MA 106: Linear Algebra

COMMON QUIZ

Day: Wednesday — Date: 20 Apr 2022

Time : 08:15-9:15 AM — Max Marks : 10

This is a question paper cum answer sheet. Record your answers in this sheet and return them to the invigilators. Answers recorded elsewhere will not be considered.

If you need to change your answer, make sure that the previous answer is unambiguously scratched out. Every ambiguity will fetch you $-1/2$.

Name: Division:

Roll number: Tutorial batch:

1. Let $\mathbf{a}$, $\mathbf{b}$ be unit vectors in $\mathbb{R}^3$ with angle between $\mathbf{a}$ and $\mathbf{b}$ being $\pi/3$. Consider the equation $\mathbf{a}\times\mathbf{x} = \mathbf{b}$ where $\mathbf{x}$ is a vector in $\mathbb{R}^3$ and $\times$ denotes the cross product. This equation is written in matrix form as $A\mathbf{x} = \mathbf{b}$. Then, $\text{rank}(A) + \text{rank}(A|\mathbf{b})$ equals _________. [1 mark]

2. Consider the three planes

$$\begin{array}{llll} 2x & -3y & +z & = 4 \\ x & +y & -2z & = 1 \\ x & +6y & -7z & = a \end{array}$$

Then, which of the following is true? Circle **ALL** correct options [2 marks]

(a) If $a = -1$, the planes pass through a line.

(b) If $a = 1$, the planes form an infinite prism.

(c) If $a = 2$, the planes have a unique point in common.

(d) For every choice of a, the system has infinitely many solutions.

3. Which of the following maps $f : \mathbb{R}^3 \mapsto \mathbb{R}^3$ are of the form $f(\mathbf{v}) = A\mathbf{v}$ where A is a 3×3 matrix and $\mathbf{v} = [x, y, z]^t$, with the superscipt t denoting transpose, is a column vector. Circle ALL the correct options. [2 marks]

(a) $f([x, y, z]^t) = [x - 1, y, z]^t$.

(b) $f([x, y, z]^t) = [y, z, 0]^t$.

(c) $f([x, y, z]^t) = [2z, y, x]^t$.

(d) $f([x, y, z]^t) = [\ |x|, z, y]^t$.

4. Let $\epsilon \in \{\pm 1\}$ be the coefficient of the term $a_{1,3}a_{2,1}a_{3,4}a_{4,2}a_{5,6}a_{6,5}$ when $\det[a_{i,j}]_{1 \le i,j \le 6}$ is expanded completely. Then the value of $6\epsilon + 15$ equals ______. [1 mark]

5. Let $\mathbf{u} = \dfrac{1}{\sqrt{6}} \begin{bmatrix} 2i \\ -1 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \dfrac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 0 \\ -2i \end{bmatrix}$. We are required to find a vector $\mathbf{z}$ in $\mathbb{C}^3$ such that the matrix $A = [\mathbf{u}|\mathbf{v}|\mathbf{z}]$ (the columns of A are the three vectors) satisfies $AA^* = I$. Then, which of the following holds? [2 marks]

   (a) Such a vector $\mathbf{z}$ does not exist.

   (b) Such a vector $\mathbf{z}$ exists and is unique.

   (c) There are precisely two such vectors $\mathbf{z}$.

   (d) There are infinitely many such vectors $\mathbf{z}$.

6. An $n \times n$ matrix with real entries can be identified naturally and in an obvious way with an element of $\mathbb{R}^{n^2}$. With this identification, which of the following are vector subspaces? Circle ALL the correct answers. [2 marks]

   (a) The set of $n \times n$ matrices with all diagonal entries being identical.

   (b) The set of 2×2 non singular matrices.

   (c) The set of 3×3 orthogonal matrices.

   (d) The set of 3×3 skew symmetric matrices.