + \dots =$

2 What is  $1 - 0.7 + (0.7)^2 - (0.7)^3 + \dots =$

# Outline

- 1 A Few Mathematical Function and Notation
- 2 Exponential Series  $e^x$  or  $(\exp(x))$
- 3 Geometric Series
- 4 Binomial Series  $(1 + x)^n$ ,  $(a + b)^n$
- 5 A Few Common Derivatives
- 6 A Few Common Integrals

# Binomial Series

# Binomial Series

Let  $x \in \mathbb{R}$  be any real number, and  $n \in \mathbb{Z}_+$  be any positive integer, then

$$(1+x)^n = \sum_{i=0}^n \binom{n}{i} x^i$$

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$$

Let  $x \in \mathbb{R}$  be any real number, and  $n \in \mathbb{Z}_+$  be any positive integer, then

$$(a+b)^n = \sum_{i=0}^n \binom{n}{i} a^i b^{n-i}$$

$$(a+b)^n = b^n + \binom{n}{1}ab^{n-1} + \binom{n}{2}a^2b^{n-2} + \cdots + \binom{n}{n-1}a^{n-1}b + \binom{n}{n}a^n$$



# Binomial Series

Let  $x \in \mathbb{R}$  be any real number, and  $n \in \mathbb{Z}_+$  be any positive integer, then

$$(1+x)^n = \sum_{i=0}^n \binom{n}{i} x^i$$

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$$

Let  $x \in \mathbb{R}$  be any real number, and  $n \in \mathbb{Z}_+$  be any positive integer, then

$$(a+b)^n = \sum_{i=0}^n \binom{n}{i} a^i b^{n-i}$$

$$(a+b)^n = b^n + \binom{n}{1}ab^{n-1} + \binom{n}{2}a^2b^{n-2} + \cdots + \binom{n}{n-1}a^{n-1}b + \binom{n}{n}a^n$$

- $(1 + y)^2 =$

- $(1 + z)^3 =$

- $(p + q)^4 =$

# Outline

- 1 A Few Mathematical Function and Notation
- 2 Exponential Series  $e^x$  or  $(\exp(x))$
- 3 Geometric Series
- 4 Binomial Series  $(1+x)^n$ ,  $(a+b)^n$
- 5 A Few Common Derivatives
- 6 A Few Common Integrals

$$\frac{dx^n}{dx} = nx^{n-1} \text{ for any integer } n.$$

$$\frac{de^x}{dx} = e^x$$

$$\frac{de^{mx}}{dx} = me^{mx} \text{ for any constant } m \in \mathbb{R}.$$

$$\frac{d \log(x)}{dx} = \frac{1}{x}.$$

Assume  $f'(x) := \frac{d}{dx} f(x)$  and  $g'(x) := \frac{d}{dx} g(x)$  for the following formula

$$\text{Product Rule: } \frac{d}{dx} \{ f(x)g(x) \} = f'(x)g(x) + f(x)g'(x)$$

$$\text{Addition Rule: } \frac{d}{dx} \{ c_1 f(x) + c_2 g(x) \} = c_1 f'(x) + c_2 g'(x) \text{ for any constant } c_1, c_2 \in \mathbb{R}.$$

$$\text{Chain Rule: } \frac{d}{dx} \{ f(g(x)) \} = f'(g(x))g'(x).$$

- $\frac{d}{dx} x^4$

- $\frac{d}{dx} x^4$

# Outline

- 1 A Few Mathematical Function and Notation
- 2 Exponential Series  $e^x$  or  $(\exp(x))$
- 3 Geometric Series
- 4 Binomial Series  $(1+x)^n$ ,  $(a+b)^n$
- 5 A Few Common Derivatives
- 6 A Few Common Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} \text{ for any integer } n, n \neq -1.$$

$$\int \frac{1}{x} dx = \log(x)$$

$$\int e^{mx} dx = \frac{e^{mx}}{m} \text{ for any nonzero real number } m \in \mathbb{R}, m \neq 0.$$

$$\int e^{-x} dx = -e^{-x}.$$

Assume  $f'(x) := \frac{d}{dx} f(x)$  and  $g'(x) := \frac{d}{dx} g(x)$  for the following formula

Integral By Parts:  $\int f(x)g(x)dx = f(x) \left( \int g(x)dx \right) - \int \left\{ f'(x) \left( \int g(x)dx \right) \right\} dx$

Addition Rule:  $\int \left\{ c_1 f(x) + c_2 g(x) \right\} dx = c_1 \int f(x)dx + c_2 \int g(x)dx$  for any constant  $c_1, c_2 \in \mathbb{R}$ .

# Discussion on Various Concepts

Log (function) Equation of Line and Regions Circles and Regions



Questions?