classmate

Date

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⇒ Laplace Transformation –

→ $t \to s = \sigma + j\omega$.

Laplace Transformation.

$$F(s) = L[f(t)] = \int_0^\infty f(t) \cdot e^{-st} \cdot dt.$$

where s = frequency.

$$F(s) = \int_0^\infty 1 \cdot e^{-st}\, dt$$

$$= \frac{e^{-st}}{-s} \Big|_0^\infty$$

$$= \frac{1}{s}$$

⇒ Unit function –

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \end{cases}$$

$f(t) = 1$.

$$F(s) = \int_0^\infty f(t) \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty 1 \cdot e^{-st} \cdot dt$$

$$= \frac{(e^{-st})_0^\infty}{-s}$$

$$= \frac{-1}{s}[0 - 1] = 1/s.$$

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$\Rightarrow$ Ramp function -

$f(t) = t$

$F(s) = ?$

$$F(s) = \int_0^\infty f(t) \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty t \cdot e^{-st} \, dt$$

$$= \left(t \frac{e^{-st}}{-s} \right)_0^\infty - \frac{1}{-s} \int_0^\infty 1 \cdot e^{-st} \cdot dt$$

$$= (0 - 0) + \frac{1}{s} \frac{\left(e^{-st} \right)_0^\infty}{-s}$$

$$= \frac{-1}{s^2} (0 - 1)$$

$$= 1/s^2 .$$

$\Rightarrow$ $f(t) = e^{-\alpha t}$.

$F(s) = ?$

$$F(s) = \int_0^\infty f(t) \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty e^{-\alpha t} \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty e^{-t(s+\alpha)} \cdot dt .$$

$$= \frac{1}{s+\alpha}$$

$\Rightarrow$ 'sin' function -

$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$f(t) = \sin\omega t$

$F(s) = ?$

$$F(s) = \int_0^\infty f(t) \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty \sin\omega t \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty \left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j} \right) e^{-st} \cdot dt$$

$$= \int_0^\infty \frac{e^{j\omega t} \cdot e^{-st}}{2j} \cdot dt - \int_0^\infty \frac{e^{-st} \cdot e^{-j\omega t}}{2j} \cdot dt$$

$$= \frac{1}{2j} \int_0^\infty e^{-t(-j\omega + s)} \cdot dt - \frac{1}{2j} \int_0^\infty e^{-t(j\omega + s)} \cdot dt$$

$$= \frac{1}{2j} \cdot \frac{1}{s - j\omega} - \frac{1}{2j} \cdot \frac{1}{s + j\omega}$$

$$= \frac{1}{2j} \left[\frac{1}{s - j\omega} - \frac{1}{s + j\omega} \right]$$

$$= \frac{1}{2j} \left[\frac{\cancel{s} + j\omega - \cancel{s} + j\omega}{s^2 - j^2\omega^2} \right]$$

$$= \frac{1}{2j} \left[\frac{2j\omega}{s^2 + \omega^2} \right] = \frac{\omega}{s^2 + \omega^2}$$

$\Rightarrow$ find $L\{f'(t)\}$.

$$F(s) = \int_0^\infty f(t) \cdot e^{-st} \, dt.$$

$$F(s) = \left(f(t) \cdot \frac{1}{s} e^{-st} \right)_0^\infty + \frac{1}{s} \int_0^\infty f'(t) \, e^{-st} \, dt.$$

$$\therefore sF(s) = -f(t) \cdot e^{-st} + \int_0^\infty f'(t) \cdot e^{-st} \, dt$$

$$\therefore sF(s) = f(0^+) + L\{f'(t)\}.$$

$$\therefore sF(s) - f(0^+) = L\{f'(t)\}.$$

$\Rightarrow$ $\cos\omega t = \frac{1}{\omega} \frac{d}{dt} (\sin\omega t)$

$$= \frac{1}{\omega} \left[sF(s) - f(0^+) \right]$$

$$= \frac{1}{\omega} \left[s \cdot \frac{\omega}{s^2 + \omega^2} \right]$$

$$= \frac{s}{s^2 + \omega^2}.$$

$\Rightarrow$ $L\{f(t)\} = \int_0^\infty f(t) \cdot e^{-st} \cdot dt.$

$\Rightarrow$ $L\{A f_1(t) + B f_2(t)\} = A F_1(s) + B F_2(s).$

$\Rightarrow$ $L\{\frac{d}{dt} f(t)\} = sF(s) - f(0^+).$

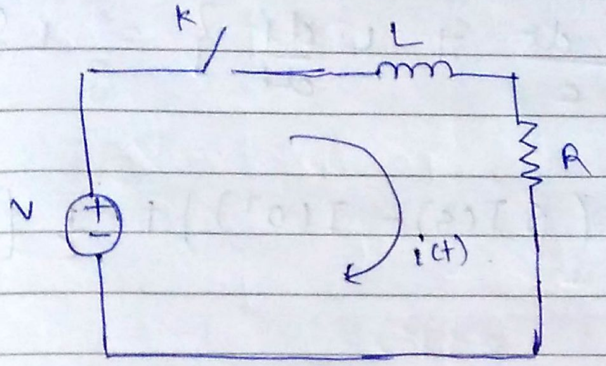
$\Rightarrow$ $L\{\int_{t_1}^{t_2} f(t) \cdot dt\} = \frac{1}{s} F(s).$

⇒ Initial condition —

$$f(0^+) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s).$$

⇒ Final condition —

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s).$$

⇒ 

Here, $Ri + L\cdot\frac{di}{dt} = v.$

Now, we will take it's Laplace,

$$L\left\{Ri + L\cdot\frac{di}{dt}\right\} = L\{v\} = v\,L\{1\}.$$

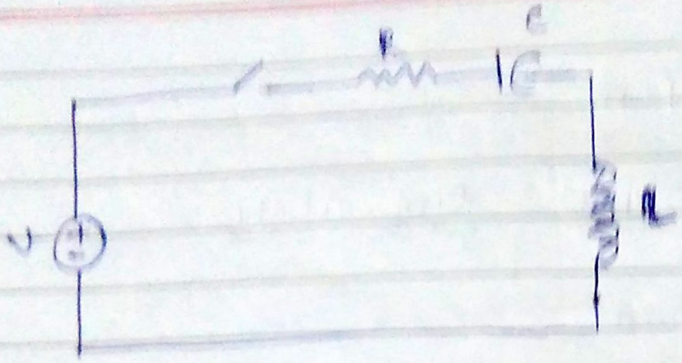
$$\therefore R\,I(s) + L\big(sI(s) - f(0^+)\big) = v/s.$$

$$\therefore R\,I(s) + sL\,I(s) - L\,I(0^+) = v/s.$$

intially zero.

$$\boxed{\therefore I(s) = \frac{v/s}{R+sL}}$$

⇒



Here, $Ri + \frac{\int i \cdot dt}{C} + L\frac{di}{dt} = v$. → time domain

$\therefore L\left\{ Ri + \frac{\int i \cdot dt}{C} + L\frac{di}{dt} \right\} = L\{v\}$.

s) domain → $\therefore RI(s) + L(sI(s) - I(0^+)) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{\overset{=0\text{ (initial)}}{q(0)}}{s}\right] = V/s$.

$\therefore RI(s) + sLI(s) + \frac{I(s)}{Cs} = V/s$.

$\therefore \frac{RCs\,I(s) + s^2LC\,I(s) + I(s)}{Cs} = \frac{V}{s}$

$\therefore I(s)\left[RCs + s^2LC + 1\right] = VC$.

$\therefore I(s) = \frac{VC}{RCs + s^2LC + 1}$

$\therefore I(s) = \frac{V/s}{R + Ls + \frac{1}{Cs}}$.

⇒ Partial fractions –

⇒ $$\frac{s-1}{s^2+3s+2} = \frac{s-1}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$\therefore \quad s-1 = A(s+1) + B(s+2).$$

If $s = -2 \Rightarrow -3 = A(-2+1)$

$$\therefore -3 = A(-1)$$

$$\therefore A = 3$$

nitial)

If $s = -1 \Rightarrow -2 = B(1)$

$$\therefore B = -2.$$

v/s.

$$\therefore \frac{s-1}{(s+2)(s+1)} = \frac{3}{s+2} + \frac{-2}{s+1}.$$

⇒ $$\frac{1}{s[s^2+6s+9]} = \frac{1}{s[s^2+3s+3s+9]}$$

$$= \frac{1}{s[s(s+3)+3(s+3)]}$$

$$= \frac{1}{s[s+3]^2} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+3)^2}$$

$$\therefore \frac{1}{s(s+3)^2} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+3)^2}$$

$$\therefore 1 = A(s+3)^2 + B\cdot(s)(s+3) + C\cdot(s)$$

if $s = -3$

$\therefore 1 = C(-3)$

$\therefore c = -1/3$.

if $s = 0$.

$\therefore 1 = A(9)$

$\therefore A = 1/9$.

if $s = -1$

$\therefore 1 = A(4) + B(-2) + C(-1)$

$\therefore 1 = \frac{4}{9} - 2B + 1/3$.

$\therefore 2B = \frac{4}{9} + \frac{3}{9} - \frac{9}{9}$

$\therefore 2B = -\frac{2}{9}$

$\therefore B = -1/9$

$\Rightarrow$ Inverse Laplace -

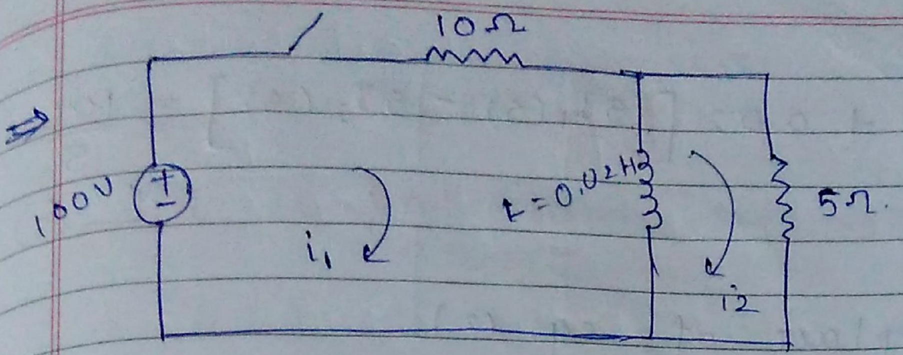
$$L^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma - j\omega}^{\sigma + j\omega} F(s) \cdot e^{st} \cdot ds = f(t)$$

$\Rightarrow$ find Laplace -

$$f(t) = \cos 4t + 2 \cdot e^{-3t} + t + \frac{1}{2} t$$

$$L\{f(t)\} = L\{\cos 4t\} + 2 L\{e^{-3t}\} + L\{t\} + \frac{1}{2} L\{t\}$$

$$= \frac{1}{4} \cdot \frac{s}{s^2 + 16} + 2 \cdot \frac{1}{s+3} + \frac{1}{s^2} + \frac{1}{2} \cdot \frac{1}{s^2}$$



$$L = \frac{R \cdot L}{R + L} = \frac{5(0.02)}{5 + 0.02} =$$

→ $100 = R_1 i_1 + L \cdot \frac{d(i_1 - i_2)}{dt}$

$\therefore 100 = 10 i_1 + 0.02 \left(\frac{di_1}{dt} - \frac{di_2}{dt} \right)$ — (1).

→ for <u>loop - 2</u>

$\therefore \cancel{100 = R_1 i_1 + R_2 i_2}$

$\therefore \cancel{100 = 10 i_1 + 5 i_2}$ — (2).

$0 = R_2 i_2 + L \frac{d}{dt}(i_2 - i_1)$

$\therefore 5 i_2 + 0.02 \cdot \frac{d}{dt} (i_2 - i_1) = 0$ — (2).

→ Now, Laplace of eq. (1).

$\therefore L\left\{ 10 i_1 + 0.02 \left(\frac{di_1}{dt} - \frac{di_2}{dt} \right) \right\} = L\{100\}$

$\therefore 10 . I_1(s) + 0.02 \left[\left(s I_1(s) - I_1(0^+) \right) - \left(s I_2(s) - I_2(0^+) \right) \right]$

$= 100 \cdot 1/s.$

$$\therefore 10 I_1(s) + 0.02 \left[s I_1(s) - s I_2(s) \right] = \frac{100}{s} \quad (3)$$

and Laplace of eq. (2),

$$\therefore \cancel{10}\; 5 \cdot I_2(s) + 0.02 \left[\left(s I_2(s) - I_2(0^+) \right) - \left(s I_1(s) - I_1(0^+) \right) \right] = 0$$

$$\therefore 5 I_2(s) + 0.02 \left[s I_2(s) - s I_1(s) \right] = 0. \quad (4)$$

from eq. (4),

$$\cancel{sL\left(I_1(s) - I_2(s) \right) = \frac{100}{s} - 10 I_1(s)}$$

$$\therefore 5 I_2(s) + sL \left[I_2(s) - I_1(s) \right] = 0.$$

$$\therefore 5 I_2(s) + sL \left(I_2(s) \right) - sL\, I_1(s) = 0$$

$$\therefore (5 + sL)\, I_2(s) = sL\; I_1(s)$$

$$\therefore I_2(s) = \frac{sL \cdot I_1(s)}{5 + sL} \quad (5).$$

put eq.(5) in eq.(3)

$$\therefore 10 I_1(s) + sL\left[I_1(s) - I_2(s) \right] = \frac{v}{s}$$

$$\therefore 10 I_1(s) + sL\left[I_1(s) - \frac{sL \cdot I_1(s)}{5 + sL} \right] = v/s$$

$$\therefore 10 I_1(s) + sL\left[\frac{5 I_1(s) + \cancel{sL I_1(s)} - \cancel{sL I_1(s)}}{5 + sL} \right] = v/s$$

$$\therefore 10 I_1(s) + \frac{5sL}{5+sL} I_1(s) = \frac{v}{s}$$

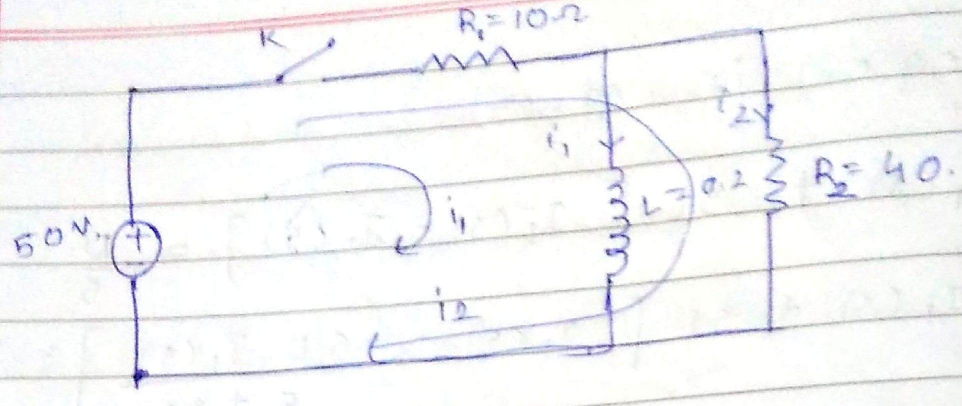
$$\therefore I_1(s)\left[10 + \frac{5sL}{5+sL} \right] = v/s$$

$$\therefore I_1(s)\left[\frac{50 + 10sL + 5sL}{5 + sL} \right] = v/s$$

$$\therefore I_1(s)\left[\frac{50 + 15sL}{5 + sL} \right] = v/s$$

$$\therefore I_1(s) = \frac{v(5 + sL)}{s(50 + 15sL)} \quad \text{— (6)}$$

Que



$\cancel{50 \mp i_1(10) \mp L \cdot \frac{d}{dt}(i_1+i_2) = 0}$

$\cancel{\therefore 50 = 10 i_1 + L \cdot \frac{di_1}{dt}}$ —— ①

→ ~~for 2nd loop,~~

$50 = 10(i_1+i_2) + L \cdot \frac{di_1}{dt}$ —— ①

→ for 2nd loop,

$50 = 10(i_1+i_2) + 40 i_2$ —— ~~②~~

$\therefore 50 = 10 i_1 + 50 i_2$ —— ②

→ Laplace of 1st loop.

$L\{50\} = L\{10(i_1+i_2)\} + L\{0.2 \cdot \frac{di_1}{dt}\}$

$\therefore 50 \cdot \frac{1}{s} = 10[I_1(s) + I_2(s)] + 0.2 \cdot [sI_1(s) - I_1(0)$

$\therefore \frac{50}{s} = 10 I_1(s) + 10 I_2(s) + 0.2[sI_1(s) - I_1(0)$

$\therefore \frac{50}{s} = (10 + Ls) I_1(s) + 10 I_2(s)$ —— ③

→ Laplace of 2nd loop -

$$L\{50\} = L\{10 i_1\} + 50 \cdot L\{i_2\}$$

$$\therefore \frac{50}{s} = 10 \cdot I_1(s) + 50\, I_2(s) \quad — (4).$$

→ ~~comparing (3) and (4).~~

$$\begin{bmatrix} 10 & 40 \\ 10+Ls & 10 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 50/s \\ 50/s \end{bmatrix}$$

$$\therefore I_1(s) = \frac{\begin{vmatrix} \frac{50}{s} & 40 \\ \frac{50}{s} & 10 \end{vmatrix}}{\begin{vmatrix} 10 & 40 \\ 10+Ls & 10 \end{vmatrix}}$$

$$= \frac{|100 - 400|}{|100 - 400 - 40Ls|}$$

$$\therefore I_1(s) = \frac{300}{300 + 40Ls}$$

→ and
$$I_2(s) = \frac{\begin{vmatrix} 10 & 50/s \\ 10+Ls & 50/s \end{vmatrix}}{|100 - 400 - 40Ls|}$$

$$= \frac{\cancel{500/s} - \cancel{\frac{500}{s}} - 50L}{300 + 40Ls}$$

$$\therefore I_2(s) = \frac{50L}{300 + 40Ls}$$

$\therefore (10 + Ls) I_1(s) + 10 I_2(s) = 10 I_1(s) + 50 I_2(s)$

$\therefore \cancel{10 I_1(s)} + Ls\, I_1(s) + 10 I_2(s) = \cancel{10 I_1(s)} + 50 I_2(s)$

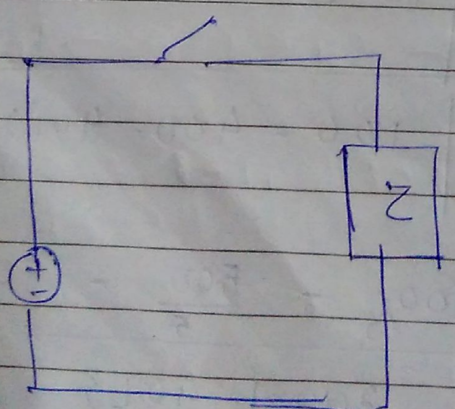
$\therefore \quad Ls\, I_1(s) - 40 I_2(s) = 0.$

$\therefore I_1(s) = \frac{5}{s + 0.625}, \quad I_2(s) = \frac{1}{s} - \frac{1}{s + 0.625}$

$L^{-1}(I_1(s)) = 5 \cdot e^{-0.625t}$

$L^{-1}(I_2(s)) = 1 - e^{-0.625t}$

⇒ Obtain the equivalent impedence of the s domain network. find total current using current div. method (same network as before)



$$Z = R_1 + \left(\frac{R_2 L}{R_2 + L}\right)$$

$$= 10 + \left(\frac{40 \times 0.2}{40.2}\right)$$

$$Z = R_1 + \left(\frac{R_2 \cdot \frac{1}{\omega L}}{R_2 + \frac{1}{\omega L}}\right)$$

$$= 10 + \frac{\left(\frac{40}{0.2s}\right)}{\left(40 + \frac{1}{0.2s}\right)} \qquad = 10 + \left(\frac{40 \times 0.2s}{40 + 0.2s}\right)$$

$$= 10 + \left(\frac{8s}{40 + 0.2s}\right)$$

$$= 10 + \frac{\frac{200}{s}}{\frac{8s + 1}{0.2s}} \qquad = \frac{400 + 2s + 8s}{40 + 0.2s}$$

$$= 10 + \frac{40}{8s + 1} \qquad = \frac{400 + 10s}{40 + 0.2s}$$

$$= \frac{80s + 10 + 40}{8s + 1}$$

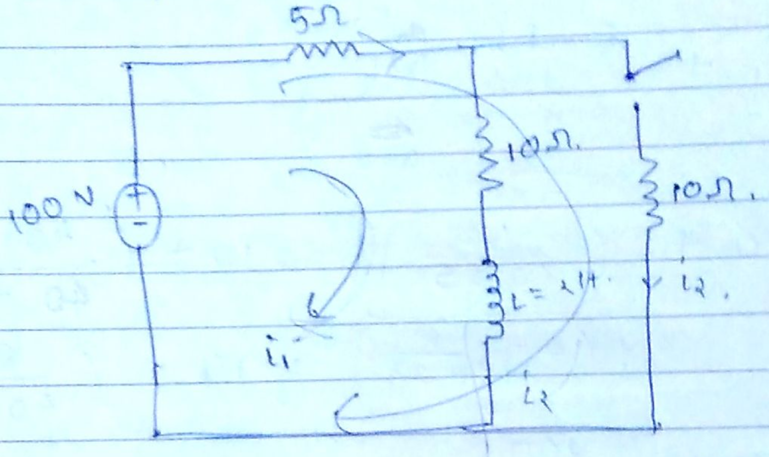
$$= \frac{80s + 50}{8s + 1}$$

$\rightarrow$ so, $I = \dfrac{50/s}{Z}$

$$= \frac{50/s\,(40 + 0.2s)}{400 + 10s}$$

$$\therefore \quad I = \frac{50(40 + 0.2s)}{s(400 + 10s)} = I_1 + I_2.$$

ue. A 100V source passing a continuously. At time $t=0$, switch is closed with 10Ω parallel resister. find the Laplace of resultant current.



Here $I(0^-) = \dfrac{100}{5+10} = \dfrac{100}{15} = 20/3 \text{ A} = I(0^+)$

→ for first loop,

$$100 - 5(i_1 + i_2) - 10 i_1 - L \cdot \frac{di_1}{dt}$$

$$\therefore 100 = 15 i_1 + 5 i_2 + L \frac{di_1}{dt} \quad \text{—(1)}$$

→ for second loop.

$$100 - 5(i_1 + i_2) - 10 i_2 = 0$$

$$\therefore 100 = 5 i_1 + 15 i_2 \quad \text{—(2)}$$

→ taking laplace of eq. (1),

$$L\{100\} = 15\,L\{i_1\} + 5\,L\{i_2\} + 2\,L\left\{\frac{di_1}{dt}\right\}$$

$$\therefore \frac{100}{s} = 15 \cdot I_1(s) + 5 \cdot I_2(s) + 2\left(sI_1(s) - I(0^+)\right)$$

$$\therefore \frac{100}{s} = 15I_1(s) + 5I_2(s) + 2sI_1(s) - 2\left(\frac{20}{3}\right)$$

$$\therefore \frac{100}{s} = (15+2s)I_1(s) + 5I_2(s) - 40/3$$

$$\therefore \frac{100}{s} + \frac{40}{3} = (15+2s)I_1(s) + 5I_2(s) \quad \text{— (3)}$$

→ taking laplace of eq. (2),

$$L\{100\} = 5\,L\{i_1\} + 15\,L\{i_2\}$$

$$\therefore \frac{100}{s} = 5I_1(s) + 15 \cdot I_2(s) \quad \text{— (4)}$$

$$\therefore \cancel{\frac{300}{s} = 15I_1(s) + 75I_2(s)}$$

$$\cancel{\frac{100}{s} + \frac{40}{3} = 15I_1(s) + 2sI_1(s) + 5I_2(s)}$$

$$\cancel{\frac{300}{s} = 15I_1(s) + 75I_2(s)}$$

$$\cancel{- \qquad - \qquad -}$$

$$\cancel{\frac{40}{3} - \frac{200}{s}}$$

→ Multiplying eq.(3) by 3,

$$\therefore \frac{300}{s} + 40 = (75 + 6s)\, I_1(s) + \cancel{15 I_2(s)}$$

$$\frac{100}{s} = 5 I_1(s) + \cancel{15 I_2(s)}$$

$$\frac{200}{s} + 40 = (70 + 6s)\, I_1(s).$$

$$\therefore I_1(s) = \frac{\frac{200}{s} + 40}{70 + 6s}$$

$$\boxed{\therefore I_1(s) = \frac{200 + 40s}{s(70 + 6s)}} \quad \text{— A}$$

and, $$\frac{100}{s} = \left[\frac{200 + 40s}{s(70 + 6s)}\right] 5 + 15 I_2(s)$$

($\because$ from eq.(4))

$$\therefore \frac{20}{s} = \frac{200 + 40s}{s(70 + 6s)} + 3 I_2(s).$$

$$\therefore \frac{20}{s} - \frac{200 + 40s}{s(70 + 6s)} = 3 I_2(s)$$

$$\therefore \frac{1400 + 120s - 200 - 40s}{s(70 + 6s)} = 3 I_2(s)$$

$$\therefore \frac{1200 + 80s}{s(70 + 6s)} = 3 I_2(s).$$

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$$\therefore I_2(s) = \frac{1}{3s}\left[\frac{1200+80s}{70+6s}\right]$$

$$\frac{200+40s}{s(70+6s)} = \frac{A}{s} + \frac{B}{70+6s}$$

$$\therefore 200+40s = A(70+6s) + Bs$$

$$s = 0 \Rightarrow 200 = A(70)$$

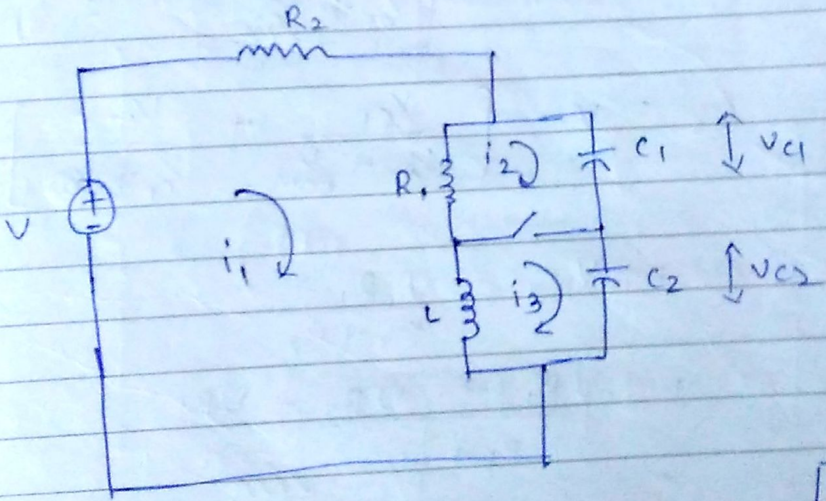
$$\therefore A = 200/70$$

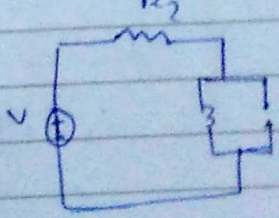
Que.

(i) initial value of current.

(ii) at $t=0^-$ current.

(iii) at $t=0$ what are the various currents.





at $t=0^-$, $i_{R_1}(0^-) = \frac{V}{R_1+R_2}$

$$\therefore i_{R_1}(0^-) = i_L(0^-) = \frac{V}{R_1+R_2}$$

$$q_1 = q_2$$

$$\therefore C_1 V_{c1} = C_2 V_{c2}$$

$$\therefore \frac{V_{c1}}{V_{c2}} = \frac{C_2}{C_1}$$

and $V_{c1} = \dfrac{1/C_1}{\frac{1}{C_1} + \frac{1}{C_2}} \left(\dfrac{V}{R_1 + R_2} \right)(R_1)$

and $V_{c2} = \dfrac{1/C_2}{1/C_1 + 1/C_2} \left(\dfrac{V}{R_1 + R_2} \right)(R_1)$

→ at $t = 0^+$

$$V - i_1 R_2 - V_{c1} - V_{c2} = 0$$

$$\therefore i_1 R_2 = V - V_{c1} - V_{c2}$$

$$\therefore i_1 R_2 = V - \left[\frac{1/C_1}{1/C_1 + 1/C_2} \left(\frac{V \cdot R_1}{R_1 + R_2} \right) \right] - \left[\frac{1/C_2}{1/C_1 + 1/C_2} \left(\frac{VR_1}{R_1 + R_2} \right) \right]$$

$$= V - \left[\frac{1/C_1}{1/C_1 + 1/C_2} + \frac{1/C_2}{1/C_1 + 1/C_2} \right] \frac{VR_1}{R_1 + R_2}$$

$$= V - \frac{VR_1}{R_1 + R_2}$$

$$= \frac{VR_1 + VR_2 - VR_1}{R_1 + R_2}$$

$$\therefore i_1 R_2 = \frac{VR_2}{R_1 + R_2}$$

$$\therefore i_1 = \frac{V}{R_1 + R_2}$$

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$\cancel{V_{C1} + R_1(i_2 - i_1)}$

$i_1(0^+) = i_1(0^+) - i_3(0^+)$

$$\frac{V}{R_1+R_2} = \frac{V}{R_1+R_2} - i_3(0^+)$$

→ at $t = 0^+$, $i_3(0^+) = 0$.

→ $R_1\,[\,i_1(0^+) - i_2(0^+)\,] = V_{C1}$.

$$\therefore R_1\left(\frac{V}{R_1+R_2} - i_2\right) = \frac{1/C_1}{1/C_1 + 1/C_2}\left(\frac{VR_1}{R_1+R_2}\right)$$

$$\therefore \frac{VR_1}{R_1+R_2} - i_2 R_1 = \frac{1/C_1}{1/C_1 + 1/C_2}\left(\frac{VR_1}{R_1+R_2}\right)$$

$$\therefore \left[1 - \frac{1/C_1}{1/C_1 + 1/C_2}\right]\frac{VR_1}{R_1+R_2} = i_2 R_1$$

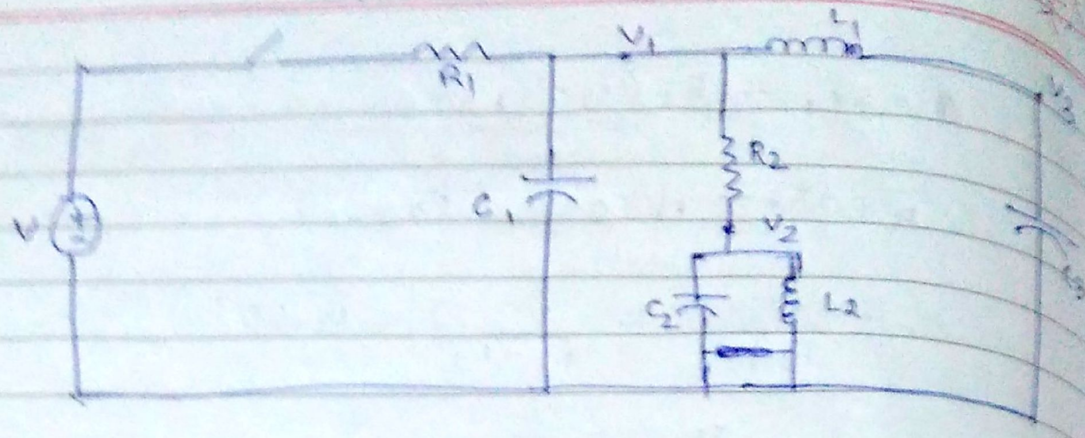
$$\therefore \left[\frac{1/C_1 + 1/C_2 - 1/C_1}{1/C_1 + 1/C_2}\right]\frac{V\cancel{R_1}}{R_1+R_2} = i_2 \cancel{R_1}$$

$$\therefore i_2 = \left(\frac{\frac{1}{C_2}}{1/C_1 + 1/C_2}\right)\frac{V}{R_1+R_2}$$

$v_1 - v_3 = L \cdot \frac{di}{dt}$

$\therefore \frac{1}{L}(v_1 - v_3) = \frac{di}{dt}$

Que.



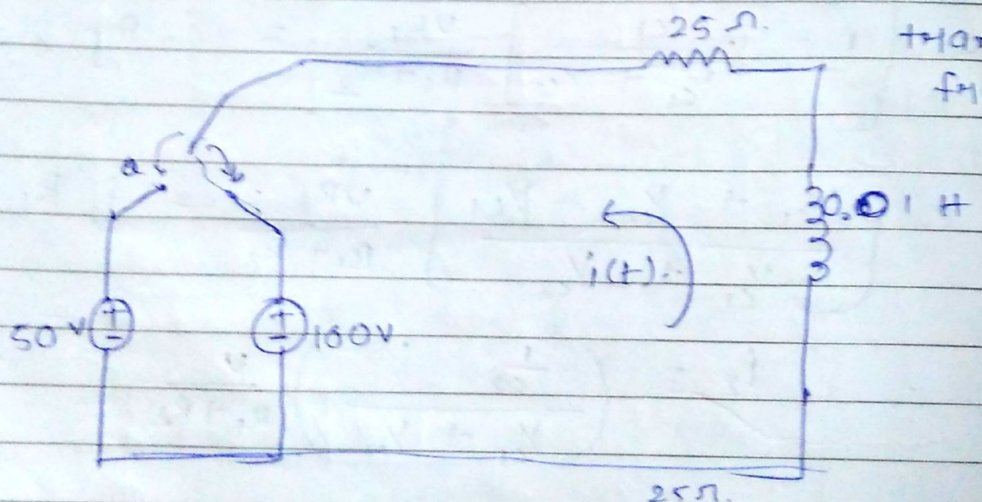
$\rightarrow \frac{v_1 - V}{R_1} + \frac{v_1 - v_2}{R_2} + C_1 \cdot \frac{dv_1}{dt} + \frac{1}{L_1}\int (v_1 - v_3)\, dt = 0$

$\rightarrow \frac{v_2 - v_1}{R_2} + C_2 \cdot \frac{dv_2}{dt} + \frac{1}{L_2}\int v_2 \cdot dt = 0$

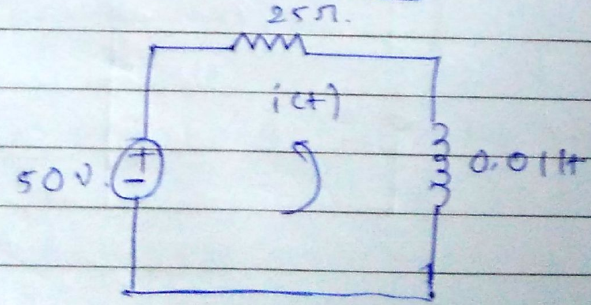
$\rightarrow C_3 \cdot \frac{dv_3}{dt} + \frac{1}{L_1}\int (v_3 - v_1) \cdot dt = 0$

Que.

switch is transferred from a to b



$\rightarrow$ at $t = 0^-$



$I(0^-) = -2\ A$

KVL for loop, for $I(0^+)$.

$$\overset{100}{\overset{\cancel{50}}{-\cancel{100}}} - L\cdot\frac{di}{dt} - 25i = 0.$$

$$\therefore -\overset{50}{\cancel{100}} = 0.01\frac{di}{dt} - 25i$$

$$\therefore L\{-\overset{50}{\cancel{100}}\} = 0.01\, L\left\{\frac{di}{dt}\right\} + 25\cdot L\{i\}$$

$$\therefore -\overset{50}{\cancel{100}}/s = 0.01\left[sI(s) - I(0^-)\right] + 25\,I(s).$$

$$\therefore -\frac{\overset{50}{\cancel{100}}}{s} = 0.01\,s\,I(s) + 25\,I(s) - 0.01\,I(0^-)$$

$$\therefore -\frac{\overset{50}{\cancel{100}}}{s} = [25 + 0.01s]\,I(s) - 0.01\times 2.$$

$(\because I(0^-) = 50/25 = -2A)$ due to opposite

$$\therefore \frac{-50/s + 0.02}{25 + 0.01s} = I(s)$$

$$\therefore \frac{-50\ \cancel{+0.02}}{s(25+0.01s)} + \frac{0.02}{25+0.01s} = I(s)$$

0.02

$$\frac{-50}{s(25+0.01s)} = \frac{A}{s} + \frac{B}{25+0.01s}$$

$$\therefore -50 = A(25+0.01s) + Bs.$$

$\Rightarrow s = 0 \Rightarrow -50 = A(25)$

$\therefore A = -2$

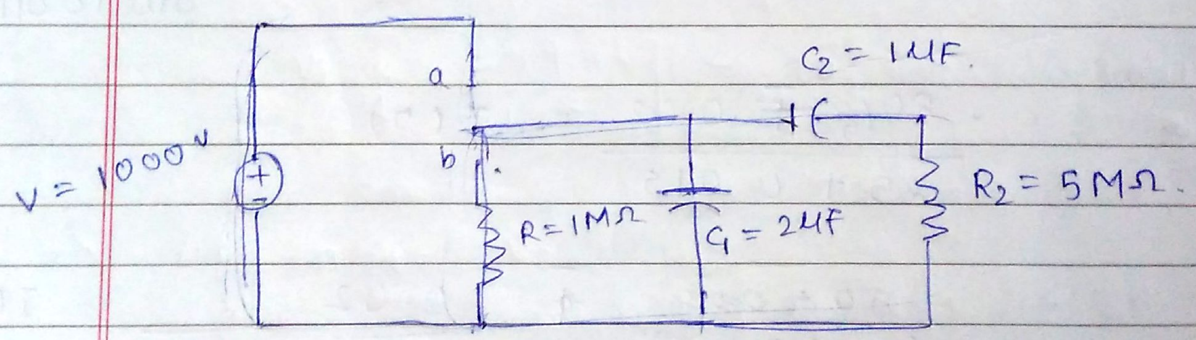
$\Rightarrow s = -2500 \Rightarrow -50 = B(-2500)$

$\therefore B = 0.02.$

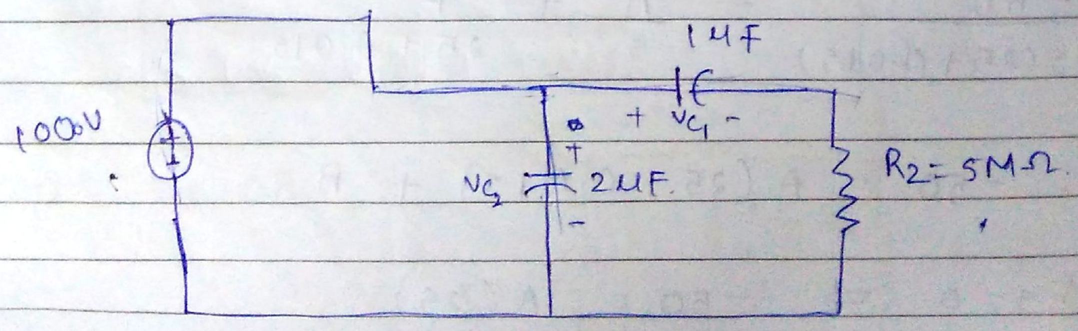
~~and~~ so, $\dfrac{-2}{s} = I(s)$

$\therefore\ i(0^-) = -2\ A.$

Que. steady state reache ~~at~~ when connected to ~~at t=0~~ a at time $t=0$.



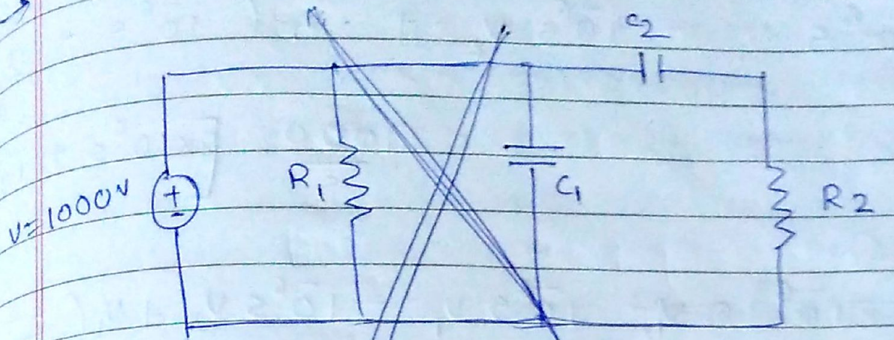
→ at $t = 0^-$



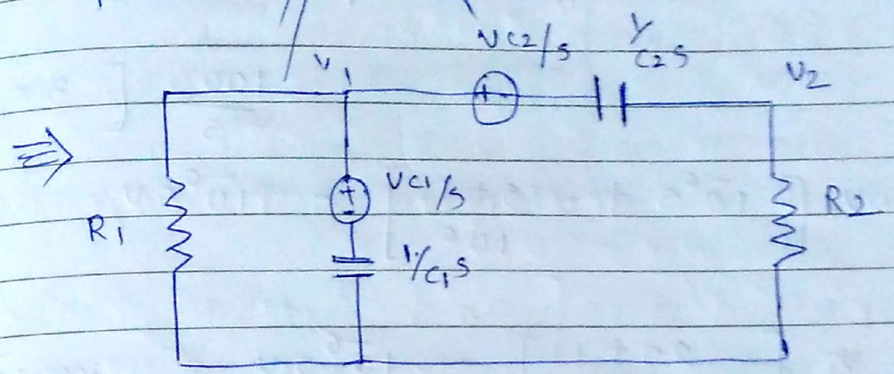
$V_{C1} = V_{C2} = V$ at $t = 0^-$

so, open circuit current $I(0^+) = 0$.

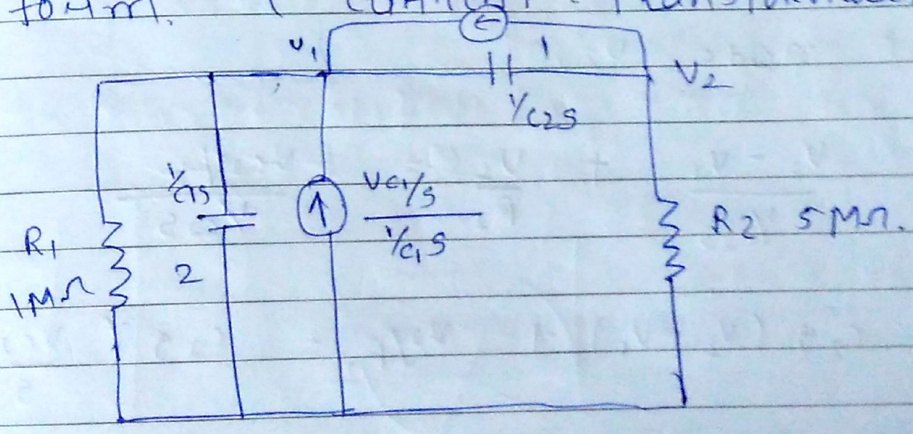
At $t = 0^+$.



$\Rightarrow$



Now, convert voltage source in current form. (current transformation)



Now, for Nodal circuit.

$$\frac{V_1 - V_2}{1/C_2s} + \frac{V_1}{R_1} + \frac{V_1}{1/C_1s} = \frac{V_{C1}/s}{1/C_1s} + \frac{V_{C2}/s}{1/C_2s}$$

$$\therefore \; C_2 s V_1 - C_2 s V_2 + V_1\left(\frac{1}{R_1} + C_1 s\right) = 1000/s \left[C_1 s + C_2 s \right]$$

$$\therefore \; 10^{-6} s \, V_1 - 10^{-6} s \, V_2 + V_1\left(10^{-6} s + \frac{1}{10^6}\right) = \frac{1000}{s}\left[2\times 10^{-6} s + 10^{-6} s\right]$$

$$\therefore \; \cancel{(2\times 10^{-6}) s V_1} \; 10^{-6} s \, V_1 - 10^{-6} s \, V_2 + V_1\left(\frac{s+1}{10^6}\right) = \frac{1000}{s}\left[3\times 10^{-6} s\right].$$

$$\therefore \; V_1\left[10^{-6} s + \frac{s+1}{10^6}\right] - 10^{-6} s \, V_2 = (3\times 10^{-6}) \frac{1000}{s}$$

$$\therefore \; V_1\left[\frac{2s+1}{10^6}\right] - 10^{-6} s \, V_2 = \frac{3000 \, 10^{-6}}{s} \quad \text{—(1)}$$

→ At node V_2,

$$\frac{V_2 - V_1}{1/C_2 s} + \frac{V_2}{R_2} = \frac{V_{C2}/s}{1/C_2 s}$$

$$\therefore \; C_2 s \, (V_2 - V_1) + V_2/R_2 = C_2 s\left(\frac{V_{C2}}{s}\right)$$

$\therefore$

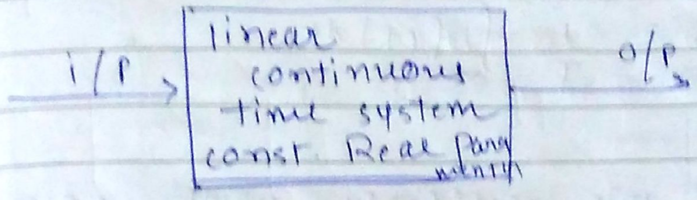
Fourier Series

→ If any signal,

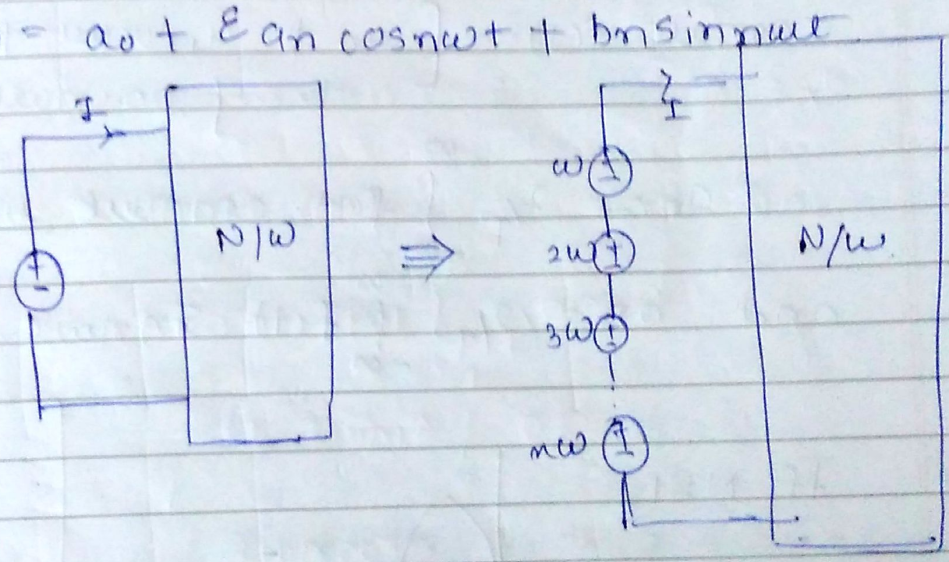
ex. $y(t) = \int e^{s(t-\tau)} d\tau = e^{st} \int_{-\infty}^{\infty} e^{-\tau s} d\tau$

eigen-function   eigen value

→ Signal should not be changed with respect to time.



→ $f(t) = a_0 + \Sigma a_n \cos n\omega t + b_n \sin n\omega t$



→ $f(t) = f(t + T_0)$, T_0 = Periodic time

$f = 1/T$ (frequency)

ω = Angular frequency.

$f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots$
$+ a_1 \sin \omega t + a_2 \sin 2\omega t + \ldots$

$\therefore \int_{-\infty}^{\infty} f(t)\,dt = \int a_0 + \int a_1 \cos \omega t + \int a_2 \cos 2\omega t + \ldots$

but Summation of Sinusoidal Signal will be zero.

$\therefore \int_{-\infty}^{\infty} f(t)\,dt = a_0 T$

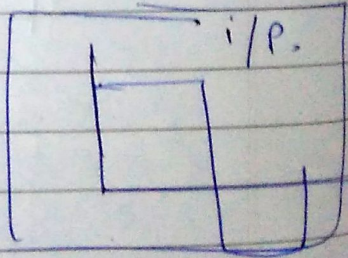
$\therefore a_0 = \frac{1}{T} \int_{-\infty}^{\infty} f(t).dt$

Now, if we multiply with $\cos n\omega t$ then,

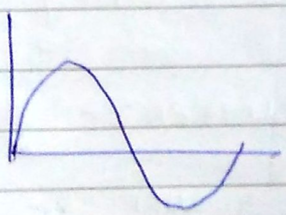
$\cos n\omega t \cdot f(t) = [a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots] \cos n\omega t$

$\therefore a_n = 2/T \int_{-\infty}^{\infty} f(t) \cdot \cos n\omega t.dt$

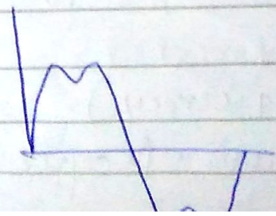
and $b_n = 2/T \int_{-\infty}^{\infty} f(t) \cdot \sin n\omega t.dt$



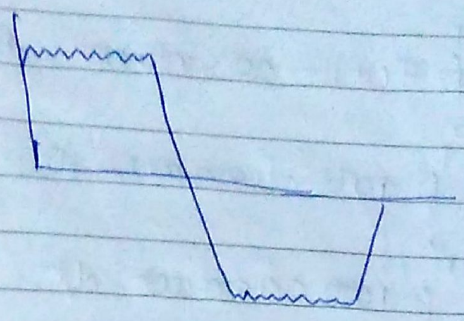
If $N=1$,



$N=2$

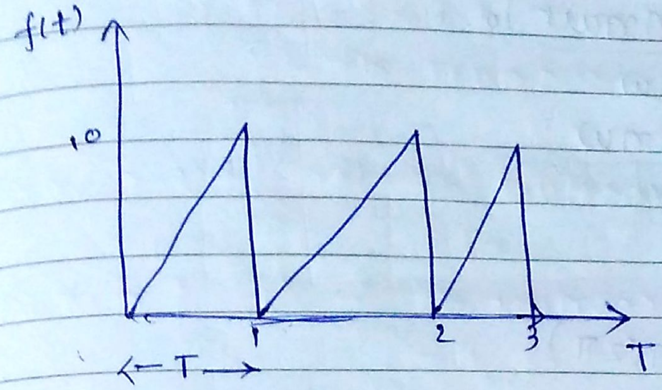


N = 41



so, at the receiving end, we can get almost same signal.

Que.



find a_0, $a_n b_n$ = ?

$f(x) = mx + c$.

here, $m = 10$ ($\because$ slope)

$$\boxed{\therefore f(t) = 10t}$$

$$\therefore f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos n\omega t + b_n \sin n\omega t \right]$$

$\therefore$ ~~$10T = a_0 + T$~~

$$\therefore a_0 = \frac{1}{T} \int_0^T f(t) \cdot dt$$

$$= \frac{1}{T} \int_0^T 10T \cdot dt$$

$\therefore a_0 = 10.$

$$\therefore a_0 = \int_0^1 10t \cdot dt$$

$$= \frac{10}{2} \cdot (t^2)_0^1$$

$$= 5(1-0)$$

$$= 5$$

$$\boxed{\therefore a_0 = 5}$$

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$$a_n = \frac{2}{T}\int_0^T f(t)\cdot \cos n\omega t \cdot dt.$$

$$= 2/T \int_0^T 10t \cos n\omega t \cdot dt.$$

$$= 2/T \int_0^1 10t \cos n\omega t \cdot dt.$$

$$= \frac{20}{T}\int_0^1 t\cos n\omega t \cdot dt$$

$$= 20 \frac{(\sin n\omega t)_0^1}{n\omega}$$

$$= \frac{20}{n\omega} \sin n\omega$$

$(\omega = 2\pi)$.

$$= \frac{20}{n(2\pi)} \sin n(2\pi)$$

$\therefore a_n = 0,$

and $b_n = 2/T \int_0^T f(t) \cdot \sin n\omega t \cdot dt.$

$$= 2/T \int_0^T 10t \sin n\omega t \cdot dt.$$

$$= 20 \int_0^1 \sin n\omega t \cdot dt$$

$$= 20 \frac{(-\cos n\omega t)_0^1}{n\omega}$$

$$= \frac{20}{n\omega}(-\cos n\omega + 1)$$

$$= \frac{20}{n\omega}(-\cos n(2\pi) + 1)$$

$$= \frac{20}{n(2\pi)} ($$

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$$= \frac{20}{T} \int_0^T t \cdot \sin n\omega t \cdot dt.$$

$$= 20 \int_0^1 t \cdot \sin n\omega t \cdot dt$$

$$= 20 \left[\left(t \int \sin n\omega t \cdot dt \right)_0^1 - \int_0^1 \left(\int \sin n\omega t \cdot dt \right) \cdot dt \right]$$

$$= 20 \left[\left(t \left(\frac{-\cos n\omega t}{n\omega} \right) \right)_0^1 - \int_0^1 -\cos n\omega t \cdot dt \right]$$

$$= 20 \left[\frac{-1}{n\omega} + \int_0^1 \cos n\omega t \cdot dt \right]$$

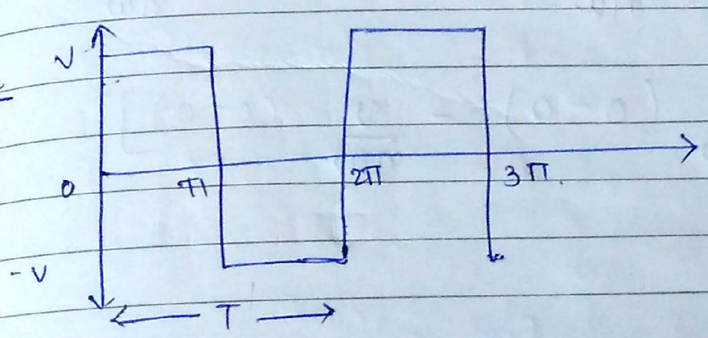
$$= 20 \left[\frac{-1}{n\omega} + \frac{(\sin n\omega t)_0^1}{n\omega} \right]$$

$$= 20 \left[\frac{-1}{n\omega} + 0 \right]$$

$$= \frac{-20}{n(2\pi)}$$

$$= \frac{-10}{n\pi}$$

Que



$a_0 = ?$

$a_n, b_n = ?$

Here, $f(t) = \begin{cases} V, & 0 < \omega t < \pi \\ -V, & \pi < \omega t < 2\pi. \end{cases}$

$$a_0 = \frac{1}{T} \int_0^T f(t) \cdot dt$$

$$= \frac{1}{T} \left[\int_0^{\pi} f(t) \cdot dt + \int_{\pi}^{2\pi} f(t) \cdot dt \right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} v \cdot dt + \int_{\pi}^{2\pi} -v \cdot dt\right]$$

$$= \frac{1}{2\pi}\left[v \cdot (t)_0^{\pi} - v\,(t)_{\pi}^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[v(\pi) - v(\pi)\right]$$

$\therefore a_0 = 0.$

$\rightarrow$ $a_n = 2/T \int_0^T f(t) \cdot \cos n\omega t \cdot dt.$

$$= 2/T\left[\int_0^{\pi} v \cdot \cos n\omega t \cdot dt + \int_{\pi}^{2\pi} -v \cdot \cos n\omega t \cdot dt\right]$$

$$= 2/T\left[v \int_0^{\pi} \cos n\omega t \cdot dt - v \int_{\pi}^{2\pi} \cos n\omega t \cdot dt\right]$$

$$= 2/T\left[v \frac{(\sin n\omega t)_0^{\pi}}{n\omega} - v \frac{(\sin n\omega t)_{\pi}^{2\pi}}{n\omega}\right]$$

$$= 2/T\left[\frac{v}{n\omega}(0-0) - \frac{v}{n\omega}(0-0)\right]$$

$\therefore a_n = 0$

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$$b_n = \frac{2}{T}\int_0^T f(t).\ \sin n\omega t\ dt$$

$$= \frac{2}{T}\left[\int_0^{\pi} v\cdot \sin n\omega t\, dt + \int_{\pi}^{2\pi} (-v)\cdot \sin n\omega t\ dt\right]$$

$$= \frac{2}{T}\left[v\int_0^{\pi} \sin n\omega t\, dt - v\int_{\pi}^{2\pi} \sin n\omega t\ dt\right]$$

$$= \frac{2}{T}\left[-v \left(\frac{\cos n\omega t}{n\omega}\right)_0^{\pi} + v \left(\frac{\cos n\omega t}{n\omega}\right)_{\pi}^{2\pi}\right]$$

$$= \frac{2}{T}\left[-v\left(\frac{\cos n\pi\omega - 1}{n\omega}\right) + \frac{v}{n\omega}\left(1 - \cos n\pi\omega\right)\right]$$

$$= \frac{2v}{n\omega}\left[-\cos n\pi\omega \cancel{-1} + \cancel{1} - \cos n\pi\omega\right]$$

$$= \frac{2v\,(2\pi)}{n(2\pi)}\left[-2\cos n\pi\omega\right].$$

$$= \frac{2v}{n}\left[-2\cos n\pi\omega\right].$$

$$= \frac{2v}{nT}\left[-\left[\frac{\cos nt}{n}\right]_0^{\pi} + \left[\frac{\cos nt}{n}\right]_{\pi}^{2\pi}\right]$$

$$= \frac{2v}{nT}\left[-\left(\cos \pi n - 1\right) + \left(\cos 2n\pi - \cos n\pi\right)\right]$$

$$\left[-\cos n\pi + 1 + 1 - \cos n\pi\right)$$

$$\frac{\cancel{2}v}{\cancel{n}\pi}\left(2 - 2\cos n\pi\right)$$

$$\frac{2v}{n\pi}\left(1 - \cos n\pi\right)$$

$\Rightarrow$ Exponencial Series of,

$$f(t) = a_0 + \Sigma [a_n \cos n\omega t + b_n \sin n\omega t].$$

$$\cancel{= a_0 + \sum_{-\infty}^{\infty}} [e^{jn\omega t}$$

$$= a_0 + a_n \left(\frac{e^{jn\omega t} + e^{-jn\omega t}}{2j} \right)$$

$$+ b_n \left(\frac{e^{jn\omega t} - e^{-jn\omega t}}{2j} \right)$$

$$= a_0 + \sum_{-\infty}^{\infty} \left[e^{jn\omega t} \left(\frac{a_n - jb_n}{2} \right) + e^{-jn\omega t} \left(\frac{a_n + jb_n}{2} \right) \right]$$

$$\therefore f(t) = A_0 + \Sigma \left[A_n e^{jn\omega t} + B_n e^{-jn\omega t} \right].$$

where, $A_0 = a_0$

$A_n = \dfrac{a_n - jb_n}{2}$ $\qquad A_{-n} = \dfrac{a_n + jb_n}{2}$

$B_n = \dfrac{a_n + jb_n}{2}$ $\qquad B_{-n} = \dfrac{a_n - jb_n}{2}$

$$A_n = \frac{1}{T} \int_{-\infty}^{\infty} f(t)\, e^{-jn\omega t}.$$

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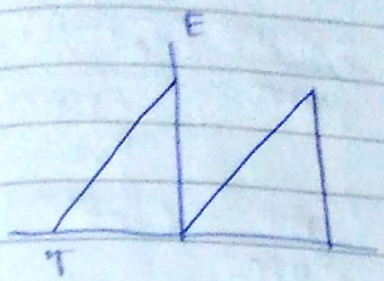

$$f(t) = \frac{E}{T} t \quad (mx)$$

$$A_0 = a_0 = \frac{1}{T}\int_0^T f(t)\cdot dt$$

$$= \frac{1}{T}\int_0^T \frac{E}{T}\cdot t \cdot dt$$

$$= \frac{E}{T^2}\int_0^T t\cdot dt$$

$$= E/T^2 \left(t^2/2\right)_0^T$$

$$= E/T^2 \cdot \frac{T^2}{2}$$

$$\therefore A_0 = E/2$$

$$A_n = \frac{a_n - jb_n}{2} = \frac{1}{T}\int_0^T f(t)\cdot e^{-jn\omega t}\cdot dt$$

$$= \frac{1}{T}\int_0^T E/T \cdot t \cdot e^{-jn\omega t}\cdot dt$$

$$= \frac{E}{T^2}\int_0^T t\cdot e^{-jn\omega t}\cdot dt$$

$$\cancel{= E/T^2 \left[\left(e^{-jn\omega t}\int t\cdot dt\right)_0^T - \int_0^T \frac{e^{-jn\omega t}}{-jn\omega}\right.}$$

$$= E/T^2 \left[\left(t\,\frac{e^{-jn\omega t}}{-jn\omega}\right)_0^T - \int_0^T \frac{e^{-jn\omega t}}{-jn\omega}\cdot dt\right]$$

$$= E/T^2 \left[\frac{e^{-jn\omega T}\cdot T}{-jn\omega} + \frac{1}{jn\omega}\cdot\left(\frac{e^{-jn\omega t}}{-jn\omega}\right)_0^T\right]$$

$$= E/T^2 \left[\frac{e^{-j\omega nT} \cdot T}{-j\omega n} + \frac{1}{j\omega n} \left(\frac{e^{-j\omega nT} - 1}{-j\omega n} \right) \right]$$

$$= E/T^2 \left[\frac{e^{-j\omega nT} \cdot T}{-j\omega n} - \frac{1}{j^2 \omega^2 n^2} \left(e^{-j\omega nT} - 1 \right) \right]$$

$$\therefore A_n = E/T^2 \left[\frac{e^{-j\omega nT} \cdot T}{-j\omega n} + \frac{1}{\omega^2 n^2} \left(e^{-j\omega nT} - 1 \right) \right].$$

$$a_n = A_n + A_{-n} \qquad \left(\because \; A_n = \frac{a_n - jb_n}{2} \right.$$

$$\text{and } b_n = j(A_n - A_{-n}). \qquad \left. A_{-n} = \frac{a_n + jb_n}{2} \right)$$

$$A_n = \frac{E}{\left(\frac{2\pi}{\omega}\right)^2} \left[\frac{e^{-\left(\frac{2\pi}{\omega}\right)\omega n \left(\frac{2\pi}{\omega}\right)} \cdot 2\pi/\omega}{-j\omega n} + \frac{1}{\omega^2 n^2} \left(e^{-j\omega n \left(\frac{2\pi}{\omega}\right)} - 1 \right) \right]$$

$$= \frac{E\omega^2}{4\pi^2} \left[\frac{e^{-\frac{4\pi^2 n}{\omega}} \cdot 2\pi}{-j\omega^2 n} + \frac{1}{\omega^2 n^2} \left(e^{-jn(2\pi)} - 1 \right) \right]$$

$$A_n = \frac{jE}{2\pi n}$$

$$a_n = A_n + A_{-n} = \frac{jE}{2\pi n} + \frac{jE}{2\pi(-n)} = 0$$

$$b_n = j(A_n - A_{-n}) = j\left(\frac{jE}{2\pi n} - \frac{jE}{2\pi(-n)} \right)$$

$$\therefore f(n) = a_0 + a_n \left(\frac{e^{jn\omega t} + e^{-jn\omega t}}{2j} \right) + b_n \left(\frac{e^{jn\omega t} - e^{-jn\omega t}}{2j} \right)$$

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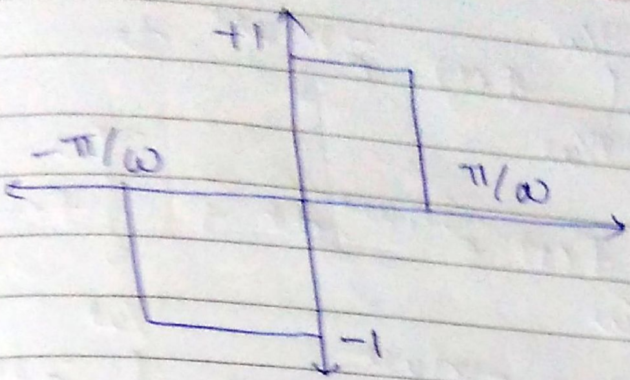
Ques



$$f(t) = \begin{cases} +1, & 0 < t < \pi/\omega \\ -1, & -\frac{\pi}{\omega} < t < 0 \end{cases}$$

$$A_0 = a_0 = \frac{1}{T}\int_0^T f(t)\cdot dt$$

$$= \frac{1}{T}\left[\int_{-\pi/\omega}^{0} f(t)\cdot dt + \int_0^{\pi/\omega} f(t)\cdot dt\right]$$

$$= 1/T\left[\int_{-\pi/\omega}^{0} -1\cdot dt + \int_0^{\pi/\omega} 1\cdot dt\right]$$

$$= 1/T\left[-(t)_{-\pi/\omega}^{0} + (t)_0^{\pi/\omega}\right]$$

$$= 1/T\left[-(0 + \pi/\omega) + \pi/\omega\right]$$

$$= \frac{1}{T}\left[0\right]$$

$$= \cancel{\frac{2\pi}{\omega}\cdot\frac{\omega}{2\pi}}$$

$\therefore a_0 = 0$. ($\because$ Area Under the curve $= 0$).

and $A_n = \frac{1}{T}\int_{-\pi/\omega}^{\pi/\omega} f(t)\cdot e^{-jn\omega t}\cdot dt$

$= \frac{1}{T}\left[\int_{-\pi/\omega}^{0} f(t)\cdot e^{-jn\omega t}\cdot dt + \int_{0}^{\pi/\omega} f(t)\cdot e^{-jn\omega t}\cdot dt\right]$

$= \frac{1}{T}\left[\int_{-\pi/\omega}^{0} -e^{-jn\omega t}\cdot dt + \int_{0}^{\pi/\omega} e^{-jn\omega t}\cdot dt\right]$

$= \frac{1}{T}\left[-\frac{\left(e^{-jn\omega t}\right)_{-\pi/\omega}^{0}}{-jn\omega} + \frac{\left(e^{-jn\omega t}\right)_{0}^{\pi/\omega}}{-j\omega n}\right]$

$= \frac{1}{T}\left[-\frac{\left(1-e^{-jn\omega\left(-\frac{\pi}{\omega}\right)}\right)}{-j\omega n} + \frac{e^{-jn\omega\left(\frac{\pi}{\omega}\right)}-1}{-j\omega n}\right]$

$= \frac{1}{T}\left[\frac{1-e^{jn\pi}}{j\omega n} + \frac{e^{-jn\pi}-1}{j\omega n}\right]$

$= \frac{1}{T(j\omega n)}\left[\cancel{1} - e^{jn\pi} + e^{-jn\pi} - \cancel{1}\right]$

$= \frac{-2}{T(j\omega n)}\left[\frac{e^{jn\pi} - e^{-jn\pi}}{2}\right]$

$= \frac{-2}{T\omega n}\left(\sin n\pi\right)$

$= \frac{-2}{\frac{2\pi}{\omega}\cdot\omega n}\left(\sin n\pi\right)$

$= -\frac{1}{\pi n}\left(\sin n\pi\right)$.

$a_n = A_n + A_n$

$= -\frac{1}{\pi n}(\sin n\pi) + \frac{-1}{\pi(-n)}(-\sin n\pi)$

$= \frac{-1}{\pi n}(\sin n\pi) - \frac{1}{\pi n}(\sin n\pi)$