CONTROL SYSTEM AND INSTRUMENTATION

NOTE:

MAKAUT course structure and syllabus of 6^{th} sem has been changed from 2021. Previously **CONTROL SYSTEM [EC 501]** was in 5^{th} semester. This subject has been totally redesigned & restructured as **CONTROL SYSTEM AND INSTRUMENTATION [EC 601]** in present curriculum. Taking special care of this matter we are providing chapterwise relevant MAKAUT university solutions of **CONTROL SYSTEM** with some model questions & answers for newly introduced topics, so that students can get an idea about university questions patterns.

INTRODUCTION

Multiple Choice Type Questions

1. A feedback control system is basically [WBUT 2011, 2015, 2018]
   a) high pass filter b) band pass filter c) low pass filter d) none of these

Answer: (c)

2. A system is stable [WBUT 2012]
   a) if bounded inputs produce bounded outputs
   b) if bounded inputs produce unbounded outputs
   c) if bounded inputs produce unbounded outputs
   d) if all bounded inputs produce bounded outputs

Answer: (a)

3. The human system is [WBUT 2018]
   a) Multivariable feedback control system b) Open loop control system
   c) Single variable control system d) Complex control system

Answer: (d)

Short Answer Type Questions

1. What are the advantages & disadvantages of closed loop control system? [WBUT 2013]

Answer:

Advantages of Closed Loop System

- Accuracy is very high.
- The effect of external disturbances signals is very small.
- Variation of Internal parameters is effectively taken care off.
- Speed of response can be greatly improved.

Disadvantages of the Closed Loop System

- They are more expensive and complex.
- Maintenance cost is higher.
- The systems may become unstable.
- Design aspects are very complex

2. Distinguish between an open loop and a closed loop system. [WBUT 2015]

Answer:

Open-loop System	Closed-loop System
(a) No feedback used.	(a) Feedback is there for comparison between desired output and reference input.
(b) Open-loop system are generally stable.	(b) Closed-loop system can become unstable under certain conditions.

Open-loop System	Closed-loop System
(c) Their accuracy is determined by the calibration of their elements. Simple to develop and cheap.	(c) They are more complex. Complicated to construct and costly.
(d) Affected by non-linearities in the system.	(d) Adjust to the effects of non-linearities present in the system.
Examples: Washing machine, fixed time traffic control system, room heater, etc.	Examples: Servomotor control, generator output voltage control system and so on.

Long Answer Type Questions

1. Write short note on Open Loop and Closed Loop Control System. [WBUT 2017]

Answer:

1. Open Loop Control System:

A control system is called an open-loop, where, the output depends explicitly on the input and an output changes when the input changes assuming the disturbances remain unaltered and output is not self-corrective to the external disturbances. Fig. 1(a) shows the block diagram of an open loop system.

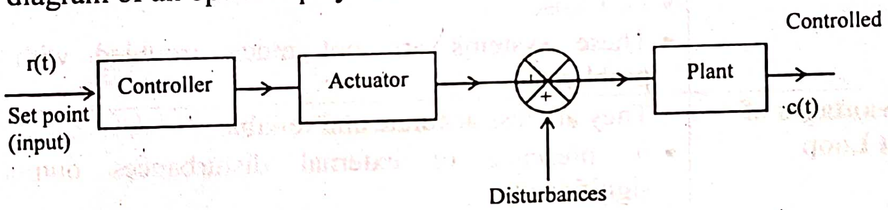


Fig: 1(a) An open loop control system

2. Closed Loop Control System:

In closed loop control system, the output is fed back to the input through a feedback element to maintain a prescribed relationship between them even in the presence of disturbances. Closed control system uses the difference or error e(t) between controlled variable (output) and set point (input) and sends this generated e(t) to the controller for necessary actions to reduce the error e(t). Thus, the controlled variable is continuously fed back and compared with the input signal. If disturbances are present, naturally the output will change and an e(t) will generate. The controller once again will try to reduce the error. So, the closed loop control system provides a self-corrective process against the variation of output.

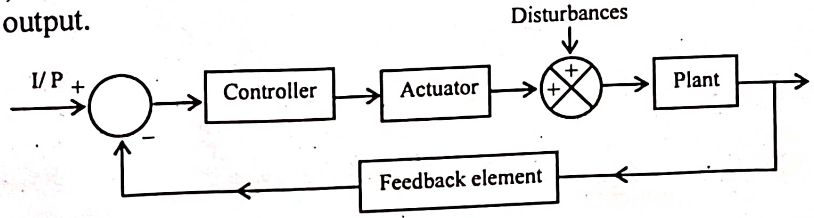


Fig: 1(b) A closed loop system

Fig. 1(b) depicts the block diagram of a closed loop control system, which is also known as a feedback control system.

3. Examples of Open Loop Systems are	• Bread toaster. • Traffic signals. • Sprinklers. • Coffee /tea server. • Electric hand drier. • Washing machines. • Electric iron etc .etc.
4. Examples of Close Loop Systems are	• Automatic voltage stabilizer. • Temperature controller. • Motor speed controller. • Water level controller.
5. Advantages of Open Loop System	• Simple in construction and design. • Easy maintenance. • Low cost. • These systems are not much troubled with stability problems.
6. Disadvantages of the Open Loop System	• They are less accurate and reliable. • In presence of external disturbances output differs significantly. • Output changes if there are variations in the system parameters.
7. Advantages of Closed Loop System	• Accuracy is very high. • The effect of external disturbances signals is very small. • Variation of Internal parameters is effectively taken care off. • Speed of response can be greatly improved.
8. Disadvantages of the Closed Loop System	• They are more expensive and complex. • Maintenance cost is higher. • The systems may become unstable. • Design aspects are very complex.

TRANSFER FUNCTION

Multiple Choice Type Questions

1. The transfer function of a system is defined as [WBUT 2012, 2014, 2017]
   a) the ratio of Laplace transform of output to Laplace transform of input considering initial conditions as zero
   b) the ratio of output to input
   c) both (a) and (b)
   d) none of these

Answer: (a)

2. A system is represented by the differential equation $M\frac{d^2x}{dt^2}+F\frac{dx}{dt}+Kx=u(t)$.
The transfer function relating $X(s)$ and $U(s)$ is [WBUT 2012]

a) $\frac{M}{(Ms^2+Fs+K)}$ b) $\frac{M}{(Fs^2+Ms+K)}$ c) $\frac{1}{(Ms^2+Fs+K)}$ d) $\frac{1}{(Fs^2+Ms+K)}$

Answer: (c)

3. The Laplace transform of $e^{-2t}\sin 2t$ is [WBUT 2014]

a) $\frac{4}{(s+2)^2+4}$ b) $\frac{4}{s^2+4}$ c) $\frac{2}{s^2+4s+8}$ d) $\frac{2}{s^2+4}$

Answer: (c)

4. Transfer function of a simple integrator where $a=1/RC$ is given by [WBUT 2015]

a) $\frac{1}{s-a}$ b) $\frac{1}{s+a}$ c) $\frac{a}{s-a}$ d) none of these

Answer: (d)

5. The Laplace transform of $e^{-3t}\sin 2t$ is [WBUT 2017]

a) $\frac{4}{(s+3)^2+4}$ b) $\frac{4}{s^2+4}$ c) $\frac{2}{s^2+4s+8}$ d) $\frac{2}{s^2+4}$

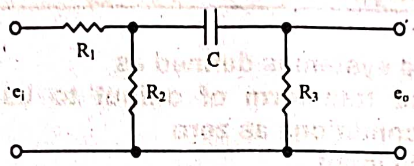
Answer: $\frac{2}{(s+3)^2+4}$

6. The transfer function of a system is its [WBUT 2018]
   a) step response
   b) ramp response
   c) impulse response
   d) square wave response

Answer: (c)

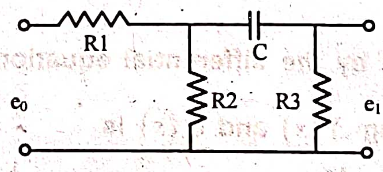
Short Answer Type Questions

1. Determine the transfer function of the network shown in figure relating $E_o(s)$ & $E_i(s)$. **[WBUT 2009]**



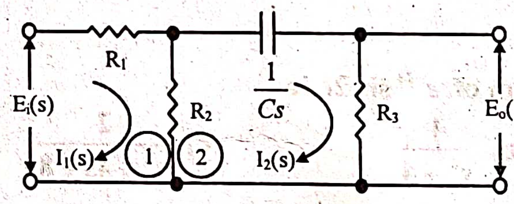
OR,

Determine transfer function of the network shown in figure relating $E_1(s)$ and $E_0(s)$. **[WBUT 2017]**



Answer:

Drawing the transformed circuit as shown below:



Applying KVL in the meshes (1) and (2) we have mesh equation as:

$$E_i(s) = (R_1 + R_2) I_1(s) - R_2 I_2(s) \qquad \text{.... (i)}$$

$$0 = \left(R_2 + R_3 + \frac{1}{Cs}\right) I_2(s) - R_2 I_1(s) \qquad \text{.... (ii)}$$

$$\Rightarrow \quad I_1(s) = \frac{\left(R_2 + R_3 + \frac{1}{Cs}\right)}{R_2} I_2(s) \qquad \text{.... (iii)}$$

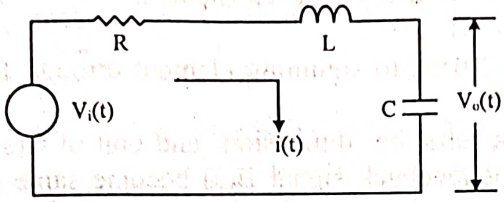
$\therefore$ From equation (i) and (iii)

$$E_i(s) = \left[\frac{(R_1 + R_2)(R_2 Cs + R_3 Cs + 1)}{R_2 Cs} - R_2\right] I_2(s)$$

But, $\quad E_o(s) = I_2(s) \cdot R_2 = \left[\dfrac{R_2^2\, Cs}{(R_1 + R_2)(R_2 Cs + R_3 Cs + 1) - R_2^2\, Cs}\right] E_i(s)$

$\therefore$ Transfer function $= \dfrac{E_o(s)}{E_i(s)} = \dfrac{R_2^2\, Cs}{(R_1 + R_2)(R_2\, Cs + R_3\, Cs + 1) - R_2^2\, Cs}$.

2. Obtain the Transfer function of the given electrical system. The symbols have their usual meaning. [WBUT 2012]



Answer:
V is the voltage source fed to an R-L-C series circuit.
Now, applying KVL in this circuit, we have Equation as:

$$V_i(t) = R_i(t) + L\frac{di(t)}{dl} + \frac{1}{c\int i} \quad \text{.... (i)}$$

And the output voltage $V_o(t) = \frac{1}{c}\int idt$ .... (ii)

Now, taking Laplace Transform on both the equations, we get:

$$V_i(s) = RI(s) + sLI(s) + \frac{1}{Cs}I(s)$$

[When, initial current in inductor and initial charge stored in capacitor of capacitance C is zero.]

And from the equation (ii) we get, $V_o(s) = \frac{1}{Cs}I(s)$

∴ The transfer function of the given electrical system

$$= \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}I(s)}{RI(s) + SLI(s) + \frac{1}{Cs}I(s)} = \frac{1}{sRC + s^2LC + 1}$$

3. Determine the transfer function of a simple closed loop system. [WBUT 2015]
Answer:

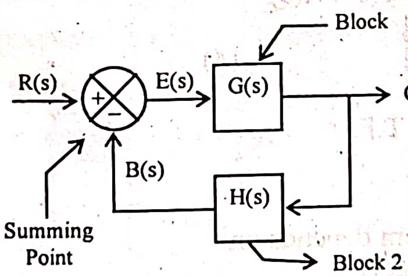


Fig: 1 Closed loop

Fig: 1 shows the block diagram of a closed loop system with negative feedback.
where R(s) ⇒ Input signal
B(s) ⇒ Feedback signal
E(s) ⇒ Error or Actuation signal

$G(s) \Rightarrow$ Forward path Transfer Function
$H(s) \Rightarrow$ transfer function of Feedback Element
$C(s) \Rightarrow$ Output signal

1. The output C(s) is fed back to summing element through feedback element having Transfer Function H(s).
2. Feedback element converts the 'dimension' and unit of C(s) in such a way that the dimension and unit of feedback signal B(s) become same as those of input signal R(s).

For example, in a pressure control system, the output signal C(s) is usually the controlled pressure.
Let R(s) has the dimension of voltage and unit volt.
The feedback element must convert the output signal (having dimension of Pressure & unit N/m^2) to a feedback signal whose dimension & unit should be voltage & volt respectively.

3. B(s) is compared with R(s) at summing point to generate an actuating signal E(s).
4. Output signal C(s) is obtained, here, by multiplying Transfer Function G(s) of Block 1 by E(s) which is Input signal to Block 1.

Evaluation of Closed Loop Transfer Function

- For Block 1,
  $C(s) = G(s).\,E(s)$ ... (1)
- For Block 2,
  $B(s) = H(s)\,.\,C(s)$ ... (2)
- For Summing Point,
  $E(s) = R(s) - B(s)$ ... (3)

From equations (1), (2) and (3), we get

$$C(s) = G(s).\,E(s) = G(s)\,[R(s) - B(s)]$$
$$= G(s)\,[R(s) - H(s)\,C(s)] = G(s)\,R(s) - H(s)\,G(s)\,C(s)$$

or, $C(s) + H(s)\,G(s)\,C(s) = G(s)\,R(s)$

or, $C(s)\,[1 + G(s)\,H(s)] = G(s)\,.\,R(s)$

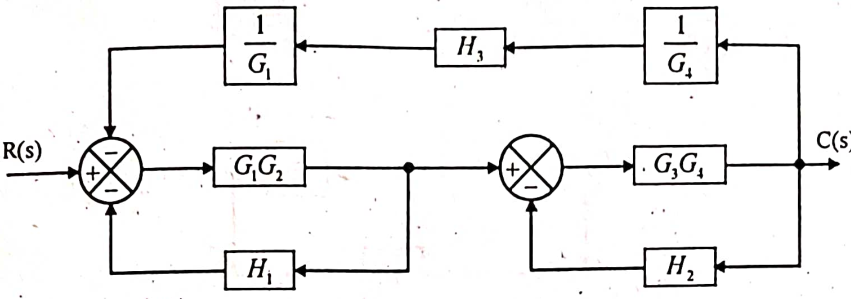
or, $$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}} \quad \text{... (4)}$$

= Closed loop T.F.

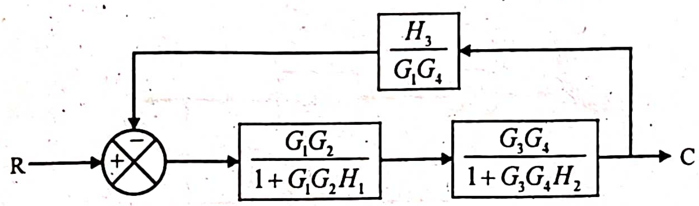
- ***Conclusions:***

Output of the closed loop system depends on
(i) nature of input signal R(s).
(ii) closed loop transfer function.

4. Find the overall transfer function of the system given below: **[WBUT 2016]**



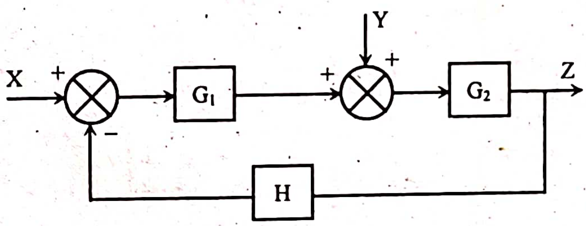
Answer:



$$\frac{C}{R}=\frac{\dfrac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}}{1+\dfrac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}\cdot\left(\dfrac{H_3}{G_1G_4}\right)}$$

$$=\frac{G_1G_2G_3G_4}{1+G_1G_2H_1+G_3G_4H_2+G_1G_2H_1G_3G_4H_2+G_2G_3H_3}$$

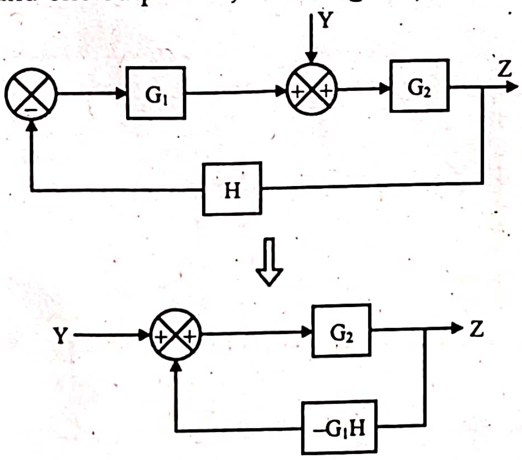
5. Determine the transfer function of the given system. **[WBUT 2018]**



Answer:

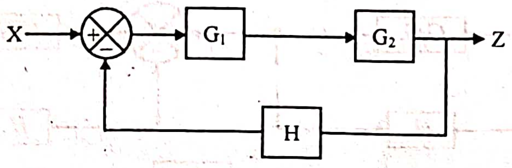
As there is two input and one output. So, we will get two different transfer function.

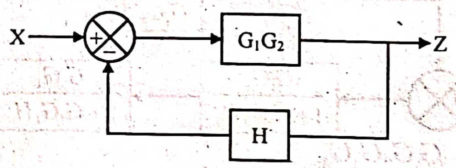
For $X=0$



$$\therefore \quad \frac{Z}{Y} = \frac{G_2}{1-(G_2)(-G_1H)} = \frac{G_2}{1+HG_1G_2}$$

For $\quad Y = 0$





$$\therefore \quad \frac{Z}{X} = \frac{G_1G_2}{1+G_1G_2H}$$

So, there will be two transfer function which are,

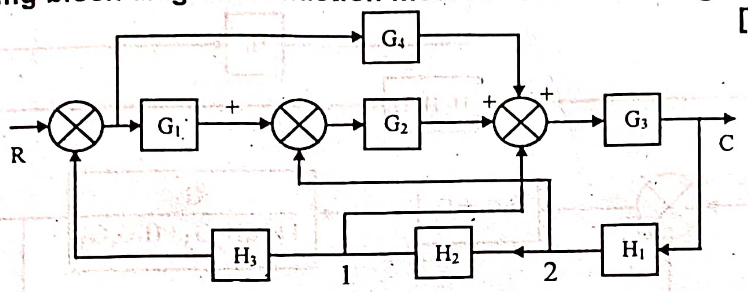
$$\frac{Z}{Y} = \frac{G_2}{1+G_1G_2H}$$

$$\frac{Z}{X} = \frac{G_1G_2}{1+G_1G_2H}$$

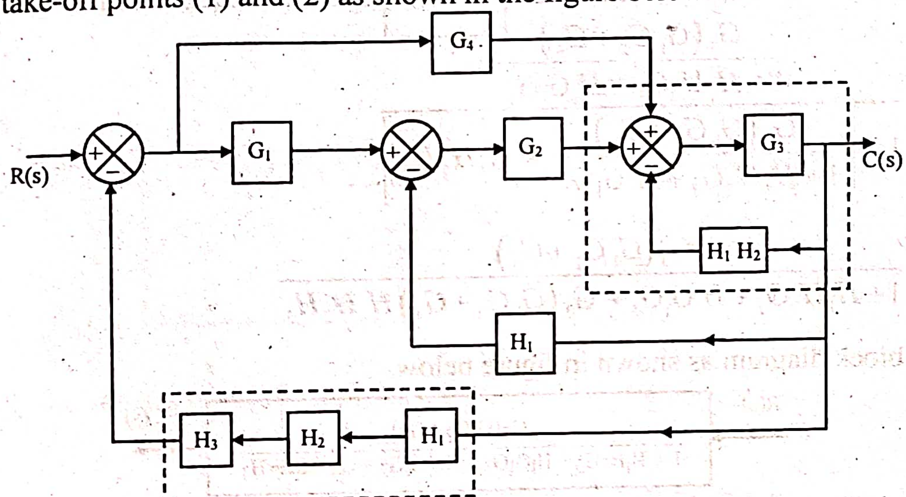
BLOCK DIAGRAM

Long Answer Type Questions

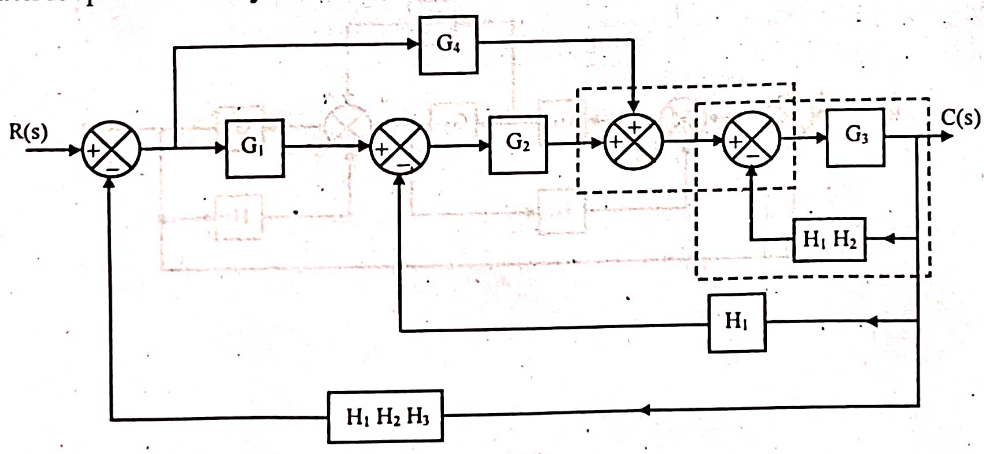
1. Find C/R using block diagram reduction method of the following diagram:
[WBUT 2007]

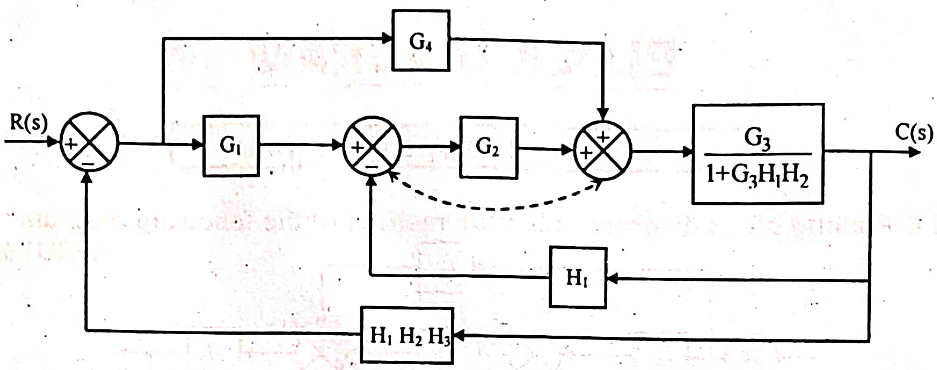


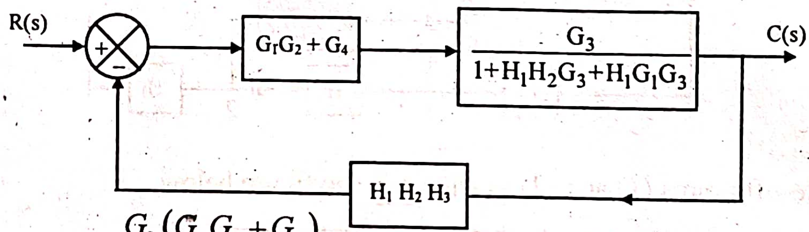
Answer:
Shift the take-off points (1) and (2) as shown in the figure below.



The comparator in the dotted box needs bifurcation. After bifurcation we find a negative feedback loop as shown by the dotted enclosures.



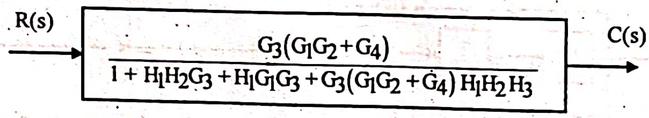




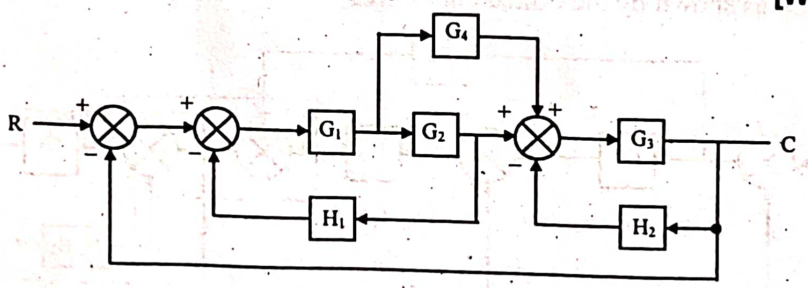
$$\frac{C(s)}{R(s)}=\frac{\dfrac{G_3\left(G_1G_2+G_4\right)}{1+H_1H_2G_3+H_1G_1G_3}}{1+\left[\dfrac{G_3\left(G_1G_2+G_4\right)}{1+H_1H_2G_3+H_1G_1G_3}\times H_1H_2H_3\right]}$$

$$=\frac{G_3\left(G_1G_2+G_4\right)}{1+H_1H_2G_3+H_1G_1G_3+G_3\left(G_1G_2+G_4\right)H_1H_2H_3}$$

Reduced block diagram as shown in figure below.



2. Using block diagram reduction technique find C/R. **[WBUT 2008]**



Answer:

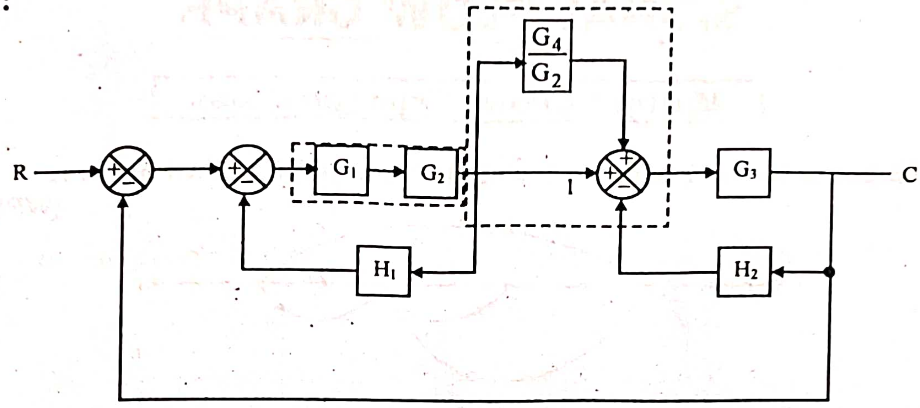


Fig: 1(a)

Using the rules for bifurcation of a comparator, cascaded blocks and parallel blocks the Fig. 1(a) may be redraw as shown in Fig. 1(b).

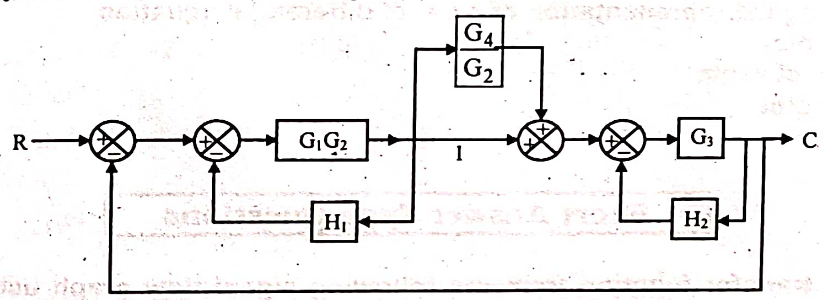


Fig: 1(b)

Eliminating the feedback loops and parallel blocks Fig. 1(b), may be re-drawn in Fig. 1(c).

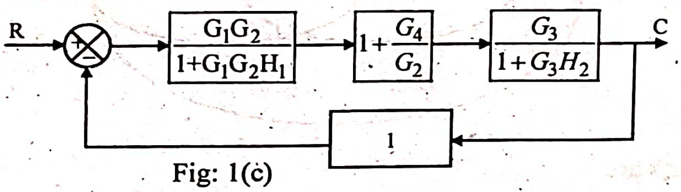


Fig: 1(c)

∴ The overall transfer function is given by

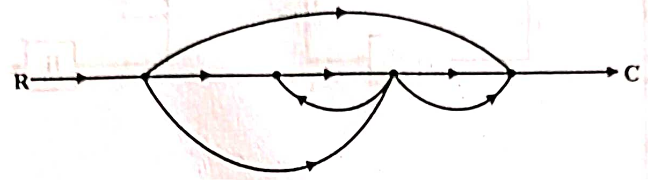
$$\frac{C}{R}=\frac{\left(\dfrac{G_1G_2}{1+G_1G_2H_1}\right)\times\left(\dfrac{G_2+G_4}{G_2}\right)\times\left(\dfrac{G_3}{1+G_3H_2}\right)}{1+\left(\dfrac{G_1G_2}{1+G_1G_2H_1}\right)\times\left(\dfrac{G_2+G_4}{G_2}\right)\times\left(\dfrac{G_3}{1+G_3H_2}\right)\cdot 1}$$

$$=\frac{G_1G_2G_3(G_2+G_4)}{G_2(1+G_1G_2H_1)(1+G_3H_2)+G_1G_2G_3(G_2+G_4)}$$

SIGNAL FLOW GRAPH

Multiple Choice Type Questions

1. The number of forward paths in the signal flow graph shown below is [WBUT 2009]



a) 1 b) 2 c) 3 d) 5

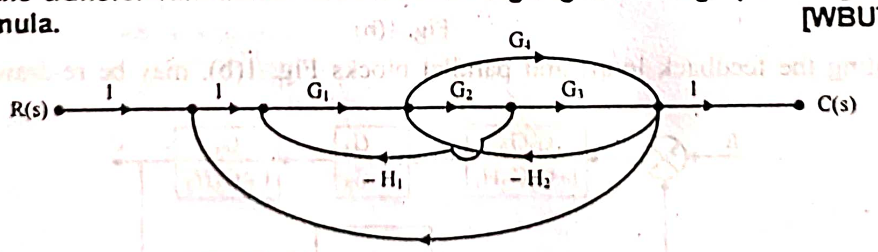
Answer: (c)

2. A signal flow graph is [WBUT 2018]
   a) Topological representation of a set of differential equation
   b) Bode plot
   c) Locus of roots
   d) Polar plot

Answer: (a)

Short Answer Type Questions

1. Find the transfer function from the following signal flow graph using Mason's gain formula. [WBUT 2009]



Answer:

Forward path gains are

$$P_1 = G_1G_2G_3$$
$$P_2 = G_1G_4$$

Loop gains are

$$L_1 = -G_1G_2H_1$$
$$L_2 = -G_2G_3H_2$$
$$L_3 = -G_4H_2$$
$$L_4 = -G_1G_2G_3$$
$$L_5 = -G_1G_4$$

No non-touching loops are there co-factors for forward paths are

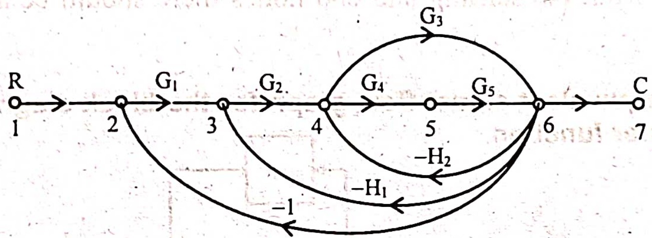
$\Delta_1 = 1,\ \Delta_2 = 1$

Using MGF, the overall transfer function is

$$\frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_4 + G_1G_2G_3 + G_4H_2 + G_2G_3H_2 + G_1G_2H_1}$$

2. Find $\frac{C}{R}$ of the following signal flow graph using Mason's gain formula.

[WBUT 2010]



Answer:

$P_1 = G_1G_2G_4G_5;\ P_2 = G_1G_2G_3$

$L_1 = -G_4G_5H_2;\ L_2 = -G_2G_4G_5H_1;\ L_3 = -G_1G_2G_4G_5$

$L_4 = -G_3H_2;\ L_5 = -G_2G_3H_1;\ L_6 = -G_1G_2G_3$

No non-touching loops are there.

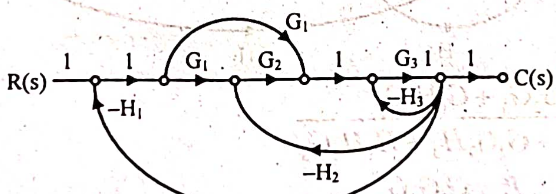
So, $\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6)$

$\Delta_1 = 1$ and $\Delta_2 = 1$

$$\therefore \frac{C}{R} = \frac{G_1G_2G_4G_5 + G_1G_2G_3}{(1 + G_1G_2G_3 + G_2G_3H_1 + G_3H_2 + G_1G_2G_4G_5 + G_2G_4G_5H_1 + G_4G_5H_2)}$$

3. Using Mason's Gain formula, determine the transfer function of the system.

[WBUT 2011]



Answer:

$P_1 = G_1G_2G_3,\ P_2 = G_1G_3$

$L_1 = -G_1G_2G_3H_1,\ L_2 = -G_2G_3H_2,\ L_3 = -G_3H_3$

$L_4 = -G_1G_3H_1$, No non-touching loops are available.

$\Delta = 1 + G_1G_2G_3H_1 + G_2G_3H_2 + G_3H_3 + G_1G_3H$

$\Delta_1 = \Delta_2 = 1$

$$\therefore \frac{C}{R} = \frac{G_1G_2G_3 + G_1G_3}{1 + G_1G_2G_3H_1 + G_2G_3H_2 + G_3H_3 + G_1G_3H}$$

4. What do you mean by the term 'Transmittance'? Differentiate between 'Self Loop' and 'Closed Loop'. [WBUT 2012]

Answer:

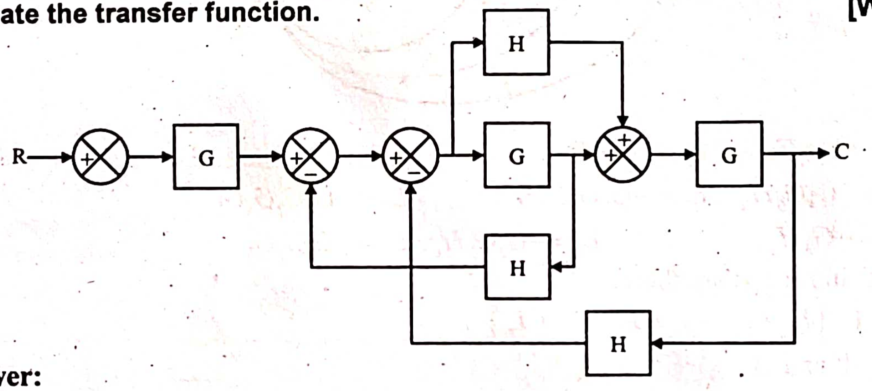
1st Part:

Transmittance is defined as the gain between two nodes.

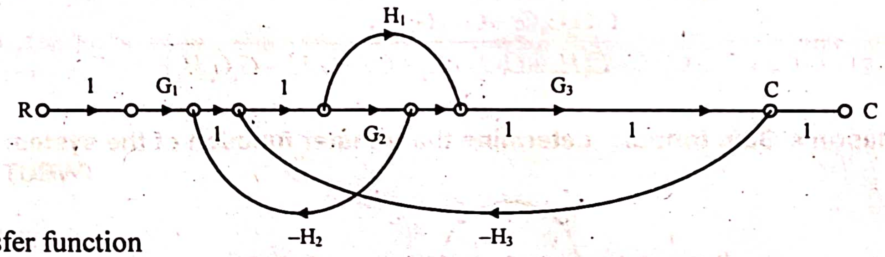
2nd Part:

In self-loop there is no other node other than the starting and terminal node. In closed loop system, apart from the starting and end nodes there should be at least one node in between.

5. Construct the equivalent signal flow graph for the block diagram show below & evaluate the transfer function. [WBUT 2013]



Answer:



Transfer function

$$\frac{C}{R}=\frac{G_1G_2G_3+G_1H_1G_3}{1+G_2H_2+G_2G_3H_3+H_1H_3G_3}$$

Forward path gains: $P_1=G_1G_2G_3$

$P_2=G_1H_1G_3$

Loop gains: $L_1=-G_2H_2$, $L_2=-G_2G_3H_3$, $L_3=-H_1H_3G_3$

No touching loops are present

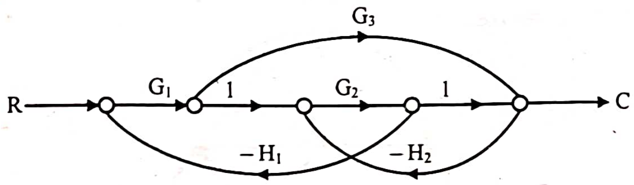
$$\Delta=1+G_2H_2+G_2G_3H_3+H_1G_3H_3$$

Co-factors: For P_1, $\Delta_1=1$

For P_2, $\Delta_2=1$

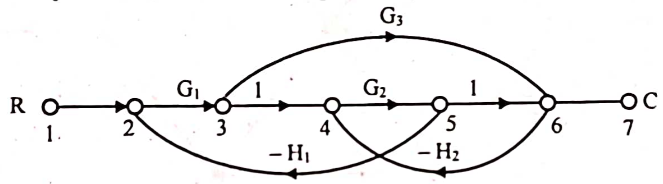
$$\therefore \quad \frac{C}{R}=\frac{G_1G_2G_3+G_1H_1G_3}{1+G_2H_2+G_2G_3H_3+H_1G_3H_3}.$$

6. Determine the transfer function relating C and R for the signal flow graph given below using Mason's gain formula. **[WBUT 2014]**



Answer:

Step I: Represent the system variables or signals by nodes



Step II: Evaluation of forward path & forward path gains:

Forward Path Identification through nodes	Forward path gains
a) 1, 2, 3, 4, 5, 6, 7	$P_1 = G_1G_2$
b) 1, 2, 3 , 6, 7	$P_2 = G_1G_3$

Step III: Evaluation of loops & loop gains:

Loop Identification through nodes	Loop gains
a) 2, 3, 4, 5, 2	$L_1 = -G_1G_2H_1$
b) 4,5,6,4	$L_2 = -G_2H_2$

Step IV: Evaluation of non-touching Loops:
Observations from loop nodes say that there is no non-touching loop.

Step V: Evaluation of Δ:

$$\therefore \quad \Delta = 1+G_1G_2H_1+G_2H_2$$

Step VI: Evaluation of co-factors:
As the no of forward path is 2, number of Co-factors will also be 2.
Cofactor for forward path 1:

$$\therefore \quad \Delta 1 = 1$$

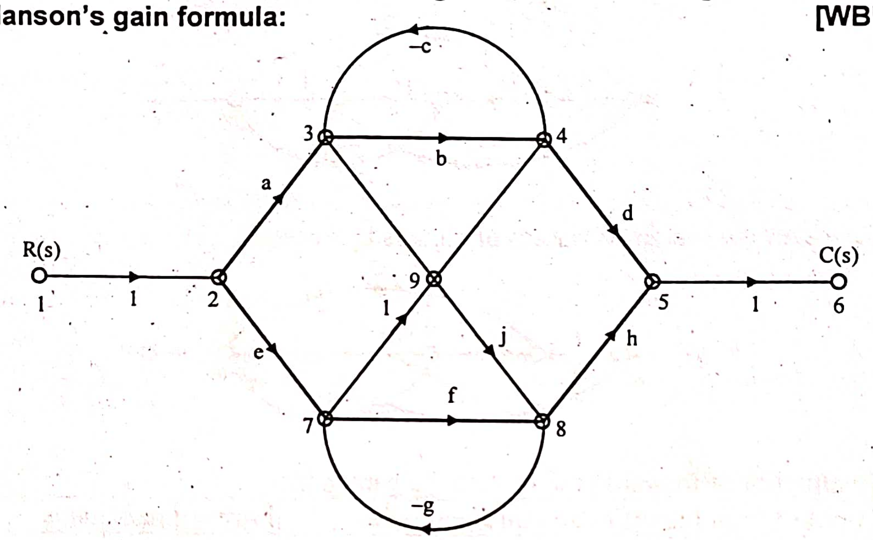
Cofactor for forward path 2:

$$\therefore \quad \Delta 2 = 1$$

Step VII: From Mason's Gain Formula:

$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_1 + G_2H_2} \text{ ...Ans.}$$

7. Determine the transfer function relating C and R for the signal flow graph given below Manson's gain formula: **[WBUT 2017]**



Answer:

Step I: Represent the system variables or signals by nodes

Step II: Evaluation of Forward path & Forward path gains:

Forward Path Identification through nodes	Forward path gains
a) 1, 2, 3, 4, 5, 6,	$P_1 = a\,b\,d$
b) 1, 2, 7, 8, 5, 6	$P_2 = e\,f\,h$
c) 1, 2, 7, 9, 8, 5, 6	$P_3 = e\,j\,h$

Step III: Evaluation of loops and loop gains:

Loop Identification through nodes	Loop gains
a) 3, 4, 3	$L_1 = -b\,c$
b) 7, 8, 7	$L_2 = -f\,g$
c) 7, 9, 8, 7	$L_3 = -1\,j\,g$

Step IV: Evaluation of non-touching loops:

Observations from loop nodes say that the non-touching loops are:

(i) Loop L_1 & L_2

(ii) Loop L_1 & L_3

So, $L_1 \times L_2 = bcfg$

$L_1 \times L_3 = bcjg$

Step V: Evaluation of Δ:

$$\therefore \quad \Delta = 1-(L_1+L_2+L_3)+(L_1\times L_2+L_1\times L_3)$$
$$=1-(-bc+(-fg)+(-jg))+(bcfg+bcjg)$$
$$=1+(bc+fg+jg)+bcfg+bcjg$$
$$=1+bc+fg+jg+bcfg+bcjg$$
$$=1+bc(1+fg+jg)+fg+jg$$

Step VI: Evaluation of co-factors:

Here number of forward path so, number of co-factor is 3.

$$\therefore \quad \Delta_1 = 1+fg+jg$$

and $\quad \Delta_2 = 1+fg$

Step VII: From Mason's Gain Formula:

Overall gain

$$T=\frac{1}{\Delta}\sum_{k=1}^{n}P_k\Delta_k=\frac{P_1\Delta_1+P_2\Delta_2}{\Delta}=\frac{abd(1+fg+jg)+efh(1+fg)}{1+bc+(c_1+fg+jg)+fg+jg} \qquad \textbf{(Ans.)}$$

Long Answer Type Questions

1. Write short note on Mason's gain formulas. **[WBUT 2005, 2013]**

OR,

Write down 'Mason's gain' formula and explain the meaning of each and every term. **[WBUT 2012]**

OR,

State Mason's Gain formula. **[WBUT 2015]**

Answer:

Mason's gain formula states that the ***overall gain*** of the system is

$$T=\frac{1}{\Delta}\sum_{k=1}^{n}P_K\Delta_K \qquad ...(1)$$

where

T = Overall transmittance of the system

Δ = 1 − (sum of individual loop gains) + (sum of gain products of all possible combinations of two non touching loops) − (sum of gain products of all possible combinations of three non touching loops) + (...) − (...) + ... = Determinant of the graph.

P_k = Gain of k^{th} forward path.

Δ_k = the cofactor value of Δ for the k^{th} forward path, with the loops touching the k^{th} forward path removed

n = Number of forward paths

Use of Mason's Gain Formula

(i) It is used to determine the overall Transfer function of a system.

(ii) The conventional block reduction approach (to get the overall transfer function) is much tedious and complicated. It provides a very simple approach to get the same.

2. a) Explain the different 'Signal Flow Graph' terminologies. **[WBUT 2012]**

Answer:

Terminologies in SFG

The following terminologies are used in SFG. Let us consider a typical signal flow graph as shown in figure 1.

(1) ***Input or Source Node:*** It is a node that has only outgoing branches. For example, x_1 is a source node in Fig (1).

(2) ***Output Node or Sink Node:*** This is a node that has only incoming branches. For example in figure 1 x_4 in an output node. e.g., x_4 in Fig. (1).

(3) ***Chain Node / Mixed Node:*** A node that has both incoming and outgoing branches is a chain node e.g., in figure 1, x_2 and x_3 are mixed nodes.

Both source nodes or output node can be having both type of branches incoming or outgoing. It can be made a dummy source or sink node as in figures 1a and 1b.

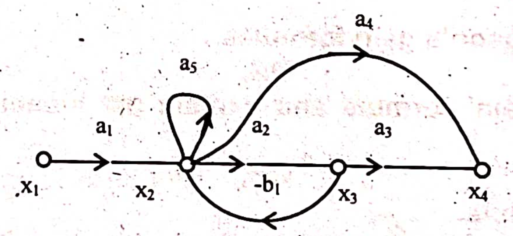


Fig: 1 Typical SFG

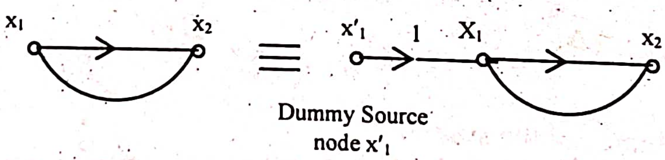


Fig: 1a

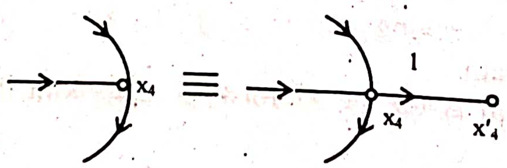


Fig: 1b

(4) ***Forward path:*** Any path *from input node to output node* is a forward path. In Fig: 1, there are two forward paths.

i) $x_1 \to x_2 \to x_3 \to x_4$
ii) $x_1 \to x_2 \to x_4$

Input modes   Output modes

Fig: 2

(5) ***Path gain:*** It is product of gains while going through a directed path. Path gain for forward path mentioned in the figure (1) are:

i) $a_1 \times a_2 \times a_3 = a_1 . a_2 . a_3$
ii) $a_1 \times a_4 = a_1 . a_4$

Path gain of forward path is the product of gains while going through a directed path.
In figure 2, forward path gains are

i) $P_1 = a_1 \times a_2 \times a_3$ (for forward path $x_1 \to x_2 \to x_3 \to x_4$)
ii) $P_2 = a_1 \times a_4$ (for forward path $x_1 \to x_2 \to x_4$)

(6) ***Feedback loops or feedback path*** *(or simply* ***Loops****):* A path, which originates and terminates on a same node, is a feedback path or loop.
For instance, there are two loops in Fig. (1).

i) $x_2 - x_3 - x_2$
ii) x_2 itself (via a_5 gain). A special name for a feedback loop that consists of only one node is self - loop. Here x_2 is a self-loop.

Now in canonical systems, feedback path is defined from output to input only; in SFG terminology used for loop is from input to output to input. The gain of SFG is specified by such relations involving loops.
Hence the slight modification is terminology for a feedback path.

(7) ***Non-touching loops:*** Loops are said to be non-touching if they do not have any common nodes.

(8) ***Loop Gain:*** Loop gain is the product of the branch transmittances of a loop.
For instance, the loops in Fig. 1

i) $x_2 - x_3 - x_2$, corresponding loop gain $-a_2 b_1$.
ii) x_2, a self-loop, corresponding loop gain is a_5

b) Compare between 'Block Diagram' and 'Signal Flow Graph' methods.
[WBUT 2012]

OR,

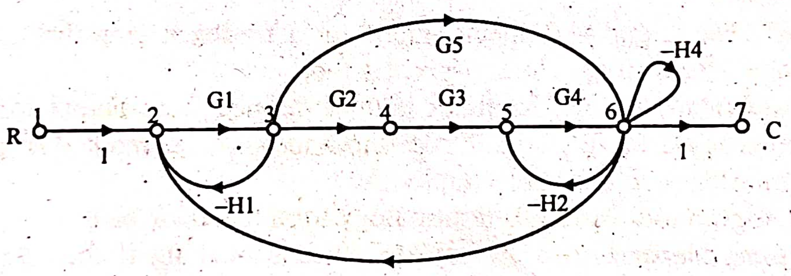
What is the difference between block diagram and signal flow graph? [WBUT 2015]

Answer:

Block Diagram	Signal Flow Graph
1. Each element is represented by block.	1. Each variable is represented by a separate node.
2. Basic importance is given to the elements and their transfer functions.	2. Basic importance is given to the variables of the systems.

Block Diagram	Signal Flow Graph
3. Transfer function of the element is shown inside the corresponding block.	3. The transfer function is shown along the branches connecting the nodes.
4. Summing points and take off points are separate.	4. Summing and take off points are not present. Any node can have any number of incoming and outgoing branches.
5. Feedback path is present from output to input.	5. In S.F.G, instead of feedback path various feedback loops are considered for the analysis.
6. In case of minor feedback loop the formula $\frac{G}{1\pm GH}$ is used.	6. In case of Gains of various for ward paths and feedback loops are just the product of associative branch gains. No such formula $\frac{G}{1\pm GH}$ is necessary.
7. Block diagram reduction rules can be used to obtain the resultant transfer function.	7. Mason's gain formula is available which can be used directly to get resultant transfer function without reduction of signal flow graph.
8. It is applicable only to linear time invariant systems	8. It is applicable to linear time invariant systems.

c) For the given signal flow graph find the C/R ratio. **[WBUT 2012]**



Answer:

Forward path gains

$P_1 = G_1G_2G_3G_4$ $\qquad P_2 = G_1G_5$

Loop gains

$L_1 = -G_1H_1$ $\qquad L_2 = -G_4H_2$

$L_3 = -G_1G_2G_3G_4H_3$ $\qquad L_4 = -G_1G_5H_3$

$L_5 = -H_4$

L_1 and L_5 are non-touching loops.

$\Delta = 1 + G_1H_1 + G_4H_2 + G_1G_2G_3G_4H_3 + G_1G_5H_3 + H_4 + G_1H_1H_4$

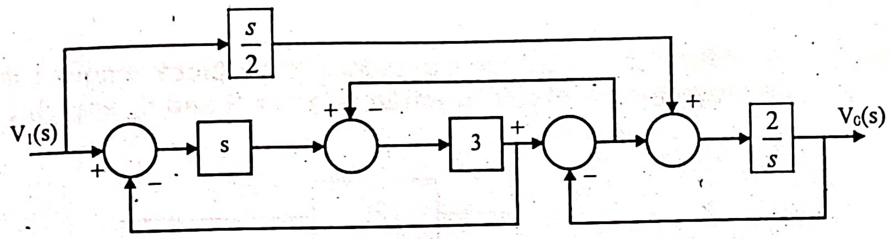
$\Delta_1 =$ Co-factor for first forward path $= 1$.

$\Delta_2 =$ Co-factor for the second forward path $= 1$

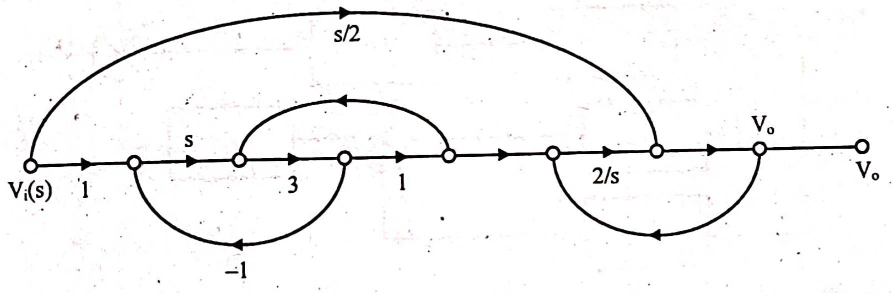
So, using Mason's gain formula,

$$\frac{C}{R}=\frac{G_1\left(G_2G_3G_4+G_5\right)}{1+H_4+G_1H_1+G_4H_2+G_1G_5H_3+G_1H_1H_4+G_1G_2G_3G_4H_3}.$$

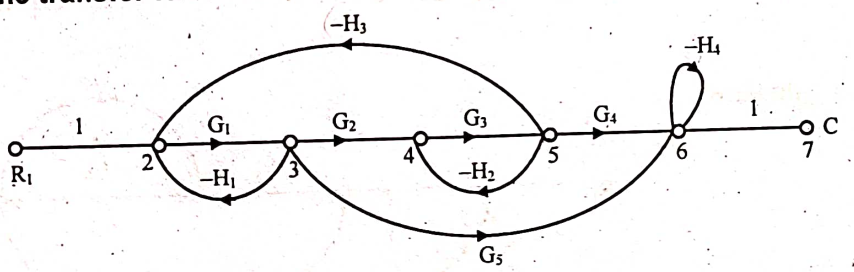
3. a) Construct the equivalent signal flow graph for the block diagram shown in below and evaluate the transfer function using Mason's gain formula. **[WBUT 2015]**



Answer:



b) Obtain the transfer function C/R form the given SFG. **[WBUT 2015]**



Answer:

For word path gains

(i) $P_1=G_1G_2G_3G_4$

(ii) $P_2=G_1G_5$

Individual loop gains:

(i) $L_1=-G_1H_1$

(ii) $L_2=-G_3H_2$

(iii) $L_3=-G_1G_2G_3H_3$

(iv) $L_4=-H_4$

Non-touching loops and gains:

(i) L_1L_4

(ii) L_2L_4

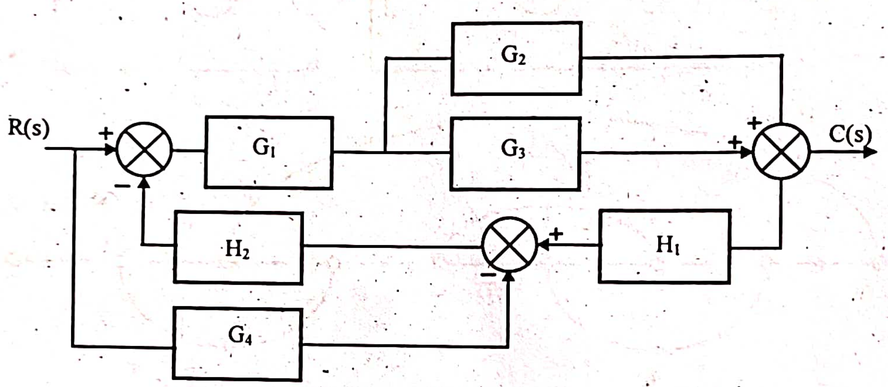
(iii) L_3L_4

$$\Delta = 1 + H_4 + G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + L_4(L_1 + L_2 + L_3)$$

Co-factors: $\Delta_1 = 1,\ \Delta_2 = 1$

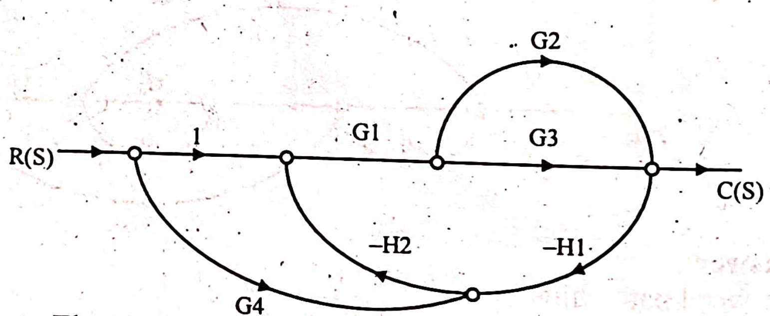
$$\therefore \quad \frac{C}{R} = \frac{G_1G_2G_3G_4 + G_1G_5}{\Delta}$$

4. Open loop transfer function of an for a system whose block diagram is given in figure and evaluate close loop transfer function relating *R* and *C*, applying Mason's Gain formula.



$G_1 = 12$, $H_1 = 1$, $G_2 = 5$, $H_2 = 0.7$, $G_3 = 0.8$, $G_4 = 3$. **[WBUT 2017]**

Answer:



Forward path gains are,

$P_1 = G_1G_2$

$P_2 = G_1G_3$

Loop gains are,

$L_1 = G_1G_3H_1H_2$

$L_2 = G_1G_2H_1H_2$

No, non-touching loops are there. The co-factors for forward paths are,

$\Delta_1 = 1,\ \Delta_2 = 1$

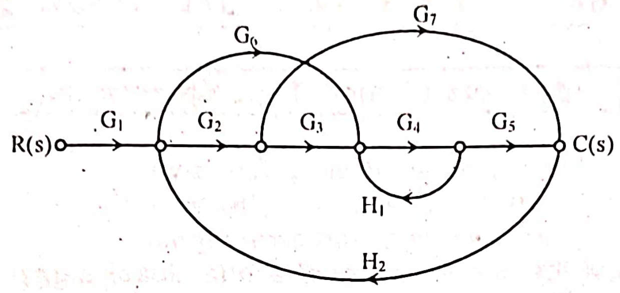
Using MFG, the overall TF,

$$\frac{C(S)}{R(S)} = \frac{G_1G_2 + G_1G_3}{1-(L_1+L_2)} = \frac{G_1G_2 + G_1G_3}{1-G_1H_1H_2(G_2+G_3)} = \frac{G_1(G_2+G_3)}{1-G_1H_1H_2(G_2+G_3)}$$

$G_1 = 12$, $H_1 = 1$, $G_2 = 5$, $H_2 = 0.7$, $G_3 = 0.8$, $G_4 = 3$

$$= \frac{12(5+0.8)}{1-12.1.0.7(5+0.8)} = \frac{69.6}{-47.72} = -1.458$$

5. Determine the transfer function of the given signal flow graph using Mason's gain formula. **[WBUT 2018]**



Answer:

$P_1 = G_1G_2G_3G_4G_5$; $\quad P_2 = G_1G_2G_7$; $\quad P_3 = G_1G_4G_5G_6$

$L_1 = G_2G_3G_4G_5H_2$; $\quad F_2 = G_4H_1$

As no non-touching loops are available

$\therefore \quad \Delta = 1 - G_2G_3G_4G_5H_2 - G_4H_1$

$\Delta_1 = 1$; $\quad \Delta_2 = 1 - G_4H_1$; $\quad \Delta_3 = 1$

$$\therefore \quad G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k P_k\Delta_k}{\Delta}$$

$$\therefore \quad G(s) = \frac{G_1G_2G_3G_4G_5 + G_1G_2G_7(1 - G_4H_1) + G_1G_4G_5G_6}{1 - G_2G_3G_4G_5H_2 - G_4H_1}$$

$$\therefore \quad G(s) = \frac{G_1G_2G_3G_4G_5 + G_1G_2G_7 - G_1G_2G_4G_7H_1 + G_1G_4G_5G_6}{1 - G_2G_3G_4G_5H_2 - G_4H_1}$$

COMPONENTS OF A CONTROL SYSTEM

Multiple Choice Type Questions

1. The error detector element in a control system gives **[WBUT 2008]**
a) the sum of the reference signal and feedback signal
b) the sum of the reference signal and error signal
c) the difference of the reference signal and feedback signal
d) the difference of the reference signal and output signal

Answer: (c)

2. The characteristic equation of an armature controlled D.C. motor is **[WBUT 2018]**
a) 1^{st} order equation
b) 0^{th} order equation
c) 2^{nd} order equation
d) 3^{rd} order equation

Answer: (c)

Short Answer Type Questions

1. Determine the transfer function of an armature control d.c. motor system. **[WBUT 2007]**

Answer:
The field circuit is activated by a constant supply (DC) or the field may be due to a permanent magnet.
The control signal is applied to the armature terminals.

Schematic Diagram

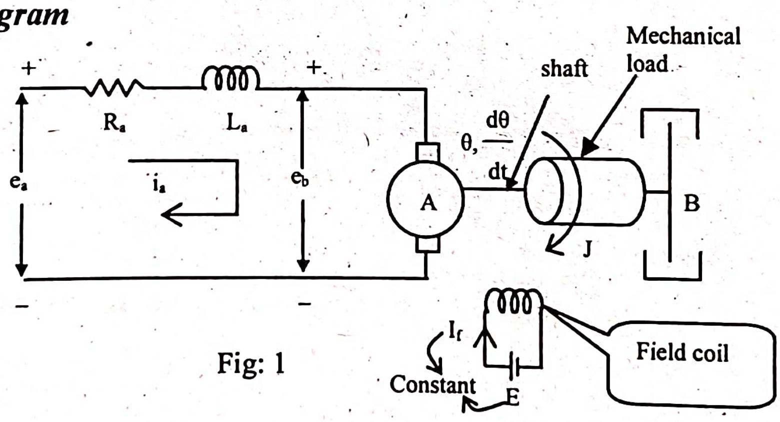


Fig: 1

In the above figure, the armature circuit is modeled as a linear network of passive elements, where

- Armature resistance R_a is connected in series with an inductance L_a (armature).
- The control signal $e_a(t)$ is applied to the armature terminals as the applied voltage.
- Due to $e_a(t)$, an armature current $i_a(t)$ flows through the armature circuit.
- The field current is constant as the supply voltage to field is constant.

- The magnetic field generated by the armature circuit interacts with magnetic field due to the field circuit. As a result a torque (T) is developed (shown in the figure 1(a)).

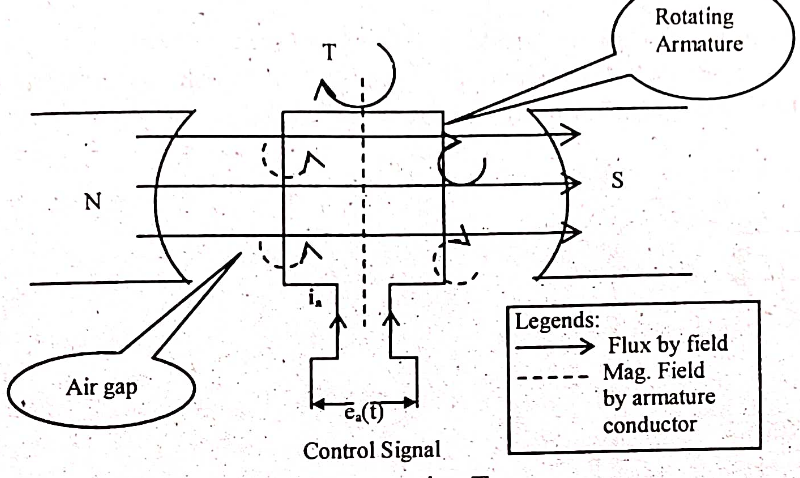


Fig: 1(a) Generation Torque

- Torque delivered by the motor is proportional to the product of armature current (ϕ) and air gap flux

i.e., $T \propto \phi . i_a$ .... (1)

But, in a d.c. servomotor field winding is operated in linear portion of magnetization curve (fig. 1(b))

$\therefore \phi \propto i_f$ [Linear part of magnetization curve] .... (2)

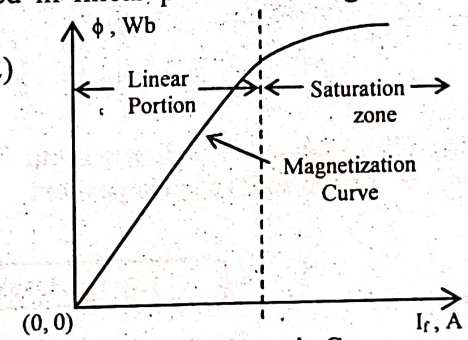


Fig. 1(b) Magnetic Curve

$\therefore T \propto i_f i_a$

or, $T = K_1 i_f . i_a$ .... (3)

But $i_f = \text{Constant} = K_2$

$\therefore T = K_1 K_2 . i_a$

$T = K i_a$ .... (4)

where K ⇒ Motor's torque constant, N-m /Amp

- As the motor rotates, a voltage proportional to the product of flux and angular velocity is induced in the armature circuit as per Faraday's laws of induction.

This voltage is known as back e.m.f. / counter e.m.f., e_b

$\therefore e_b \propto \phi . \frac{d\theta}{dt}$, in armature controlled d.c. servomotor ϕ is constant.

$$\therefore \quad e_b \propto \frac{d\theta}{dt}$$

or, $$e_b = K_b \frac{d\theta}{dt} \quad .... (5)$$

where, K_b ⇒ back e.m.f. constant., Volt-sec / rad.

Applying KVL in armature circuit

$$L_a \frac{di_a(t)}{dt} + R_a i_a(t) + e_b(t) = e_a(t) \quad \text{.... (6)}$$

- The motor's torque is now transmitted to the mechanical load.

From Free body diagram of the load (Mechanical system), we get the mathematical expression for the mechanical system is written as

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T = K_a i_a. \quad \text{.... (7)}$$

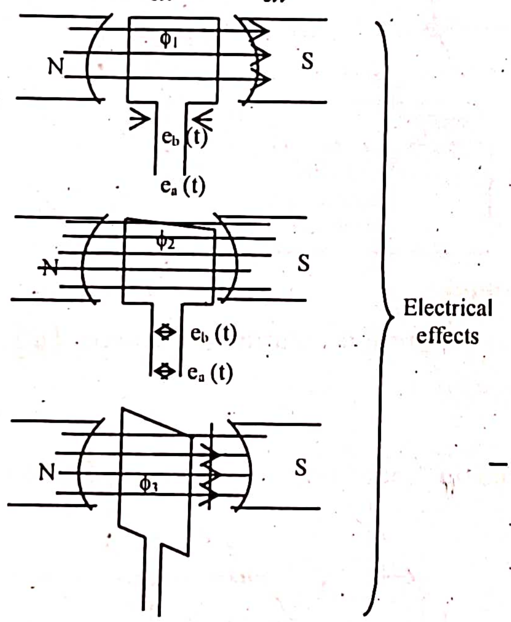


Fig. 1(c) Variation in flux linkage with the variation in coil's angular position

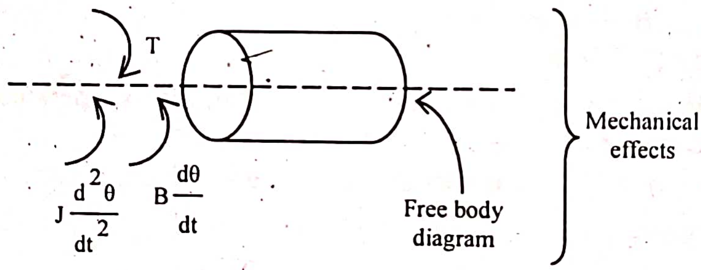


Fig. 1(d): Free body diagram of the load

Long Answer Type Questions

1. Write short notes on the following:
a) Servo Motors **[WBUT 2015, 2017]**
b) Synchros and Position encoders **[WBUT 2018]**

Answer:

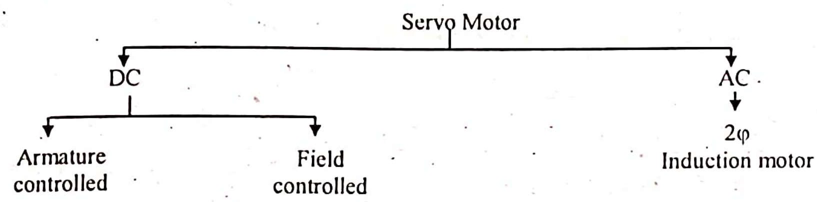
a) Servo Motors:

The word 'Servo' has basically been derived from the word 'Servant', who follows the instructions given by the master. Hence, a DC servomotor is used as an actuator in the control loop to drive a load as instructed by the controller.

It is usually a DC motor of low power rating, and having high ratio of Torque to Inertia (T/J) so bearing a faster dynamic response.

A Servomotor is a component of a control system in which the controlled variable is a mechanical angular position or rate of change of angular position. It converts electrical signal into equivalent mechanical system.

- **Classification**



Armature Controlled DC Motor
Refer to Question No. 1 of Short Answer Type Questions.

- ***Block Diagram presentation***

Step 1: We take Laplace transform of equation (5), (6) and (7) to have

$$E_b(s) = K_b s H(s) \quad \text{... (8)}$$

$$L_a s I_a(s) + R_a I_a(s) + E_b(s) = E_a(s)$$

$$\text{or,} (R_a + L_a s) I_a(s) + E_b(s) = E_a(s) \quad \cdots (9)$$

$$Js^2 H(s) + Bs H(s) = T(s) = K_a I_a(s)$$

$$\text{or,} (Js + B) s H(s) = K_a I_a(s) \quad \cdots (10)$$

Rearranging equation above equation, we get

$$E_a(s) - E_b(s) = (R_a + L_a s) I_a(s) \quad \text{... (11)}$$

Step 2: *Development of blocks:*

From equation (8),

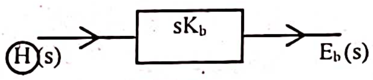


Fig: 1e

From equation (11),

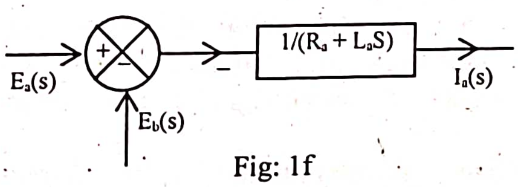


Fig: 1f

From equation (8),

$$\frac{H(s)}{I_a(s)} = \frac{K_a}{s(Js+B)}$$

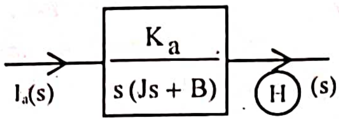


Fig: 1g

Step 3: Recombine the blocks of figures 1 (e), (f) and (g)

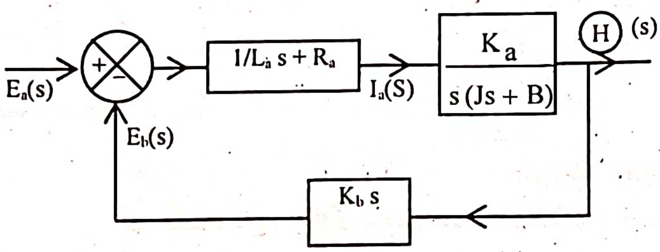


Fig: 1h Block diagram of the armature controlled DC motor

- ***Evaluation of Transfer Function***

We know,

$$\text{CLTF} = \frac{H(s)}{E_a(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\text{where } G(s) = \text{Forward path T.F.} = \frac{K_a}{s(Js+B)(L_a s+R_a)} \qquad \dots (12)$$

$$H(s) = \text{Feedback path T.F.} = \mathrm{K_b} s \qquad \dots (13)$$

$$\frac{H(s)}{E_a(s)} = \frac{\dfrac{K_a}{s(Js+B)(L_a s+R_a)}}{1+\dfrac{K_a}{s(Js+B)(L_a s+R_a)} \times K_b s} = \frac{K_a}{s\left[JL_a s^2 + (L_a B + R_a J)s\right] + K\ K_b s}$$

$$= \frac{K_a}{s\left[JL_a s^2 + (R_a J + L_a B)s + K_a K_b\right]} \qquad \dots (14)$$

- ***Effect of back e.m.f.***

A DC motor is basically an open-loop system.
But, the basic block diagram (figure 1(h)) shows that motor has an ***inherent*** negative feedback loop which is due to the back e.m.f. (counter e.m.f.).
Physically, back e.m.f. represents a feedback signal which is proportional to the speed of the motor.
We know that negative feedback improves the stability of a system.
In armature controlled DC servo motor, e_b (back e.m.f.) causes an inherent negative feedback. This improves the stability of the DC motor. As a result, the performance of the servo motor improves.

Field Control DC Servo

Here, in figure 2,

i) armature terminal is connected to a constant DC source (e_a) with very high internal impedance(R). As a result, armature current (i_a) remains constant.

ii) control signal (from controller) is fed to the field windings with resistance R_f and inductance L_f.

iii) armature is mechanically coupled to the load.

- **Operation**

Torque T developed by the motor is proportional to the product of air gap flux ϕ_f and armature current i_a

$\therefore \quad T \propto \phi_f i_a$

As i_a is constant,

$\therefore \quad T \propto \phi_f$

In linear part of magnetization curve, $\phi_f \propto i_f$

$\therefore \quad T \propto i_f$

or, $\quad T = K_f . i_f \qquad \dots (15)$

where, K_f = Motor's field torque constant, N – m/Amp.

Schematic diagram of a field controlled dc motor is as shown in figure 2.

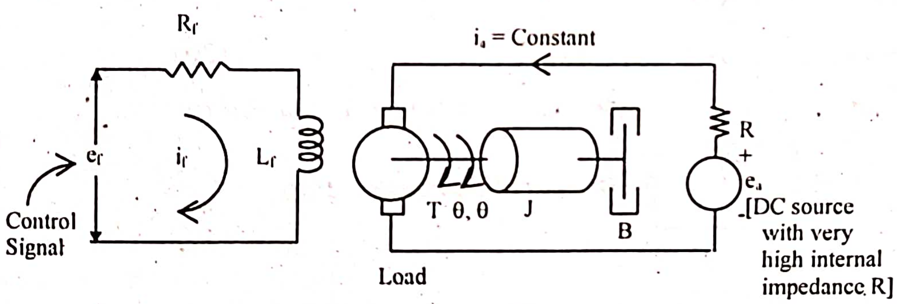


Fig: 2 Field control DC servo

Here, R_f = Field winding resistance, ohm

L_f = Field winding inductance, henry

i_f = Field current in amp.

e_f = Applied field voltage, volt

R = Internal Impedance of the source, ohm

i_a = Armature current in amp.

T = Torque developed by the motor, N – m

J = Equivalent moment of inertia of rotor of motor & load, Kg – m^2

B = Equivalent viscous friction of the rotor of motor & load, Nm/rad/sec.

From field circuit we have, $\quad e_f = R_f i_f + L_f \frac{d.i_f}{dt} \qquad \dots (16)$

From the mechanical system, we draw the free body diagram (figure 2a).

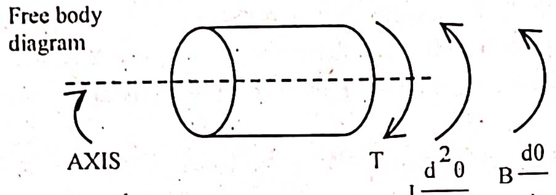


Fig: 2a Free body diagram

From the free body diagram, balancing the different forces

$$T-\left(J\frac{d^2\theta}{dt^2}+B\frac{d\theta}{dt}\right)=0 \qquad \dots (17)$$

or, $$T=J\frac{d^2\theta}{dt^2}+B\frac{d\theta}{dt}=K_f i_f \qquad \dots (18)$$

[From equation (15)]

- ***Generation of Block diagram***

Step 1: Take Laplace transformation of time-domain differential equations. We thus get,

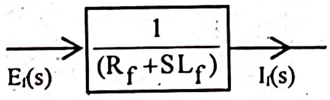
$$E_f(s)=R_f I_f(s)+sL_f I_f(s)$$

or, $$E_f(s)=(R_f+sL_f)I_f(s) \qquad \dots (19)$$

and $$Js^2 H(s)+Bs H(s)=K_f I_f(s)$$

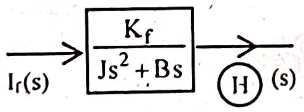
or, $$H(s)\left[Js^2+Bs\right]=K_f I_f(s) \qquad \dots (20)$$

Step 2: Draw blocks from individual equation in Laplace form



[From equation (19)]

Fig: 2b



[From equation (20)]

Fig: 2c

Step 3: Combine the blocks in figures 2 (b) and 2 (c)

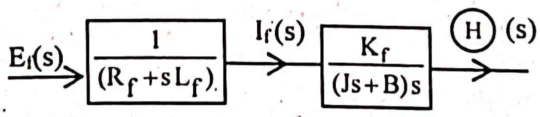


Fig: 2d

Step 4: Apply block diagram reduction technique in figure 2 (d)

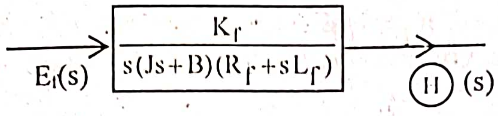


Fig: 2e Represents the block diagram of a field controlled DC serve-motor

N.B.: (a) It is a type-1, 3rd order system.
(b) Like armature controlled DC motor, it does not provide any inherent feedback path. So, from stability point of view it does not provide a good option.

b) Synchros and Position encoders:
A **Synchro,** named also as Selsyn (a word made up from self-synchronizing) or Autosyn (a word made up from **automatically-synchronizing**), is an electromagnetic transducer, which produces an electrical signal in response to the angular displacement.
A Synchro is basically consists of two sections

1. Synchro transmitter
2. Synchro receiver

Synchro Transmitter
The synchro transmitter converts the angular position of its rotor (mechanical input) into an electrical output signal. (Fig. 1)

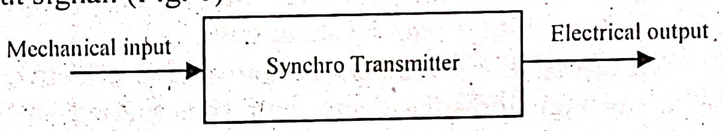


Fig: 1

- **Construction**

Synchro Transmitter by construction is similar to a Y-connected 3-phase alternator, having a stator part and a rotor part.

- **Stator part**

The Stator, which is stationary, is made up of laminated silicon steel and is slotted to wind a **balanced 3-phase** winding which is of concentric coil type.
The axes of the coils are displaced 120° apart with each other and Y connected. (Fig. 2)
The stator windings provide electrical output.

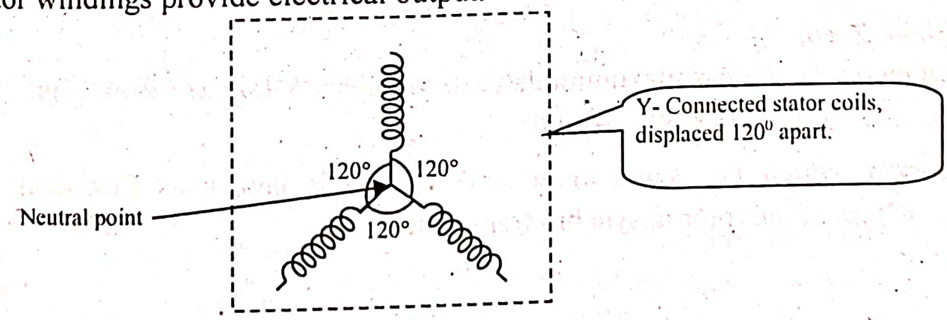


Fig: 2

- **Rotor part:**

The rotor is of dumb bell shaped. It is a salient pole type wound with concentric coils. Through the ship rings, an a.c. voltage is fed to the rotor winding.

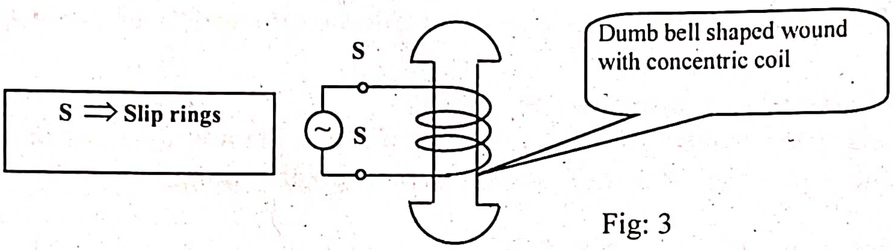


Fig: 3

- **Principle of operation**

The synchro transmitter is based on Faraday's laws of inductions and acts as a transformer with

- rotor as primary side.
- stator as secondary side where the windings are displaced 120° apart from each other.

When an ac voltage is applied to the rotor of a synchro transmitter, the following events occur

- an alternating current produces an ac magnetic field around the rotor windings.
- the lines of force cut through the windings of the three stator coils and, by transformer action, induce voltage into the stator coils.
- when the maximum effective coil voltage is known, the effective voltage induced into a stator coil at any angular displacement can be determined.

Let us consider, ac voltage applied to the rotor is

$$V_r(t) = A\sin\omega t$$

and induced voltages in the stator windings are

$$\left.\begin{aligned} V_{S1}(t) &= KA\sin\omega t\cos\theta \\ V_{S2}(t) &= KA\sin\omega t\cos(120+\theta) \\ V_{S3}(t) &= KA\sin\omega t\cos(240+\theta) \end{aligned}\right\} \qquad \ldots (1)$$

- ***Electrical Zero:***

In Fig. 4 when θ = 0, V_{S1} has maximum value of voltage $= KA\sin\omega t$ (from Eqn. 1)

and $V_{L2} = \sqrt{3}KA\sin\omega t \cdot \sin(180+\theta) = 0$

The position at which V_{S1} is maximum and $V_{L2} = 0$ is known as Electrical zero or reference position of the rotor in synchro transmitter.

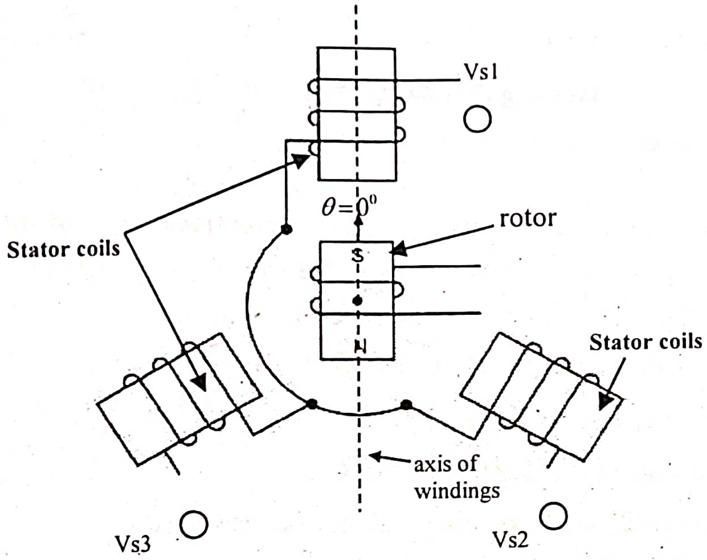


Fig: 4 Electrical zero in synchro transmitter

Synchro Receiver / Control Transformer

When the stator windings (output) of a synchro transmitter is coupled to a synchro control transformer as shown in the Fig. 5, the complete system is called Synchro Error Detector or simply Synchro.

Unlike the synchro transmitter, the receiver has an electrical input to its stator and a mechanical output from its rotor.

The synchro receiver's function is to convert the electrical signal at its stator from the transmitter, back to a mechanical angular position through the movement of its rotor.

Synchro Error Detector compares the angular positions of the two rotors.

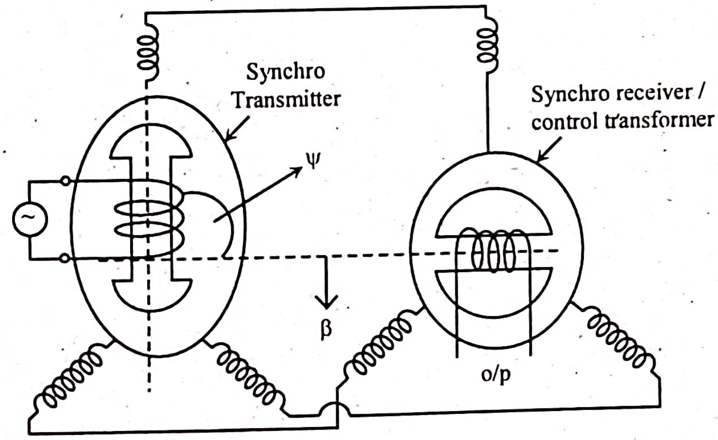


Fig: 5 Synchro

- **Working**

Since stators of both Synchro transmitter and Synchro control transformer are identical and output signal from synchro transmitter is fed as the input to the stator of the control transformer, the flux patterns are identical in both the systems.

A voltage will be induced in the rotor of the control transformer. The induced voltage will be proportional to the cosine of the angle between two rotors

$\therefore \quad e(t) = K_1 A \sin\omega t \cos\Psi$

where, $\Psi \Rightarrow$ angular displacement between the rotors.

and $\quad K_1 \Rightarrow$ proportionality constant

When $\Psi = 90^\circ$, $e(t) = 0$ This position is known as **electrical zero of the error detector**.

Let us consider, $\theta_1 \Rightarrow$ Angular displacement of the rotor of transmitter

and $\quad \beta \Rightarrow$ Angular displacement of the control transformer

$\therefore$ Net angular $e(t) = K_1 A \sin\omega t \cos(90 + \theta - \beta)$

$$= K_1 A \sin\omega t \sin(\beta - \theta) \qquad \text{.... (2)}$$

For $(\beta - \theta)$ to be small, $\sin(\beta - \theta) \to (\beta - \theta)$ .... (3)

So, from Eqns. (2) and (3) we have $e(t) = K_1 A \sin\omega t \cdot (\beta - \theta)$

$\Rightarrow \quad e(t) \propto (\beta - \theta)$

$\Rightarrow$ Synchro-transmitter and control transformer pair acts as an Error detector.

STABILITY ANALYSIS AND ROUTH STABILITY CRITERION

Multiple Choice Type Questions

1. A system has a single pole at origin. Its impulse response will be [WBUT 2007]
   a) constant b) ramp c) decaying exponentially d) oscillatory

Answer: (a)

2. The Routh-Hurwitz criterion gives [WBUT 2008, 2012, 2016]
   a) Relative stability b) Absolute stability
   c) Gain margin d) Phase margin

Answer: (b)

Short Answer Type Questions

1. The characteristic equation of a system is given by [WBUT 2005, 2008]

$$s^3+3Ks^2+(K+2)s+4=0$$

Find the range of k for which the system is stable.

Answer:

First write the characteristic equation $s^3+3Ks^2+(K+2)s+4=0$

While checking necessary condition found hold good. In order check the check the sufficient condition we form Routh's array.

	Col 1	Col 2
Row 1 s^3	1	K + 2
Row 2 s^2	3K	4
Row 3 s^1	$\frac{3K(K+2)-4}{3K}$	0
Row 4 s^0	4	

To have a stable system there should not be any change in sign of Routh elements in the first column of Routh array, i.e.,

$$3K>0 \quad \text{(From row 2)}$$

$$\Rightarrow \quad K>0$$

$$\& \quad \frac{3K(K+2)-4}{3K}>0 \quad \text{(From row 3)}$$

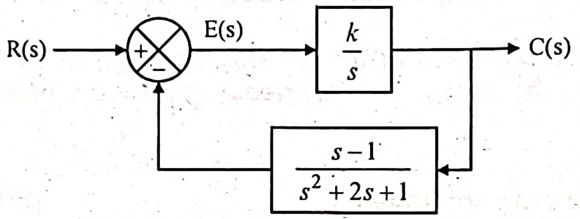
$$\Rightarrow \quad 3K(K+2)-4>0$$

$$\Rightarrow \quad 3K^2+6K-4>0$$

i.e., $K_1, K_2 = \dfrac{-6 \pm \sqrt{36+48}}{6} = \dfrac{-6 \pm \sqrt{84}}{6} = 0.53 - 2.52 = -1.99$

But $K = -2.52$ is not acceptable. Thus the range of K for stability is $0<K<0.53$.

2. Find the range of *k* to keep the system shown in figure to be stable. [WBUT 2009]



Answer:

The characteristic equation is $1+G(s)H(s)=0$

$$1+\frac{k}{s}\cdot\frac{s-1}{s^2+2s+1}=0$$

$\Rightarrow \quad s\left(s^2+2s+1\right)+k(s-1)=0$

$\Rightarrow \quad s^3+2s^2+s+ks-k=0$

$\Rightarrow \quad s^3+2s^2+s(1+k)-k=0$

$$\begin{array}{c|cc} s^3 & 1 & (1+k) \\ s^2 & 2 & -k \\ s^1 & \dfrac{2(1+k)+k}{2} & \\ s^0 & -k & \end{array}$$

$$\frac{2(1+k)+k}{2}>0$$

$\Rightarrow \quad 2(1+k)+k>0$

$\Rightarrow \quad 2+2k+k>0$

$\Rightarrow \quad 3k>-2$

or, $\quad k>\dfrac{-2}{3} = -k>0=k<0$

The range of stability $-\dfrac{2}{3}<k<0$.

3. For a system with $F(s)=s^4+22s^3+10s^2+s+k=0$, obtain the marginal value of *k* & the frequency of oscillation for that value of *k*. **[WBUT 2010]**

Answer:

1st Part:

Forming Routh's array

$$\begin{array}{c|ccc} s^4 & 1 & 10 & k \\ s^3 & 22 & 1 & 0 \\ s^2 & \frac{219}{22} & k & 0 \\ s^1 & \dfrac{\frac{219}{22}-22k}{219/22} & 0 & 0 \\ s^0 & k & 0 & 0 \end{array}$$

For marginal value of k

$$\frac{219}{22}-22k=0 \quad \Rightarrow \quad k=\frac{219}{22\times 22}=0.4525.$$

2nd Part:

Auxiliary equation $=\frac{219}{22}s^2+k=0$

$$\Rightarrow \quad s^2=\frac{-22}{219}k=\frac{-22}{219}\times\frac{219}{22\times 22}=-\omega^2$$

$$\therefore \quad \omega=\sqrt{\frac{1}{22}} \text{ rad./sec.}$$

4. Using Routh criterion investigate the stability of the unity feedback control system whose open loop transfer function is $G(s)=\frac{e^{-sT}}{s(s+2)}$. For what value of T will be system be stable? **[WBUT 2011]**

Answer:

$$G(s)=\frac{e^{-sT}}{s(s+2)}$$

[For low frequency higher terms in the expansion of e^{-sT} may be neglected]

$$\therefore 1+GH=1-\frac{1-sT}{s(s+2)}=s^2+(2+T)s-1=0$$

Forming Routh array

$$\begin{array}{c|cc} s^1 & 1 & -1 \\ s & 2+T & 0 \\ s^0 & 1 & 0 \end{array}$$

For stability, $2+T>0 \quad \Rightarrow T>-2$.

5. The characteristics equation of a feedback system is $s^4+4s^3+16s^2+16s+48=0$. Check whether the system is oscillatory. If so, determine the frequency of oscillations. **[WBUT 2013]**

Answer:

Step 1: Forming Routh array

$$\begin{array}{cccc}
s^4 & 1 & 16 & 48 \\
s^3 & \cancel{4}^{1} & \cancel{16}^{4} & 0 \\
s^2 & \cancel{12}^{1} & \cancel{48}^{4} & 0 \\
s^1 & 0 & 0 & 0
\end{array}$$

Step 2: Forming auxiliary equation

s^1 row becomes zero row.

s^1 row is therefore formed by the procedure below:

$$12s^2+48=0$$

or, $\quad s^2+4=0$

$\Rightarrow \quad s^2=-4$

$\Rightarrow \quad s=\pm j\sqrt{2}$.

6. Find the stability of the system whose characteristic equation is given by $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$ **[WBUT 2015]**

Answer:

	Col 1	Col 2	Col 3	Col 4
Row 1 s^7	1	7	11	2
Row 2 s^6	3	10	11	6
Row 3 s^5	$\frac{21-10}{3}=\frac{11}{3}$	$\frac{33-11}{3}=\frac{22}{3}$	$\frac{6-6}{3}=0$	0
Row 4 s^4	$\frac{\frac{11}{3}\cdot 10-22}{\frac{11}{3}}$ $=\frac{\frac{110}{3}-\frac{66}{3}}{\frac{11}{3}}=4$	$\frac{\frac{11}{3}\cdot 11-0}{\frac{11}{3}}=11$	$\frac{\frac{11}{3}\cdot 6-0}{\frac{11}{3}}=6$	0
Row 5 s^3 Sign change	$\frac{4\times\frac{22}{3}-11\times\frac{11}{3}}{4}$ $=\frac{\frac{88}{3}-\frac{121}{3}}{4}=-\frac{11}{4}$	$\frac{4\cdot 0-6\cdot\frac{11}{3}}{4}=-\frac{11}{2}$	0	0
Row 6 s^2	$\frac{\frac{-121}{4}+22}{\frac{-11}{4}}=3$	6	0	0
Row 7 s^1	0	0	0	0
Row 8 s^0	0	0	0	0

Auxiliary equation $A(s) = 3s^2 + 6 = 0$

$$\frac{dA(s)}{ds} = 6s$$

Row 7 s^1	6	0	0	0
Row 8 s^0	6	0	0	0

There are two sign changes in the first column i.e., 4 to −11/4 and −11/4 to 3. Therefore, two of the seven roots are in the right half of s-plane.

From the auxiliary equation

$$3s^2 + 6 = 0$$

or, $s^2 + 2 = 0$

$\Rightarrow$ $s^2 = -2$

$\Rightarrow$ $s = \pm j1.41$

The solution of the auxiliary equation say that the system has two roots lying on the imaginary axis of s-plane.

So number of roots lying left side of s-plane is three and two roots line on the $j\omega$ -axis.

So the system is unstable.

7. Utilize the Routh table to determine the number of roots of the following polynomial in the right half of s plane. Comment on the stability of the system: **[WBUT 2016]**

$$s^5 + 6s^4 + 15s^3 + 30s^2 + 44s + 24$$

Answer:

$$s^5 + 6s^4 + 15s^3 + 30s^2 + 44s + 24$$

Formation of Routh array:

s^5	1	15	44
s^4	6	30	24
s^3	$\frac{90-30}{6} = 10$	$\frac{130\times 44 - 15\times 24}{30} = \frac{1320-360}{30} = 32$	0
s^2	$\frac{300-32\times 6}{10} = 19.2$	$\frac{32\times 24 - 0}{32} = 24$	0
s^1	$\frac{32\times 19.2 - 240}{19.2} = 19.5$	0	0
s^0	24	0	0

As the signs of all the elements of the first column of the Routh array are positive, the roots of the characteristic equation in the left half of s-plane. So the system is stable. So number of roots lying right side of s-plane is zero.

8. $s^3+5s^2+6s+30=0$. **Test the stability of the given system.** **[WBUT 2018]**

Answer:

Given system characteristic equation:

$$s^3+5s^2+6s+30=0$$

In order to check the stability, we form Routh's array:

		Col 1	Col 2
Row 1:	s^3	1	6
Row 2:	s^2	5	30
Row 3:	s^1	0	0
Row 4:	s^0	30	

As in the 1st column of the Routh's array have no sign change, so, the system is stable.

Long Answer Type Questions

1. a) Mention the difficulties that may arise in applying Routh stability criterion. What do you mean by relative stability? **[WBUT 2007, 2013, 2015]**

Answer:

1st Part:

1. Routh stability criterion gives the idea of absolute stability only. It is very difficult and tedious job to get the idea of relative stability of a system with such an approach.
2. It does not give any idea about the pole locations on the s-plane if the system is unstable.
3. Cannot handle a system with time delay. Solution demands a low frequency approximation.
4. Cannot handle a system having characteristics equation with imaginary and exponential co-efficients.
5. The Routh criterion can be used on small systems (Small here means systems of low order. One hopefully wouldn't use Routh's approach on systems with 10 poles).

2nd Part:

Relative stability is the degree of stability, i.e. how better the system is from the standpoint of stability.

b) The open lop transfer function of a unity feedback control system is given by

$$G(s)=\frac{K(s+1)}{2s^3+as^2+2s+1}$$

The above system oscillates with frequency ω, if it has poles on $s=+j\omega$ and $s=-j\omega$ and no poles in the right half s-plane. Determine the values of K and a, so that the system oscillates at a frequency of 2 radian/sec. **[WBUT 2007]**

Answer:

Characteristic equation is given by, $2s^3 + as^2 + s(K+2) + (K+1) = 0$

The Routh array is formed as

$$\begin{array}{c|cc} s^3 & 2 & (K+2) \\ s^2 & a & (K+1) \\ s^1 & \dfrac{a(K+2)-2(K+1)}{a} & 0 \\ s^0 & K+1 & \end{array}$$

Now, two poles are on the imaginary axis

$\therefore$ The auxiliary equation is given by $\quad as^2 + K + 1 = 0$

$$\Rightarrow \quad s^2 = -\frac{K+1}{a} \quad \Rightarrow s = j\omega = j\sqrt{\frac{K+1}{a}}$$

But frequency of oscillation is 2 rad./ sec.

$$\therefore \quad \sqrt{\frac{K+1}{a}} = 2 \quad \Rightarrow \frac{K+1}{4} = a \qquad \cdots(i)$$

Again, $\dfrac{a(K+2)-2(K+1)}{a} = 0$

$$\Rightarrow \quad a = 2\left(\frac{K+1}{K+2}\right) \qquad \cdots(ii)$$

Combining equation (i) and (ii)

$$\frac{K+1}{4} = \frac{2(K+1)}{K+2}$$

$$\Rightarrow \quad K+2 = 8$$

$$\Rightarrow \quad K = 6$$

$$\therefore \quad a = \frac{K+1}{4} = \frac{7}{4} = 1.75$$

2. a) State the R-H criteria for stability. Find the range of open-loop gain (K) for which the system given below will be stable. Consider unity negative feedback.

$$G(s) = \frac{K}{s^5 + 2s^4 + 3s^2 + 4s}$$

[WBUT 2014]

Answer:

$$G(s) = \frac{K}{s^5 + 2s^4 + 3s^2 + 4s}$$

$$H(s) = -1$$

The characteristics equation is

$$1 + GH = 0$$

or, $\quad 1+\dfrac{K}{s^5+2s^4+3s^2+4s}\cdot(-1)=0$

or, $\quad s^5+2s^4+3s^2+4s-K=0$

The Routh's array is

$$\begin{array}{llll} s^5 & 1 & 3 & -K \\ s^4 & 2 & 4 & \\ s^3 & \dfrac{6-4}{2}=1 & \dfrac{-4K-0}{4}=-K & \\ s^2 & 4-2K & 0 & \end{array}$$

For stability

$4-2K>0$

or, $\quad 2K<4$

or, $\quad K<2$

b) A dynamic system has the characteristic equation $s^5+s^4+2s^3+2s^2+3s+5=0$. Check whether the system is stable or not. **[WBUT 2014]**

Answer:

Routh's array is

$$\begin{array}{l|lll} s^5 & 1 & 2 & 3 \\ s^4 & 1 & 2 & 5 \\ s^3 & d & -2 & \\ s^2 & \dfrac{2d+2}{d} & 5 & \\ s^1 & \dfrac{-4d-4-5d^2}{2d+2} & & \\ s^0 & 5 & & \end{array}$$

The first term of the fifth row has a value of -2 as $d\to 0$. Thus, there are sign changes in the first column making the system unstable.

3. For the unity feedback system whose open loop transfer function is

$$G(s)=\frac{k}{s(s+1)(s+2)(s+5)}$$

[WBUT 2015]

(i) Find the range of k for stability of the system.

(ii) Find the value of k if the system is marginally stable.

(iii) Find the actual location of the closed loop poles when the system is marginally stable.

Answer:

Step 1: Development of characteristic equation.

$1+G(s)H(s)=0$

$\Rightarrow \quad s^4+8s^3+17s^2+10s+K=0 \qquad \cdots(i)$

Step 2: *To check the necessary conditions.*

Necessary conditions hold good.

Step 3: To check the sufficient condition.

For this form Routh array:

	Col 1	Col 2	Col 3
Row 1 s^4	1	17	K
Row 2 s^3	8	10	0
Row 3 s^2	15.75	K	0
Row 4 s^1	$\dfrac{157.5-8K}{15.75}$	0	0
Row 5 s^0	K		

Step 4:

Part A:

For the stability of the system:

- $K > 0$
- $157.5 - 8K > 0$

$\Rightarrow \quad K < 19.7$

$\therefore$ Range of K for the stability of the system is $0 < K < 19.7$.

Part B:

For marginal stability:

$K = 19.7$

Part C:

The auxiliary equation:

$$15.75s^2 + K = 0 \qquad \cdots(ii)$$

$$\Rightarrow \quad 15.75s^2 + 19.7 = 0$$

$$\Rightarrow \quad s^2 + 1.25 = 0$$

$$\Rightarrow \quad s = \pm j1.1$$

So, the two poles are at $+j1.1$ and $-j1.1$.

Since the order of the system = 4

$\therefore$ Two more poles are there for K = 19.7 whose locations are to be determined as

$$
\begin{array}{r|l}
 & s^2 + 8s + 15.75 \\
\hline
s^2 + 1.25 & s^4 + 8s^3 + 17s^2 + 10s + 19.7 \\
 & s^4 + 1.25s^2 \\
\hline
 & 8s^3 + 15.75s^2 + 10s + 19.7 \\
 & 8s^3 + 10s \\
\hline
 & 15.75s^2 \quad + 19.7 \\
 & 15.75s^2 \quad + 19.7 \\
\hline
 & 0 \qquad\qquad 0
\end{array}
$$

$\therefore s^2 + 8s + 15.75$ must be a factor of the characteristic equation.

$\therefore \qquad s^2 + 8s + 15.75 = 0$

$\Rightarrow \qquad s_{1,2} = \dfrac{-8 \pm \sqrt{64-63}}{2} = \dfrac{-8 \pm 1}{2} = -3.5 \ \& \ -4.5$

So, actual pole locations for K = 19.7 are $+j1.1$, $-j1.1$, -3.5 & -4.5.

4. A closed loop system have $G(s) = \dfrac{k}{s}$ and $H(s) = \dfrac{(s-1)}{1+2s+s^2}$. Find the range of k so that the given system is stable. **[WBUT 2018]**

Answer:

$$G(s)H(s) = \frac{k(s-1)}{s(1+2s+s^2)} = \frac{k(s-1)}{(s^3+2s^2+s)}$$

$\therefore$ Characteristics equation $= 1 + \dfrac{k(s-1)}{(s^3+2s^2+s)} = s^3 + 2s^2 + s + ks - k = 0$

$\Rightarrow \qquad s^3 + 2s^2 + (1+k)s - k = 0$

Forming Routh's array

$$\begin{array}{c|cc} s^3 & 1 & 1+k \\ s^2 & 2 & -k \\ s^1 & \dfrac{2(1+k)+k}{2} & 0 \\ s^0 & -k & \end{array}$$

For stability $k < 0$ [From s^0th row]

and $3k + 2 > 0$ [From s^1 th row]

$\Rightarrow \qquad k > -\dfrac{2}{3} \Rightarrow k > -0.66$

$\therefore$ Range of k should be $-0.66 < k < 0$

5. Write short notes on the following:

a) Routh array **[WBUT 2013]**

b) Relative stability & Routh's stability criterion. **[WBUT 2013]**

OR,

Relative and absolute stability of a control system **[WBUT 2015]**

Answer:

a) A Routh stability criterion is an approach or a tool to find out the stability of a linear time-invariant differential equation system by considering the characteristic equation of the system. We know that for a system to be stable, the poles must lie in the left half of s-plane. For systems with lower order (say up to third order), one can find the pole locations; but for higher order systems, it becomes difficult to find the pole locations and to know about the stability of the system. Routh stability method provides an answer to this problem.

Routh's ***necessary conditions*** for a stable system
For the ***characteristics equation*** of a system

(i) There should not be any missing power of s.
(ii) All the co-efficients of the polynomial should be real.
(iii) All the co-efficients of the polynomial should have same sign.

Sufficient condition for a stable system
There should be no change in sign in the elements of the first column of Routh array.

Procedure in Routh Stability Criteria
Step 1: Develop the characteristics equation.
Step 2: Check for necessary conditions of RSC. If the necessary conditions fail, then no more need to proceed for checking sufficient condition. As by obvious reason, the roots of the characteristic equation will lie either on the imaginary axis or R.H.P.
Step 3: Check for sufficient condition. For this, form Routh array.
Step 4: Check for any change in sign in the Routh elements of *first column* of Routh array.
Step 5: Draw conclusion about stability as

(i) No change in sign → system is stable.
(ii) Change in sign → unstable system. Number of change in sign of Routh elements in first column will indicate the Number of roots with positive real part.

Procedure of Forming Routh Array
Step 1: Let the characteristic equation is $a_0s^n + a_1s^{n-1} + \ldots\ldots + a_{n-1}s + a_n = 0$. n is an integer, may be odd or even.
Step 2: If n, the highest power in s, is even then (n –1) will be odd and vice-versa.

Arrange the co-efficients of the polynomial in s in rows and column as follows:

s^n	a_0	a_2	a_4	a_6	......
s^{n-1}	a_1	a_3	a_5	a_7	......
s^{n-2}	$\frac{a_1a_2 - a_0a_3}{a_1} = b_1$	$\frac{a_1a_4 - a_0a_5}{a_1} = b_2$			......
s^{n-3}	$c_1 =$	c_2			......
	$c_1 = \frac{b_1a_3 - b_2a_1}{b_1}$				
..	......				
..	......				
s^0	......				

The process of forming rows continues until we reach s^0.

The total number of rows formed will be (n +1).
The complete array of co-efficients will be triangular.

b) By the term relative stability we mean the degree of stability i.e. how better the stability of a system with respect to a standard one or how far or close the system is to instability.
By applying Routh Stability criterion, which basically finds for the presence of all roots in the left half of the s-plane, we can say whether a system is stable or not i.e. the absolute stability of the system.
If there are no roots neither on the imaginary axis nor in the right half of s-pane, we say that the system is absolutely stable.

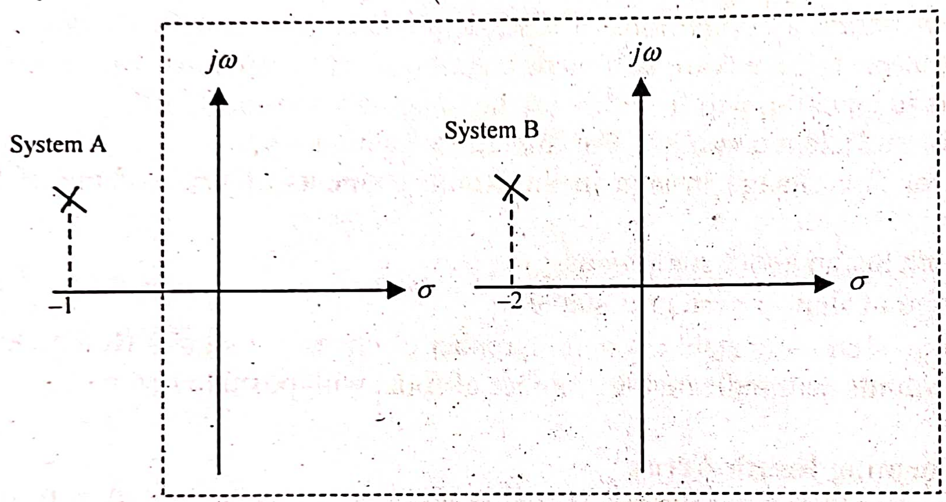


Fig: a Dominant pole locations relative to $j\omega$ axis
System B is more stable relative to the system A

Hence it is necessary that poles nearest to the $j\omega$ -axis called the dominant poles of the system, are reasonably away from the $j\omega$- axis. The systems with more negative real parts of the dominant poles will be more stable than the systems with less negative real parts of the dominant poles as shown in Fig: a
Hence we can say that the distance of the negative real parts of the dominant poles from the $j\omega$-axis is a *measure of the relative stability.*
A useful approach for examining the relative stability is to shift the $j\omega$-axis of s-plane to form a new plane, say it as z-plane and apply Routh's stability criterion.

- Substitute $s = z - a$ (a = constant) into the characteristic equation of the system,
  - write the polynomial in terms of z.
  - apply Routh's stability criterion to the new polynomial in z.
- The number of changes of sign in the first column of the array developed for the polynomial in z is equal to the number of roots which are located to the right of the vertical line $s = -a$.

If this number is zero then we can say the dominant pole of the system is having more negative real parts than $-a$.

In this way test is carried out till we get the change in sign in the first column of the Routh array. This way we can measure the distance of the negative real part of the dominant pole and hence the relative stability.

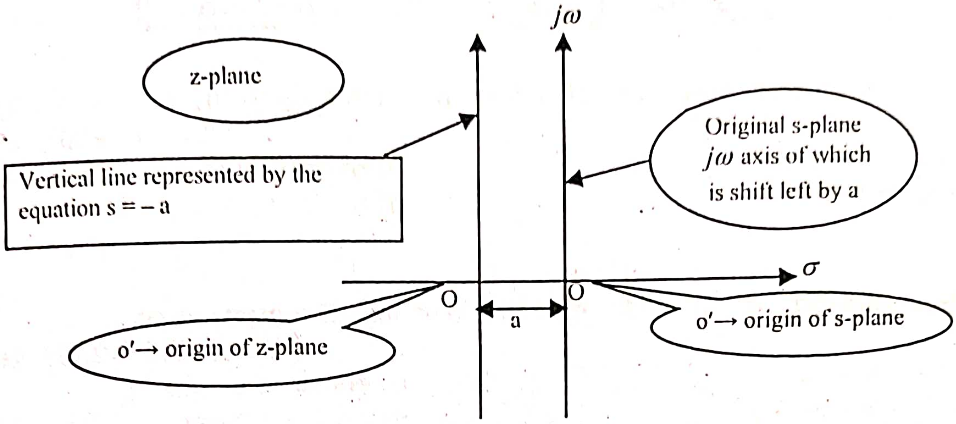


Fig: b Formation of z-plane from s-plane

TIME DOMAIN ANALYSIS

Multiple Choice Type Questions

1. The characteristic equation of a second order system is $s^2+6s+25=0$. The system is [WBUT 2007, 2012, 2016]

a) under damped
b) over damped
c) undamped
d) critically damped

Answer: (a)

2. The type number of a transfer function denotes the number of [WBUT 2007, 2008, 2009, 2017]

a) poles at origin
b) zeros at origin
c) poles at infinity
d) none of these

Answer: (a)

3. The response of a control system, having damping factor as unity will be [WBUT 2007, 2012]

a) oscillatory
b) underdamped
c) critically damped
d) none of these

Answer: (c)

4. Area under a unit impulse function is [WBUT 2007, 2012, 2016]

a) infinity b) zero c) unity d) none of these

Answer: (c)

5. A second order control system with $\xi=0$ is always [WBUT 2008, 2012]

a) Marginally stable b) Stable c) Unstable d) None of these

Answer: (a)

6. A system having transfer function $G(s)=\dfrac{1}{2(s+0.5)}$ is subjected to a unit step input, the steady value of the output is [WBUT 2009]

a) 1 b) 2 c) $\frac{1}{2}$ d) $\frac{1}{10}$

Answer: (a)

7. The natural frequency of oscillations of the output for the equation $\dfrac{d^2x}{dt^2}+1.5\dfrac{dx}{dt}+4x=1$ is [WBUT 2009]

a) 0 rad/sec b) 1.5 rad/sec c) 2 rad/sec d) 4 rad/sec

Answer: (c)

8. The steady state error for a unity feedback system having open loop transfer function as $G(s)=\frac{9}{s(0.2s+1)}$ when subjected to a unit step input will be [WBUT 2009]

a) 0.1 b) 1/9 c) 0.2 d) 0

Answer: (d)

9. The settling time of a second order system on 2% basis is given by [WBUT 2009]

a) $t_s=\frac{4}{\zeta\omega_n}$ b) $t_s=\frac{\zeta\omega_n}{4}$ c) $t_s=\frac{4\zeta}{\omega_n}$ d) $t_s=4\zeta\omega_n$

Answer: (a)

10. Integral error control [WBUT 2009, 2011]

a) increases the order of the system b) decreases the order of the system
c) increases the steady state error d) does not affect the steady state error

Answer: (a)

11. Addition of poles to the closed loop transfer function [WBUT 2010, 2011, 2017]

a) increases rise time b) decreases rise time
c) increases overshoot d) has no effect

Answer: (c)

12. A system has a pole at origin, its impulse response will be [WBUT 2010, 2018]

a) constant b) ramp
c) decaying exponentially d) oscillatory

Answer: (a)

13. If the maximum overshoot is 100%, the damping ratio is [WBUT 2011, 2015, 2018]

a) 1 b) 0 c) 0.5 d) ∞

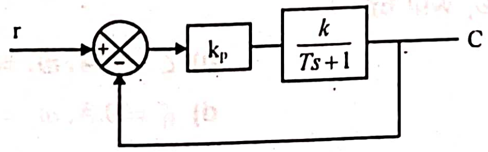
Answer: (b)

14. If the, characteristic equation of a system is $s^2+8s+25=0$, the value of ξ_n and ω_n will be [WBUT 2011]

a) 0.8, 5 rad/s b) 0.8, 0.5 rad/s c) 0.5, 8 rad/s d) 5, $\sqrt{8}$ rad/s

Answer: (a)

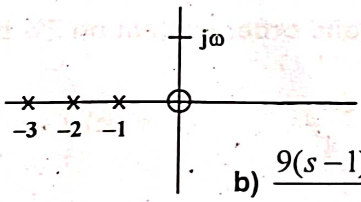
15. The steady-state error due to a step input for the system shown below is [WBUT 2011]



a) zero b) $\frac{1}{1+kk_p}$ c) $\frac{kk_p}{1+kk_p}$ d) infinite

Answer: (b)

16. The transfer function of a system having a gain of 9 and a pole zero map as in figure below is [WBUT 2011]



a) $\frac{9(s+1)(s+2)(s+3)}{s}$ b) $\frac{9(s-1)(s-2)(s-3)}{s}$

c) $\frac{9s(s+1)}{(s+2)(s+3)}$ d) $\frac{9s}{(s+1)(s+2)(s+3)}$

Answer: (d)

17. If the system has multiple poles of imaginary axis the system is [WBUT 2013]

a) stable b) unstable
c) marginally stable d) none of these

Answer: (b)

18. A feedback control system has transfer function given by $G(s)=\frac{6(s+1)(s+6)}{s^3(s+2)(s+4)}$. It is a [WBUT 2013]

a) Type-5 system b) Type-3 system c) Type-2 system d) Type-0 system

Answer: (b)

19. What happens to the time constant of a system if a negative feedback is inserted? [WBUT 2013]

a) Time constant is increased
b) Time constant is decreased
c) Time constant is unaffected
d) Time constant is increased 10 times

Answer: (b)

20. For the system $\frac{C(s)}{R(s)}=\frac{25}{s^2+6s+25}$ the damping factor ζ and damped frequency of oscillations ω_4 will be [WBUT 2014]

a) $\zeta=0.6, \omega_d=5$ b) $\zeta=0.4, \omega_d=6$

c) $\zeta=0.5, \omega_d=3$ d) $\zeta=0.3, \omega_d=5$

Answer: (a)

21. A closed loop control system has $G(s)=\frac{100}{s^2}$ and $H(s)=(s+1)$. The steady state output for a unit step input is [WBUT 2014, 2017]

a) 1.0 b) 1/100 c) 1/10 d) 10

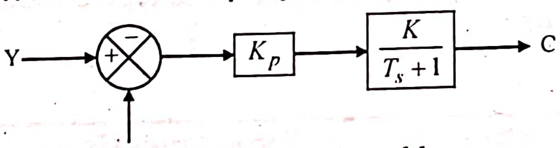
Answer: (a)

22. The settling time of a second order system 2% basis is given by [WBUT 2014]

a) $t_s=\frac{4}{\xi\omega_n}$ b) $t_s=\frac{\xi\omega_n}{4}$ c) $t_s=\frac{4\xi}{\omega_n}$ d) $t_s=4\xi\omega_n$

Answer: (a)

23. The steady-state error due to a step input for the system shown below is [WBUT 2014]



a) zero b) $\frac{1}{1+kk_p}$ c) $\frac{kk_p}{1+kk_p}$ d) infinite

Answer: (b)

24. If the poles of a control system lie on the imaginary axis in s-plane the system will be [WBUT 2015]

a) unstable
b) stable
c) conditionally
d) marginally stable

Answer: (d)

25. The poles of a system lie on –4 and –20 the system is [WBUT 2015]

a) overdamped
b) under damped
c) critically damped
d) undamped

Answer: (a)

26. The settling time of a second order linear system is [WBUT 2015]

a) 4 times the time constant of the system
b) 3 times the time constant of the system
c) 1/4 times the time constant of the system
d) none of these

Answer: (a)

27. If there is no overshoot in a system, then the damping ratio is [WBUT 2016]

a) 1 b) 0 c) 0.5 d) ∞

Answer: (b)

28. Velocity error of a system occurs due to [WBUT 2016]

a) unit step input
b) unit ramp input
c) unit impulse input
d) unit parabolic input

Answer: (a)

29. For the system $\frac{C(s)}{R(s)} = \frac{36}{s^2+6s+36}$ the damping factor ζ and damped frequency of oscillations ω_d will be, [WBUT 2017]

a) $\zeta = 0.6,\ \omega_d = 5$
b) $\zeta = 0.4,\ \omega_d = 6$
c) $\zeta = 0.5,\ \omega_d = 3$
d) $\zeta = 0.3,\ \omega_d = 5$

Answer: (c)

30. The steady state error of unit ramp input in the type 2 systems is [WBUT 2018]

a) 0
b) 1
c) 5
d) infinity

Answer: (a)

Short Answer Type Questions

1. A feedback control system is described as $G(s) = \frac{50}{s(s+2)(s+5)}$, $H(s) = \frac{1}{s}$

Evaluate the static error constants K_p, K_v & K_a for the system. [WBUT 2007]

Answer:

$$G(s) = \frac{50}{s(s+2)(s+5)} \quad H(s) = \frac{1}{s}$$

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{50}{s^2(s+2)(s+5)} = \infty \quad \text{(Ans.)}$$

$$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{50}{s(s+2)(s+5)} = \infty \quad \text{(Ans.)}$$

$$K_a = \lim_{s\to 0} s^2G(s)H(s) = \lim_{s\to 0} \frac{50}{(s+2)(s+5)} = 5 \quad \text{(Ans.)}$$

2. A feedback control system is described as $G(s) = \frac{50}{s(s+2)(s+5)} \cdot H(s) = \frac{1}{5}$

Evaluate static error constants K_p, K_v and K_q for the system. [WBUT 2008]

Answer:

$$G(s) = \frac{50}{s(s+2)(s+5)}$$

$$H(s) = \frac{1}{5}$$

$$G(s)H(s)=\frac{50}{s(s+2)(s+5)}\cdot\frac{1}{5}=\frac{10}{s(s+2)(s+5)}$$

$$K_p=\lim_{s\to 0}G(s)H(s)=\lim_{s\to 0}\frac{10}{s(s+2)(s+5)}=\infty$$

$$K_v=\lim_{s\to 0}s\cdot G(s)H(s)=\lim_{s\to 0}s\cdot\frac{10}{s(s+2)(s+5)}=1/\text{sec}$$

$$K_a=\lim_{s\to 0}s^2\cdot G(s)\cdot H(s)=\lim_{s\to 0}s^2\cdot\frac{10}{s(s+2)(s+5)}=0/\text{sec}^2$$

3. A unity feedback heat treatment system has open loop transfer function $G(s)=\frac{10000}{(1+s)(1+0.5s)(1+0.02s)}$. The output set point is 500°C. What is the steady state temperature? **[WBUT 2009, 2016]**

Answer:

$$\frac{C(s)}{R(s)}=\frac{10000/(1+s)(1+0.5s)(1+0.02s)}{1+\frac{10000}{(1+s)(1+0.5s)(1+0.02s)}}$$

$$=\frac{10000}{(1+s)(1+0.5s)(1+0.02s)+10000}$$

$$\therefore\quad C(s)=\left[\frac{10000}{(1+s)(1+0.5s)(1+0.02s)+10000}\right]R(s)$$

$$=\left[\frac{10000}{(1+s)(1+0.5s)(1+0.02s)+10000}\right]\times\frac{500}{s}$$

$$\therefore\ \text{Steady state temperature}=\lim_{s\to 0}sC(s)=\frac{10000}{1000}\times 500=500^\circ\text{C}$$

4. For a unity feedback system having open loop transfer function as $G(s)=\frac{k(s+2)}{s^2(s^2+7s+12)}$, determine (a) number of types of the system, (b) error constants and (c) steady state error for parabolic input. **[WBUT 2010]**

Answer:

a) Type number is 2

b) $$k_p=\lim_{s\to 0}GH=\lim_{s\to 0}\frac{k(s+2)}{s^2(s^2+7s+12)}=\infty$$

$$k_v = \lim_{s\to 0} sGH = \infty/\sec$$

$$k_a = \lim_{s\to 0} s^2 GH = \frac{k\cdot 2}{12} = \frac{k}{6}/\sec^2$$

c) $$e_{ss} = \frac{1}{k_a}$$ [As input is parabolic]

$$= \frac{6}{k}\sec.$$

5. What do you mean by 'Transient response' and 'Steady State response' of a system? Obtain an expression for 'Unit impulse response' of first order system. **[WBUT 2012]**

OR,

What is the difference between transient response & steady state response of a system? **[WBUT 2013]**

Answer:

1st Part:

Transient response is that part of the time response that tends to a constant value as time becomes large, theoretically infinity. It is the response of the signal when the input signal changes from one state to other. As the transient response is a function of time is called also as the ***dynamic response.***

The nature of the transient response depends on the nature of excitation and the transfer function.

Steady state response: Steady state response is that part of the time response, which remains constant as the time tends to infinity. It depends on the nature of the input, type number of the system and the pole locations. Since the steady state response in independent of time it is called also as static response of the system

If c(t) is the system response then

$$c(t) = c_{tr}(t) + c_{sr}(t)$$

2nd Part:

A 1st order system is given by the differential equation

$$a_0\frac{dc(t)}{dt} + a_1 c(t) = br(t) \qquad \text{.... (i)}$$

where, $r(t)$ = The input variable in time domain

$c(t)$ = The output variable in time domain

and, a_0, a_1, b = coefficient of the system parameters.

Now, taking Laplace transform of the equation we get

$$a_0\left[sC(s) - C_0\right] + a_1 C(s) = bR(s)$$

$$\therefore \quad [a_0 s + a_1]C(s) - a_0 c_0 = bR(s)$$

$$\therefore \quad C(s) = \frac{bR(s)}{a_0 s + a_1} + \frac{a_0 c_0}{a_0 s + a_1} \qquad \text{.... (ii)}$$

Now, for time response due to unit impulse $r(t) = \delta(t)$ input and initial condition $C'_0 = 0$

$\therefore$ The Laplace transform of a unit impulse is $R(s) = L[r(t)] = L[\delta(t) = 1$

Then from equation (ii) the output $C(s)$ we get,

$$\therefore \quad C(s) = \frac{b}{a_0 s + a_1} \quad \text{or,} \quad C(s) = \frac{\frac{b}{a_0}}{s + \frac{a_1}{a_0}}$$

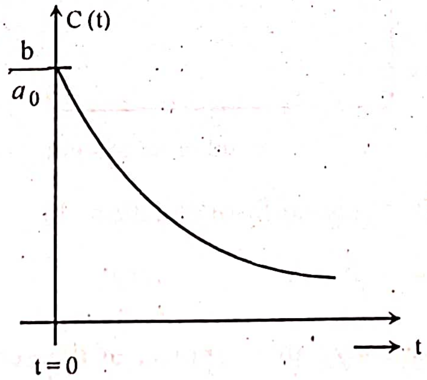
Now, taking inverse Laplace transform of $C(s)$ we have,

$$C(t) = L^{-1}[C(s)] = L^{-1}\left[\frac{\frac{b}{a_0}}{s + \frac{a_1}{a_0}}\right]$$

$$\therefore \quad C(t) = \frac{b}{a_0} e^{-\frac{a_1}{a_0}t} = \frac{b}{a_0} e^{-\frac{t}{T}}$$

where, $T = \frac{a_0}{a_1}$ = time constant.

$\therefore$ The time response curve of a first-order system due to unit impulse input is given below.



6. What is 'Damping ratio'? Obtain an expression for 'Unit step response' of a second order system when the damping ratio is unity. [WBUT 2012]

Answer:

1st Part:

Damping ratio (ξ): The ratio of the damping coefficient B of a second-order system and value of the damping coefficient B_C required for critical damping is called the damping ratio ξ

$$\therefore \qquad \xi = \frac{B}{B_C}$$

When $\xi = 0 =$ System is no damping
$\xi < 1 =$ System is under damping
$\xi = 1 =$ System is critical damping
$\xi > 1 =$ System is over damping

2nd Part: ***Transient Response of a Second Order System***

Let us consider (refer to figure 1)

a) a unity feedback control system i.e. H(s) = 1

b) forward path transfer function $G(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

where $\omega_n \Rightarrow$ natural frequency(rad./sec)
$\xi \Rightarrow$ damping ratio $\Big\}$ of the system

$\therefore$ Closed loop T.F. of the second order system $= \dfrac{G(s)}{1+G(s)H(s)}$

$$\text{or,} \qquad \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad \cdots(1)$$

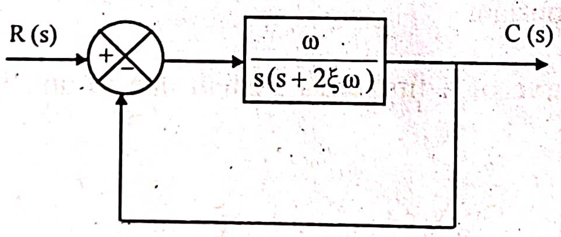


Fig: 1 A second order system

For unit step function input R(s) = 1/s, so from equation (1)

$$C(s) = \frac{1}{s} \times \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad \cdots(2)$$

For a constant ω_n (natural frequency), the response of the second order system depends on the value of ξ.

So, the response of the second order system is classified as

i) Under damped response, for $0 < \xi < 1$.
ii) Critically damped response, for $\xi = 1$.
iii) Over damped response, for $\xi > 1$.
iv) Undamped response, for $\xi = 0$.

In order to find the response of the second order system in time domain, for different cases, we take inverse Laplace transformation at both sides of equation. (2).

For Critically Damped System ($\xi = 1$)

Putting $\xi = 1$ in equation 1, we have

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2.1.\omega_n.s + \omega_n^2}$$

with step input $R(s) = \frac{1}{s}$

$$\Rightarrow \quad C(s) = \frac{\omega_n^2}{s(s^2 + 2.1.\omega_n.s + \omega_n^2)} = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

By partial fraction method,

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{(s+\omega_n)} + \frac{C}{(s+\omega_n)^2}$$

Solving for A, B and C, we get

$$A = 1, \quad B = -1, \quad C = -\omega_n$$

$$\therefore \quad C(s) = \frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n^2}{s(s+\omega_n)^2}$$

$$\therefore \quad c(t) = 1 - e^{-\omega_n t} - \omega_n \cdot te^{-\omega_n t}$$

Values of c(t) for two extreme values of t (i.e. $t = 0$ and infinity) are being tabulated in following table. Figure 2 shows the pattern of the step response. The response shows no overshoot and undershoot.

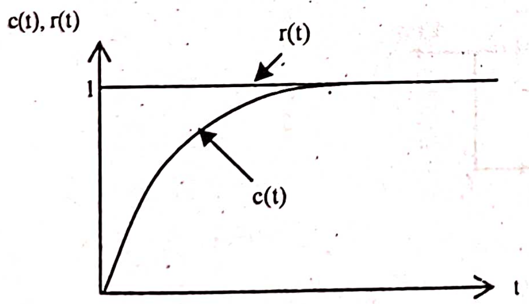


Table-

t	c(t)
0	0
∞	1

Fig. 2: step response for critically damped second order system

7. Define the following: (i) Rise time; (ii) Delay time; (iii) Settling time; (iv) Overshoot; (v) Steady state error. [WBUT 2014]

Answer:

1. ***Rise time (t_r):*** It is the time needed for the response to reach 100% of the final value *for the first time.*
2. ***Delay time (t_d):*** It is defined as the time needed for the response to reach 50% of the final value.

3. ***Settling time (t_s):*** It is the time needed for the system step response to reach & stay within a predefined tolerance limit / band.
t_s is usually taken as +/- 2% to +/-5%
4. ***Maximum Overshoot (M_p):***

$$M_p = \frac{c(t_p)-c(\infty)}{c(\infty)}\times 100\%$$

5. ***Steady state Error (e_{ss}):*** It is defined as

$$e_{ss} = \lim\left[r(t)-c(t)\right]$$

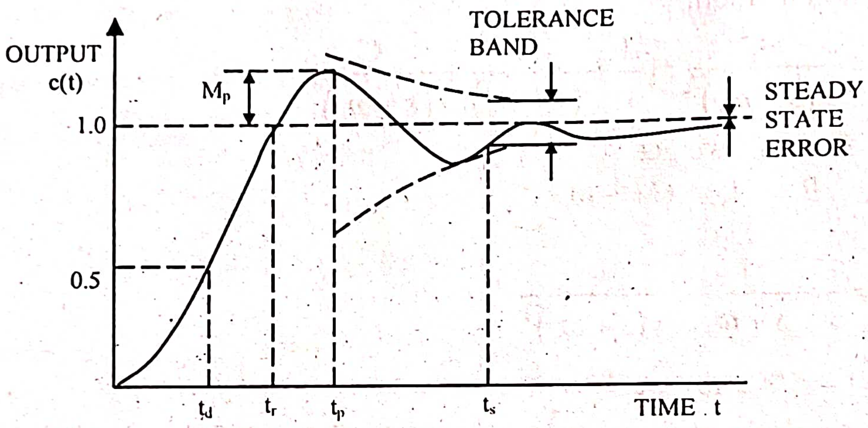
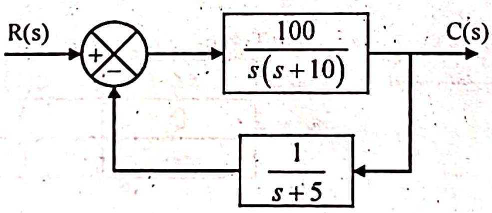


Fig: Time Domain Performance parameters

8. Find the error constants K_p, K_v, K_a of the following system: **[WBUT 2016]**



Answer:

$$G(s)H(s) = \frac{100}{s(s+10)}\cdot\frac{1}{s+5}$$

$$\therefore \quad K_p = \underset{s\to 0}{Lt}\, G(s)H(s) = \underset{s\to 0}{Lt}\, \frac{100}{s(s+5)(s+10)} = \infty$$

$$K_v = \underset{s\to 0}{Lt}\, s\, a(s)\, H(s) = \underset{s\to 0}{Lt}\, s\cdot\frac{100}{s(s+5)(s+10)} = \frac{100}{50} = 2$$

$$K_a = \underset{s\to 0}{Lt}\, s^2\, a(s)\cdot H(s) = \underset{s\to 0}{Lt}\, s^2\cdot\frac{100}{s(s+5)(s+10)} = 0$$

9. The maximum overshoot for a unity feedback control system having its forward path transfer function as $G(s)=\frac{K}{s(sT+1)}$ is to be reduced from 60% to 20%. The system input is a unit step function. Determine the factor by which K should be reduced to achieve aforesaid reduction. **[WBUT 2017]**

Answer:

$$G(s)=\frac{K}{s(1+sT)}$$

Let the value of damping ratio is ξ_1 when the peak overshoot is 60% and ξ_2 when peak overshoot is 20%.

$$M_p=\exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)$$

for $M_p=60\%$

$$\log\left(\frac{60}{100}\right)=\frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \quad \log\left(\frac{3}{5}\right)=\frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \quad -0.2218=\frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \quad \left(\frac{0.2218}{\pi}\right)^2=\frac{\xi^2}{1-\xi^2}$$

$$\Rightarrow \quad \left(\frac{\pi}{0.2218}\right)^2+1=\frac{1}{\xi^2}$$

$$\Rightarrow \quad \xi^2=4.9597\times10^{-3}$$

$$\Rightarrow \quad \xi=\xi_1=0.0704$$

$$\text{Closed loop transfer function}=\frac{G(s)}{1+G(s)H(s)}=\frac{K}{Ts^2+s+K}=\frac{C(s)}{R(s)}$$

or, $$\frac{C(s)}{R(s)}=\frac{\frac{K}{T}}{s^2+\frac{1}{T}s+\frac{K}{T}}$$

Therefore, $\omega_n=\sqrt{\frac{K}{T}}$ and $2\xi\omega_n=\frac{1}{T}$

Let the value of $K=K_1$ when $\xi=\xi_1$ and $K=K_2$ when $\xi=\xi_2$.

Since $2\xi\omega_n = \frac{1}{T}$

When $M_p = 20\%$

$$\log\left(\frac{20}{100}\right) = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \quad -0.6989 = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \quad \left(\frac{0.6989}{\pi}\right)^2 = \frac{\xi^2}{1-\xi^2}$$

$$\Rightarrow \quad \left(\frac{\pi}{0.6989}\right)^2 = \frac{1-\xi^2}{\xi^2}$$

$$\Rightarrow \quad \left(\frac{\pi}{0.6989}\right)^2 = \frac{1}{\xi^2} - 1$$

$$\Rightarrow \quad \left(\frac{\pi}{0.6989}\right)^2 + 1 = \frac{1}{\xi^2}$$

$$\Rightarrow \quad \xi^2 = 0.04715$$

$$\Rightarrow \quad \xi = \xi_2 = 0.21715$$

$$\xi = \frac{1}{2T\omega_n} = \frac{1}{2\sqrt{KT}}$$

We get, $\frac{\xi_1}{\xi_2} = \sqrt{\frac{K_2}{K_1}}$

or, $\frac{K_2}{K_1} = \left(\frac{0.0704}{0.21715}\right)^2 = 0.1051$

or, $K_2 = 0.1051K_1$

The amplifier gain has to be reduced by a factor $= \frac{1}{0.1051} = 9.5142$

10. Determine error co-efficient corresponding to step, ramp and parabolic inputs. **[WBUT 2018]**

Answer:

Classification of Error Constants

Depending upon the nature of excitation signal, error constants / co-efficients are classified as

- Position Error Constant
- Velocity Error Constant
- Acceleration Error Constant

(1) Position Error Constant (K_p)

Position Error Constant is defined for a unit step input i.e. $R(s)=\frac{1}{s}$. The steady state error of the system for a unit step input is called Position Error. K_p is defined as

$$K_p = \lim_{s\to 0} G(s)H(s)$$

For a unit step input, $R(s)=\frac{1}{s}$

$$\therefore \qquad e_{ss} = \lim_{s\to 0}\frac{s\cdot R(s)}{1+G(s)H(s)} = \lim_{s\to 0}\frac{1}{1+G(s)H(s)}\cdot s\cdot\frac{1}{s}$$

$$= \lim_{s\to 0}\frac{1}{1+G(s)H(s)} = \frac{1}{1+\lim_{s\to 0}G(s)H(s)}$$

Using the definition of K_p

$$e_{ss} = \frac{1}{1+K_p} \qquad \text{.... (1)}$$

The Eqn. 1 says

- As K_p increases e_{ss} increases and accuracy of the system increases
- At $K_p = \infty$, $e_{ss} = 0$

(2) Velocity Error Constant (K_v)

Velocity Error Constant is defined for unit ramp, i.e. $R(s)=\frac{1}{s^2}$

$$\text{Now,} \qquad e_{ss} = \lim_{s\to 0}\frac{s\cdot R(s)}{1+G(s)H(s)} = \lim_{s\to 0}\left[\frac{s}{1+G(s)H(s)}\right]\cdot\frac{1}{s^2}$$

$$= \lim_{s\to 0}\frac{1}{s+sG(s)H(s)} = \frac{1}{\lim_{s\to 0} sG(s)H(s)}$$

$$\Rightarrow \qquad e_{ss} = \frac{1}{K_v} \qquad \text{...... (2)}$$

where, $K_v = \lim_{s\to 0} sG(s)H(s)$ is defined as Velocity Error Co-efficient.

We conclude that

- $K_v \uparrow$ $e_{ss} \downarrow$ and accuracy $\uparrow$
- $e_{ss} \to 0$ as $K_v \to \infty$

$\boxed{\text{The Unit of } K_v \text{ is sec}^{-1}}$

(3) Acceleration Error Constant (K_a)

Acceleration Error Constant is defined for unit parabolic input i.e.,

$$R(s)=\frac{1}{s^3}$$

Now, $$e_{ss}=\lim_{s\to 0}\frac{s\cdot R(s)}{1+G(s)H(s)}=\lim_{s\to 0}\left[\frac{s}{1+G(s)H(s)}\right]\cdot\frac{1}{s^3}$$

$$=\lim_{s\to 0}\frac{1}{\left(s^2+s^2G(s)H(s)\right)}=\frac{1}{\lim\limits_{s\to 0}s^2G(s)H(s)}=\frac{1}{K_a}$$

where, $\boxed{K_a=\lim\limits_{s\to 0}s^2G(s)H(s)}$ is defined as Acceleration Error Co-efficient.

$\boxed{\text{The Unit of } K_a \text{ is sec}^{-2}}$

11. A unity feedback system has an open-loop transfer function $G(s)=\dfrac{25}{s(s+8)}$. Determine its damping ratio, peak overshoot and peak time. **[WBUT 2018]**

Answer:

$$G(s)=\frac{25}{s(s+8)}$$

So, close loop transfer function $T(s)=\dfrac{G(s)}{1\pm G(s)H(s)}$

∴ As the system have unit feedback

$$\therefore \quad T(s)=\frac{25}{s(s+8)\pm 25}=\frac{25}{s^2+8s\pm 25}$$

We have the general expression of a 2nd order transfer function is

$$T(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Comparing both the equation $\omega_n=5$

$$2\xi\omega_n=8$$

$$\xi=\frac{8}{2\times 5}=0.8$$

So, damping ratio $(\xi)=0.8$

Peak overshoot $(M_p)=\exp\left(\dfrac{-\pi\xi}{\sqrt{1-\xi^2}}\right)=1.015$

Peak time $t_p=\dfrac{\pi}{\omega_d}=\dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}=1.05s$

Long Answer Type Questions

1. For a unity feedback system G(s) given below, find the time domain specification for a unit step input $G(s)=\frac{200}{s(s+2)}$. **[WBUT 2008]**

Answer:

$$G(s)=\frac{200}{s(s+2)}$$

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}$$

But, $H(s)=1$ and $G(s)=\frac{200}{s(s+2)}$.

Therefore $\frac{C(s)}{R(s)}=\frac{200}{s^2+2s+200}$

Therefore $\omega_n^2=200$ $\qquad \omega_n=14.142$ rad./sec.

$2\xi\omega_n=2$

$$\xi=\frac{2}{2\times\sqrt{200}}=\frac{1}{\sqrt{200}}=0.07$$

$$\omega_d=\omega_n\sqrt{1-\xi^2}=14.142\sqrt{1-0.07^2}=14.10 \text{ rad./sec.}$$

$$\beta=\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}=\tan^{-1}\frac{\sqrt{1-0.07^2}}{0.07}=\tan^{-1}\frac{0.998}{0.07}=\tan^{-1}14.25=85.985°=1.5 \text{ rad.}$$

1) Rise time $\quad t_r=\frac{\pi-\beta}{\omega_d}=\frac{3.14-1.5}{14.10}=3.033$ sec.

2) Peak time $\quad t_p=\frac{\pi}{\omega_d}=\frac{3.14}{14.1}=0.2226$ sec.

3) Peak overshoot $M_p=e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}=e^{-\frac{3.14\times0.07}{\sqrt{1-0.07^2}}}=e^{-0.22}=80.25\%=0.8025$

4) Setting time (2% tolerance band) $=\frac{4}{\xi\omega_n}=\frac{4}{0.07\times14.142}=4.04$ sec.

4% tolerance band $=\frac{3}{0.07\times14.142}=3.03$ sec.

2. Prove that for a standard second order system defined by $G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$, the peak overshoot due to a step input depends on ξ only. Prove further that the constant ξ lines pass through the origin of the $(\sigma i j\omega)$ plane. [WBUT 2011]

Answer:

1st part:

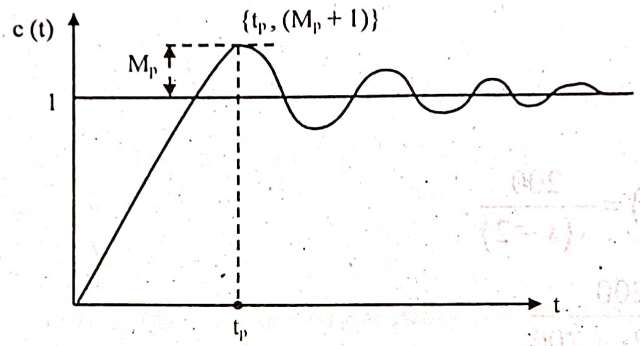


Fig: 1 Step response of a underdamped second order system

From fig. 1, at $t = t_p$, i.e., at peak time

$$c(t)\big|_{t=t_p} = 1 + M_p$$

$$= 1 - e^{-\frac{\sigma\pi}{\omega_d}}\left(\cos\omega_d \frac{\pi}{\omega_d} + \frac{\sigma}{\omega_d}\sin\pi\right)$$

$$= 1 - e^{-\frac{\sigma\pi}{\omega_d}}\left[\cos\pi + \frac{\sigma}{\omega_d}.0\right] \quad = 1 + e^{-\frac{\sigma\pi}{\omega_d}} = 1 + M_p$$

$$M_p = e^{\frac{-\xi\omega_n \cdot \pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}, \text{ for } 0 \le \xi < 1 \qquad \cdots\cdots(a)$$

Percent peak overshoot

$$= \%M_p = \left[e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\right] \times 100 \qquad \cdots\cdots(b)$$

Hence peak overshoot depends on ξ only.

2nd Part:

When $\xi < 1$, the output response equation $C(s) = \frac{\omega_n^2}{s\left(s^2 + 2\xi\omega_n s + \omega_n^2\right)}$ for a unit step input is written as

$$C(s)=\frac{A}{s}+\frac{B}{s+\xi\omega_n-j\omega_n\sqrt{1-\xi^2}}+\frac{C}{s+\xi\omega_n+j\omega_n\sqrt{1-\xi^2}} \quad \text{....}(1)$$

The location of the two conjugate poles is shown in figure 1.

From figure 1, we get

$$\cos\alpha=-\xi \quad \text{....}(2)$$

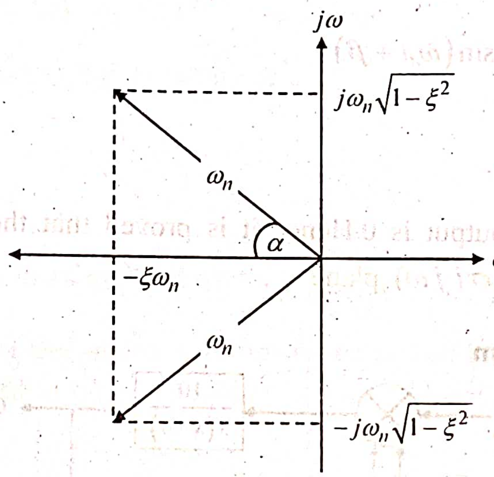


Fig: 1

$$\sin\alpha=\sqrt{1-\xi^2} \quad \text{....}(3)$$

Using these relations constants A, B and C are found out as

$A=1$

$N=2 \quad \because\ P=0 \quad \therefore\ Z=N+P=2$

$\omega=0.4$

Therefore equation (1) can be written as

$$\phi=-180^\circ+\tan^{-1}\frac{\omega}{0.25}-\tan^{-1}\omega-\tan^{-1}\frac{\omega}{0.5}$$

$$c(t)=1+\frac{e^{-j\alpha}}{2j\sin\alpha}e^{-\left(\xi\omega_n-j\omega_n\sqrt{1-\xi^2}\right)t}-\frac{e^{j\alpha}}{2j\sin\alpha}e^{-\left(\xi\omega_n-j\omega_n\sqrt{1-\xi^2}\right)t}$$

$$c(t)=1+\frac{e-\xi\omega_n t}{\sqrt{1-\xi^2}}\frac{e^{j\left(\omega_n t\sqrt{1-\xi^2}-\alpha\right)}-e^{-j\left(\omega_n t\sqrt{1-\xi^2}-\alpha\right)}}{2j}$$

or, $$c(t)=1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_n\sqrt{1-\xi^2}\,t-\alpha\right) \quad \text{....}(4)$$

From equations (2) and (3)

$$\frac{\sin\alpha}{\cos\alpha}=\frac{\sqrt{1-\xi^2}}{-\xi}$$

$$\alpha=-\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

Also, putting $\omega_d = \omega_n\sqrt{1-\xi^2}$, where ω_d is called a damped natural of frequency, equation (4) can be written as

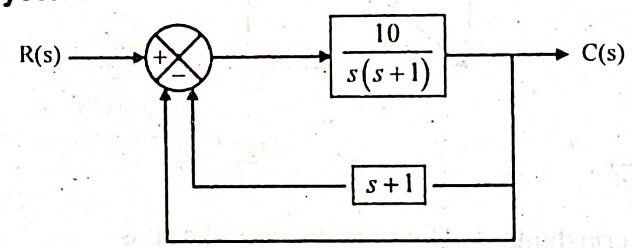
$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

or, $$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \beta)$$

where $\beta = \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi}$.

If we put $t=0$ then the output is 0.Hence it is proved that the constant ξ lines pass through the origin of the $(\sigma i\, j\omega)$ plane.

3. a) For the given system **[WBUT 2013]**



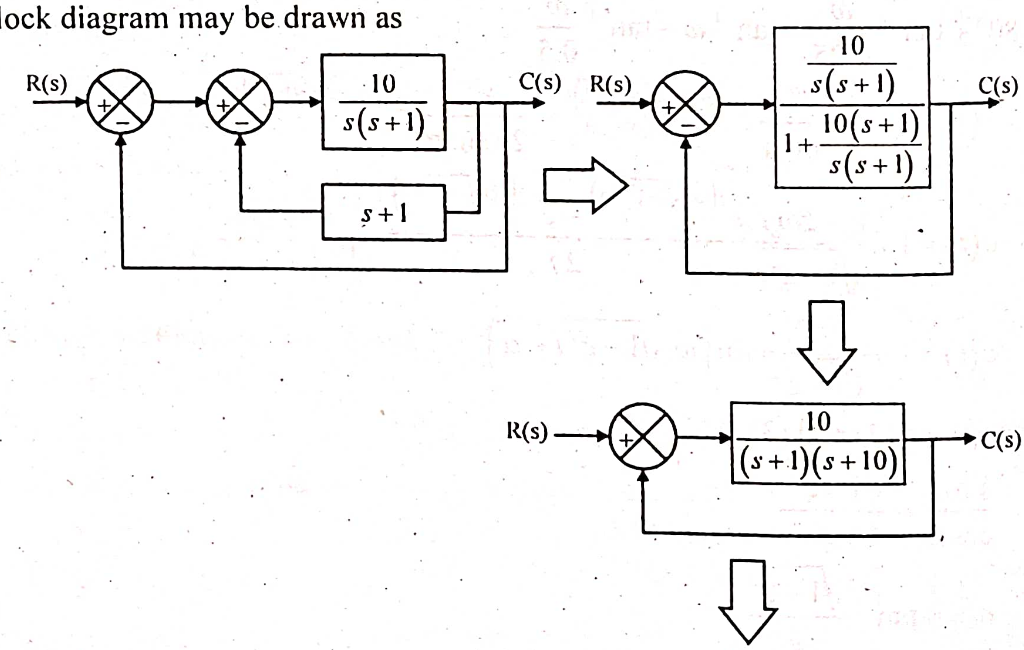
i) **Determine k_p, k_v & k_a.**

ii) **Find the steady state error for input $5t^2u(t)$.**

iii) **State the 'type' number of the system.**

Answer:

The block diagram may be drawn as



$$G(s)H(s)=\frac{10}{(s+1)(s+10)} \qquad [\text{As } H(s)=1]$$

(a) $K_p = \lim_{s\to 0} G(s)H(s) = 1$

$K_v = \lim_{s\to 0} sG(s)H(s) = 0/\sec$

$K_a = \lim_{s\to 0} s^2 G(s)H(s) = 0/\sec^2$.

(b) $e_{ss} = \frac{1}{K_v}$ [As input is ramp $= 5t^2u(t)$]

$= \infty$.

(c) Type number is 0.

b) Consider the closed loop system given by $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$. Determine the value of ζ & ω_n so that the system response to a step input with approximate 5% overshoot & with a settling time of 2 sec. **[WBUT 2013]**

Answer:

$$0.05 = e^{-\frac{\xi\times\pi}{\sqrt{1-\xi^2}}}$$

$$t_s = \frac{4}{\xi\omega_n}$$

$$0.05 = e^{-\frac{3.14\xi}{\sqrt{1-\xi^2}}}$$

$$\Rightarrow 2 = \frac{4}{\xi\omega_n}$$

$$\log 0.05 = \frac{-3.14\xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \quad -1.301 = \frac{-3.14\xi}{\sqrt{1-\xi^2}}$$

Squaring both sides we get

$$\Rightarrow \quad (1.301)^2 = \frac{3.14^2\xi^2}{1-\xi^2}$$

$$\Rightarrow \quad 1.692601 = \frac{9.8596\xi^2}{1-\xi^2},$$

$$\Rightarrow \quad \frac{\xi^2}{1-\xi^2} = \frac{1.692601}{9.8596} = 0.1716$$

$$\Rightarrow \quad \xi^2 = 0.1716 - 0.1716\xi^2$$

$$\Rightarrow \quad 1.1716\xi^2 = 0.1716$$

$$\Rightarrow \quad \xi^2 = \frac{0.1716}{1.1716} = 0.1465$$

$$\xi = 0.3827$$

$$2 = \frac{4}{\xi\omega_n}$$

$$\Rightarrow \quad \omega_n = \frac{4}{2\xi} = \frac{4}{2\times 0.3827} = 5.22 \text{ rad/sec}$$

$$\xi = 0.3827$$

$$\omega_n = 5.22 \text{ rad/sec.}$$

4. a) Define the following terms, Steady state error, Settling time, Peak Overshoot, Type and Order of a Control system. **[WBUT 2015, 2017]**

Answer:

Steady state Error (e_{ss}): It is defined as

$$e_{ss} = \lim\left[r(t) - c(t)\right]$$

Settling time (t_s): It is the time needed for the system step response to reach & stay within a predefined tolerance limit / band.

t_s is usually taken as +/- 2% to +/-5%

Maximum Overshoot (M_p):

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

The type number of an open loop system is defined as the number of poles at origin in s-plane.

The definition can be understood through the table.

For the systems:

Table:

$G(s)H(s)$	Type	Order
$\dfrac{(s+1)}{(s+3)(s+4)}$	0	2
$\dfrac{(s+1)(s+2)}{s(s+3)(s+4)}$	1	3
$\dfrac{(s+1)(s+2)}{s^2(s+3)(s+4)}$	2	4

b) Sketch the transient response of a 2nd order system and derive the expression for rise time and peak overshoot. **[WBUT 2015, 2017]**

Answer:

Let us consider (refer to figure 1)

a) a unity feedback control system i.e. H(s) = 1

b) forward path transfer function $G(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

where $\omega_n \Rightarrow$ natural frequency(rad./sec) } of the system
$\xi \Rightarrow$ damping ratio

$\therefore$ Closed loop T.F. of the second order system $= \dfrac{G(s)}{1+G(s)H(s)}$

or, $$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad \cdots (1)$$

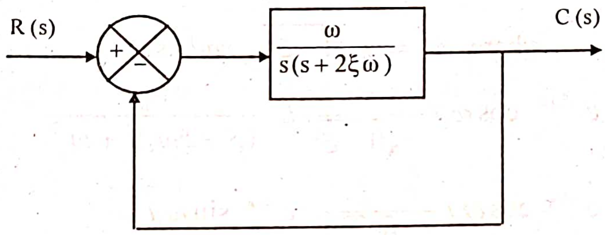


Fig: 1 A second order system

For unit step function input R(s) = 1/s, so from equation (1)

$$C(s) = \frac{1}{s} \times \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad \cdots (2)$$

For a constant ω_n (natural frequency), the response of the second order system depends on the value of ξ.

So, the response of the second order system is classified as

i) Under damped response, for $0 < \xi < 1$.

ii) Critically damped response, for $\xi = 1$.

iii) Over damped response, for $\xi > 1$.

iv) Undamped response, for $\xi = 0$.

In order to find the response of the second order system in time domain, for different cases, we take inverse Laplace transformation at both sides of equation. (2).

For Underdamped System, ($\xi < 1$)

$$c(t) = L^{-1}\left[\frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}\right]$$

$$= L^{-1}\left[\frac{1}{s} - \frac{s+2\xi\omega_n}{(s^2+2\xi\omega_n s+\omega_n^2)}\right]$$

$$= L^{-1}\left[\frac{1}{s} - \left\{\frac{s+\xi\omega_n}{(s^2+2\xi\omega_n s+\omega_n^2)} + \frac{\xi\omega_n}{(s^2+2\xi\omega_n s+\omega_n^2)}\right\}\right]$$

$$= L^{-1}\left[\frac{1}{s} - \frac{s+\xi\omega_n}{(s^2+2\xi\omega_n s+(\xi\omega_n)^2-(\xi\omega_n)^2+\omega_n^2)}\right.$$

$$\left. - \frac{\xi\omega_n}{s^2+2\xi\omega_n s+(\xi\omega_n)^2-(\xi\omega_n)^2+\omega_n^2)}\right]$$

$$= L^{-1}\left[\frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}\right]$$

where, $\omega_d = \omega_n\sqrt{1-\xi^2}$, $rad./\sec.$

$$= 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}} L^{-1}\frac{\omega_d}{(S+\xi\omega_n)^2+\omega_d^2}$$

$$= 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}\cdot e^{-\xi\omega_n t}\sin\omega_d t$$

$$= 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}\sin\omega_d t\right]$$

$$= 1 - e^{-\sigma t}\left[\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right] \quad \cdots(3)$$

where $\sigma = \xi\omega_n$, $rad./\sec.$, called as damping co-efficient.

ω_d is called as damped natural frequency, which is equal to the natural frequency when $\xi = 0$.

Now, from the knowledge of Trigonometry, we know,

$$A\sin\theta + B\cos\theta = C\cos(\theta-\beta) \quad \cdots(4)$$

where, $A = C\sin\beta$ and $B = C\sin\beta$

Comparing equations (3) and (4)

$$c(t) = 1 - e^{-\sigma t}C\left[\cos(\omega_d t-\beta)\right] \quad \cdots(5)$$

where, $B = 1$, $A = \dfrac{\sigma}{\omega_d}$, $\theta = \omega_d t$

As, $C = \sqrt{A^2+B^2}$, so we can re-write the equation (5)

$$\therefore \quad c(t) = 1 - e^{-\sigma t}\left(\sqrt{\left(\frac{\sigma}{\omega_d}\right)^2 + 1}\right)\cdot\cos(\omega_d t - \beta)$$

$$c(t) = 1 - e^{-\xi\omega_n t}\left(\sqrt{\left(\frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}\right)^2 + 1}\right)\cdot\cos(\omega_d t - \beta) \qquad \cdots(6)$$

Figure (1) shows the graphical presentation of c(t) w.r.t. time (t)

Now, at maximum value of c(t), $\frac{d}{dt}c(t) = 0 \qquad \cdots(7)$

or, $\frac{d}{dt}\left[1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right)\right] = 0$ (From equations 6 and 7)

or, $+\sigma e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right) - e^{-\sigma t}\left(-\omega_d t\sin\omega_d t + \sigma\cos\omega_d t\right) = 0$

or, $\sigma e^{-\sigma t}\cos\omega_d t + e^{-\sigma t}\frac{\sigma^2}{\omega_d}\sin\omega_d t + e^{-\sigma t}\omega_d t\sin\omega_d t - e^{-\sigma t}\sigma\cos\omega_d t = 0$

or, $e^{-\sigma t}\left(\frac{\sigma^2}{\omega_d}\sin\omega_d t + \omega_d\sin\omega_d t\right) = 0 \qquad \cdots(8)$

This happens, when $\sin\omega_d t = 0$

or, $\sin\omega_d t_p = \sin n\pi$

where, $n = 1, 2, 3, \cdots$

or, $\omega_d t_p = n\pi$

or, $t_p = \frac{n\pi}{\omega_d} \qquad \cdots(9)$

or, $t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}} \qquad \cdots(10)$

In equation (10), when $n = 1$, we get the first overshoot. Second overshoot is obtained for $n = 3$ and so on. Similarly, when $n = 2$, we get first under short etc.
In general, for all odd values of n, i.e. $n = 1, 3, 5, \cdots$, we get overshoots and for all even values of n, i.e. $n = 2, 4, 6, \cdots$, we get undershoots.

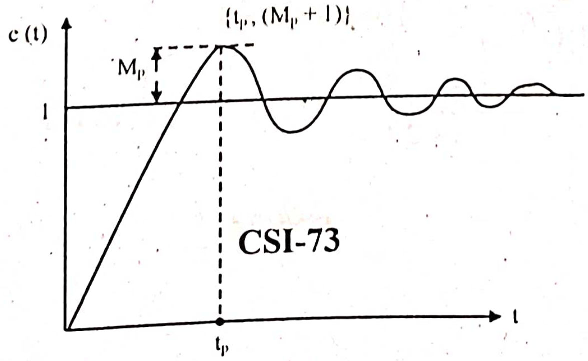


Fig: 2 Step response of a underdamped second order system

Evaluation of Peak Overshoot M_P

From figure 2, at $t = t_p$, i.e., at peak time

$$c(t)\Big|_{t=t_p} = 1 + M_p$$

$$= 1 - e^{-\frac{\sigma\pi}{\omega_d}}\left(\cos\omega_d\frac{\pi}{\omega_d} + \frac{\sigma}{\omega_d}\sin\pi\right) \text{ [from equation 3]}$$

$$= 1 - e^{-\frac{\sigma\pi}{\omega_d}}\left[\cos\pi + \frac{\sigma}{\omega_d}.0\right] \qquad = 1 + e^{-\frac{\sigma\pi}{\omega_d}} = 1 + M_p$$

$$M_p = e^{\frac{-\xi\omega_n\cdot\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}, \text{ for } 0 \le \xi < 1 \qquad \cdots(11)$$

Percent peak overshoot

$$= \%M_p = \left[e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\right] \times 100 \qquad \cdots(12)$$

c) The forward path transfer function of a unity feedback control system is given by $G(s) = \frac{2}{[s(s+3)]}$. Obtain the expression for unit step response of the system.

[WBUT 2015, 2017]

Answer:

$$\text{CLTP} = \frac{\frac{2}{s(s+3)}}{1 + \frac{2}{s(s+3)}} = \frac{C(s)}{R(s)}$$

$$\Rightarrow \quad \frac{C(s)}{R(s)} = \frac{2}{s^2 + 3s + 2} = \frac{2}{(s+2)(s+1)}$$

$$\therefore \quad C(s) = \frac{2}{(s+1)(s+2)} \cdot \frac{1}{s} \quad \left[\text{As } R(s) = \frac{1}{s}\right]$$

$$\Rightarrow \quad C(s) = 2\left[\frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}\right] \qquad \ldots(1)$$

Solving, we get $A=\frac{1}{2}$, $B=-1$ and $C=\frac{1}{2}$

$\therefore$ From Eqn. (1)

$$C(s)=\frac{1}{s}-\frac{2}{s+1}+\frac{1}{s+2}$$

$$\therefore \quad C(t)=1-2\cdot e^{-t}+e^{-2t}.$$

5. a) Consider the closed loop transfer function (CLTF) given by $\frac{C(s)}{R(s)}=\frac{W_n^2}{s^2+2\varepsilon W_n+W_n^2}$. Determine the value of ε and W_n so that the system responds to a step input with approximately 5% overshoot and with a settling time of 2 sec.

b) The open loop transfer function of a unity feedback control system is given by $G(s)=\frac{k(s+2)}{s^3+\beta s^2+4s+1}$. Determine the value of k and β such that the closed loop unit step response has $W_n=3$ rad/sec & $\varepsilon=0.2$. **[WBUT 2016]**

Answer:

a) $$\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2}$$

Setting time $T_s=2$ sec

$$\frac{4}{\xi\omega_n}=2 \qquad \text{.... (1)}$$

$$\therefore \quad \xi\omega_n=2$$

$$\% \ M_p=5$$

$$5=100e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\Rightarrow \quad e^{-\pi\xi/\sqrt{1-\xi^2}}=\frac{5}{100}$$

$$-\frac{\pi\xi}{\sqrt{1-\xi^2}}=\ln(.05)$$

$$\Rightarrow \quad \frac{-\pi\xi}{\sqrt{1-\xi^2}}=-2.99$$

$$\Rightarrow \quad \frac{\xi}{\sqrt{1-\xi^2}}=\frac{2.99}{3.14}$$

$$\Rightarrow \quad \xi^2=.95^2(1-\xi^2)$$

$$\Rightarrow \quad \xi^2=.91(1-\xi^2)$$

$\Rightarrow \quad \xi^2 + .91\xi^2 = .91$

$\Rightarrow \quad 1.91\xi^2 = .91$

$\Rightarrow \quad \xi^2 = \dfrac{.91}{1.91}$

$\Rightarrow \quad \xi = .69$

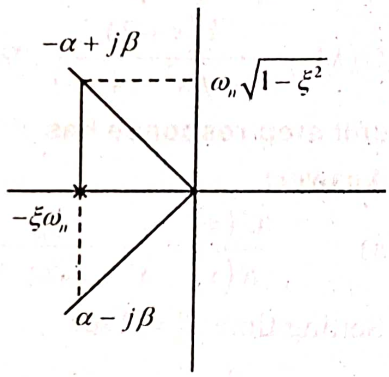
Putting the value in (1) we get

$$\frac{4}{\xi\omega_n} = 2$$

$\Rightarrow \quad \xi\omega_n = 2$

$\Rightarrow \quad \omega_2 = \dfrac{2}{.69} = 2.89 \text{ rad/sec}$

b) $G(s) = \dfrac{k(s+2)}{s^3 + \beta s^2 + 4s + 1} = \dfrac{k(s+2)}{s^2(s+\beta) + 1(4s+1)}$



The characteristic equation is

$$s^3 + \beta s^2 + (k+4)s + (2k+1) = 0$$

$$\cos\theta = \xi = 0.2$$

$$\omega_n = 3 \text{ rad/sec}$$

The poles are

$$s + (0.6 - j2.32)$$

$$s + (0.6 + j2.32)$$

$3\xi^2\omega_n = \beta$

$\Rightarrow \quad 3\cdot(0.2)^2\cdot 3 = \beta$

$\Rightarrow \quad \beta = 9 \times 0.04 = 0.36$

$2k + 1 = \omega_n^{\ 3}$

$\Rightarrow \quad 2k + 1 = 3^3$

$\Rightarrow \quad 2k = 27 - 1 = 26$

$\Rightarrow \quad k = 13$

6. The open loop transfer function of a unity feedback system is given by $G(s) = \dfrac{108}{s^2(s+4)(s^2+3s+12)}$. Find the static error co-efficient and steady state error of the system when subjected to an input given by $r(t) = 2 + 5t + 2t^2$.

[WBUT 2018]

Answer:

For

i) Step input $= 2$, $K_p = \alpha$, therefore, $e_{ss} = \dfrac{1}{(1+\alpha)} = 0$

ii) Ramp input $= 5t$, $K_v = \alpha$, therefore, $e_{ss} = \dfrac{1}{\alpha} = 0$

iii) For parabolic input $= 2t^2 = 4\cdot\left(\dfrac{1}{2}\right)t^2$, $K_a = \dfrac{108}{48} = 2.25$, $e_{ss} = 4(2.25) = 9$

Therefore, the steady state error for input $r(t) = 2 + 5t + 2t^2 = 0 + 0 + 9 = 9$ units

7. Write short notes on the following:
a) Type and Order of a system **[WBUT 2012]**
b) Transient Response and Steady state response **[WBUT 2012]**
c) Static error coefficients **[WBUT 2014]**
d) Time domain specifications **[WBUT 2018]**
e) Effects of feedback **[WBUT 2018]**

Answer:

a) Type and Order of a system:

The open loop transfer function $G(s)H(s)$ of a system may be written by two forms as shown in the table 1.

Table: 1

Form	Representation
Time Constant Form	$\dfrac{K(T_1 s+1)(T_2 s+1)\cdots\cdots}{s^n (T_a s+1)(T_b s+1)\cdots\cdots}$
Pole-zero form	$\dfrac{K_1(s+z_1)(s+z_2)\cdots\cdots}{s^n (s+p_1)(s+p_2)\cdots\cdots}$

Both of the forms contain a term s^n in the denominator representing number of poles at origin in s-plane.
where $n \Rightarrow$ type number of the open loop system.
If $n = 0$, the open loop system has no pole at origin and called type zero system.
If $n = 1$, then open loop system has one pole at origin and called type one (type-1) system.
The order of an open loop system is defined as the number of poles at origin in s-plane.
The definition can be understood through the table.
For the systems:

Table: 2

$G(s)H(s)$	Type	Order
$\dfrac{(s+1)}{(s+3)(s+4)}$	0	2

$\dfrac{(s+1)(s+2)}{s(s+3)(s+4)}$	1	3
$\dfrac{(s+1)(s+2)}{s^2(s+3)(s+4)}$	2	4

b) Transient response & Steady state response:

Types of Time Response:

Time Response is classified into two parts:

(i) Transient Response $c_{tr}(t)$

(ii) Steady-State Response $c_{ss}(t)$

Transient response & Steady state response:

Refer to Question No. 5 of Short Answer Type Questions.

Let us say

(i) a system is stable, i.e., the system exhibits a steady state

(ii) now the system is excited.

(iii) as a result, the steady state of the system gets disturbed.

(iv) the system exhibits a new steady state.

Transient response is defined as the part of the response between two steady states and which died out (goes to zero) as time becomes large

i.e., $\underset{t\to\infty}{Lt}\ c_{tr}(t)=0$

Diagrammatically, it is represented as

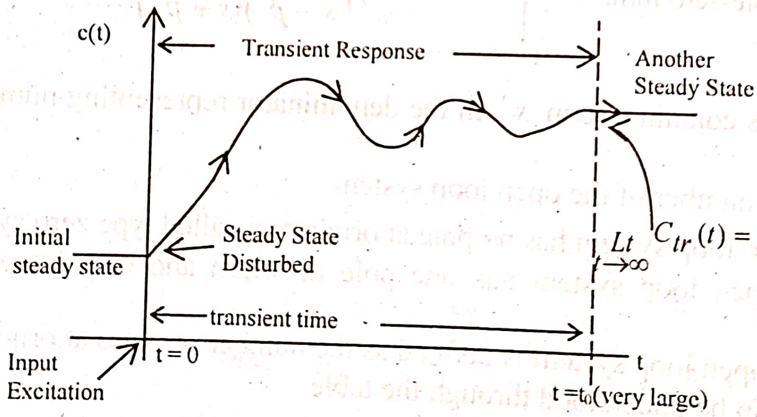


Fig: 1 Nature of c(t) with respect to time

c) Static error coefficients:

Error constants are figure of merit of a steady state error and are the measure of steady state error (e_{ss}).

Error constants give an idea that how the steady state error can be reduced or eliminated to improve the steady state response of a system.

The steady state error (e_{ss}) is the difference between the desired value (input) and the output of a closed loop system as time tends to infinity after the excitation of the system for a known input $r(t)$ [Figs. (a) and (b)].

Mathematically,

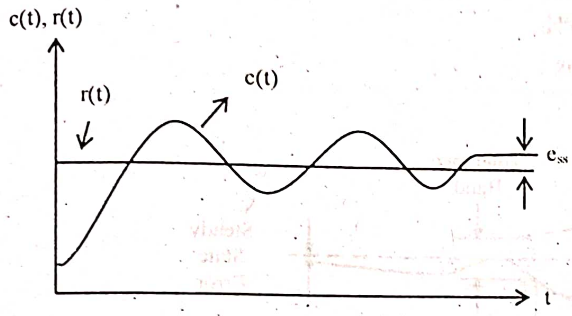


Fig: (a) Depicting the steady state error (e_{ss})

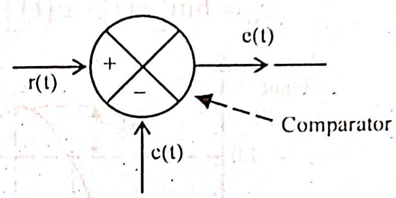


Fig: (b) Error signal e(t) from the comparator

$$e_{ss} = \underset{t\to\infty}{Lt}\left[r(t)-c(t)\right] = \underset{t\to\infty}{Lt}\, e(t)$$

From Final Value Theorem, $e_{ss} = \underset{t\to\infty}{Lt}\, e(t) = \underset{s\to 0}{Lt}\, s \cdot E(s)$.

The steady state error is a measure of the ***accuracy*** of the control system and is evaluated for the following standard test signals.

- Unit step signal
- Unit ramp signal
- Unit parabolic signal.

d) Time - Domain Performance Specifications:

These are indicative of the performance of the closed-loop system in terms of its time response.

This response is achieved most commonly with step response.

Since a control system is almost always required to function in real time, time - domain performance criteria / specifications are a direct way of evaluating the system.

The various time domain specifications are:

1. ***Delay time (t_d):*** It is defined as the time needed for the response to reach 50% of the final value.
2. ***Rise time (t_r):*** It is the time needed for the response to reach 100% of the final value **for the first time.**
3. ***Peak time (t_p):*** It is the time needed for the response to reach the first Peak of the overshoot.
4. ***Maximum Overshoot (M_p):***

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

5. ***Settling time (t_s):*** It is the time needed for the system step response to reach & stay within a predefined tolerance limit / band.
   t_s is usually taken as +/–2% to +/–5%
6. ***Steady state Error (e_{ss}):*** It is defined as

$$e_{ss} = \lim\left[r(t) - c(t)\right]$$

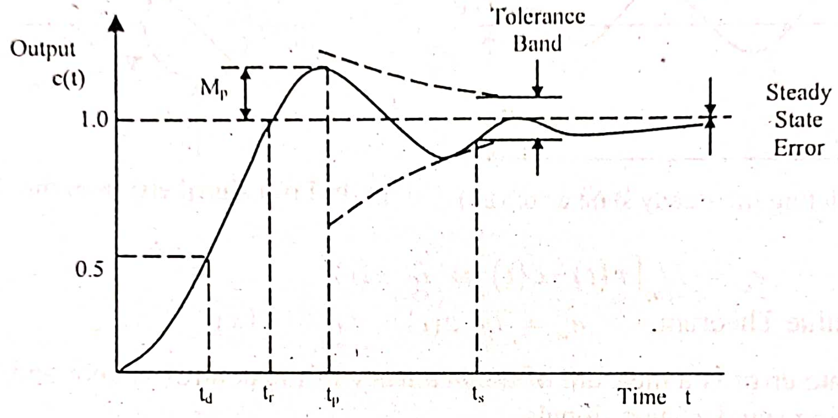


Fig: 1 Time Domain Performance parameters

e) Effects of feedback:

Feedback is the process by which a fraction of the output signal, either a voltage or a current, is used as an input. If this feedback fraction is opposite in value or phase ("anti-phase") to the input signal, then the feedback is said to be **Negative Feedback**, or **Degenerative feedback.**

Negative feedback opposes or subtracts from the input signals giving it many advantages in the design and stabilization of control systems. For example, if the systems output changes for any reason, then negative feedback affects the input in such a way as to counteract the change.

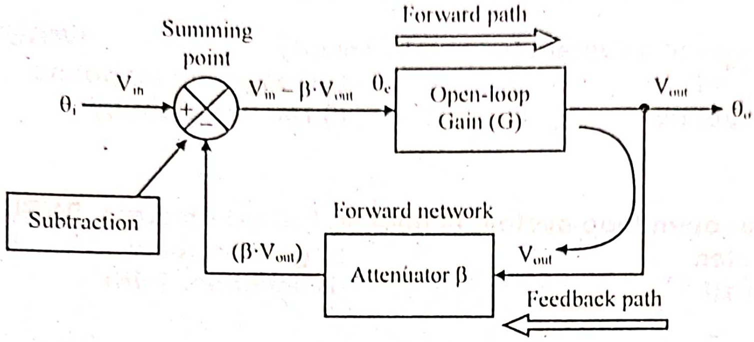
Feedback reduces the overall gain of a system with the degree of reduction being related to the systems open-loop gain. Negative feedback also has effects of reducing distortion, noise, sensitivity to external changes as well as improving system bandwidth and input and output impedances.

Feedback in an electronic system, whether negative feedback or positive feedback is unilateral in direction. Meaning that its signals flow one way only from the output to the input of the system. This then makes the loop gain, G of the system independent of the load and source impedances.

As feedback implies a closed-loop system it must therefore have a summing point. In a negative feedback system this summing point or junction at its input subtracts the feedback signal from the input signal to form an error signal, β which drives the system.

If the system has a positive gain, the feedback signal must be subtracted from the input signal in order for the feedback to be negative as shown.

Negative Feedback Circuit



The circuit represents a system with positive gain, G and feedback, β. The summing junction at its input subtracts the feedback signal from the input signal to form the error signal $V_{in} - \beta G$, which drives the system.

FREQUENCY DOMAIN ANALYSIS

Multiple Choice Type Questions

1. The phase margin of a system is used to specify [WBUT 2010, 2018]
   a) time response
   b) frequency response
   c) absolute stability
   d) relative stability

Answer: (d)

2. If the gain of an open loop system is double, the gain margin [WBUT 2010, 2017]
   a) is not affected
   b) gets doubled
   c) becomes half
   d) becomes 1/4th

Answer: (c)

3. The location of the closed loop conjugate pair of poles on the $j\omega$ axis indicates that the system is [WBUT 2010, 2017]
   a) stable
   b) unstable
   c) marginally stable
   d) critically stable

Answer: (c) and (d)

4. The gain of a system is 10. In terms of dB it is [WBUT 2010]
   a) 0 dB
   b) 1 dB
   c) 20 dB
   d) 100 dB

Answer: (c)

5. If the phase margin is negative, it indicates the system is [WBUT 2011, 2014, 2015, 2017]
   a) highly stable
   b) unstable
   c) oscillatory
   d) it has nothing to do with stability

Answer: (b)

6. For a system defined by $G(s)=\frac{e^{-0.15}}{s}$, the phase crossover frequency is [WBUT 2011]
   a) $\frac{\pi}{2}$
   b) $\frac{\pi}{10}$
   c) $\frac{\pi}{0.2}$
   d) $\frac{\pi}{4}$

Answer: (c)

7. For a stable liner control system [WBUT 2013]
   a) gain crossover frequency is greater than phase crossover frequency
   b) gain crossover frequency is less than phase crossover frequency
   c) gain crossover frequency is equal to phase crossover frequency
   d) both (a) & (b) are possible

Answer: (d)

8. By increasing the gain, k of the system, the steady state error of the system [WBUT 2013, 2015]
a) increases
b) decreases
c) remains unaltered
d) none of these

Answer: (b)

9. For $\xi = 0$, resonant frequency (ω_r) is equal to [WBUT 2013]

a) 0
b) ω_n
c) $\frac{\omega_n}{\sqrt{1-2\xi^2}}$
d) $\omega_n\sqrt{1-3\xi^2}$

Answer: (b)

10. For the transfer $G(s)H(s) = \frac{1}{(1+sT)}$, what is the error at the corner frequency in log magnitude plot? [WBUT 2014]
a) –1 dB
b) –2 dB
c) –3 dB
d) –4 dB

Answer: (c)

11. Additional of a pole to the close loop transfer function [WBUT 2015]
a) increases rise time
b) decreases rise time
c) increases overshoot
d) has no effect

Answer: (c)

12. The phase crossover frequency (W_p) is a frequency at which the angle of $G(jW_p)$ should be equal to [WBUT 2016]

a) 0°
b) 90°
c) 180°
d) 270°

Answer: (c)

13. Gain margin is the reciprocal of the gain at the frequency at which the phase angle is [WBUT 2013]
a) 90 degree
b) 180 degree
c) –180 degree
d) 270 degree

Answer: (c)

Short Answer Type Questions

1. What do you understand by the terms gain and phase cross over frequency? [WBUT 2014]

Answer:
The frequency at which, the phase of open loop transfer functions is called phase cross over frequency ω_{pc}.

The gain cross over frequency ω_{gc} is the frequency at which the magnitude of the open loop transfer function is unity.

Long Answer Type Questions

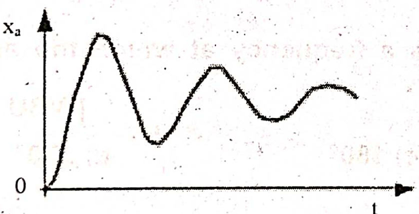
1. a) Define the following terms: **[WBUT 2017]**
Stable system, Critically Stable, Unstable system, Marginally stable system.
b) For a unity feedback system having Open Loop transfer function

$$G(s)H(s)=\frac{k}{[s(1+0.6s)(1+0.4s)]}$$

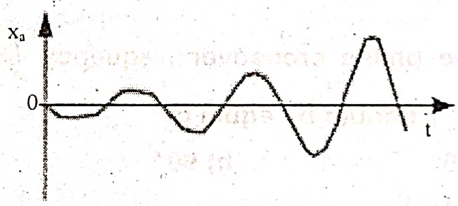
Determine the range of k for stability, marginal value of k and frequency of sustained oscillations.
c) Determine the Steady state error for a unity feedback system having forward path transfer function $G(s)=\frac{50}{[s(s+10)]}$. For the input $r(t)$ given by $r(t)=1+2t+t^2$.

Answer:

a) A linear time-invariant system is said to be a stable one if for a bounded input to a system the system output is also finite/bounded, or not continuously growing up or down when time is measured from time of excitation to infinity.
In the following diagram, the system (a) is stable and system (b) is unstable for a bounded input.



(a)



(b)

The system is said to be unstable when the output is unbounded for a bounded input. In the above diagram, the system (b) is unstable for a bounded input as the response is continuously growing up.

A linear time-invariant system is **marginally stable** if it is neither asymptotically **stable** nor unstable. Sometimes, marginal stability refers to a critically stable system. Here, for a bounded input, the system shows a sustained oscillatory response.

b) Given open loop transfer function:

$$G(s)H(s)=\frac{k}{[s(1+0.6s)(1+0.4s)]}$$

Step 1: To develop the characteristic equation:

$$1+\frac{k}{s(1+0.6s)(1+0.4s)}=0 \quad \text{[For unity feedback]}$$

$\Rightarrow \quad s(1+0.4s)(1+0.6s)+k=0$

$\Rightarrow \quad (s+0.4s^2)(1+0.6s)+k=0$

$\Rightarrow \quad s+0.6s^2+0.4s^2+0.24s^3+k=0$

$\Rightarrow \quad s+s^2+0.24s^3+k=0$

$\Rightarrow \quad 0.24s^3+s^2+s+k=0$

Step 2: Here, all the necessary conditions hold good.

Step 3: To check for sufficient condition:

For this, we form Routh array:

	Col 1	Col 2
Row 1 s^3	0.24	1
Row 2 s^2	1	k
Row 3 s^1	$\frac{1-0.24k}{1}$	0
Row 4 s^0	k	0

Step 4: For stability,

$$\frac{1-0.24k}{1}>0$$

$\Rightarrow \quad 1-0.24k>0$

$\Rightarrow \quad k<4.1666$

and $\quad 0.24k>0$

$\Rightarrow \quad k>0$

Step 5: Conclusion:

The system will be stable if the range of k is $\quad 0<k<4.1666$

c) $$k_p=\lim_{s\to0}G(s)=\lim_{s\to0}\frac{50}{s(s+10)}=\infty$$

$$k_v=\lim_{s\to0}G(s)=\lim_{s\to0}\frac{50}{s+10}=5\,/\text{sec}.$$

$$k_a=\lim_{s\to0}G(s)=\lim_{s\to0}\frac{50s^2}{s(s+10)}=0\,/\text{sec}^2.$$

$$r(t)=1+2t+t^2$$

$$\therefore \quad e_{ss}=\frac{R_1}{1+k_p}+\frac{R_2}{k_v}+\frac{R_3}{k_a}=\frac{1}{1+\infty}+\frac{2}{10}+\frac{1}{0}=0+\frac{1}{5}+\infty=\infty.$$

2. Write short notes on the following: **[WBUT 2013, 2017]**

a) Gain margin & Phase margin

b) Absolute and Relative stability **[WBUT 2018]**

Answer:

a) Gain margin & Phase margin:

Gain margin:

It is the reciprocal of the magnitude of $[G(j\omega).H(j\omega)]$, i.e.,$|G(j\omega).H(j\omega)|$, at the frequency at which the phase angle is $-180°$.

Mathematically, Gain Margin = $K_{GM} = \dfrac{1}{|G(j\omega).H(j\omega)|} \quad \cdots(i)$

In terms of decibels,

$K_{GM}\,(dB) = 20\log_{10} K_{GM} = -20\log_{10}|G(j\omega)H(j\omega)|. \quad ...(ii)$

It is a measure of the relative stability. It says how much gain can be increased to cause system instability.

Phase margin:

It is that amount of additional phase lag at the gain crossover frequency required to bring the system to the verge of instability.

It is a measure of the relative stability. It says the phase angle can be increased to make the system unstable from a stable condition.

b) Absolute and Relative stability:

Absolute stability implies whether a system is stable or unstable. Relative Stability says about the degree of stability. It says how close the system is towards instability.

If a system under equilibrium is excited by the momentary disturbance and the system returns to equilibrium then the system is called as an absolutely stable system. If the effect of disturbance persists indefinitely after it is removed, the system is said as an absolutely unstable system.

Methodically, in order to assess the absolute stability of a system the positions of the poles on s-plane are obtained. If all the poles are left-handed relative to the imaginary axis of s-plane, then the system is absolutely stable.

Two general methods of determining the presence of unstable roots without actually finding their numerical values are:

1. Routh Stability Criterion: This method works with the closed-loop system characteristic equation in an algebraic fashion.
2. Nyquist Stability Criterion: This method is a graphical technique based on the open-loop frequency response polar plot.

However, to assess the relative stability of a system, we have various tools in time domain and frequency domain. Root locus is a time domain based tool for determining the relative stability of a system. If the root locus moves away from the imaginary axis of s-plane the degree of stability of the said system is better.

Bode-plots, Polar plot are the basic tools in frequency domain, which are adapted to find gain margin, phase margin, etc. which are the frequency domain specifications for assessing the relative stability. Higher the values of the margins, better is the degree of stability.

ROOT LOCUS

Multiple Choice Type Questions

1. Root loci of a system has three asymptotes, the system can have [WBUT 2005, 2016]

a) Three poles and one zero
b) Four poles and two zeros
c) Five poles and two zeros
d) Six poles and four zeros

Answer: (c)

2. If the root locus branches cross the imaginary axis, the system becomes [WBUT 2009, 2012]

a) overdamped
b) underdamped
c) oscillatory
d) sustained oscillations

Answer: (d)

3. Root locus technique is applicable to [WBUT 2010]

a) single loop system
b) multiple loop system
c) single as well as multiple loop system
d) not more than two loop systems

Answer: (c)

4. The root locus is symmetrical w.r.t [WBUT 2011, 2014, 2015]

a) negative real axis
b) positive real axis
c) imaginary axis
d) positive & negative real axis

Answer: (d)

5. Root locus is the variation of open loop .... as open loop gain, K, is varied from zero to infinity. [WBUT 2013]

a) poles to zeros b) zeros to poles c) origin to zeros d) poles to origin.

Answer: (a)

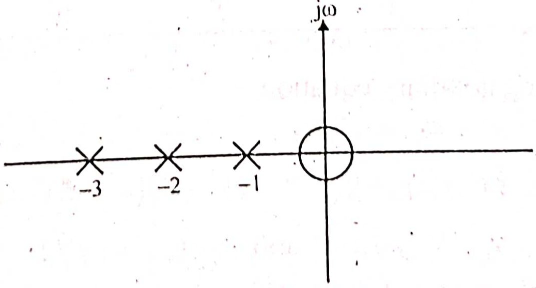
6. The transfer function of a system having a gain of 9 and a pole zero map as in figure is [WBUT 2014]

a) $\dfrac{9(s+1)(s+2)(s+3)}{s}$

b) $\dfrac{9(s-1)(s-2)(s-3)}{s}$

c) $\dfrac{9s(s+1)}{(s+2)(s+3)}$

d) $\dfrac{9s}{(s+3)(s+2)(s+1)}$



Answer: (d)

7. A root is symmetrical w. r. T [WBUT 2017]
a) Negative real axis
b) Positive real axis
c) Imaginary axis
d) Positive and negative real axis

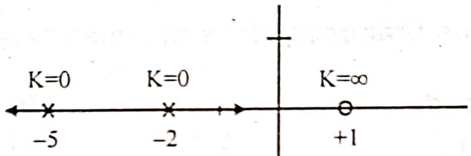
Answer: (c)

Short Answer Type Questions

1. For the system defined by $G(s)=\dfrac{5(s-1)}{(s+2)(s+5)}$ draw the approximate root-locus diagram. [WBUT 2011]

Answer:

Number of Loci = 2
Number of asymptotes = 1
Angle of asymptote = 180°



One Locus will initiate from pole at –2 and terminal at the zero at +1. No break-away point is obtained as the loci initiating from open loop poles at –2 and –5 can not approach towards each other. Second locus initiating from –5 will move towards the zero lying at ∞ on negative real axis.

2. Determine the stability of a closed loop control system whose characteristic equation is

$$s^6+3s^5+6s^4+12s^3+12s^2+12s+8=0$$ [WBUT 2017]

Answer:

Forming Routh's array

s^6	1	6	12	8	
s^5	$\cancel{3}$	$\cancel{12}$	$\cancel{12}$	0	[3 as common]
	↓	↓	↓		
s^5	1	4	4		
s^4	$\cancel{2}$	$\cancel{8}$	$\cancel{8}$		[Taking 2 as common]
	↓	↓	↓		
s^4	1	4	4		
s^3	0	0	0		

Forming auxiliary equation

$$s^4+4s^2+4=0$$

$$\left(s^2+2\right)^2=0 \Rightarrow \left(s^2+2\right)\left(s^2+2\right)=0$$

$$\Rightarrow \quad s_{1,2}=\pm j\sqrt{2} \quad \text{and} \quad s_{3,4}=\pm j\sqrt{2}$$

Since the system has repetition roots on imaginary-axis, system is unstable.

Long Answer Type Questions

1. The open loop transfer function of a unity feedback control system is given by $G(s)=\dfrac{K}{s(s^2+6s+25)}$ **[WBUT 2007]**

Find:

- **i) the number, angle and centroid of the asymptotes**
- **ii) angle of departure**
- **iii) the break-way point**
- **iv) the condition for marginal stability**

OR,

Obtain the root locus for a unity feedback system with open loop transfer function $G(s)=\dfrac{k}{s\left(s^2+6s+25\right)}$**. Show all relevant steps. [WBUT 2009]**

Answer:

(i) Number of asymptotes: No. of open loop poles (m) – no. of finite open loop zeros (n) $=3-0=3$

$$\text{Angle of departure}=\frac{(2q+1)}{(m-n)}\times 180^\circ \quad \text{where, } q=0,1,2$$

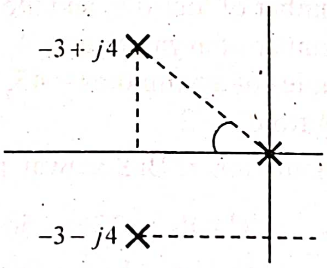
$$=60^\circ, 180^\circ \text{ and } 300^\circ$$

$$\text{Centroid}=\frac{-3-3}{3}=-3$$

(ii) The poles are located as shown in the pole-zero map.

Angles contributed by the all other poles at $-3+j4$ are $90^\circ+\tan^{-1}\left(\dfrac{3}{4}\right)=233.13^\circ$.

$\therefore$ Angle of departure $=233.13^\circ$.



(iii) $$1+\frac{K}{s\left(s^2+6s+25\right)}=s^3+6s^2+25s+K=0$$

$$\Rightarrow \quad K=-s^3-6s^2+25s$$

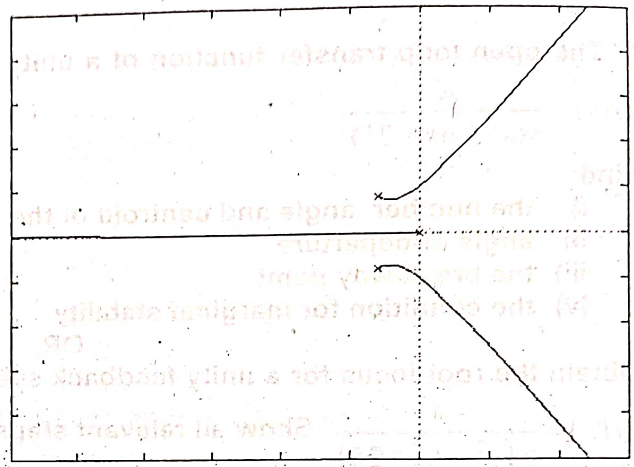
$$\therefore \quad \frac{dK}{sa}=0=3s^2+12s+25$$

$$\therefore \quad s_2, s_2=\frac{-12\pm\sqrt{144-800}}{2}=\frac{-12\pm j12.48}{2}=-6\pm j6.24$$

Moreover, since real axis proceeds to ∞, no valid breakaway point will be found.

(iv) Forming Routh's array

s^3	1	25
s^2	6	K
s^1	$150-K$	0
s^0	K	0



Making $150-K=0$

We get, $K=150$

Forming auxiliary equation

We have $6s^2+K=0$

$\Rightarrow \quad s^2=\dfrac{-150}{6}$

$\therefore \quad s=j\omega=j5$

$\therefore$ For Marginal stability $0<K<150$.

2. A unity feedback control system has an open loop transfer function

$$G(s)=\frac{K}{s(s+2)(s^2+6s+25)}, \quad K\ge 0$$

Sketch the root locus of the system mentioning relevant steps.
Also find the value of K so that the system has a damping factor of 0.707.

[WBUT 2008]

Answer:

1st Part:

Number of finite poles = 4, poles are at $0, -2, -3+4j, -3-4j$

Number of finite zeros = 0,

Number of loci = 4, and the number of loci terminating at ∞ = 4

Number of asymptotes = 4

Angles of asymptotes = 45, 135°, 225°, 315°

Centroid = –2

Calculation of Break away point $s(s+2)(s^2+6s+25)+K=0$

$\Rightarrow \quad s^4+8s^3+37s^2+50s+K=0$

$\Rightarrow \quad K=-s^4-8s^3-37s^2-50s$

hence, $\dfrac{dK}{ds}=-4s^3-24s^2-74s^2-50$

$\therefore$ For Break away point

$4s^3+24s^2+74s+50=0$ we except break away point between 0 and –2, so, try –1 as first trial, $\Rightarrow \quad 2s^3+12s^2+37s+25=0$

$$\begin{array}{r|rrrr} -1 & 2 & 12 & 37 & 25 \\ & \downarrow & -2 & -10 & -27 \\ \hline & 2 & 10 & 27 & \mathbf{-2} \end{array}$$

Try with -1.5

$$\begin{array}{r|rrrr} -1.5 & 2 & 12 & 37 & 25 \\ & \downarrow & -3 & -13.5 & -36.25 \\ \hline & 2 & 9 & 23.5 & \mathbf{-10.25} \end{array}$$

Try with -0.5

$$\begin{array}{r|rrrr} -0.5 & 2 & 12 & 37 & 25 \\ & \downarrow & -1 & -5.5 & -16.25 \\ \hline & 2 & 11 & 32.5 & \mathbf{8.75} \end{array}$$

Try with -0.9

$$\begin{array}{r|rrrr} -0.5 & 2 & 12 & 37 & 25 \\ & \downarrow & -1.8 & -9.18 & -25.092 \\ \hline & 2 & 10.2 & +27.88 & \mathbf{-0.92} \end{array}$$

$\therefore$ Break away point may be considered as -0.899

$\Rightarrow \quad 4s^3 + 24s^2 + 74s + 50 = 0$

$\Rightarrow \quad 2s^3 + 12s^2 + 37s + 25 = 0$

Roots are $-4.05, \ -0.39, \ -1.544$

Imaginary axis crossover:

Characteristic equation $s^4 + 8s^3 + 37s^2 + 50s + K = 0$ using Routh-Hurwitz criterion

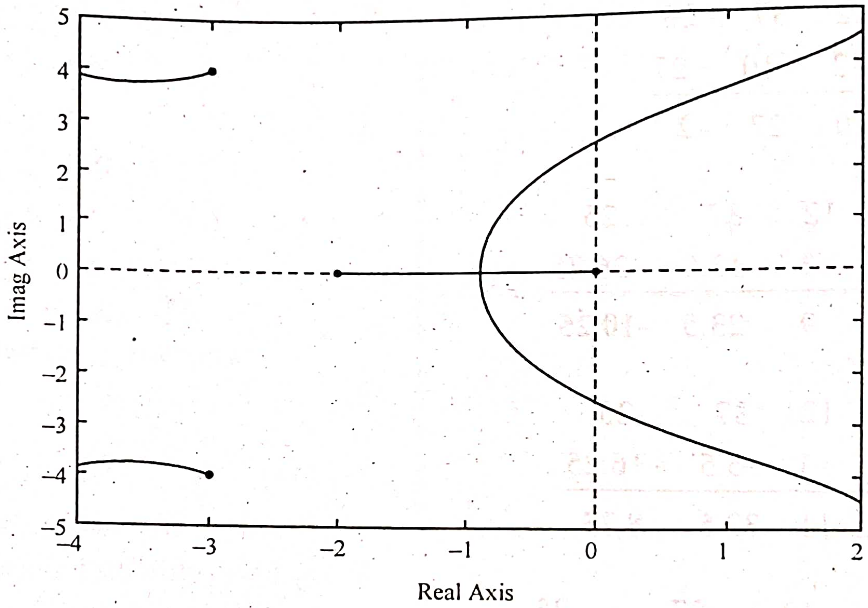
$$\begin{array}{c|ccc} s^4 & 1 & 37 & K \\ s^3 & 8 & 50 & 0 \\ s^2 & 30.75 & K & \\ s^1 & \dfrac{1537.5 - 8K}{30.75} & & \\ s & K & & \end{array}$$

$\therefore$ For sustained oscillation $1537.5 - 8K = 0$

$\Rightarrow \quad K = 192.1875.$

$\therefore$ Auxilliary equation $= 30.75s^2 + K = 0$

$\Rightarrow \quad s^2 - 6.25 = \pm j2.5$



2nd Part:

For $\xi = 0.707$,

$\cos\phi = \xi = 0.707 \Rightarrow \phi = 45°$

So, draw a line which intersects the locus at $s = -0.3 + 0.3j$

$\therefore$ From magnitude condition

$\Rightarrow \quad K = 17.032$

Angle of departure at s = –1 + j2 is calculated graphically. Refer Fig. 1(a).

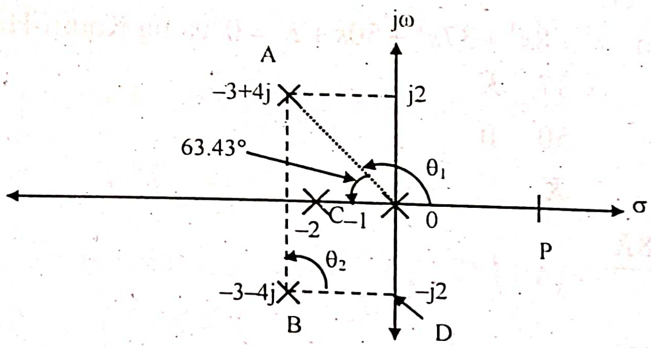


Fig: 1(a) Calculations for angular departure

Poles	Angular Contribution
0	$\angle AOC = \tan^{-1}\frac{4}{3} = 53.13°$ $\therefore \angle POA = 180 - 53.13 = 126.87° = \theta_1$
B	$\angle ABD = 90° = \theta_2$
C	$180° - 63.43° = 116.57$

$\sum \phi_P = 333.44°$

and, as there is no zero

$\therefore \quad \sum \phi_z = 0°$

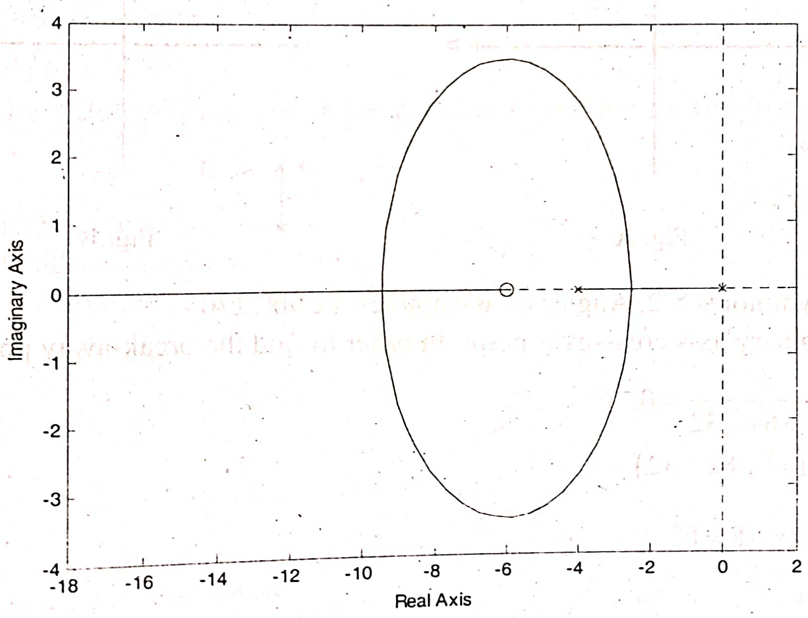
$\therefore \quad \phi_D = 180° + \sum \phi_z - \sum \phi_P = 180 - 333.44° = -153.44°$

$\therefore \quad \phi_D$ at complex conjugate pole $s = -3 - j4$ is $+153.44°$.

3. The loop transfer function of a feedback control system is given by $G(s)H(s) = \dfrac{k(s+6)}{s(s+4)}$.

a) Sketch the root locus plot with *k* as a variable parameter & show that loci of complex roots are part of a circle.
b) Determine the break away/break in points if any.
c) Determine the range of *k* for which the system is under-damped. [WBUT 2010]

Answer:



Break in point = – 9.46 at K = 14.9
Break away point = – 2.54 at K = 1.07
At break away and break in points the damping ratio is 1 i.e. the system is critically damped.
The range of K for which the system is underdamped is given by

$$1.07 < K < 14.5.$$

All the points on the loci lying on the complex plane are meant for underdamped system.

4. a) A unity feedback control system has an open loop transfer function

$$G(s)=\frac{k}{(s^2+8s+32)}$$

Sketch the root locus of the system and deduce how the peak overshoot varies with increasing k if the loop is closed.

b) What kind of controller would you recommend for this system? [WBUT 2011]

Answer:

a) 1[st] part:

Pole locations are $-4\pm j4$

$$\left[s^2+8s+32=0,\ s_1,s_2=\frac{-8\pm\sqrt{64-128}}{2}=\frac{-8\pm j8}{2}=-4\pm j4\right]$$

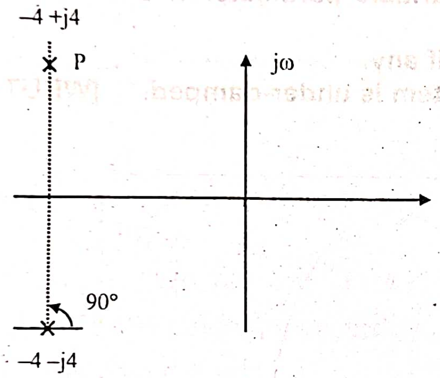


Fig: A

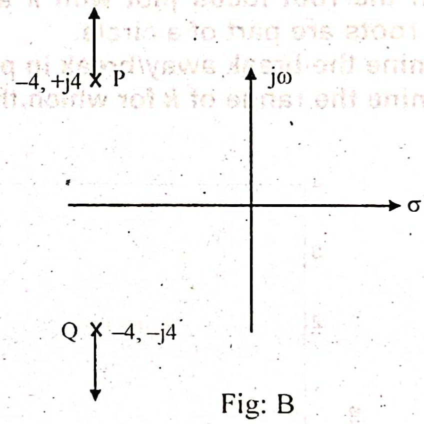


Fig: B

Number of asymptotes = 2, Angles of asymptotes are $90°, 270°$.

No finite imaginary axis cross-over point. In order to find the break-away point,

$$1+\frac{k}{s^2+8s+32}=0$$

$$\Rightarrow \quad k=-\left(s^2+8s+32\right)$$

$$\therefore \quad \frac{dk}{ds}=2s+8=0$$

$$\Rightarrow \quad s=-4$$

Angle of departure (ϕ_0) of the locus from pole P can be found from Fig. A.

$\phi_0=180°-\phi_p=180°-90°=90°$ and angle of departure (ϕ_0) of the locus initiating from pole Q $=-90°$

2[nd] Part:

Characteristic polynomial

$$=s^2+8s+32+k=0$$

$$\Rightarrow \quad s^2+2.4s+(32+k)=0$$

$\Rightarrow \quad \omega_n = \sqrt{32+k}$

$$\xi = \frac{4}{\sqrt{32+k}}$$

$$\therefore \quad M_p = \frac{-\xi\pi}{e^{\sqrt{1-\xi^2}}}$$

So, as $k\uparrow, \xi\uparrow, M_p\uparrow$

b) P-I controller is used for the system. For the given system the steady state error is zero.
The transient effects of P-I controller are observed: i) overshoot increases ii) stability decreases iii) settling time increases iv) rise time increases .These observations are followed by P-I controller. Hence P-I controller is used for the given system.

5. a) What are the angle & magnitude conditions for root locus?
b) Draw the root locus for the control system having the open loop transfer function with unity feedback.

$$G(s) = k/s(s+1)(s^2+2s+2).$$

c) Determine the value of K at the point on root locus where the damping factor $\xi = 0.5$. **[WBUT 2013]**

Answer:

a) Let us consider a system (figure 1) with

- forward path transfer function = G(s)
- feedback path transfer function = H(s)

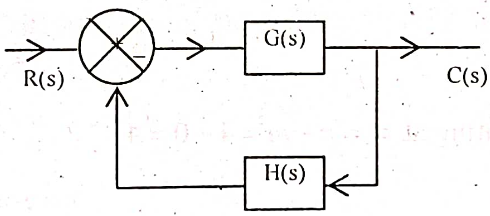


Fig: 1

The closed loop transfer function is

$$T(s) = \frac{G(s)}{1+G(s)H(s)} \quad \cdots\cdots(1)$$

The characteristic equation of the system is

$$1+G(s)H(s)=0 \quad \cdots\cdots(2)$$

$$\Rightarrow \quad \boxed{G(s)H(s) = -1 = -1+j0 \quad \cdots\cdots(3)}$$

Since s is a complex variable, therefore, $G(s)H(s)$ may be expressed as

$$G(s)H(s) = |G(s)H(s)| \underline{|G(s)H(s)}$$
$$= M\underline{|\phi}$$

where, $M = |G(s)H(s)| = \sqrt{(-1)^2 + (0)^2}$

$\Rightarrow \quad M = 1$ (Magnitude Criterion) ……(4)

and $\quad \phi = \underline{|G(s)H(s)} = \tan^{-1}\left(\frac{0}{-1}\right)$

$\Rightarrow \quad \phi = \pm 180(2p+1),\ p = 0,1,2$ (Angle Criterion) ……(5)

Any point on root locus must satisfy angle criterion.

The Evan's criteria and corresponding applications are listed in Table:

Table:

Criterion	Application
Angle	To plot the root locus.
Magnitude	To get the value of K corresponding to a point on the root locus.

b) & c)

$$G(s) = \frac{k}{s(s+1)(s^2+2s+2)}$$

Pole locations for the factor

$s^2 + 2s + 2$ are $(s+1+j)(s+1-j)$

Here $\quad n = 4 \quad m = 0$

Step 1:
1. Number of loci terminating at $\infty = n - m = 4 - 0 = 4$

Step 2: Real axis loci are
i) Present for $-1 \le \sigma \le 0$
ii) Absent for $-\infty < \sigma < -1$

Step 3: Number of asymptotes $= n - m = 4$
Angles of asymptotes

$$\theta = \frac{(2q+1)180°}{n-m},\ q = 0,1,2,3$$
$$\theta = 45°, 135°, 225°, 315°$$

Step 4: Centroid σ_c is

$$\sigma_c = \frac{\sum(0-1-1-1)-\sum 0}{n-m} = \frac{-3}{4} = -0.75$$

Step 5: Breakaway point

$$k = \frac{-s(s+1)(s^2+2s+2)}{1} \qquad \text{[Arranging the characteristic equation]}$$

$$k = \frac{-(s^2+5)(s^2+2s+2)}{1} = \frac{s^2+s^3+2s^3+2s^2+2s^2+2s}{1} = \frac{s^4+3s^3+4s^2+2s}{1}$$

$$\frac{dk}{ds} = \frac{d}{ds}\left[-\left(s^4+3s^3+4s^2+2s\right)\right] = 0$$

or, $\quad 4s^3+9s^2+8s+2=0$

Since, breakaway point is between Trial Takes $= -1 = -0.4$

Step 6: To find ϕ_D at $s=-1-j$

Angular contribution at the pole $[-1-j]$ due to all other poles

Poles	Angular condition
1	225°
2	315°
3	270°

$$\sum \phi_s = 225° + 270° + 315° = 810° = 90°[810° - 2.360°]$$

and at there is no zero.

$\therefore \quad \sum \phi_{sp} = 0°$

$\therefore$ Angle of departure of the locus from the point $s=-1-j$ is

$$\phi_D = 180° + 0° - 90° = 90°.$$

$\therefore$ Angle of departure of the locus from the point $s=-1+j$ is $-90°$

Step 7: To find $j\omega$ crossover

$$1+G(s)H(s)=0$$

$\Rightarrow \quad s^4+3s^3+4s^2+2s+k=0$

The Routh Array is

s^4	1	4	k
s^3	3	2	
s^2	$\frac{12-2}{3} = \frac{10}{3}$	k	

$$\begin{array}{c|cc} s^1 & \dfrac{\frac{20}{3}-3k}{\frac{10}{3}} & 0 \\ s^0 & k & \end{array}$$

$$\frac{\frac{20}{3}-3k}{\frac{10}{3}}=0 \text{ gives } j\omega \text{ crossover}$$

$\therefore \quad \frac{20}{3}-3k=0$

or, $\quad 3k=\frac{20}{3}$

$\Rightarrow \quad k=\frac{20}{9}$

Auxiliary equation for s^2 row is

$$\frac{10}{3}s^2+k=0$$

$\Rightarrow \quad 10s^2+3k=0$

$\Rightarrow \quad 10s^2=-3k$

$\Rightarrow \quad s^2=-3\times\frac{20}{9}\times\frac{1}{10}$

$\Rightarrow \quad s^2=\frac{-2}{3}$

$\Rightarrow \quad s=\pm 0.9036$

$j\omega$ axis crossover at $=\pm j0.9036$

To find out k at $\xi=0.5$

Draw line A-B at $\theta=60°$. As $\cos\theta=\xi$, $\theta=60°=\cos^{-1}0.5$

Intersects at root locus at $P(-0.65, j0.75)$.

Step 8:

$$\left|\frac{k}{(-0.65+j0.75)(-0.65+j0.75+1)(0.65+j0.75+1+j)(0.65+j0.75+1-j)}\right|=1$$

or, $$\frac{k}{|(0.65+j0.75)||(35+j0.75)||(1.65+j1.75)||(1.65-j0.25)|}=1$$

$\Rightarrow$ $$\frac{k}{\sqrt{0.65^2+0.75^2}\sqrt{0.35^2+0.75^2}\sqrt{1.65^2+1.75^2}\sqrt{1.65^2+0.25^2}}=1$$

$$\Rightarrow \frac{k}{\sqrt{0.4225+0.5625}\sqrt{0.1225+0.5625}\sqrt{2.7225+3.0625}\sqrt{2.7225+0.0625}}=1$$

$$\Rightarrow \frac{k}{0.9924\times1.056\times2.405\times1.668}=1$$

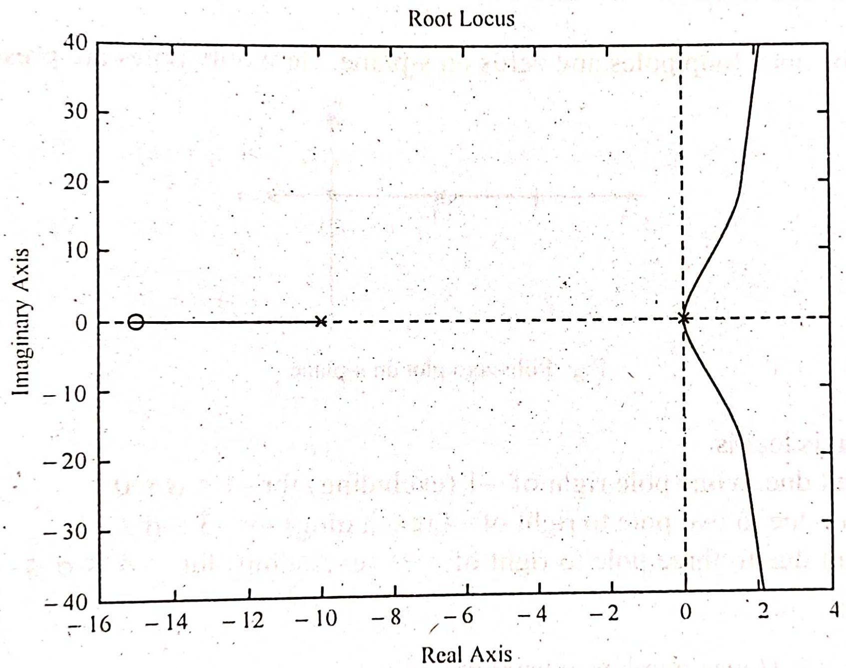
$$\Rightarrow k=0.9924\times1.056\times2.405\times1.668=4.1720$$ **Ans.**

6. The open-loop transfer function is given by $G(s)=\frac{K(s+15)}{s^2(s+10)}$.

Sketch the root loci for the system when K is varied from 0 to infinity.

[WBUT 2014]

Answer:



No. of Loci	3
No loci will be present between 0 and -10	Two loci will terminate at infinity as two zeros are at there.
No. of asymptotes	
Angles of asymptotes	90° and 270°
Centroid	2.5
Break away point	Origin, as two of the three loci are meeting and then diverging for the s-plane.
Finite Imaginary axis cross over point	Nil

7. Sketch the root locus for the open loop transfer function of a unity feedback control system given below and determine, (i) the value of K for $\xi = 0.5$, (ii) the value of k for marginal stability. **[WBUT 2015, 2017]**

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

Answer:

Step 1: *Finding pole –zero details*

n = 3 and located at 0, –1, – 3

m = 0 and no finite zero is there.

∴ Number of loci = n = 3

Number of loci ending at ∞ = n – m = 3

Step 2: Plot the open loop poles and zeros on s-plane. Here only poles are present.

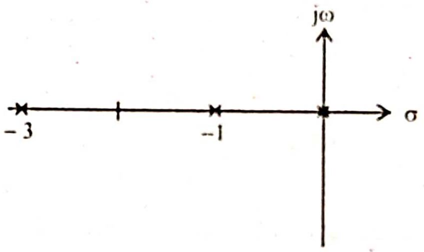


Fig: Pole-zero plot on s-plane

Step 3: Real axis loci is

(i) Present due to one pole right of –1 (excluding) for $-1 < \sigma < 0$

(ii) Absent due to two pole to right of –4 (excluding) for $-3 \le \sigma \le -1$

(iii) Present due to three pole to right of $-\infty$ (excluding) for $-\infty \le \sigma \le -3$ as shown in Fig.

Step 4: n – m = 3. Hence number of asymptotes = 3

$$\therefore \text{ Angle of asymptotes} = \frac{(2q+1)180°}{3} = \theta,\ q = 0,\ 1,\ 2$$

$\therefore \theta = 60°, 180°, 300°$ are the angles of asymptotes.

Step 5: $\sigma_c = \dfrac{\sum \text{real part of OLTF poles} - \sum \text{real part of OLTF zero}}{n-m}$

Σ real part of poles = (–0 – 1 – 3) = – 4

Σ real part of zeros = 0

$$\sigma_c = \frac{\sum(0-1-3)-\sum(0)}{3} = -\frac{4}{3} = -1.33$$

Step 6: Of three loci to ∞, last one from s = – 3 proceeds on real axis via 180° asymptote. Loci from s = 0 and –1 must breakaway between 0 and –1.

For. σ_b.

$$k = \frac{-s(s+1)(s+3)}{1} = -\left[s^3 + 4s^2 + 3s\right]$$

$$\frac{dk}{ds} = 0 = \left(3s^2 + 8s + 3\right) = 0$$

Quadratic solution gives s = – 0.451 and –2.215.

Since σ_b is such that $-1 < \sigma_b < 0$

$\therefore \qquad \sigma_b = -0.451$

(The value of $\sigma_b = -2.215$ is for k < 0), hence not valid.

Step 7: As complex poles/zeros are not there, this step is not needed.

Step 8: As locus crosses jω axis, value of k and s can be found out at this cross over by Routh Stability Criterion.

The characteristic equation is

$s(s+1)(s+3)+k=0$

$s^3 \quad + \quad 4s^2 \quad + \quad 3s \quad + \quad k \quad = 0$

s^3	1	3
s^2	4	k
s^1	$\frac{12-k}{4}$	0
s^0	k	

Put. $\quad \frac{12-k}{4} = 0$

or, $\quad k = 12.$

At $k = 12$, cross over takes place. Auxiliary Equation for s^2 row is

$$4s^2 + k = 0 \quad \therefore 4s^2 = -k$$

$$4s^2 = -12 \quad \text{or, } s^2 = -3 \quad \text{or, } s = \pm\sqrt{3}j$$

$s = \pm\sqrt{3}j$ gives point of cross-over. Root loci are drawn in the figure below.

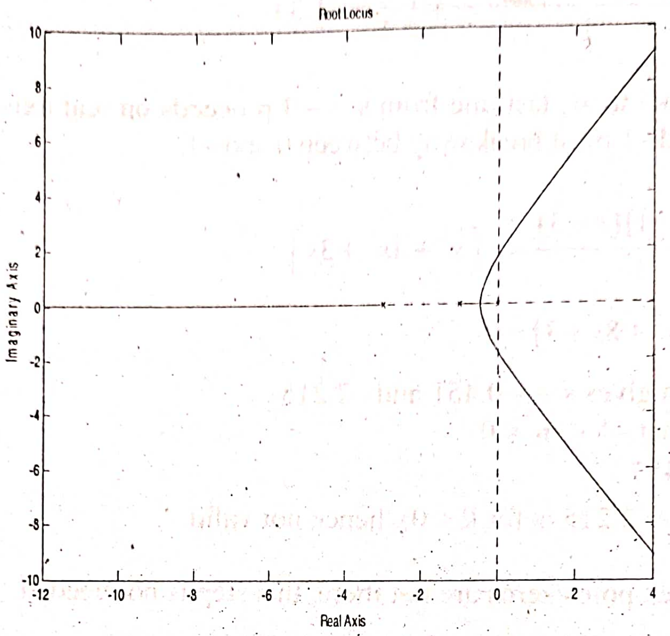


Fig: Root Loci

8. Sketch the root locus diagram as k is varied from zero to infinity for the system whose open loop transfer function is given by $G(s)H(s)=\dfrac{k}{s(s+4)(s^2+4s+20)}$. Evaluate the value of k at the point where the root locus crosses the imaginary axis. Also determine the frequency at this point. **[WBUT 2016]**

Answer:

Solution:

Step 1: Here number of poles = n = 4

Number of zeros = m = 0

Number of loci = 4 as n > m

∴ Number of loci ending at ∞ = n – m = 4

Also from $s^2 + 4s + 20$

$$s = \frac{-4 \pm \sqrt{16-80}}{2} = \frac{-4 \pm j8}{2} = -2 \pm j4$$

i.e., $(s + 2 - j4)(s + 2 + j4)$

Step 2: Plot the open loop poles and zeros on s-plane Fig. 1(a)

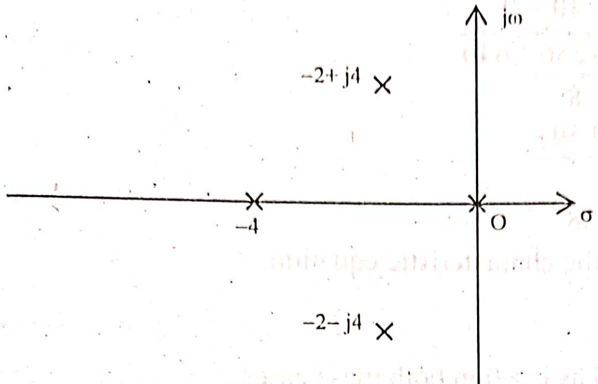


Fig: 1(a) Pole-zero map

Step 3: Real axis loci are

(i) Present $-4 \leq \sigma \leq 0$

(ii) Absent for $-\infty < \sigma < -4$

Complex factors are not taken, as they do not affect real axis loci.

Step 4: The number of asymptotes = n − m.

Angles of asymptotes,

$$\therefore \quad \theta = \frac{(2q+1)180}{n-m}, \quad q = 0, 1, 2, 3$$

$\therefore \quad \theta = 45°, 135°, 225°, 315°$ are the angles of the asymptotes.

Step 5: Centroid

$$= \sigma_c = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{n-m} = \frac{(0-2-2-4)-0}{4} = -\frac{8}{4} = -2$$

Step 6: To evaluate breakaway point.

$$s(s+4)(s^2+4s+20)+k=0$$

$$\therefore \quad k = -s(s+4)(s^2+4s+20) = -\left[s^4+8s^3+36s^2+80s\right]$$

$$\therefore \quad \frac{dk}{ds} = 0 \text{ gives}$$

$$-(4s^3+24s^2+72s+80)=0$$

Since breakaway point is between 0 and −4, try s = −2 as starting point.

−2	4	24	72	80
		−8	−32	−80
	4	16	40	(0)

So, $s = -2$ is a valid breakaway point.

Other breakaway points are given by solving

$4s^2 + 16s + 40 = 0$

i.e., $s = \dfrac{-16 \pm \sqrt{256-640}}{8}$

$= \dfrac{-16 \pm 19.59j}{8}$

$= -2 \pm j2.48$

Putting these values of s in the characteristic equation.

For $s = -2$, $k = 64$

For $s = -2 \pm j2.45, k = 100$.

Hence these points are valid as $k > 0$ in both these cases.

Step 7: ϕ_D at $s = -2 + j4$ is calculated graphically. Refer to Fig. 1(b)

$\sum \phi_{zeros} = \sum \phi_Z = 0$

$\sum \phi_{poles} = \theta_1 + \theta_2 + \theta_3$

To find, θ_1:

$\angle BOC = \tan^{-1}\dfrac{4}{2} = 63.43°$

$\therefore \quad \angle AOC = 180° - \angle BOC = 116.57° = \theta_1$

To find θ_2:

$\angle FEC = 90°$

To find θ_3:

$\angle BDC = \tan^{-1}\dfrac{4}{2} = 63.43°$

$\therefore \quad \sum \phi_Z - \sum \phi_P = 0 - 270° = -270°$

$\therefore \quad \phi_D = 180 - 270° = -90°$ at $s = -2 + j4$

$\therefore \quad \phi_D$ at $s = -2 - j4 = 90°$

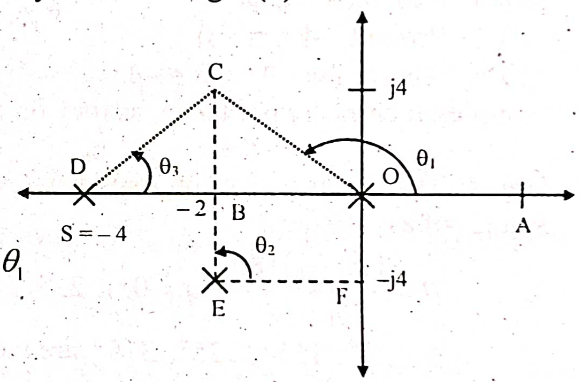


Fig: 1(b) Angle of departure

Step 8: Cross-over at jω- axis is determined as

$1 + G(s)H(s) = 0$ gives

$$1 + \frac{k}{s(s+4)(s^2+4s+20)} = 0$$

$\therefore \quad s^4 + 8s^3 + 36s^2 + 80s + k = 0$

s^4	1	36	k
s^3	8	80	
s^2	26	k	

s^1	$\frac{2080-8k}{26}$	0
s^0	k	

Cross over at $j\omega$- axis is at

$$\frac{2080-8k}{26}=0$$

$\therefore \quad 8k = 2080$

$\therefore \quad k = 260$

The auxiliary equation at s^2 row is

$$26s^2 + k = 0$$

$\therefore \quad 26s^2 + 260 = 0$

$\therefore \quad s^2 = -10$

$\therefore \quad s = \pm j3.16$

The Root Locus is drawn in the Fig. 1(c).

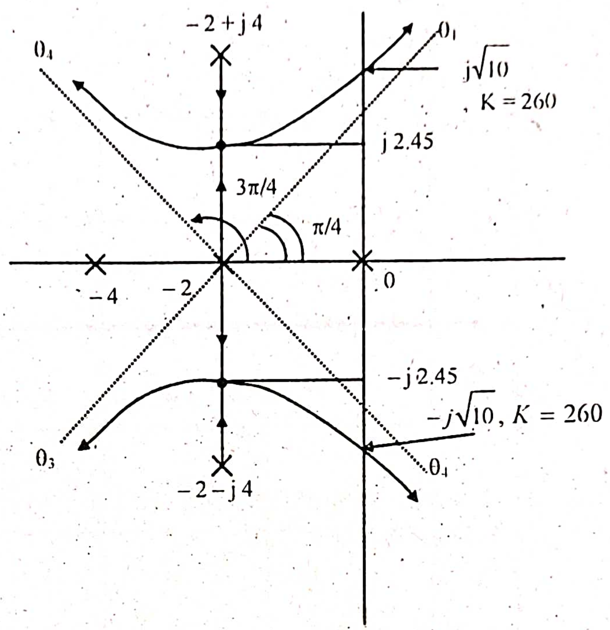


Fig: 1(c) Root Loci

9. Sketch the root locus of the system having open loop transfer function

$$G(s)H(s)=\frac{K(s+1)}{13+4s+s^2}$$ **[WBUT 2018]**

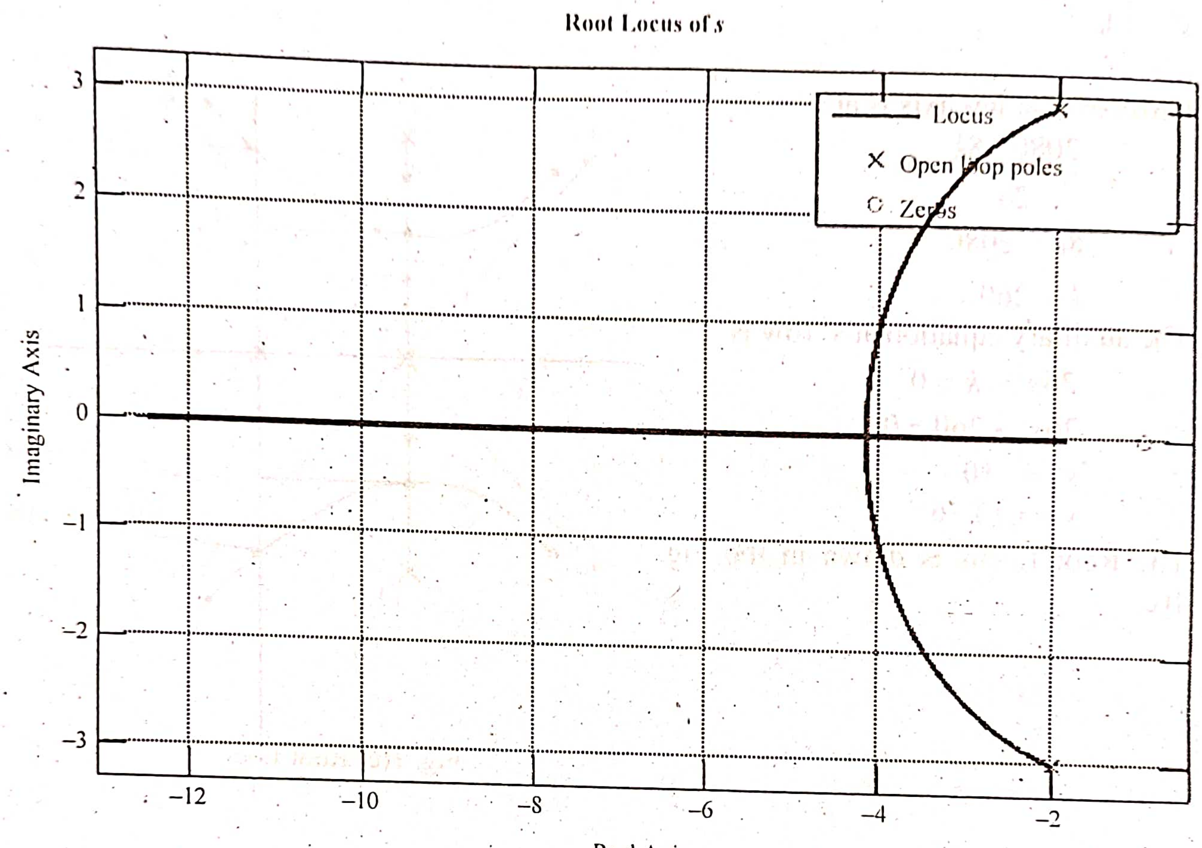
Answer:

$n=[1\ 1];$

$d=[1\ 4\ 13];$

$s=tf(n,\ d)$

Root locus (s)



Gain = [0, 10.3574]

BODE PLOT

Multiple Choice Type Questions

1. In terms of Bode plot, the system is stable if [WBUT 2007, 2012]
   a) PM = GM
   b) PM & GM both are positive
   c) PM and GM both are negative
   d) PM negative but GM positive

Answer: (b)

2. For a stable system [WBUT 2008, 2012]
   a) the gain crossover occurs before phase crossover
   b) the gain crossover occurs after phase crossover
   c) the gain crossover and phase crossover frequencies are very close to each other
   d) the gain crossover and phase crossover frequencies are same

Answer: (a)

3. The initial slope of the Bode plot gives an indication of [WBUT 2009, 2012, 2017]
   a) type of the system
   b) nature of the system time response
   c) system stability
   d) gain margin

Answer: (a)

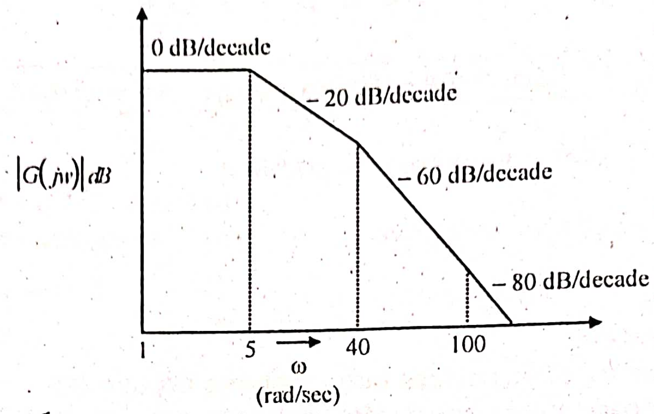
4. The initial slope of Bode plot for a transfer function having single pole at origin is [WBUT 2013, 2015]
   a) 20 dB/decade
   b) – 40 dB/decade
   c) 40 dB/decade
   d) – 20 dB/decade

Answer: (d)

5. What is the effect of transportation lag, $e^{-j\omega t}$ on Bode plot and Nyquist plot of a system? [WBUT 2014]
   a) There is no change in magnitude plot of Bode plot but the critical point of Nyquist shifts from $-1+j0$ to the locus of $-e^{j\omega t}$
   b) There is some change in magnitude plot of Bode plot but the critical point of Nyquist remains at $-1+j0$
   c) The phase plot of Bode plot undergoes a phase shift of $-\omega T$ rad but the critical point of Nyquist remains at $-1+j0$
   d) The phase plot of Bode plot undergoes a phase shift of $-\omega T$ rad but the critical point of Nyquist shifts to the origin

Answer: (c)

6. What is the transfer function of the following magnitude plot of the given Bode plot? [WBUT 2014]



a) $G(s)H(s)=\dfrac{1}{s^2(1+s)}$

b) $G(s)H(s)=\dfrac{10^5}{(1+0.2s)(1+0.01s)(1+0.025s)}$

c) $G(s)H(s)=\dfrac{1}{s^3(1+s)(2+s)(3+s)}$

d) $G(s)H(s)=\dfrac{1}{(1+s)^2}$

Answer: (b)

7. For a type 3 system the asymptote at a lower frequency will have a slope of [WBUT 2018]
a) –6dB/octave
b) –18dB/octave
c) –24dB/octave
d) –40dB/octave

Answer: (b)

Short Answer Type Questions

1. Sketch the Bode plots showing the magnitude in decibels and phases angle in degrees as a function of log frequency for the given transfer function: $G(s)=\dfrac{10(s+10)}{s(s+2)(s+5)}$. [WBUT 2014]

Answer:

$$G(s)=\frac{10(1+0.1s)}{s(1+0.5s)(1+0.2s)} \quad \text{.... (i)}$$

Step 1: Getting the Magnitude plot:

Step A: Forming the table

Factors	Corner frequency (rad/sec)	Slope contributed by each factor (dB/dec)	Cumulative slope (dB/dec)	The straight line Starts at ω	Ends at ω
10	None	0	0		
$\frac{1}{s}$	None	−20	−20	0.1 rad/sec	2 rad/sec
$\frac{1}{(1+0.5s)}$	$\frac{1}{0.5}=2$	−20	−40	2	5
$\frac{1}{(1+0.2s)}$	$\frac{1}{0.2}=5$	−20	−60	5	10
$(1+0.1s)$	$\frac{1}{0.1}=10$	+20	−40	10	Extend the line

Step B: Getting the starting point of the Magnitude plot:

- Form the transfer function with the factors which do not contain any corner frequency.

$$\text{T.F.}\big|_{1}=\frac{10}{s} \quad \text{(Factors which do not have corner frequencies)}$$

- Find M of the obtained transfer function in dB

$$M=\left|\frac{10}{j\omega}\right|=\frac{10}{\omega},$$

$$\therefore \text{M in dB}=20\log_{10}10-20\log_{10}\omega \qquad \text{.... (ii)}$$

- Put the starting value of ω in the expression for M in dB, say = 0.1 rad/sec.

$$\text{M in dB}\big|_{\omega=0.1}=20+20=40\text{dB}$$

Step 2: Getting the Phase plot

Step A: Express the phase angle (ϕ) from the given expression of the Transfer function as a function of ω.

$$\phi=-90°-\tan^{-1}0.5\omega-\tan^{-1}0.2\omega+\tan^{-1}0.1\omega \qquad \text{.... (iii)}$$

Step B: Form the table

ω rad/sec	ϕ
0.1	−93°
0.5	− 107°
1	− 122°
5	−176.6°

ω rad/sec	φ
10	−186°
20	−187°

Bode plots are shown in Figure below.

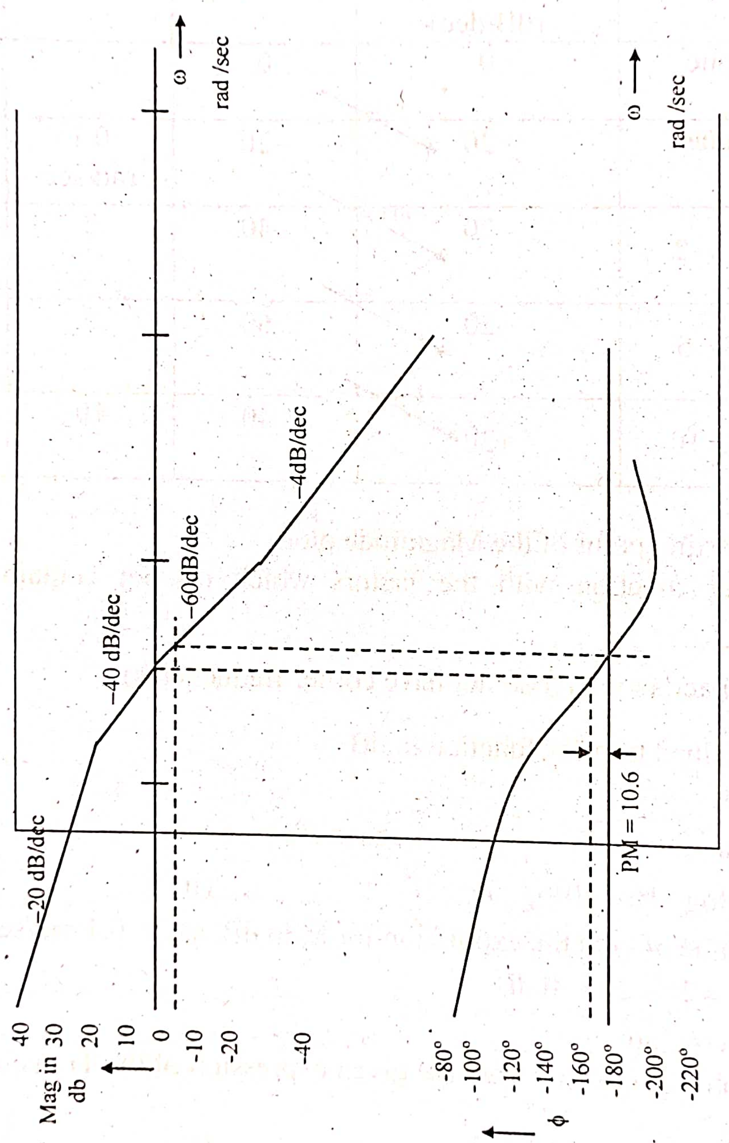
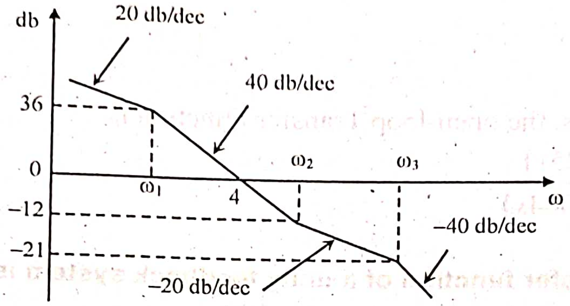


Fig: Bode plots

From the Bode plots, we get
1. Gain Margin = 7.36 dB at 5.7 rad/sec
2. Phase Margin = 10.6 at 4.9 rad/sec
3. Stability:
Since gain and phase margin are positive, so the system is absolutely stable.

Long Answer Type Questions

1. Determine the transfer function of the system whose Bode plot is shown below: **[WBUT 2007]**



Answer:

Referring to the figure between ω_1 and $\omega_4 = 4$ rad/sec

Change in log Magnitude is −36dB

$$\therefore \quad -36 = -40(\log 4 - \log \omega_1)$$

$$\left[\text{As } \frac{\text{Change in dB}}{\text{Number of decades between two frequencies}} = \text{Slope}\right]$$

or, $\omega_1 = 0.5\,\text{rad/sec}$

Calculation of 'K'

$$20\log K = 36 + 20\log 0.5$$

or, $K = 31.62$

Calculation of 'ω_2'

$$-12 = -40(\log\omega_2 - \log 4)$$

or, $\omega_2 = 8\,rad/\text{sec}$

Calculation of 'ω_3'

$$-21 + 12 = -20(\log\omega_3 - \log 8)$$

or, $\omega_3 = 22.5\,\text{rad/sec}$

First line has a slope of −20 dB/dec indicating a term $\frac{1}{s}$ and since it is not passing through $\omega = 1$ rad/sec, the term is $\frac{K}{s}$ or $\frac{31.62}{s}$.

At $\omega_2 = 8$ rad/sec, slope changes to −20dB/dec indicating a term $\left(1+\frac{s}{8}\right)$ or $(1+0.125s)$.

At $\omega_3 = 22.5$ rad/sec, slope changes to –40dB/dec indicating a term $\dfrac{1}{\left(1+\dfrac{s}{22.5}\right)}$ or $\dfrac{1}{(1+0.044s)}$.

Combining all the terms, the open-loop Transfer Function is

$$G(s) = \frac{31.62(1+0.125s)}{s(1+2s)(1+0.044s)}.$$

2. The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{200}{s(s+4)(s+10)}$$

Construct the Bode plot and determine

a) Gain crossover frequency
b) Phase crossover frequency
c) Gain margin
d) Phase margin
e) Comment on the stability of the system. **[WBUT 2008]**

Answer:

$$G(s)H(s) = \frac{200}{s\cdot 4\left(1+\frac{s}{4}\right)\cdot 10\left(1+\frac{s}{10}\right)} = \frac{5}{s\left(1+\frac{s}{4}\right)\left(1+\frac{s}{10}\right)}$$

Factors	Slope (dB/sec)	Corner frequency	Start point	End point
$\dfrac{5}{s}$	–20	Nil	0.1	4
$\dfrac{1}{\left(1+\dfrac{s}{4}\right)}$	–20	4	4	10
$\dfrac{1}{\left(1+\dfrac{s}{10}\right)}$	–20	10	10	Extended to ∞

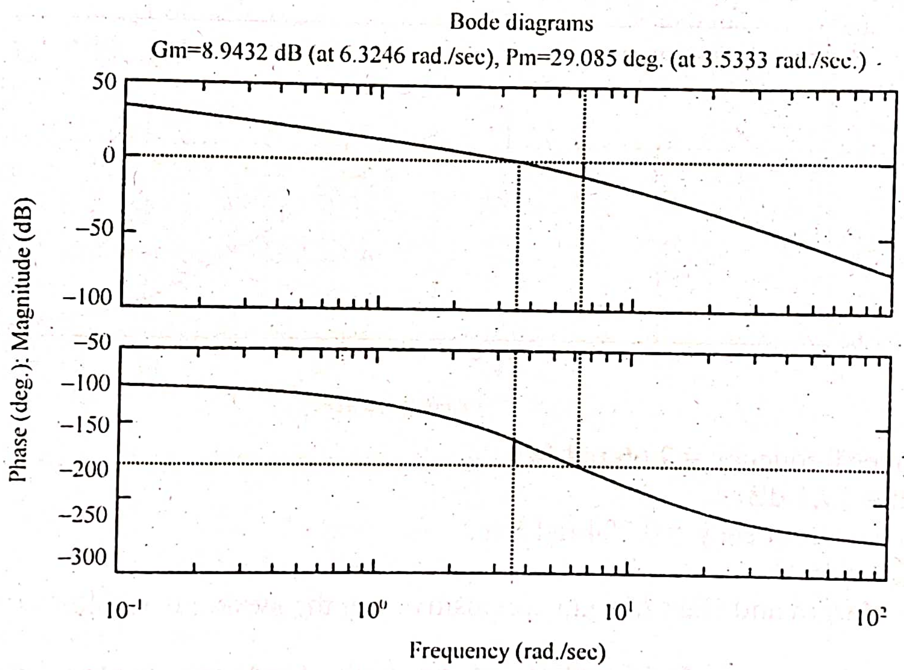
Starting point $\quad M = 20\log_{10}\dfrac{5}{\omega} = 20\log_{10}\left(\dfrac{5}{0.1}\right) = 34\,\text{dB}$.

Phase $\quad \phi = -90° - \tan^{-1}\dfrac{\omega}{4} - \tan^{-1}\left(\dfrac{\omega}{10}\right)$

ω (rad./sec)	ϕ (degree)	ω (rad./sec)	ϕ (degree)
1	−109.7	10	−203.1
2	−127.8	0.1	−92
3	−143.5	0.4	−98
4	−156.8	0.5	−99
5	−167.9	11	−207.7
6	−177.2	12	−211.7
7	−185.5		
8	−192		
9	−198		

(a) Gain crossover frequency = 4.5 rad./sec
(b) Phase crossover frequency = 6.1 rad./sec
(c) Gain margin = 5 dB
(d) Phase margin $= 180° + \phi = 180° - 162° = 18°$

As gain margin and phase margin are both positive, so, the system is stable.



3. Draw the Bode plot of the system whose open loop transfer function is given by $GH(s) = \dfrac{k}{s(1+s)(1+0.1s)(1+0.02s)}$**. Determine the value of** k **for the gain margin of 10 dB.** **[WBUT 2009]**

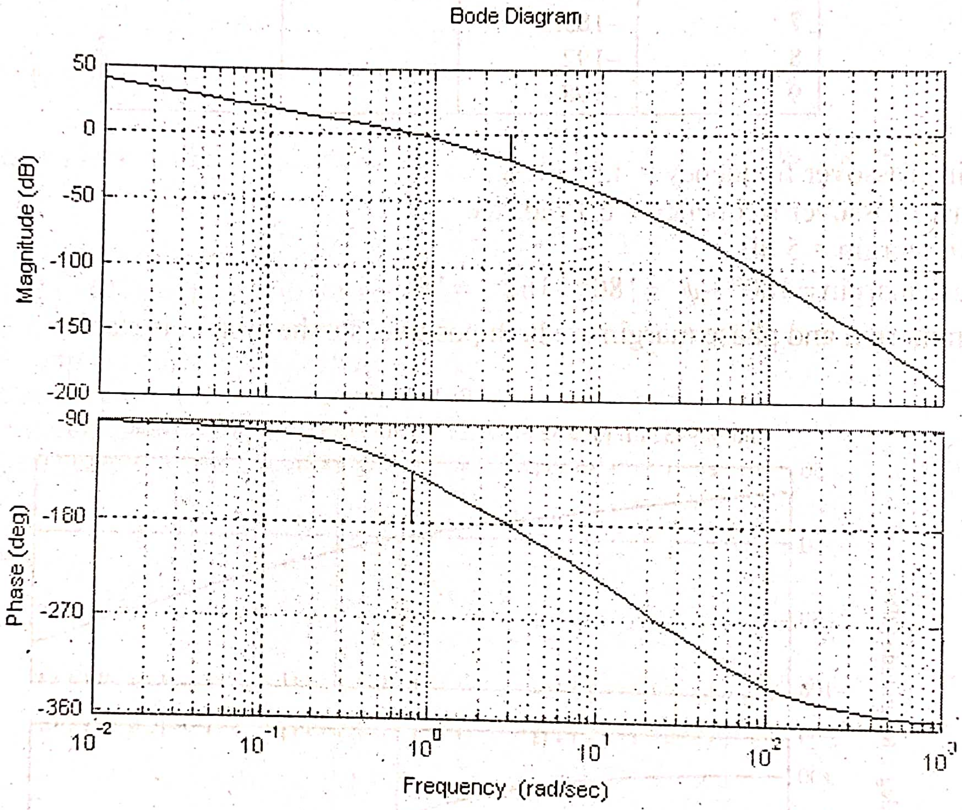
Answer:

Here, gain margin is 19.1 dB

To have a gain margin of 10 dB, the magnitude plot has to be lifted up by $(19.1-10)\,\text{dB} = 9.1\,\text{dB}$.

or, $20\log k = 9.1$

or, $k = 10^{9.1/20} = 10^{0.455} = 2.85$



Gain Crossover frequency = 2.66 rad./sec.

Gain Margin = 19.1 dB

Phase Crossover frequency = 0.784 rad./sec.

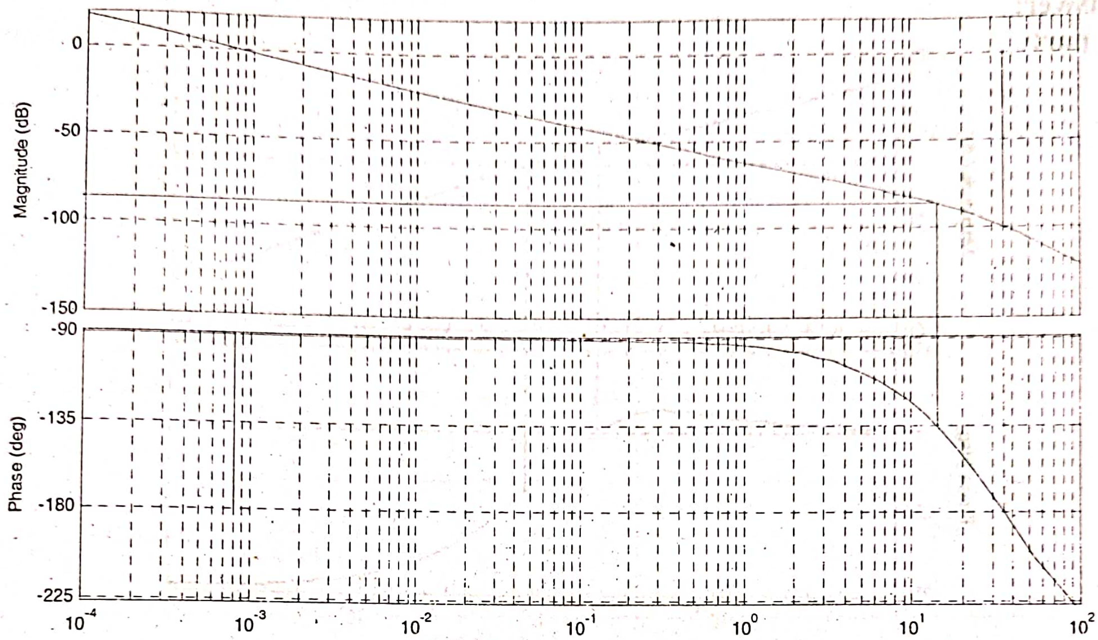
Phase Margin = 46.5°

Since Phase Margin and Gain Margin are positive, so, the system is stable.

4. The open loop transfer function of an unity feedback system is given by $G(s) = \dfrac{k}{s(1+0.02s)(1+0.04s)}$. Draw the Bode plot. Find the gain margin & phase margin. Hence find the values of open loop gain so that the system has a phase margin of 45°.

[WBUT 2010]

Answer:



From the Bode plots we have
Gain Margin = 99dB
Phase Margin = 90 deg.
In order to have a phase margin of 45 deg. The phase angle should be

$$PM = 180° + \phi \Rightarrow 45° = 180° + \phi \Rightarrow \phi = -135°$$

The magnitude plot is to be shifted up by 87 dB
So, $87 = 20\log_{10} K$

$\Rightarrow \log_{10} K = 4.35$

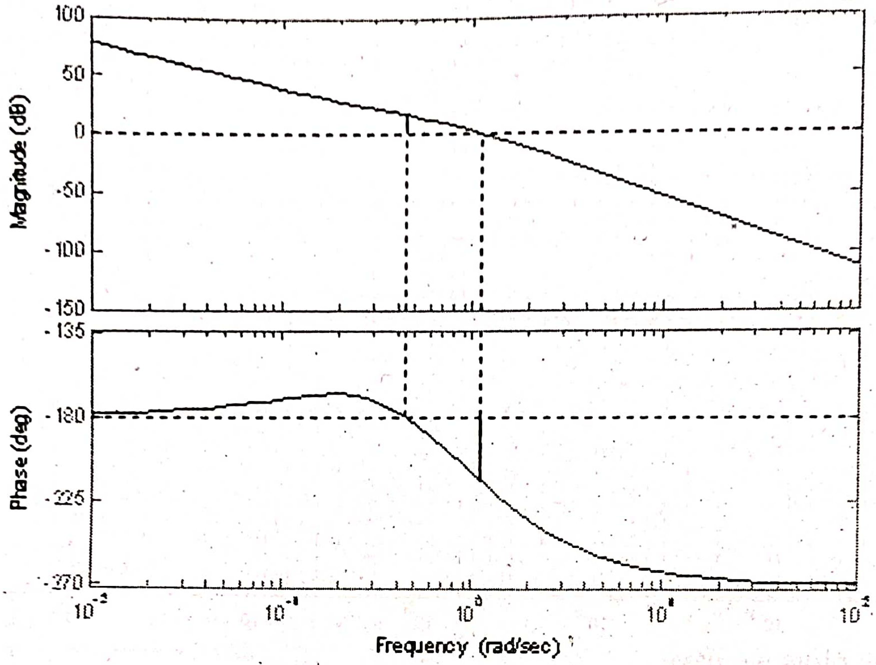
$\Rightarrow K = 22387.$

5. Sketch the Bode Plot for the transfer function defined by **[WBUT 2011]**

$$G(s)H(s) = \frac{2(s+0.2)}{s^2+(s+1)(s+0.5)}$$ **Determine**

a) **phase cross-over frequency**
b) **gain cross-over frequency**
c) **grain margin**
d) **phase margin.**

Answer:
1[st] part:



2[nd] Part:
From the Bode plots we get

a) Phase cross-over frequency	0.447 rad./sec
b) Gain cross over frequency	1.11 rad./sec
c) Gain margin	-16.5 dB
d) Phase margin	-34.1degrees

6. The open loop transfer function of an unity feedback system, $G(s)=\dfrac{10(s+3)}{s(s+2)(s^2+4s+100)}$. Draw the Bode Plot and determine (i) gain crossover frequency (ii) phase crossover frequency (iii) gain margin (iv) phase margin.
[WBUT 2015]

Answer:
Step 1: Converting the T.F. into Time Constant form

$$G(s)=\frac{10.3(1+0.33s)}{100.2s(1+0.5s)(1+0.04s+0.01s^2)}$$

$$=\frac{0.15(1+0.33s)}{s(1+0.5s)(1+0.04s+0.01s^2)}$$

$$\therefore \quad G(j\omega)=\frac{0.15(1+j0.33\omega)}{j\omega(1+j0.55\omega)(1-0.01\omega^2+j0.04\omega)} \quad \cdots(i)$$

Step 2: Getting the Magnitude plot

Step A: Creating the Table (T-1)

Table: T-1

Factors	Corner frequency (rad/sec)	Slope contributed by each factor (dB/dec)	Cumulative slope (dB/dec)	The straight line Starts at ω	Ends at ω
0.15	None	0	0		
$\frac{1}{s}$	None	−20	−20	0.1	2
$\frac{1}{1+0.5s}$	$\frac{1}{0.5}=2$	−20	−40	2	3
$\frac{1}{1+0.33s}$	$\frac{1}{0.33}\approx 3$	+20	−20	3	10
$\frac{1}{1+0.04s+0.01s^2}$	$\frac{1}{\sqrt{0.01}}=\sqrt{100}$ $=10$	−40	−60	10	Extend the line

Step B: Getting the starting point of the Magnitude plot:

$$G_1(j\omega)=\frac{0.15}{j\omega}$$

$$M=\left|\frac{0.15}{j\omega}\right|=\frac{0.15}{\omega},$$

$$\therefore \quad M \text{ in dB} = 20\log_{10}0.15-20\log_{10}\omega \qquad \cdots(ii)$$

$$M \text{ in dB}\Big|_{\omega=0.1}=20\log_{10}0.15+20=-16.5+20=+3.5\text{dB}$$

Step 3: Getting the Phase plot

Step A:

$$\phi=-90^\circ+\tan^{-1}\frac{\omega}{3}-\tan^{-1}\frac{\omega}{2}-\tan^{-1}\frac{4\omega}{\left(100-\omega^2\right)} \qquad \cdots(iii)$$

Step B:

Table: T-2

ω (rad/sec)	1	8	9	9.5	10
ϕ (degree)	−100	−137	−151	−172	−185

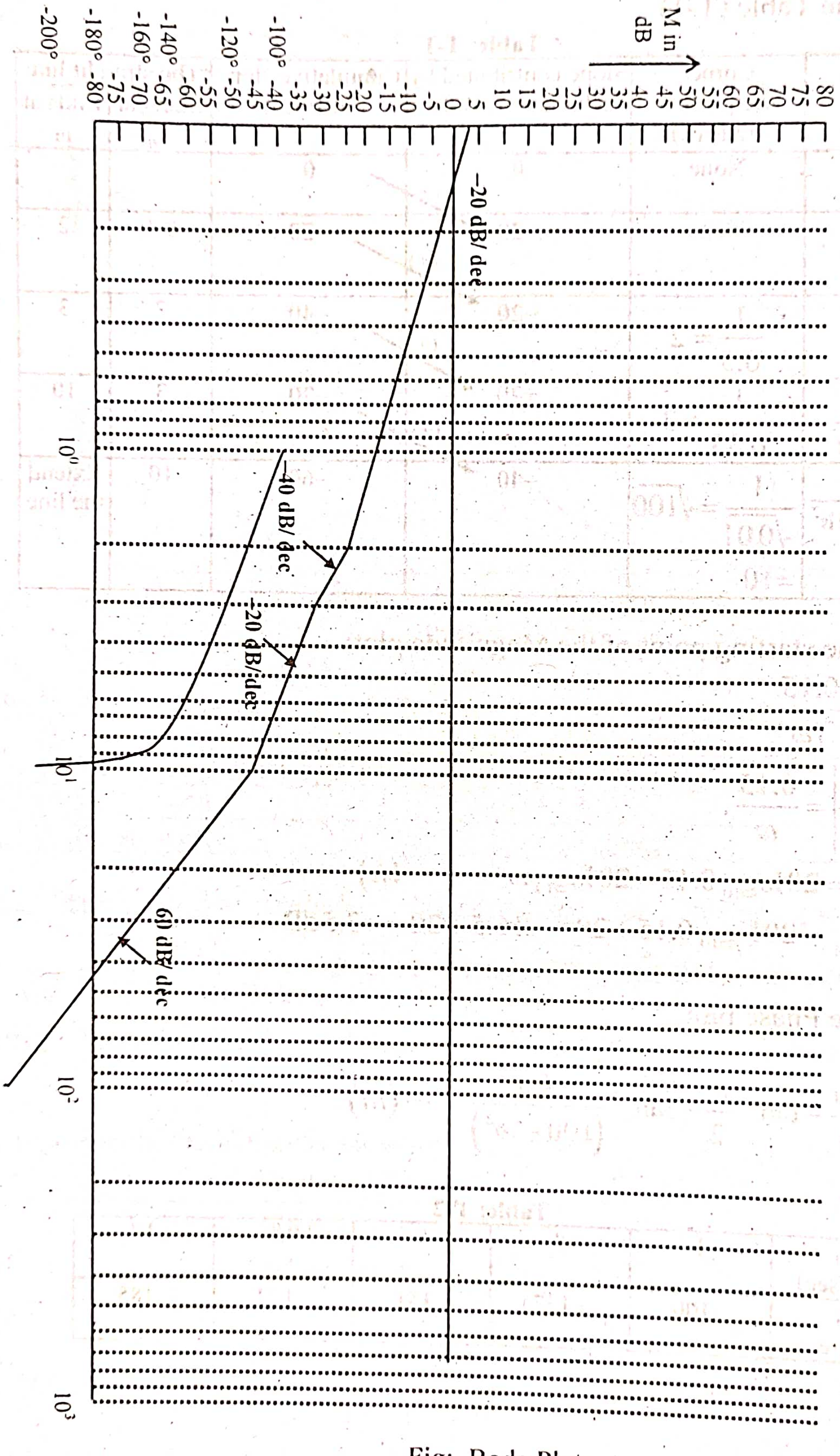


Fig: Bode Plots

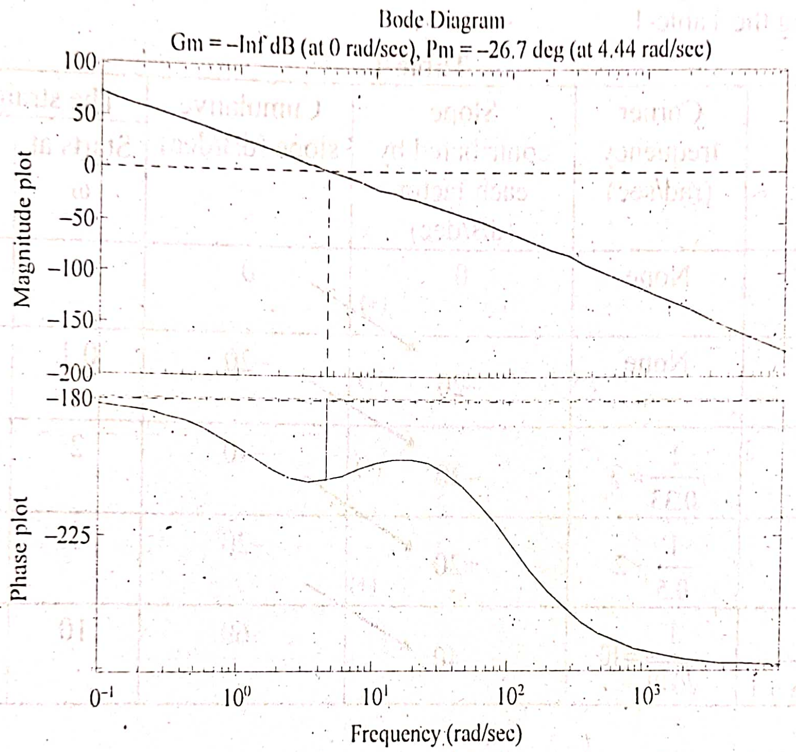
7. a) Construct the Bode Plot for a unity feedback control system having $G(s)=\dfrac{36(0.2s+1)}{s^2(0.5s+1)(0.01s+1)}$.

b) From the plot obtain the gain margin, phase margin, gain crossover frequency, phase crossover frequency.

c) Comment on the closed loop stability of the system. [WBUT 2016]

Answer:

a)



b) Gain margin $= -\infty$ dB

Phase margin $= -26.7°$

Gain crossover frequency $= 0$ rad/sec

Phase crossover frequency $= 4.44$ rad/sec

c) The system is unstable.

8. The open loop transfer function of an unity feedback system, [WBUT 2017]

$$G(s)=\frac{10(s+2)}{s(s+3)(s^2+4s+100)}.$$

Draw the Bode Plot and determine:

i) gain crossover frequency
ii) phase crossover frequency
iii) gain margin
iv) phase margin

Answer:

Step 1: Converting the T.F. into Time Constant form

$$G(s)=\frac{10.2\,(0.5s+1)}{3.100.\,s(0.33s+1)(0.01s^2+0.04s+1)}$$

$$\therefore \quad G(j\omega)=\frac{0.066(0.5j\omega+1)}{j\omega(0.33j\omega+1)(0.04j\omega-0.01\omega^2+1)} \qquad \text{.... (i)}$$

Step 2: Getting the Magnitude plot

Step A: Creating the Table-1

Table-1

Factors	Corner frequency (rad/sec)	Slope contributed by each factor (dB/dec)	Cumulative slope (dB/dec)	The straight line Starts at ω	Ends at ω
0.0667	None	0 (+)	0		
$\frac{1}{s}$	None	−20 (+)	−20	0.1	2
$\frac{1}{1+0.33s}$	$\frac{1}{0.33}\approx 3$	−20 (+)	−40	2	3
$\frac{1}{1+0.5s}$	$\frac{1}{0.5}\approx 2$	+20 (+)	−20	3	10
$\frac{1}{0.01s^2+0.04s+1}$	$\frac{1}{\sqrt{0.01}}=10$	−40	−60	10	Extend the line

Step B: Getting the starting point of the Magnitude plot:

$$G_1(j\omega)=\frac{0.066}{j\omega}$$

$$\text{M}=\left|\frac{0.066}{j\omega}\right|=\frac{0.066}{\omega},$$

$$\text{M in dB}\Big|_{\omega=0.1}=20\log_{10}0.066-20\log_{10}\omega=-23.517+20=-3.517\text{ db}$$

Step 3: Getting the Phase plot

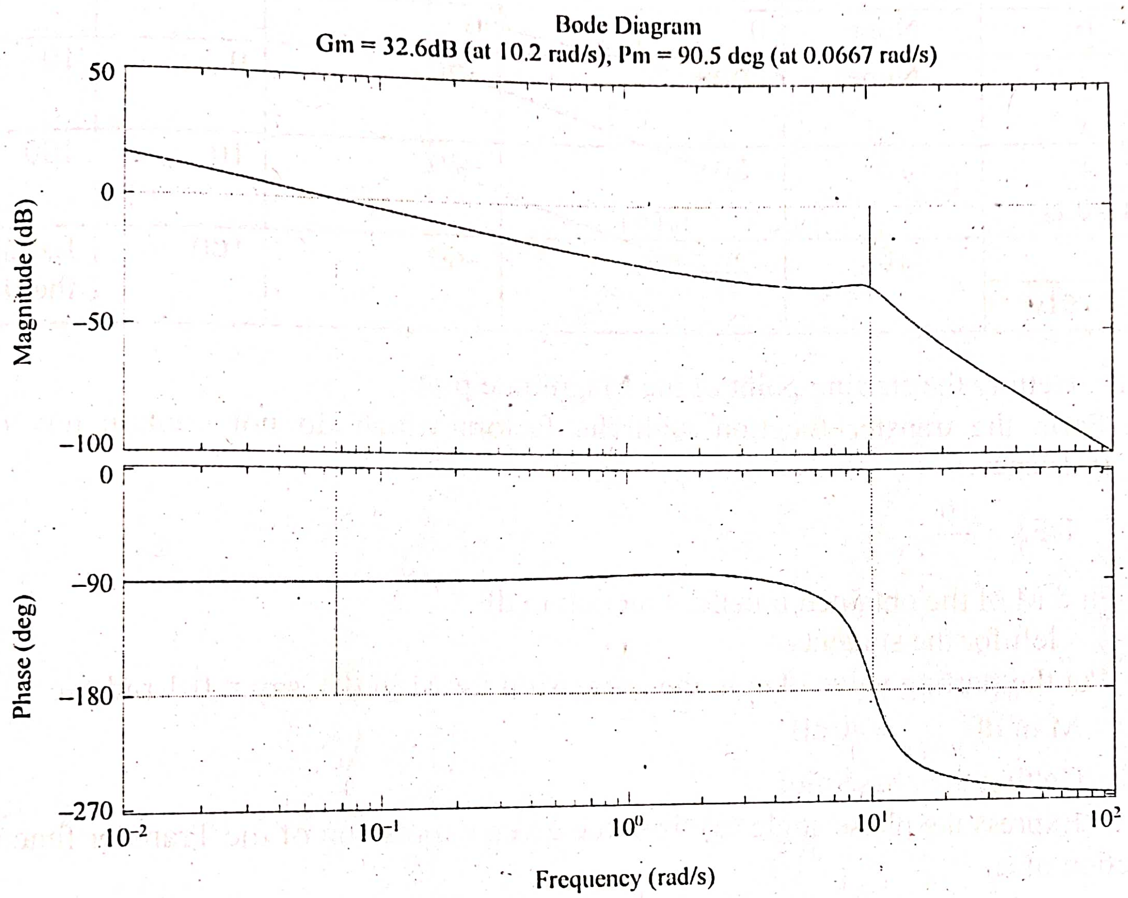
Step A:

$$\phi=-90°+\tan^{-1}\frac{\omega}{2}-\tan^{-1}\frac{\omega}{3}-\tan^{-1}\frac{4\omega}{\left(100-\omega^2\right)}$$

Step B:

Table-2

ω (rad/sec)	1	8	9	9.5	10
ϕ (degree)	−84.1	−125	−147	−160	−181



Gain Margin = 32.6 dB
Phase Margin = 90.5 deg
Gain crossover frequency = 0.0667 rad/sec
Phase crossover frequency = 10.2 rad/sec
Gain margin and phase margin are positive.
Hence the system is stable.

9. Draw the Bode plot for the negative unity feedback system having **[WBUT 2018]**

$$G(s)=\frac{10}{s(1+0.01s)(1+0.1s)}$$

Determine GCF, PCF, GM and PM.

Answer:

Step 1: Getting the Magnitude plot:

Step A: Forming the table (1)

Table: 1

Factors	Corner frequency (rad/sec)	Slope contributed by each factor (dB/dec)	Cumulative slope (dB/dec)	The straight line	
				Starts at ω	Ends at ω
10	None	0 (+)	0		
$\frac{1}{s}$	None	−20 (+)	−20	0.1	10
$\frac{1}{(1+0.1s)}$	5	−20 (+)	−40	10	100
$\frac{1}{(1+0.01s)}$	100	−20	−60	100	Extend the line

Step B: Getting the starting point of the Magnitude plot:

- Form the transfer function with the factors which do not contain any corner frequency.

  $$\text{T.F.}\big|_1 = \frac{10}{s}$$

- Find M of the obtained transfer function in dB
  < left for the students >
- Put the starting value of ω in the expression for M in dB, say = 0.1 rad/sec.

  $$\text{M in dB}\big|_{\omega=0.1} = 40\text{dB}$$

Step 2: Getting the phase plot

Step A: Express the phase angle (ϕ) from the given expression of the Transfer function as a function of ω.

$$\phi = -90° - \tan^{-1} 0.1\omega - \tan^{-1} 0.01\omega$$

Step B: Forming the table (2)

Table: 2

ω rad/sec	ϕ
0.1	− 91.2°
0.5	− 96°
1	− 102°
5	− 138°
10	− 159°
20	− 177°
50	− 200.8°
100	− 222°

Bode plots are shown in Fig. 1

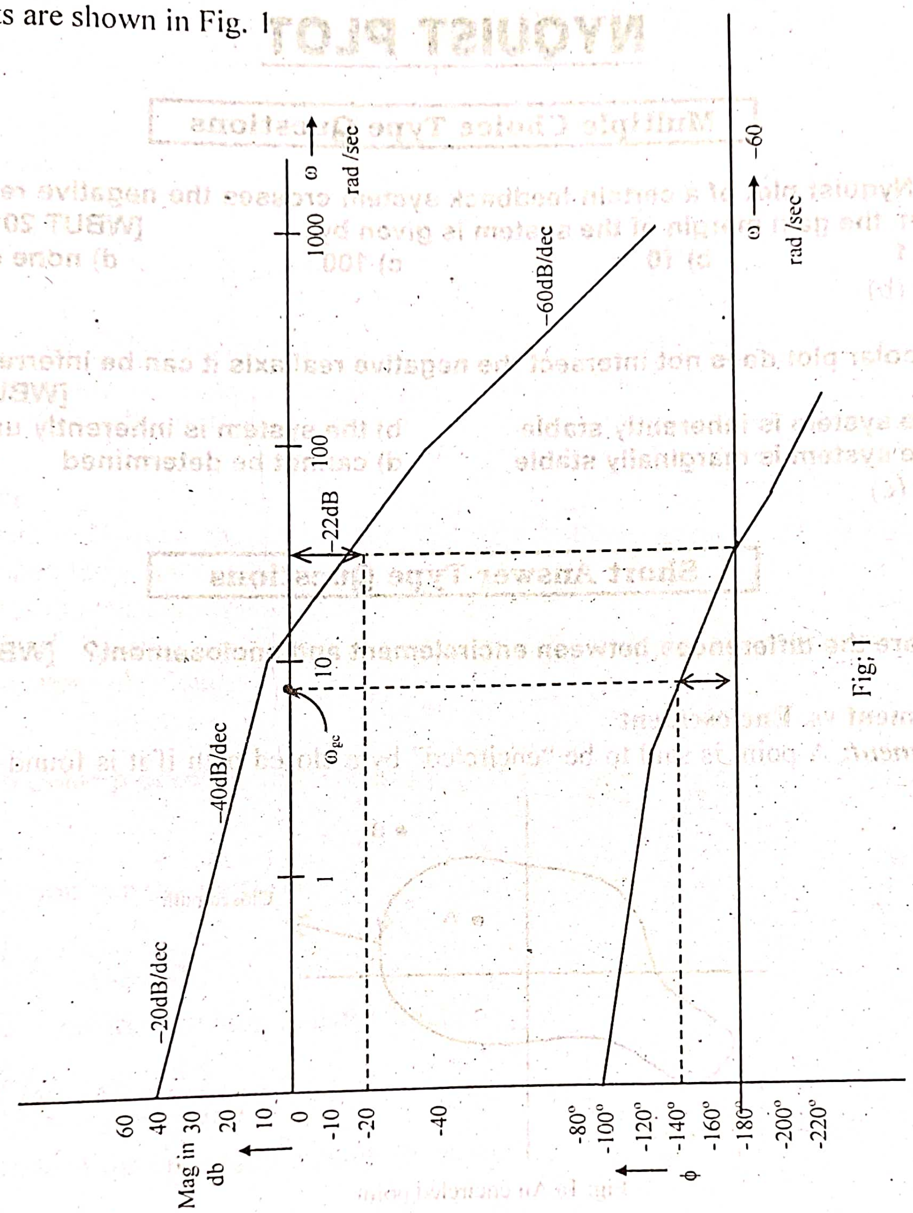


Fig. 1

From the Bode plots, we get

1. Gain Crossover frequency = 7 rad/sec
2. Phase Crossover frequency = 23 rad/sec
3. Gain Margin = 22 dB
4. Phase Margin = 36°
5. Stability of closed loop:

Since gain and phase margin are positive, so the system is absolutely stable.

NYQUIST PLOT

Multiple Choice Type Questions

1. If the Nyquist plot of a certain feedback system crosses the negative real axis at -0.1 point, the gain margin of the system is given by [WBUT 2011, 2016]
a) 0.1 b) 10 c) 100 d) none of these

Answer: (b)

2. If the polar plot does not intersect the negative real axis it can be inferred that [WBUT 2013]
a) the system is inherently stable
b) the system is inherently unstable
c) the system is marginally stable
d) cannot be determined

Answer: (c)

Short Answer Type Questions

1. What are the differences between encirclement and enclosement? [WBUT 2014]

Answer:

Encirclement vs. Enclosement

Encirclement: A point is said to be "encircled" by a closed path if it is found inside the path.

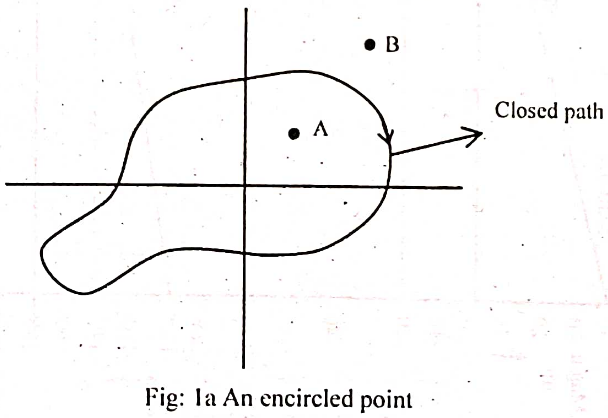


Fig: 1a An encircled point

Point A is said to be encircled, as it is within the closed path & Point B is outside the path.

Enclosed: A point or region is said to be enclosed by a closed path, if it found to the right of the path when the path is traversed in a specified direction.

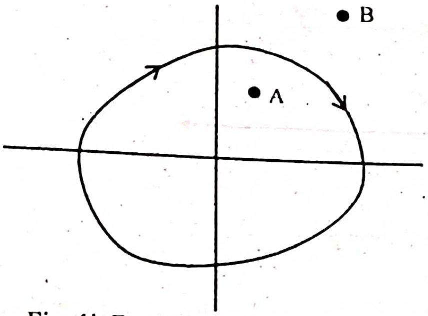


Fig: 1b Enclosement of point A

Here, point A is enclosed. But point B is not; as A falls on the right of the path.

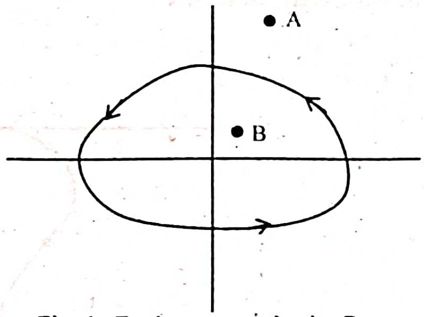


Fig: 1c Enclosement of point B

Here, point B is enclosed. But point A is not.

Closed path

A closed path in Nyquist approach of analysis implies the frequency domain plot or polar plot of an open loop transfer function.

The closed path is usually specified by a direction.

In Fig 1a and 1b, the paths have clockwise directions and in Fig 1c anti-clockwise direction has been provided.

2. Draw the polar plot of the system having $OLTF = \frac{1}{s(sT+1)}$. **[WBUT 2018]**

Answer:

Given open loop transfer function

$$G(s) = \frac{1}{s(1+sT)}$$

Substitute $s = j\omega$ in open loop transfer function

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)}$$

The magnitude of the open loop transfer function

$$M = \frac{1}{\omega\sqrt{1+\omega^2T^2}}$$

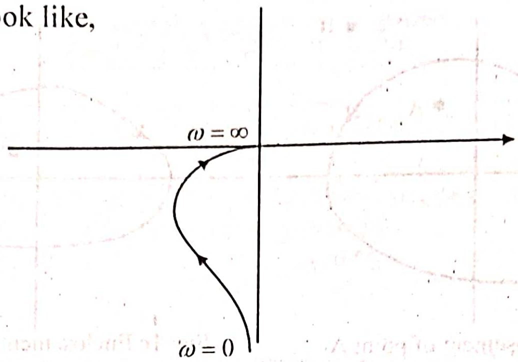
The phase angle of the loop transfer function

$$\phi = -90° - \tan^{-1}\omega T$$

Following table shows the magnitude and phase angle of the open loop transfer function

Frequency (rad/s)	Magnitude	Phase Angle
0	∞	$-90°$ or $270°$
10	$\frac{1}{10\sqrt{1+10^2T^2}}$	$-90° - \tan^{-1}10T$
∞	0	$-180°$

So, the polar plot will look like,



Long Answer Type Questions

1. State the 'Principle of argument' and its extension to Nyquist criterion. **[WBUT 2005]**

OR,

What do you mean by principle of argument? State and explain Nyquist criterion of a control system. **[WBUT 2007, 2013]**

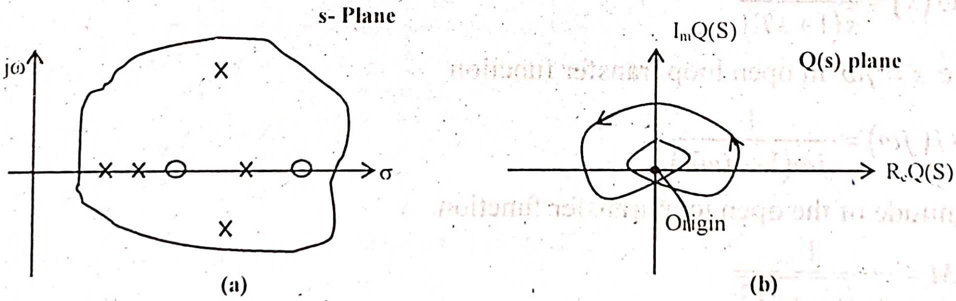
OR,

Write short note on Nyquist plot. **[WBUT 2005, 2007]**

Answer:

1st Part:

Consider the s-plane contour encloses P poles and Z zeros of Q(s) in s-plane as shown in the figure (a). Then, corresponding Q(s) plane contour must encircle the origin (z+p) times in the anti-clockwise direction or (z–p) times in the clockwise direction as shown in the figure (b).



(a) (b)

This relation between the enclosure of poles and zeros of Q(s) in the s-plane contour to the encirclement of the origin by the Q(s)-plane contour is known as 'Principle of Argument'.

2nd Part:

Let us consider a system whose forward path transfer function is $G(s)$ and that of feedback path is $H(s)$ as shown in Fig: (a).

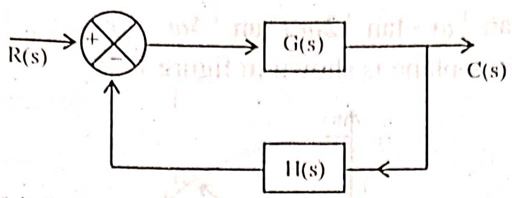


Fig: (a) A closed loop system with forward path T.F = G(s) and feedback path T.F =H(s)

The closed loop transfer function

$$T.F. = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

and the characteristic polynomial $= F(s) = 1 + G(s)H(s)$

From our basic knowledge, if all the roots of the characteristic equation lie in the left half of s-plane then the system in stable.

Now, assume

$$G(s)H(s) = \frac{K(s+z_1)(s+z_2)\cdots\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots\cdots(s+p_n)}; \quad n \geq m \qquad \ldots \text{(i)}$$

and $$F(s) = \frac{(s+z_1')(s+z_2')\cdots\cdots(s+z_n')}{(s+p_1)(s+p_2)\cdots\cdots(s+p_n)} \qquad \ldots \text{(ii)}$$

From the equation (i) and (ii)

We can say,

Open loop poles are same. As the closed loop poles i.e. poles of $G(s)\ H(s)$ are same as that of $F(s)$. Zeros of the characteristic polynomial $F(s)$ are the roots of the characteristic equation (i.e. $F(s) = 0$. For stability, the roots of characteristic equation of or zeros of $F(s)$ should lie in left half of s-plane. If a zero of $F(s)$ is found in right half side (with positive real part) then the system is stable. With this fact and taking the help of Cauchy's principle of argument Nyquist constructed a closed path or contour in s-plane such that entire right half of s-plane is encircled to find the presence of poles and zeros of $F(s)$. For stability, $F(s)$ will not encircle the origin.

2. A unity feedback control system has open loop transfer function.

$$G(s)H(s) = \frac{(4s+1)}{s^2(s+1)(2s+1)}$$

Determine closed loop stability by Nyquist plot. **[WBUT 2007]**

Answer:

$$M = \left|G(j\omega)H(j\omega)\right| = \frac{\sqrt{1+16\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

and $\phi = -180° - \tan^{-1}\omega - \tan^{-1} 2\omega + \tan^{-1} 4\omega$

The Nyquist contour in s-plane is shown in figure 1.

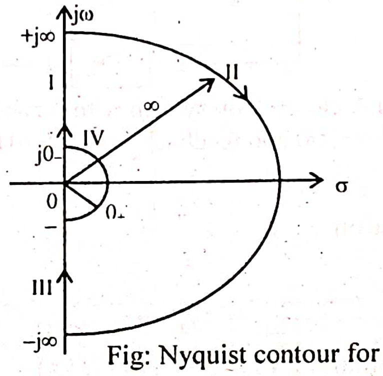
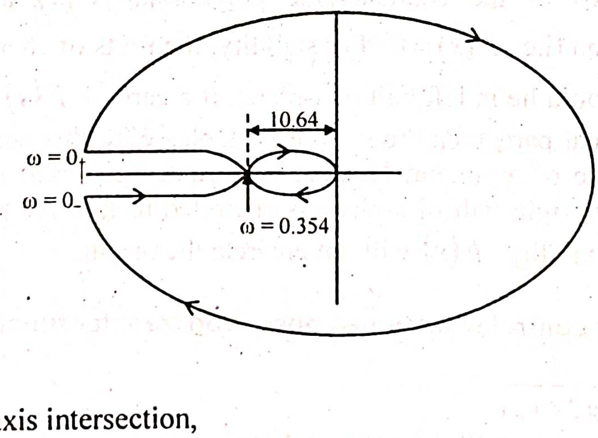


Fig: Nyquist contour for Type-1 higher system

Forming the table

Sections	ω	M	ϕ
I	0_+ ∞	∞ 0	$-180°$ $-270°$
II	∞ $-\infty$	0 0	$-270°$ $+270°$
III	$-\infty$ 0_-	0 ∞	$+270°$ $+180°$
IV	0_-	∞ ∞	$+180°$ $-180°$



In order to get real axis intersection,

$\phi = -180° - 180° - \tan^{-1}\omega - \tan^{-1} 2\omega + \tan^{-1} 4\omega$

$\Rightarrow \quad \tan^{-1}\omega + \tan^{-1} 2\omega = \tan^{-1} 4\omega$

$\Rightarrow \quad \dfrac{\omega + 2\omega}{1 - 2\omega^2} = 4\omega$

$\Rightarrow \quad 4-8\omega^2 = 3 \quad [\because \omega \neq 0]$

$\therefore \quad \omega = \sqrt{\frac{1}{8}} = 0.354$ rad./sec.

Again putting $\omega = 0.354$ in M to have $M = 10.64$.
The Nyquist plot in GH-plane looks like
From the curve we can say, $N = 2$
$\because P = 0, \therefore Z = N + P = 2$
So closed loop system has two roots lying on the imaginary axis.
So, the closed loop system is unstable.

3. The open loop transfer function of a unity feedback control system is given by

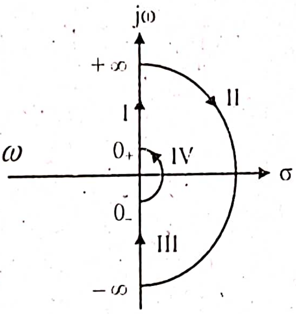
$$G(s) = \frac{s+0.25}{s^2(s+1)(s+0.5)}$$

Determine the closed-loop stability by applying Nyquist criterion.

[WBUT 2011, 2014]

Answer:

$$G(s) = \frac{s+0.5}{s^2(s+1)(s+0.5)}$$

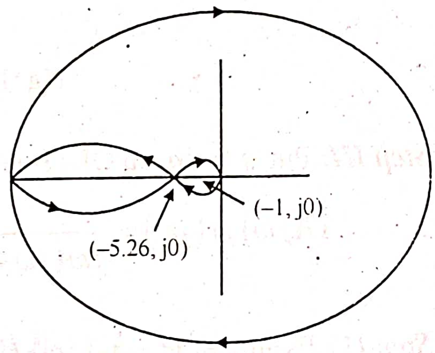


Transfer function says $P = 0$ and type no. = 2
So the control in s-plane, as proposed by Nyquist is putting $s = j\omega$

$$G(j\omega) = \frac{j\omega + 0.25}{(j\omega)^2(j\omega+1)(j\omega+0.5)}$$

$$\therefore \quad M = \frac{\sqrt{\omega^2 + (0.25)^2}}{\omega^2\sqrt{\omega^2+1}\sqrt{\omega^2+(0.5)^2}}$$

$$\phi = -180° + \tan^{-1}\frac{\omega}{0.25} - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{0.5}$$



In order to evaluate the negative real axis crossover, we put $\phi = -180°$ in the above expression of ϕ and get $\omega = 0.4$ rad./sec.
Then putting $\omega = 0.4$ in M, we get $M = 5.26$
The Nyquist plot is obtained as shown which says

$N = 2 \quad \because P = 0 \quad \therefore Z = N + P = 2$

So, two closed loop poles will lie R.H.S. of s-plane.
So, system is unstable.

4. The open loop transfer function of a unity feedback system is given by Nyquist criteria for, $G(s)H(s)=\dfrac{5}{s(s+1)(s+2)}$. Draw the Nyquist plot and hence find out whether the system is stable or not. **[WBUT 2014]**

Answer:

Step I: From the given open loop T.F.

$$P=0 \qquad \ldots\ldots (i)$$

Step II: There is a pole at origin, so the Nyquist contour in s-plane should not pass through the origin and the contour should be as shown in figure (1).

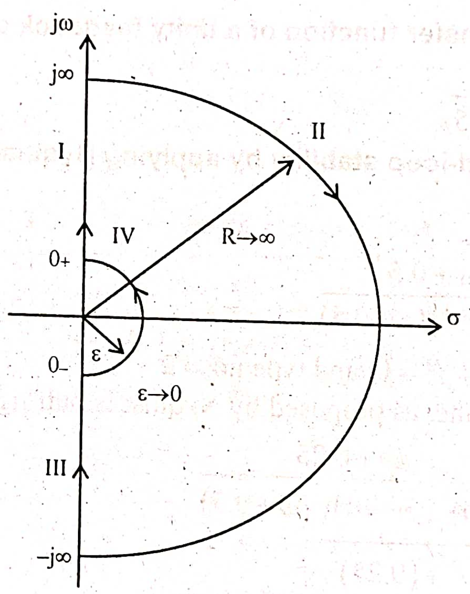


Fig: 1 Nyquist contour in s-plane

Step III: Put $s=j\omega$ in OLTF $G(s)\,H(s)$

$$\therefore \quad G(j\omega)\,H(j\omega)=\frac{5}{j\omega(j\omega+1)(j\omega+2)}$$

Step IV: Evaluate $M=|G(j\omega)\,H(j\omega)|$

$$=\frac{5}{\omega\sqrt{(1+\omega^2)}\sqrt{4+\omega^2}} \qquad \ldots\ldots (ii)$$

Step V: Evaluate $\phi=\angle G(j\omega)\,H(j\omega)$

i.e. $$\phi=-90^\circ-\tan^{-1}\omega-\tan^{-1}\frac{\omega}{2} \qquad \ldots\ldots (iii)$$

Step VI: Form Table T-1

Table: T-1

Sections in s-plane	ω rad./sec.	M [from equation (ii)]	ϕ [from equation (iii)]
I begins from $\omega = 0_+$ and ends at $\omega = +\infty$	0_+	∞	$-90°$
	$+\infty$	0	$-270°$
II begins from $\omega = +\infty$ and terminates at $-\infty$ through a semicircular path of infinite radius $(R \to \infty)$	$+\infty$	0	$-270°$
	$-\infty$	0	$+270°$
III begins from $\omega = -\infty$ to $\omega = 0_-$ along the negative $j\omega$ axis	$-\infty$	0	$+270°$
	0_-	∞	$90°$
IV begins from $\omega = 0_-$ and ends at $\omega = 0_+$ through a circular path of radius ε where $F \to 0$	0_-	∞	$90°$
	0_+	∞	$-90°$

Step VII: Real axis intersection of $G(j\omega)H(j\omega)$

For this put $\phi = -180°$ in equation (iii)

$$\therefore \quad -180° = -90° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$\Rightarrow \quad \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2} = 90°$$

$$\Rightarrow \quad \tan\left[\tan^{-1}\omega + \tan^{-1}\frac{\omega}{2}\right] = \tan 90°$$

$$\Rightarrow \quad \frac{\omega + \frac{\omega}{2}}{1 - \frac{\omega^2}{2}} = \infty = \frac{1}{0}$$

$$\Rightarrow \quad 1 - \frac{\omega^2}{2} = 0 \quad \Rightarrow \quad \omega^2 = 2$$

$$\therefore \quad \omega_5 = \sqrt{2}\ \text{rad/sec} \qquad \left(\omega = -\sqrt{2}\ \text{rad./sec discarded}\right)$$

The magnitude of $G(j\omega)H(j\omega)$ at $\omega = \sqrt{2}$ is obtained from equation (ii)

$$\therefore \qquad M\Big|_{\omega=\sqrt{2}} = \frac{1}{\sqrt{2}\sqrt{1+2}\sqrt{4+2}} = \frac{1}{\sqrt{2}\sqrt{3}\sqrt{6}} = \frac{1}{6} = 0.167 < 1$$

Step VIII: To draw Nyquist plot following table T-1.

For section I

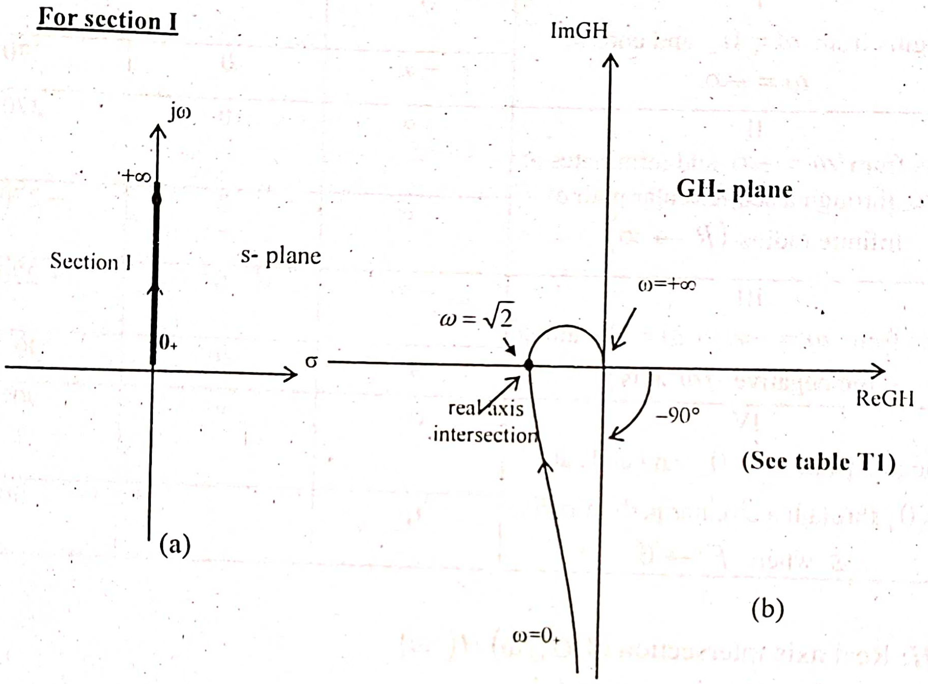


Fig: 2 (a) section I of s-plane contour (b) mapping of GH for section-I on GH-plane

For section-II

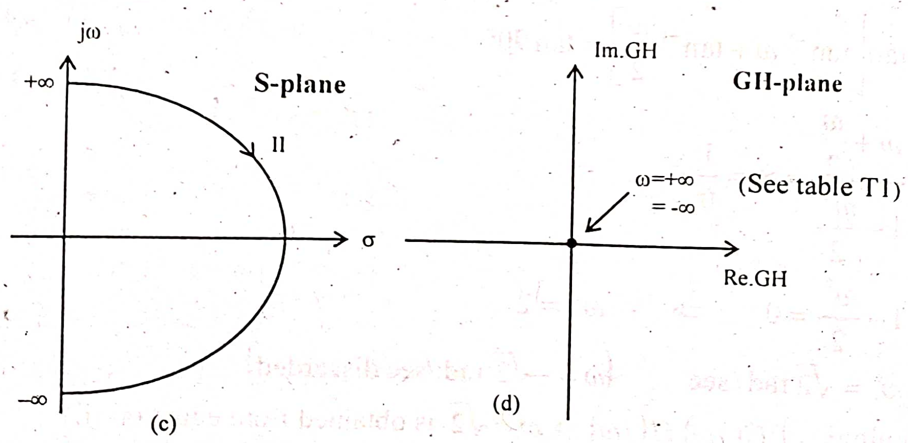


Fig: 2 (c) section II of s-plane contour (d) mapping of GH for section II on GH-plane

For section III

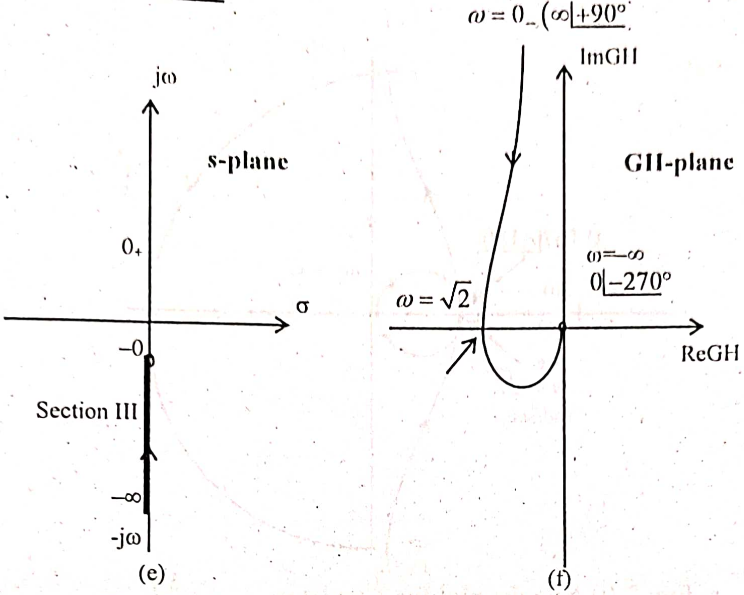


Fig: 2 (e) section III of s-plane contour (f) mapping of GH for section III on GH-plane

For section IV

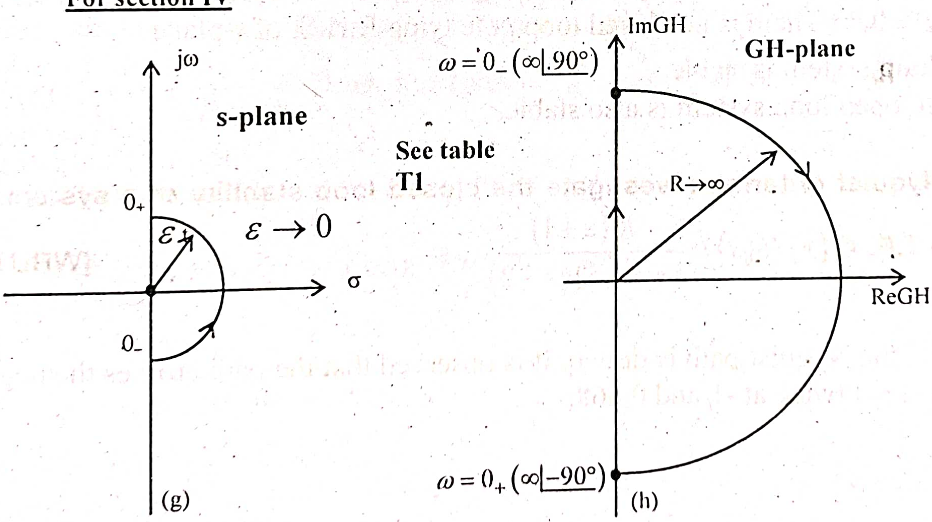


Fig: 2 (g) section IV of s-plane contour (h) mapping of GH for section IV on GH-plane

The complete Nyquist plot is as shown in figure 2(i)

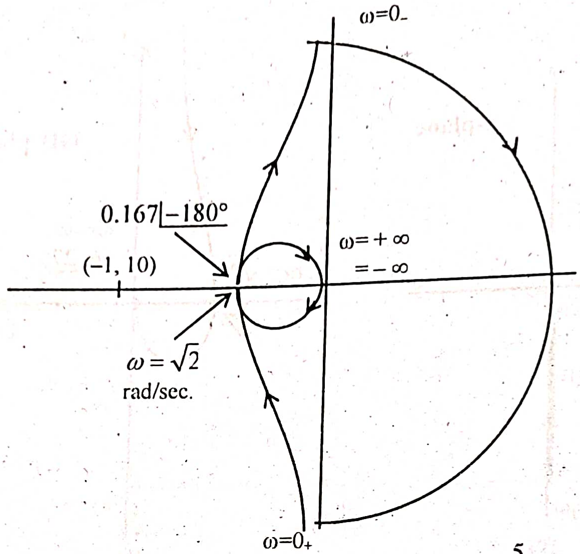


Fig: 2 (i) Nyquist plot for $G(s)H(s)=\dfrac{5}{s(s+1)(s+2)}$

Step IX: $\because\ P=0$ [From equation (i)]

and $N=0$ [From the Fig. 2(i)]

$\therefore\ Z=N+P=0 \Rightarrow$ There is no closed loop pole lying R.H.S. of s-plane.

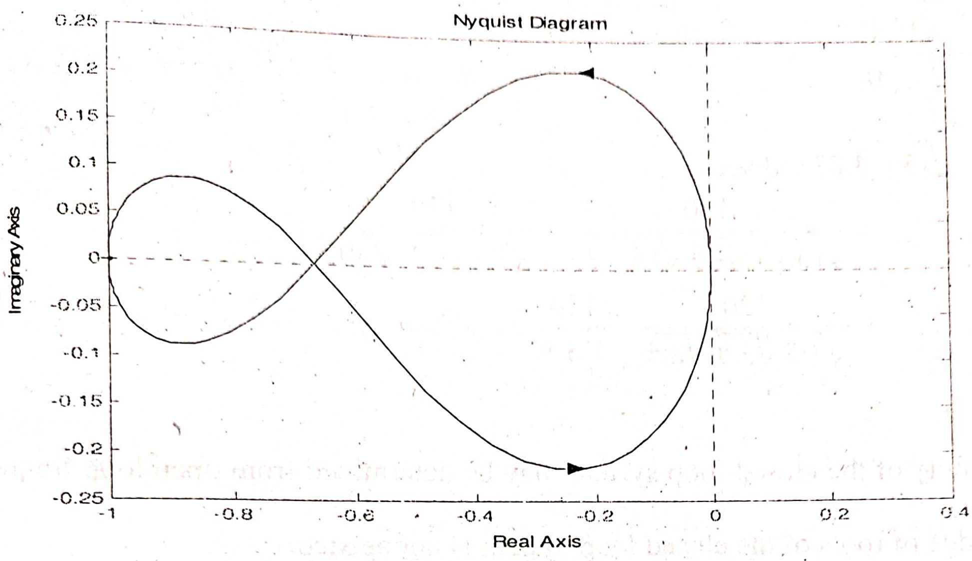
$\therefore$ Closed loop system is stable.

$\because\ P=0$, so open loop system is also stable.

5. Using Nyquist criterion investigate the closed loop stability of a system whose open-loop T.F. $G(s)H(s)=\dfrac{K(s+1)}{(s+0.5)(s-2)}$. **[WBUT 2015]**

Answer:

Taking K=1, the Nyquist path is drawn. It is observed that the path crosses the negative real axis at -1+j0 twice at -1 and 0.668.



With K added in the given transfer function, the crossover points will be -K and -0.633K.
Moreover, P = 1
So for stability 0.633K > 1 i.e. K > 1.57

6. a) State and explain Nyquist criteria for study of control system.
b) The open loop transfer function of closed loop system is $G(s)H(s)=\dfrac{120}{s(s+3)(s+5)}$. Draw the Nyquist plot and hence find out whether the system is stable or not.
c) What are the advantages of Nyquist plot? **[WBUT 2016]**

Answer:

a) ***Refer to Question No. 1 of Long Answer Type Questions.***

b) Step I: $P=0$

Step II: $M=\dfrac{120}{\omega\sqrt{\omega^2+9}}\cdot\dfrac{1}{\sqrt{\omega^2+25}}$

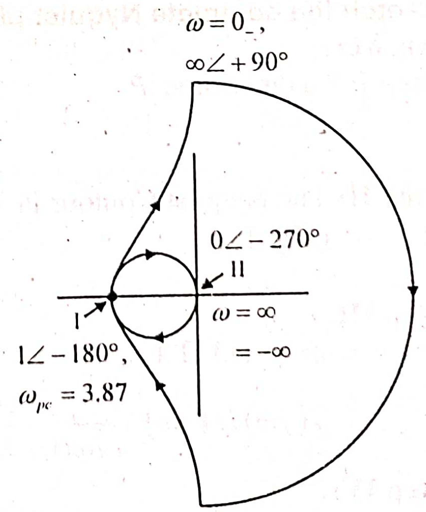
Step III: $\phi=-90°-\tan^{-1}\dfrac{\omega}{3}-\tan^{-1}\dfrac{\omega}{5}$

Step IV: At negative real axis crossover

$$-180°=-90°-\tan^{-1}\frac{\omega}{3}-\tan^{-1}\frac{\omega}{5}$$

$$\Rightarrow \quad \tan^{-1}\frac{\omega}{3}+\tan^{-1}\frac{\omega}{5}=90°$$

Taking tan both sides



$$\frac{\frac{\omega}{3}+\frac{\omega}{5}}{1-\frac{\omega^2}{15}}=\frac{1}{0}$$

$\Rightarrow \qquad \omega=\sqrt{15}=3.87$ rad/sec

Step V: $M\big|_{\text{at } \phi=-180^\circ} = \dfrac{120}{\sqrt{15}\sqrt{15+9}\sqrt{15+25}} = \dfrac{120}{\sqrt{15}\cdot\sqrt{24}\cdot\sqrt{40}}$

$$= \frac{120}{\sqrt{3.5}\sqrt{3.8}\sqrt{8.5}} = \frac{120}{3\cdot5\cdot8} = 1$$

c)
1) The stability of the closed-loop system may be determined from open loop frequency response.
2) Knowledge of roots of the closed loop system is not needed.
3) Often we do not get any mathematical model of the physical system. We often get the frequency response.

7. a) State the Nyquist stability criteria. **[WBUT 2018]**

Answer:

Nyquist stability criterion states the number of encirclements about the critical point $(1+j0)$ must be equal to the poles of characteristic equation, which is nothing but the poles of the open loop transfer function in the right half of the 's' plane.

b) The open loop transfer function of a unity feedback control system is

$$G(s)=\frac{K(1+2s)}{s(1+s)(1+s+s^2)}$$ **[WBUT 2018]**

Sketch the complete Nyquist plot and find out the value of K for stability.

Answer:

Step I: To determine P

$P=0$

Step II: The Nyquist Contour in s-plane having detour at $s=0$ (Fig. 1)

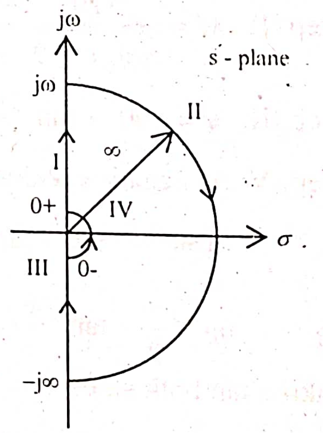


Fig: 1 Nyquist contour in s-plane

Step III:

Put $s=j\omega$ in O.L.T.F.

$$G(j\omega)H(j\omega)=\frac{K(1+j2\omega)}{j\omega(1+j\omega)(1+j\omega-\omega^2)}$$

Step IV:

Evaluate $|G(j\omega)| = M$

$$M = \frac{K\sqrt{1+4\omega^2}}{\omega\sqrt{1+\omega^2}\sqrt{(1-\omega^2)^2+4\omega^2}}$$

Step V:

Evaluate $\phi = \angle\{G(j\omega)H(j\omega)\}$

$$\phi = \tan^{-1}2\omega - 90° - \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right)$$

Step VI:

Formation of table 1

Table: 1

Section in s-plane	ω	M	ϕ
I begins from $\omega = 0_+$ and ends at $\omega = +\infty$	$0+$	∞	$-90°$
	$+\infty$	0	$-270°$
	1	$K\sqrt{5/2}$	$-161.5°$
	0.5	2.8K	$-105.5°$
II begins from $\omega = +\infty$ and terminates at $-\infty$ through a semicircular path of infinite radius $(R \to \infty)$	$+\infty$	0	$-270°$
	$-\infty$	0	$+270°$
III $-\infty \le \omega \le 0$	$-\infty$	0	$+270°$
	$0-$	∞	$+90°$
IV $0- \le \omega \le 0+$	$0-$	∞	$+90°$
	$0+$	∞	$-90°$

Step VII: To evaluate Negative Real axis intersection of Nyquist path.

$$G(j\omega)H(j\omega) = \frac{K(1+j2\omega)}{j\omega(1+j\omega)\{(1-\omega^2)+j\omega\}}$$

Separating into real and imaginary parts and equating the imaginary part to zero. We get $1+2\omega^2-2\omega^4=0$.

Now putting $\omega^2 = x$, in the above expression,

$$1+2x-2x^2=0 \Rightarrow 2x^2-2x-1=0$$

$$x_{1,2} = \frac{2\pm\sqrt{4+8}}{4} = \frac{2\pm2\sqrt{3}}{4} = \frac{1\pm\sqrt{3}}{2} = 0.5+0.866 = 1.366$$

$\omega = 1.17$ rad / sec.

$$\therefore \quad M\Big|_{\text{at } \omega=1.17} = \frac{K\sqrt{1+(2\times 1.17)^2}}{1.17\sqrt{1+(1.17)^2}\sqrt{(1-1.17^2)^2+1.17^2}} = 1.152K$$

Step VIII:
To draw Nyquist path (Fig. 2)

Step IX:
Conclusion:

$N = 1$

$\therefore \quad Z = N + P$

Part (b)

$$GM = 20\log\frac{1}{1.15K}$$

$$3 = -20\log_{10}1.15 - 20\log_{10}K$$

$\Rightarrow \quad K = 0.615$

For stability, $N = 0$ as $z = N + P = 0 + 0 = 0$

$\Rightarrow \quad 1.15K > 1 \quad \Rightarrow \quad K > 0.8695$

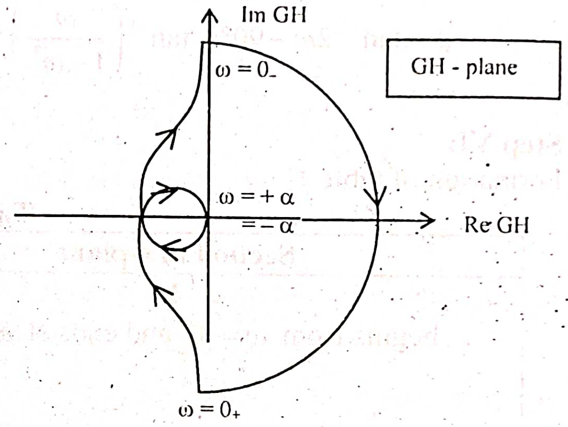


Fig: 2 Nyquist path in GH-plane

Part: (b)

$$GM \text{ in dB} = 20\log_{10}\left(\frac{1}{a}\right) = -20\log_{10}a$$

$\therefore \quad 3 = -20\log_{10}a$

$\Rightarrow \quad \log_{10}a = -0.15$

$\Rightarrow \quad a = 0.707$

But, $\quad a = 1.152K = 0.707$

$\Rightarrow \quad K = 0.614$

8. Write short note on Polar plot. **[WBUT 2007, 2012, 2015]**

Answer:

The polar plot of a sinusoidal transfer function $G(j\omega)$ is a plot of $|G(j\omega)|$ versus phase angle of $G(j\omega)$ on polar co-ordinates as 'ω' is varied from zero to infinity. It is also called Nyquist plot. The plot is plotted on a complex plane. As ω is varied, magnitude and phase angle change and if magnitude is plotted for varying phase angles, locus obtained is the polar plot. It is easier to construct and at any desired frequency, ready information of magnitude and phase angle can be obtained. However calculations are tedious and if a system is modified by adding a pole or zero, the polar plot has to be constructed again.

DESIGN OF CONTROL SYSTEM

Multiple Choice Type Questions

1. The lead-lag compensation will improve [WBUT 2007]
   a) transient response b) transient response and steady state response
   c) none of these

Answer: (b)

2. The lead compensator network is considered to [WBUT 2008]
   a) high-pass filter b) low-pass filter c) equalizer d) none of these

Answer: (a)

3. A phase lead compensator is responsible for [WBUT 2013]
   a) fast vanish of transient b) improvement of steady state error
   c) both (a) & (b) d) neither (a) nor (b)

Answer: (c)

4. The transfer function of an integral compensator is given by [WBUT 2015]
   a) $\frac{1}{s}$ b) $\frac{1}{s^2}$ c) $\frac{k}{s}$ d) ks

Answer: (d)

5. The transfer function of a lag compensator is $D(s)=\frac{1+\alpha\tau s}{1+\tau s}$, $\tau>0$. The value of α is given by [WBUT 2016]
   a) $\alpha=1$ b) $\alpha>1$ c) $\alpha<1$ d) α is a constant

Answer: (b)

Short Answer Type Questions

1. Design a lag compensator using R-C network. [WBUT 2015]

Answer:

Realization of Lag Network

The basic Lag network is shown in Fig. 1

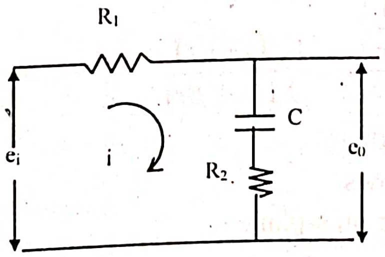


Fig: 1 A lag network

Step 1: Mathematical equations are developed first

$$e_i = R_1 i + iR_2 + v_c \quad = (R_1 + R_2)i + \frac{Q}{C}$$

$$= (R_1 + R_2)i + \frac{\int idt}{C} \quad \text{.... (1)}$$

$$e_o = iR_2 + v_c = iR_2 + \frac{\int idt}{C} \quad \text{.... (2)}$$

Taking Laplace transform at both side (assuming initial conditions to be zero)

$$E_i(s) = (R_1 + R_2)I(s) + \frac{I(s)}{sC} \quad \text{.... (3)}$$

$$E_i(s) = R_2 I(s) + \frac{I(s)}{Cs} \quad \text{.... (4)}$$

or, $$E_i(s) = \left[(R_1 + R_2) + \frac{1}{Cs}\right] I(s)$$

or, $$I(s) = \frac{E_i(s)}{\left[(R_1 + R_2) + \frac{1}{Cs}\right]} \quad \text{.... (5)}$$

Putting this value of $I(s)$ in

$$E_0(s) = \left[R_2 + \frac{1}{Cs}\right] I(s) = \frac{(R_2 Cs + 1)}{Cs} \times \frac{E_i(s)}{\frac{(R_1 + R_2)(Cs) + 1}{Cs}} = \frac{(R_2 Cs + 1)}{(R_1 + R_2)(Cs) + 1} \cdot E_i(s)$$

$\therefore$ Transfer function of the Lag Network

$$= G_c(s) = \frac{E_o(s)}{E_i(s)} = \frac{(R_2 C + 1)}{(R_1 + R_2)(Cs) + 1} \quad \text{.... (6)}$$

Now, put $R_2 C = \tau$

$$\frac{(R_1 + R_2)}{R_2} = \beta > 1$$

$$\therefore \quad (R_1 + R_2)Cs = \frac{(R_1 + R_2)}{R_2} \cdot (R_2 \cdot C)s = \beta \cdot \tau \cdot s$$

$$\therefore \quad G_c(s) = \frac{(\tau s + 1)}{\beta \tau s + 1} = \frac{\tau\left(s + \frac{1}{\tau}\right)}{\beta\tau\left(s + \frac{1}{\beta\tau}\right)} = \frac{1}{\beta} \cdot \frac{(s + 1/\tau)}{(s + 1/\beta\tau)} \quad \text{.... (7)}$$

The above transfer function shows

(i) A pole at $-1/\beta\tau$ on s-plane

(ii) A zero at $-1/\tau$ on s-plane

Long Answer Type Questions

1. Write short notes on the following:
a) Compensation techniques **[WBUT 2007]**
b) Effect of Lag-Lead Compensator **[WBUT 2014]**
c) Lead-Lag compensation **[WBUT 2015, 2017]**

Answer:

a) Compensation techniques:

An uncompensated system may be compensated by cascade compensation approach or through feedback compensation approach as shown in the figure below.

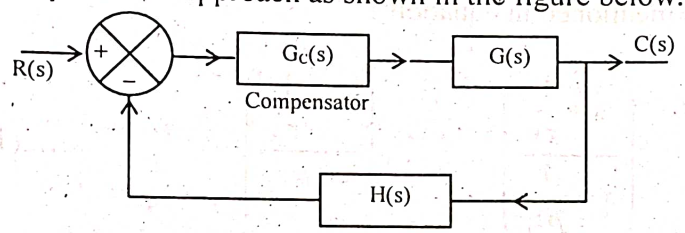


Fig: Cascade Compensation

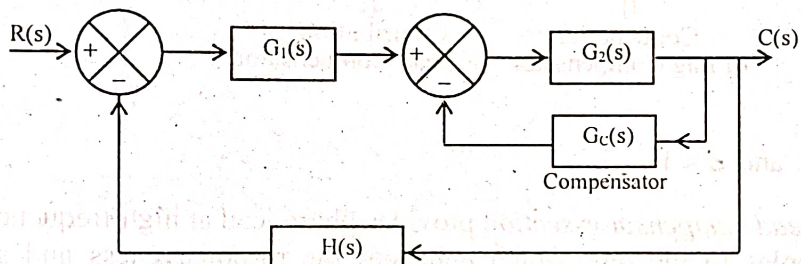


Fig: Feedback Compensation

Types of Compensator in these approaches we use
Lag Compensator, Lead Compensator and Lag-Lead Compensator
Lag Compensator is used

1. To improve steady-state behavior.
2. To preserve a satisfactory transient Response.

We know that higher the order of the system lower is its stability. The system which are type-2 or higher, are usually unstable i.e., bear lower margin of stability.
As the Lead compensator increases the margin of stability. So we will use Lead Compensator for such system having lower degree of stability.
Lead-Lag compensator is selected when

1) both the transient and steady state responses (in time domain) need improvement.
2) the complex poles in Forward path Transfer function are close to the $j\omega$-axis.

b) Effect of Lag-Lead Compensator:
To get fast response and good static accuracy, a lag-lead compensator is used. It also increases the low frequency gain which improves the steady state. Since it increases the

bandwidth of the system, the system response becomes very fast. In general, the phase lead portion of this compensator provides large bandwidth and hence shorter rise time and settling time, while the phase lag portion provides the major damping of the system.

c) Lead-Lag compensation:

- A lead-lag compensator consists of a lead compensator cascaded with a lag compensator.
- Both lead compensators and lag compensators introduce a pole-zero pair into the open loop transfer function.
- The General form of the overall transfer function of such a compensator can be written as mentioned in equation 1

$$G_C(s) = \underbrace{\left(\frac{s+\frac{1}{\tau_1}}{s+\frac{1}{\beta\tau_1}}\right)}_{\substack{\text{Contribution} \\ \text{of Lag Compensator}}} \cdot \underbrace{\left(\frac{s+\frac{1}{\tau_2}}{s+\alpha\tau_2}\right)}_{\substack{\text{Contribution} \\ \text{of Lead Compensator}}} \quad \ldots\ldots(1)$$

where, $\beta > 1$ and $\alpha < 1$

- *The lead compensator section* provides phase lead at high frequencies. This shifts the poles to the left, which enhances the responsiveness and stability of the system.
- The *lag compensator section* provides phase lag at low frequencies reducing the "steady state" error.

Selection Criteria

a) When both the transient and steady state responses (in time domain) need improvement.

b) When the complex poles in Forward path Transfer function are close to the $j\omega$-axis.

Lag-Lead Network

Fig. 1 shows an electrical circuit representing a Lag-Lead network

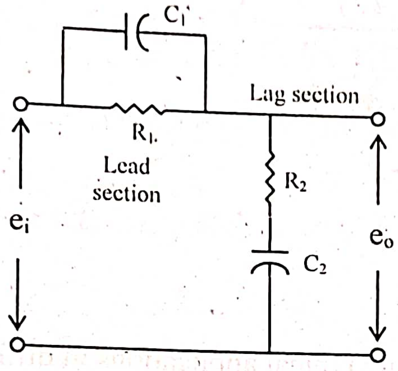


Fig: 1 Lag -Lead network

- ***Transfer Function:*** Applying potential divider rule

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 + \dfrac{1}{sC_2}}{R_2 + \dfrac{1}{sC_2} + \dfrac{R_1/sC_1}{R_1 + \dfrac{1}{sC_1}}} \quad \text{.... (1)}$$

After Simplification,

$$= \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s}$$

$$= \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_2 C_2 R_1 C_1 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

$$= \frac{\left(s + \dfrac{1}{R_1 C_1}\right)\left(s + \dfrac{1}{R_2 C_2}\right)}{\left[s^2 + \left(\dfrac{1}{R_1 C_1} + \dfrac{1}{R_2 C_2} + \dfrac{1}{R_1 C_2}\right)s + \dfrac{1}{R_1 R_2 C_1 C_2}\right]} \quad \text{.... (2)}$$

Comparing equations (1) and (2), we get

$$R_1 C_1 = \tau_1, \qquad R_2 C_2 = \tau_2$$

$$R_1 R_2 C_1 C_2 = \alpha \beta \tau_1 \tau_2 \qquad \Rightarrow \alpha \beta = 1$$

and $$\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_1 C_2}\right) = \frac{1}{\beta \tau_1} + \frac{1}{\beta \tau_2}$$

∴ Equation (1) may be re-written as

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right)}{\left(s+\frac{1}{\beta\tau_1}\right)\left(s+\frac{1}{\alpha\tau_2}\right)} \qquad \text{.... (3)}$$

where, $\beta = \frac{z_{C_1}}{p_{C_1}}$ and $\alpha = \frac{p_{C_2}}{z_{C_2}}$

Applications:

Lead-lag compensators are having typical applications in different disciplines such as

- Satellite, robotics control.
- Automobile diagnostics.
- Laser frequency stabilization, and many more.
- They are an important building block in analog control systems, and can also be used in digital control.

STATE SPACE ANALYSIS

Multiple Choice Type Questions

1. State variable approach converts a n^{th} order system into [WBUT 2005, 2008, 2009]

a) n first order differential equation
b) n second order differential equation
c) Two differential equation
d) A lower order system

Answer: (a)

2. A is an $n \times n$ matrix. Then the system to be controllable, the rank of the controllability matrix should be [WBUT 2007, 2016]

a) n b) $> n$ c) $\geq n$ d) $\leq n$

Answer: (a)

3. The state transition matrix $\phi(t)$ is given by [WBUT 2009]

a) $[sI]-[A]$
b) $\{[sI]-[A]\}^{-1}$
c) $L^{-1}\{[sI]-[A]\}^{-1}$
d) $L^{-1}\{[sI]-[A]\}$

Answer: (c)

4. The transfer function for the state variable representation $\frac{dx}{dt} = Ax + Bu$, $Y = Cx + Du$ is given by [WBUT 2010]

a) $D + C(SI - A)^{-1} B$
b) $B(SI - A)^{-1} C + D$
c) $B(SI - A)^{-1} B + C$
d) $C(SI - A)^{-1} D + B$

Answer: (a)

5. A system described by the state equation $\dot{x} = Ax + Bu$, & $y = Cx$ where $A = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C = [1 \quad 0]$. The transfer function $G(s)$ of the system is [WBUT 2013]

a) $\frac{s}{s^2 + 5s + 7}$ b) $\frac{1}{s^2 + 5s + 7}$ c) $\frac{s}{s^2 + 3s + 2}$ d) $\frac{1}{s^2 + 3s + 2}$

Answer: (a)

6. The state transition matrix $\phi(t)$ possesses which of the following properties? [WBUT 2016]

a) $\phi(0) = I$

b) $\phi^{-1}(t) = \phi(-t)$

c) $\phi(t_2 - t_1)\phi(t_1 - t_0) = \phi(t_2 - t_0)$ for any t_0, t_1, t_2

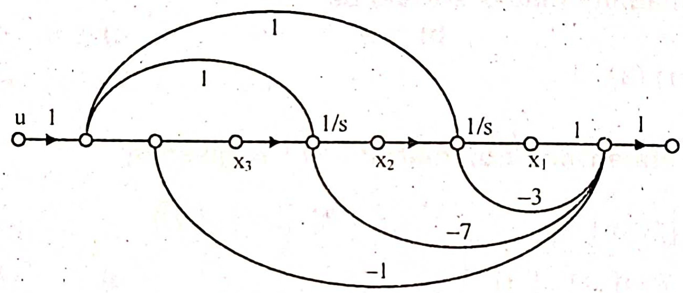
d) all of these

Answer: (d)

Short Answer Type Questions

1. Obtain state variable model of the system whose transfer function is given by $\dfrac{Y(s)}{U(s)} = \dfrac{s+1}{s^3 + 3s^2 + 7s + 1}$. [WBUT 2007]

Answer:



$y = x_1$

$\dot{x}_1 = x_2 - 3y + 0$

$= x_2 - 3x_1$

$\dot{x}_2 = x_3 - 7y + u$

$= x_3 - 7x_1 + u$

$\dot{x}_3 = u - y = u - x_1$

Therefore the state model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -7 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

2. Construct the state model for a system characterized by the differential equation $\ddot{y} + 5\dot{y} + 6y = 4$ [WBUT 2009, 2014]

Answer:

Let $y = x_1 = 1 \cdot x_1 + 0 \cdot x_2 + 0 \cdot u$

$\dot{y} = \dot{x}_1 = x_2 = 0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot u$

$\ddot{y} = \dot{x}_2 = -6y - 5y + 4u = -6x_1 - 5x_2 + 4u$

State equation is written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u$$

The output equation may be expressed as

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot [u].$$

3. A system is represented by the state & output equations given below. Find:
a) Characteristic equation **[WBUT 2010]**
b) The poles.

$$\dot{X} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 4 \\ 1 & 3 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$Y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} X.$$

Answer:

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 3 & 4 \\ 1 & 3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \quad D = (0)$$

Characteristic equation

$$= \left| (\lambda I - A) \right| = 0$$

where, $\lambda =$ Eigen values

$I =$ Identity matrix

$$= \left| \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} - \begin{pmatrix} 0 & 1 & 2 \\ 0 & 3 & 4 \\ 1 & 3 & 2 \end{pmatrix} \right| = 0 \text{ sb}$$

$$= \begin{vmatrix} \lambda & -1 & -2 \\ 0 & \lambda - 3 & -4 \\ -1 & -3 & \lambda - 2 \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda[(\lambda-3)(\lambda-2)-12]-(-1)[0-4]+(-2)[0+(\lambda-3)]=0$

$\Rightarrow \quad \lambda[(\lambda-3)(\lambda-2)-12]-4-2(\lambda-3)=0$

$\Rightarrow \quad (\lambda-3)(\lambda-2)\lambda-12\lambda-4-2\lambda+6=0$

$\Rightarrow \quad \lambda^3-5\lambda^2-8\lambda+2=0$

$\Rightarrow \quad (\lambda+1.45)(\lambda-0.22)(\lambda-6.23)=0$

(a) The characteristics equation is $\lambda^3-5\lambda^2-8\lambda+2=0$

(b) The poles are $-1.45, 0.22, 6.23$

4. A system is described by $\dot{X}=\begin{bmatrix}1 & -1\\1 & -1\end{bmatrix}X+\begin{bmatrix}0\\1\end{bmatrix}u$

$Y=\begin{bmatrix}1 & 0\end{bmatrix}X$.

Check the controllability & observability of the system. **[WBUT 2010]**

Answer:

For controllability:

$$Q_c=\left[B:AB\right]=\begin{bmatrix}0 & \vdots & \begin{pmatrix}1 & -1\\1 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\\1 & & \end{bmatrix}$$

$$\Rightarrow \qquad Q_c=\begin{bmatrix}0 & -1\\1 & -1\end{bmatrix}$$

$\therefore$ det. of $\qquad Q_c=1\neq 0$

So, system is controllable.

For observability:

$$Q_o=\left[C^T:A^TC^T\right]=\begin{bmatrix}1 & \vdots & \begin{pmatrix}1 & 1\\-1 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\\0 & & \end{bmatrix}=\begin{bmatrix}1 & 1\\0 & -1\end{bmatrix}$$

$$\therefore \text{ det. of } Q_o=\begin{vmatrix}1 & 1\\0 & -1\end{vmatrix}=-1\neq 0$$

So, the system is observable.

5. A system is represented by **[WBUT 2011]**

$X=\begin{bmatrix}-3 & -2\\-1 & -2\end{bmatrix}X+\begin{bmatrix}1\\1\end{bmatrix}U(t)$

$Y=[1 \quad 2]X$

Find the poles of the system. Assume D = 0.

Answer:

Characteristic equation

$$|(sI-A)|=\left|\begin{pmatrix}s & 0\\0 & s\end{pmatrix}-\begin{pmatrix}-3 & -2\\-1 & -2\end{pmatrix}\right|=0$$

$$\Rightarrow\begin{vmatrix}(s+3) & 2\\1 & (s+2)\end{vmatrix}=0$$

$$\Rightarrow (s+3)(s+2)-2=0$$

$$\Rightarrow s^2+5s+6-2=0$$

$$\Rightarrow s^2+5s+4=0$$

$$\Rightarrow s^2+4s+s+4=0$$

$\Rightarrow s(s+4)+1(s+4)=0$

$\Rightarrow (s+4)(s+1)=0$

$\therefore$ Poles are at -4 and -1.

6. Construct the state model for a system characterized by the different equation $\ddot{y}+3\dot{y}+2y=4$. **[WBUT 2017]**

Answer:

Let $y = x_1 = 1\cdot x_1 + 0\cdot x_2 + 0\cdot u$

$\dot{y} = \dot{x}_1 = x_2 = 0\cdot x_1 + 1\cdot x_2 + 0\cdot u$

$\ddot{y} = -2y - 3\dot{y} + 4u = -2x_1 - 3x_2 + 4u$

State equation is written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u$$

The output equation may be expressed as

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0\cdot[u].$$

Long Answer Type Questions

1. Obtain the eigenvalues and eigenvectors for a system described by:

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} X.$$ **[WBUT 2007]**

Answer:

Eigen values

$$(\lambda I - A) = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix} = \begin{bmatrix} \lambda & -1 & 0 \\ -3 & \lambda & -2 \\ 12 & 7 & \lambda+6 \end{bmatrix}$$

Characteristic equation is

$\lambda\{\lambda(\lambda+6)-(-2)7\}-(-1)\{-3(\lambda+6)-2(-2)(12)\}=0$

or, $(\lambda+1)(\lambda+2)(\lambda+3)=0$

$\therefore$ Eigen values are $\lambda = -1, -2, -3$

2. A single input single output system is given by **[WBUT 2007]**

$$\dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t) \text{ and } y(t) = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} x(t).$$

Test for controllability and observability.

Answer:

For controllability test

$$Q_c = \left[B : AB : A^2B\right] \text{ when } \quad B = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

$$\therefore \quad Q_c = \left[\begin{matrix} 1 \\ 0 \\ 0 \end{matrix} : \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} : \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}^2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right]$$

The matrix is $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 0 & 0 & 0 \end{pmatrix}$

Determinant of the above matrix is, $1\begin{vmatrix} -2 & 4 \\ 0 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & 4 \\ 0 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 0 & 0 \end{vmatrix} = 0$

The system is not controllable.

For observability:

$$Q_o = \left[C^T : A^T C^T : \left(A^T\right)^2 C^T\right] \text{ where, } \quad C = [1 \quad 0 \quad 2], \; C^T = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$A^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -6 \end{pmatrix}$$

$$\left(A^T\right)^2 C^T = A^T C^T \cdot A^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 18 \end{bmatrix}$$

The matrix is $\begin{vmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 2 & -6 & 18 \end{vmatrix}$

The determinant of the above matrix

$$1\begin{vmatrix} 0 & 0 \\ -6 & 18 \end{vmatrix} + 1\begin{vmatrix} 0 & 0 \\ 2 & 18 \end{vmatrix} + 1\begin{vmatrix} 0 & 0 \\ 2 & -6 \end{vmatrix} = 0$$

The system is not observability.

3. A control system is described by the state equation: **[WBUT 2008]**

$$\begin{bmatrix} \dot{x}_i \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Obtain the transfer function of the system.

Answer:

Here, we have $A = \begin{bmatrix} -5 & -1 \\ -3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $D = 0$

Then, transfer function, $G(s) = \dfrac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D$

We take

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$sI = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix}$$

Now, we will find $(sI - A)^{-1}$

Cofactors of $(sI - A) = \begin{bmatrix} s+1 & 3 \\ -1 & s+5 \end{bmatrix}$

So, transpose of cofactors of $(sI - A) = \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}$

$$(sI - A) = \begin{vmatrix} (s+5) & 1 \\ -3 & (s+1) \end{vmatrix} = (s+5)(s+1) - (-3)(1)$$

$$= s^2 + 6s + 5 + 3 = s^2 + 6s + 8 = (s+2)(s+4)$$

$$\therefore (sI - A)^{-1} = \frac{\text{Transpose of co-factors of } (sI\text{-A})}{\Delta(sI - A)}$$

$$=\frac{\begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}}{(s+2)(s+4)}=\begin{bmatrix} \frac{s+1}{(s+2)(s+4)} & \frac{-1}{(s+2)(s+4)} \\ \frac{3}{(s+2)(s+4)} & \frac{s+5}{(s+4)(s+2)} \end{bmatrix}$$

Now, $C=[1\ \ 2]$

$$\therefore C[sI-A]^{-1}=[1\ \ 2]\begin{bmatrix} \frac{s+1}{(s+2)(s+4)} & \frac{-1}{(s+2)(s+4)} \\ \frac{3}{(s+2)(s+4)} & \frac{s+5}{(s+4)(s+1)} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{s+1}{(s+2)(s+4)}+\frac{6}{(s+2)(s+4)} & \frac{-1}{(s+2)(s+4)}+\frac{2(s+5)}{(s+4)(s+2)} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{s+7}{(s+2)(s+4)} & \frac{2s+9}{(s+2)(s+4)} \end{bmatrix}$$

Now, $C(sI-A)^{-1}B$ is

$$\begin{bmatrix} \frac{s+7}{(s+2)(s+4)} & \frac{2s+9}{(s+2)(s+4)} \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix}=\begin{bmatrix} \frac{(s+7)2}{(s+2)(s+4)}+\frac{5(2s+9)}{(s+2)(s+4)} \end{bmatrix}$$

$$=\frac{2s+14+10s+45}{(s+2)(s+4)}=\frac{12s+59}{(s+2)(s+4)}$$

$$\therefore G(s)=C(sI-A)^{-1}B$$

$$\therefore G(s)=\frac{12s+59}{(s+2)(s+4)}$$

4. a) Obtain state transition matrix $\phi(t)$ from non-homogeneous state equation of a linear time invariant control system and list the properties of it.

[WBUT 2008, 2012, 2013, 2014]

Answer:

Let us consider an unforced system,

i.e., $U=0$

$\therefore$ State Equation of this system may be written as

$$\dot{X}=AX+BU=AX+B.0 \quad \text{.... (1)}$$

Taking Laplace Transformation of equation (1)

$$sX(s)-X(0)=AX(s)$$

where $x(t)$ is the state variable and $X(s)$ is the Laplace Transform of x. $x(0)$ = initial condition of state vector X.

$$sX(s) - AX(s) = X(0)$$

or, $X(s) = [sI - A] = X(0)$

or, $X(s) = [sI - A]^{-1} X(0)$ .... (2)

Taking inverse Laplace transform, of equation (2) we have,

$$x(t) = L^{-1}\left[(sI - A)^{-1}\right] X(0) \quad .... (3)$$

$$\phi(t) = L^{-1}\left[(sI - A)^{-1}\right]$$

$$= e^{AT} = STM$$

$$\text{STM} = e^{AT} = L^{-1}\left[(sI - A)^{-1}\right]$$

Here, x(t) is the unforced or non-homogeneous response of the system.
Let us have a first order scalar differential equation

$$\frac{dx}{dt} = ax, \; x(0) = x_0 \quad .... (4)$$

or, $\frac{dx}{x} = adt$

or, $\log_e x = at + c$

At $t = 0, \; x = x_0$

∴ $\log_e x_0 = c$

∴ Solution of the above equation

$$x(t) = e^{at} x_0 = \left(1 + at + \frac{1}{2!}a^2t^2 + \frac{1}{3!}a^3t^3 + \cdots\right) x_0 \quad .... (5)$$

With this analogy with the scalar case, let us assume a solution

$$x(t) = a_0 + a_1 t + a_2 t_2 + \cdots + a_k t^k + \cdots$$

where, a_i are vector co-efficient

∴ $\dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2 + \cdots$

$$= A(a_0 + a_1 t + a_2 t^2 + \cdots +) \quad .... (6)$$

where,

$$a_1 = Aa_0$$

$$a_2 = \frac{1}{2}Aa_1 = \frac{1}{2!}Aa^2{}_0$$

$$a_3 = a_2 t^2 = \frac{1}{3}.\frac{1}{2!}A^3 a_0 = \frac{1}{3!}A^3 a_0$$

. . . .

$$a_k = \frac{1}{k!}A^k a_0$$

∴ Solution of $x(t)$ can be written as

$$x(t) = \left(I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \cdots\right)x(0) \qquad \text{.... (7)}$$

Each of the term inside the Brackets is an n × m matrix.

∴ We can write similarly

$$e^{AT} = \left(I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \cdots\right) \qquad \text{.... (8)}$$

∴ Solution $x(t) = e^{AT} \cdot x(0)$ .... (9)

From this equation we can say that the initial state x_0 at $t = 0$ is driven to a state $x(t)$ at time t.

This transition in state is carried out by the matrix exponential e^{AT}

Because of this property, e^{AT} is known as **Transition Matrix.**

Now, for a **Forced or Homogeneous System**, State Equation can be written as

$$\dot{x}(t) = Ax(t) + Bu(t),\ x(0) = X_0 \qquad \text{.... (10)}$$

Rewrite this equation in the form.

$$\dot{x}(t) - Ax(t) = Bu(t)$$

Multiplying both sides by e^{AT}, we have

$$e^{-AT}\left[\dot{x}(t) - Ax(t)\right] = e^{-AT} \cdot Bu(t) = \frac{d}{dt}\left[e^{-AT}x(t)\right]$$

or, $\quad d\left[e^{-AT}x(t)\right] = e^{-AT}Bu(t)dt$ .... (11)

Integrating both sides with respect to τ and having limit from 0 to t, we get

$$e^{-AT}\, x(t)\Big|_0^t = \int_0^t e^{-A\tau}Bu(\tau)d\tau$$

$$\text{or, }\left[e^{-AT}x(t) - e^{-A.0}.x(0)\right] = \int_0^t e^{-A\tau}Bu(\tau)d\tau$$

$$\text{or, } e^{-AT}x(t) = x(0) + \int_0^t e^{-A\tau}Bu(\tau)d\tau$$

Multiplying both sides by e^{+AT}, we have

$$\underbrace{x(t) = e^{AT}x(0)}_{\text{Unforced Solution}} = \underbrace{\int_0^t e^{A(t-\tau)}.B.u(\tau)d\tau}_{\text{Forced Solution}} \quad \text{.... (12)}$$

Properties of State Transition Matrix

(1) $\phi(0) = e^{A0} = 1$

(2) $\phi(t) = e^{At} = [\phi(-t)]^{-1}$ i.e., $\phi^{-1}(t) = \phi(-t)$

(3) $\phi(t_1 + t_2) = e^{A(t_1+t_2)} = e^{At_1}.e^{At_2} = \phi(t_1)\phi(t_2)$

(4) $[\phi(t)]^n = \phi(nt)$

(5) $\phi(t_2 - t_1)\phi(t_1 - t_0) = \phi(t_2 - t_0)$

- The state transition matrix gives idea about the progress of state from x(0), i.e., initial value of x(t).
- It gives the free response of the system.

b) The state model of the following system is given below: **[WBUT 2008]**

$$\begin{bmatrix} \dot{x}_i \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Determine the following:

i) The state transition matrix

ii) Test controllability of the system

iii) Test observability of the system.

Answer:

i) Identify matrix $= I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}_{2\times 2} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}_{2\times 2} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}_{2\times 2}$$

$$\therefore \quad [sI - A]^{-1} = \frac{1}{(s+1)(s+2)}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} = \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$\therefore$ State transition matrix $\phi(t) = L^{-1}\left[(sI - A)^{-1}\right]$

$$= L^{-1}\left[\left(\begin{matrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{matrix}\right)\right] \quad \text{.... (i)}$$

$$= L^{-1}\begin{bmatrix} \frac{A}{s+1}+\frac{B}{s+2} & \frac{C}{s+1}+\frac{D}{s+2} \\ \frac{E}{s+1}+\frac{F}{s+2} & \frac{G}{s+1}+\frac{H}{s+2} \end{bmatrix} \qquad \text{.... (ii)}$$

[Expressing the elements in shaded portions in the expression (i) as a sum of partial fractions]

Evaluating the co-efficients A, B, C, ... In the expression (ii) {Left for the students/readers] we have $\phi(t) = \begin{bmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{-2t} \end{bmatrix}_{2\times 2}$.

ii) Test controllability of the system

The controllability matrix $= Q_c = [B : AB]$

where $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$.

$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix}$ The determinant of the matrix $= -1 \neq 0$, so the system is controllable.

iii) Test observability of the system

The observability matrix is given by $Q_o = \left[C^T : A^T C^T\right]$

where $C = [1 \quad 0]$ So, $C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ So, $A^T = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

So, $A^T C^T = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad Q_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The determinant of the matrix $= 1 \neq 0$, so the system is observable.

5. a) Obtain the transfer function for the system which is represented in state space representation as follows: **[WBUT 2009]**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u. \quad Y = [0 \quad 1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Answer:

We have $T(s) = C(sI - A)^{-1}B + D$

Given $D = 0$.

Therefore $T(s) = C(sI - A)^{-1}B$

$$A=\begin{bmatrix}-2 & 1 & 0\\ 0 & -3 & 1\\ -3 & -4 & -5\end{bmatrix}$$

$$sI-A=s\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}-\begin{bmatrix}-2 & 1 & 0\\ 0 & -3 & 1\\ -3 & -4 & -5\end{bmatrix}=\begin{bmatrix}s+2 & -1 & 0\\ 0 & s+3 & -1\\ 3 & 4 & s+5\end{bmatrix}$$

$$(sI-A)^{-1}=\frac{\text{adj}(sI-A)}{\det(sI-A)}$$

$$\text{adj}(sI-A)=\begin{bmatrix}s^2+8s+19 & s+5 & 1\\ -3 & s^2+7s+10 & s+2\\ -3(s+3) & -4s-11 & s^2+5s+6\end{bmatrix}$$

$$\det(sI-A)=(s+2)\left[(s+3)(s+5)+4\right]+1[3]$$

$$=(s+2)(s^2+8s+19)+3\ =s^3+10s^2+35s+41$$

$$\therefore\quad (sI-A)^{-1}=\frac{1}{s^3+10s^2+35s+41}\begin{bmatrix}s^2+8s+19 & s+5 & 1\\ -3 & s^2+7s+10 & s+2\\ -3(s+3) & -4s-11 & s^2+5s+6\end{bmatrix}$$

$$T(s)=C(sI-A)^{-1}B$$

$$=\frac{1}{s^3+10s^2+35s+41}\begin{bmatrix}-3\\ s^2+7s+10\\ s+2\end{bmatrix}^T\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}.$$

b) A linear time invariant system is characterised by the state variable model. Comment on the controllability and observability of the system: **[WBUT 2009]**

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u.$$

Answer:

$$A=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\qquad B=\begin{bmatrix}0\\ 1\end{bmatrix}$$

The controllability matrix $Q_c=[B:AB]$

where $AB=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}0\\ -2\end{bmatrix}$

$$\therefore \quad Q_c = \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix}$$

Determinant of $Q_c = 0$

So, the system is not controllable.

For evaluation of observability, adequate date is not available.

6. a) Write down the advantages and disadvantages of state space techniques. **[WBUT 2010, 2012, 2013, 2015]**

Answer:

Advantages of State Space Approach

1) This approach is applicable for both linear and non-linear systems.
2) This approach is applicable for both time invariant and variant systems.
3) This approach is applicable for both SISO and MIMO systems.
4) Internal state of the system parameters are taken care, thus improving the accuracy of the control system.

Disadvantages of State Space Approach

1) Initial conditions are to be known.
2) Techniques are complex.
3) Complicated and tedious computations are needed.

b) Realize $H(s)$ in cascade form: **[WBUT 2010]**

$$H(s) = \frac{s(s+2)}{(s+1)(s+3)(s+4)}.$$

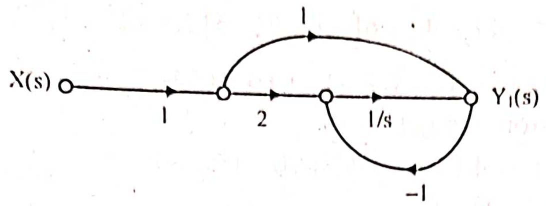
Answer:

Rearranging $H(s)$ as

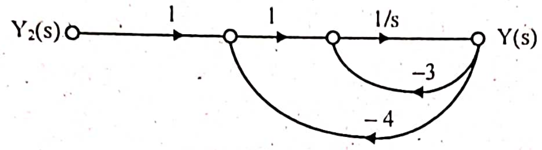
$$H(s) = \frac{s^2\left(1+\frac{2}{s}\right)}{s^3\left(1+\frac{1}{s}\right)\left(1+\frac{3}{s}\right)\left(1+\frac{4}{s}\right)} = \frac{\left(1+\frac{2}{s}\right)}{\left(1+\frac{1}{s}\right)} \cdot \frac{1}{s\left(1+\frac{3}{s}\right)\left(1+\frac{4}{s}\right)}$$

$$= \frac{\left(1+\frac{2}{s}\right)}{\left(1+\frac{1}{s}\right)} \cdot \frac{s^{-1}}{\left(1+\frac{3}{s}\right)\left(1+\frac{4}{s}\right)} = G_1(s) \cdot G_2(s)$$

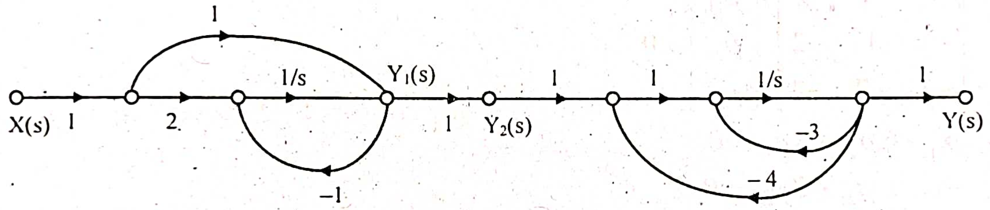
S.F.G. for $G_1(s) = \dfrac{\left(1+\frac{2}{s}\right)}{\left(1+\frac{1}{s}\right)}$ is drawn as shown below



S.F.G. for $G_2(s)=\dfrac{s^{-1}}{\left(1+\dfrac{3}{s}\right)\left(1+\dfrac{4}{s}\right)}$ is drawn as shown below.



So the cascaded form of $H(s)$ is drawn by inserting a unity transmittance between each S.F.G.



c) Obtain the eigenvalues and eigenvectors for a system described by

$$\dot{X}=\begin{bmatrix}0 & 6 & -5\\ 1 & 0 & 2\\ 3 & 2 & 4\end{bmatrix}X+\begin{bmatrix}1\\0\\1\end{bmatrix}U \text{ and } Y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}X.$$ **[WBUT 2010, 2012, 2014]**

Answer:

Let us consider, $A=\begin{bmatrix}0 & 6 & -5\\ 1 & 0 & 2\\ 3 & 2 & 4\end{bmatrix}$

We find eigen values first $\lambda I-A=\begin{bmatrix}\lambda & -6 & 5\\ -1 & \lambda & -2\\ -3 & -2 & \lambda-4\end{bmatrix}$

Solving, we get

$$=\lambda\begin{bmatrix}\lambda & -2\\ -2 & \lambda-4\end{bmatrix}-(-6)\begin{bmatrix}-1 & -2\\ -3 & \lambda-4\end{bmatrix}+5\begin{bmatrix}-1 & \lambda\\ -3 & -2\end{bmatrix}$$

$$=\lambda\left[\lambda(\lambda-4)-4\right]+6\left[-\lambda+4-6\right]+5\left[2+3\lambda\right]$$

$$= \lambda\left[\lambda^2 - 4\lambda - 4\right] + 6\left[-\lambda - 2\right] + 5\left[2 + 3\lambda\right]$$
$$= \lambda^3 - 4\lambda^2 - 4\lambda - 6\lambda - 12 + 10 + 15\lambda$$

From the above expression, we get

$$\lambda^3 - 4\lambda^2 - 4\lambda - 6\lambda - 12 + 10 + 15\lambda = 0$$
$$(\lambda - 1)(\lambda - 2)(\lambda - 1) = 0$$

or, $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 1$

Thus eigen value at 1 is repeated twice.

Let eigen vector for $\lambda_1 = 1$ be p_1

$$p_1 = \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \end{bmatrix}$$

$\therefore$ $[\lambda_1 I - A]\ [P_1] = 0$

Putting $\lambda = 1$ in $[\lambda . 1 - A] . [p_1]$

$$\begin{bmatrix} 1 & -6 & 5 \\ -1 & 1 & -2 \\ -3 & -2 & -3 \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \end{bmatrix} = 0$$

$$p_{11} - 6p_{21} + 5p_{31} = 0$$
$$-p_{11} + p_{21} - 2p_{31} = 0$$
$$-3p_{11} - 2p_{21} - 3p_{31} = 0$$

We Take $p_{11} = 1$.

i.e., $-1 - 6p_{21} + 5p_{31} = 0$ .... (1)

$-1 + p_{21} - 2p_{31} = 0$ .... (2)

$-3 - 2p_{21} - 3p_{31} = 0$

From the equation (2), we get,

$$p_2 = 2p_3 + 1$$

substituting the value of p_{21}, we get,

$$p_{31} = 1 \text{ and } p_{21} = 3$$

$$\therefore \quad [p_1] = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Now let $[p_2] = \begin{bmatrix} p_{21} \\ p_{22} \\ p_{32} \end{bmatrix}$

Again, it gives

$$\begin{bmatrix} 2 & -6 & 5 \\ -1 & 2 & -2 \\ -3 & -2 & -2 \end{bmatrix} \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \end{bmatrix} = 0$$

$$2p_{12} - 6p_{22} + 5p_{32} = 0$$

$$-p_{12} + 2p_{22} - 2p_{32} = 0 \quad \text{.... (3)}$$

$$-3p_{12} - 2p_{22} - 2p_{32} = 0 \quad \text{.... (4)}$$

Solving equations (3) and (4), we get,

$$p_{12} = p_{32}$$

$\therefore$ $p_{12} = 1$ and $p_{32} = 1$

Putting the value of p_{12} and p_{32}

$\therefore$ $p_{22} = 1.5$

$$\therefore \quad [p_2] = \begin{bmatrix} 1 \\ 1.5 \\ 1 \end{bmatrix}$$

Again taking the value of $\lambda_3 = 1$ we get, $[p_3] = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$.

The eigen values are 1, 2, 1

The eigen vectors are $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1.5 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$.

7. a) Determine the transfer matrix for a system whose A, B,C matrices are

$$A = \begin{bmatrix} 1 & -2 \\ 4 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad C = [1 \ 1]$$

b) Is the system stable?

c) Is the system controllable? Assume D = 0. **[WBUT 2011]**

Answer:

$$A = \begin{bmatrix} 1 & -2 \\ 4 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad C = [1 \quad 1] \quad D = 0$$

$$G(s) = C(sT - A)^{-1} B$$

$$C = [1 \quad 1] \qquad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad [sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 4 & -5 \end{bmatrix}$$

$$[sI-A]^{-1}=\frac{\begin{bmatrix} s+5 & -2 \\ 4 & s-1 \end{bmatrix}}{(s-1)(s+5)-(-8)}=\begin{bmatrix} s-1 & 2 \\ -4 & s+5 \end{bmatrix}$$

$$=\frac{\begin{bmatrix} s+5 & -2 \\ 4 & s-1 \end{bmatrix}}{s^2-s+5s-5+8}=\frac{\begin{bmatrix} s+5 & -2 \\ 4 & s-1 \end{bmatrix}}{s^2+4s+3}=\frac{\begin{bmatrix} s+5 & -2 \\ 4 & s-1 \end{bmatrix}}{(s+1)(s+8)}$$

$$=\begin{bmatrix} \frac{s+5}{(s+1)(s+3)} & \frac{-2}{(s+1)(s+3)} \\ \frac{4}{(s+1)(s+3)} & \frac{s-1}{(s+1)(s+3)} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{2}{s+1}-\frac{1}{s+3} & \frac{1}{s+3}-\frac{1}{s+1} \\ 2\left\{\frac{1}{(s+1)}-\frac{1}{(s+3)}\right\} & \frac{2}{(s+3)}-\frac{1}{s+1} \end{bmatrix}$$

Taking inverse Laplace Transform we get

$$\phi(T)=L^{-1}\begin{bmatrix} L^{-1}\left(\frac{2}{s+1}-\frac{1}{s+3}\right) & L^{-1}\left(\frac{1}{s+3}-\frac{1}{s+1}\right) \\ L^{-1}\left(\frac{2}{s+1}-\frac{2}{s+3}\right) & L^{-1}\left(\frac{2}{s+3}-\frac{1}{s+1}\right) \end{bmatrix}$$

$$=\begin{bmatrix} 2e^{-t}-e^{-3t} & e^{-3t}-e^{-t} \\ 2e^{-t}-2e^{-3t} & 2e^{-3t}-e^{-t} \end{bmatrix}$$

= State Transfer Matrix

Controllability:

$$B=\begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad AB=\begin{bmatrix} 1 & -2 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}=\begin{bmatrix} 1\times2-2\times1 \\ 2\times4-5\times1 \end{bmatrix}=\begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$[B:AB]=\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}=(6-0)=6\neq0$$

The determinant is not zero, hence the rank is 2.
So, the system is completely controllable.

Observability:

$$A=\begin{bmatrix} 1 & -2 \\ 4 & -5 \end{bmatrix} \qquad B=\begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad C=[1 \quad 1]$$

$$A^T = \begin{bmatrix} 1 & 4 \\ -2 & -5 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 1 & 4 \\ -2 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+4 \\ -2-5 \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$$

$$\left[C^T : A^T C^T\right] = \begin{bmatrix} 1 & 5 \\ 1 & -7 \end{bmatrix}$$

Now the determinant is

$$[-7-5] = -12 \neq 0$$

Since determinant is not equal to zero hence rank is 2 and the system is completely observable.

The given system is controllable as well as observable. So the given system is stable.

8. Check for controllability and observability of a system having following coefficient matrices. **[WBUT 2013]**

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \text{ and } C^T = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 10 & 5 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The composite matrix for the controllability of the system is given by

$$Q_C = \left[B : AB : A^2 B\right]$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0+0 \\ 0+0+1 \\ 0+0+-6 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}$$

$$A^2 B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 0+1+0 \\ 0+0+-6 \\ 0-11+36 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 25 \end{bmatrix}$$

$$\therefore \quad Q_C = \left[\begin{pmatrix}0\\0\\1\end{pmatrix} : \begin{pmatrix}0\\1\\-6\end{pmatrix} : \begin{pmatrix}1\\-6\\25\end{pmatrix}\right] = \begin{bmatrix}0 & 0 & 1\\0 & 1 & -6\\1 & -6 & 25\end{bmatrix}$$

$$= 0\begin{vmatrix}1 & -6\\-6 & 25\end{vmatrix} - 0\begin{vmatrix}0 & -6\\1 & 25\end{vmatrix} + 1\begin{vmatrix}0 & 1\\1 & -6\end{vmatrix}$$

$$= 0 + 0 + 1 = 1 \neq 0$$

Hence rank is 3.

So the system is completely controllable.

The composite matrix for the observability of the system is given by

$$Q_0 = \left[C^T : A^T C^T : A^{2T} C^T\right]$$

$$C^T = \begin{bmatrix}10 & 5 & 1\end{bmatrix}^T = \begin{bmatrix}10\\5\\1\end{bmatrix}$$

$$A^T C^T = \begin{bmatrix}0 & 0 & -6\\1 & 0 & -11\\0 & 1 & -6\end{bmatrix}\begin{bmatrix}10\\5\\1\end{bmatrix} = \begin{bmatrix}0+0+-6\\10+0-11\\0+5-6\end{bmatrix} = \begin{bmatrix}-6\\-1\\-1\end{bmatrix}$$

$$A^{2T} C^T = \begin{bmatrix}0 & 0 & -6\\1 & 0 & -11\\0 & 1 & -6\end{bmatrix}\begin{bmatrix}-6\\-1\\-1\end{bmatrix} = \begin{bmatrix}0+0+6\\-6+0+11\\0-1+6\end{bmatrix} = \begin{bmatrix}6\\5\\5\end{bmatrix}$$

$$Q_0 = \left[\begin{pmatrix}10\\5\\1\end{pmatrix} : \begin{pmatrix}-6\\-1\\-1\end{pmatrix} : \begin{pmatrix}6\\5\\5\end{pmatrix}\right] = \begin{bmatrix}10 & -6 & 6\\5 & -1 & 5\\1 & -1 & 5\end{bmatrix}$$

$$= 10\begin{vmatrix}-1 & 5\\-1 & 5\end{vmatrix} + 6\begin{vmatrix}5 & 5\\1 & 5\end{vmatrix} + 6\begin{vmatrix}5 & -1\\1 & -1\end{vmatrix}$$

$$= 10\left|-5+5\right| + 6\left|25-5\right| + 6\left|-5+1\right|$$

$$= 0 + 120 - 24 = 96 \neq 0$$

Hence the 3rd order system has rank 3, hence the rank of $Q_0 = 3$, so the system is completely observavable.

9. Determine the transfer matrix for a system whose A, B, C matrices are

$$A = \begin{bmatrix}1 & -2\\4 & -5\end{bmatrix},\; B = \begin{bmatrix}1\\1\end{bmatrix} \text{ and } C = \begin{bmatrix}1 & 0\end{bmatrix}.$$

[WBUT 2013]

Answer:

$$A=\begin{bmatrix}1 & -2\\ 4 & -5\end{bmatrix}$$

$$\phi(t)=L^{-1}[sI-A]^{-1}$$

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$sI=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}$$

$$[sI-A]=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}1 & -2\\ 4 & -5\end{bmatrix}=\begin{bmatrix}s-1 & 2\\ -4 & s+5\end{bmatrix}$$

Now find $[sI-A]^{-1}$

$$\text{Cofactor}=\begin{bmatrix}s-1 & 2\\ -4 & s+5\end{bmatrix}$$

$$\text{Transpose of cofactor}=\begin{bmatrix}s+5 & -2\\ 4 & s-1\end{bmatrix}$$

$$\Delta=(s+5)(s-1)-(-8)$$
$$=s^2+5s-s-5+8=s^2+4s+3=(s+3)(s+1)$$

$$[sI-A]^{-1}=\frac{\text{Transpose of co-factor}}{\Delta}$$
$$=\frac{1}{(s+3)(s+1)}\begin{bmatrix}s+5 & -2\\ 4 & s-1\end{bmatrix}$$
$$=\begin{bmatrix}\dfrac{s+5}{(s+3)(s+1)} & \dfrac{-2}{(s+3)(s+1)}\\ \dfrac{4}{(s+3)(s+1)} & \dfrac{s-1}{(s+3)(s+1)}\end{bmatrix}=\begin{bmatrix}\dfrac{2}{s+1}-\dfrac{1}{s+3} & \dfrac{1}{s+3}-\dfrac{1}{s+1}\\ \dfrac{2}{s+1}-\dfrac{2}{s+3} & \dfrac{2}{s+3}-\dfrac{1}{s+1}\end{bmatrix}$$

For $L^{-1}[sI-A]^{-1}$, we take individual inverse Laplace and get

$$\therefore\qquad \phi(t)=L^{-1}[sI-A]^{-1}=\begin{bmatrix}2e^{-t}-3^{-3t} & e^{-3t}-e^{-t}\\ 2e^{-t}-2e^{-3t} & 2e^{-3t}-e^{-t}\end{bmatrix}$$

10. A linear time-invariant system is characterized by the non-homogeneous state equation, $\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix}=\begin{bmatrix}-1 & 1\\ 0 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}U$ **and** $\begin{bmatrix}x_1 & (0)\\ x_2 & (0)\end{bmatrix}=\begin{bmatrix}-1\\ 0\end{bmatrix}$**. Determine the non-homogeneous solution.** **[WBUT 2014]**

Answer:

To evaluate state transition matrix $\phi(t)$

$$\phi(t) = L^{-1}[sI - A]^{-1}$$

where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = unit matrix

$$sI = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

$$\therefore \quad [sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} s+1 & 0-1 \\ 0 & s-2 \end{bmatrix} = \begin{bmatrix} s+1 & -1 \\ 0 & s-2 \end{bmatrix}$$

Cofactor of $[sI - A] = \begin{bmatrix} s+1 & -1 \\ 0 & s-2 \end{bmatrix}$

Transpose of cofactor $= \begin{bmatrix} s-2 & 1 \\ 0 & s+1 \end{bmatrix}$

Determinant of the transpose of cofactor $= \Delta = (s-2)(s+1) - 0$

$$= s^2 - 2s + s - 2 = s^2 - s - 2$$

Inverse of $[sI - A] = [sI - A]^{-1} = \dfrac{\text{Transpose of co-factor}}{\Delta}$

$$= \frac{\begin{bmatrix} s-2 & 1 \\ 0 & s+1 \end{bmatrix}}{(s-2)\ (s+1)} = \begin{bmatrix} \dfrac{s-2}{(s-2)(s+1)} & \dfrac{1}{(s-2)(s+1)} \\ 0 & \dfrac{s+1}{(s-2)(s+1)} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{s+1} & \dfrac{1}{3}\left(\dfrac{1}{s-2} - \dfrac{1}{s+1}\right) \\ 0 & \dfrac{1}{s-2} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{s+1} & \dfrac{1}{3}\left(\dfrac{1}{s-2} - \dfrac{1}{s+1}\right) \\ 0 & \dfrac{1}{s-2} \end{bmatrix}$$

For $L^{-1}[sI - A]^{-1}$ we take individual inverse Laplace and get

$$\phi(t) = L^{-1}[sI - A]^{-1} = \begin{bmatrix} e^{-t} & \frac{1}{3}\left(e^{2t} - e^{-t}\right) \\ 0 & e^{2t} \end{bmatrix}$$

Evaluation of the solution $x(t)$

$$x(t) = \phi(t)x(0) + \int_0^t \phi(t-\tau)Bud\tau$$

$$=\begin{bmatrix} e^{-t} & \frac{1}{3}\left(e^{2t}-e^{-t}\right) \\ 0 & e^{2t}\end{bmatrix}\begin{bmatrix}-1\\0\end{bmatrix}+\int_0^t\begin{bmatrix} e^{-(t-\tau)} & \frac{1}{3}\left(e^{2(t-\tau)}-e^{(t-\tau)}\right) \\ 0 & e^{2(t-\tau)}\end{bmatrix}d\tau\begin{bmatrix}-1\\0\end{bmatrix}u$$

$$=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\int_0^t\begin{bmatrix} e^{-(t-\tau)} & \frac{1}{3}\left(e^{2(t-\tau)}-e^{-(t-\tau)}\right) \\ 0 & e^{2(t-\tau)}\end{bmatrix}d\tau\begin{bmatrix}-1\\0\end{bmatrix} \quad [u=\text{unit step}]$$

$$=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\begin{bmatrix} e^{-t}\int_0^t e^{\tau}d\tau & \frac{1}{3}\left(e^{2t}\int_0^t e^{-2\tau}d\tau-e^{-t}\int_0^t e^{\tau}d\tau\right) \\ 0 & e^{2t}\int_0^t e^{-2\tau}d\tau\end{bmatrix}\begin{bmatrix}-1\\0\end{bmatrix}$$

$$=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\begin{bmatrix} e^{-t}e^{\tau}\Big|_0^t & \frac{1}{3}\left(e^{2t}\frac{e^{-2x}}{2}\Big|_0^t-e^{-t}e^{\tau}\Big|_0^t\right) \\ 0 & e^{2t}\frac{e^{-2\tau}}{2}\Big|_0^t\end{bmatrix}$$

$$=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\begin{bmatrix} e^{-t}\left(e^{t}-1\right) & \frac{1}{3}e^{2t}\left(\frac{e^{-2t}}{2}-\frac{1}{2}\right)-e^{-t}\left(e^{t}-1\right) \\ 0 & e^{2t}\left(\frac{e^{-2t}}{2}-\frac{1}{2}\right)\end{bmatrix}\begin{bmatrix}-1\\0\end{bmatrix}$$

$$=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\begin{bmatrix} e^{-t}\left(e^{t}-1\right) & \frac{1}{3}\left[e^{2t}\left(\frac{e^{-2t}}{2}-\frac{1}{2}\right)-\left(e^{-t}\cdot e^{t}-e^{-t}\right)\right] \\ 0 & e^{2t}\left(\frac{e^{-2t}}{2}-\frac{1}{2}\right)\end{bmatrix}\begin{bmatrix}-1\\0\end{bmatrix}$$

$$=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\begin{bmatrix} e^{-t}\cdot e^{t}-e^{-t} & \left(\frac{1}{3}\left[\frac{e^{2t}\cdot e^{-2t}}{2}-\frac{\frac{2t}{e}}{2}-e^{-t}\cdot e^{t}+e^{-t}\right]\right) \\ 0 & \frac{e^{2t}\cdot e^{-2t}}{2}-\frac{e^{2t}}{2}\end{bmatrix}\begin{bmatrix}-1\\0\end{bmatrix}$$

$$=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\begin{bmatrix} 1-e^{-t} & \frac{1}{3}\left[\frac{1}{2}-\frac{e^{2t}}{2}-1+e^{-t}\right] \\ 0 & \frac{1}{2}-\frac{e^{2t}}{2}\end{bmatrix}\begin{bmatrix}-1\\0\end{bmatrix}$$

$$=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\begin{bmatrix}1-e^{-t} & \frac{1}{3}\left[-\frac{1}{2}-\frac{e^{-2t}}{2}+e^{-t}\right]\\0 & \frac{1}{2}-\frac{e^{2t}}{2}\end{bmatrix}\begin{bmatrix}-1\\0\end{bmatrix}$$

$$=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\begin{bmatrix}-1+e^{-t}-0+e^{-t}\\0\end{bmatrix}=\begin{bmatrix}-e^{-t}\\0\end{bmatrix}+\begin{bmatrix}-1+e^{-t}\\0\end{bmatrix}$$

$$=\begin{bmatrix}-e^{-t}-1+e^{-t}\\0\end{bmatrix}=\begin{bmatrix}-1\\0\end{bmatrix}\quad \textbf{(Ans.)}$$

11. a) Compute state transition matrix $A=\begin{bmatrix}0 & 1\\-3 & -4\end{bmatrix}$. **[WBUT 2016]**

b) A system is characterized by transfer function $G(s)=\frac{Y(s)}{U(s)}=\frac{2}{s^3+6s^2+11s+6}$.
Find the state equation & output equation in matrix form. Test the controllability & observability of the system.

Answer:

a) $A=\begin{bmatrix}0 & 1\\-3 & -4\end{bmatrix}$

$\phi(t)=L^{-1}[sI-A]^{-1}$ where $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$sI=\begin{bmatrix}s & 0\\0 & s\end{bmatrix}$

$\therefore\quad [sI-A]=\begin{bmatrix}s & 0\\0 & s\end{bmatrix}-\begin{bmatrix}0 & 1\\-3 & -4\end{bmatrix}=\begin{bmatrix}s & -1\\3 & s+4\end{bmatrix}$

Now we find $[SI-A]^{-1}$

$\text{Cofactor}=\begin{bmatrix}s+4 & -3\\1 & s\end{bmatrix}$

$\text{Transpose of cofactor}=\begin{bmatrix}s+4 & -3\\1 & s\end{bmatrix}$

$$\Delta=s(s+4)-(-3)=s^2+4s+3=(s+3)(s+1)$$

$$\therefore [SI-A]^{-1} = \frac{\text{Transpose of factor}}{\Delta} = \frac{1}{(s+3)(s+1)}\begin{bmatrix} s+4 & -3 \\ 1 & s \end{bmatrix} = \begin{bmatrix} \frac{s+4}{(s+3)(s+1)} & \frac{-3}{(s+3)(s+1)} \\ \frac{1}{(s+3)(s+1)} & \frac{s}{(s+3)(s+1)} \end{bmatrix}$$

$$L^{-1}\frac{s+4}{(s+3)(s+1)} = \frac{3}{s+1} - \frac{1}{s+3} = \frac{1}{2}\left(3e^{-t} - e^{-3t}\right)$$

$$L^{-1}\frac{-3}{(s+3)(s+1)} = \left[\frac{1}{s+3} - \frac{1}{s+1}\right] = \frac{3}{2}e^{-3t} - \frac{3}{2}e^{-t}$$

$$\phi(t) = L^{-1}[sI-A]^{-1} = \begin{bmatrix} \frac{3}{2}\left(e^{-t} - e^{-3t}\right) & \frac{3}{2}\left(e^{-3t} - e^{-t}\right) \\ \frac{1}{2}\left(e^{-t} - e^{-3t}\right) & \frac{3}{2}e^{-3t} - \frac{1}{2}e^{t} \end{bmatrix},$$

b) The given transfer function may be written as $(s^3 + 6s^2 + 11s + 6)Y(s) = U(s)$

In time-domain the above expression may be written as

$$\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y = u(t)$$

$$\therefore \quad \dot{X} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}U$$

$$\therefore \quad A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix}; \qquad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Now, $$AB = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -6 \end{pmatrix}.$$

12. A linear time invariant system is characterized by the state variable model. Comment controllability and observability of the system.

$$\frac{d_{x_1}(t)}{d_t} = -x_1(t), \quad \frac{dx_2(t)}{dt} = -2x_2(t) + u(t),$$

$$Y(t) = x_1(t) + 2x_2(t)$$

[WBUT 2017]

Answer:

Controllability: ***Refer to Question No. 5(b) of Long Answer Type Questions.***

Observability:

The observability matrix: $Q_c = \left[C^T : A^T C^T\right]$

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$\therefore A^T . C^T = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\therefore Q_c = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Determinant of $Q_c = 0$

Hence, the system is not observable.

13. Check the controllability and observability of the given equation:

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} u;\ y = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix} x$$ **[WBUT 2018]**

Answer:

We know the normal state variable equation

$$\dot{X}(t) = Ax(t) + Bu$$

$$Y(t) = Cx(t)$$

So, comparing with the given equation

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix}$$

The composite matrix for the controllability of the system is given by

$$Q_c = \left[B : AB : A^2 B\right]$$

$$AB = \begin{bmatrix} 0+0+0 \\ 0+0+(-1) \\ -2+0+3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$A^2 B = \begin{bmatrix} 0+1+0 \\ 0+0+1 \\ 0-4-3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -7 \end{bmatrix}$$

$$\therefore \quad Q_c = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & -7 \end{bmatrix}$$

$|Q_c| \neq 0$

$\therefore$ The system is controllable.

The composite matrix for the observability of the system is given by,

$$Q_0 = \left[C^T : A^T C^T : A^{T^2} C^T\right]$$

$$A^T C^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix}^T \begin{bmatrix} 0 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 0 & -4 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0+0+2 \\ 0+0+4 \\ 0+1+3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$$A^{T^2} C^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix}^T \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 0 & -4 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ -1 \end{bmatrix}$$

$$\therefore \quad Q_0 = \begin{bmatrix} 0 & 2 & -2 \\ 1 & 4 & 3 \\ -1 & 4 & -1 \end{bmatrix}$$

$A|Q_0| \neq 0$, so the system is completely observable.

14. Write short notes on the following:
a) Kalman's Test **[WBUT 2014, 2015]**
b) Eigen value and Eigen vectors **[WBUT 2014, 2015]**
c) S.T.M. (State Transition Matrix) **[WBUT 2014]**

Answer:

a) Kalman's Test for Controllability

Consider n^{th} order multiple input linear time invariant system represented by its state equation as,

$$\dot{X} = AX(t) + BU(t) \qquad \text{.... (1)}$$

where A has order $n \times n$ matrix.

and $U(t)$ is $m \times 1$ vector i.e., there are m inputs.

$X(t)$ is $n \times 1$ state vector.

The necessary and sufficient condition for the system to be completely state controllable is that the rank of the composite matrix Q_c is 'n'.

The composite matrix Q_c is given by,

$$\boxed{Q_c = \left[B : AB : A^2B : \ldots\ A^{n-1}B\right]} \qquad \text{.... (2)}$$

In this composite matrix Q_c, B, AB, A^2B... are the various columns.

Kalman's Test for Observability

Consider n^{th} order multiple input multiple output linear time invariant system, represented by its state equation as,

$$\dot{X} = AX(t) + BU(t) \qquad \text{.... (3)}$$

and $\quad Y(t) = CX(t)$

where $\quad Y(t) = p \times 1$ output vector

and $\quad C = 1 \times n$ matrix

The system is completely observable if an only is the rank of the composite matrix Q_o is 'n'.

The composite matrix Q_o is given by,

$$\boxed{Q_o = \left[C^T : A^T C^T : \text{........} \left(A^T\right)^{n-1} C^T\right]}$$

where $\quad C^T$ = Transpose of matrix C

and $\quad A^T$ = Transpose of matrix A

Thus if, rank of $Q_o = n$, then system is completely observable.

b) Eigen value and Eigen vectors:

We know that

$$\frac{Y(s)}{Y(s)} = C(sI - A)^{-1} + D = C\frac{\text{adjoint}(sI - A)}{|(sI - A)|} + D \qquad \text{.... (1)}$$

= Transfer function of the system under study.

By setting the denominator transfer function to zero, we have characteristic equation as

$$|sI - A| = 0 \qquad \text{.... (2)}$$

But, $|sI - A|$ is basically an n^{th} order polynomial in s with roots $\lambda_1, \lambda_2, \cdots\cdots \lambda_n$, so equation (2) may be represented as

$$|sI - A| = s^n + a_{n-1}s^{n-1} + \cdots\cdots + a_1 s + a_0$$

$$= (s - \lambda_1)(s - \lambda_2)\cdots\cdots(s - \lambda_n) = 0 \qquad \text{.... (3)}$$

where, $\lambda_1, \lambda_2, \cdots\cdots \lambda_n$ are the eigen values of ***matrix A (n×n)***. If λ_i is the i^{th} root, then we have $|\lambda_i I - A| = 0$ .... (4)

where, $i = 1$ to n

c) S.T.M. (State Transition Matrix):

Let us consider an unforced system,

i.e., $\quad \dot{U} = 0$

∴ State Equation of this system may be written as

$$X = AX + BU = AX + B.0 \qquad \text{.... (1)}$$

Taking Laplace Transformation of equation (1)

$$sX(s) - X(0) = AX(s)$$

where x(t) is the state variable and $X(s)$ is the Laplace Transform of x. $x(0)$ = initial condition of state vector X.

$$sX(s) - AX(s) = X(0)$$

or, $$X(s) = [sI - A] = X(0)$$

or, $$X(s) = [sI - A]^{-1} X(0) \quad \text{.... (2)}$$

Taking inverse Laplace transform, of equation (2) we have,

$$x(t) = L^{-1}\left[(sI - A)^{-1}\right] X(0) \quad \text{.... (3)}$$

$$\phi(t) = L^{-1}\left[(sI - A)^{-1}\right]$$

$$= e^{AT} = STM$$

Remember: $\boxed{\text{STM} = e^{AT} = L^{-1}\left[(sI - A)^{-1}\right]}$

Here, x(t) is the unforced or non-homogeneous response of the system.
Let us have a first order scalar differential equation

$$\frac{dx}{dt} = ax, \; x(0) = x_0 \quad \text{.... (4)}$$

or, $$\frac{dx}{x} = adt$$

or, $$\log_e x = at + c$$

At $$t = 0, \; x = x_0$$

$\therefore$ $$\log_e x_0 = c$$

$\therefore$ Solution of the above equation

$$x(t) = e^{at} x_0$$

$$= \left(1 + at + \frac{1}{2!}a^2t^2 + \frac{1}{3!}a^3t^3 + \cdots\right)x_0 \quad \text{.... (5)}$$

With this analogy with the scalar case, let us assume a solution

$$x(t) = a_0 + a_1 t + a_2 t_2 + \cdots + a_k t^k + \cdots$$

where, a_i are vector co-efficients

$\therefore$ $$\dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2 + \cdots$$

$$= A(a_0 + a_1 t + a_2 t^2 + \cdots +) \quad \text{.... (6)}$$

where,

$$a_1 = Aa_0$$

$$a_2 = \frac{1}{2}Aa_1 = \frac{1}{2!}Aa^2{}_0$$

$$a_3 = a_2 t^2 = \frac{1}{3}\cdot\frac{1}{2!}A^3 a_0$$

$$= \frac{1}{3!}A^3 a_0$$

. . . .

$$a_k = \frac{1}{k!}A^k a_0$$

$\therefore$ Solution of $x(t)$ can be written as

$$x(t) = \left(I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \cdots \right)x(0) \quad \text{.... (7)}$$

Each of the term inside the Brackets is an n × m matrix.

$\therefore$ We can write similarly

$$e^{AT} = \left(I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \cdots \right) \quad \text{.... (8)}$$

$\therefore$ Solution $x(t) = e^{AT} \cdot x(0)$ .... (9)

From this equation we can say that the initial state x_0 at $t = 0$ is driven to a state $x(t)$ at time t.

This transition in state is carried out by the matrix exponential e^{AT}

Because of this property, e^{AT} is known as **Transition Matrix.**

Now, for a **Forced or Homogeneous System,** State Equation can be written as

$$\dot{x}(t) = Ax(t) + Bu(t), \; x(0) = X_0 \quad \text{.... (10)}$$

Rewrite this equation in the form

$$\dot{x}(t) - Ax(t) = Bu(t)$$

Multiplying both sides by e^{AT}, we have

$$e^{-AT}\left[\dot{x}(t) - Ax(t)\right] = e^{-AT} \cdot Bu(t) = \frac{d}{dt}\left[e^{-AT}x(t)\right]$$

or, $d\left[e^{-AT}x(t)\right] = e^{-AT}Bu(t)dt$ .... (11)

Integrating both sides with respect to τ and having limit from 0 to t, we get

$$e^{-AT}x(t)\Big|_0^t = \int_0^t e^{-A\tau}Bu(\tau)d\tau$$

$$\text{or,} \left[e^{-AT} x(t) - e^{-A.0}.x(0) \right] = \int_0^t e^{-A\tau} Bu(\tau) d\tau$$

$$\text{or,} \, e^{-AT} x(t) = x(0) + \int_0^t e^{-A\tau} Bu(\tau) d\tau$$

Multiplying both sides by e^{+AT}, we have

$$x(t) = \underbrace{e^{AT} x(0)}_{\text{Unforced Solution}} = \underbrace{\int_0^t e^{A(t-\tau)}.B.u(\tau) d\tau}_{\text{Forced Solution}} \quad \text{.... (12)}$$

Properties of State Transition Matrix

(1) $\phi(0) = e^{A0} = 1$

(2) $\phi(t) = e^{At} = [\phi(-t)]^{-1}$ i.e., $\phi^{-1}(t) = \phi(-t)$

(3) $\phi(t_1 + t_2) = e^{A(t_1+t_2)} = e^{At_1}.e^{At_2} = \phi(t_1)\phi(t_2)$

(4) $[\phi(t)]^n = \phi(nt)$

(5) $\phi(t_2 - t_1)\phi(t_1 - t_0) = \phi(t_2 - t_0)$

- The state transition matrix gives idea about the progress of state from x(0), i.e., initial value of x(t).
- It gives the free response of the system.

CONTROL ACTION

Multiple Choice Type Questions

1. By the use of PD control to the second order system, the rise time [WBUT 2007]
   a) decreases b) increases c) remains same d) none of these

Answer: (a)

2. In z plane, the unit circle corresponds to [WBUT 2008]
   a) Imaginary axis of s-plane
   b) Positive real axis of s-plane
   c) Negative real axis of s-plane
   d) Origin of s-plane

Answer: (a)

3. The transfer function of an integral compensator is given by [WBUT 2009]
   a) $\frac{1}{s}$ b) $\frac{1}{s^2}$ c) $\frac{k}{s}$ d) ks

Answer: (a)

4. The TF of a network $\frac{1+0.5s}{2+s}$ is known as [WBUT 2011]
   a) High pass system
   b) Lead network
   c) Lag network
   d) Proportional controller

Answer: (d)

5. A PD controller is inserted in a system to [WBUT 2013]
   a) fasten the response
   b) decrease steady state error
   c) slow down the response
   d) increase steady state error

Answer: (b)

6. Derivative error control [WBUT 2016]
   a) increases the overshoot
   b) decreases the overshoot
   c) increases the steady state error
   d) decreases the steady state error

Answer: (b)

7. A potentiometer converts linear rotational displacement into [WBUT 2018]
   a) Current b) Power c) Torque d) Voltage

Answer: (d)

8. By the use of PID controller to a second order system, the rise time [WBUT 2018]
   a) increases b) decreases c) remains same d) has no effect

Answer: (b)

Short Answer Type Questions

1. Derive the transfer function of a PID controller. What is the advantage of PID control over other control actions? **[WBUT 2013]**

Answer:

1st Part: ***Refer to Question No. 1 of Long Answer type Questions.***

2nd Part:

A PID controller (Proportional plus Integral plus Derivative controller) produces an output signal consisting of three terms — one proportional to error signal, another one proportional to integral of error signal and the third one proportional to derivative of error signal.

The combination of proportional control action, integral control action and derivative control action is called PID control action. The combined action has the advantage of each of the three individual control actions.

The proportional controller stabilizes the gain but produces a steady-state error. The integral controller reduces or eliminates the steady-state error. The derivative controller reduces the rate of change of error.

2. a) Why PI and PID controllers are used in a closed loop control system? **[WBUT 2014]**

Answer:

P controller is mostly used in first order processes with single energy storage to stabilize the unstable process. The main usage of the P controller is to decrease the steady state error of the system. As the proportional gain factor K increases, the steady state error of the system decreases. However, despite the reduction, P control can never manage to eliminate the steady state error of the system. P-I controller is mainly used to eliminate the steady state error resulting from P controller. However, in terms of the speed of the response and overall stability of the system, it has a negative impact. This controller is mostly used in areas where speed of the system is not an issue.

P-I-D controller has the optimum control dynamics including zero steady state error, fast response (short rise time), no oscillations and higher stability. The necessity of using a derivative gain component in addition to the PI controller is to eliminate the overshoot and the oscillations occurring in the output response of the system. One of the main advantages of the P-I-D controller is that it can be used with higher order processes including more than single energy storage.

b) Draw the operational Amplifier realization of P-I, P-D controller and hence find the expression for gain. **[WBUT 2014]**

Answer:
Realization of P + I Controller

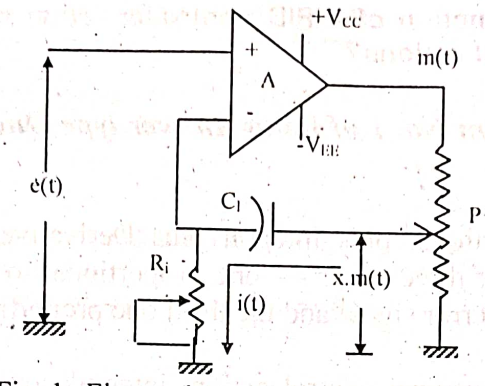


Fig: 1a Electronic circuit of a PI controller

- *The Basic Components*

Components	Functions
Op-Amp(A)	acts as non-inverting amplifier.
Potentiometer (P)	acts as a variable voltage source. The wiper position determines the gain for P-action.
R_I and C_I	These passive circuit elements determine the integral time constant, i.e., $T_I = R_I.C_I$

- *Circuit Explanation*

In figure 1a,
Step 1: The error signal *e(t)* is fed to the Non-inverting Terminal of the OP-AMP.
Step 2: The Output of the Op-Amp is fed to a potentiometer P, which acts as a variable voltage source.
Step 3: The wiper of the potentiometer (P) feds a fraction of the output voltage *m(t)* to $R_I.C_I$ network.
Step 4: The inverting terminal of the OP-AMP is connected to the junction of R_I and C_I.

- *Verification*

Wiper of the potentiometer provides a fraction of the output *m(t)*, i.e., *x.m(t)*, where, $x < 1$.
From the characteristics of Op-Amp, potential of inverting terminal is *e(t)*.
$\therefore$ Using formula for the capacitor C_I

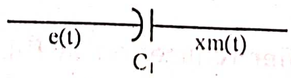


Fig: 1b Potentials at capacitor terminals

$q(t) = C_I[xm(t) - e(t)]$ $\quad [\because (m(t) >> e(t)$

(refer to figure 1b) $\quad \therefore\ xm(t) > e(t)$

as $x \neq 0]$

q(t)⇒ charge accumulated by the capacitor

or, $\frac{dq(t)}{dt} = C_I \frac{d}{dt}[x\,m(t) - e(t)] = i(t)$ ... (1)

This current $i(t)$ will flow through R_I also.

$\therefore \quad \frac{e(t)}{R_I} = i(t)$ ... (2)

∴ From equations (1) and (2)

$$\frac{d}{dt}[x\,m(t) - e(t)] = \frac{e(t)}{R_I C_I}$$

$$\Rightarrow \quad d[x\,m(t) - e(t)] = \frac{e(t)}{R_I C_I} dt$$

$$\Rightarrow \quad x\,m(t) - e(t) = \frac{\int e(t)\,dt}{R_I C_I}$$

$$\Rightarrow \quad m(t) = \frac{e(t)}{x} + \frac{1}{x}\frac{\int e(t)\,dt}{T_I} L \quad \text{.... (3)}$$

where $T_I = R_I C_I$ = integral time sec.

$$\because \; x < 1, \; \frac{1}{x} > 1$$

$$\therefore \text{Say, } \frac{1}{x} = K_C \quad \text{.... (4)}$$

$$\therefore \quad m(t) = K_C e(t) + \frac{K_c}{T_i}\int e(t)\,dt \quad \text{... (5)}$$

This is actually the mathematical expression for P + I action.

Realization of P + D Controller

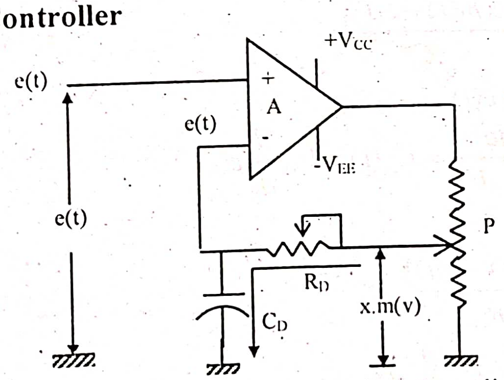


Fig: 2 Electronic circuit of a PD-controller

- ***The Basic Components***

Components	Functions
Op-Amp (A)	acts as non-Inverting amplifier
Potentiometer (P)	acts as a variable voltage source. The wiper position determines the gain for P-action
R_D & C_D	these passive circuit elements determine derivative time constant of the derivative action as $T_D \Rightarrow R_D.C_D$ (sec.)

- ***Circuit Explanation***

In figure 2,
Step 1: The error signal e(t) is fed to the non-inverting terminal of the OP-AMP.
Step 2: The O/P of the OP-AMP is fed to a potentiometer P, which acts as a variable voltage source.
Step 3: The wiper of the potentiometer (P) feeds a fraction of the output voltage m(t) to R_D, C_D network.
Step 4: The inverting terminal of the OP-AMP is connected to the junction of R_D and C_D.

- ***Verification***

The wiper of the potentiometer provides a fraction of the output $m(t)$, i.e., $x.m(t)$, where, $x < 1$.
From the characteristic of Op-Amp potential at inverting terminal is $e(t)$.
So, potential difference across $R_D = \upsilon_{R_D} = [xm(t) - e(t)]$

$\therefore$ Current through, $R_D = I_{R_D}(t)$

$$= \frac{\upsilon_{R_D}}{R_D} \quad [\text{as } m(t) >> e(t),\ xm(t) > e(t)]$$

$$= \frac{[xm(t) - e(t)]}{R_D} \qquad \ldots \text{(i)}$$

Now, Voltage across $C_D = e(t)$

$$\therefore \text{ Current through } C_D = \frac{dq(t)}{dt} = i_{C_D}(t) = C_D \frac{d}{dt} e(t) \qquad \ldots \text{(ii)}$$

$$\because \quad i_{C_D}(t) = i_{R_D}(t)$$

$$\therefore \quad C_D \frac{d}{dt} e(t) = \frac{[xm(t) - e(t)]}{R_D}$$

$$\Rightarrow \quad [xm(t) - e(t)] = R_D C_D \frac{d}{dt} e(t)$$

$$\Rightarrow \quad xm(t) = e(t) + R_D C_D \frac{d}{dt} e(t)$$

$$\Rightarrow \quad m(t) = \frac{e(t)}{x} + \frac{R_D C_D}{x}\frac{d}{dt}e(t)$$

$\because$ x < 1, $\therefore$ $\frac{1}{x} > 1$

Say $\frac{1}{x} = K_C$ & $R_D C_D = T_D$

We have, from the above equation $m(t) = K_C e(t) + K_C T_D \frac{d}{dt} e(t)$

This actually represents the mathematical expression or structure for P + D action.

Long Answer Type Questions

1. Explain the function of a PID controller enumerating the benefits of using a PID controller. [WBUT 2005]

Answer:

The combination of proportional, integral and derivative actions is terms as PID control action and has the advantages of each of the three individual control actions.

Mathematical Structure

$$m(t) = K_c e(t) + \frac{K_c}{T_i}\int e(t)dt + K_c T_D \frac{d}{dt} e(t) \qquad \text{.... (i)}$$

where $m(t) \Rightarrow$ PID Controller's output

$K_C \Rightarrow$ proportional sensitivity

$T_i \Rightarrow$ integral time; sec.

$e(t) \Rightarrow$ error Signal

$T_D \Rightarrow$ derivative time; sec

Laplace Transformed Form

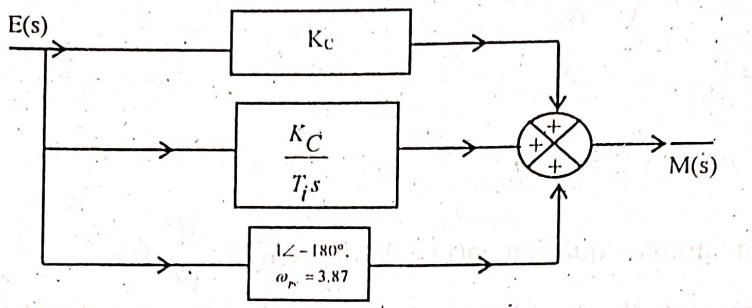
Laplace transforming equation (i), we get

$$M(s) = K_c E(s) + \frac{K_c}{T_i}\frac{E(s)}{s} + K_c T_D s E(s) = K_c\left(1 + \frac{1}{T_i s} + T_D s\right)E(s)$$

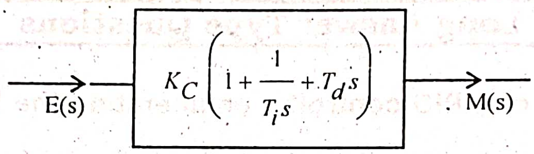
$\Rightarrow$ Transfer Function

$$\frac{M(s)}{E(s)} = K_c\left(1 + \frac{1}{T_i s} + T_D s\right)$$

Block Diagram Representation of a PID Controller



Reducing the above block diagram using the rule of parallel path we get



Characteristic Curve

Let the error signal be defined as

$e(t) = t \quad$ for $\quad t \geq 0$

Putting this error signal in equation

$$m(t) = K_c t + \frac{K_c}{T_i}\int t dt + K_c T_D \frac{d}{dt} t \qquad = \frac{K_c}{T_i}\frac{t^2}{2} + K_c t + K_c T_D$$

which represents the equation of a parabola.

For opamp realization
Circuit Diagram

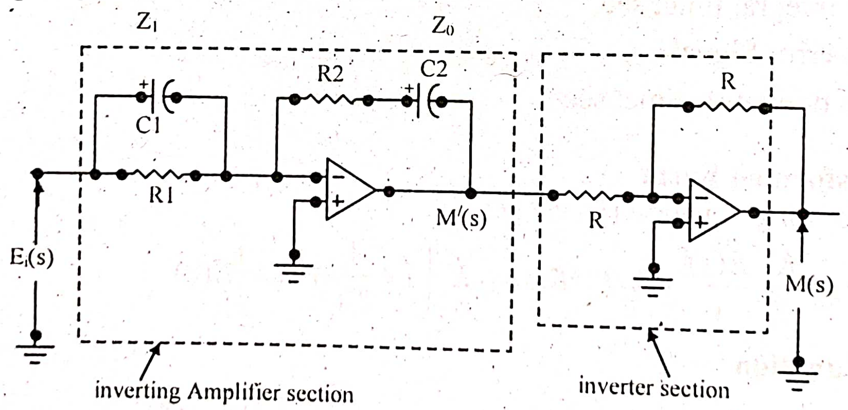


Fig: 1 Electronic circuit of a PID controller

Transfer function for the amplifier section (fig.1) is given by

$$\frac{M'(s)}{E_i(s)} = -\frac{z_0}{z_i} \qquad \dots (1)$$

where, $Z_0 = R_2 + \frac{1}{sC_2} = \frac{(R_2 C_2 s + 1)}{sC_2}$

and $Z_i = \frac{R_1 \cdot \frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} = \frac{R_1}{(R_1 C_1 s + 1)}$

$$\therefore \quad \frac{M'(s)}{E_i(s)} = -\frac{(R_2 C_2 s + 1)/sC_2}{\frac{R_1}{(R_1 C_1 s + 1)}}$$

$$\Rightarrow \quad \frac{M'(s)}{E_i(s)} = -\frac{(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{sR_1 C_2} \qquad \ldots (2)$$

T.F. for inverter section (Fig. 1) is given by

$$\frac{M(s)}{M'(s)} = -1 \qquad \ldots (2a)$$

$$\therefore \text{T.F.}|_{\text{PID}} = \frac{M(s)}{E_i(s)}$$

$$T.F.\Big|_{PID} = \frac{M(s)}{E_i(s)} = \frac{M(s)}{M'(s)} \times \frac{M'(s)}{E_i(s)}$$

$$= \frac{(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{sR_1 C_2}; \text{combining equations 2 and } 2a$$

$$= \frac{R_2}{R_1} \times \frac{(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{R_2 C_2 s} = \frac{R_2}{R_1}\left[\frac{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2)s + 1}{R_2 C_2 s}\right]$$

$$= \frac{R_2}{R_1}\left[R_1 C_1 s + \frac{(R_1 C_1 + R_2 C_2)}{R_2 C_2} + \frac{1}{R_2 C_2 s}\right]$$

$$= \frac{R_2}{R_1} \times \frac{(R_1 C_1 + R_2 C_2)}{R_2 C_2}\left[1 + \frac{R_2 C_2}{(R_1 C_1 + R_2 C_2) R_2 C_2 s} + \frac{R_1 C_1 R_2 C_2}{R_1 C_1 + R_2 C_2} \cdot s\right]$$

$$\Rightarrow \frac{M(s)}{E_i(s)} = \frac{R_1 C_1 + R_2 C_2}{R_1 C_2}\left[1 + \frac{1}{(R_1 C_1 + R_2 C_2)} \cdot \frac{1}{s} + \left(\frac{R_1 C_1 R_2 C_2}{R_1 C_1 + R_2 C_2}\right)s\right] \qquad \ldots (3)$$

The equation 2 may be expressed as

$$\frac{M(s)}{E_i(s)} = K_P\left[1 + \frac{1}{T_i s} + T_D s\right]$$

which satisfies the basic mathematical structure of a PID control action where,

$$K_P = \frac{(R_1C_1 + R_2C_2)}{R_1C_2},\ T_i = (R_1C_1 + R_2C_2) \text{ and } \quad T_D = \frac{R_1\,C_1\,R_2\,C_2}{R_1\,C_1 + R_2\,C_2}.$$

2. What is compensation? What is a compensated system? What is a compensator? **[WBUT 2016]**

Answer:

1st part:

Compensation is the modification of dynamics of the control system by introducing some subsystems to the original uncompensated system.

2nd Part:

- To have desired performance from the control system, proper selection of performance specifications is necessary
- The desired behaviour is specified in terms of
  (i) Transient Response measures
  (ii) Steady – State Error.
- Transient Response is specified in terms of
  (ii) Peak Overshoot
  (iii) Settling time
  (iv) Rise time
  (v) Peak time etc.

In practice, if a system is to be redesigned so as to meet the required specifications, it is necessary to alter the system by adding an external device to it. Such a redesign or alteration of system using an additional suitable device is called compensation of a control system.

3rd Part:

The combination of subsystems and original system together is called a compensated system and the subsystems as a whole is called a compensator.

3. Write short notes on the following:
a) P controller **[WBUT 2016]**
b) PI controller **[WBUT 2016]**
c) PID controller **[WBUT 2005, 2007, 2010, 2012, 2015, 2016, 2017, 2018]**

Answer:

a) P Controller:

Mathematical Structure

$$m(t) = K_c e(t) \qquad \ldots (1)$$

where, $m(t) \Rightarrow$ P-controller's output

$K_C \Rightarrow$ proportional sensitivity

$e(t) \Rightarrow$ error signal

Laplace Transformed Form
Laplace transforming the equation (1)

$$M(s) = K_c E(s)$$

$$\Rightarrow \textbf{Transfer Function} = \frac{M(s)}{E(s)} = K_c \quad \ldots (2)$$

Block Diagram Presentation

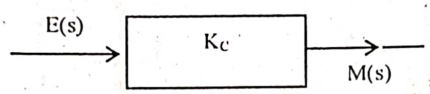


Fig: 1 Block diagram of a P-controller

Characteristic Curve

Let the error signal be defined as

$$\left.\begin{aligned} e(t) &= A \quad \text{for } t > 0 \\ &= 0 \quad \text{for } t < 0 \end{aligned}\right\} \quad \ldots (3a)$$

Putting this error signal in equation (2),

$$m(t) = K_C (A) \quad \text{for } t > 0 \quad \ldots (3b)$$

The equation (3b) says output $m(t)$ is K_C times greater than the error signal; $e(t)$. It means the proportional controller is an amplifier of gain K_C

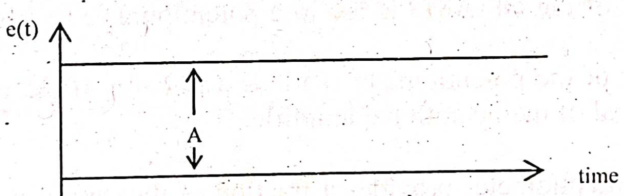


Fig: 2a: Error signal

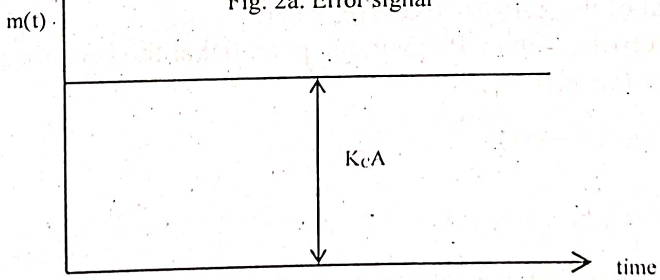


Fig: 2b Step response of a P-controller

The characteristic curve in figure 2b supports the fact.

Realization of P-controller

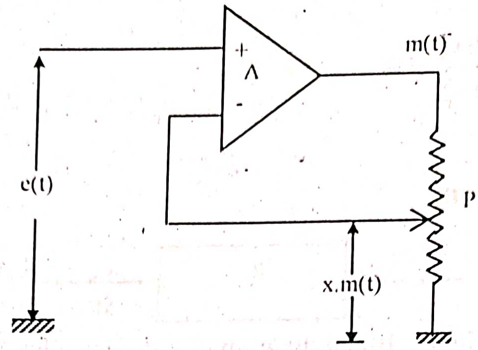


Fig: 3Circuit diagram of a P-controller

- ***The Basic Components***

Components	Functions
Op-Amp (A)	acts as a non-inverting amplifier
Potentiometer (P)	Acts as a variable voltage source. The wiper position determines the gain for P-action.

- ***Circuit Explanation***

Step 1: The error signal *e(t)* is fed to the Non-inverting Terminal of the OP-AMP.
Step 2: The output of the OP-AMP is fed to a potentiometer P, which acts as a variable voltage source.
Step 3: The wiper of the potentiometer (P) feds a fraction of the output voltage *m(t)* to the inverting terminal of the operational amplifier.

- ***Verification***

The wiper of the potentiometer provides a fraction of the output $m(t)$, i.e., $x.m(t)$, to the inverting terminal of the OP-AMP where, $x < 1$.
From the characteristic of the OP-AMP, the potential at the inverting terminal is $e(t)$. So,

$$x.m(t) = e(t)$$

$$\Rightarrow \quad m(t) = \frac{1}{x}e(t) \qquad \text{.... (4)}$$

Since $x < 1$, so $\frac{1}{x} = k_C$ (say) > 1.

So, rewriting equation (1b), we get $m(t) = k_C.e(t)$ .... (5)
which satisfies the equation (1).

b) PI Controller: ***Refer to Question No. 2(b) of Short Answer Type Questions.***

c) PID Controller: ***Refer to Question No. 1 of Long Answer Type Questions.***

CRO

Multiple Choice Type Questions

1. In a CRT the focusing anode is located [MODEL QUESTION]
   a) between pre-accelerating and accelerating anodes
   b) after accelerating anode
   c) before pre-accelerating anode
   d) none of these

Answer: (a)

2. An aquadag is used in a CRO to collect [MODEL QUESTION]
   a) primary electrons
   b) secondary emission electrons
   c) both primary and secondary emission electrons
   d) none of these

Answer: (b)

3. The term "Lissazous pattern" is associated with [MODEL QUESTION]
   a) CRO b) Digital multimeter c) Galvanometer d) Thermocouple

Answer: (a)

4. A virtual amplifier for a CRO can be designed for [MODEL QUESTION]
   a) only a high gain
   b) only a broad band
   c) a constant gain time bandwidth product
   d) all of these

Answer: (c)

5. The deflection system of an oscilloscope works on the principle of [MODEL QUESTION]
   a) electrostatic b) electromagnetic
   c) thermionic d) magnetic induction

Answer: (a)

6. The source of emission of electrons in a CRT is [MODEL QUESTION]
   a) PN junction diode
   b) a barium and strontium oxide coated cathode
   c) accelerating anodes
   d) post accelerating anodes

Answer: (b)

7. Which one of the following measuring devices has minimum loading effect on the quantity under measurement? [MODEL QUESTION]
   a) PMMC b) CRO c) Hot wire d) Electrodynamometer

Answer: (b)

8. LVDT is used to measure [MODEL QUESTION]
a) displacement b) temperature c) pH value d) intensity of light
Answer: (a)

9. LVDT is a [MODEL QUESTION]
a) capacitive transducer b) resistive transducer
c) inductive transducer d) none of these
Answer: (c)

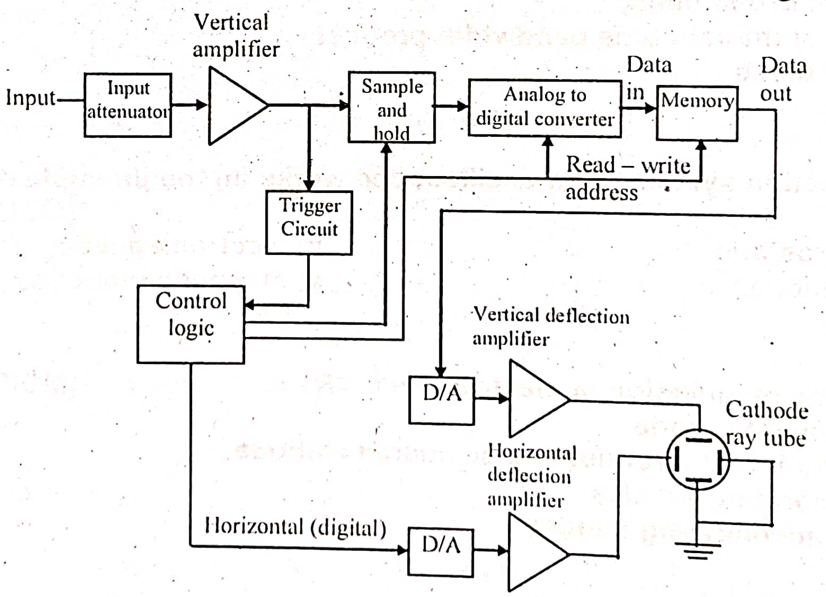
10. Swamping resistance is a resistance which added to the moving coil of a meter to [MODEL QUESTION]
a) reduce the full scale current b) reduce the temperature error
c) increase the sensitivity d) none of these
Answer: (b)

11. When the strain of a wire gauge changes, it results in a change of [MODEL QUESTION]
a) pressure b) temperature c) inductance d) resistance
Answer: (d)

Short Answer Type Questions

1. Explain the functional block diagram of CRO with neat diagram. [MODEL QUESTION]

Answer:
The block diagram of digital storage oscilloscope is shown in the following figure.



The input signal is applied to the amplifier arid attenuator section, as done in all oscilloscopes. The oscilloscope uses same type of amplifier, attenuator circuitry as the conventional oscilloscopes. The vertical amplifier then gets the attenuated signal.

The main requirement of A/D converter in the digital storage oscilloscope is its speed, while in digital voltmeters accuracy and resolution were the main requirements. The digitized output needed only in the, binary form and not in BCD. The successive approximation type of A/D converter is most often used in the digital storage oscilloscopes.

The digitizing the analog input signal means taking samples at periodic intervals of the input signal. The rate of sampling should be at least twice as fast as the highest frequency present in the input signal, according to sampling theorem. This ensures no loss of information. The sampling rates as high as 100,000 samples per second is used. This requires very fast conversion rate of A/D converter.

The sampling rate and memory size are selected depending upon the duration and the waveform to be recorded.

After the input signal is sampled, the A/D converter digitizes it. This signal is then stored in the memory. Once it is stored in the memory, multi manipulations are possible as memory can be read out without being erased.

Single shot events, like the waveform of an explosion are transient in nature and very quickly lost. Unless the waveform is photographed or stored the observer cannot see such events. These events can be stored in memory of digital storage oscilloscope. After reading the memory rapidly and repetitively the continuous waveform can be obtained.

The digital storage oscilloscope has three modes of operation

(i) ***Roll mode:*** In this mode very fast varying signals are displayed clearly. The input signal is not triggered at all. The fast varying signal is displayed as if it is changing slowly, on the screen in this mode.

(ii) ***Store mode:*** The input initiates trigger circuit. With trigger pulse memory write cycle starts. When memory is full, write cycle stops. This is called refresh. Then using digital to analog converter, the stored signal is converted to analog and displayed. When next trigger occurs the memory is refreshed.

(iii) ***Hold or save mode;*** This is automatic refresh mode. When new sweep signal is generated by time base generator, the old contents get over written by new one. By pressing hold or save button, overwriting can be stopped and previously saved signal gets locked.

2. A Lissajous pattern on an oscilloscope is stationary and has 5 horizontal tangencies and 2 vertical tangencies. The frequency of horizontal input is 1000Hz. Determine the frequency of vertical input. [MODEL QUESTION]

Answer:

Frequency of horizontal input, f_x =1000 Hz

Frequency of tangency to vertical line = 2

Points of tangency to horizontal line = 5

Frequency of vertical input $f_y = f_x \times$ (horizontal tangencies/vertical tangencies)

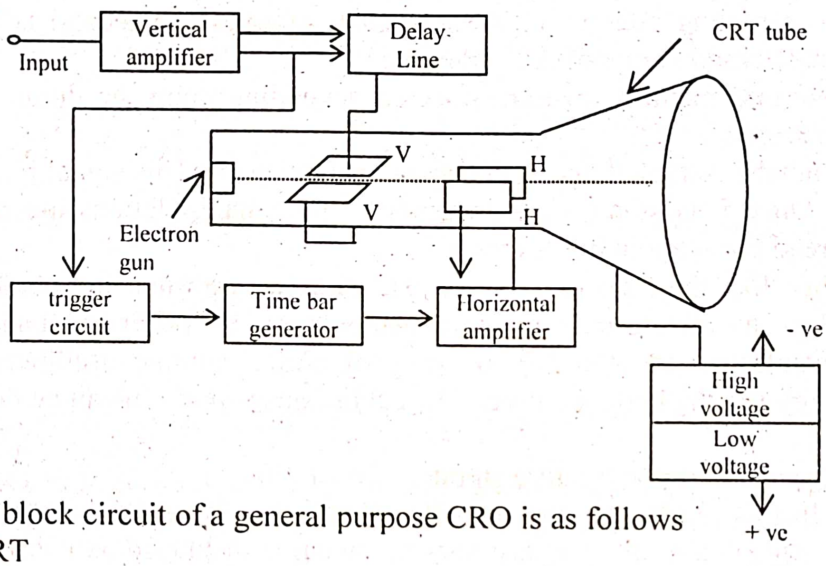
$= 1000 \times (5/2) = 2500$ Hz

Long Answer Type Questions

1. a) Draw a diagram showing the interval structure of a cathode ray tube.
b) What are 'Lissajous' pattern?
c) Show either graphically or analytically that two sinusoidal signals of the same amplitude and frequency but with a phase shift of 90° gives a circular Lissajous pattern. **[MODEL QUESTION]**

Answer:

a)



The major block circuit of a general purpose CRO is as follows
1) CRT
2) Horizontal Amplifier
3) Vertical Amplifier
4) Delay Line
5) Power Supply
6) Time base
7) Trigger Circuit

1) CRT: This is the cathode ray tube which emits electrons that strikes the phosphor search internally to provide a visual display at signal.
2) Horizontal Amplifier: It amplifies the sawtooth voltage before it is applied to horizontal deflection plates.
3) Vertical Amplifier: The sensitivity and bandwidth of an oscilloscope is determined by the vertical amplifiers. The gain of the vertical amplifier determines the smallest signal that the oscilloscope can satisfactorily reproduce on the CRT screen.
The sensitivity of oscilloscope is directly proportional to gain of the vertical amplifier.
4) Delay Line: In vertical sections, delay line is used to delay the signal for a certain span of time. The part of the signal gets lost when the delay line is not used,. Then the input signal is not applied directly to the vertical plates but is delayed by sometime using a delay line circuit. The sweep generator output gets enough time to reach to the Horizontal plates when the signal is delayed.

There are two types at delay lines used on CRO.

a) ***Lamped parameter delay line:*** Lamped parameter delay line counts at number of cascaded symmetrical LC networks.

b) ***Distributed parameter Delay line:*** It is basically a transmission line constructed with a wound helical coil on a mandrel and extruded insulation between it.

5) Power Supply: The power supply block provides the voltage required by CRT to generate and accelerate on electron beam and voltage required by other circuits at the oscilloscope like Horizontal Amplifier, vertical amplifier etc.

There are two sections at a power supply block. The High voltage section (HV) and Low voltage section (LV). The High voltages of the order of 1000 volt to1500 volt are requited by CRT. Such high negative voltages are used for CRT.

The negative high voltage has following advantages:

a) Accelerating orders and the deflection plates are clean to ground potential. This ground potential protects the operator from shocks.

b) The deflection voltages are measured with respect to ground hence blocking on coupling capacitors are not necessary.

c) Insulation between controls and chains is less.

6) Time base: Time base generates the sawtooth voltage required to reflect the beam to the horizontal section. This voltage deflects the spot at a constant time dependent rate. Thus the Y–axis on the screen can be represented as time, which helps to display and analyze time varying signal.

7) Trigger Circuit: To synchronise the input signal and the sweep frequency trigger circuit is used. Trigger circuit converts the incoming signal into trigger pulse. Trigger circuit is necessary that horizontal deflection starts at the same point of the input vertical signal each time sweeps.

b) When sinusoidal voltages are simultaneously applied to horizontal and vertical plates, the patterns that appear on the screen of CRT are called "Lissajous pattern".

c)

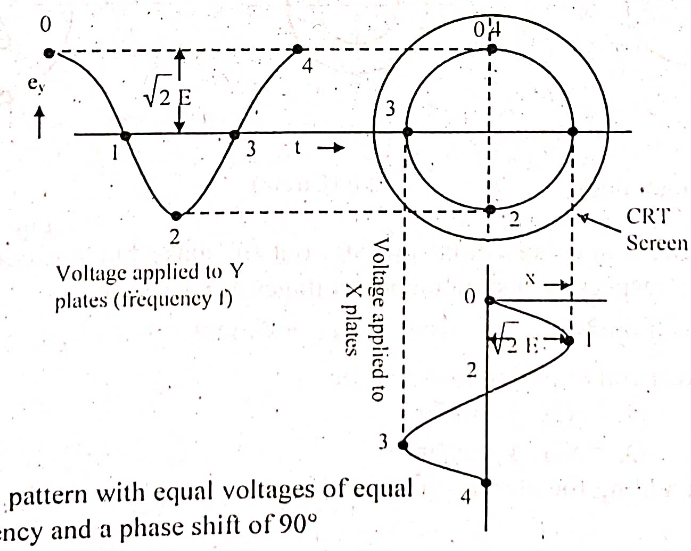


Fig.: Lissajous pattern with equal voltages of equal frequency and a phase shift of 90°

The various figures that appear on the screen of the oscilloscope, when sinusoidal voltages are simultaneously applied to horizontal and vertical plates are called Lissajous figures or pattern.

(i) Let the voltage e_x applied to X-plates be $\sqrt{2}E_x \sin 2\pi ft$. Then the voltage e_y (having the same frequency and phase) applied to Y-plates will be $\sqrt{2}E_y \sin 2\pi f$, deflection of the spot is proportional to the voltage at any instant and therefore horizontal since the deflection is given by,

$$D_x = \sqrt{2}C_1 E_x \sin 2\pi ft$$

And the vertical deflection will be

$$D_y = \sqrt{2}C_1 E_y \sin 2\pi ft$$

Where C_1 is a constant.

$$\text{Therefore, } \frac{D_y}{D_x} = \frac{E_y}{E_x} = C_2 \qquad \text{.... (1)}$$

Here C_2 is another constant.

Equation (1) can be written as, $D_y = C_2 D_x$

This is the equation of a straight-line passing through the origin as shown in Figure (a) whose slope will be,

$$\tan\alpha = \frac{D_y}{D_x} = \frac{E_y}{E_x} \qquad \text{.... (2)}$$

This shows that when two sinusoidal voltage of the same frequency and in phase with each other applied to the CRO the pattern appearing on the screen is a straight line as shown in Figure (a).

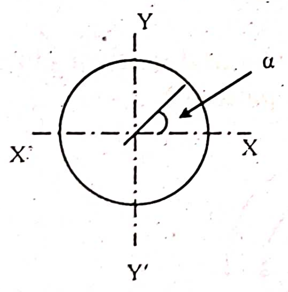


Fig: a (Straight line)

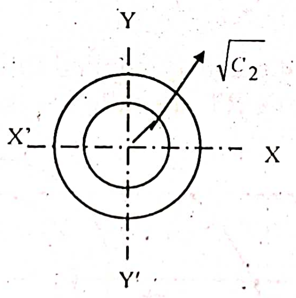


Fig: b (Circle)

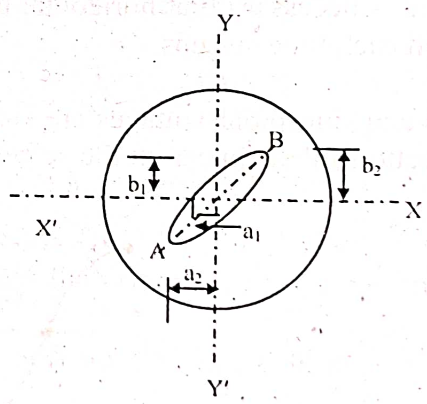


Fig: c (Ellipse)

(ii) If the vertical and horizontal are 90° out of phase but are equal in magnitude and frequency, the respective instantaneous voltages are given by,

$$e_x = \sqrt{2}E \sin 2\pi ft \qquad \text{and} \qquad e_y = \sqrt{2}E \cos 2\pi ft.$$

The respective deflections will be

$$D_x = \sqrt{2}C_1 E \sin 2\pi ft$$

$$D_y = \sqrt{2}C_1 E \cos 2\pi ft$$

Squaring and adding the above equation

$$D_x^2 + D_y^2 = 2C_1^2E^2 = C_2 \quad \text{(constant)}$$

This is the equation of the circle with its center at the origin and whose radius is $\sqrt{C_2}$ as shown in Figure (b).
Thus when two voltages of equal frequency but 90° out of phase are applied to the CRO, the trace on screen is a circle. However if the two voltages are not equal and/or out of phase, an ellipse is formed as shown in Figure (c).
(iii) When two equal voltages of equal frequency but with a phase difference of Φ (neither 0 nor 90°) are applied to CRO, the trace on the screen is parabola. The parabola is also observed when unequal voltages of the same frequency are applied to CRO.

2. a) What are the functions of the following in a CRO: [MODEL QUESTION]
i) Time-base generator, ii) Focussing anode, and iii) Delay line in a CRO?

Answer:

i) Time base:
Time base generates the sawtooth voltage required to reflect the beam to the horizontal section. This voltage deflects the spot at a constant time dependent rate. Thus the Y–axis on the screen can be represented as time, which helps to display and analyze time varying signal.

ii) Focussing anode:
It serves the dual purpose of attracting electrons from the area of the control grid and focusing the electrons into a beam. This is achieved by applying positive potential with reference to the cathode.

iii) Delay line:
In vertical sections, delay line is used to delay the signal for a certain span of time. The part of the signal gets lost when the delay line is not used. Then the input signal is not applied directly to the vertical plates but is delayed by sometime using a delay line circuit. The sweep generator output gets enough time to reach to the Horizontal plates when the signal is delayed.
There are two types at delay lines used on CRO.

a) ***Lamped parameter delay line:*** Lamped parameter delay line counts at number of cascaded symmetrical LC networks.
b) ***Distributed parameter Delay line:*** It is basically a transmission line constructed with a wound helical coil on a mandrel and extruded insulation between it.

b) How frequency can be measured from Lissajous pattern on a CRO?
[MODEL QUESTION]

Answer:

Measurement of frequency:
The signal whose frequency is to be measured is applied to the Y-plates. With internal sweep generator switched off an accurately calibrated standard variable frequency source is used to supply voltage to X-plates. The standard frequency is adjusted until the pattern

appears as a circle or an ellipse indicating that both signals are of the same frequency. Then it can be shown that,

$$\frac{f_x}{f_y} = \frac{\text{No. of times tangent touches top or bottom}}{\text{No. of times tangent touches either side}} = \frac{\text{No. of horizontal tangencies}}{\text{No. of vertical tangencies}}$$

where f_y is the signal frequency applied to Y-plates
f_x is the signal frequency applied to X-plates.

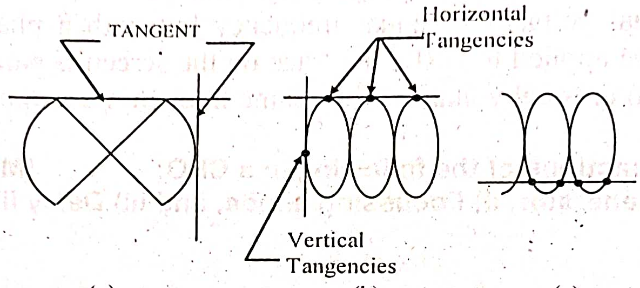


Fig.: Lssajous Patterns

The counting of number of tangencies are as shown in Fig: (a). The fig shows the Lissajous pattern for frequency ratio 2 : 1 in which $f_y = 2$ and $f_x = 1$. Similarly Fig: (b) shows the Lissajous patterns for frequency ratio 3 : 1 in which $f_y = 3$ and $f_x = 1$.

3. How do we measure phase of a.c. quantity with the help of a CRO? [MODEL QUESTION]

Answer:

Measurement of Phase: A simple means of finding phase of difference between two voltages regardless of the two amplitudes of the applied voltage is provided by the ellipse. Referring to Fig (c), the sine of the phase angle ϕ between the voltage is given by

$$\sin\phi = \frac{b_1}{b_2} = \frac{a_1}{a_2}$$

For convenience, the gains of the vertical and horizontal amplifiers are adjusted so that the ellipse fits exactly into a square as marked by the lines on the screen of the oscilloscope.

If the major axis AB of the ellipse lies in the first and third quadrants i.e., its slope is positive the phase angle ϕ is either between 0 to 90° or between 270° to 370°. When the major axis of ellipse lies on second and fourth quadrants, i.e., is negative, the phase angle ϕ is either between 90° to 130° or between 180° to 270°.

4. What are the functions of Time-base generator in CRO? [MODEL QUESTION]

Answer:

Time Base Generator

If the horizontal deflecting plates, X-plates are not energized, the electron beam would draw a vertical line on the screen.

- The function of the time base generator is to drive the beam at a steady speed across the screen and, when it reaches the right-hand side of the screen, the beam is made to fly back to the starting position on the left-hand side of the screen. The time base generator produces a saw-tooth wave (Fig.) of the same frequency as the input signal to the Y-plates.

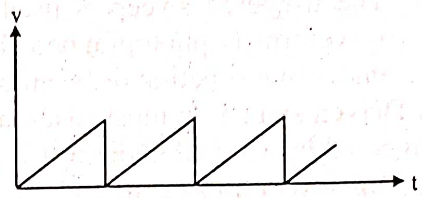


Fig: Saw-tooth voltage wave

- The horizontal deflection of the beam is proportional to the instantaneous voltage of the saw-tooth wave, therefore, the beam sweeps at a uniform rate across the screen in the horizontal direction, running to the starting position almost instantaneously and sweeping again.
- The time base generator gives an output sweep signal of adjustable frequency so that the Y input signals of a wide range of frequencies may be displayed on the screen.
- There are many applications of a CRO in which a time base generator is not needed and it is necessary to apply an external input signal to the X-plates (through a switch).

Types of Sweeps
There are four types of sweeps:

1. Free running sweep:

- In very basic oscilloscopes, the sweep generator a continuously charging and discharging a capacitor. One ramp voltage is followed immediately by another, hence the sawtooth patterns appears. A sweep generator operating in this manner is said to be "free running".
- In order to present a stationary display on the screen, the sweep generator signal must be forced to run in synchronization with a vertical input signal. In basic oscilloscopes, this is accomplished by carefully adjusting the sweep frequency to a value very close to the exact frequency of the vertical input signal, or submultiple of this frequency. With signals at the same frequency, an internal synchronization pulse will lock sweep generator into the vertical input signal. This method of synchronization has some serious limitation is probably the inability of the instrument to maintain synchronization when the amplitude or frequency of the vertical signal is not constant, such as voice or music signals,

2. Triggered sweep:

- A waveform to be observed on the CRO may not be periodic but may perhaps occur at irregular intervals. In this case, it is desirable that the sweep circuit remain imperative and the sweep be initiated by the waveform under examination. In some cases, the waveform may be periodic, but it may be that the interesting part of the waveform is of very short duration compared to the period of the waveform. Under such cases, a triggered switch is used.
- In triggered sweep or single sweep, the spot is swept once across the screen in response to a trigger signal.

- The triggered sweep is used for examination of transient or one time signals and waveform is photographed for record. The trigger can be obtained from the signal under investigation or by an external source.

3. Driven sweep. In most cases, a 'driven sweep' is used where the sweep is recurrent but triggered by a signal under test.

4. Non-sawtooth sweep:

- For some applications like comparison of two frequencies or finding phase shift between two voltages non-sawtooth sweep voltages are utilised for the sweep circuit.
- Sweep frequencies vary with the type of the oscilloscope.

A laboratory oscilloscope may have sweep frequencies upto several MHz; a simple oscilloscope for audio work has an upper limit 100 kHz. Most T.V. services require a sweep voltage frequency upto 1 MHz.

5. a) What are the differences between dual beam CRO & dual trace CRO?
[MODEL QUESTION]

Answer:

Dual Trace CRO	Dual Beam CRO
i) single beam is used for producing two different wave forms.	i) two separate electron beam use for producing different wave forms.
ii) it can not capture two fast transient events.	ii) It can capture two fast transient events as it can display two signals simultaneously
iii) Signal loss in case of Dual Trace CRO about 50% of each signal.	iii) No loss occurs during dual-beam display.
iv) Two operating modes under this CRO a) alternate & b) chop	iv) Two operating modes under this CRO a) double gun tube. & b) split beam

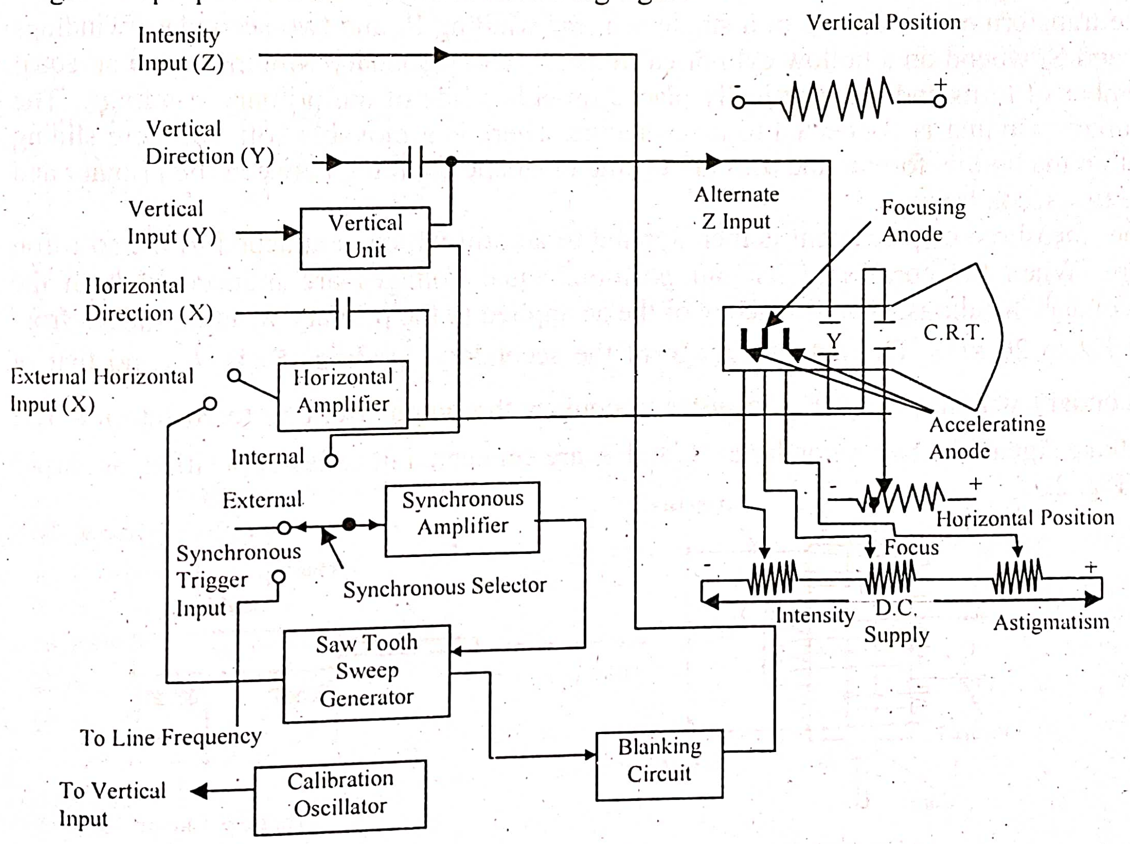
b) Draw and explain different blocks of a CRO. **[MODEL QUESTION]**

Answer:

The signal to be examined are usually applied to the vertical or y-plates through an input attenuator and a number of amplifier stages. When high voltage signals are to be examined, they are attenuated to bring them within the range of vertical amplifiers. The vertical amplifier output is also applied to the synchronizing amplifier through synchronizer selector switch in the internal position. This permits the horizontal sweep circuit to be triggered by the signal being investigated.

The horizontal or x-deflection plates are fed by a sweep voltage that provides a time base. The horizontal plates are supplied through an amplifier, but they can be fed directly when voltages are of sufficient magnitude. When external signals are to be applied to the horizontal plates, they can also be fed through the horizontal plates. The horizontal amplifier through the sweep selector switch in the external position. When the sweep selector switch is in the internal position the horizontal amplifier receives an input from the sawtooth sweep generator which is triggered by the synchronizing amplifier.

The sweep must be synchronized with the signal being measured. Synchronization is done to obtain a stationary pattern. This requires that time base be operated at a sub-multiple frequency of the signal under measurement.
The saw tooth sweep voltage applied to the horizontal plates moves the beam across the cathode ray tube in a straight horizontal line from left to right during the sweep or trace time. A comparatively slow movement of the spot will appear as a solid line, provided the rate of movement exceeds the threshold of persistence of vision. Below this threshold limit, a moving spot is perceived. On the other hand, the comparatively rapid movement of spot will appear as a thin and dim line, or may be invisible. If the retrace or flyback time is very small, the spot remain invisible. In an ideal case the flyback time is zero and hence the spot while moving from right to left remains invisible. In practice the flyback time is not zero and hence the retrace causes confusion. Thus the retrace should be eliminated or blanked out, the circuit which provides this is called blanking circuit, which is achieved by applying a negative voltage triggered by sweep generator. .
The positioning of the trace on the screen is done by applying a small independent internal d.c. voltages to the deflecting plates and control can be exercised by varying the voltage with the help of external potentiometers. This arrangement is called positioning control.
A general purpose C.R.O. shown in following figure



6. a) Explain with a neat sketch the operating principle of Linear variable differential transformer (LVDT). **[MODEL QUESTION]**

Answer:
The differential transformer is a passive inductive transformer. It is also known as a Linear Variable Differential transformer (LVDT). The basic construction is as shown in Fig. 1.

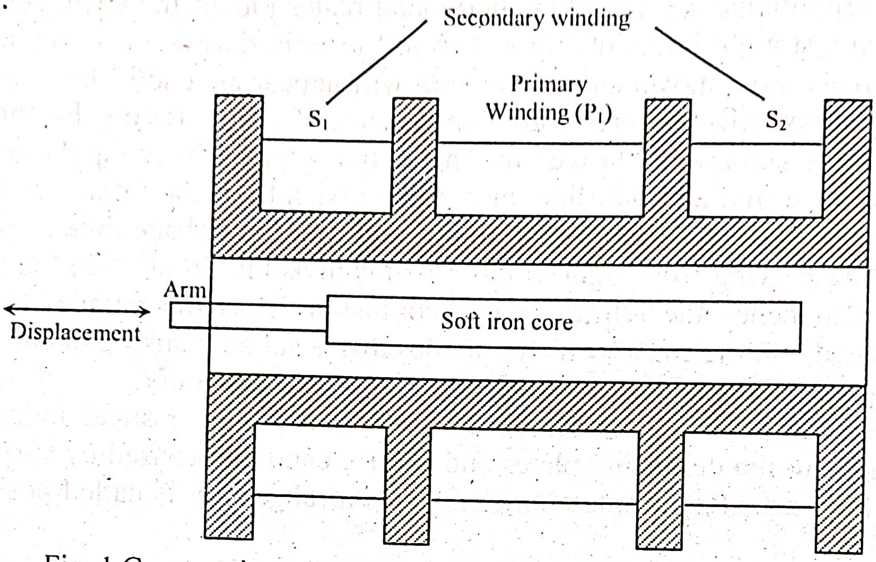


Fig: 1 Construction of a linear variable differential transducer (LVDT)

The transformer is made up of a single primary winding P_1 and two secondary windings S_1 and S_2 wound on a hollow cylindrical former. The secondary windings have an equal number of turns and are identically placed on either side of the primary windings. The primary winding is connected to as ac source. There is a movable soft iron core sliding within the hollow former and thus affects the magnetic coupling between the primary and the two secondaries.

The measured displacement is then applied to an arm which is attached to the soft iron core. When the core is in its null position, equal voltages are induced in both the secondary windings. The frequency of the ac applied to the primary winding ranges from 50 Hz to 20 kHz. The output voltage of the secondary windings S_1 is E_{S_1} and that of secondary winding S_2 is E_{S_i}. In order to convert the output from S_1 to S_2 into a single voltage signal, the two secondaries S_1 and S_2 are connected in series opposition, as shown in Fig. 2.

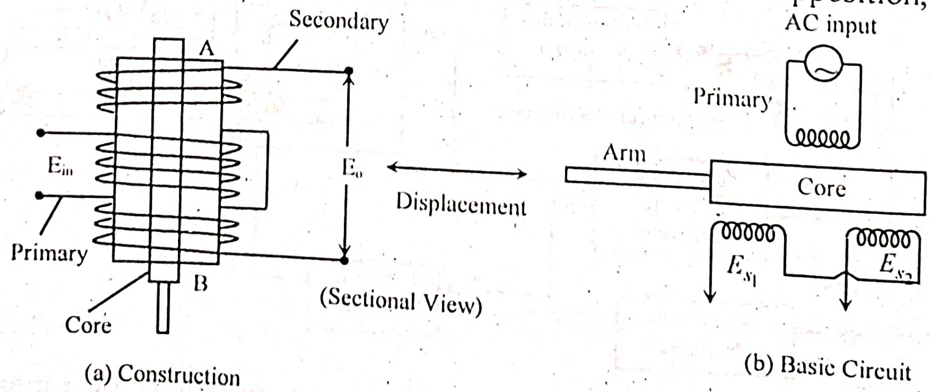


Fig: 2 Secondary winding connected for differential output

Hence the output voltage of the transducer is the difference of the two voltages. Therefore the differential output voltage $E_o = E_{S_1} \sim E_{S_i}$ When the core is at its normal position, the flux linking with both secondary windings is equal, and hence in both of them equal emfs are induced. Hence, at null position $E_{S_1} = E_{S_2}$. Since the output voltage of the transducer is the difference of the two voltages, the output voltage E_o is zero at null position. Now, when the core is moved to the left of the null position, more flux links with winding S_1 and less with winding S_2. Hence, output voltage E_{S_1} of the secondary winding S_1 is greater than E_{S_i}. The magnitude of the output voltage of the secondary is then $E_{S_1} - E_{S_2}$, in phase with E_{S_1} (the output voltage of secondary winding S_1).Similarly, when the core is moved to the right of the null position the flux linking with winding S_2 becomes greater than that linked with winding S_1. This results in E_{S_2} becoming larger than E_{S_1}. The output voltage in this case is $E_o = E_{S_2} - E_{S_1}$ and is in phase with E_{S_i}

The amount of change in voltage in either of the secondary windings is proportional to the amount of movement of the core. Hence, we get an indication of the amount of linear motion. Output is increasing or decreasing, the direction of motion can be determined. The output ac voltage inverts as the core passes the center position. The farther the core moves from the center, the greater the difference in value between E_{S_1} *and* E_{S_2} and consequently the greater the value of E_o. Hence, the amplitude becomes the function of the distance the core has moved, and the polarity or phase indicates the direction of motion, as shown in Fig. 8. As the core moves in one direction from the null position, the difference voltage, i.e. The difference of the two secondary voltages increases, while maintaining an in-phase relation with the voltage from the input source. In the other direction from the null position, the difference voltage is increased but is 180° out of phase with the voltage form the source.

By comparing the magnitude and the phase of the difference output voltage to that of the source, the amount and direction of the movement of the core and hence of the displacement may be determined. For the determination of the displacement the amount of output voltage may be measured. The output signal can also be applied to a recorder or directly to a controller that can restore the moving system to its normal position.

The output voltage of an LVDT is a linear function of the core displacement within a limited range of motion (say 5 mm from the null position). Figure 3(d) displays the variation of the output voltage against the displacement for various position of the core. This is a linear curve for small displacements (up to 5 mm). Beyond this range, the curve starts to deviate. The diagram of Figs 3(a), (b) and (c) shows the core of an LVDT at three different positions. In Fig. 3 (b), the core is at 0, which is the central zero or null position. Therefore $E_{S_1} = E_{S_2}$ and $E_o = 0$.

When the core is moved to the left, as in Fig. 3(a) and is at A, E_{S_1} is more than E_{S_2} *and* E_o is positive. This movement represents a positive value and therefore the phase angle, is $\phi = 0°$. After moving the core in the right direction towards B, E_{S_2} is greater than E_{S_1} and hence E_0 is negative. Therefore, S_2 the output voltage is 180° out of phase with the voltage which is obtained when the core is moved to the left. The characteristics are linear form O – A and O – B, but after that they become non-linear. One advantage is that over the inductive bridge type LVDT produces higher output voltage for small changes in core position. Several commercial models that produce 50 mV/mm to 300 mV/mm are also available. 300 mV/mm implies that a 1 mm displacement of the core produces a voltage output of 300 mV.

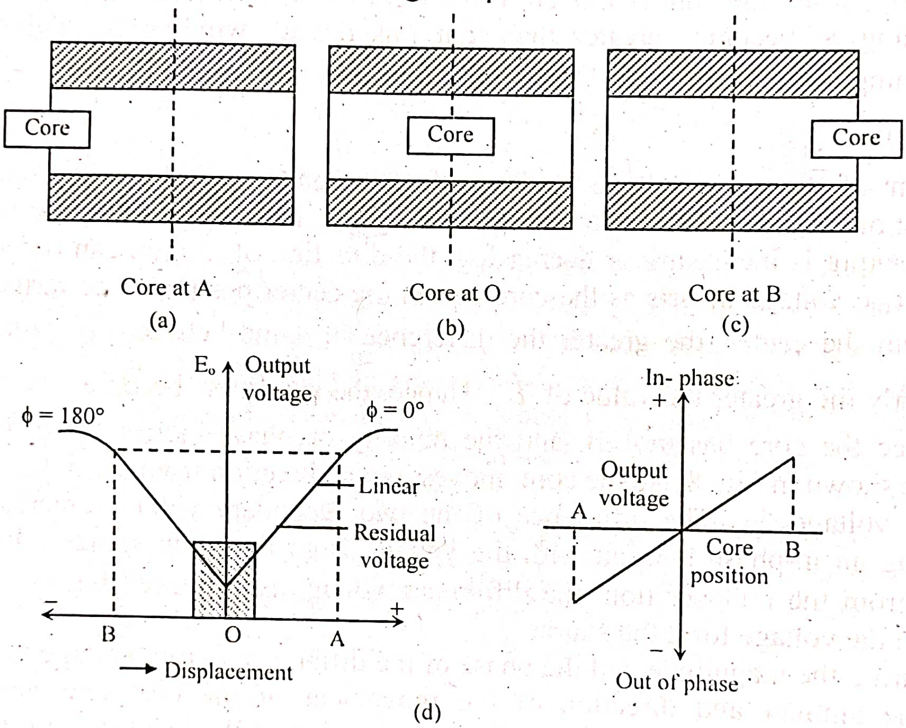


Fig: 3 (a), (b), (c) Various core position of LVDT (d) Variation of output voltage vs Displacement

LVDTs are available with ranges as low as ± 0.05 in. to as high as ± 25 in. and are sensitive enough to be used to measure displacements of well below 0.001 in. They can be obtained for operation at temperatures as low as –265°C and as high as +600°C and are also available in radiation resistance designs for nuclear operations.

b) What are the advantages and disadvantages of LVDT? [MODEL QUESTION]

Answer:

Advantages of LVDT

1. ***Ruggedness:*** These transducers can usually tolerate a high degree of vibration and shock.
2. ***Less friction:*** There are no sliding contacts.
3. ***Infinite resolution:*** The change in output voltage is stepless. The effective resolution thus depends more on the test equipment than on the transducer.
4. ***Low power consumption***: Most LVDTs consume less than 1 W of power.
5. ***High sensitivity:*** The transducer is very sensitive as high as 40 V/mm.
6. ***High output***: It gives appreciable output (therefore there is frequently no need for intermediate amplification devices).
7. ***Linearity:*** Due to the displacement the output voltage of the transducer is practically linear for displacements up to 5 mm (a linearity of 0.05% is available in commercial LVDTs).
8. ***Low hysteresis:*** This transducer has a low hysteresis, hence repeatability is excellent under all conditions.

Disadvantages

1. The dynamic response is limited mechanically by the mass of the core and electrically by the applied voltage.
2. Large displacements are required for appreciable differential output.
3. Transducer is effected by the temperature.
4. The instrument which is used to receive must be selected so as to operate on ac signals, or a demodulator network must be used if a dc output is required.
5. They are sensitive to stray magnetic fields (but shielding is possible)

c) The output of an LVDT is connected to a 5V voltmeter through an amplifier of amplification factor 250. The voltmeter scale has 100 divisions and the scale can be read to 1/5th of a division. An output of 2mV appears across the terminals of the LVDT when the core is displaced through a distance of 0.5 mm.
Calculate –
i) the sensitivity of the LVDT
ii) that of the whole setup
iii) the resolution of the instrument in mm. [MODEL QUESTION]

Answer:

i) The output voltage of LVDT$\left(V_{out}\right)$=2mV

$$\text{Displacement} = 0.5\text{mm}$$

$$\text{Sensitivity of LVDT}=\frac{V_{out}}{\text{Displacement}}=\frac{2\text{mv}}{0.5\text{mv}}=4\text{mv/mm.}$$

ii) Sensitivity of the entire set up = Amplification factor × Sensitivity of LVDT

$$= 250 \times 4\,\text{mV/mm} = 1000\,\text{mV/mm} = 1\text{V/m}$$

iii) full scale of voltmeter $= 0 - 5V$
No. of divisions on voltmeter scale $= 100$

$$1 \text{ scale division} = \frac{5}{100} = 0.05\text{V or } 50\text{mV}$$

$$\text{Minimum voltage that can be read on voltmeter} = \frac{50\text{mV}}{5} = 10\text{mV}$$

$$\text{Resolution of instrument} = \frac{10\text{mV}}{1000\text{mV/mm}} = 0.01\text{mm}$$

7. Write short notes on the following: **[MODEL QUESTION]**
a) Semiconductor strain gauge
b) Piezoelectric transducer
c) Temperature transducers
d) Strain gauge
e) Double beam CRO

Answer:

a) Semiconductor strain gauge:

Semiconductor strain gauges are required for a very high gauge factor. They have a gauge factor 50 times as high as wire strain gauges. The resistance of the semiconductor changes with change in applied strain.

The piezo resistive effect affects the semiconductor strain gauges for their action, i.e. change in value of the resistance due to change in resistivity, unlike metallic gauges where change in resistance is mainly due to the change in dimension when strained. Semiconductor materials such as germanium and silicon are used as resistive materials. A typical strain gauge consists of a strain material and leads that are placed in a protective box as shown in Fig. 1.. Semiconductor wafer or filaments, which have a thickness of 0.05 mm, are the ones that are used. They are bonded on suitable insulating substrates such as Teflon:

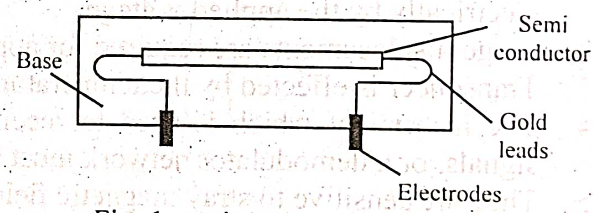


Fig: 1 semiconductor strain gauge

For the making contacts gold leads. are generally used These strain gauges can be fabricated along with an Instrumentation Op Amp, which can act as a pressure sensitive of resistance approximately 50 times higher than that for resistive gauges. Hence, a semiconductor strain gauge is as stable as the metallic type, but has a much higher output.

Advantages of Semiconductor Strain Gauge

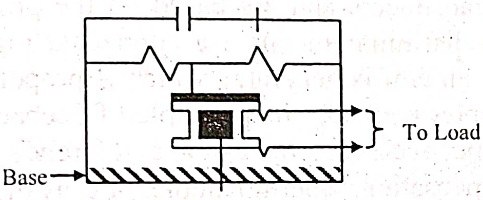
1. Hysteresis characteristics of semiconductor strain gauges are most appropriate, i.e. less than 0.05%.
2. Semiconductor strain gauges also have a high gauge factor of about + 130. This allows measurement of very small strains, of the order of 0.01 micro strain.
3. This strain gauges are small also, ranging in length from 0.7 to 7.0 mm.

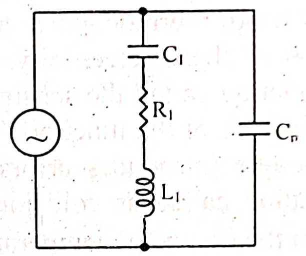
Disadvantages of Semiconductor Strain Gauge

1. They are highly sensitive in temperature.
2. Linearity of semiconductor strain gauges is poor.
3. They are more expensive.

b) Piezoelectric Transducer

A symmetrical crystalline materials (Quartz, RoChelle Salt, Barium titanate) produce an emf when they are placed under stress. This properly is used in piezoelectric transducers, where a crystal is placed a solid base and the force summing as shown in following figure.





Equivalent circuit of crystal

An externally applied force, entering the transducers through its pressure port, applies pressure to the top of a crystal. This produces an emf across the crystal proportional to the magnitude of applied pressure.

This device needs no external power source and is therefore self generating. The disadvantage is that it cannot measure static conditions. The output voltage is also affected by temperature variation of the crystal. The basic expression for output voltage E is given by

$$E = \frac{Q}{C_P} \quad \text{[where } Q = \text{Generated charge, } C_P = \text{Shunt capacitance]}$$

For a piezoelectric element under pressure, part of the energy is converted to an electric potential that appears on opposite faces of the element, analogous to a charge on the plates of a capacitor. The rest of the applied energy is converted to mechanical energy, analogous to a compressed spring. When the pressure is removed, it returns to its original shape and losses its electric change from these relationship the following formulas have been derived for the coupling coefficient.

$$k = \frac{\text{Mechanical energy converted to electrical energy}}{\text{Applied mechanical energy}}$$

$$k = \frac{\text{Electrical energy converted to mechanical energy}}{\text{Applied electrical energy}}$$

The disadvantage is that voltage will be generated as long as the pressure applied to the piezoelectric element changes.

c) Temperature transducers

Temperature is one of the most widely measuring and controlling parameter in the process industry, as a lot of products during manufacturing require controlled temperature at various stages of processing. A wide variety of temperature transducers and temperature measurement system have been developed for different applications requirements. There are many variety in temperature transducer. Most of the temperature transducers are Resistance Temperature Detectors (RTD), Thermistors or Thermocouples. Of these, RTDs and Thermistors are passive devices whose resistance changes with temperature hence needing an electrical supply to give a voltage output. On the other hand, thermocouples are active transducers and are based on the principle of generation of thermoelectricity, when two dissimilar metals are connected together to form a junction called the sensing junction, an emf is generated which is proportional to the temperature of the junction. Thermocouples maintain the principle of Seeback effect. Thermocouple introduces errors and can be overcome by using a reference junction compensation called as cold junction compensation. Thermocouples are available that span from the cryogenic temperatures to 2000°C temperature range. RTD commonly uses platinum, Nickel or any resistance wire that has resistance and that varies with temperature and has a high intrinsic accuracy. Platinum is the most widely used RTD for its high stability and large operating range.

Temperature measurement is also possessed through the use of thermistor. A thermistor is a thermally sensitive resistor that exhibits change in electrical resistance with change in temperature. Thermistors made up of oxides exhibit a negative temperature coefficient (NTC), that, their resistance decreases with increase in temperature. Thermistors are also available with positive temperature-coefficient (PTC), but PTC thermistor are seldom used for measurement because they have poor sensitivity. There are different thermistors according to their different sizes and shapes such as breads, rods, discs, washers and in the form of probes.

d) Strain gauge:

The strain gauge is an example of a passive transducer that uses the variation in electrical resistance in wires to sense the strain produced by a force on the wires.

Types of Strain Gauges

The following types of strain gauges are the most important:

i) Wire strain gauges
ii) Foil strain gauges
iii) Semiconductor strain gauges

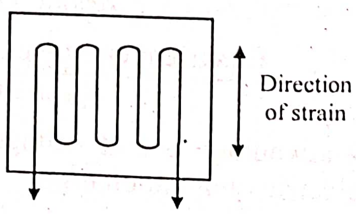


Fig: 1 Grid Type strain gauge

Wire Strain Gauge

1. Grid type
2. Rosette type
3. Torque type
4. Helical type

The grid arrangement of the wire element in a bonded strain gauge creates problem .But unbounded strain gauge does not create this problem. The wire element must measure strain along one axis to be useful as a strain gauge. When the direction and magnitude of stress are known the complete and accurate analysis of strain in rigid member is possible. The measuring axis of a strain gauge is along its longitudinal axis, which is parallel to the wire demand, as shown in Fig. 1 So there is an error in the response to the gauge. Error affects the strain being measured parallel to the longitudinal axis as well as the transverse axis of the gauge.

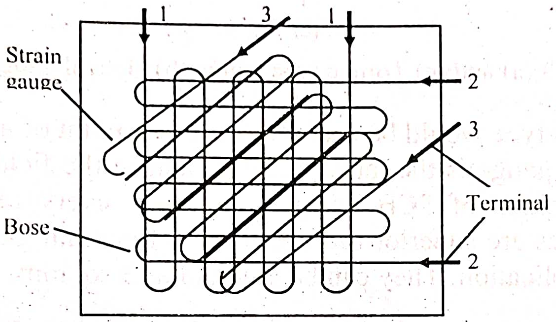
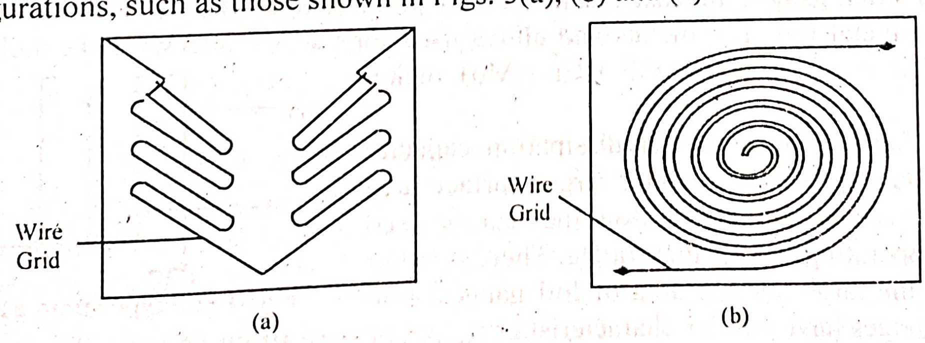


Fig: 2 Rossette Gauge

In most applications, some degree of strain is present along the transverse axis and the transverse sensitivity must be considered in the final gauge output. Transverse sensitivity cannot be completely eliminated, and in highly accurate measurements the resultant gauge error must be compensated for the error caused. Strain gauges may be used for the determination of the exact direction when the axis of the strain of a component is unknown. The standard procedure is to place several gauges at a point o the member's surface, with known angles between them. The magnitude of strain in each individual gauge is measured, and used in the geometrical determination of the strain in the member. Figure 2 shows a three-element strain gauge, called a Rosette gauge, in which the angle between any two longitudinal gauge axes is 45°.

The most popular is the 45° Rosette gauge. It has different shapes and sizes of strain gauges for various purposes.

Serving a similar, but not specialized, purpose are gauges with specially modified grid configurations, such as those shown in Figs. 3(a), (b) and (c).



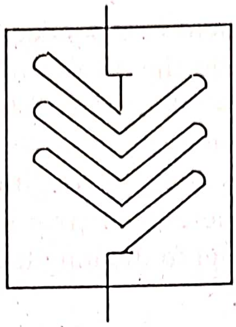

(c)

Fig: 3 (a) and (c) Torque type gauge (b) Helical gauge

A measurement of this type would be useful at the cross-point of an X-shaped frame. The etched foil strain gauge is the latest development in the field of strain gauges. This device uses the technique of PCB design. In almost every respect its physical and electrical characteristics are superior to the bonded wire strain gauges. The size of strain gauges varies with application. They can be as small as 3 sq. mm.

Characteristic of Wire Strain Gauge

1. Hysteresis effect should not be present in the strain gauge response.
2. It has a high value of gauge factor (this is because a high value of gauge factor indicates a large change in resistance for particular strain and thus resulting in high sensitivity).
3. Using leads are must be of materials which have low and stable resistivity and low resistance temperature coefficient.
4. Strain gauges are frequently used for dynamic measurements and so their frequency response should be very good. Linearity should be maintained within specified accuracy limits over the entire frequency range.

The strain gauge should have a low resistance temperature coefficient. This is necessary to minimize errors on account of temperature variation, which affects the accuracy of measurements.

Foil Strain Gauge

This class of strain gauges increase resistance wire strain gauge. The strain is sensed with the help of a metal foil. The metals and alloys used for the foil and wire are nichrome, constantan (Ni + Cu), isoelastic (Ni + Cr + Mo), nickel and platinum.

Foil gauges have a much greater dissipation capacity than wire wound pages as for their larger surface area for the same volume. For this reason, they can be used for a higher operating temperature range. There is better bonding for the large surface area of foil gauges. Foil type strain gauges have similar characteristics to that of wire strain gauges. Their gauges factors are typically the same. The advantage of foil type strain gauges is that they can be fabricated on a large scale, and in any shape. The foil can also be etched on a carrier.

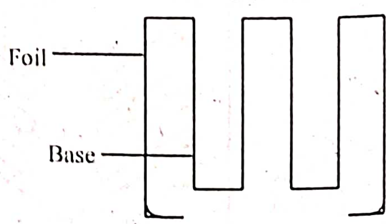
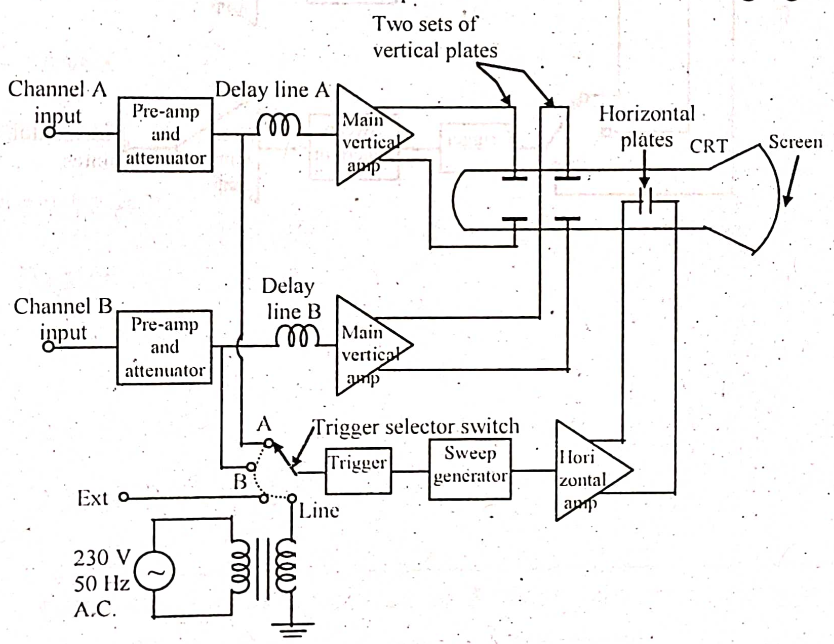


Fig: 4 Foil type strain gauge.

This method of construction enables etched foil strain gauges to be made rather than comparable wire units, as shown in Fig. 4. This characteristics, along with a greater degree of flexibility, makes it mount for the etched foil a more remote and restricted placed and on a wide range of curved surfaces.
The sensitivity of the foil gauge, which is longitudinal is approximately 5% greater than that of similar wire elements. The transverse strain sensitivity of the gauge is smaller 1/3 to ½ of same types of wire gauges. The hysteresis of the foil gauge is also 1/3 to ½ of a wire strain gauge.
The resistance film formed is typically 0.2mm thick. The resistance value of commercially available foil gauges is between 50 and 1000Ω. The resistance films are vacuum coated with ceramic film and deposited on a plastic backing for insulation.

e) Dual Beam Oscilloscope:
The block diagram of dual beam oscilloscope is shown in the following figure.



The oscilloscope has two vertical deflection plates and two separate channels A and B for the two separate input signals. Each channel consists of a preamplifier and an attenuator. A delay line, main vertical amplifier and a set of vertical deflection plates together forms a single channel.
There is a single set of horizontal plates and single time base circuit.
The sweep generator drives the horizontal amplifier which in turn drives the plates. The horizontal plates sweep both the beams across the screen at the same rate.
The sweep generator can be triggered internally by the channel A signal or channel B signal.
Similarly it can also be triggered from an external signal or line frequency signal. This is possible with the help of trigger selector switch, a front panel control.
Such an oscilloscope may have separate time base circuit for separate channel.

This allows different sweep rates for the two channels but increases the size and weight of the oscilloscope.
The dual beam oscilloscope with separate time base circuits is shown in the following figure.