Assignment 2

STAT 230
UAEU

There are a total of 9 problems. You may review **Unit3 slides** while answering the questions. **Show your steps to get entire credit for your solutions.**

1. Let A, B, C be three events. Use the basic properties of probabilities to show that
    (a) (2.5 points) $P(A \cup B) \leq P(A) + P(B)$
    (b) (2.5 points) $P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$
    (c) (2.5 points) $P(A \cap B) \geq P(A) + P(B) - 1$
    (d) (2.5 points) $P(A \cap B \cap C) \geq P(A) + P(B) + P(C) - 2$

2. A car repair can be performed either on time or late and either satisfactorily or unsatisfactorily. The probability of a repair being on time and satisfactory is 0.30. The probability of a repair being on time is 0.75. The probability of a repair being satisfactory however it is performed late is 0.20.
    (a) (8 points) What is the probability of a repair being late and unsatisfactory?
    (b) (7 points) What is the probability that the car repair is unsatisfactory?

3. A student answers a multiple-choice examination question that offers four possible answers. Suppose the probability that the student knows the answer to the question is .8 and the probability that the student will guess is .2. Assume that if the student guesses, the probability of selecting the correct answer is .25. If the student correctly answers a question, what is the probability that the student really knew the correct answer?

4. (5 points) **Two** A large group of people is to be checked for two common symptoms of a certain disease. It is thought that 20% of the people possess symptom A alone, 30% possess symptom B alone, 10% possess both symptoms, and the remainder have neither symptom. For one person chosen at random from this group, find these probabilities:
    (a) The person has neither symptom.
    (b) The person has at least one symptom.
    (c) The person has both symptoms, given that he has symptom B.

5. Medical case histories indicate that different illnesses may produce identical symptoms. Suppose that a particular set of symptoms, denoted H, occurs only when any one of three illnesses, I_1, I_2, or I_3, occurs. Assume that the simultaneous occurrence ofmore that one of these illnesses is impossible and that $P(I_1) = .01, P(I_2) = .005, P(I_3) = .02$. The probabilities of developing the set of symptoms H, given each of these illnesses, are known to be $P(H \mid I_1) = .90, P(H \mid I_2) = .95, P(H \mid I_3) = .75$.

   (a) (5 points) Assuming that an ill person exhibits the symptoms, H, what is the probability that the person has illness I1?

6. Let A and B be any two events. Which of the following statements, in general, are false?

   (a) (3 points) $P(A \mid B) + P(A \mid B) = 1$.
   (b) (4+3 points) $P(A \mid B) + P(A \mid B) = 1$.?
   (c) $P(A \mid B) + P(A \mid B) = 1$.

7. (5 points) A business tax form is either filed on time or late, is either from a small or a large business, and is either accurate or inaccurate. There is an 11% probability that a form is from a small business and is accurate and on time. There is a 13% probability that a form is from a small business and is accurate but is late. There is a 15% probability that a form is from a small business and is on time. There is a 21% probability that a form is from a small business and is inaccurate and is late.

   (a) If a form is from a small business and is accurate, what is the probability that it was filed on time?
   (b) What is the probability that a form is from a large business?

8. (5 points) A warning light in the cockpit of a plane is supposed to indicate when a hydraulic pump is inoperative. If the pump is inoperative, then there is a probability of 0.992 that the warning light will come on. However, there is a probability of 0.003 that the warning light will come on even when the pump is operating correctly. Furthermore, there is a probability of 0.996 that the pump is operating correctly. If the warning light comes on, what is the probability that the pump really is inoperative?

9. The marketing division of a company profiles its potential customers and grades them as either likely or unlikely purchasers. Overall, 16% of the potential customers are graded as likely purchasers. In reality, 81% of the potential customers graded as likely purchasers actually make a purchase, while only 9% of the potential customers graded as unlikely purchasers actually make a purchase. If somebody made a purchase, what is the probability that they had been graded as a likely purchaser?