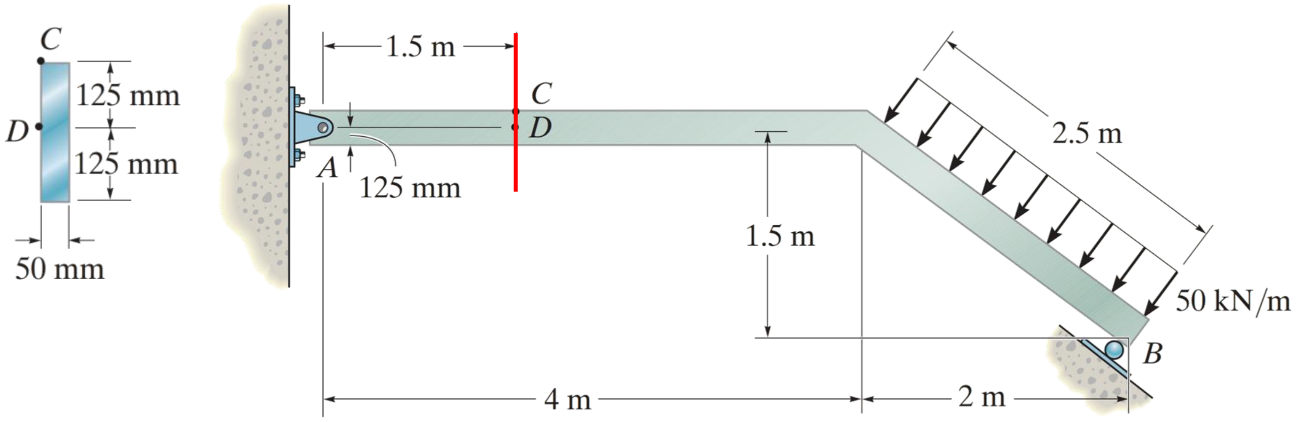
COMBINED LOADING

[and Stress Transformation]

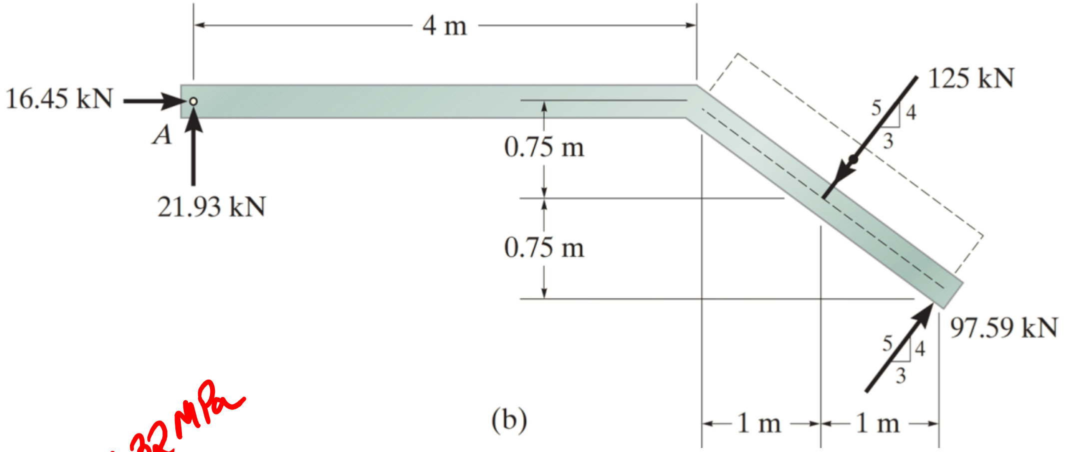
MECHANICS OF MATERIAL (SOLID MECHANICS)

Find state of stress at different points of a section 1.50 m from left end of the beam

COMBINED LOADING



(a)



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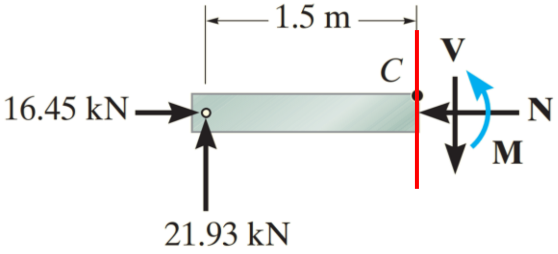
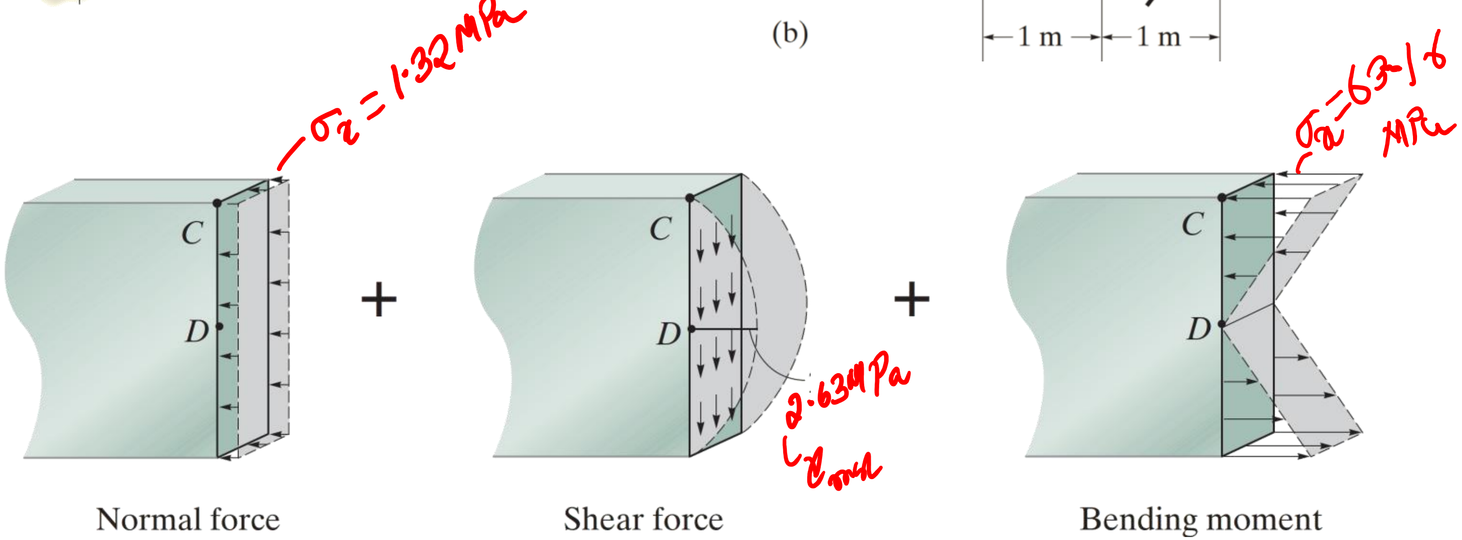
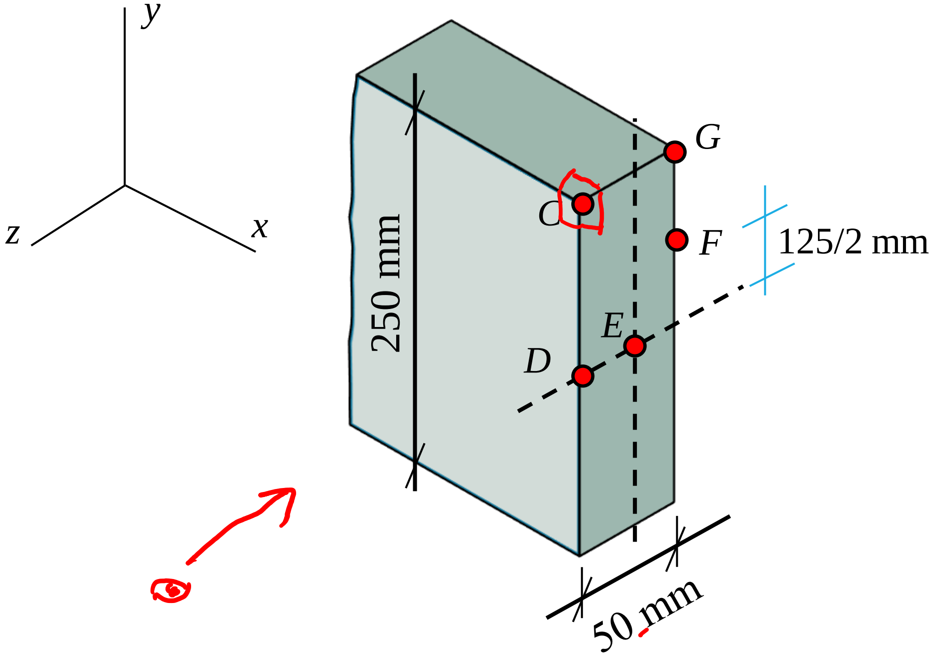
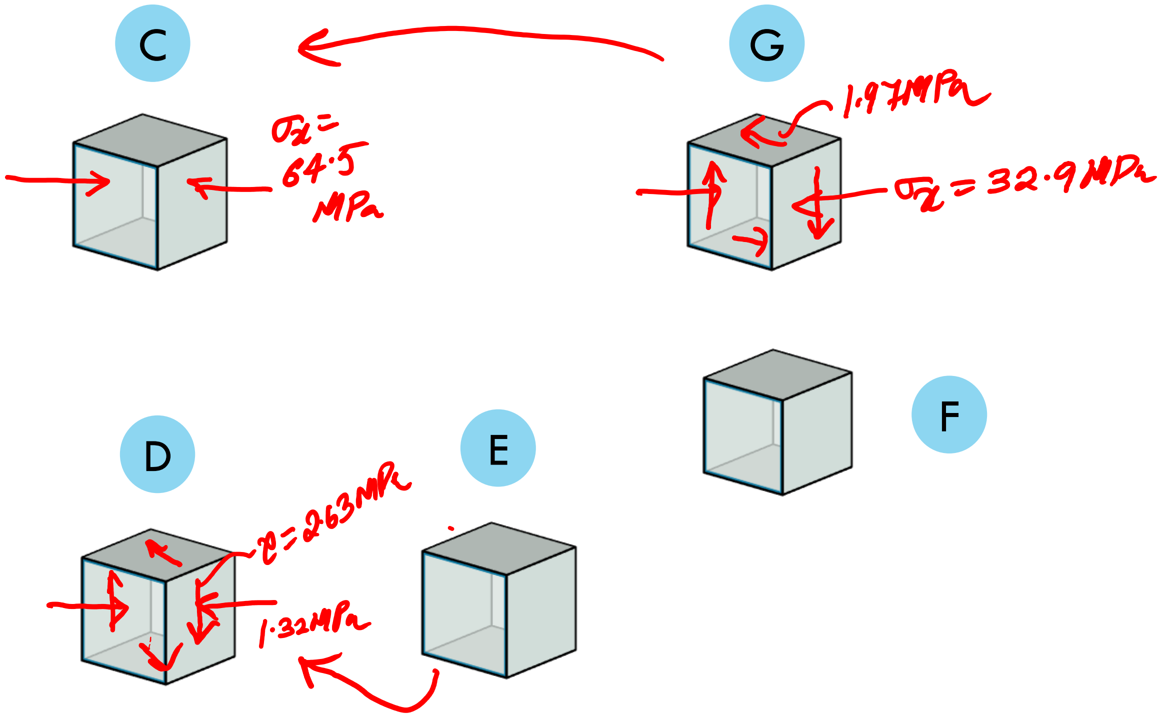


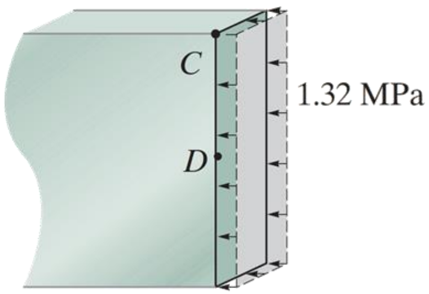


State of Stress

Find state of stress at the points shown

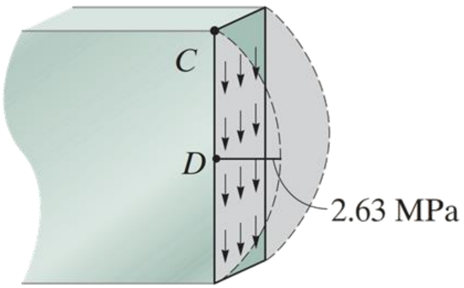






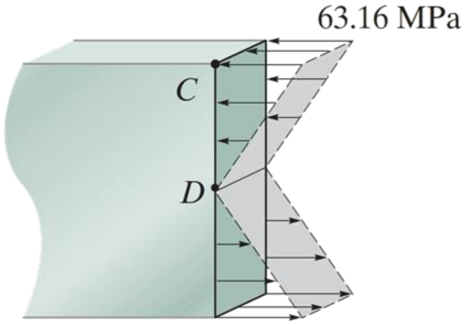
Normal force

+



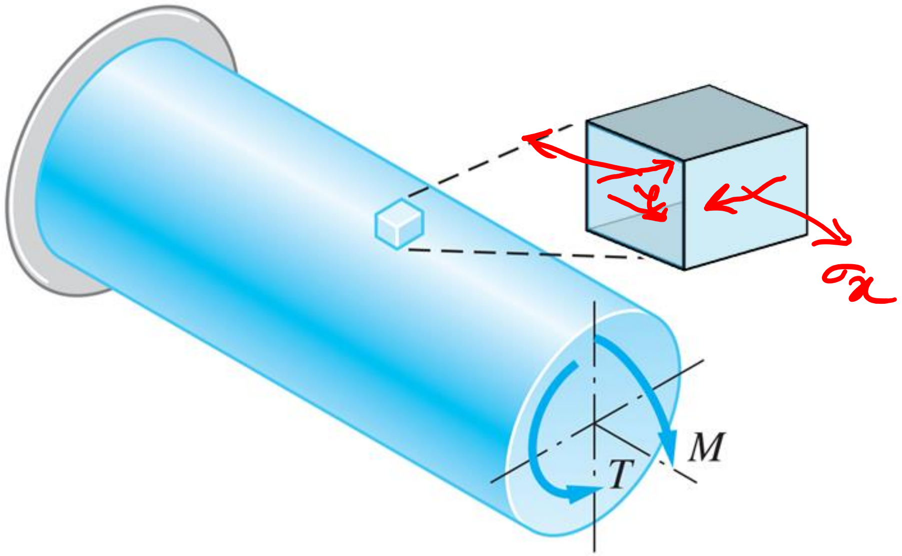
Shear force

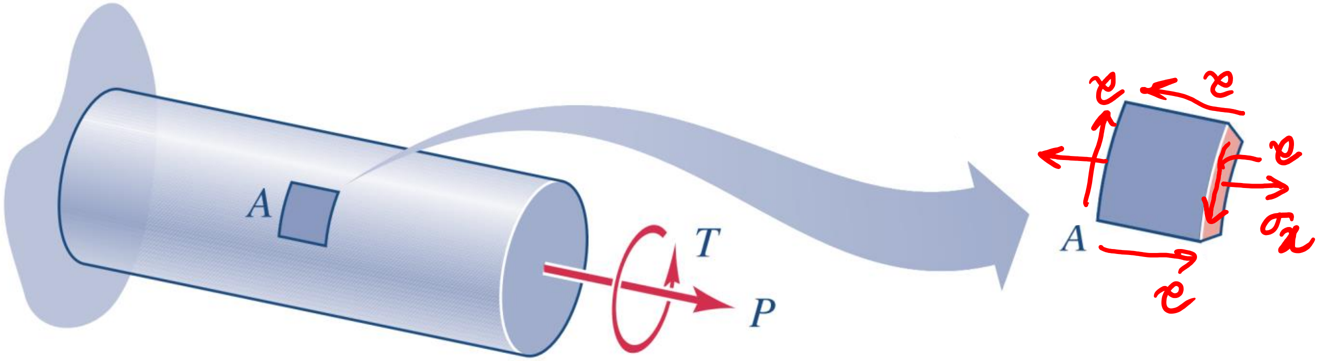
+



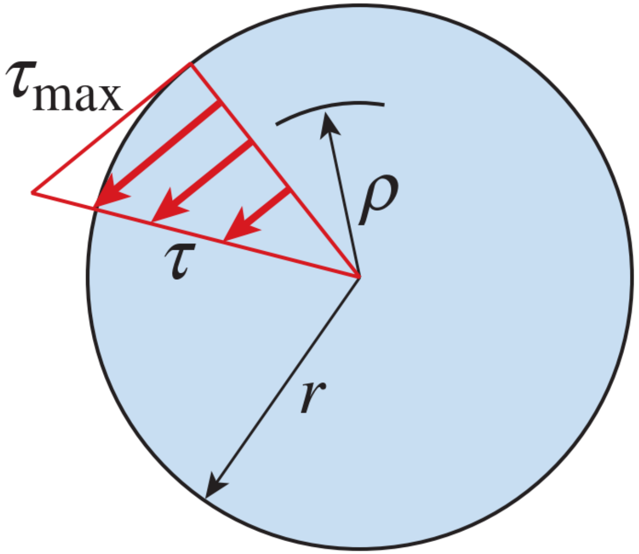
Bending moment

HOW ABOUT THESE ONES?





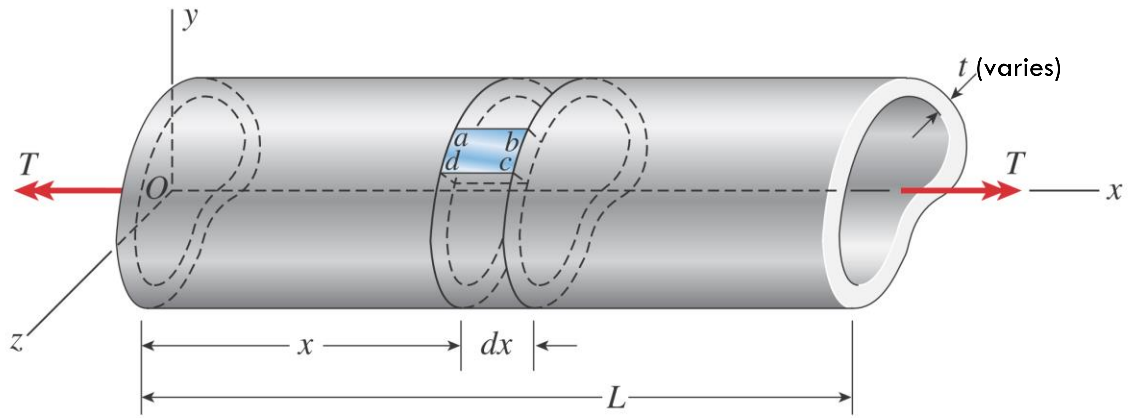
TORSION

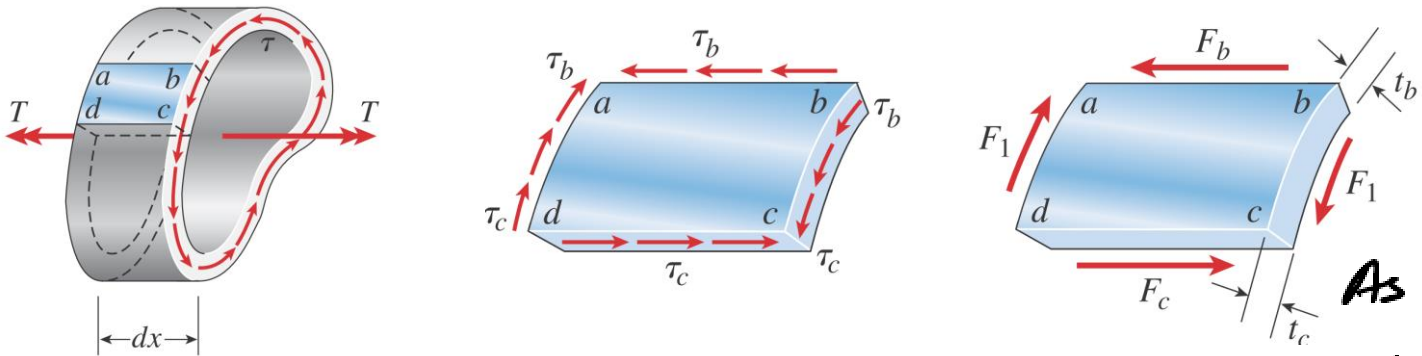


TORSION

- Shear Stress and <u>Shear Flow</u>

Wall is thin – Assume shear stress is constant through the thickness





$F_b = \tau_b \times t_b \times dx$ —— ①

$F_c = \tau_c \times t_c \times dx$ —— ②

$\Sigma F_x = 0$

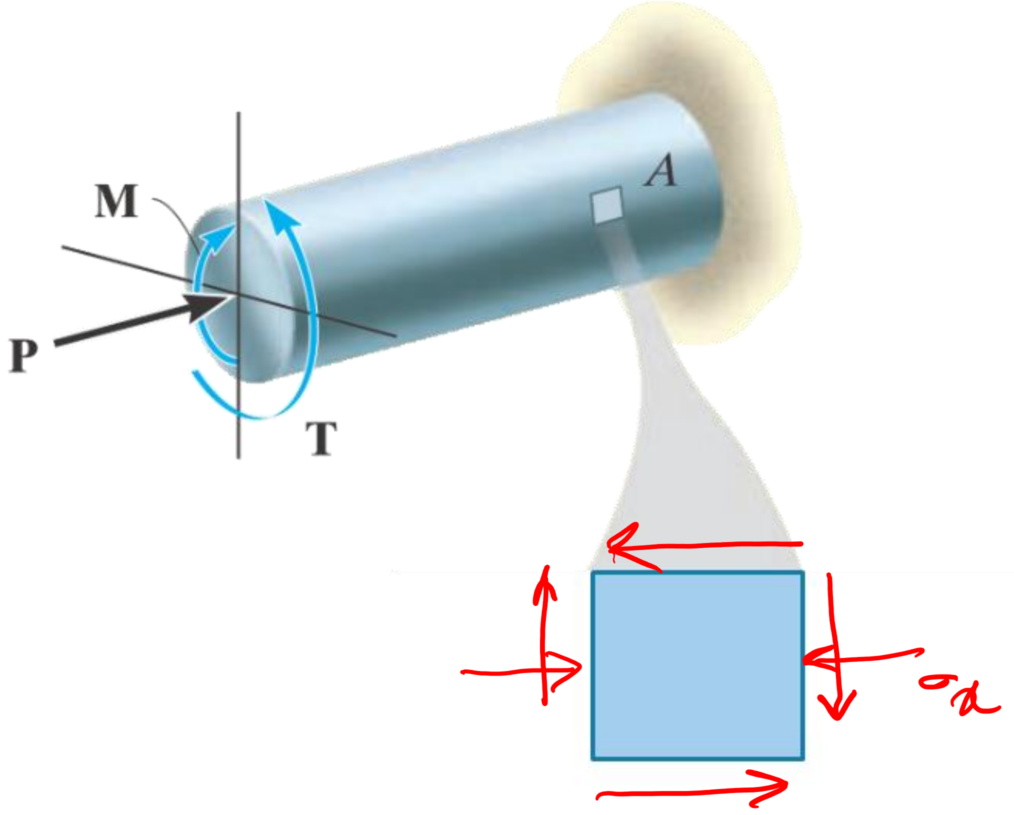
$\Rightarrow \tau_b \times t_b \times \cancel{dx} = \tau_c \times t_c \times \cancel{dx}$

$\Rightarrow \boxed{\tau_b t_b = \tau_c t_c = q}$

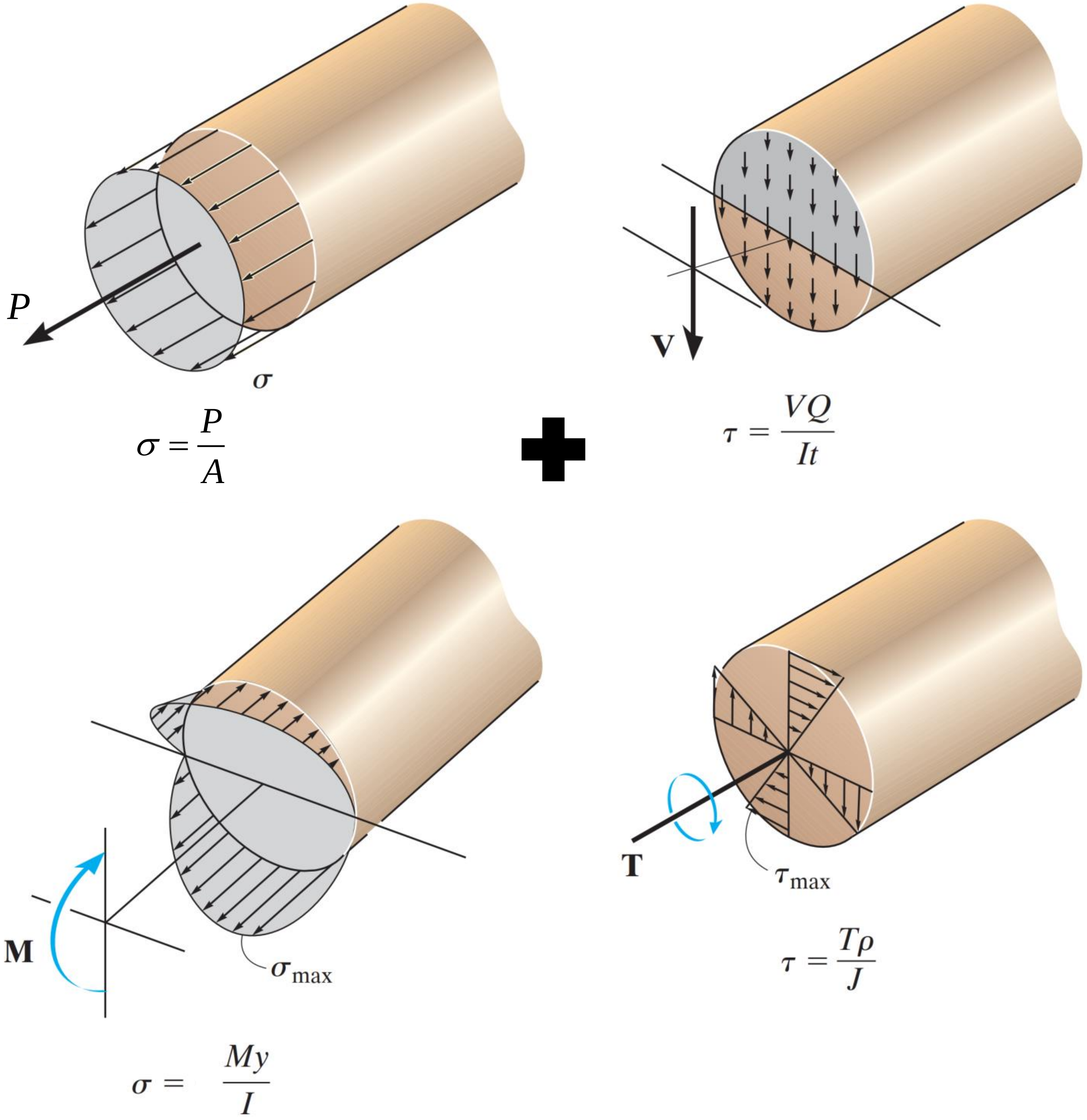
Shear Flow

As long as the thickness is small, product of τ & t $\rightarrow$ constant

HOW ABOUT THESE ONES?

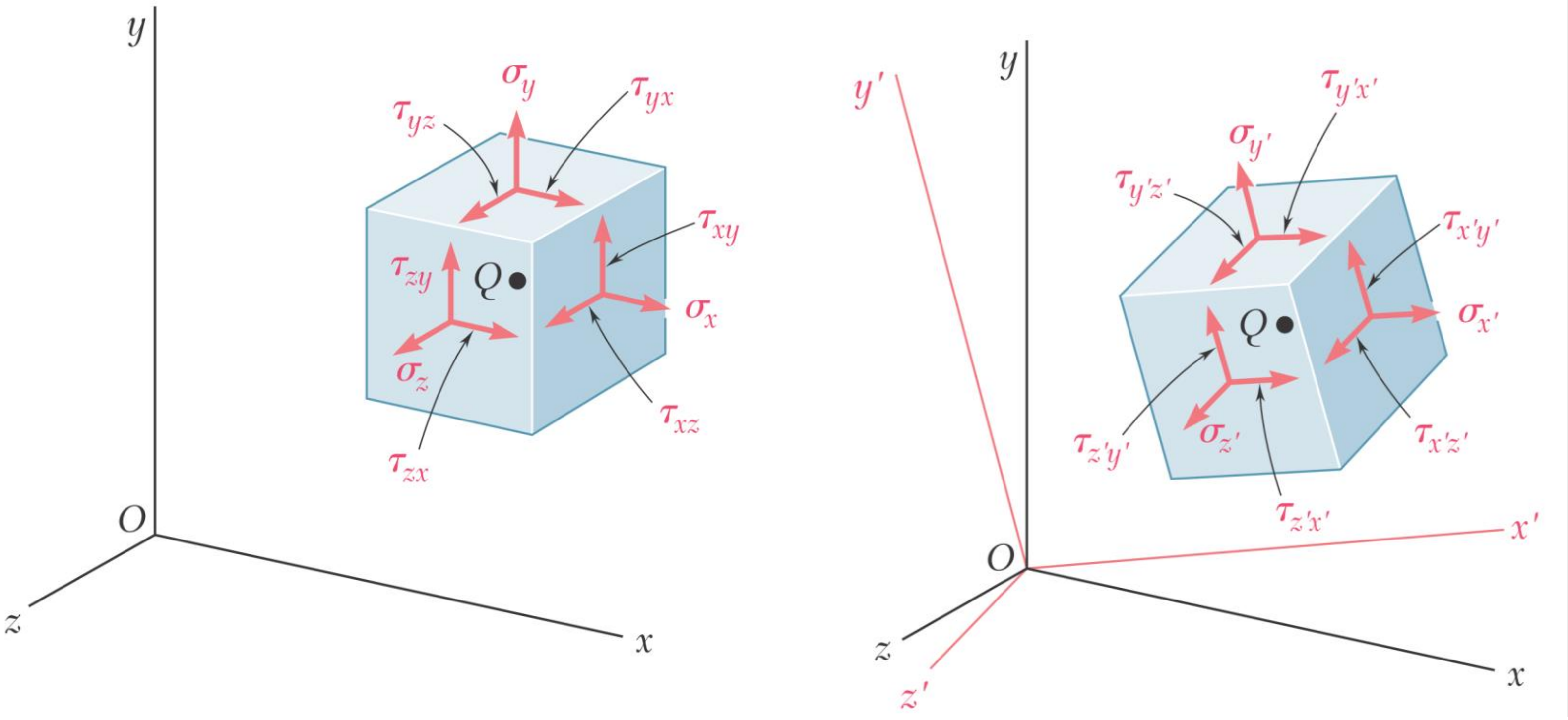


UNDER COMBINED LOADING



DOG AFTER CHEWING ON PHILOSOPHY TEXTBOOK

TRANSFORMATION OF STRESSES



<u>What we are trying to find</u>

How do stresses (and strains) in an element get transformed about a rotated set of axis?

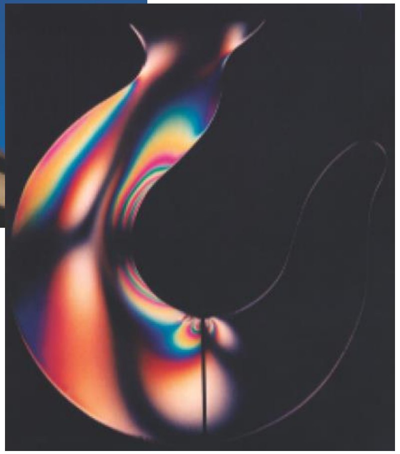
TRANSFORMATION OF STRESSES

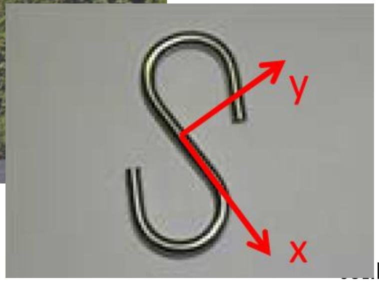
<u>Why is it important?</u>

Because maximum stresses and failures may happen along directions different from the applied forces and chosen section cuts

<u>What we are trying to find</u>

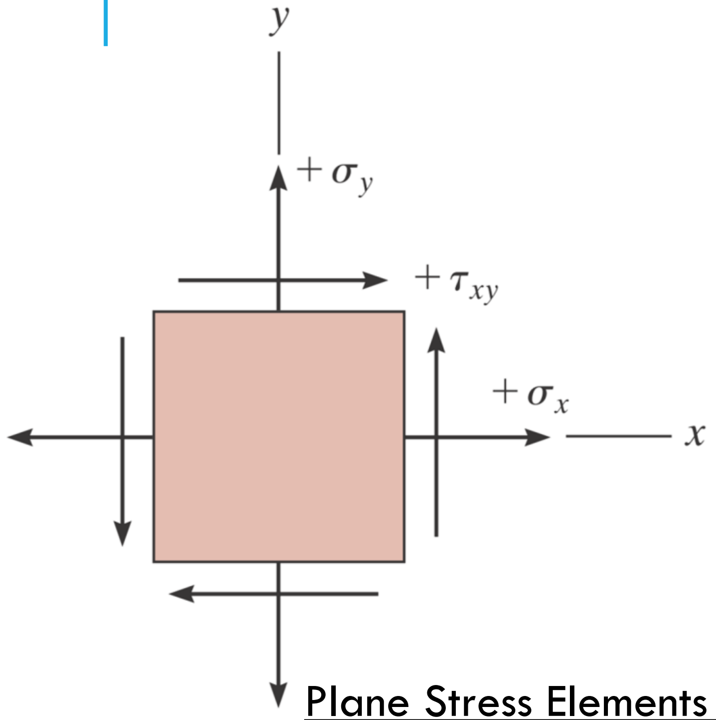
How do stresses (and strains) in an element get transformed about a rotated set of axis?

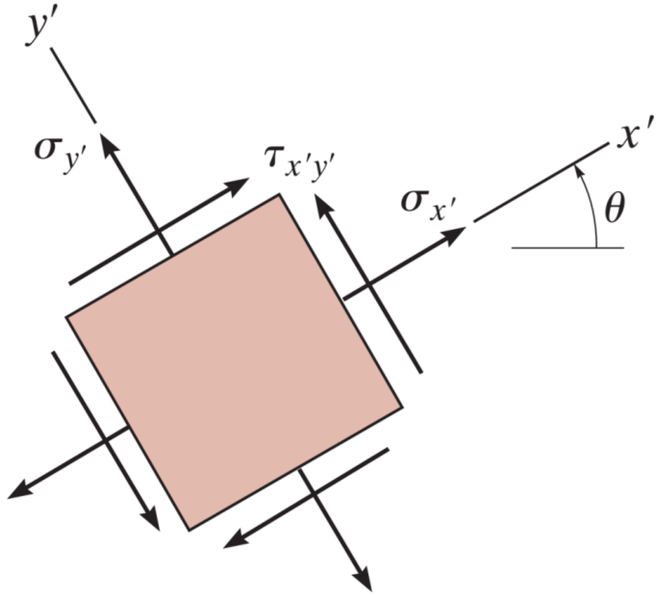


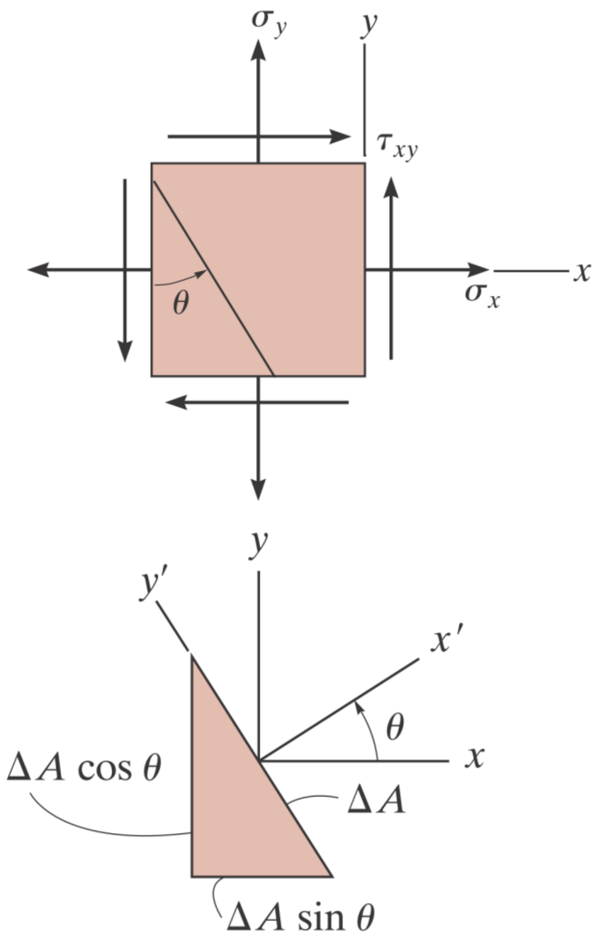


PART 1: TRANSFORMATION OF STRESSES

TRANSFORMATION OF STRESSES







<u>Plane Stress Elements (2D)</u>

<u>Rotated Plane Stress Elements (2D)</u>

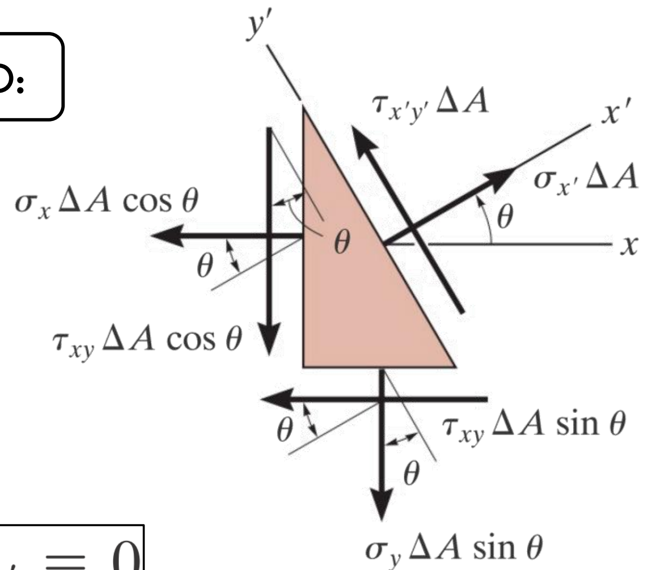
σ_x, σ_y → +ve Tensile

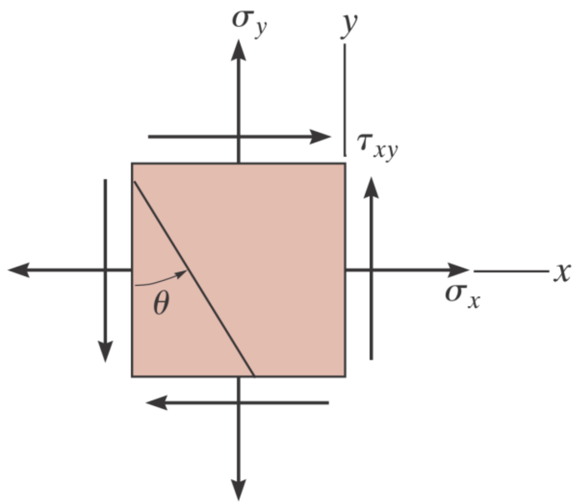
θ → +ve anti-clockwise

τ_{xy}
- → +ve when generating on a +ve face and acting along +ve direction
- → +ve when generating on a -ve face and acting along -ve direction
- → -ve otherwise

TRANSFORMATION OF STRESSES

FBD:





$$\boxed{\Sigma F_{x'} = 0}$$

$$\sigma_{x'}\Delta A - (\tau_{xy}\Delta A\sin\theta)\cos\theta - (\sigma_y\Delta A\sin\theta)\sin\theta$$
$$- (\tau_{xy}\Delta A\cos\theta)\sin\theta - (\sigma_x\Delta A\cos\theta)\cos\theta = 0$$
$$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \tau_{xy}(2\sin\theta\cos\theta)$$

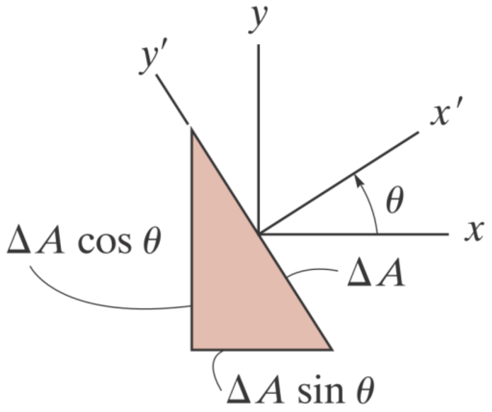
<u>Using identities</u>

$$\cos^2\theta = (1 + \cos 2\theta)/2$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

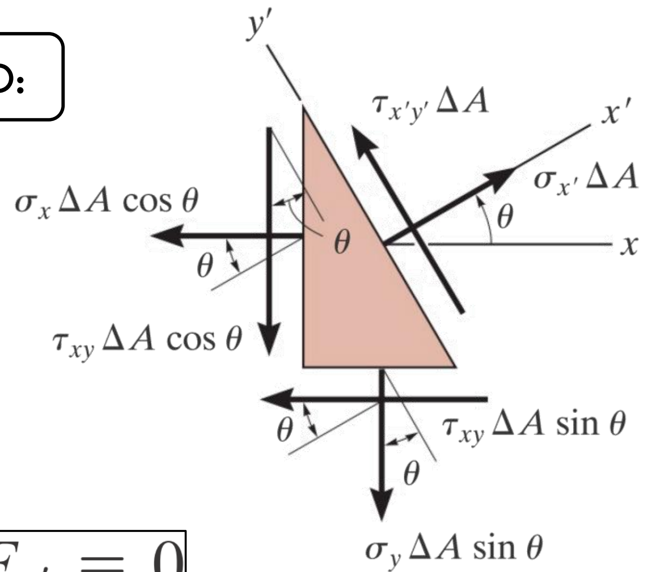
$$\sin^2\theta = (1 - \cos 2\theta)/2$$

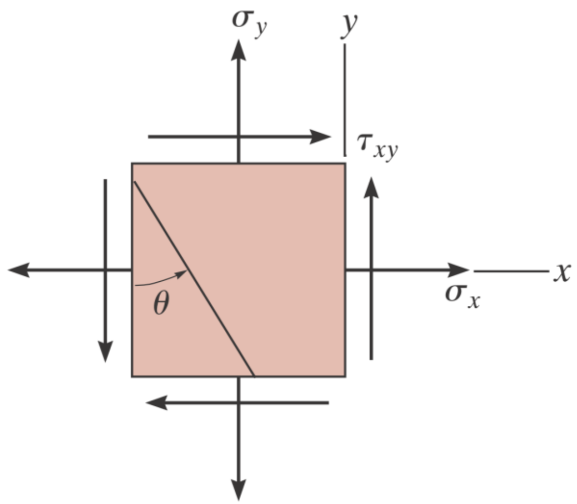
$$\boxed{\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta}$$



TRANSFORMATION OF STRESSES

FBD:





$$\boxed{\Sigma F_{y'} = 0}$$

Using identities

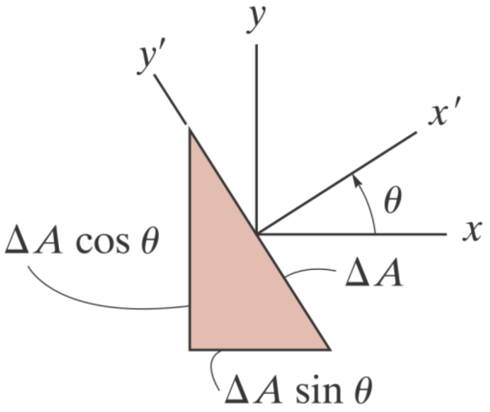
$$\cos^2\theta = (1 + \cos 2\theta)/2$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

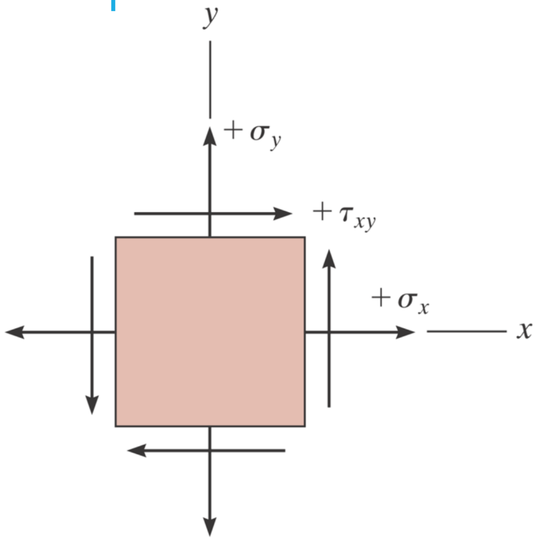
$$\sin^2\theta = (1 - \cos 2\theta)/2$$

$$\tau_{x'y'}\Delta A + (\tau_{xy}\Delta A\sin\theta)\sin\theta - (\sigma_y\Delta A\sin\theta)\cos\theta$$
$$- (\tau_{xy}\Delta A\cos\theta)\cos\theta + (\sigma_x\Delta A\cos\theta)\sin\theta = 0$$
$$\tau_{x'y'} = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\boxed{\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta}$$



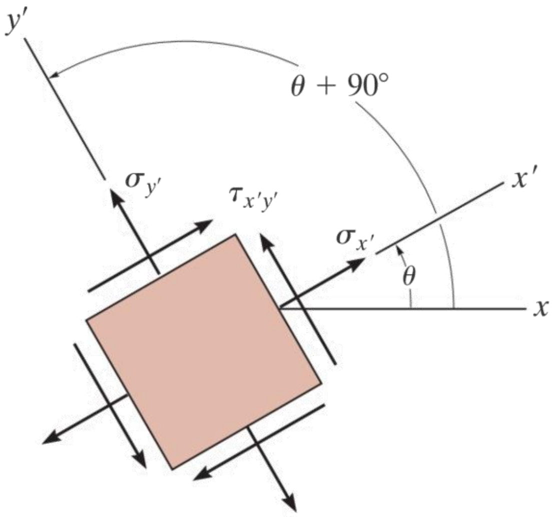
TRANSFORMATION OF STRESSES



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

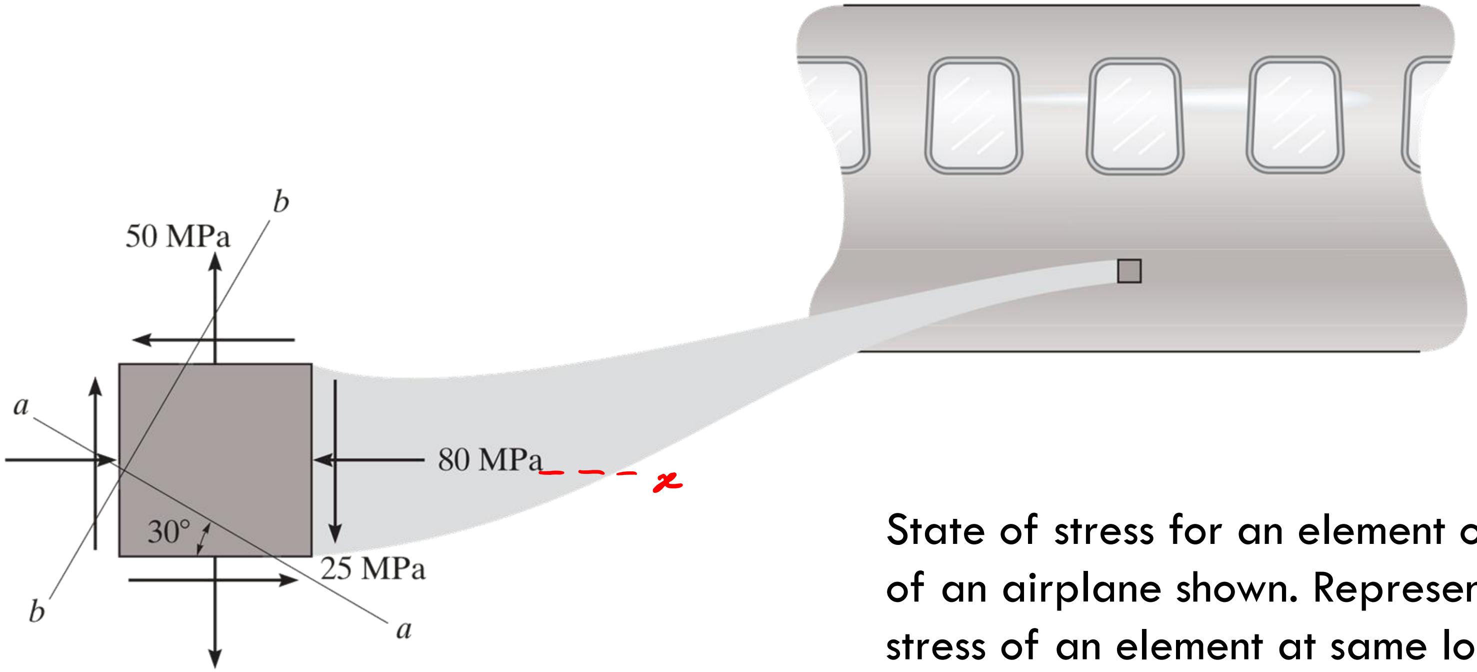


Note the following observation

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$$

**Sum of normal stresses remains constant regardless of θ !

EXAMPLE



State of stress for an element on the body of an airplane shown. Represent state of stress of an element at same location oriented at 30° clockwise

SOLUTION

$\theta = -30^{\circ}$. <u>Given</u>: $\sigma_x = -80\,MPa$, $\sigma_y = +50\,MPa$, $\tau_{xy} = -25\,MPa$

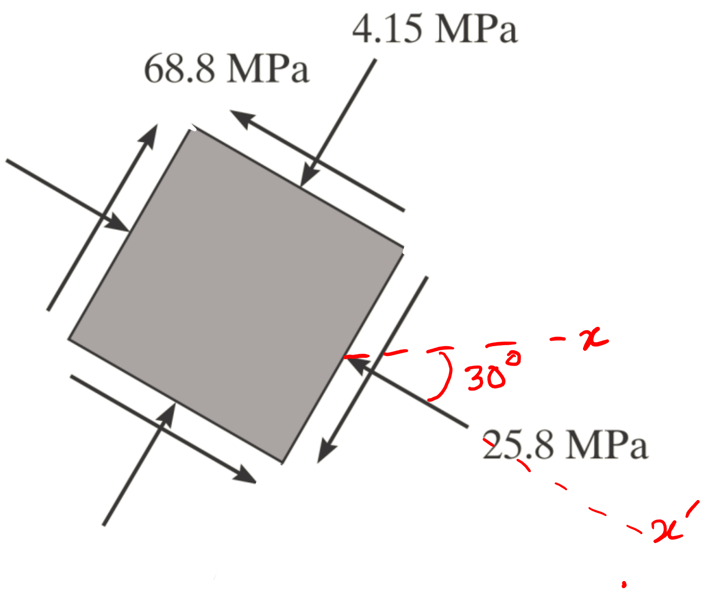
$$\therefore \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

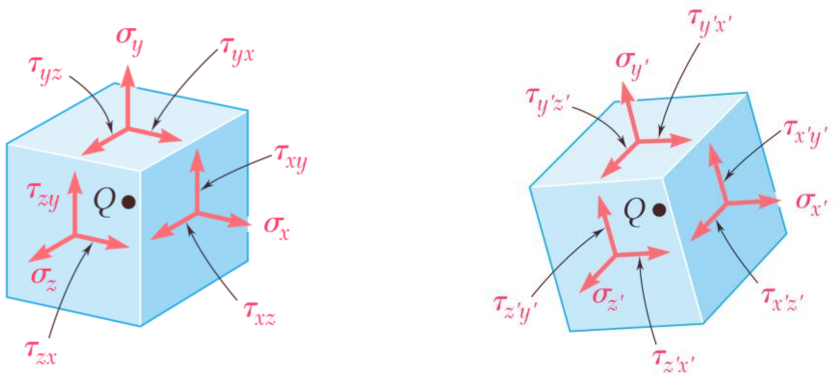
$$= -25.8\,MPa.$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= -4.15\,MPa$$

$$\tau_{x'y'} = -68.8\,MPa$$





STRESS-STRAIN TRANSFORMATION

[Principal Directions and Maximum Shear Planes]

MECHANICS OF MATERIAL (SOLID MECHANICS)

PRINCIPAL STRESSES AND MAXIMUM IN-PLANE SHEAR

There must be some angle $\theta = \theta_p$ at which the stresses $\sigma_{x'}$ and $\sigma_{y'}$ reach their maximum and minimum values respectively

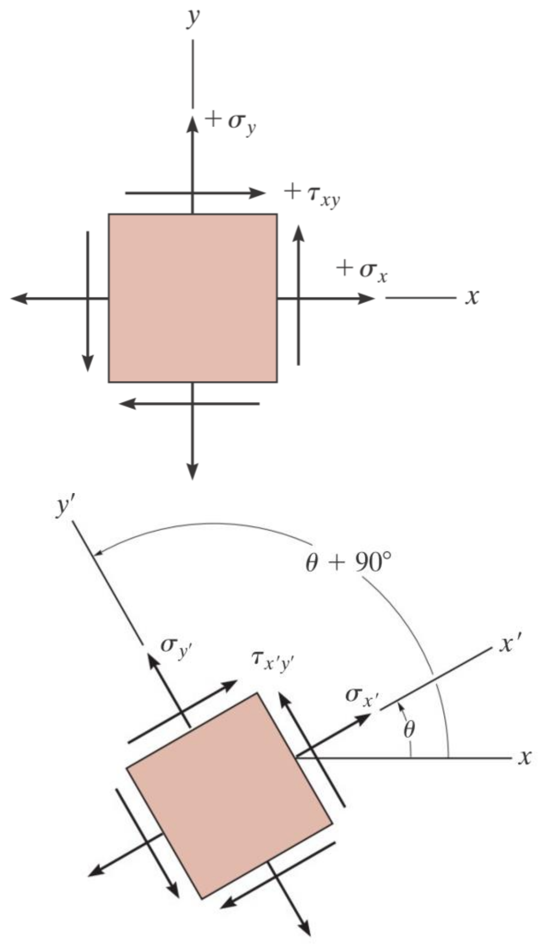
Differentiation!

Stress components along the rotated x', y' axis

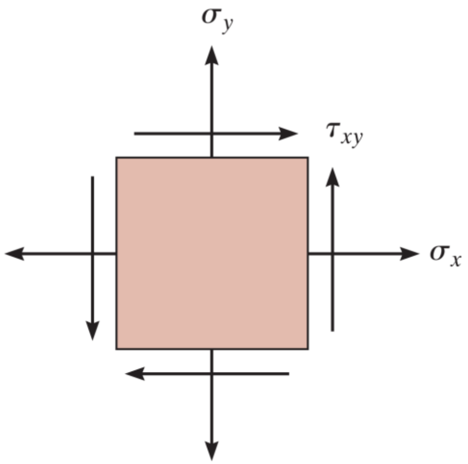
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

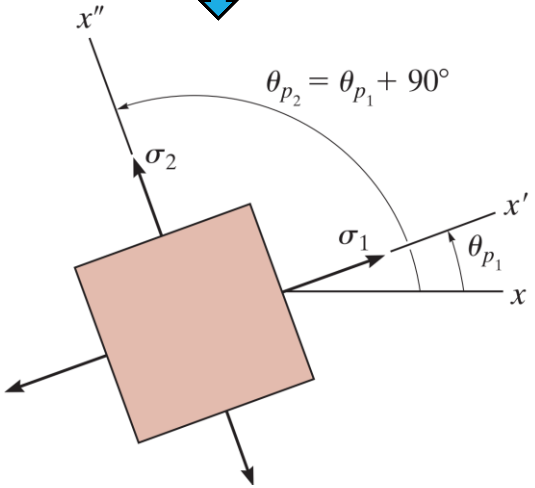
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$



PRINCIPAL STRESSES AND MAXIMUM IN-PLANE SHEAR





$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\frac{d\sigma_{x'}}{d\theta} = -\frac{\sigma_x - \sigma_y}{2}(2\sin 2\theta) + 2\tau_{xy}\cos 2\theta = 0$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

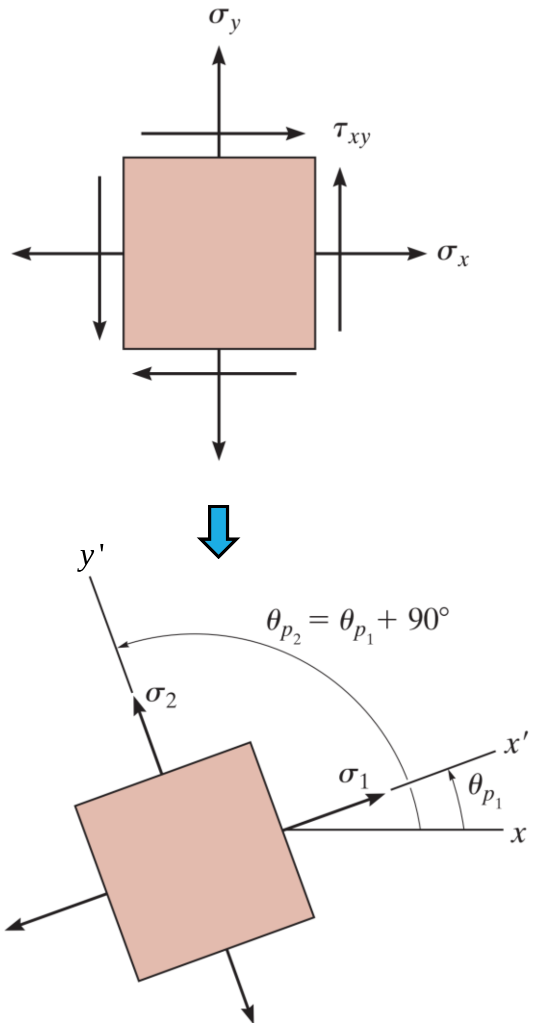
(Two angles $2\theta_p$ separated by 180 degrees)

Subscript p refers to “principal” where max. stress (σ_1) and min. stress (σ_2) occurs

$\therefore$ Actual angle of rotation, i.e. θ_p therefore rotates by $90°$

(After substituting)

PRINCIPAL STRESSES AND MAXIMUM IN-PLANE SHEAR



Maximum and Minimum (principal) stresses:

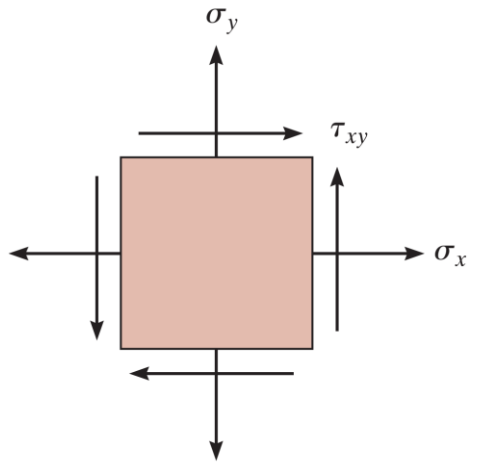
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

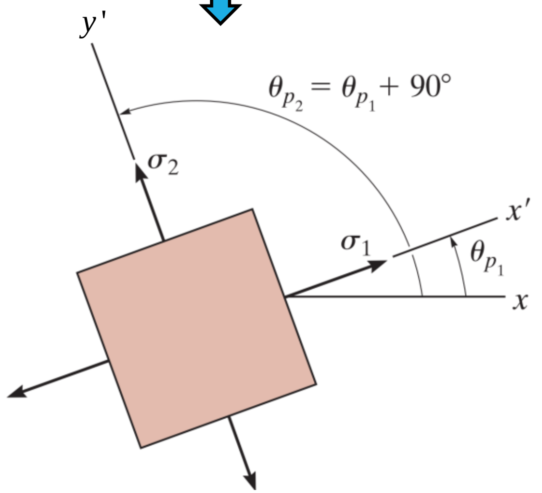
$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

(Two angles $2\theta_p$ separated by 180 degrees)
Subscript *p* refers to "principal" where max and min occurs

$\therefore$ Actual angle of rotation, i.e. θ_p therefor rotates by $90°$

PRINCIPAL STRESSES AND MAXIMUM IN-PLANE SHEAR





Maximum and Minimum (principal) stresses:

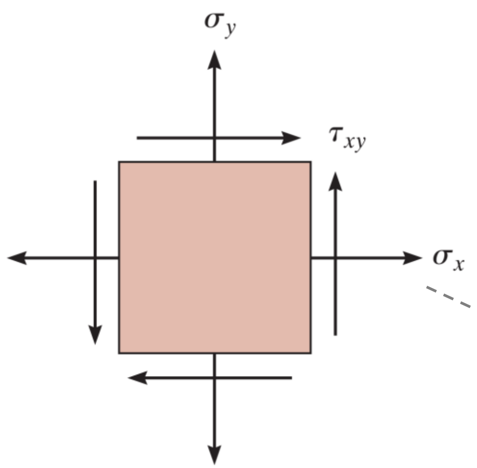
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

The planes where these maximum stresses act are called principal planes

PRINCIPAL STRESSES

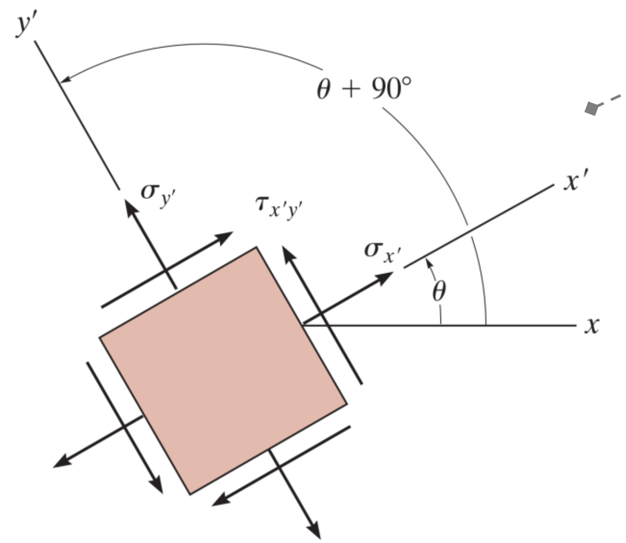
Spot the Difference



General angle θ

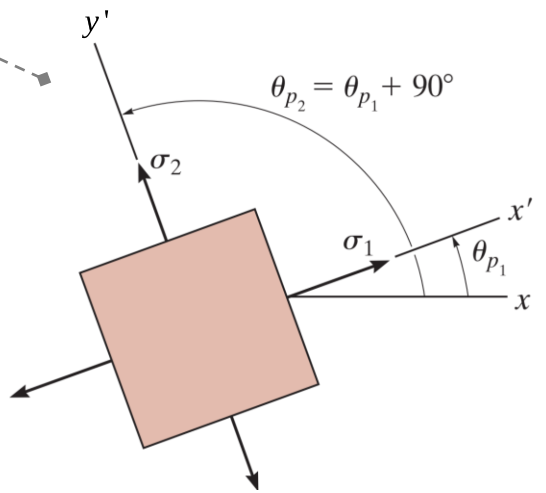
Rotation $\theta = \theta_p$

Principal Planes



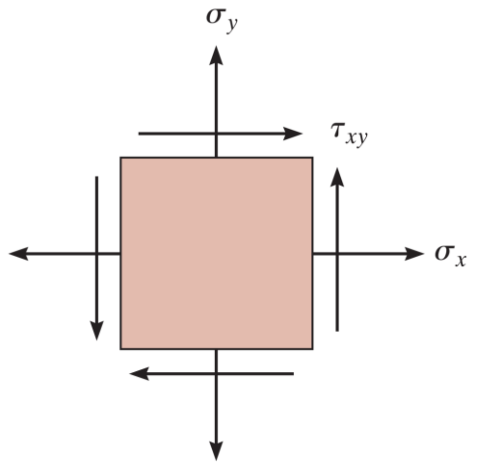
Rotated at a general angle θ

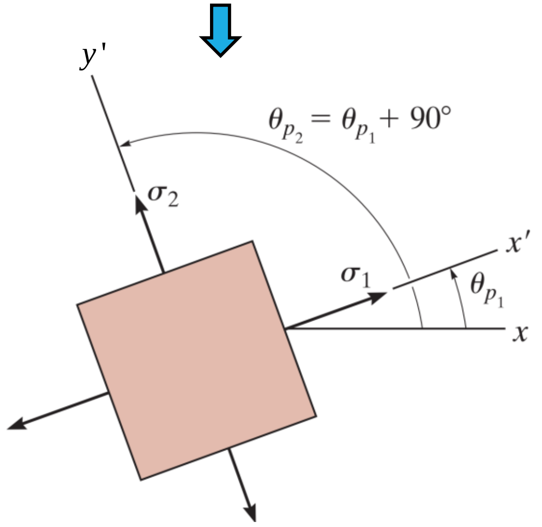
$\tau_{x'y'}$ vanishes at the principal planes,
i.e. $\tau_{x'y'} = 0$ at θ_{p_1} and θ_{p_2}



Rotated at $\theta = \theta_p$

PRINCIPAL STRESSES AND MAXIMUM IN-PLANE SHEAR





- <u>In summary</u>: Three important things to remember:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

→ Gives principal stresses (σ_1, σ_2)

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

→ Gives principal planes $(\theta_{p_1}, \theta_{p2})$

$\tau_{x'y'} = 0$ on principal planes → No shear at θ_{p_1} and θ_{p_2}

PRINCIPAL STRESSES AND MAXIMUM IN-PLANE SHEAR

There must be some angle θ_s at which the stresses $\tau_{x'y'}$ reaches the maximum value

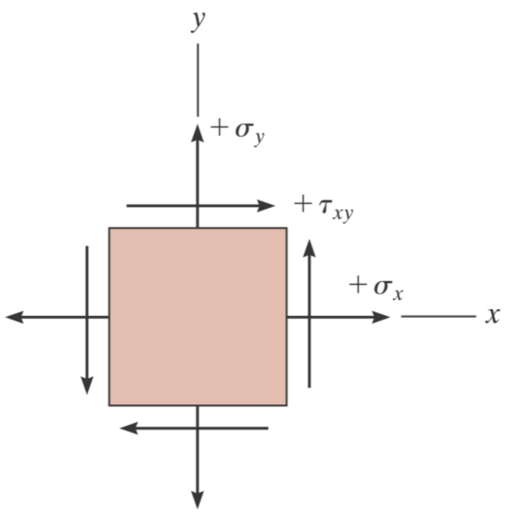
Differentiation!

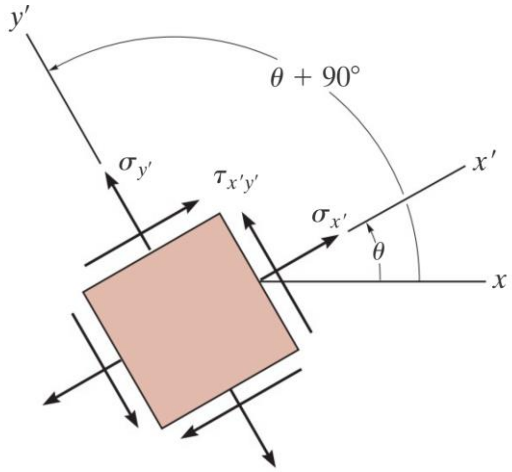
Stress components along the rotated x', y' axis

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$





PRINCIPAL STRESSES AND MAXIMUM IN-PLANE SHEAR

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\frac{d}{d\theta}\tau_{x'y'} = 0$$

Compare with principal stress directions

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

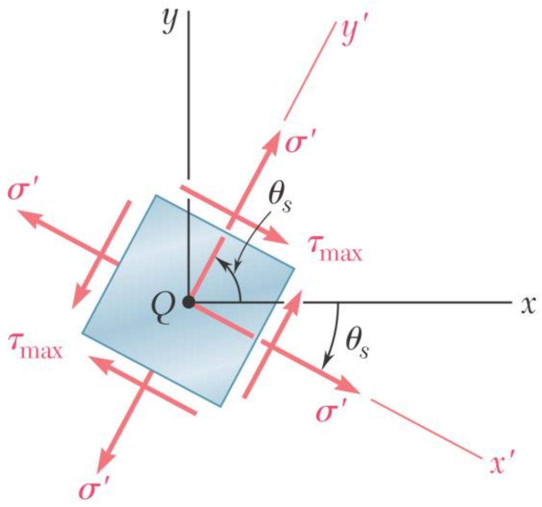
(Two angles $2\theta_s$ separated by $180°$)

(negative reciprocal)

θ_s and θ_p separated by $45°$

$\therefore$ Plane of Maximum Shear is located at $45°$ from the principal plane

PRINCIPAL STRESSES AND MAXIMUM IN-PLANE SHEAR



$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\frac{d}{d\theta}\tau_{x'y'} = 0$$

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

<u>At maximum shear planes:</u>

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

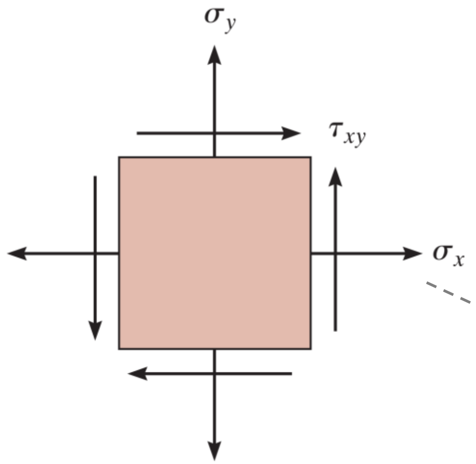
and

$$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2}$$

∴ At planes of maximum shear, normal stresses still exist!

MAXIMUM IN-PLANE SHEAR

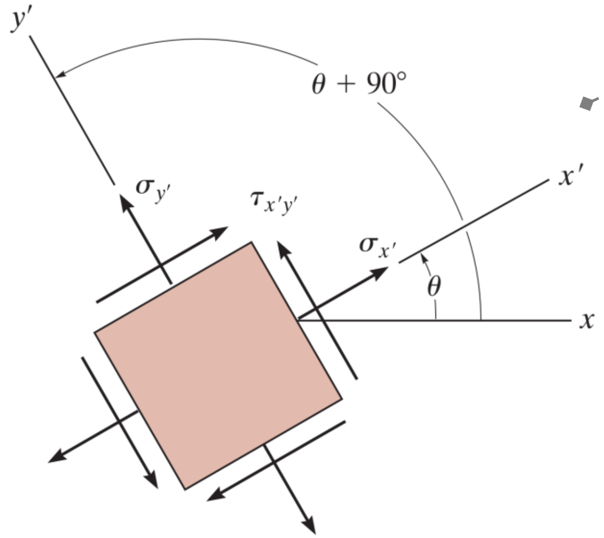
Spot the Difference



General angle θ

Rotation $\theta = \theta_s$

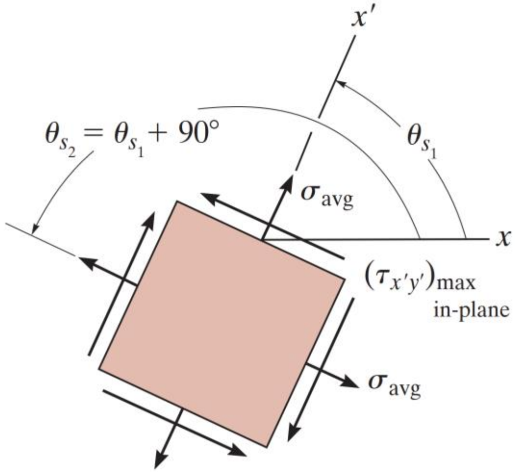
Plane of Max. Shear



Rotated at a general angle θ

$$\tau_{\substack{\max \\ \text{in-plane}}} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$



Maximum in-plane shear stresses

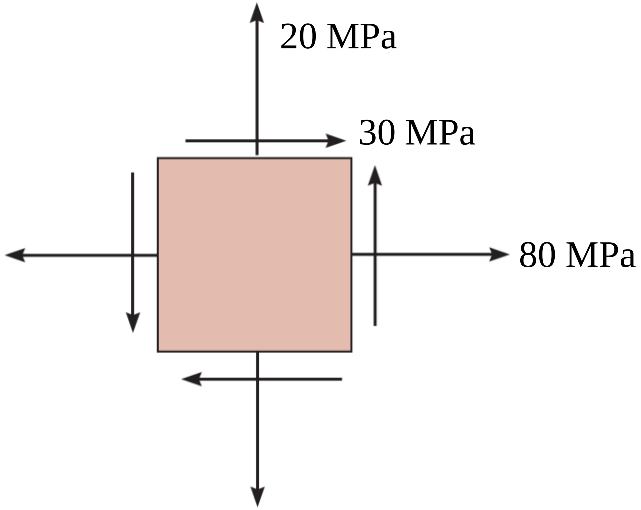
Rotated at $\theta = \theta_s$

KEY POINTS TO REMEMBER IN STRESS TRANSFORMATION

Important Points

- The *principal stresses* represent the maximum and minimum normal stress at the point.
- When the state of stress is represented by the principal stresses, *no shear stress* will act on the element.
- The state of stress at the point can also be represented in terms of the *maximum in-plane shear stress*. In this case an *average normal stress* will also act on the element.

EXAMPLE:



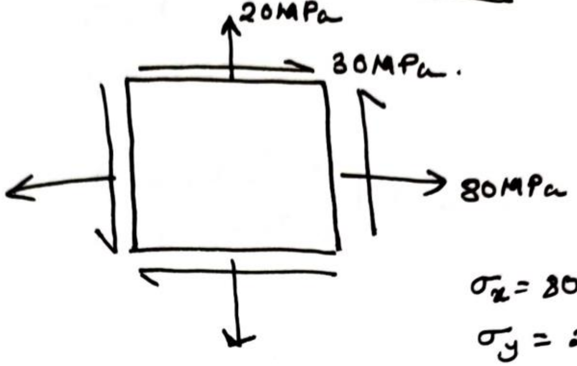
State of stress at failure point

Solve this using the analytical formulations

- Find principal planes and principal stresses. Show diagram

(Solution – Next Page)

Analytical Approach



$\sigma_x = 80\text{ MPa}$
$\sigma_y = 20\text{ MPa}$
$\tau_{xy} = 30\text{ MPa}$
} all +ve

Principal stresses

Basic Calculations:

- $\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{80+20}{2} = 50\text{ MPa}.$

- $\dfrac{\sigma_x - \sigma_y}{2} = \dfrac{80-20}{2} = 30\text{ MPa}$

- $\tau_{xy} = 30\text{ MPa}.$

$$\therefore \sigma_{1,2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}.$$

$$\Rightarrow \sigma_{1,2} = 50 \pm \sqrt{30^2 + 30^2}$$

$$\Rightarrow \left\{\begin{array}{l} \sigma_1 = 92.42\text{ MPa} \\ \sigma_2 = 7.6\text{ MPa} \end{array}\right\}$$ Principal stresses (maximum & minimum)

Also, $\tan 2\theta_p = \dfrac{\tau_{xy}}{\left(\frac{\sigma_x-\sigma_y}{2}\right)} = \dfrac{30}{30} = 1$

↳ Two roots $2\theta_{p_1}$ & $2\theta_{p_2}$ that are 180° apart. $\therefore \theta_{p_1}$ & θ_{p_2} are 90° apart. | $2\theta_p = \tan^{-1}(1) = 45^\circ$

Summary

Principal stresses:

$\sigma_1 = 92.42\text{ MPa}$
$\sigma_2 = 7.6\text{ MPa}.$

Principal Planes:

$\theta_{p_1} = 22.5^\circ,$
$\theta_{p_2} = 112.5^\circ$
| θ_p's $\rightarrow 22.5^\circ$ & 112.5°

But which θ_p corresponds to which σ? Need ~~top~~ to plug in the basic transformation equations.

$$\sigma_{x}' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

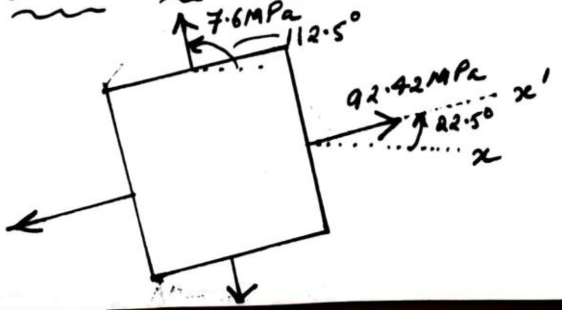
↑ substitute $\theta = \theta_{p_1} = 22.5^\circ$

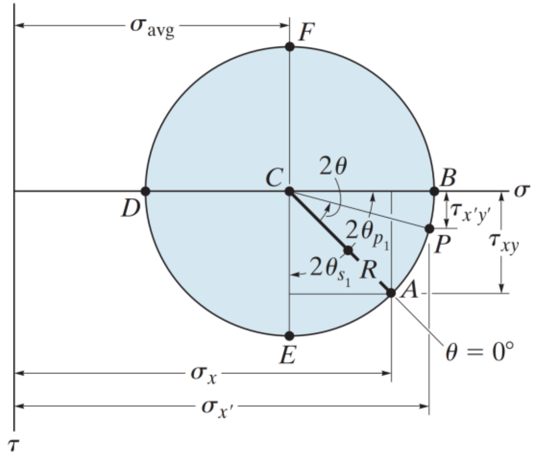
$$\Rightarrow \sigma_x' = 50 + 30\cos(2\times 22.5^\circ) + 30 \sin(2\times 22.5^\circ)$$

$$\Rightarrow \sigma_x' = 92.42\text{ MPa} = \sigma_1$$

$\therefore$ the plane for $\theta_{p_1} = 22.5^\circ$ corresponds to $\sigma_1 = 92.42\text{ MPa}.$

Element oriented along principal directions:





STRESS-STRAIN TRANSFORMATION

MOHR'S CIRCLE

MECHANICS OF MATERIAL
(SOLID MECHANICS)

MOHR'S CIRCLE

- A graphical tool that makes it (incredibly) easy to visualize stress transformations.
- Rearrange the stress transformation equations (Slide 14) to get:

$$\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{and} \quad \tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Eliminate θ as:

$$\left[\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\Downarrow$$

$$\boxed{(\sigma_{x'} - \sigma_{\text{avg}})^2 + \tau_{x'y'}^2 = R^2}$$

Equation of a circle centered at σ_{avg} and $\tau_{x'y'}$

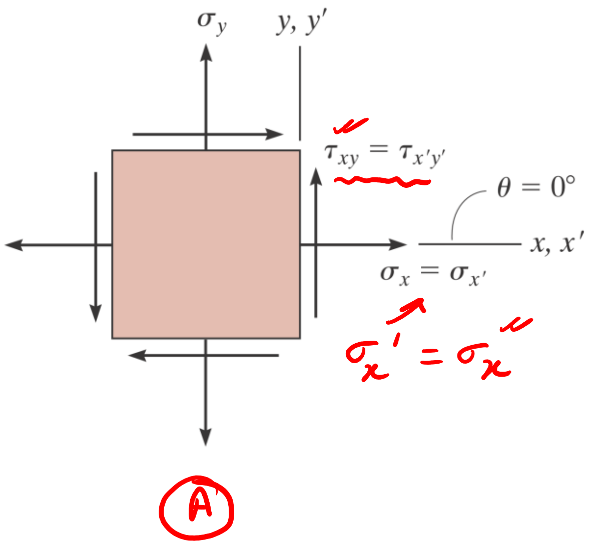
where $\sigma_{\text{avg}} = \dfrac{\sigma_x + \sigma_y}{2}$ and $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

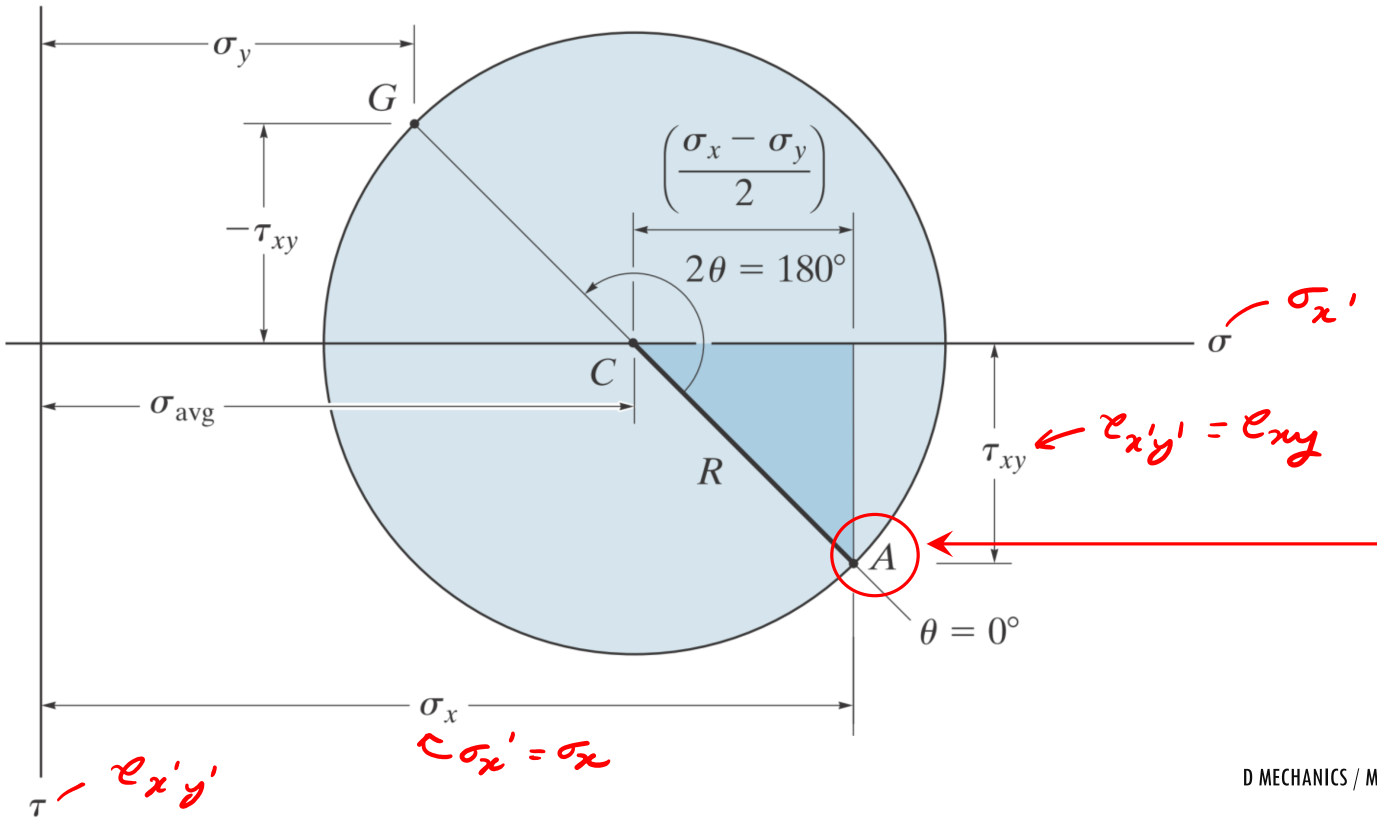
MOHR'S CIRCLE

<u>Example</u>

Imagine x' axis is coincident with x axis. i.e. $\theta = 0$. $\therefore \sigma_{x'} = \sigma_x$ and $\tau_{x'y'} = \tau_{xy}$

Point A





MOHR’S CIRCLE

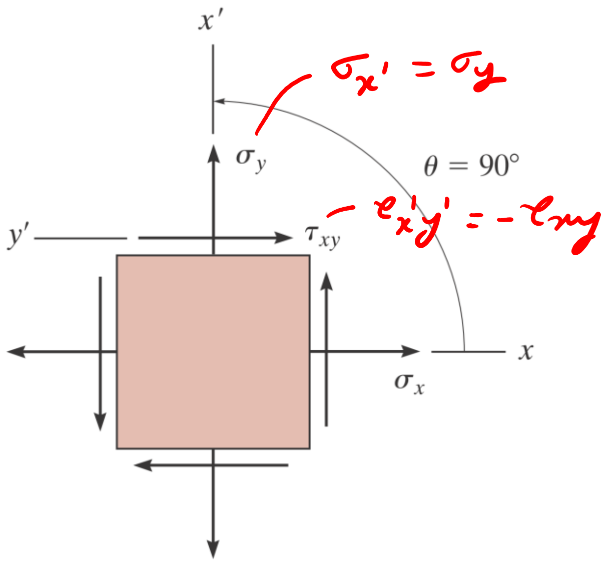
Since it acts along $-\,y'$ axis !

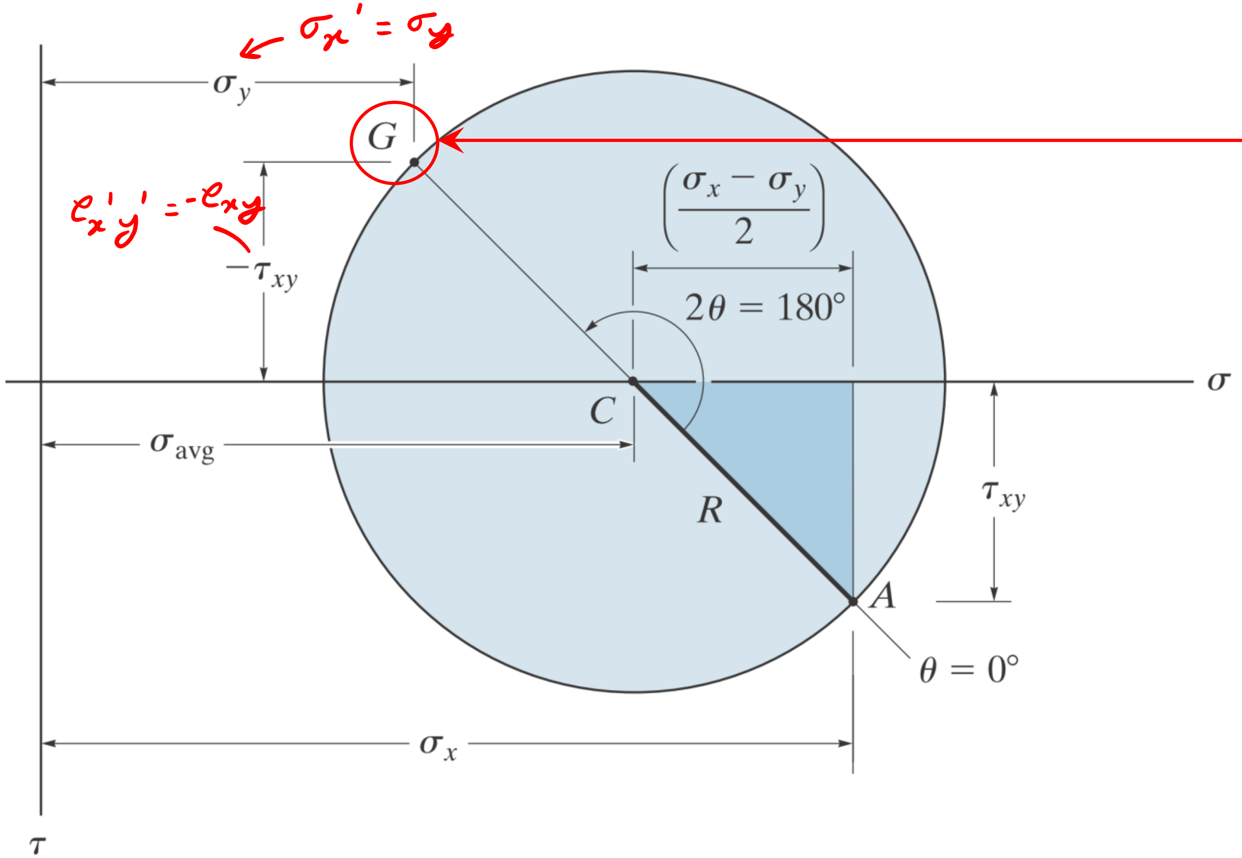
Example

Imagine x' is rotated by $\theta = 90°$. $\therefore$ Now $\sigma_{x'} = \sigma_y$ and $\tau_{x'y'} = -\tau_{xy}$

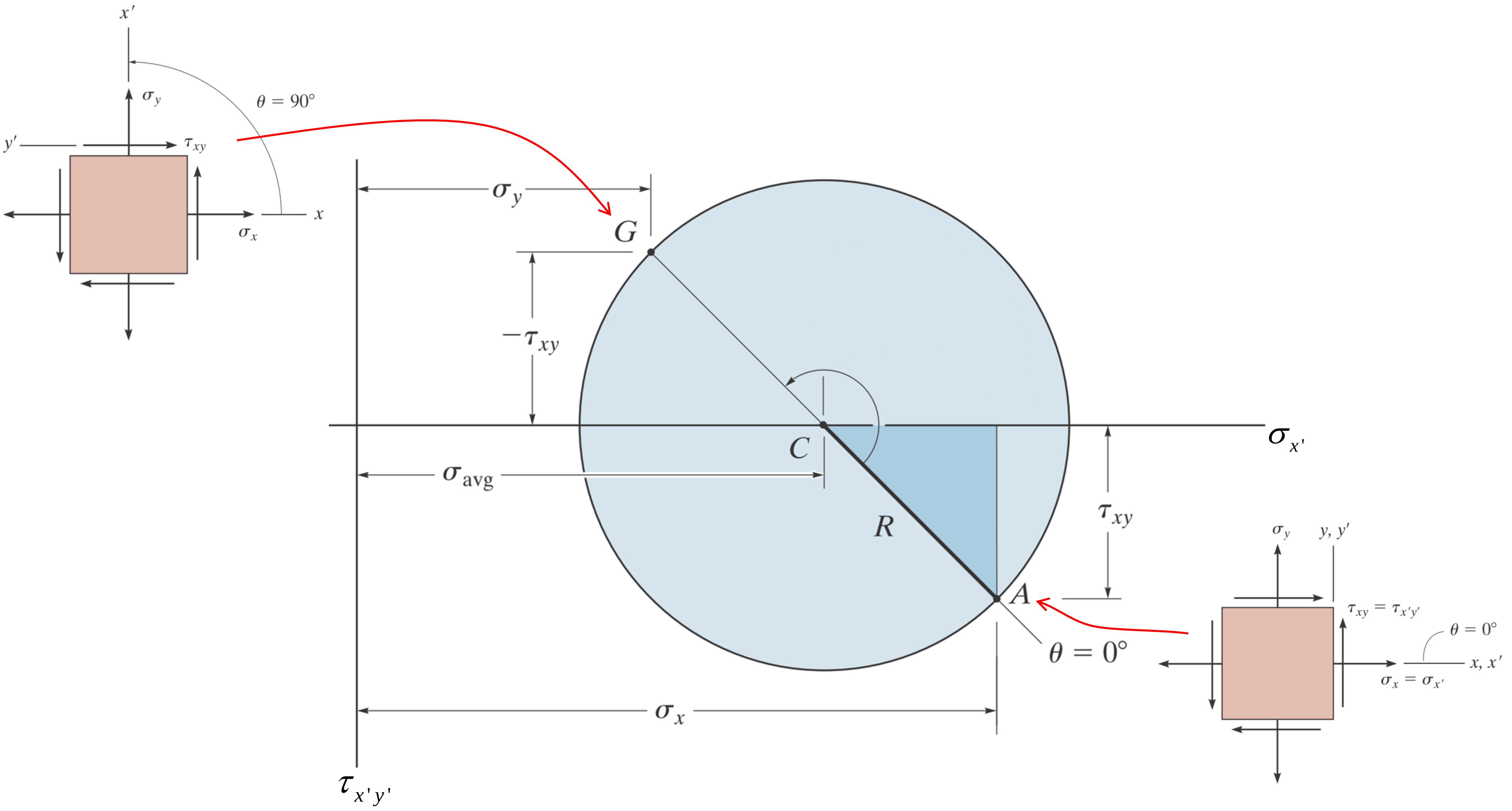
Point G

(Diametrically opposite to A)





Rotation θ in original element, leads to rotation 2θ in Mohr's plane

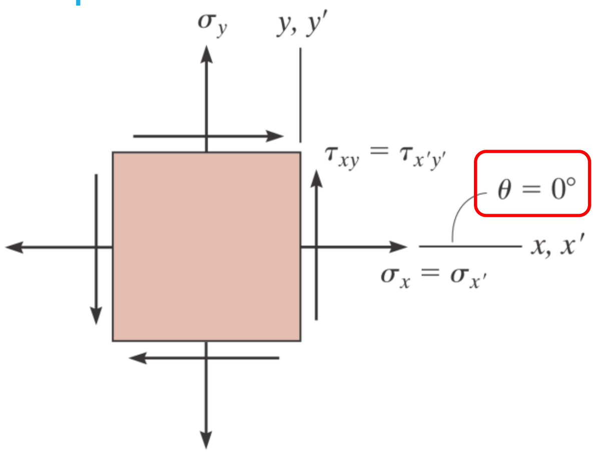


x′
θ = 90°
σ_y
y′
τ_xy
x
σ_x
σ_y
G
−τ_xy
σ_x'
C
σ_avg
R
τ_xy
A
θ = 0°
σ_x
τ_x'y'
σ_y
y, y′
τ_xy = τ_x′y′
θ = 0°
x, x′
σ_x = σ_x′

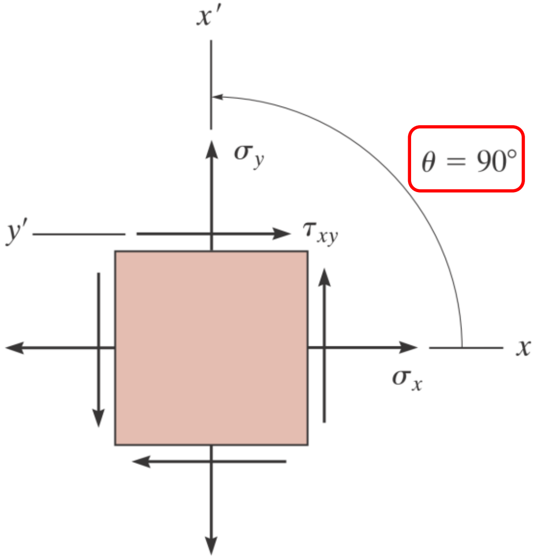
MOHR'S CIRCLE

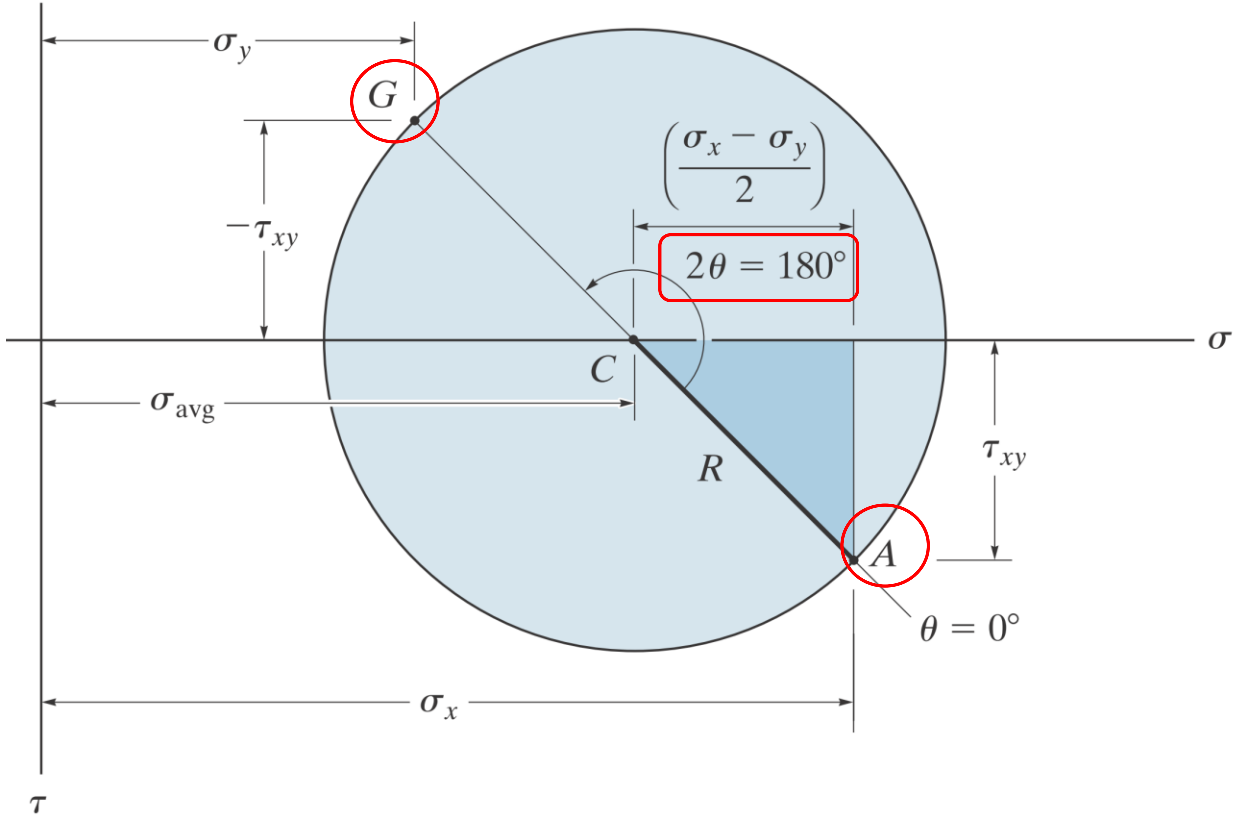
1. A rotation of θ in the physical plane corresponds to rotation of 2θ in the Mohr plane!
2. Direction of rotation remains the same. Shown here: an anti-clockwise rotation in the physical plane, leads to an anticlockwise rotation in the Mohr plane.

(Corresponds to Point A)



(Corresponds to Point G)





MOHR'S CIRCLE: CONSTRUCTION STEPS

1. Establish coordinate system
   (σ +ve right, τ +ve down)

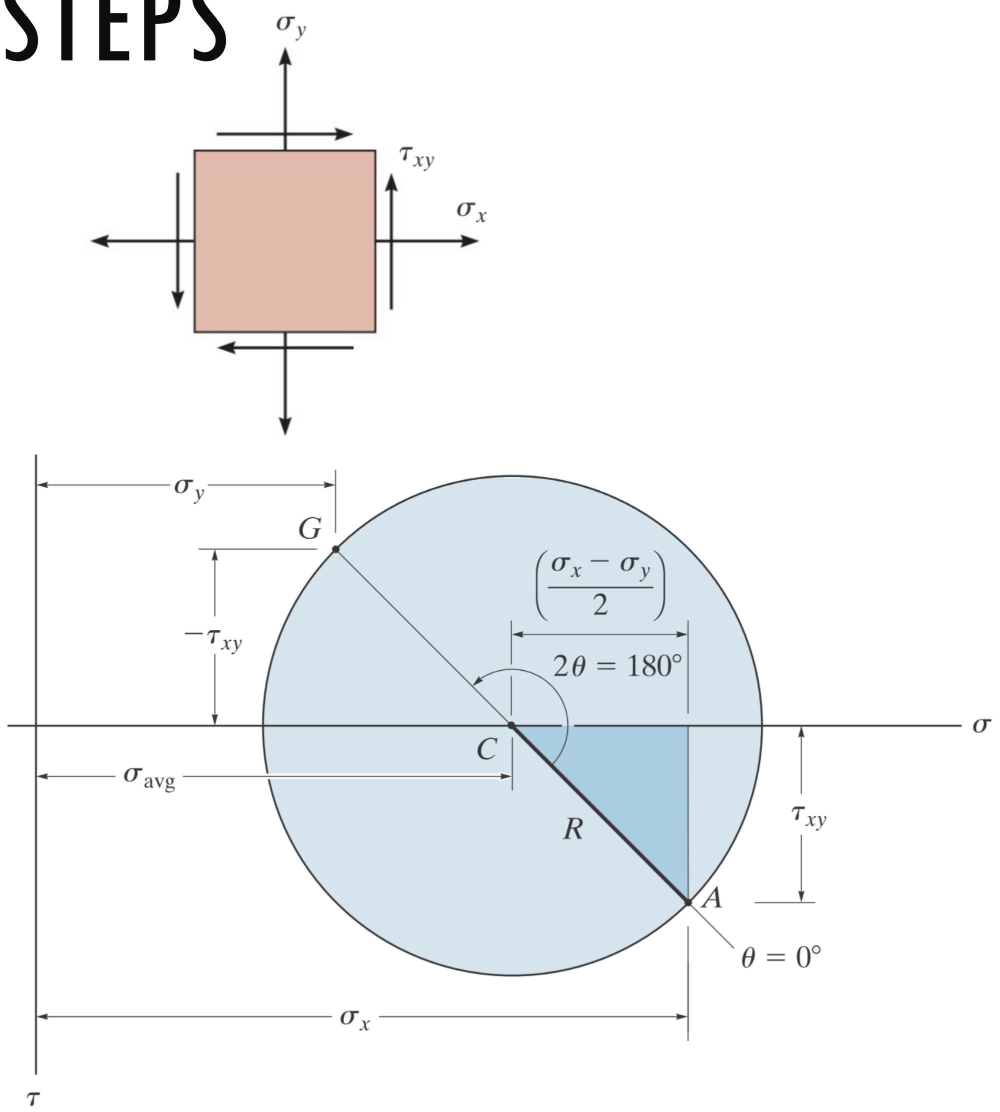
2. Plot center $C = \frac{\sigma_x+\sigma_y}{2}$ $= \sigma_{avg}$.

3. Locate reference point $A\ (\sigma_x, \tau_{xy})$
   (Point A represents $\theta = 0°$)

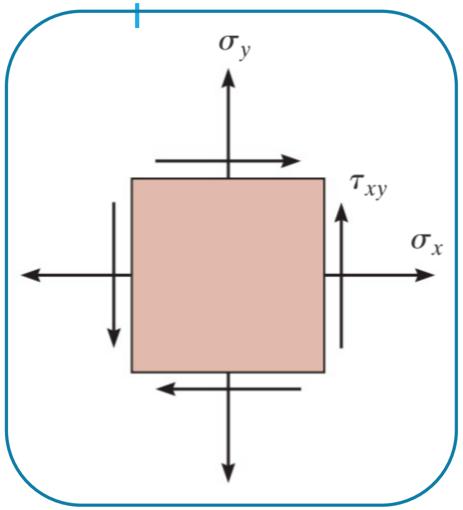
4. Connect point A with point C to get radius R

5. Note that $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

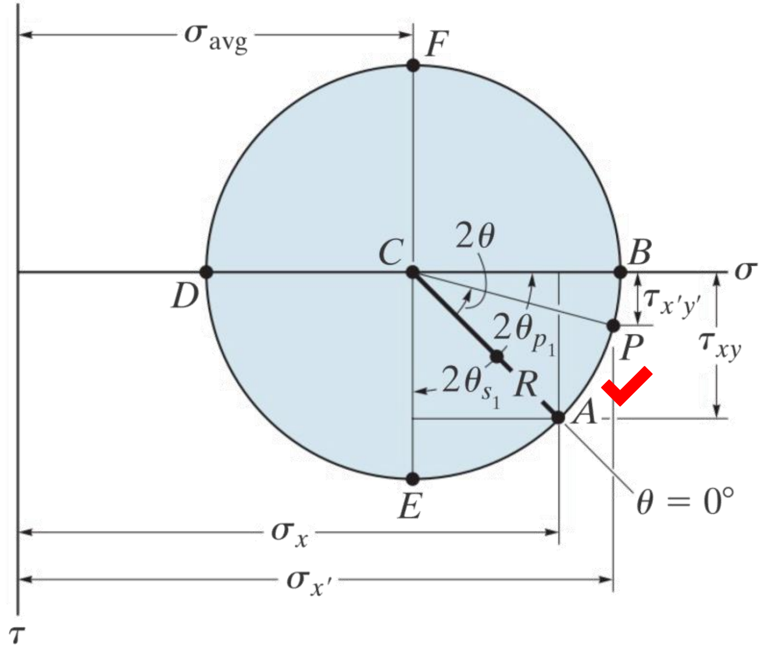
6. Sketch the Circle



MOHR'S CIRCLE: SALIENT FEATURES



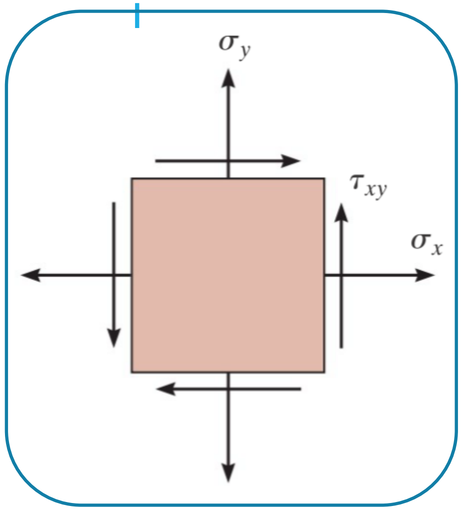
Original Element (Point A)



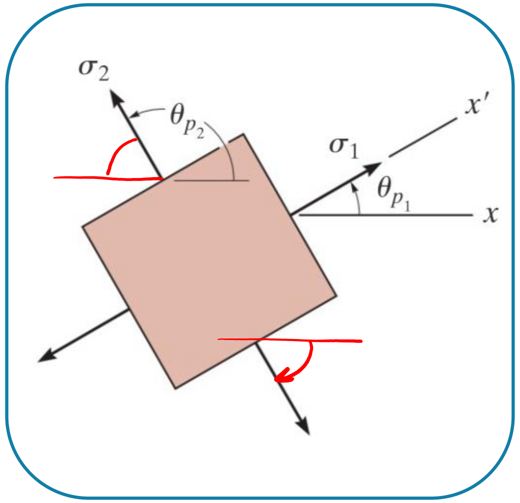
Original

Original state of stress refers to point A. Note, we had established this point while sketching the circle

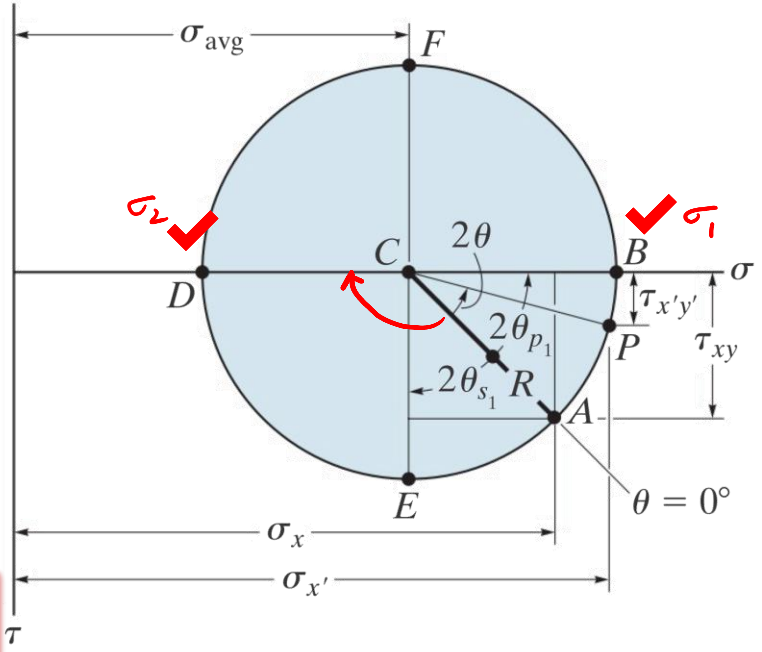
MOHR'S CIRCLE: SALIENT FEATURES



Original Element (Point A)



Element rotated along principal plane (Points B and D)



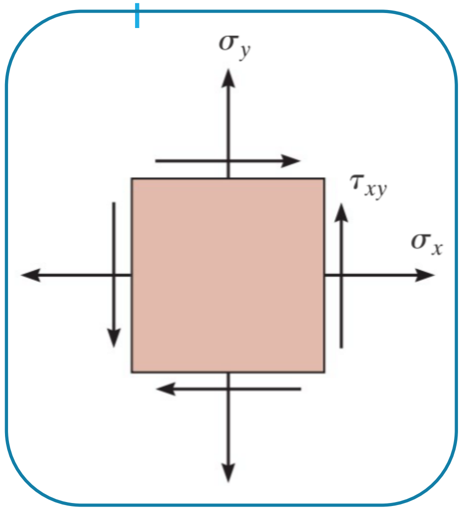
Original

Original state of stress refers to point A. Note, we had established this point while sketching the circle

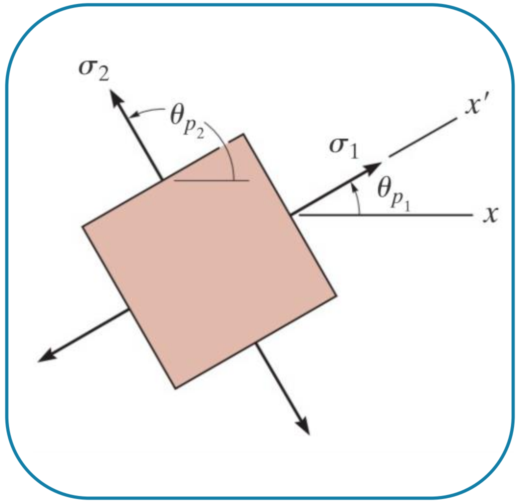
Principal Stresses

- σ_1 and σ_2 - coordinates of points B and D, where circle intersects the σ axis, i.e. $\tau = 0$
- These stresses act on physical plane defined by angles θ_{p1} and θ_{p2}, i.e. $2\theta_{p1}$ and $2\theta_{p2}$ in Mohr plane. As shown, it is measured from reference line CA to CB
- Direction of rotation is same (here anticlockwise)

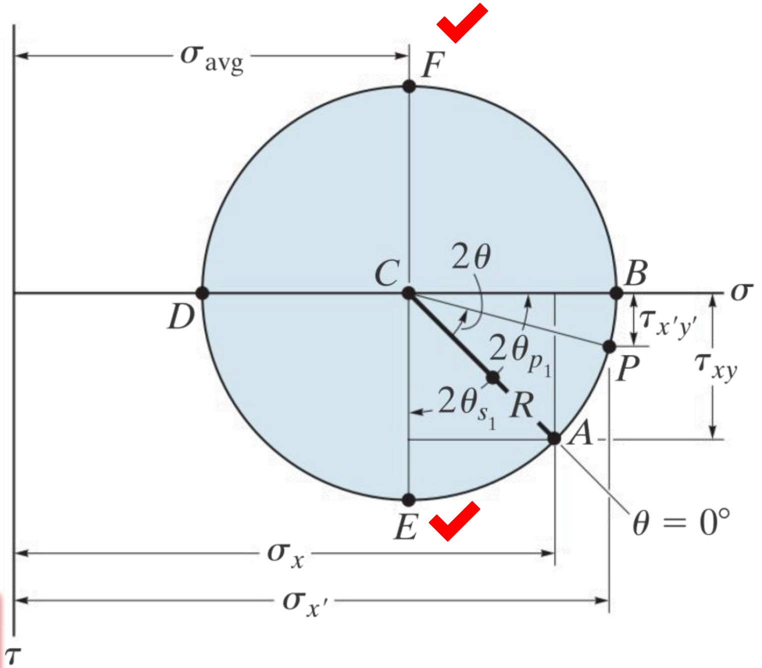
MOHR'S CIRCLE: SALIENT FEATURES

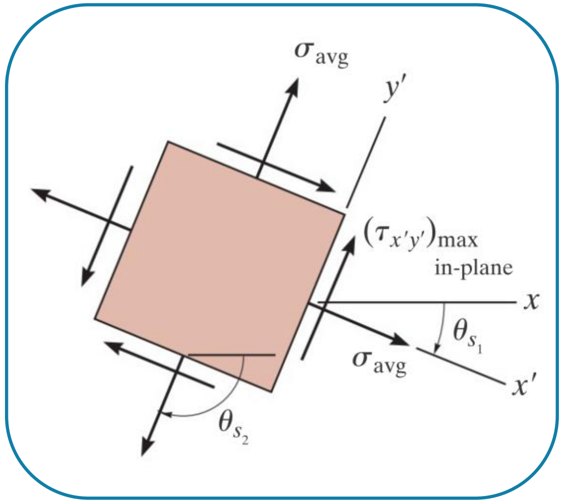


Original Element (Point A)



Element rotated along principal plane (Points B and D)





Element rotated along max. shear plane (Points F and E)

Original

Original state of stress refers to point A. Note, we had established this point while sketching the circle

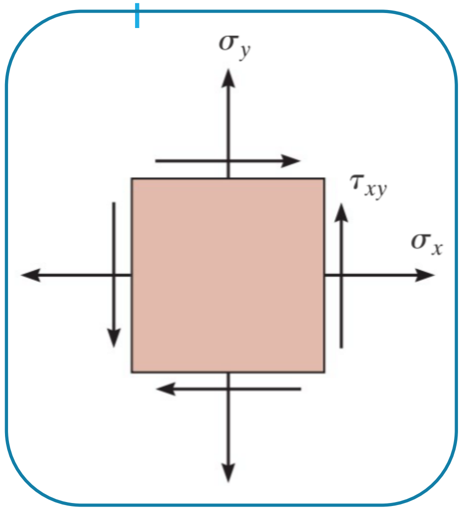
Principal Stresses

- σ_1 and σ_2 - coordinates of points B and D, where circle intersects the σ axis, i.e. $\tau = 0$
- These stresses act on physical plane defined by angles θ_{p1} and θ_{p2}, i.e. $2\theta_{p1}$ and $2\theta_{p2}$ in Mohr plane. As shown, it is measured from reference line CA to CB
- Direction of rotation is same (here anticlockwise)

Max. Shear

- σ_{avg} and max $\tau_{x'y'}$ coordinates of points E and F
- Angles θ_{s1} and θ_{s2}, i.e. $2\theta_{s1}$ and $2\theta_{s2}$ in Mohr plane. As shown, it is measured from reference line CA to CE
- Direction of rotation is same (here clockwise)

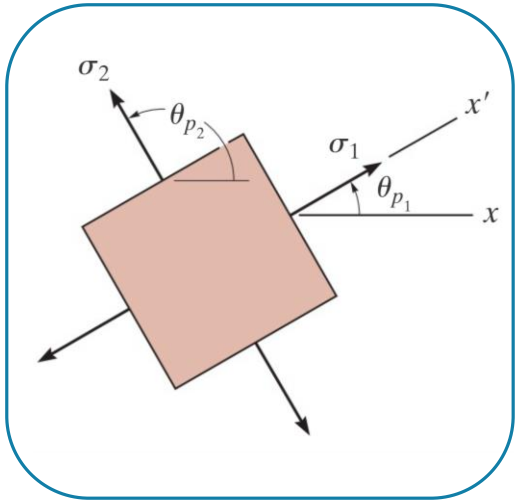
MOHR'S CIRCLE: SALIENT FEATURES



Original Element (Point A)

Original

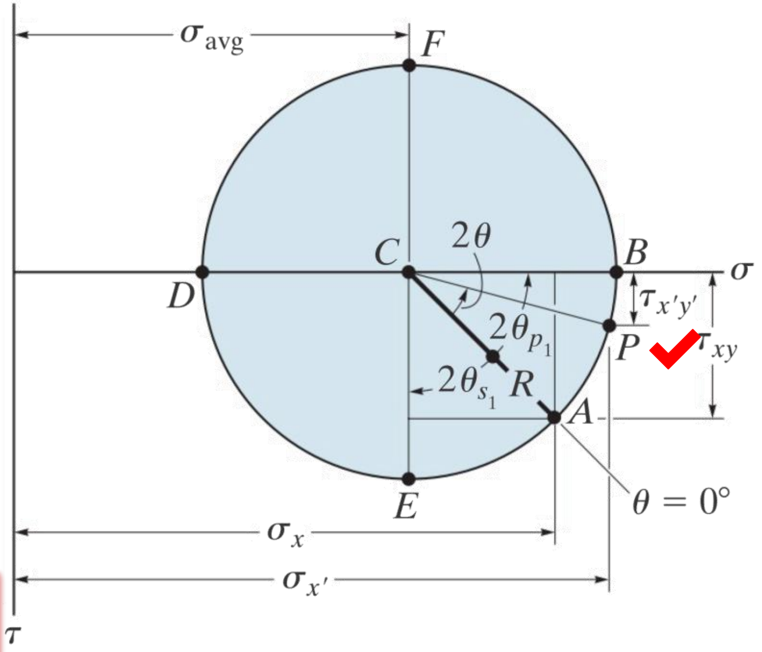
Original state of stress refers to point A. Note, we had established this point while sketching the circle

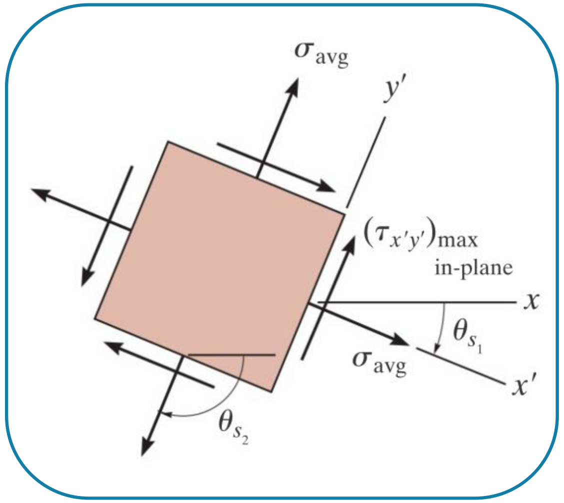


Element rotated along principal plane (Points B and D)

Principal Stresses

- σ_1 and σ_2 - coordinates of points B and D, where circle intersects the σ axis, i.e. $\tau = 0$
- These stresses act on physical plane defined by angles θ_{p1} and θ_{p2}, i.e. $2\theta_{p1}$ and $2\theta_{p2}$ in Mohr plane. As shown, it is measured from reference line CA to CB
- Direction of rotation is same (here anti-clockwise)



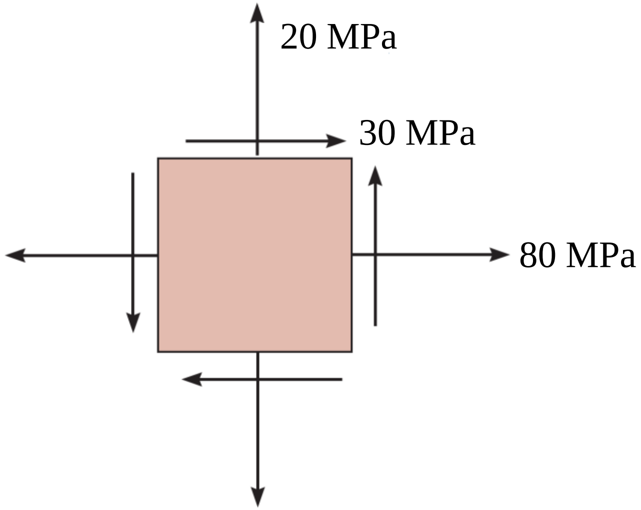


Element rotated along max. shear plane (Points F and E)

Max. Shear

- σ_{avg} and max $\tau_{x'y'}$ coordinates of points E and F
- Angles θ_{s1} and θ_{s2}, i.e. $2\theta_{s1}$ and $2\theta_{s2}$ in Mohr plane. As shown, it is measured from reference line CA to CE
- Direction of rotation is same (here clockwise)

EXAMPLE:



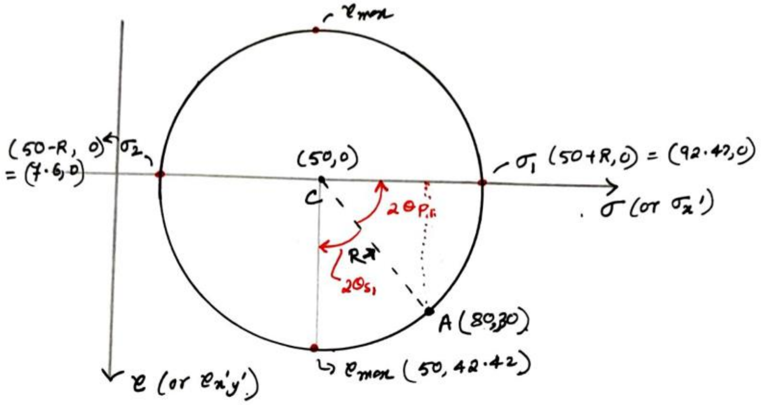
State of stress at failure point

Solve this using Mohr's Circle

- Find principal planes and principal stresses. Show diagram
- Find plane of maximum in-plane shear. Show diagram

(Solution – Next Page)

Graphical Approach [Mohr's Circle]



Steps:

① Locate center C $(\sigma_{avg}, 0)$. ~~Find~~ Find R (radius). Sketch circle.

→ Here, $\sigma_{avg} = 50$ MPa & $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 42.42$ MPa.

② Locate point A where when $\theta = 0$; $\sigma_{x'} = \sigma_x = 80$ MPa

and $\tau_{x'y'} = \tau_{xy} = 30$ MPa

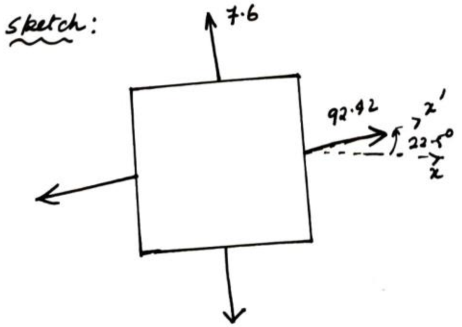
③ Principal plane: $\tan 2\theta_{p_1} = \frac{(80-50)}{30} \Rightarrow 2\theta_{p_1} = 45^\circ$, $\theta_{p_1} = 22.5^\circ$ to σ_1.

↳ sense of rotation: anticlockwise.

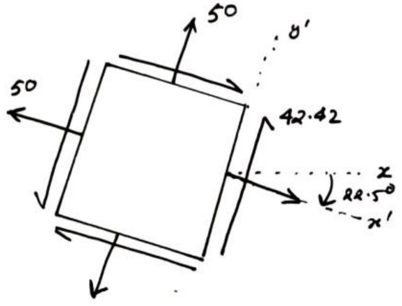
④ Plane of max shear: $2\theta_{s_1} = 90^\circ - 2\theta_{p_1} = 45^\circ$; $\therefore \theta_{s_1} = 22.5^\circ$.

↳ sense of rotation: clockwise.

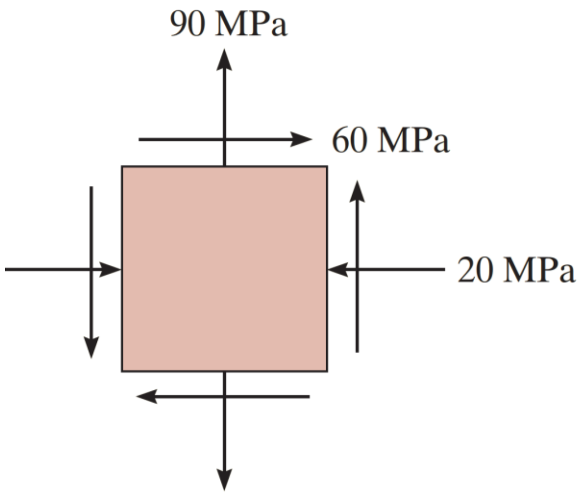
Sketch:



Principal Stress Element



Max. Shear Element

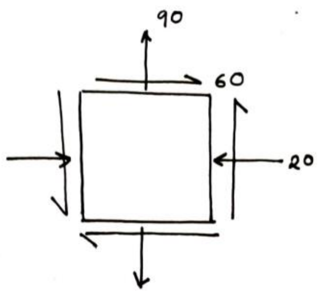


<u>Solve this using Mohr's Circle</u>

- Find principal planes and principal stresses. Show diagram
- Find plane of maximum in-plane shear. Show diagram

(Solution – Next Page)

Problem



$\sigma_x = -20$ MPa

$\sigma_y = 90$ MPa

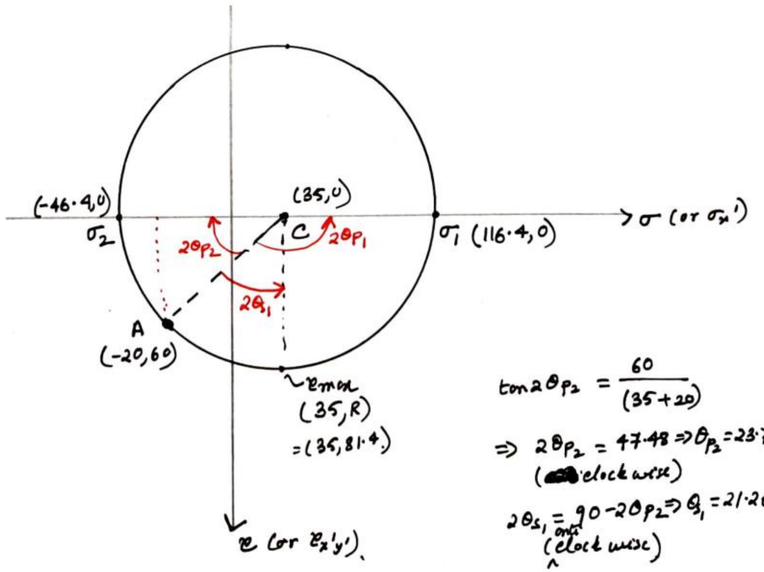
$\tau_{xy} = 60$ MPa

Mohr's Circle

- Center C : $\left(\frac{\sigma_x + \sigma_y}{2}, 0 \right) = (35, 0)$
- Radius : $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 81.4$
- Point A : $(\sigma_{x'} = \sigma_x , \tau_{x'y'} = \tau_{xy}) = (-20, 60)$

$\sigma_1 = C + R = 35 + 81.4 = 116.4$ MPa

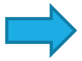
$\sigma_2 = C - R = 35 - 81.4 = -46.4$ MPa.



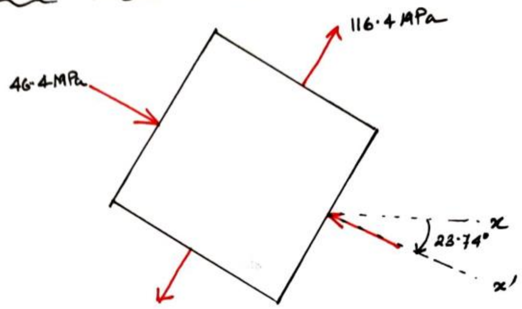
$\tan 2\theta_{p_2} = \frac{60}{(35+20)}$

$\Rightarrow 2\theta_{p_2} = 47.48 \Rightarrow \theta_{p_2} = 23.74°$ (clockwise)

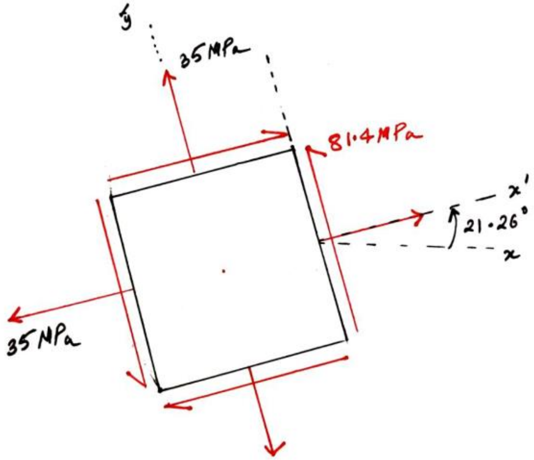
$2\theta_{s_1} = 90 - 2\theta_{p_2} \Rightarrow \theta_{s_1} = 21.26°$ (anti clockwise)



Principal Stress Element

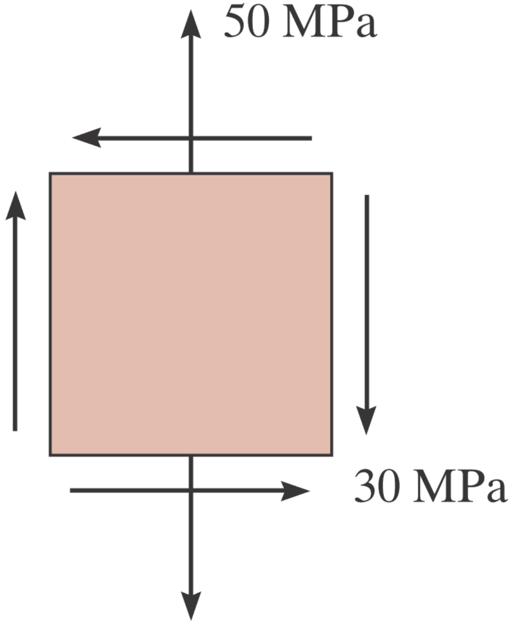


Max. Shear Element



Another Problem:

$$\sigma_{avg} = \frac{0+50}{2} = 25\ MPa; R = \sqrt{12.5^2 + 30^2} = 39.05\ MPa$$



$\sigma_1 = 25 + 39.05 = 64.1\ \text{MPa}$ **Ans.**

$\sigma_2 = 25 - 39.05 = -14.1\ \text{MPa}$ **Ans.**

$\tan 2\theta_p = \dfrac{30}{25 - 0} = 1.2$

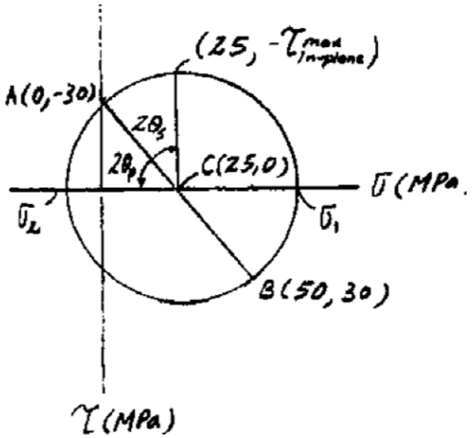
$\theta_{p2} = 25.1°$ **Ans.**

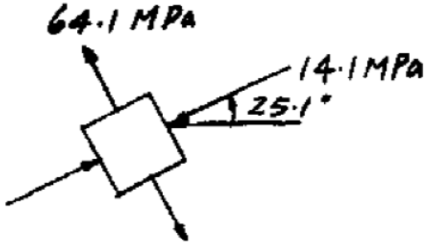
$\sigma_{\text{avg}} = 25.0\ \text{MPa}$ **Ans.**

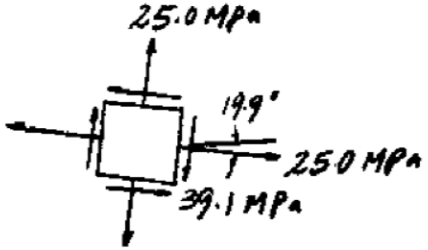
$\tau_{\substack{\max \\ \text{in-plane}}} = R = 39.1\ \text{MPa}$ **Ans.**

$\tan 2\theta_s = \dfrac{25 - 0}{30} = 0.8333$

$\theta_s = -19.9°$ **Ans.**

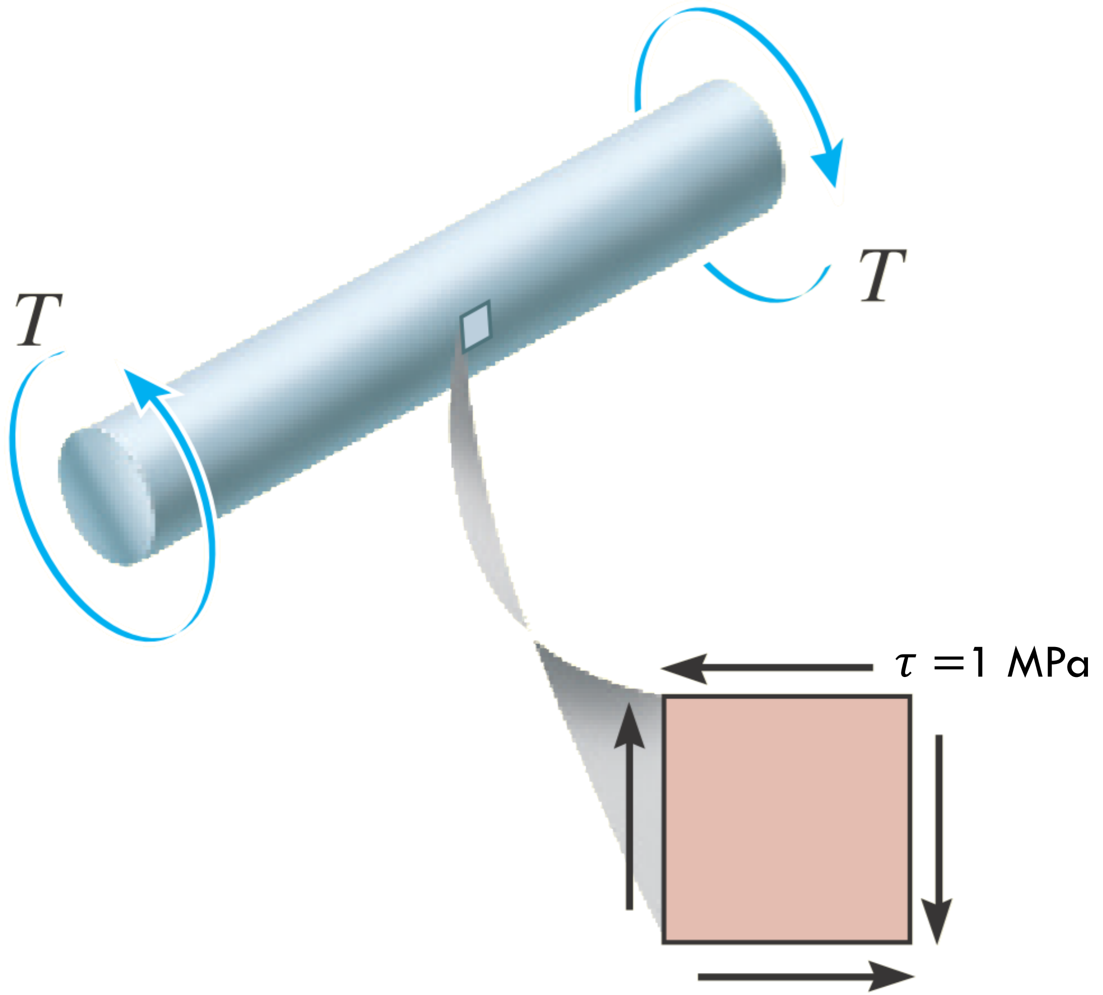






A piece of chalk under pure torsion T develops pure shear condition on surface elements as shown.

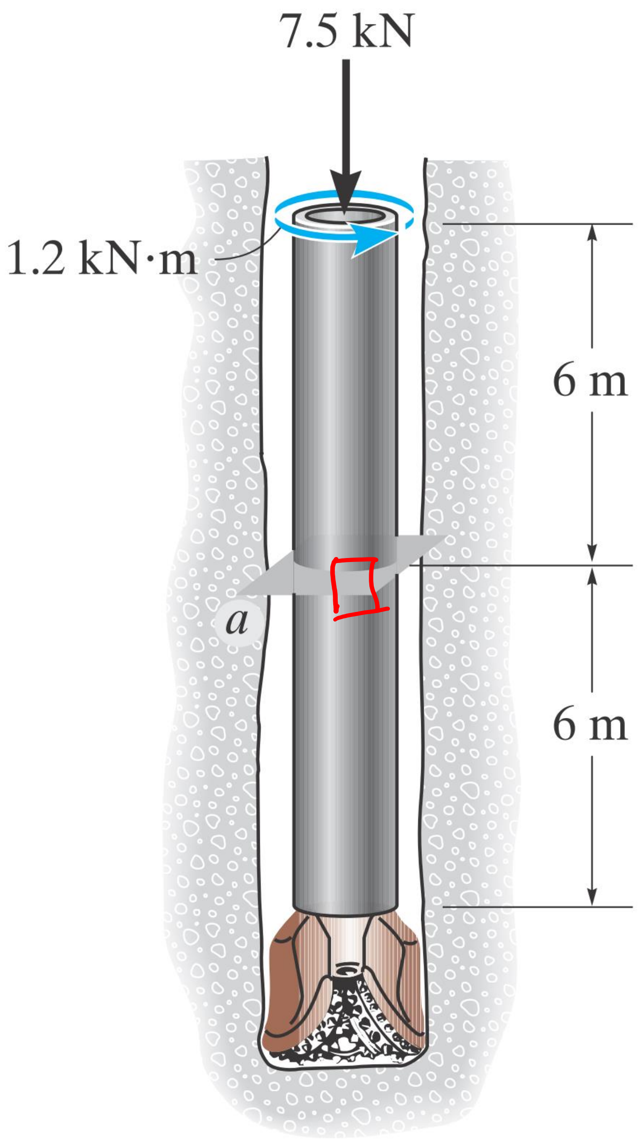
For chalk, $\sigma_{allowable} = 0.8 \text{ MPa}$ and $\tau_{allowable} = 1.2 \text{ MPa}$. Will the chalk fail? If yes, how?



*9–28. The drill pipe has an outer diameter of 75 mm, a wall thickness of 6 mm, and a weight of 0.8 kN/m. If it is subjected to a torque and axial load as shown, determine (a) the principal stress and (b) the maximum in-plane shear stress at a point on its surface at section *a*.

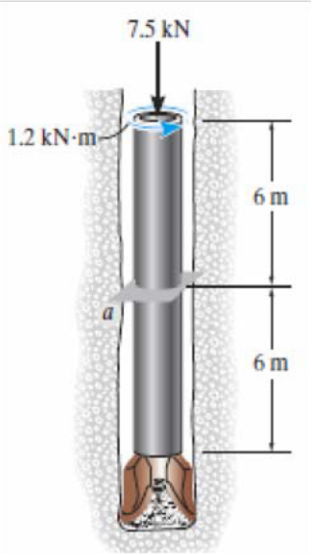
Given: $[A = 1300.61\ mm^2; J = 1559767\ mm^4; I = 779884\ mm^4]$

(Solution – Next Page)



Prob. 9–28

The drill pipe has an outer diameter of 75 mm, a wall thickness of 6 mm, and a weight of 0.8 kN/m. If it is subjected to a torque and axial load as shown, determine (a) the principal stresses and (b) the maximum in-plane shear stress at a point on its surface at section *a*.



Given: $d_o := 75\text{mm}$ $\quad t := 6\text{mm}$ $\quad L := 6\text{m}$

$P := 7.5\text{kN}$ $\quad M_x := 1.2\text{kN}\cdot\text{m}$ $\quad w := 0.8\dfrac{\text{kN}}{\text{m}}$

Solution:

Internal Force and Moment : At section *a:*

$\Sigma F_x=0;\quad N + P + w\cdot L = 0 \qquad N := -P - w\cdot L$

$\Sigma M_x=0;\quad T - M_x = 0 \qquad T := M_x$

Section Property : $d_i := d_o - 2t$

$$A := \frac{\pi}{4}\cdot\left(d_o^{\,2} - d_i^{\,2}\right) \qquad J := \frac{\pi}{32}\cdot\left(d_o^{\,4} - d_i^{\,4}\right)$$

Normal Stress: $\sigma := \dfrac{N}{A}$ $\qquad \sigma = -9.457\text{ MPa}$

Shear Stress :

$c := 0.5d_o \qquad \tau := \dfrac{T\cdot c}{J} \qquad \tau = 28.850\text{ MPa}$

a) ***In-plane Principal Stresses:***

$\sigma_x := 0 \qquad \sigma_y := \sigma \qquad \tau_{xy} := \tau$

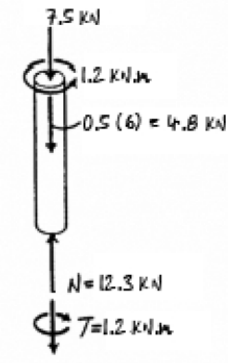
for any point on the shaft's surface. Applying Eq. 9-5,

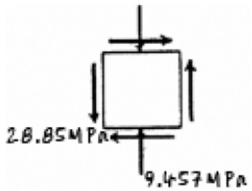
$$\sigma_1 := \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}} \qquad \sigma_1 = 24.51\text{ MPa} \quad \textbf{Ans}$$

$$\sigma_2 := \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}} \qquad \sigma_2 = -33.96\text{ MPa} \quad \textbf{Ans}$$

b) ***Maximum In-plane Shear Stress:*** Applying Eq. 9-7,

$$\tau_{max} := \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}} \qquad \tau_{max} = 29.24\text{ MPa} \quad \textbf{Ans}$$



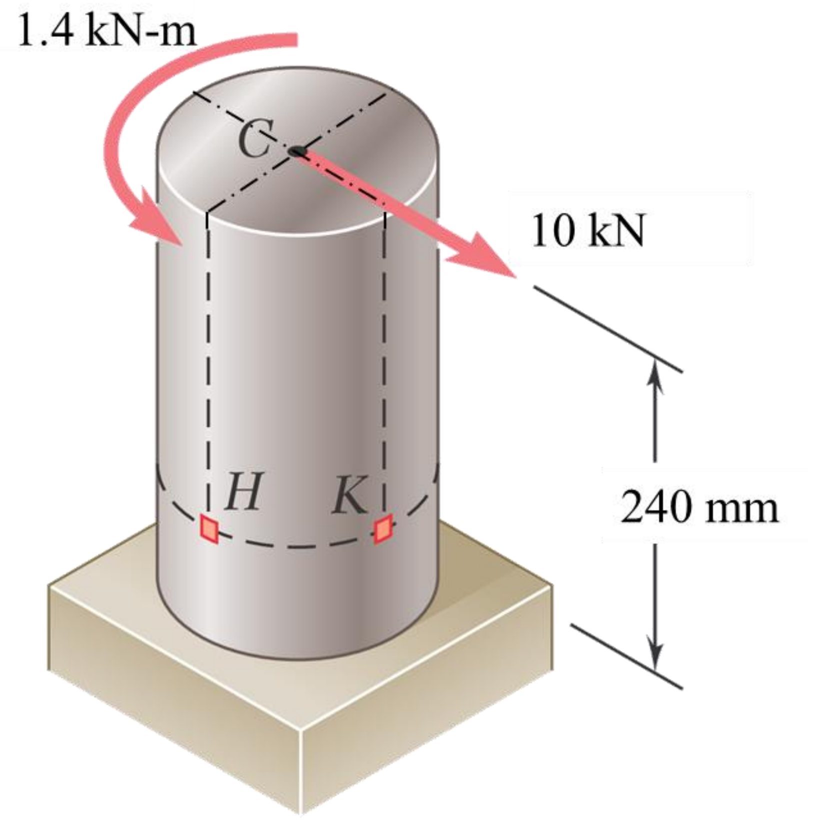


As we did in class, you can compute the principal stresses using Mohr's circle approach by calculating C(σ_{avg}, 0) and R. Then compute σ_1 and σ_2 as $C + R$ and $C - R$.

Also, $\tau_{max} = R$.

A 10-kN force and a 1.4-kN · m couple are applied at the top of the 65-mm diameter brass post shown. Determine the principal stresses and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

Given: $[A = 3318.31\ mm^2; J = 1752481\ mm^4; I = 876240\ mm^4]$



(Solution – Next Page)

SOLUTION

At the section containing points H and K,

$$V = 10\text{ kN} = 10 \cdot 10^3\text{ N}$$

$$M = (10 \cdot 10^3)(240 \cdot 10^{-3}) = 2.4 \cdot 10^3\text{ N}\cdot\text{m}$$

$$T = 1.4 \cdot 10^3\text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 32.5\text{ mm} = 0.0325\text{ m}$$

$$J = \frac{\pi}{2}c^4 = 1.75248 \cdot 10^{-6}\text{ m}^4$$

$$I = \frac{1}{2}J = 0.87624 \cdot 10^{-6}\text{ m}^4$$

For a semicircle, $Q = \frac{2}{3}c^3 = \frac{2}{3}(0.0325)^3 = 22.885 \cdot 10^{-6}\text{ m}^3$

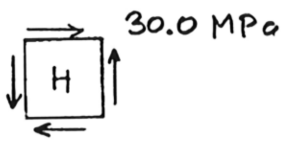
(a) Stresses at point H. H lies on the neutral axis: $\sigma = 0$

Due to torque: $\tau = \dfrac{Tc}{J} = \dfrac{(1.4 \cdot 10^3)(0.0325)}{1.75248 \cdot 10^{-6}}$

$= 25.963$ MPa

Due to shear: $\tau = \dfrac{VQ}{It} = \dfrac{(10 \cdot 10^3)(22.885 \cdot 10^{-6})}{(0.87624 \cdot 10^{-6})(0.065)}$

$= 4.018$ MPa



Total at H: $\tau = 30.0$ MPa

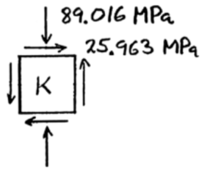
$\sigma_{ave} = 0, \quad R = 30.0$ MPa

$\sigma_{max} = \sigma_{ave} + R$ $\qquad \sigma_{max} = 30.0$ MPa ◀

$\sigma_{min} = \sigma_{ave} - R$ $\qquad \sigma_{min} = -30.0$ MPa ◀

$\tau_{max} = R$ $\qquad \tau_{max} = 30.0$ MPa ◀

(b) Stresses at point K.



Due to shear: $\tau = 0$

Due to torque: $\tau = \dfrac{Tc}{J} = 25.963$ MPa

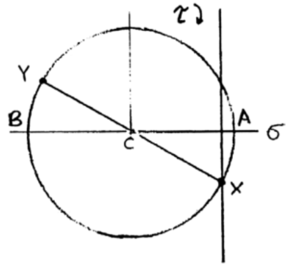
Due to bending: $\sigma = -\dfrac{Mc}{I} = -\dfrac{(2.4 \cdot 10^3)(0.0325)}{(0.87624 \cdot 10^{-6})}$

$= -89.016$ MPa

$$\sigma_{ave} = \frac{-89.016}{2} = -44.508\text{ MPa}$$

$$R = \sqrt{\left(\frac{89.016}{2}\right)^2 + (25.963)^2}$$

$= 51.527$ MPa

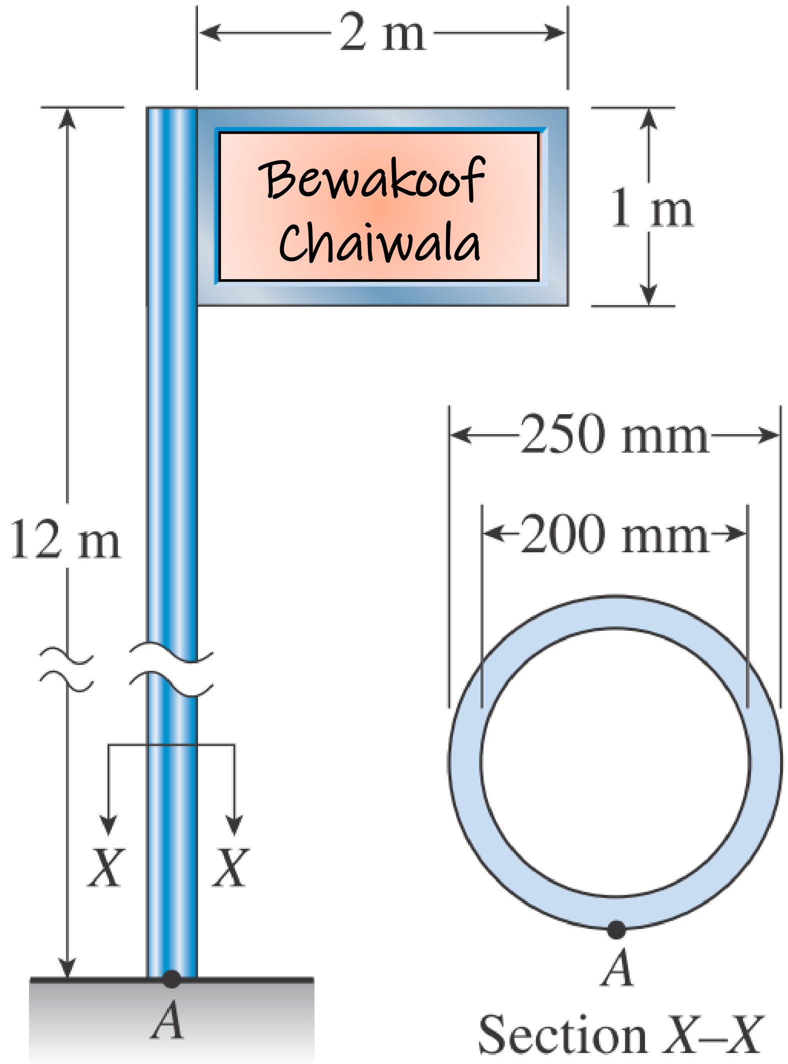


$\sigma_{max} = \sigma_{ave} + R$ $\qquad \sigma_{max} = 7.02$ MPa ◀

$\sigma_{min} = \sigma_{ave} - R$ $\qquad \sigma_{min} = -96.0$ MPa ◀

$\tau_{max} = R$ $\qquad \tau_{max} = 51.5$ MPa ◀

Answers



- Hollow pole 12 m high
- Pole weight = 18 kN
- Sign board weight = 2.2 kN
- Wind pressure = 1.5 kPa

Given: $[A = 17671.5\ mm^2; J = 226415564\ mm^4; I = 113207782 mm^4]$

Determine:

i. Sketch the stress element at A on the outer surface in front of the pole

ii. Principal Stresses at A

$\boxed{\sigma_1 = 37 \text{ MPa}}$ $\boxed{\sigma_2 = -0.094 \text{ MPa}}$

iii. Maximum in-plane Shear at A

$\boxed{\tau_{\max} = 18.57 \text{ MPa}}$