Moment Generating Function

Chebyshev inequality

Theorem 1 *If X is a random variable with mean μ and standard deviation σ, then for any positive number k*

$$P\{|X-\mu| < k\sigma\} \geq 1 - \frac{1}{k^2}.$$

Example

Example 1 *Let X be a random variable with density function*

$$f(x) = \begin{cases} |x-1| & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the exact value of $P\{|X-\mu| > 1.5 * \sigma\}$.
b) Use Chebyshev's inequality to find a bound for the above probability.

Definition

Definition 1 *The moment generating function of a random variable X is given by $M_X(t) = E[e^{tX}]$, which is:*

$$M_X(t) = \sum_i e^{ti} P[X = i]$$

for discrete random variables and

$$M_X(t) = \int_{-\infty}^{+\infty} e^{tx} f(x) dx$$

for continuous random variables.

Why the name !!!

We define the rth non-centered moment of the r.v. X by

$$\mu'_r = E(X^r).$$

Theorem 2 *Let X be a r.v. with rth non-centered moment μ'_r and m.g.f. $M_X(t)$, then*

$$\mu'_r = M_X^{(r)}(0) = \left.\frac{d^r M_X(t)}{dt^r}\right|_{t=0}.$$

Properties

Theorem 3 *Let* $Y = a + bX$ *then*

$$M_Y(t) = e^{ta} M_X(bt).$$

Moment generating function

Example 2 *Suppose X and Y are r.v. with prob. dist.*

X	1	2	3	4	5	6
$p(x)$	0	1/4	1/2	0	0	1/4

Y	1	2	3	4	5	6
$p(y)$	1/4	0	0	1/2	1/4	0

a) Show that $E(X) = E(Y) = 7/2$ and $Var(X) = Var(Y) = 9/4$.
b) Compare the m.g.f. of X and Y.

Examples

Example 3 *Find the moment generating functions for X and Y defined in Example 2.*

Example 4 *Let Y be a cont. r.v. with a Gamma(1,4) (i.e. Exponential(4)) density function. Find the moment generating function of Y.*

Links

Virtual Library/Expectation