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Mathematics-III (Probability & Statistics)
Module-I

Probability Distribution

1. A random variable X has the density function

$$f(x) = \begin{cases} \frac{x}{2}, 0 \le x \le 1 \\ \frac{1}{2}, 1 < x \le 2 \\ \frac{1}{2}(3-x), 2 < x \le 3 \end{cases}$$

Find the mean and variance of X.

Ans: $\frac{3}{2}, \frac{5}{12}$

2. Show that the function $f(x) = \begin{cases} |x|, -1 < x < 1 \\ 0, \text{otherwise} \end{cases}$ is a probability density function and hence find the corresponding distribution function.

Ans: $F(x) = \begin{cases} 0, -\infty < x \le -1 \\ \frac{1}{2}(1-x^2), -1 < x \le 0 \\ \frac{1}{2}(1+x^2), 0 < x \le 1 \\ 1, 1 < x < \infty \end{cases}$

3. For what values of a, P(x) will be a pmf. of a discrete random variable X, whose probability distribution is given below:

x	0	1	2	3	4
P(x):	a	2a	$7a^2$	2a	a

Hence find (i) $P(X > 1)$ (ii) $P(X < 3/X \ge 1)$.
Ans: $a = \frac{1}{7}$(i) $\frac{4}{7}$ (ii) $\frac{2}{3}$

4. Let X is a random variable whose density function f forms an isosceles triangle above the unit interval $I = [0, \ 1]$ and 0 elsewhere. Then prove that
   a) Height of the triangle is 2.
   b) Formula for pdf is $f(x) = \begin{cases} 4x, \ 0 \le x \le \frac{1}{2} \\ -4x + 4, \ \frac{1}{2} \le x \le 1 \\ 0, \ \text{Elsewhere} \end{cases}$
   c) The mean of X is $\frac{1}{2}$.
5. The diameter of an electric cable, say X is assumed to be a continuous random variable with pdf $f(x) = 6x(1-x), 0 \le x \le 1$
   i) Check that $f(x)$ is a pdf
   ii) Determine a number b such that $P(X < b) = P(X \ge b)$.
   Ans: (ii) $b = \frac{1}{2}$.
6. A random variable X has the following probability mass function:

x:	-2	-1	0	1	2	3
$f(x)$	0.1	k	0.2	2k	0.3	3k

i) Find k

ii) Evaluate $P(X < 2), P(X \leq 2), P(-2 < X < 2)$.

iii) Determine the distribution function $F(x)$.

Answer: i) $\frac{1}{15}$ ii) 0.5, 0.8, 0.4

iii) $F(x) = \begin{cases} 0; -\infty < x < -2 \\ 0.1; -2 \leq x < -1 \\ \frac{1}{6}; -1 \leq x < 0 \\ \frac{11}{30}; 0 \leq x < 1 \\ \frac{1}{2}; 1 \leq x < 2 \\ \frac{4}{5}; 2 \leq x < 3 \\ 1; x \geq 3 \end{cases}$

7. The pdf of a random variable X is $f(x) = \left\{\frac{1}{4};\ -2 \leq x \leq 2\right.$, find

   i) $P(X < 1)$,

   ii) $P\left(|X - 1| \geq \frac{1}{2}\right)$

   Answer: i) $\frac{3}{4}$ ii) $\frac{3}{4}$

8. The pdf of a random variable X is given by $f(x) = \frac{3}{4}x(2 - x), 0 < x < 2$. Compute Mean and Variance.

   Answer: $1, \frac{1}{5}$

9. Let $f(x) = ke^{-\alpha x}(1 - e^{-\alpha x}), x \geq 0$. Find k such that $f(x)$ is a density function. Find also the corresponding distribution function.

   Answer: $2\alpha, 1 - 2e^{-\alpha x} + e^{2\alpha x}$

10. For a random variable X, $Var(X) = 1$, find $Var(2X + 3)$.

    Answer: 4

11. The length of life of tyre manufactured by a company follows a continuous distribution given by the density function $f(x) = \begin{cases} \frac{k}{x^3}; 1000 \leq x \leq 1500 \\ 0; \text{Elsewhere} \end{cases}$. Find k and find the probability that a randomly selected tyre would function for at least 1200 hours.

    Answer: 36×10^5, 0.45

12. The pdf of a continuous random variable X is given by $f(x) = \begin{cases} \frac{2(\beta + x)}{\beta(\alpha + \beta)};\ -\beta \leq x < 0 \\ \frac{2\alpha(\alpha - x)}{(\alpha + \beta)};\ 0 \leq x < \alpha \end{cases}$. Find the Expectation and Variance of X. Given $\alpha > \beta > 0$.

    Answer: $\frac{1}{3}(\alpha - \beta), \frac{1}{18}(\alpha^2 + \alpha\beta + \beta^2)$

13. Find the value of the constant k such that $f(x) = \begin{cases} kx(1 - x), 0 < x \leq 1 \\ 0, Else\ where \end{cases}$ is a possible density function and compute $P\left(X > \frac{1}{2}\right)$. Also find $E(X)$.

    Ans: (i) $k = 6$, (ii) 0.5, (iii) 0.5

14. The distribution function of a random variable X is $F(x) = \begin{cases} cx^3, 0 \leq x < 3 \\ 1, x \geq 3 \\ 0, x < 0 \end{cases}$. If $P(X = 3) = 0$ find

(i) The constant c

(ii) The density function.

(iii) $P(X > 1)$

(iv) $P(1 < X \leq 2)$

(v) $P(3X + 2 < 8)$

Ans: (i) 1/27 (ii) $f(x) = \frac{x^2}{9}, 0 \leq x < 3$ (iii) 26/27 (iv) 7/27 (v) 8/27

15. The pdf of a random variable X is $f(x) = cx^2, 0 \leq x \leq 1$. Find

(i) C

(ii) $P\left(0 \leq X \leq \frac{1}{2}\right)$

(iii) $P(4X > 3)$

(iv) $P(1 < 4X < 3)$

(v) Mean and S. D.

(vi) $E(5X - 8), Var(4X + 6)$

Ans: (i) 3, 1/8, 37/64, 13/32, 3/5

16. The pdf of a R. V. X is given by $f(x) = k(x - 1)(2 - x), 1 \leq x \leq 2$. Determine

(i) The value of k

(ii) The distribution function

(iii) $P\left(\frac{5}{4} \leq X \leq \frac{3}{2}\right)$

(iv) Mean and Variance

(v) $E(3X - 9)$

(vi) $Var(4X + 6)$

Ans: (i) 6 (ii) $F(x) = 5 - 12x + 9x^2 - 2x^3, 1 \leq x < 2$(iii) 11/32 (iv) 3/2, 1/20 (v) $-\frac{9}{2}$ (vi) $\frac{4}{5}$