STAT230: Principles of Probability

Unit 1: Set Theory

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Objectives

- Definition of sets
- Relationship between sets
- Venn Diagrams
- Operations on sets

Sets

- A Set is any well defined collection of *objects*.
- The elements of a set are the objects in a set.
- Usually we denote sets with upper-case letters, elements with lower-case letters.
- The following notation is used to show set membership.
  1. $x \in A$ means that x is a member of the set A.
  2. $x \notin A$ means that x is not a member of the set A.

Ways of Describing Sets

- List the elements:

$$A = \{1, 2, 3, 4, 5, 6\}$$

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- Give a verbal description:
  - *A is the set of all integers from 1 to 6, inclusive*.
- Give a mathematical inclusion rule:

$$A = \{\text{All integers} | 1 \leq x \leq 6\}$$

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Special Sets

- The Null Set (or Empty Set) is a set with no elements, often symbolized by $\emptyset$.
- The Universal Set is the set of all elements currently under consideration, and is often symbolized by Ω.
- The universal set contains all of the elements relevant to a given discussion.

Membership relationships

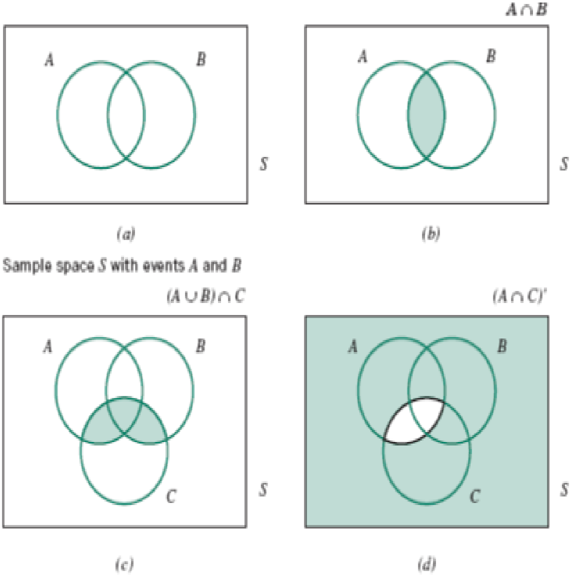
- Subsets: $A \subseteq B$, A is a subset of B.
- We say "A is a subset of B" if $x \in A \Rightarrow x \in B$, i.e., all the members of A are also members of B.
- The notation for subset is very similar to the notation for "less than or equal to" and means, in terms of the sets, "included in or equal to".
- Proper Subset: $A \subset B$, A is a proper subset of B.
- We say "A is a proper subset of B" if all the members of A are also members of B, but in addition there exists at least one element c such that $c \in B$ but $c \notin A$.
- The notation for subset is very similar to the notation for "less than", and means, in terms of the sets, "included in but not equal to".
- $A \nsubseteq B$ means A is not included in B.

Operations on sets

- "A union B" denoted $A \cup B$ is a set of all elements that are in A, or B, or both. The logical operator is OR.
- "A intersection B" denoted $A \cap B$ is a set of all elements that are in both A and B. The logical operator is AND.
- "A complement" denoted $\bar{A}$ is the set of elements that are NOT in A. The logical operator is NOT. We have also $\bar{\bar{A}} = A$.
- The set difference "A minus B", denoted $A - B$ is the set of elements that are in A, with those that are in B subtracted out. Another way of putting it is, it is the set of elements that are in A, and not in B, $A - B = A \cap \bar{B}$.

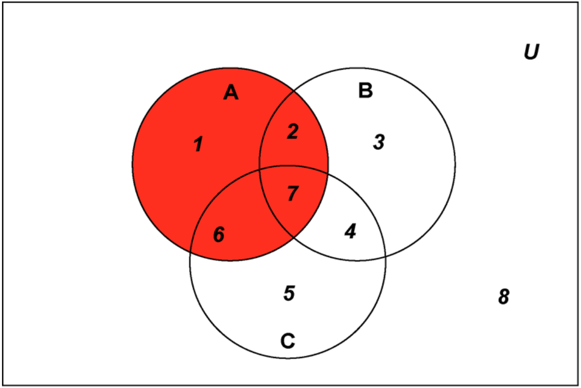
Examples using Venn Diagrams

Venn Diagrams use topological areas to stand for sets.



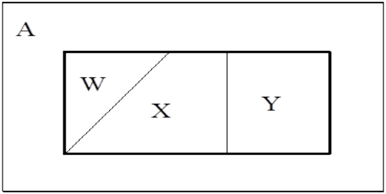
Examples using Venn Diagrams

Consider $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 6, 7\}$, $B = \{2, 3, 4, 7\}$, $C = \{4, 5, 6, 7\}$.



Mutually Exclusive, Exhaustive and Partition Sets

- We say that two sets A and B are *mutually exclusive* if $A \cap B = \emptyset$, that is, the sets have no elements in common.
- We say that a group of sets is *exhaustive* of another set if their union is equal to that set. For example, if $A \cup B = C$ we say that A and B are exhaustive with respect to C.
- We say that a group of sets *partitions* another set if they are mutually exclusive and exhaustive with respect to that set. When we partition a set, we break it down into mutually exclusive and exhaustive regions, i.e., regions with no overlap. The Venn diagram below should help you get the picture. In this diagram, the set A (the rectangle) is partitioned into sets W,X, and Y.



Test Questions

- $A \cup \emptyset =?$.
- $A \cup \bar{A} =?$.
- $A \cap \emptyset =?$.
- $A - \bar{A} =?$.
- $A \cap \bar{A} =?$.
- $A \cup \Omega =?$.
- $A \cap \Omega =?$.
- if $A \subset B$ then $A \cap B =?$.
- if $A \subset B$ then $A \cup B =?$.