NUMERICAL METHODS
(OEC - IT601A)

3/1/2024

Syllabus

1. Introduction, error terms, error analysis (MCQ/SAQ) (marks: 5-6)
2. Interpolation (marks ≥ 25)
   - equal spacing
   - unequal spacing
3. Numerical Integration
4. Soln of Linear Eq.
5. Numerical Soln of algebric & transcendental Eq.
6. " " of O.D.E

Books —
- (i) S.A. Mollah **
- (ii) E. Balaghoswami
- (iii) Dutta & Jana

MODULE - 1 → ERROR

Error terms
- → inherent (error present in parent/data set)
- → computational (error during calculation)
  - → truncation error (eg. $3.14\underbrace{312}_{\times} \to 3.14$) → (inf series to finite terms only)
  - → rounding off " (eg. 24.3836 correct upto 3 significant digits ↓ 24.4)

eg. inherent error

a data set is given & it is told to use $\pi = 3.14$. (actual value of $\pi > 3.14$)

NUMBERS

Numbers
- → Exact (eg. $5, \frac{2}{3}, \sqrt{3}, \pi$)
- → Approximate (approx. of exact numbers) (eg. $\pi = 3.14, \sqrt{3} = 1.732$)

SIGNIFICANT FIGURES / DIGIT

24.1234 → 6
0.1032 → 4
0.003 → 1
0.0102 → 3
12.102 → 5

} no. of sig. digits

37.19631 Round off upto 3 sig digits? → 37.2

SIGNIFICANT ERROR

$a = 12.3718$ (6 sig. digit)
$b = 12.3715$ (")
$c = a - b = 0.0003$ (1 sig digit)

5 sig. digits are lost
$\Rightarrow$ significant error

ERROR ANALYSIS

- Absolute Error $(E_A) = |V_T - V_A|$

true value (V_T)   approx. value (V_A)

- Relative Error $(E_R) = \dfrac{E_A}{V_T}$

- Relative percentage error $(E_p) = E_R \times 100\%$

Q. Write the approx. value of $\frac{2}{3}$ (correct upto 4 sig. digits) & find E_A, E_R and E_p

$V_T = \frac{2}{3}$

$V_A = 0.6667$

$E_A = |V_T - V_A| = \cancel{0.00003333333} = \dfrac{0.0001}{3}$

$E_R = \dfrac{E_A}{V_T} = \dfrac{0.0001}{\cancel{3} \times \frac{2}{\cancel{3}}} = 0.00005$

$E_p = 0.005$

GENERAL FORMULA FOR ERROR

9/1/24

$u = f(x, y, z)$

- $E_A = \Delta u = \left|\dfrac{\partial f}{\partial x} \cdot \Delta x\right| + \left|\dfrac{\partial f}{\partial y} \cdot \Delta y\right| + \left|\dfrac{\partial f}{\partial z} \cdot \Delta z\right|$

- $E_R = \dfrac{\Delta u}{u}$

- $E_p = E_R \times 100\% = \dfrac{\Delta u}{u} \times 100\%$

Q1. Find relative max^m error in the func. $u = \dfrac{5xy^2}{z^3}$ at $x = y = z = 1$ with $\Delta x = \Delta y = \Delta z = 0.001$

$$E_A = \left|\frac{5y^2}{z^3}\Delta x\right| + \left|\frac{10xy}{z^3}\Delta y\right| + \left|\frac{-15xy^2}{z^4}\Delta z\right|$$

$$= |5 \times 0.001| + |10 \times 0.001| + |-15 \times 0.001|$$

$$= 0.005 + 0.01 + 0.015$$

$$= 0.03$$

$$E_R = \frac{\Delta u}{u} = \frac{0.03}{5} = 0.006 \text{ (Ans)}$$

Q2. If $y = 4x^6 - 5x$ find % error in y at $x = 1$, if the error in $x = 0.04$

$$\Delta y = \left|\frac{\partial y}{\partial x} \cdot \delta x\right| = \left|(24x^5 - 5) \times 0.04\right|$$

$$= 19 \times 0.04$$

$$= 0.76.$$

$$\% \text{ error} = \left|\frac{\Delta y}{y}\right| \times 100\% = \left|\frac{0.76}{-1}\right| \times 100\%$$

$$= 76\%$$

Q. If $f(x) = 4\cos x - 6x$, find the relative % error in $f(x)$, for $x = 0$, if the error in $x = 0.005$

let $y = f(x) = 4\cos x - 6x$

$$\Delta y = \left|\frac{\partial y}{\partial x} \cdot \delta x\right| = \left|(-4\sin x - 6) \times 0.005\right|$$

$$= 0.03$$

$$\% \text{error} = \frac{\Delta y}{y} \times 100\% = \frac{0.03}{4} \times 100\% = 0.0075 \times 100\%$$

$$= 0.75\%$$

MODULE – 2

INTERPOLATION

The method or technique for estimating unknown values from a given set of observations is known as interpolation.

Types

Interpolation (based on $h \rightarrow$ interval in data set)

- Equal interval
  - $\rightarrow$ Newton forward interpolation
  - $\rightarrow$ Newton backward interpolation
  - $\rightarrow$ central tendency
  - $\rightarrow$ Gauss forward
  - $\rightarrow$ Gauss backward
  - $\rightarrow$ stirling eqns.
  - $\rightarrow$ Bassel's eqns.
- unequal interval
  - $\rightarrow$ Lagrange Interpolation
  - $\rightarrow$ Newton divided difference diff.

Newton Forward Interpolation

imp terms

- $h \rightarrow$ interval
- a dimensionless quantity, u = phase $= \dfrac{x - x_0}{h}$
- $x \rightarrow$ unknown (asked for)

$$f(x) = y_0 + u.\Delta y_0 + \frac{u.(u-1).\Delta^2 y_0}{2!} + \frac{u.(u-1).(u-2).\Delta^3 y_0}{3!} + \ldots$$

Eg.

Population dataset

	$f(x) = y$
1891	46
1901	66
1911	81
1921	93
1931	101

1895 = ??

$\therefore x = 1895$

[x is at start of table $\Rightarrow$ Newton forward

x is at end of table $\Rightarrow$ Newton backward]

Sol.

$h = 10$

$\Delta y = f(x+h) - f(x)$

difference table

x	$f(x) = y$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	46				
		66-46=20			
1901	66		15-20=-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

$$u = \frac{x - x_0}{h} = \frac{1895 - 1891}{10} = 0.4$$

$$f(1895) = y_0 + u.\Delta y_0 + \frac{u(u-1)\Delta^2 y_0}{2!} + \frac{u(u-1)(u-2)\Delta^3 y_0}{3!} + \frac{u(u-1)(u-2)(u-3)\Delta^4 y_0}{4!}$$

$$= 46 + 0.4 \times 20 + \frac{0.4 \times (0.4-1) \times (-5)}{2} + \frac{0.4(0.4-1)(0.4-2) \times 2}{6} + \frac{0.4(0.4-1)(0.4-2)(0.4-3) \times (-3)}{24}$$

$= \underline{54.8528}$. (b/w 46 & 66)

$\simeq 55$

10/1/24

Steps

1. check h (interval) $\begin{cases} \text{equal} \\ \text{unequal} \end{cases}$
2. check location of unknown (x) $\begin{cases} \text{at start of table} \rightarrow \text{Newton Forward} \\ \text{at end of table} \rightarrow \text{Newton Backward} \end{cases}$
3. calculate u (phase value)
4. create difference table
5. use general formula to find f(x)

Q2.

x	f(x)=y	Δy	$\Delta^2 y$	$\Delta^3 y$		
1	1	5	14	2	9	-33
2	6	19	16	11	-24	
3	25	35	27	-13		
4	60	62	14			
5	122	76				
6	198					

find $f(1.5) = ?$

$h = 1$

$$u = \frac{1.5 - 1}{1} = 0.5$$

$$f(1.5) = 1 + 0.5 \times 5 + \frac{0.5 \times (0.5-1) \times 14}{2!} + \frac{0.5 \times (0.5-1) \times (0.5-2) \times 2}{3!} + \frac{0.5 \times (0.5-1) \times (0.5-2) \times (0.5-3) \times 9}{4!} + \frac{0.5 \times (0.5-1) \times \ldots \times (0.5-4) \times (-33)}{5!}$$

$= 0.6211$

Q3.

Find out the frequency for the salary range 10k- 15k with the help of given data

Salary	0—10	10 — 20	20— 30	30—40
Freq	9	30	35	42

~~h = 10~~

~~10—15 → $\frac{15+10}{2} = 12.5$~~

x	$f(x)=y$	Δy	$\Delta^2 y$	$\Delta^3 y$
below 10	9			
below 20	39	30	5	2
below 30	74	35	7	
below 40	116	42		

$h = 10$

$u = \frac{15-10}{10} = 0.5$

$f(15) = 9 + 0.5 \times 30 + \frac{0.5 \times (0.5-1) \times 5}{2!} + \frac{0.5 \times (0.5-1) \times (0.5-2) \times 2}{3!}$

$= 23.5$

$\simeq 24$

$\therefore$ below 15 = 24

$\therefore$ in range 10—15 = 24 − 9 = 15

16/1/24

Q4. Find the lowest degree polynomial $y(x)$ that ~~find~~ fit the given data & find $f(5)$

x	0	2	4	6	8
y	5	9	61	209	501

$h = 2$

$u = \frac{x - x_0}{2} = \frac{x-0}{2} = \frac{x}{2}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	5				
2	9	4	48	48	0
4	61	52	96	48	
6	209	148	144		
8	501	292			

$y(x)$

$$= f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)\,\Delta^2 y_0}{2!} + \frac{u(u-1)(u-2)\Delta^3 y_0}{3!} + \frac{u(u-1)(u-2)(u-3)\,\Delta^4 y_0}{4!}$$

$$= 5 + \frac{x}{\cancel{2}}\cdot\cancel{4}^{2} + \frac{\frac{x}{2}\left(\frac{x}{2}-1\right)\cdot\cancel{48}^{24}}{\cancel{2}} + \frac{\frac{x}{2}\left(\frac{x}{2}-1\right)\left(\frac{x}{2}-2\right)\cdot\cancel{48}\,8}{\cancel{6}} + 0$$

$$= 5 + 2x + \frac{x}{\cancel{2}}\cdot\frac{x-2}{\cancel{2}}\cdot\cancel{24}^{6} + \frac{x}{\cancel{2}}\left(\frac{x-2}{\cancel{2}}\right)\left(\frac{x-4}{\cancel{2}}\right)\cdot\cancel{8}$$

$$= 5 + 2x + 6x^2 - 12x + (x^2-2x)(x-4)$$

$$= 5 + 2x + \cancel{6x^2} - 12x + x^3 - \cancel{4x^2} - \cancel{2x^2} + 8x$$

$= 5 - 2x + x^3 \rightarrow$ required polynomial.

$$\therefore f(5) = 5^3 - 2\times 5 + 5$$
$$= 125 - 10 + 5$$
$$= 120 \text{ (Ans)}$$

Q5. Find from the following table, the value of y when $\underbrace{x = 1.22}_{\text{mid of table}}$ & $\underbrace{x = 1.54}_{\text{Backward interpolation}}$

x	1	1.1	1.2	1.3	1.4	1.5
y	24	21	19	17	14	12

Sol.

$h = 0.1$

$$u = \frac{1.22 - 1.2}{0.1} = \frac{0.02}{0.1} = 0.2$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	24	-3				
1.1	21		1			
$x_0 \rightarrow$ 1.2	19 $\leftarrow y_0$	-2 Δy_0	0	-1	0	3
		-2	-1 $\leftarrow \Delta^2 y_1$	-1	3	
1.3	17					
1.4	14	-3	1	2 $\Delta^3 y_2$		
1.5	12	-2				

1.22 is close to 1.2

$$y(1.22) = 19 + 0.2\cdot(-2) + \frac{0.2(0.2-1)\cdot(-1)}{2!} + \frac{0.2(0.2-1)(0.2-2)\,2}{3!}$$

$$= 18.776$$

Newton Backward Interpolation

$x \to$ unknown

$x_n \to$ the data placed at end of table.

h = interval

$$u = \text{phase} = \frac{x - x_n}{h}$$

$$f(x) = y_n + u\nabla y_n + \frac{u(u+1)\nabla^2 y_n}{2!} + \frac{u(u+1)(u+2)\nabla^3 y_n}{3!} + \cdots$$

$$\nabla f(x) = f(x) - f(x-h)$$

Eg — Population dataset

Q1.

1891	1901	1911	1921	1931
46	66	81	93	101

Find $x = 1925 \to$?

	y	Δy	$\Delta^2 y$	$\Delta^3 y$	
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
$x_n \to$ 1931	101 (y_n)				

$h = 10$

$$u = \frac{1925 - 1931}{h} = -0.6$$

$$f(1925) = 101 + (-0.6)8 + \frac{(-0.6)(-0.6+1)(-4)}{2!} + \frac{(-0.6)(-0.6+1)(-0.6+2)(-1)}{3!} + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-3)}{4!}$$

$$= 96.8368 \simeq 97 \text{ (Ans)}$$

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Q2. Find the value of $f(7.5) = ?$ for the dataset

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1				
		7			
2	8		12		
		19		6	
3	27		18		0
		37		6	
4	64		24		0
		61		6	
5	125		30		0
		91		6	
6	216		36		0
		127		6	
7	343		42		
		169			
8	512				

$$\therefore u = \frac{x - x_n}{h} = \frac{7.5 - 8}{1} = -0.5$$

$$f(7.5) = 512 + (-0.5)\times 169 + \frac{(-0.5)(-0.5+1)\,42}{2!} + \frac{(-0.5)(-0.5+1)(-0.5+2)\times 6}{3!}$$

$$= 512 - 84.5 - 5.25 - 0.375$$

$$= 421.875 \text{ (Ans)}$$

Q3. Find the cubic polynomial ~~which takes~~

x	0	1	2	3
y	1	2	1	10

Sol

$$u = \frac{x-3}{1} = (x-3)$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
1	2	1	-2	
2	1	-1	10	12
3	10	9		

$$\therefore f(x) = 10 + (x-3)9 + \frac{(x-3)(x-2)\times 10}{2!} + \frac{(x-3)(x-2)(x-1)\times 12}{3!}$$

$$= 10 + 9x - 27 + \frac{(x^2-5x+6)\times \cancel{10}^{5}}{\cancel{2}} + \frac{(x^3 - 5x^2 + 6x - x^2 + 5x - 6)\times \cancel{12}^{2}}{\cancel{6}}$$

$$= 10 + 9x - 27 + 5x^2 - 25x + 30 + 2x^3 - 10x^2 + 12x - 2x^2 + 10x - 12$$

$$= 2x^3 - 7x^2 + 6x + 1$$

Q4. (mix category)

The table gives the dist. (in nautical miles) of the visible horizon for the given heights (in feets) above the earth's surface.

x (height)	100	150	200	250	300	350	400
y (dist.)	10.63	13.03	15.04	16.81	18.42	19.91	21.27

Find y when (i) $x = 160$ ft

(ii) $x = 410$ ft.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
100	10.63						
		2.4					
150	13.03		−0.39				
		2.01		0.15			
200	15.04		−0.24		−0.07		
		1.77		0.08		0.02	
250	16.81		−0.16		−0.05		0.02
		1.61		0.03		0.04	
300	18.42		−0.13		−0.01		
		1.48		0.02			
350	19.9		−0.11				
		1.37					
400	21.27						

(i) $x = 160$

$$u = \frac{160-150}{50} = \frac{1\cancel{0}}{5\cancel{0}} = 0.2$$

$$f(160) = 13.03 + 0.2 \times 2.01 + \frac{(0.2)(0.2-1) \times (-0.24)}{2} + \frac{0.2(0.2-1)(0.2-2)\,0.08}{6}$$

$$+ \frac{0.2(0.2-1)(0.2-2)(0.2-3) \times (-0.05)}{24} + \frac{0.2(0.2-1)\cdots(0.2-4) \times 0.04}{120}$$

$$= 13.46$$

(ii) $x = 410$

$$u = \frac{410-400}{50} = 0.2$$

$$f(410) = 21.27 + (0.2 \times 1.37) + \frac{(0.2)(0.2+1) \times (-0.11)}{2} + \frac{0.2(0.2+1)(0.2+2)(0.02)}{6}$$

$$+ \frac{0.2(0.2+1)(0.2+2)(0.2+3)(-0.01)}{24} + \frac{0.2(0.2+1)\cdots(0.2+4) \times 0.04}{120}$$

$$+ \frac{0.2(0.2+1)\cdots(0.2+5) \times 0.02}{720}$$

$$= 21.53$$

Q5. In the table below, the values of y are consecutive term of a series of which 23.6 is the 6th term. Find the first & 10th term of the series.

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	51.8	73.9

Ans — $f(1) = 3.1$

$f(10) = 100$

24/01/24

Lagrange Interpolation Method.

Scope – used for unequal spacing category $[x_1 - x_0 \neq x_2 - x_1]$
(h is varying)

obj – to find the value of y for the given value of x
(which is not listed in the provided dataset)

x – given value for which y is asked.

x_0 – start of data set

x_n – end " "

Eg –

$x_0 \rightarrow y_0$

$x_1 \rightarrow y_1$

$\vdots$

$x_n \rightarrow y_n$

x is given

asked

$f(x) = y = ?$

General formula

$$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} \times f(x_0) + \cdots\cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} \times f(x_n)$$

Q1. Calculate the value of y, when $x = 10$ by using Lagrange's interpolation for the given dataset.

	x_0	x_1	x_2	x_3
x	5	6	9	11
y	12	13	14	16

Sol.

$$f(x) = f(10) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$+ \cdots\cdots + \cdots\cdots$$

$$= \frac{\overset{4}{(10-6)}\overset{1}{(10-9)}\overset{-1}{(10-11)}}{\underset{-1}{(5-6)}\underset{-4}{(5-9)}\underset{-6}{(5-11)}} \times 12 + \frac{\overset{5}{(10-5)}\overset{1}{(10-9)}\overset{-1}{(10-11)}}{\underset{1}{(6-5)}\underset{-3}{(6-9)}\underset{-5}{(6-11)}} \times 13$$

$$+ \frac{\overset{5}{(10-5)}\overset{4}{(10-6)}\overset{-1}{(10-11)}}{\underset{4}{(9-5)}\underset{3}{(9-6)}\underset{-2}{(9-11)}} \times 14 + \frac{\overset{5}{(10-5)}\overset{4}{(10-6)}\overset{1}{(10-9)}}{\underset{6}{(11-5)}\underset{5}{(11-6)}\underset{2}{(11-9)}} \times 16$$

$$= \frac{24}{24} \times 12 + \left(-\frac{5}{15} \times 13\right) + \left(\frac{20}{24} \times 14\right) + \left(\frac{20}{60} \times 16\right)$$

$$= 2 - \frac{13}{3} + \frac{35}{3} + \frac{16}{3}$$

$$\simeq 14.67$$

Q2. Find the polynomial $f(x)$ by using lagrange's formula for the given data set.

x	0	1	2	5
y	2	3	12	147

find $f(3)$?

$$f(x) = \frac{(x-1)(x-2)(x-5)}{\underset{-1}{(0-1)}\,\underset{-2}{(0-2)}\,\underset{-5}{(0-5)}} \times 2 + \frac{(x-0)(x-2)(x-5)}{\underset{1}{(1-0)}\,\underset{-1}{(1-2)}\,\underset{-4}{(1-5)}} \times 3$$

$$+ \frac{(x-0)(x-1)(x-5)}{\underset{2}{(2-0)}\,\underset{1}{(2-1)}\,\underset{-3}{(2-5)}} \times 12 + \frac{(x-0)(x-1)(x-2)}{\underset{5}{(5-0)}\,\underset{4}{(5-1)}\,\underset{3}{(5-2)}} \times 147$$

$$= \frac{(x^2-3x+2)(x-5)}{-\cancel{10}\ 5} \times \cancel{2} + \frac{x(x^2-7x+10)}{4} \times 3$$

$$+ \frac{x(x^2-6x+5)}{-\cancel{6}} \times \overset{2}{\cancel{12}} + \frac{x(x^2-3x+2)}{\cancel{60}\ 20} \times \overset{49}{\cancel{147}}$$

$$= -\frac{x^3-3x^2+2x-5x^2+15x-10}{5} + \frac{3x^3-\overset{21}{\cancel{9}}x^2+30x}{4}$$

$$- \frac{2x^3-12x^2+10x}{1} + \frac{49x^3-147x^2+98x}{20}$$

$$= \frac{-4x^3+12x^2-8x+20x^2-60x+40+15x^3-105x^2+150x-40x^3+240x^2-200x+49x^3-147x^2+98x}{20}$$

$$= \frac{20x^3+20x^2-20x+40}{20}$$

$$= x^3+x^2-x+2$$

$f(3) = 27+9-3+2$

$= 35$

Q3. A car passes through the points $(0,18)$, $(1,10)$, $(3,-18)$, $(6,90)$. Find the slope of the curve at $x=2$.

Sol.

x	0	1	3	6
y	18	10	-18	90

$$y = f(x) = \frac{(x-1)(x-3)(x-6)}{(0-1)(0-3)(0-6)} \times 18 + \frac{(x-0)(x-3)(x-6)}{(1-0)(1-3)(1-6)} \times 10$$

$-1 \quad (-3) \quad (-6) \qquad\qquad 1 \quad -2 \quad -5$

$$+ \frac{(x-0)(x-1)(x-6)}{(3-0)(3-1)(3-6)} \times (-18) + \frac{(x-0)(x-1)(x-3)}{(6-0)(6-1)(6-3)} \times 90$$

$3 \quad 2 \quad -3 \qquad\qquad 6 \quad 5 \quad 3$

$$= \frac{(x^2-4x+3)(x-6)}{\cancel{-18}} \times \cancel{18} + \frac{x(x^2-9x+18)}{\cancel{10}} \times \cancel{10} + \frac{x(x^2-7x+6)}{\cancel{-18}} \times \cancel{(-18)}$$

$$+ \frac{x(x^2-4x+3)}{\cancel{90}} \times \cancel{90}$$

$$= -(x^3 + 4x^2 - 3x + 6x^2 - 24x + 18) + x^3 - 9x^2 + 18x +$$

$$\cancel{x^3} - 7x^2 + 6x + x^3 - 4x^2 + 3x$$

$$= 2x^3 - 10x^2 + 18$$

$$\frac{dy}{dx} = 6x^2 - 20x$$

$$\left.\frac{dy}{dx}\right|_{x=2} = 6 \times 4 - 20 \times 2$$

$$= 24 - 40$$

$$= -16$$

Q4. calculate $f(0.3)$ by using lagrange's interpolation

x	0	1	3	4	7
y	1	3	49	129	813

Ans -1.831 (app)

$$f(0.3) = \frac{(0.3-1)(0.3-3)(0.3-4)(0.3-7)}{(0-1)(0-3)(0-4)(0-7)} \times 1 + \frac{(0.3-0)(0.3-3)(0.3-4)(0.3-7)}{(1-0)(1-3)(1-4)(1-7)} \times 3$$

$$+ \frac{(0.3-0)(0.3-1)(0.3-4)(0.3-7)}{(3-0)(3-1)(3-4)(3-7)} \times 49 + \frac{(0.3-0)(0.3-1)(0.3-3)(0.3-7)}{(4-0)(4-1)(4-3)(4-7)} \times 129$$

$$+ \frac{(0.3-0)(0.3-1)(0.3-3)(0.3-4)}{(7-0)(7-1)(7-3)(7-4)} \times 813$$

$$= \frac{(-0.7)(-2.7)(-3.7)(-6.7)}{(-1)(-3)(-4)(-7)} \times 1 + \frac{0.3(-2.7)(-3.7)(-6.7)}{1.(-2)(-3)(-6)} \times 3$$

$$+ \frac{0.3(-0.7)(-3.7)(-6.7)}{3.2.(-1)(-4)} \times 49 + \frac{0.3(-0.7)(-2.7)(-6.7)}{4.3.1.(-3)} \times 129$$

$$+ \frac{0.3(-0.7)(-2.7)(-3.7)}{7.6.4.3} \times 813$$

$$= -1.831$$

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Inverse Lagrange's Interpolation

$$x = \frac{(y-y_1)(y-y_2)\cdots(y-y_n)}{(y_0-y_1)(y_0-y_2)\cdots(y_0-y_n)} \times x_0 + \cdots\cdots\cdots$$

*Q4. Using Lagrange's formula express the func. $\dfrac{3x^2+x+1}{(x-1)(x-2)(x-3)}$ as a sum of partial functions.

find the value of y at $\begin{cases} x=1 \\ x=2 \\ x=3 \end{cases}$

Sol.

$y = 3x^2+x+1$

x	1	2	3
y	5	15	31

$$\therefore\ y = \left[\frac{(x-2)(x-3)}{(1-2)(1-3)} \times 5\right] + \left[\frac{(x-1)(x-3)}{(2-1)(2-3)} \times 15\right] + \left[\frac{(x-1)(x-2)}{(3-1)(3-2)} \times 31\right]$$

$$y = 2.5(x-2)(x-3) - 15(x-1)(x-3) + 15.5(x-1)(x-2)$$

$$\therefore\ \frac{3x^2+x+1}{(x-1)(x-2)(x-3)} = \frac{2.5}{(x-1)} - \frac{15}{(x-2)} + \frac{15.5}{(x-3)}$$

Q5. Find the distance moved by a particle & its acc^n at the end of 4s, if the time vs velocity data is as follows:

t	0	1	3	4
v	21	15	12	10

$\left.\begin{matrix} s = ? \\ a = ? \end{matrix}\right]$ at $t = 4s$.

$s = 54.9$

$a|_{t=4} = -3.4$

$v = \dfrac{ds}{dt}$, $a = \dfrac{dv}{dt}$

$$v = \frac{(t-1)(t-3)(t-4)}{\underset{-1}{(0-1)}\underset{-3}{(0-3)}\underset{-4}{(0-4)}} \times 21 + \frac{(t-0)(t-3)(t-4)}{\underset{1}{(1-0)}\underset{-2}{(1-3)}\underset{-3}{(1-4)}} \times 15$$

$$+ \frac{(t-0)(t-1)(t-4)}{\underset{3}{(3-0)}\underset{2}{(3-1)}\underset{(-1)}{(3-4)}} \times 12 + \frac{(t-0)(t-1)(t-3)}{\underset{4}{(4-0)}\underset{3}{(4-1)}\underset{1}{(4-3)}} \times 10$$

$$= -\frac{7}{4}(t^2-4t+3)(t-4) + \frac{5}{2}t(t^2-7t+12) - 2t(t^2-5t+4)$$

$$+ \frac{5}{6}t(t^2-4t+3)$$

$$= -\frac{7}{4}(t^3-4t^2-4t^2+16t+3t-12) + \frac{5}{2}(t^3-7t^2+12t) - 2(t^3-5t^2+4t)$$

$$+ \frac{5}{6}(t^3-4t^2+3t)$$

$\frac{21}{12}\ \frac{7}{4}$

$\frac{15}{6}\ \frac{5}{2}$

$\frac{10}{12}\ \frac{5}{6}$

$$= \frac{-21t^3 + 168t^2 - 399t + 252 + 30t^3 - 210t^2 + 360t - 24t^3 + 128t^2 - 96t + 10t^3 - 40t^2 + 30t}{12}$$

$$= \frac{1}{12}\left(-5t^3 + 38t^2 - 105t + 252\right)$$

$$v = \frac{ds}{dt} \Rightarrow s = \int v\,dt$$

$$= \frac{1}{12}\int \left(-5t^3 + 38t^2 - 105t + 252\right) dt$$

$$= \frac{1}{12}\left(-5\frac{t^4}{4} + 38\frac{t^3}{3} - 105\frac{t^2}{2} + 252t\right) + c$$

$$= -\frac{5t^4}{48} + \frac{\cancel{38}^{19}t^3}{\cancel{36}_{18}} - \frac{\cancel{105}^{35}t^2}{\cancel{24}_{8}} + \frac{\cancel{252}^{21}t}{\cancel{12}} + c$$

$$= \left[-\frac{5t^4}{48} + \frac{19t^3}{18} - \frac{35t^2}{8} + 21t\right]_0^4$$

$$= -\frac{5 \times \cancel{256}^{16}}{\cancel{48}_{3}} + \frac{19 \times \cancel{64}^{32}}{\cancel{18}_{9}} - \frac{35 \times \cancel{16}^{2}}{\cancel{8}} + 21 \times 4$$

$$= 54.889$$

$$\simeq 54.9$$

$$a = \frac{dv}{dt} = \frac{1}{12}\left(-15t^2 + 76t - 105\right)$$

$$\Rightarrow \left.\frac{dv}{dt}\right|_{t=4} = \frac{1}{12}\left(-15 \times 16 + 76 \times 4 - 105\right)$$

$$= -\frac{41}{12}$$

$$= -3.4167$$

Inverse Lagrange's interpolation

Q1. Find the value of x corresponding to $y = \cancel{12}\ 7$ using Lagrange's technique, for the following table.

x	1.2	2.1	2.8	~~4.1~~	~~4.9~~	~~6.2~~
y	4.2	6.8	9.8	~~13.4~~	~~15.5~~	~~19.6~~

$$\therefore \cancel{x = \frac{(12-6.8)(12-9.3)(12-13.4)(4.2-15.5)(12-19.6)}{(4.2-6.8)}}$$

$$x = \frac{(7-6.8)(7-9.8)}{(4.2-6.8)(4.2-9.8)} \times 1.2 + \frac{(7-4.2)(7-9.8)}{(6.8-4.2)(6.8-9.8)} \times 2.1$$

$$+ \frac{(7-4.2)(7-6.8)}{(9.8-4.2)(9.8-6.8)} \times 2.8$$

$$= \frac{0.2 \times (-2.8)}{(-2.6)(-5.6)} \times 1.2 + \frac{(2.8)(-2.8)}{2.6 \times (-3)} \times 2.1$$

$$+ \frac{2.8 \times 0.2}{5.6 \times 3} \times 2.8 \qquad 0.0933$$

$$= 2.1579 \simeq 2.16$$

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Q2. Apply Lagrange formula inversely to obtain the root of the eq $f(x) = 0$, given that $f(30) = -30$, $f(34) = -13$, $f(38) = 3$, & $f(42) = 18$.

Sol.

	x_0	x_1	x_2	x_3
x	30	34	38	42
y	-30	-13	3	18
	y_0	y_1	y_2	y_3

$y = 0$

$$x = \frac{(0-(-13))(0-3)(0-18)}{(-30-(-13))(-30-3)(-30-18)} \times 30 + \frac{(0-(-30))(0-3)(0-18)}{(-13-(-30))(-13-3)(-13-18)} \times 34$$

$$+ \frac{(0-(-30))(0-(-13))(0-18)}{(3-(-30))(3-(-13))(3-18)} \times 38 + \frac{(0-(-30))(0-(-13))(0-3)}{(18-(-30))(18-(-13))(18-3)} \times 42$$

$$= 37.233$$

Newton Divided Difference Method

First prepare the Divided difference table from the given data set.

Then apply general formula & fit the values collecting from the prepared table.

$$f(x) = y = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + \dots$$
$$\dots\dots + (x-x_0)(x-x_1)\dots(x-x_n)\Delta^n y_0$$

Q1. Find out the value of $f(10)$ by using the given dataset following Newton Divided Diff. method.

	x_0	x_1	x_2	x_3
x	5	6	9	11
y	12	13	14	16
	y_0	y_1	y_2	y_3

$$\boxed{\Delta y_0 = \frac{y_1 - y_0}{x_1 - x_0}} \qquad \boxed{\Delta^2 y_0 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0}}$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
5	12	$\frac{13-12}{6-5} = 1$	$\frac{\frac{1}{3}-1}{9-5} = -\frac{1}{6}$	$\frac{\frac{2}{15}-\left(-\frac{1}{6}\right)}{11-5} = \frac{1}{20}$
6	13	$\frac{14-13}{9-6} = \frac{1}{3}$	$\frac{1-\frac{1}{3}}{11-6} = \frac{2}{15}$	
9	14	$\frac{16-14}{11-9} = 1$		
11	16			

$$f(10) = 12 + \left[(10-5)\,1\right] + \left[(10-5)(10-6)\left(-\tfrac{1}{6}\right)\right] + \left[(10-5)(10-6)(10-9) \times \tfrac{1}{20}\right]$$

$$= 12 + 5 + \left(-\frac{10}{3}\right) + 1$$

$$= \frac{44}{3} \approx 14.67$$

Q2. Evaluate $f(9)$ using Newton divided diff formula for the given dataset

x	5	7	11	13	17
$f(x) = y$	150	392	1452	2366	5202

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
5	150	$\frac{392-150}{7-5} = 121$	$\frac{265-121}{11-5} = 24$	$\frac{32-24}{13-5} = 1$	0
7	392	$\frac{1452-392}{11-7} = 265$	$\frac{457-265}{13-7} = 32$	$\frac{42-32}{17-7}$ 1	
11	1452	$\frac{2366-1452}{13-11} = 457$	$\frac{709-457}{17-11} = 42$		
13	2366	$\frac{5202-2366}{17-13} = 709$			
17	5202				

$$f(9) = 150 + (9-5)\times 121 + (9-5)(9-7)\times 24 + (9-5)(9-7)(9-11)\times 1$$

$$= 150 + 484 + 192 + (-16)$$

$$= 810$$