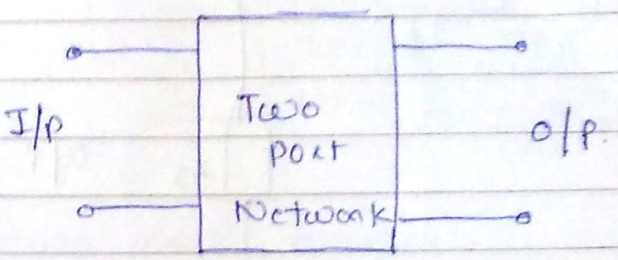
⇒ Two port Network —

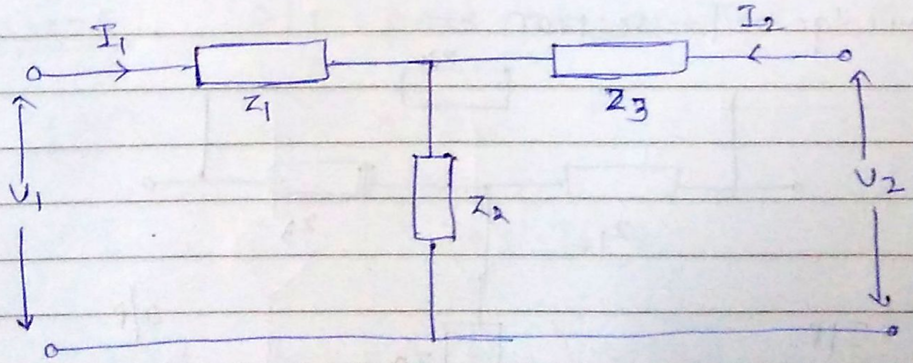


→ To analyse any network four parameters are required.

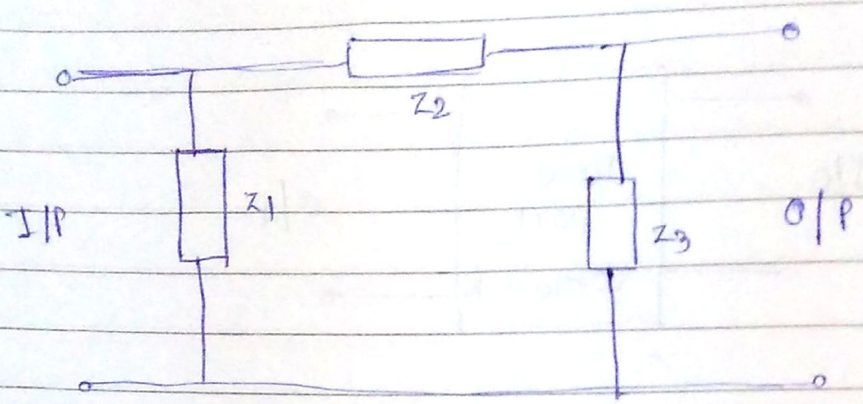
(1) Z - parameters (Impedence para. - o.c.)
(2) Y - parameters (Admittance para. - s.c.)
(3) h - parameters (hybrid parameters)
(4) ABCD parameters (Transmission)

⇒ Configurations :-

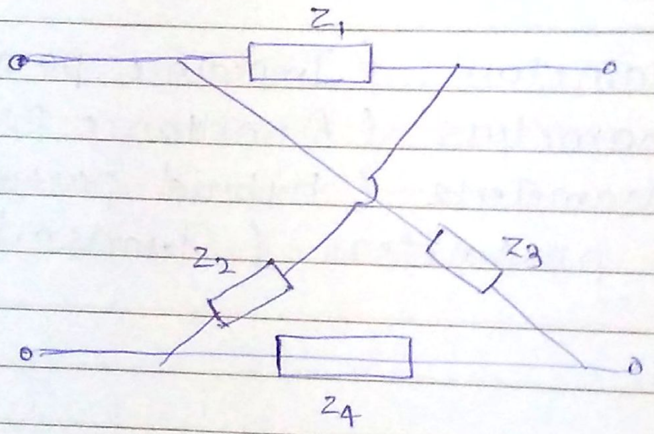
(1) T section -



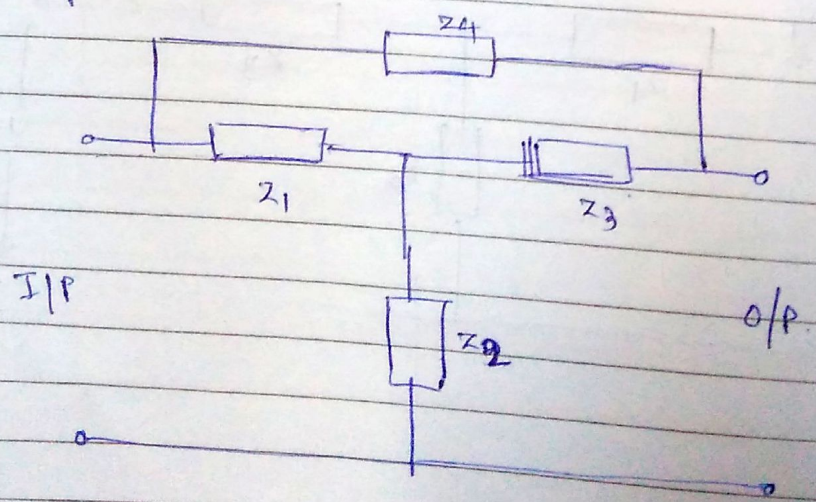
(2) π - Section -



(3) Lattice Section -



(4) Bridge T- section -



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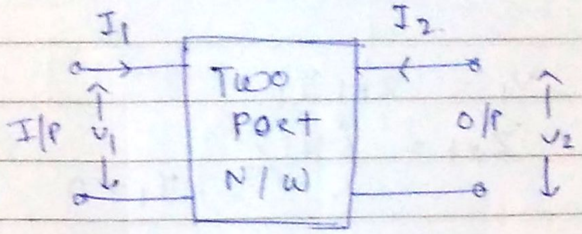
(1) Z-parameters –



→ For a two port network i/p and o/p voltages can be expressed in terms of i/p and o/p currents respectively.

$$[V] = [Z][I]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\therefore V_1 = I_1 Z_{11} + Z_{12} I_2 \quad \text{—(1)}$$

$$\therefore V_2 = I_1 Z_{21} + I_2 Z_{22} \quad \text{—(2)}$$

<u>Case-1:</u> O/P port is opened, $I_2 = 0$.

$$\therefore V_1 = I_1 Z_{11}$$

$$\therefore Z_{11} = \left(\frac{V_1}{I_1}\right)_{I_2=0}$$

and $Z_{21} = \left(V_2/I_1\right)_{I_2=0}$

Case-2: I/P port is opened, $I_1 = 0$

$\therefore V_1 = Z_{21} I_2$

$\therefore Z_{21} = \left(V_1/I_2\right)_{I_1=0}$

and $V_2 = Z_{22} I_2$

$\therefore Z_{22} = \left(V_2/I_2\right)_{I_1=0}$

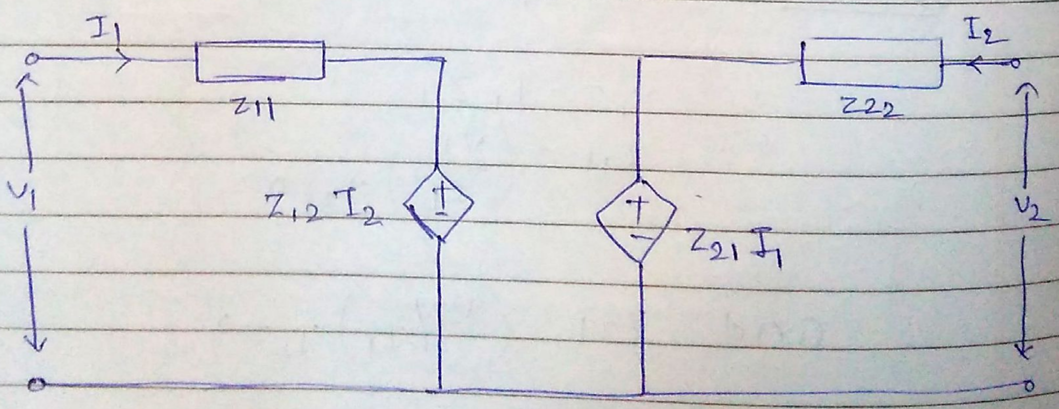
→ Z_{11} = Input driving point impedence

→ Z_{22} = Output driving point impedence

→ Z_{21} = Forward transfer impedence

→ Z_{12} = Reverse transfer impedence

⇒ Equivalent circuit of 2-port network interms of Z parameters —



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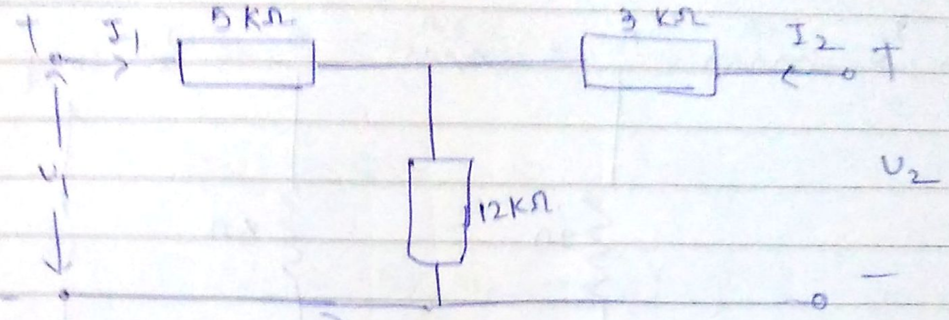


~~Here, $Z_{11} = 5\,k\Omega$~~
~~$Z_{22} = 3\,k\Omega$~~

and $v_1 = I_1 Z_{11} + Z_{12} I_2$.

$v_1 - 5 I_1 - 12(I_2 + I_1) = 0$

$\therefore v_1 = 5 I_1 + 12 I_1 + 12 I_2$

$\therefore v_1 = 17 I_1 + 12 I_2 \longrightarrow (1)$

and $v_2 - 3 I_2 - 12(I_1 + I_2) = 0$

$\therefore v_2 = 3 I_2 + 12 I_1 + 12 I_2$

$\therefore v_2 = 12 I_1 + 15 I_2 \longrightarrow (2)$

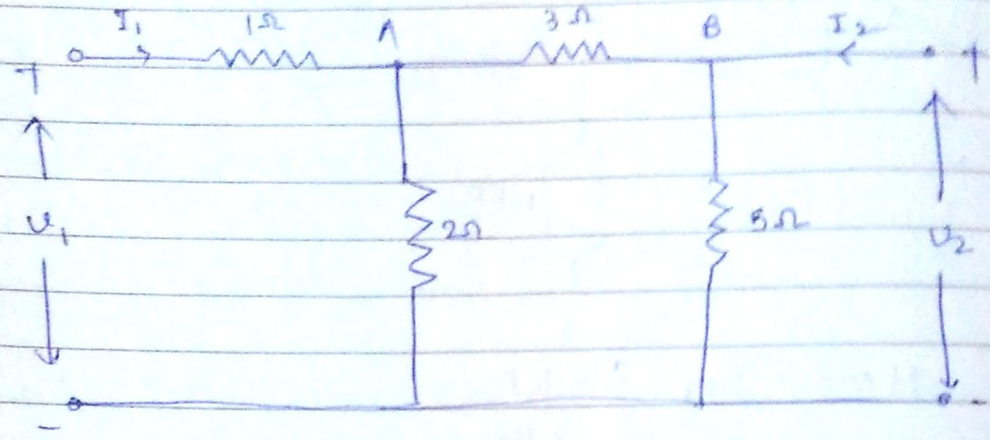
but, $v_1 = I_1 Z_{11} + I_2 Z_{12}$.

so, $\underline{Z_{11} = 17\,k\Omega}$ and $\underline{Z_{12} = 12\,k\Omega}$

Similarly, $v_2 = I_1 Z_{21} + I_2 Z_{22}$

$\therefore \underline{Z_{21} = 12\,k\Omega}$ and $\underline{Z_{22} = 15\,k\Omega}$

Que.



Here, $R_{eq} = 3 \parallel 5 \parallel 2$

$= \frac{15}{8} \parallel 2$

$\therefore R_{eq} = \dfrac{15/4}{15/8 + 2} = \dfrac{15/4}{31/8} = 30/31\ \Omega$

~~and~~ Using Nodal Analysis,

→ at Node A,

$$\frac{V_A}{2} + \frac{V_A - V_2}{3} + \frac{V_A - V_1}{1} = 0$$

$$\therefore \frac{3V_A + 2V_A - 2V_2 + 6V_A - 6V_1}{6} = 0$$

$$\therefore 11V_A - 6V_1 - 2V_2 = 0 \quad \text{—— (1)}$$

→ at Node B,

$$\frac{V_B}{5} + \frac{V_B - V_A}{3} = I_2$$

$$\therefore \frac{V_2}{5} + \frac{V_2 - V_A}{3} = I_2$$

$\therefore \quad 3V_2 + 5V_2 - 5V_A = 0$

$\therefore \quad 8V_2 - 5V_A = 0.$

$\therefore \quad V_2 = 5/8\, V_A \quad \text{—— (2)}$

so, $\quad 11V_A - 6V_1 - 2\left(\frac{5}{8 4} V_A\right) = 0$

$\therefore \quad 11V_A - 6V_1 - \frac{5}{4} V_A = 0.$

$\therefore \quad V_A\left(11 - \frac{5}{4}\right) - 6V_1 = 0$

$\therefore \quad V_A\left(\frac{39}{4}\right) = 6V_1$

$\therefore \quad V_A = \frac{24}{39} V_1.$

but $\quad \frac{V_A - V_1}{1} = I_1.$

$\therefore \quad V_A = V_1 + I_1$

$\rightarrow$

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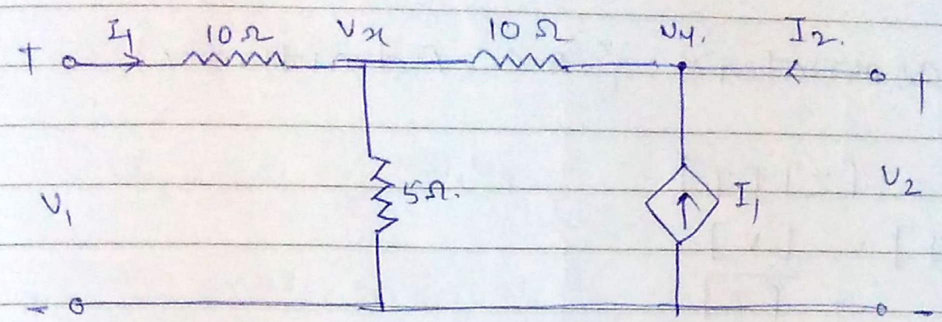

→ for Node V_x,

$$\frac{V_x}{5} + \frac{V_x - V_2}{10} = I_1$$

$$\therefore \frac{2V_x + V_x - V_2}{10} = I_1$$

$$\therefore 3V_x - V_2 = 10I_1 \quad \text{— (1)} \Rightarrow V_x = \frac{10I_1 + V_2}{3}$$

$3V_x - V_2 = 10i_1$ ~~(1)~~

but $V_x = V_1 - 10i_1$

$\therefore 3V_1 - V_2 = 40i_1$ — (1)

and $V_2 - V_1 = 10i_2$ — (2)

$\therefore 3V_1 - V_2 = 40i_1$

$V_2 - V_1 = 10i_2$

$2V_1 = 40i_1 + 10i_2$

$\therefore V_1 = 20i_1 + 5i_2$

→ for Node V_y,

$$\therefore \frac{V_2 - V_x}{10} = I_1 + I_2 \quad \text{— (2)}$$

$$\therefore \frac{V_2 - \left(\frac{10I_1 + V_2}{3}\right)}{10} = I_1 + I_2$$

$$\therefore \frac{3V_2 - 10I_1 - V_2}{30} = I_1 + I_2$$

$$\therefore V_2 - 10I_1 = 30I_1 + 30I_2$$

$$\therefore V_2 = 40I_1 + 30I_2$$

$\therefore Z_{21} = 40\ \Omega$

$\therefore Z_{22} = 30\ \Omega$

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⇒ Y - Parameters (S.C Admittance).

$$[V] = [Z][I]$$

$$\therefore [I] = \frac{[V]}{[Z]}$$

$$\therefore [I] = [V][Y]$$

$$\therefore \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}.$$

Case-1 If Output port is short-circuited.

$V_2 = 0.$

$\therefore I_2 = Y_{21} V_1 + Y_{22} V_2.$ and $I_1 = Y_{11} V_1 + Y_{12} V_2$

$\therefore I_2 = Y_{21} V_1$

$\therefore V_1 = \frac{I_2}{Y_{21}} \Big|_{V_2 = 0.}$

$\therefore Y_{21} = \frac{I_2}{V_1} \Big|_{V_2 = 0}$

↑

Forward transfer admittance.

$\therefore I_1 = Y_{11} V_1$

$\therefore Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$

↑

I/P driving point

case-2 $V_1 = 0$

$\therefore I_1 = V_1 Y_{11} + V_2 Y_{12}$

$\therefore I_1 = V_2 Y_{12}$

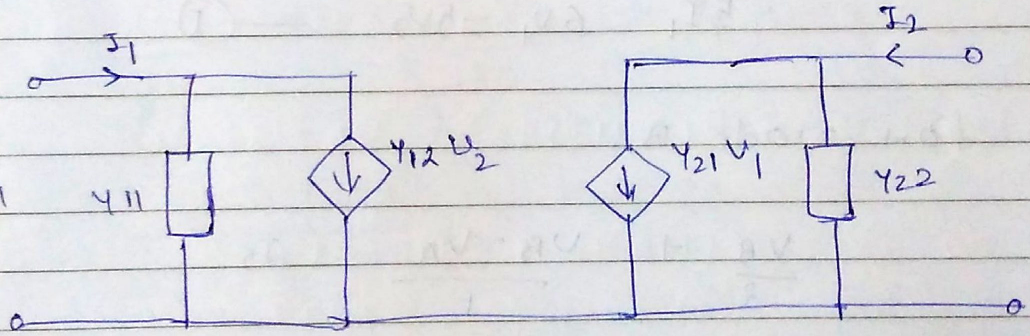
$\therefore Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$ (Reverse transfer admittance)

and $I_2 = Y_{21} V_1 + Y_{22} V_2$

$\therefore I_2 = Y_{22} V_2$

$\therefore Y_{22} = \frac{I_2}{V_2}$ (o/p driving point admittance)

⇒ Eq. circuit –



admittance.

→ Use Nodal Analysis for y-parameter.
Use Mess Analysis for z-parameter.

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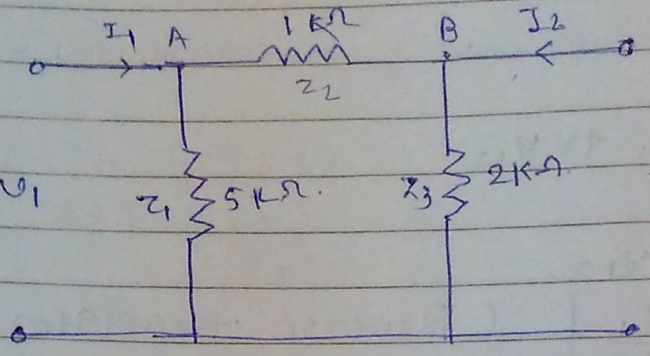

→ for node A,

$$\frac{V_A}{5k\Omega} + \frac{V_A - V_B}{1k\Omega} = I_1$$

$$\therefore V_A + 5V_A - 5V_B = 5I_1$$

$$\therefore 5I_1 = 6V_A - 5V_B.$$

$$\therefore 5I_1 = 6V_1 - 5V_2. \quad \text{—— (1)}$$

→ for node B,

$$\frac{V_B}{2} + \frac{V_B - V_A}{1} = I_2.$$

$$\therefore V_B + 2V_B - 2V_A = 2I_2$$

$$\therefore 3V_B - 2V_A = 2I_2$$

$$\therefore 2I_2 = -2V_A + 3V_B$$

$$\therefore 2I_2 = -2V_1 + 3V_2.$$

so, $y_{11} = 6/5k\ ℧$, $y_{12} = -1/k\ ℧$

$y_{21} = -1/k\ ℧$, $y_{22} = 3/2k\ ℧$

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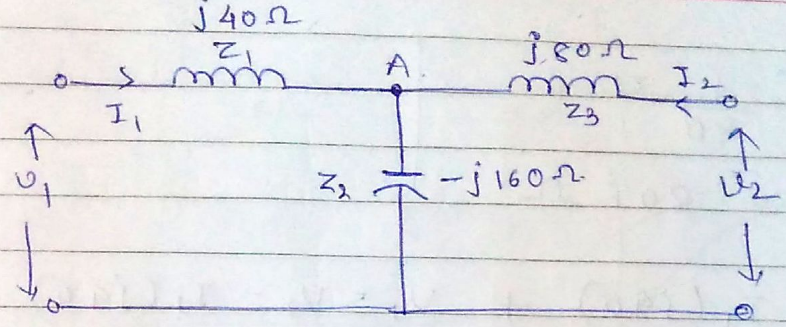

$$\frac{v_A - v_1}{j40} + \frac{v_A}{-j160} + \frac{v_A - v_2}{}$$

$$\therefore \frac{v_A}{-j160} = I_1 + I_2$$

and $I_2 = \dfrac{v_A - v_2}{j80}$

$$\therefore \frac{v_A}{-j160} = I_1 - \left(\frac{v_A - v_2}{j80}\right)$$

$$\therefore \frac{v_A}{-j160} + \frac{v_A - v_2}{j80} = I_1$$

$$\therefore \frac{v_1 - I_1(j40)}{-j60} + \frac{v_A - v_2}{j80} = I_1$$

$$\therefore \frac{v_A}{-j60} + \frac{4}{6}I_1 + \frac{v_A - v_2}{j80} = I_1$$

$$\therefore \frac{v_1}{-j60} + \frac{v_1 - v_2}{j80} = \left(1 - \frac{4}{6}\right) I_1$$

$$\therefore \frac{v_1}{-j60} + \frac{v_1}{j80} - \frac{v_2}{j80} = \frac{1}{3} I_1$$

$$\therefore v_1\left(\frac{-j}{240}\right) - \frac{v_2}{j80} = \frac{1}{3} I_1$$

$$\therefore v_1\left(-\frac{j}{80}\right) - v_2\left(\frac{3}{j80}\right) = I_1$$

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$$y_{11} = \frac{-j}{80} \;℧$$

$$\therefore Z_{11} = 80i\ \Omega$$

$$\therefore \frac{V_1 - I_1(j40)}{-j60} + \frac{V_1 - V_2 - I_1(j40)}{-j80} = I_1$$

$$\therefore \frac{V_1}{-j60} + \frac{4}{6}I_1 + \frac{V_1}{-j80} + \frac{V_2}{j80} + I_1\left(\frac{1}{2}\right) = I_1$$

$$\therefore \frac{V_1}{-j60} + \frac{V_1}{-j80} + \frac{V_2}{j80} = \frac{I_1}{2} - \frac{4}{6}I_1$$

$$\frac{v_A}{-j160} = i_1 + i_2$$

$$v_A = (-j160)i_1 + (-j160)i_2$$

but $v_A = v_1 - i_1(j40) = v_2 - (j80)i_2$

so, $v_1 = (-j120)i_1 + (-j160)i_2$

$v_2 = (-j160)i_1 + (-j80)i_2$

$$[Z] = \begin{bmatrix} -j120 & -j160 \\ -j160 & -j80 \end{bmatrix}$$

$$\therefore [Y] = [Z]^{-1} = \frac{\mathrm{Adj}[Z]}{|Z|} = \frac{\begin{bmatrix} -j80 & j160 \\ j160 & -j120 \end{bmatrix}}{\begin{vmatrix} -j120 & -j160 \\ -j160 & -j80 \end{vmatrix}}$$

$$\therefore [Y] = \begin{bmatrix} -j/200 & j/100 \\ j/100 & -j/133.33 \end{bmatrix}$$

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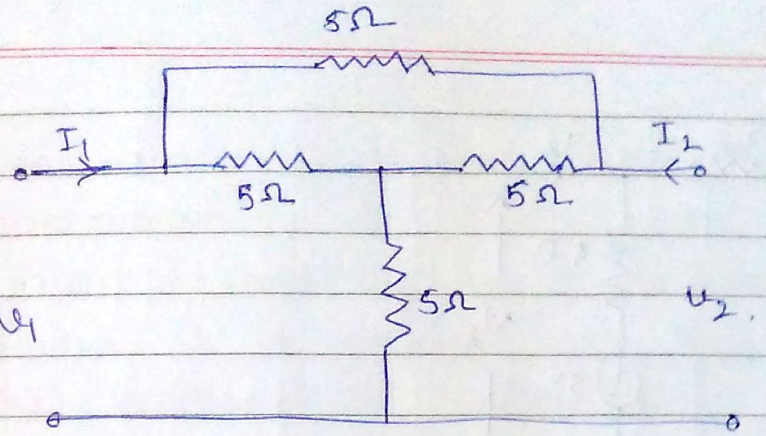
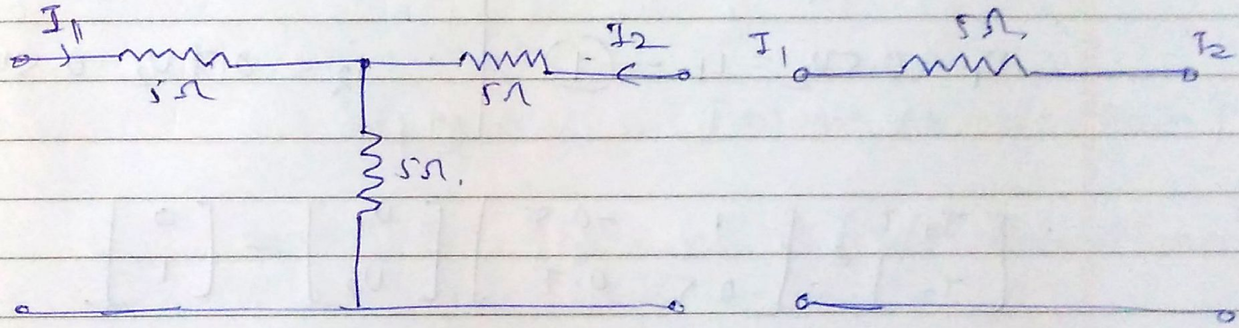


Find y parameters.

→ There are two seperate circuits.



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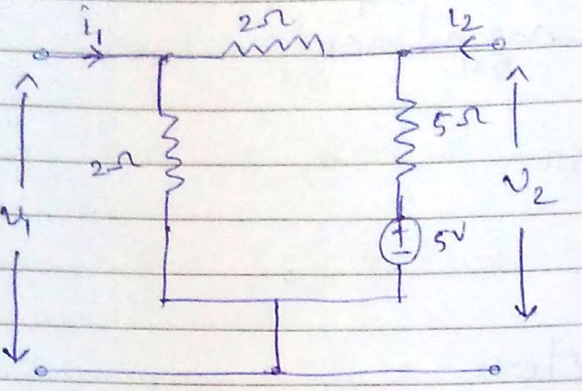


$$\frac{v_1}{2} + \frac{v_1 - v_2}{2} = i_1$$

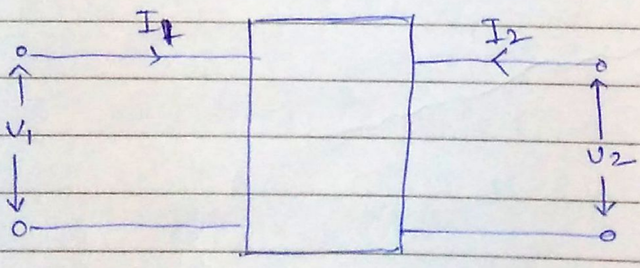
$$\therefore v_1 - 0.5 v_2 = i_1 \quad \text{--- (1)}$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - 5}{5} = i_2$$

$$\therefore i_2 = 0.7 v_2 - 0.5 v_1 - 1.$$

$$\therefore \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 0.7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$\Rightarrow$ H-parameters — (Hybrid-parameters)



$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

→ It is widely used for modelling of electronic components. As both open circuit and short-circuit conditions are utilized Hence it is known as <u>hybrid parameters</u>.
The voltage of input port and current of Output port are expressed in terms of input current and output voltage.

(i) I/P Open, $I_1 = 0$.

$\therefore V_1 = h_{12} V_2$
$\therefore h_{12} = V_1/V_2$ (Reverse Voltage gain)

and $I_2 = h_{22} V_2$ ~~R~~ (O/P Admittance).
$\therefore h_{22} = I_2/V_2$.

(ii) O/P S.C., $V_2 = 0$.

$h_{11} = V_1/I_1$. (Input Impedence)

$h_{21} = I_2/I_1$ (Current gain)

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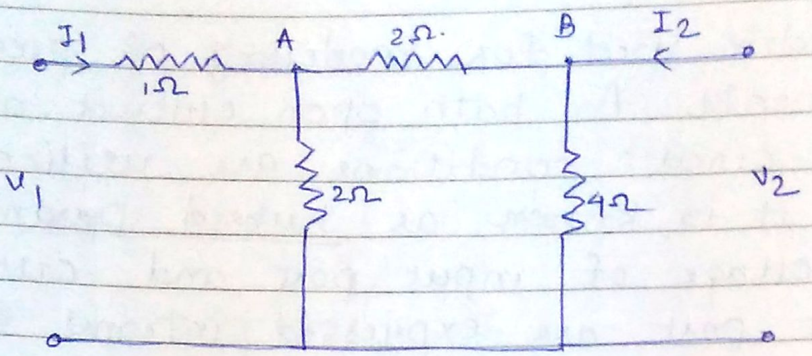

→ for Node A,

$$\frac{V_A}{2} + \frac{V_A - V_2}{2} = I_1$$

$$\therefore 2V_A + 2V_A - 2V_2 = 4I_1$$

$$\therefore 4V_A - 2V_2 = 4I_1$$

but $V_1 - I_1 = V_A$.

$$\therefore 4(V_1 - I_1) - 2V_2 = 4I_1$$

$$\therefore 4V_1 - 4I_1 - 2V_2 = 4I_1$$

$$\therefore 4V_1 - 2V_2 = 8I_1$$

$$\therefore 4V_1 = 8I_1 + 2V_2$$

$$\therefore V_1 = \frac{8}{4} I_1 + \frac{1}{2} V_2$$

$h_{11} = \cancel{4}$, $h_{12} = 1/2$

$= 2\,\Omega$

→ for Node B.

$$\frac{V_2}{4} + \frac{V_2 - V_A}{2} = I_2.$$

$\therefore\ 2V_2 + 4V_2 - 4V_A = 8I_2$

$\therefore\ 6V_2 - 4V_A = 8I_2$

$\therefore\ 8I_2 = 6V_2 - 4(V_1 - I_1)$

$\therefore\ 8I_2 = 6V_2 - 4V_1 + 4I_1$

but $V_1 = 2I_1 + \frac{1}{2}V_2.$

$\therefore\ 8I_2 = 6V_2 - 4\left(2I_1 + \frac{1}{2}V_2\right) + 4I_1$

$\therefore\ 8I_2 = 6V_2 - 8I_1 - 2V_2 + 4I_1$

$\therefore\ 8I_2 = 4V_2 - 4I_1$

$\therefore\ \cancel{I_2 = -\frac{1}{4}I_1 + \frac{1}{2}V_2}$

$\therefore\ I_2 = -\frac{1}{2}I_1 + \frac{4V_2}{8}$

$\therefore\ h_{21} = -\frac{1}{2},\quad h_{22} = \frac{1}{2}\ \mho.$

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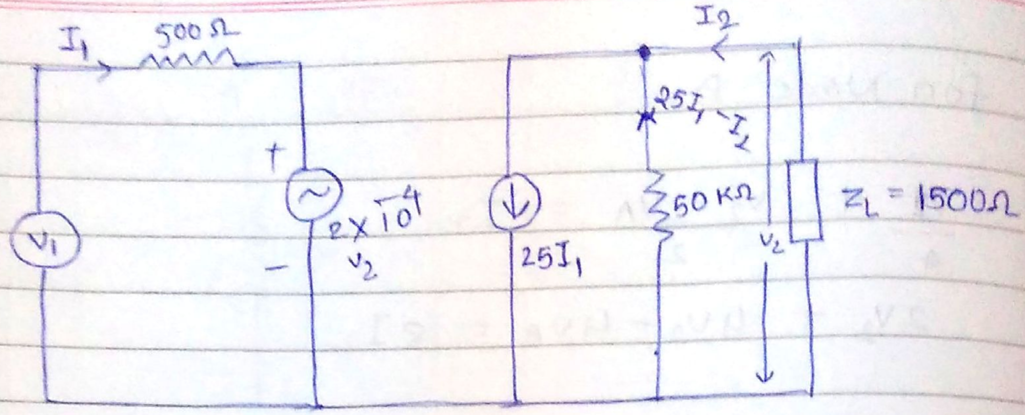

find current gain and voltage gain

$$v_1 - 500 I_1 - (2 \times 10^{-4} v_2) = 0$$

$$\therefore 500 I_1 = v_1 - (2 \times 10^{-4} v_2)$$

$$\therefore I_1 = \frac{v_1 - (2 \times 10^{-4} v_2)}{500} \quad \text{—— (1)}$$

KCL for 2nd loop,

$$I_2 = \frac{v_2}{50} + 25 I_1 \quad \text{—— (2)}$$

Using current division Rule,

$$I_2 = \frac{25 I_1 \times 50}{50000 + 1500}$$

$$\therefore I_2 = \frac{25 I_1 \times 50}{51500}$$

$$\therefore \frac{I_2}{I_1} = \frac{25 \times 50 \times 10^3}{51500} = 0.0242 \times 10^3$$

$$= 24.2$$

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but $\frac{V_2}{1500} + I_2 = 0.$

$\therefore I_2 = \frac{-V_2}{1500}.$

putting value of I_2 in eq. (2).

so. $\frac{-V_2}{1500} = \frac{V_2}{50000} + 25 I_1$

$\therefore \frac{-V_2}{1500} = \frac{V_2}{50000} + 25\left(\frac{V_1 - (2\times10^{-4} V_2)}{500}\right)$

$\therefore V_2\left(\frac{1}{50000} - \frac{1}{1500}\right) = \frac{V_1 - 2\times10^{-4} V_2}{20}$

~~$\therefore V_2\left(\frac{1450}{75000}\right) = \frac{V_1 - 2\times10^{-4} V_2}{20}$~~

~~$\therefore V_2(0.3866) = V_1 - 2\times10^{-4} V_2$~~

~~$\therefore V_1 = V_2(0.3868)$~~

~~$\therefore \frac{V_2}{V_1} =$~~

$\therefore V_2\left(\frac{-48500}{75000000}\right) = \frac{V_1 - 2\times10^{-4} V_2}{20}$

$\therefore V_2(-0.0129) = V_1 - 2\times10^{-4} V_2$

$\therefore \frac{V_2}{V_1} = -78.53.$

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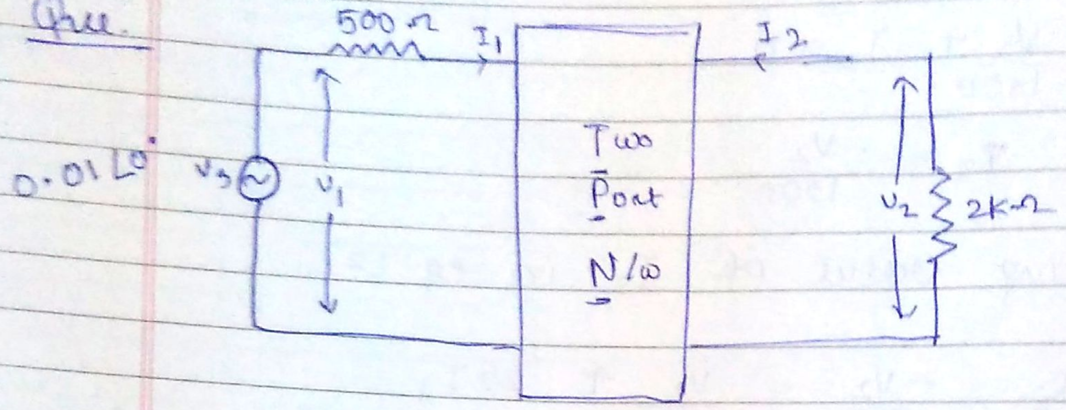


$h_{11} = 1\,k\Omega$

$h_{12} = 0.003$

$h_{21} = 100$

$h_{22} = 50 \times 10^{-6}$ ℧. Find $V_2 = ?$

$V_1 = h_{11} I_1 + h_{12} V_2$ —— (1)

$I_2 = h_{21} I_1 + h_{22} V_2$ —— (2)

Nodal ~~KVL~~ for loop-2,

$\cancel{V_2 + 2I_2 = 0}$

$\therefore V_2 = -2I_2$

$\therefore V_2 = -2000 I_2$

so, from eq. (2),

$I_2 = h_{21} I_1 + h_{22} (-2000 I_2)$

$\therefore 1 = h_{21} \dfrac{I_1}{I_2} + h_{22}(-2000)$

$\therefore h_{21} \dfrac{I_1}{I_2} = 1 + 2000 h_{22}$

$$\therefore I_2 = \frac{h_{21} I_1}{1 + 2000 h_{22}} \quad \text{--- (2)}$$

$$\therefore I_2 = \frac{100 I_1}{1 + 2000(50 \times 10^{-6})}$$

$$\therefore I_2 = \frac{100 I_1}{1.1} \quad \text{--- (3)}$$

~~From loop-1,~~

$$\cancel{V_1 - 500 I_1 = 0.}$$

$$\cancel{\therefore V_1 = 500 I_1}$$

$$\cancel{\therefore I_1 = \frac{V_1}{500}}$$

$$V_1 = h_{11} I_1 + h_{12}(-2000 I_2)$$

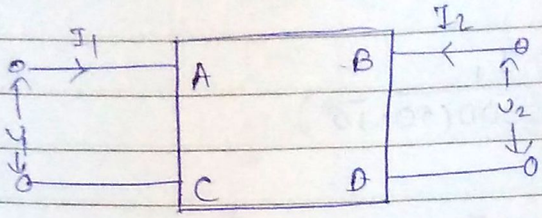
$$\therefore V_1 = h_{11} I_1 - h_{12}(2000)\left(\frac{100 I_1}{1.1}\right)$$

$$\therefore V_1 = 1000 I_1 - 0.003(2000)\left(\frac{100 I_1}{1.1}\right)$$

$$\therefore V_1 = 1000 I_1 - 545 I_1$$

$$\therefore V_1 = 454.54 I_1$$

⇒ ABCD Parameters –



→ ABCD Parameters also known as transmission parameters are used for the modelling of transmission lines in electrical power system. The Input voltage and current is expressed interms of Output voltage and current. $V_1 I_1$ – Sending end quantities, $V_2 I_2$ – Receiving end quantities.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} \qquad B = \frac{V_1}{-I_2}\Big|_{V_2=0}$$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} \qquad D = \frac{I_1}{-I_2}\Big|_{V_2=0}$$

A = Reverse voltage Gain.
B = short-circuited impedence.
C = Open-circuited Admittance.
D = Reverse Current gain.

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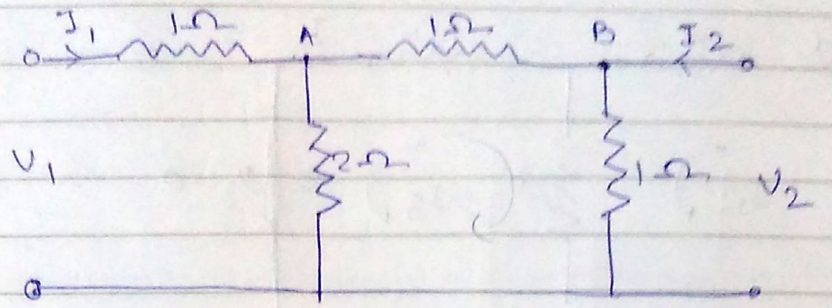


case 1: $I_2 = 0$.

from loop 1 : I_1

from loop 2 : I_3

$V_1 - I_1 + 2(I_3 - I_1) = 0$

$\therefore V_1 = I_1 - 2I_3 + 2I_1$

$\therefore V_1 = 3I_1 - 2I_3$ —— (1)

$-I_3 + 2(I_1 - I_3) - I_3 = 0$

$\therefore -2I_3 + 2I_1 - 2I_3 = 0$

$\therefore 2I_1 - 4I_3 = 0$

$\therefore 2I_1 = 4I_3$

$\therefore I_3 = I_1/2$ —— (2)

so, $V_1 = 2I_1$ —— (3)

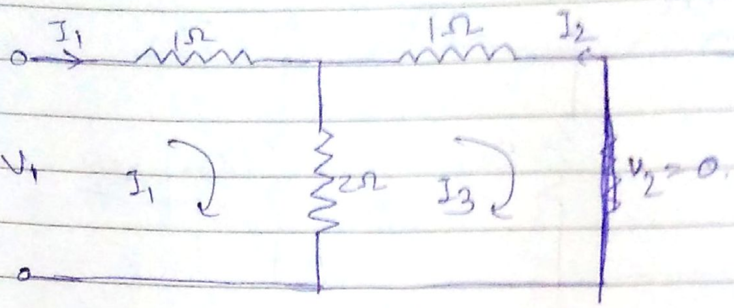
and $V_2 = I_3 \times 1$

$\therefore V_2 = I_3 = I_1/2$

and $A = \frac{V_1}{V_2} = \frac{2I_1}{I_1/2} = 4$.

and $C = \frac{I_1}{V_2} = \frac{I_1}{I_1/2} = 2\,\mho$.

Case-2:



$V_1 - I_1 + 2(I_3 - I_1) = 0$

$\therefore V_1 = I_1 - 2I_3 + 2I_1$

$\therefore V_1 = 3I_1 - 2I_3$ — (5)

and $2(I_1 - I_3) - I_3 = 0$

$\therefore 2I_1 - 3I_3 = 0$

and $I_3 = -I_2$.

$\therefore V_1 = 3I_1 + 2I_2$.

and $2I_1 = -3I_2$.

$\therefore I_2 = -2/3\, I_1$

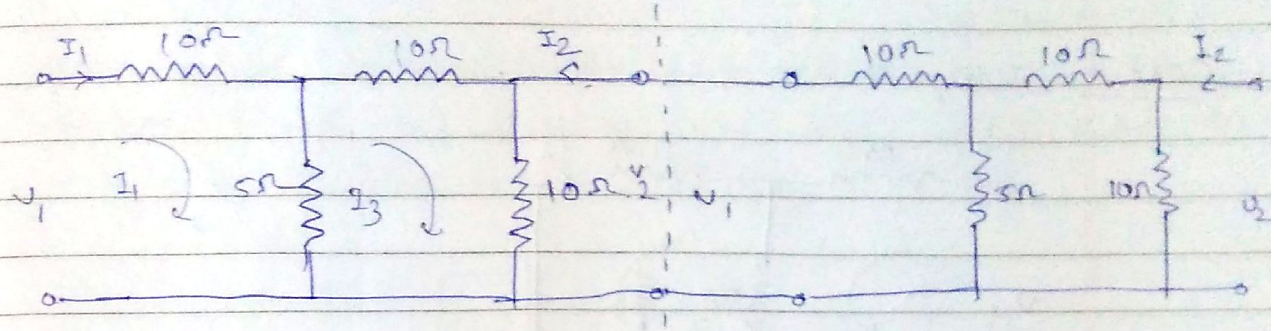
$\therefore \frac{I_1}{I_2} = 3/2 = D$

and $V_1 = 3(-3/2) I_2 + 2I_2$

$\therefore \frac{V_1}{-I_2} = B = 5/2$

This is Cascade not series

Que.



case-1: $I_2 = 0$.

$v_1 - 10I_1 + 5(I_3 - I_1) = 0$

$\therefore v_1 = 10I_1 - 5I_3 + 5I_1$

$\therefore v_1 = 15I_1 - 5I_3$ —— (1)

and $-10I_3 + 5(I_1 - I_3) - 10I_3 = 0$

$\therefore -10I_3 + 5I_1 - 5I_3 - 10I_3 = 0$

$\therefore -25I_3 + 5I_1 = 0$

$\therefore \frac{5I_1}{25} = I_3$

so, $v_1 = 15I_1 - 5\left(\frac{5}{25}I_1\right)$

$= 15I_1 - I_1$

$\therefore v_1 = 14I_1$

and $v_2 = I_3 \times 10$

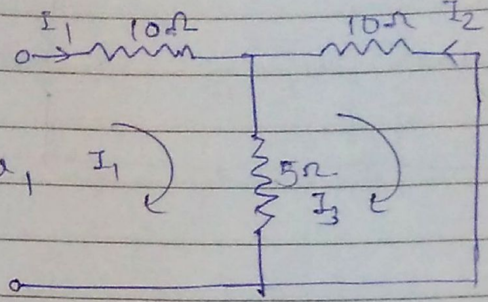
$= \frac{5}{25}I_1 \times 10$

$\therefore v_2 = 2I_1$

$\therefore A = \frac{v_1}{v_2} = 7$

and $C = \frac{I_1}{v_2} = \frac{I_1}{2I_1} = 1/2$

Case-2 $\quad v_2 = 0$



$v_1 - 10I_1 + 5(I_3 - I_1) = 0$

$\therefore v_1 = 10I_1 - 5I_3 + 5I_1$

$\therefore v_1 = 15I_1 - 5I_3$

and $\quad 5(I_1 - I_3) - 10I_3 = 0$

$\therefore 5I_1 - 5I_3 - 10I_3 = 0$

$\therefore 5I_1 - 15I_3 = 0$

$\therefore 5I_1 = 15I_3 \Rightarrow I_1 = \frac{15}{5}(-I_2) = -3I$

and $\quad I_3 = -I_2$

$\therefore v_1 = 15\left(\frac{\cancel{15}^{3}}{5} I_3\right) - 5I_3$

$\therefore v_1 = 40 I_3 = -40I_2$

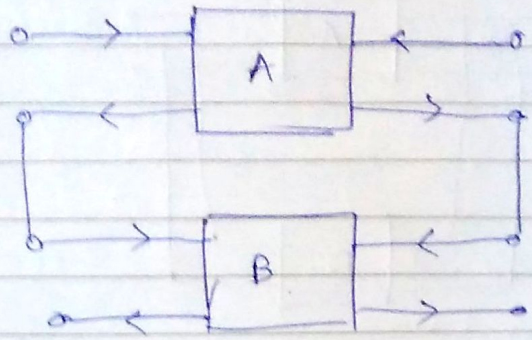
and $\quad B = \frac{v_1}{-I_2} = 40$

and $\quad D = \frac{I_1}{I_2} = \frac{-3I_2}{-I_2} = 3.$

$H^9 \times 20$

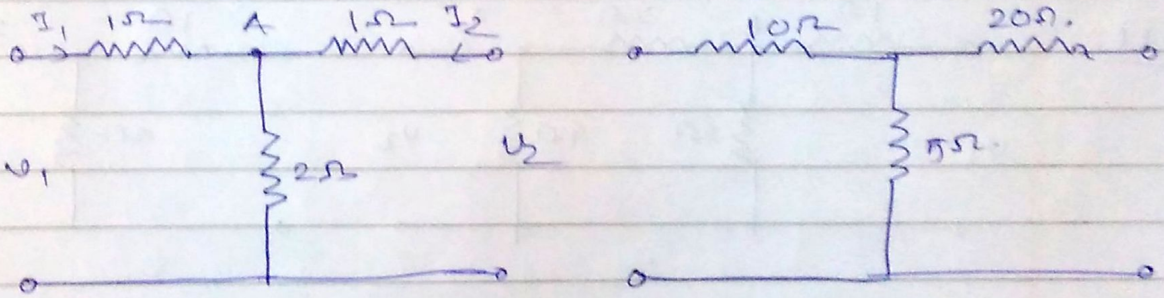
$$\begin{bmatrix} 7 & 40 \\ \frac{1}{2} & 3 \end{bmatrix}\begin{bmatrix} 7 & 40 \\ \frac{1}{2} & 3 \end{bmatrix} = \begin{bmatrix} 69 & 400 \\ 5 & 29 \end{bmatrix}.$$

Note: When we ~~cascade~~ have two networks in series at that ~~them~~ time Z parameter will be Addition of two Matrices.



Series.

Que



$\cancel{V_1 = I_1}$

$$\frac{V_A}{2} + \frac{V_A - V_1}{1} + \frac{V_A - V_2}{1} = 0$$

$$\therefore \frac{V_A}{2} + 2V_A - V_1 - V_2 = 0$$

$$\therefore V_A + 4V_A - 2V_1 - 2V_2 = 0$$

$$\therefore 5V_A = 2V_1 + 2V_2$$

but $V_1 - I_1 = V_A$, $V_2 - I_2 = V_A$

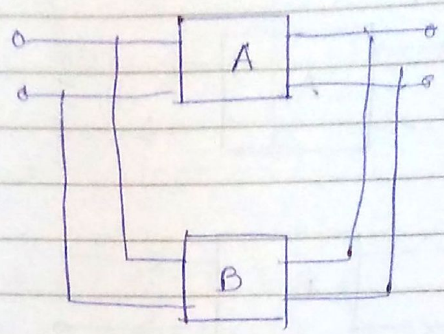
$\therefore 5V_1 - 5I_1 = 2V_1 + 2V_2$ $\qquad$ $\therefore 5V_2 - 5I_2 = 2V_1 + 2V_2$

$\therefore 3V_1 - 2V_2 = 5I_1$ —(1) $\qquad$ $\therefore 3V_2 - 2V_1 = 5I_2$

$\cancel{\therefore 6V_1 - 4V_2 = 10 I_1}$

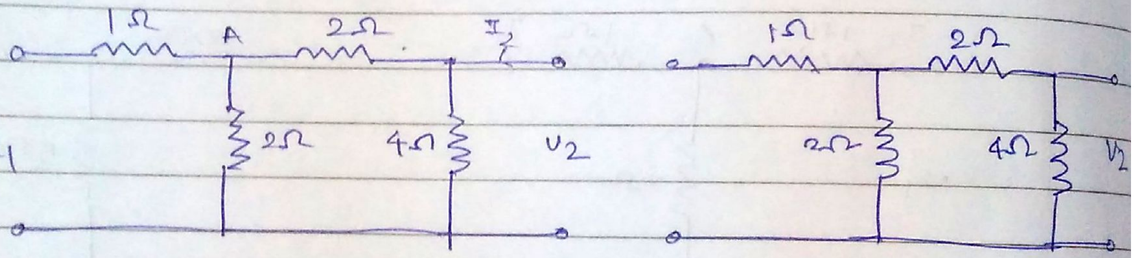
$\cancel{6V_2 -}$

Note: When two n/w are connected in parallel then their y parameters add each other



$$[Y] = [Y_A + Y_B]$$

Que.



$$\frac{V_A}{2} + 2\frac{(V_A - V_1)}{1} + \frac{V_A - V_2}{2} = 0$$

$\therefore V_A + 2V_A - 2V_1 + V_A - V_2 = 0$

$\therefore 4V_A - 2V_1 - V_2 = 0$

$\therefore 4(V_1 - I_1) = 2V_1 + V_2$

$\therefore 4V_1 - 4I_1 = 2V_1 + V_2$

$\therefore 2V_1 - V_2 = 4I_1$ —— (1)

for Node B,

$$\frac{V_2}{4} + \frac{V_2 - V_A}{2} = I_2$$

$$\therefore \frac{V_2}{4} + \frac{2(V_2 - V_1 + I_1)}{4} = I_2$$

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$\cancel{= 4V_2 + 8}$

$\therefore V_2 + 2V_2 - 2V_1 + 2I_1 = 4I_2$

$\therefore 3V_2 - 2V_1 = 4I_2 - 2I_1$

$\therefore 6V_2 - 4V_1 = 8I_2 - \cancel{4I_1}$

$\therefore 2V_1 - V_2 = \cancel{4I_1}$

$5V_2 - 2V_1 = 8I_2$

$\therefore I_2 = \left(\frac{-2}{8}\right) V_1 + \left(\frac{5}{8}\right) V_2$

$Y_{21} = \frac{-1}{4}, \quad Y_{22} = 5/8.$

and $3V_2 - 2V_1 = 4\left(\frac{-1}{4} V_1 + \frac{5}{8} V_2\right) - 2I_1$

$\therefore 3V_2 - 2V_1 = -V_1 + \frac{5}{2} V_2 - 2I_1$

$\therefore 2I_1 = V_1 + \frac{5}{2} V_2 - 3V_2$

$\therefore 2I_1 = V_1 + V_2\left(5/2 - 3\right)$

$\therefore 2I_1 = V_1 + V_2\left(-1/2\right)$

$\therefore I_1 = \frac{1}{2} V_1 + \left(-\frac{1}{4}\right) V_2$

$Y_{11} = 1/2, \quad Y_{12} = -1/4.$

$$\begin{bmatrix} 1/2 & -1/4 \\ -1/4 & 5/8 \end{bmatrix} + \begin{bmatrix} 1/2 & -1/4 \\ -1/4 & 5/8 \end{bmatrix} = \begin{bmatrix} 1 & -1/2 \\ -1/2 & 5/4 \end{bmatrix}.$$

⇒ Interrelationship b/w parameters —

(a) Z-parameters —

(i) ABCD —

$$V_1 = AV_2 - BI_2 \quad \text{— (1)}$$
$$I_1 = CV_2 - DI_2 \quad \text{— (2)}$$

$$V_2 = \frac{I_1 + DI_2}{C}$$

$$\therefore V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \quad \text{— (3)}$$

putting the value of V_2 in eq. (1).

$$\text{so,} \quad V_1 = A\left(\frac{1}{C} I_1 + \frac{D}{C} I_2\right) - BI_2$$

$$= A/_C \, I_1 + \frac{AD}{C} I_2 - BI_2$$

$$\therefore V_1 = A/_C \, I_1 + \left(\frac{AD}{C} - B\right) I_2 \quad \text{— (4)}$$

so, $Z_{11} = A/_C$, $Z_{12} = \frac{AD}{C} - B$, $Z_{21} = 1/_C$, $Z_{22} = D/$

(ii) h-parameters —

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{— (1)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{— (2)}$$

$$\text{so,} \quad V_2 = \frac{I_2 - h_{21} I_1}{h_{22}} \quad \text{— (3)}$$

$$\therefore V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad \text{— (3)}$$

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putting the value of V_2 in eq. (1).

$$\text{so, } V_1 = h_{11} I_1 + h_{12}\left(\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \right)$$

$$\therefore V_1 = h_{11} I_1 - \frac{(h_{12})(h_{21})}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$\therefore V_1 = \left[h_{11} - \frac{(h_{12})(h_{21})}{h_{22}} \right] I_1 + \left[\frac{h_{12}}{h_{22}} \right] I_2 \quad \text{—(4)}$$

(iii) Y-parameters —

$$[Z] = [Y]^{-1}.$$

	[Z]		[Y]		[h]		ABCD	
[Y]	$\frac{Z_{22}}{\Delta Z}$	$\frac{-Z_{12}}{\Delta Z}$	—		$\frac{1}{h_{11}}$	$\frac{-h_{12}}{h_{11}}$	$\frac{D}{B}$	$\frac{\Delta[ABCD]}{B}$
	$\frac{-Z_{21}}{\Delta Z}$	$\frac{Z_{11}}{\Delta Z}$			$\frac{h_{21}}{h_{11}}$	$\frac{\Delta h}{h_{11}}$	$-\frac{1}{B}$	A/B
[h]	$\frac{\Delta Z}{Z_{22}}$	$\frac{Z_{12}}{Z_{22}}$	$\frac{1}{Y_{11}}$	$\frac{-Y_{12}}{Y_{11}}$	—		B/D	$\frac{\Delta[ABCD]}{D}$
	$\frac{-Z_{21}}{Z_{22}}$	$\frac{1}{Z_{22}}$	$\frac{Y_{21}}{Y_{11}}$	$\frac{\Delta Y}{Y_{11}}$			$-1/D$	C/D
[ABCD]	$\frac{Z_{11}}{Z_{21}}$	$\frac{\Delta Z}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	$\frac{-1}{Y_{21}}$	$\frac{-\Delta h}{h_{21}}$	$\frac{-h_{11}}{h_{21}}$		
	$\frac{1}{Z_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-\Delta Y}{Y_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	$\frac{-h_{22}}{h_{21}}$	$\frac{-1}{h_{21}}$		

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	[z]	[Y]		[h]		[A…
[z]	—	$\frac{y_{22}}{\Delta y}$	$\frac{y_{21}}{\Delta y}$	$\frac{h_{11}-(h_{12})(h_{21})}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	A/C
		$\frac{-y_{12}}{\Delta y}$	$\frac{y_{11}}{\Delta y}$	$\frac{-h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$1/C$

<u>Que</u> $[z] = \begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix}$

$h = ?\ ,\quad y = ?\ ,\quad ABCD = ?$

$$[h] = \begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{175}{20} & \frac{5}{20} \\ -\frac{5}{20} & \frac{1}{20} \end{bmatrix} = \begin{bmatrix} 8.75 & 0.25 \\ -0.25 & 0.05 \end{bmatrix}$$

$$[y] = \begin{bmatrix} 20 & -5 \\ -5 & 10 \end{bmatrix} = \cancel{225}\ 175$$

$$[ABCD] = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} 2 & \frac{175}{5} \\ \frac{1}{5} & \frac{20}{5} \end{bmatrix}$$

Que $h_{11} = 1\Omega$

$h_{12} = 2$

$h_{21} = -2$

$h_{22} = 1℧$.

$$\Delta h = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = 1 + 4 = 5$$

find z and y = ?

$$[Z] = \begin{bmatrix} \frac{h_{11} - (h_{12})(h_{21})}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1 - (2)(-2)}{1} & \frac{2}{1} \\ -\left(\frac{-2}{1}\right) & \frac{1}{1} \end{bmatrix}$$

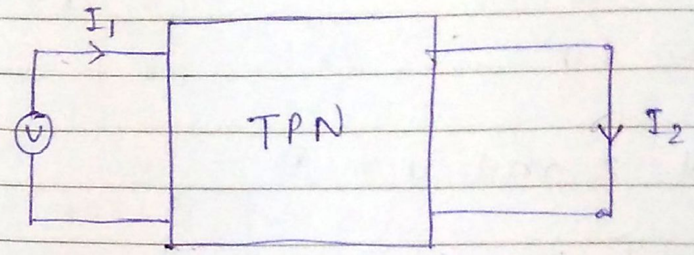
$$= \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

= 0

⇒ Condition for Reciprocity and Symmetry –



Statement: A Network is said to be reciprocal, if the ratio of responce variable to the excitation variable remains identical even if the positions of responce and excitation in the network are interchanged.

→ For Symmetrical – A two port N/w is said to be symmetrical if the i/p and o/p ports can be interchanged without changing the port voltages and current.

⇒ for z parameters – (Reciprocity)

$$V_1 = Z_{11} I_1 - Z_{12} I_2 \quad (\because I_2 = -I_2) \quad —(1)$$

$$0 = Z_{21} I_1 - Z_{22} I_2 \quad (\because V_2 = 0) \quad —(2)$$

$\therefore$ ~~$Z_{21} I_1 = Z_{22} I_2$~~

$\therefore$ ~~$I_1 = \left(\frac{Z_{22}}{Z_{21}}\right) I_2$~~

$$Z_{11} I_1 = V_1 + Z_{12} I_2$$

$$\therefore I_1 = \frac{V_1}{Z_{11}} + \left(\frac{Z_{12}}{Z_{11}}\right) I_2$$

putting in eq. (2),

$\therefore \quad Z_{21} I_1 = Z_{22} I_2$

$\therefore \quad Z_{21} \left[\frac{V_1}{Z_{11}} + \left(\frac{Z_{12}}{Z_{11}}\right) I_2 \right] = Z_{22} I_2$

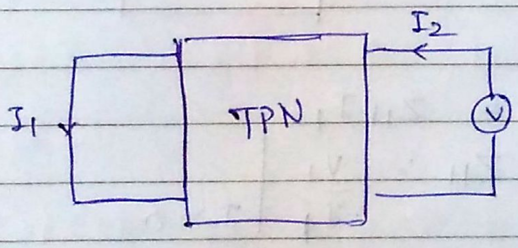
$\therefore \quad \frac{Z_{21}}{Z_{11}} V_1 + \frac{(Z_{21})(Z_{12})}{Z_{11}} I_2 = Z_{22} I_2$

$\therefore \quad \frac{Z_{21}}{Z_{11}} V_1 = I_2 \left[\frac{Z_{11} Z_{22} - (Z_{12})(Z_{21})}{Z_{11}}\right]$

$\therefore \quad Z_{21} V_1 = I_2 \Delta Z$

$\therefore \quad I_2 = \frac{Z_{21} V_1}{\Delta Z} \quad \text{—} \; (4)$

Case-2



$0 = -Z_{11} I_1 + Z_{12} I_2 \quad \text{—} \; (1)$

$V_2 = -Z_{21} I_1 + Z_{22} I_2 \quad \text{—} \; (2)$

$\therefore \quad I_2 = \frac{V_2 + Z_{21} I_1}{Z_{22}} \quad \text{—} \; (3)$

putting eq. (3) in eq. (1),

$\therefore \quad Z_{11} I_1 = Z_{12} \left[\frac{V_2 + Z_{21} I_1}{Z_{22}}\right]$

$\therefore \quad Z_{11} I_1 = \frac{Z_{12}}{Z_{22}} V_2 + \frac{(Z_{21})(Z_{12})}{Z_{22}} I_1$

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$$\therefore I_1\left(Z_{11} - \frac{(Z_{12})(Z_{21})}{Z_{22}}\right) = \frac{Z_{12}}{Z_{22}} V_2$$

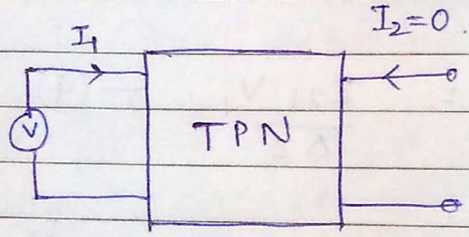
$$\therefore I_1\left(Z_{22}Z_{11} - (Z_{12})(Z_{21})\right) = Z_{12}V_2$$

$$\therefore I_1 = \frac{V_2 Z_{12}}{\Delta Z} \quad — (4)$$

→ For Reciprocity, $\boxed{Z_{12} = Z_{21}}$

⇒ For symmetry –

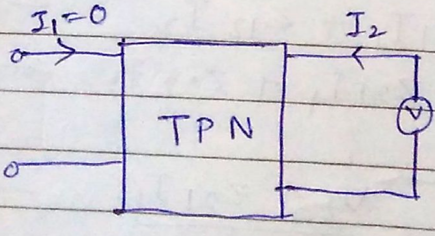
Case-I



$$V_1 = Z_{11} I_1$$

$$\therefore Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}.$$

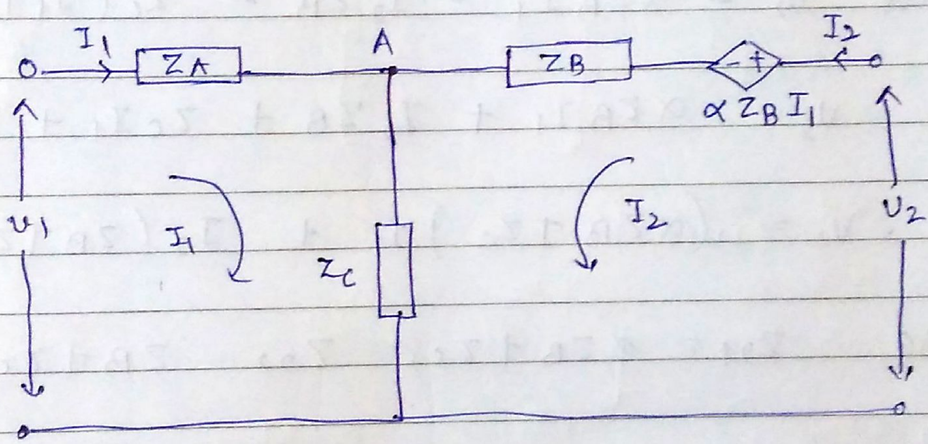
Case-II



$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}.$$

	Reciprocity	Symmetry.
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$.
h	$h_{12} = -h_{21}$	$\Delta h = 1$.
ABCD	$AD - BC = 1$	$A = D$.

Que



Here, Nodal Analysis for node A,

$$\frac{V_A - V_1}{Z_A} + \frac{V_A}{Z_C} + \frac{V_A + \alpha Z_B I_1 - V_2}{Z_B} = 0.$$

$$\therefore \frac{V_A - V_1}{Z_A} + \frac{V_A}{Z_C} \neq I_2.$$

but $V_A = V_1 - Z_A I_1$

$\therefore V_1 -$

→ Using Mesh Analysis,

$$V_1 - I_1 Z_A - Z_C(I_1 + I_2) = 0$$

$$\therefore V_1 = I_1 Z_A + Z_C I_1 + Z_C I_2$$

$$\therefore V_1 = I_1(Z_A + Z_C) + I_2 Z_C.$$

so, $Z_{11} = Z_A + Z_C$, $Z_{12} = Z_C$.

and $V_2 - \alpha Z_B I_1 - I_2 Z_B - Z_C(I_1 + I_2) = 0$

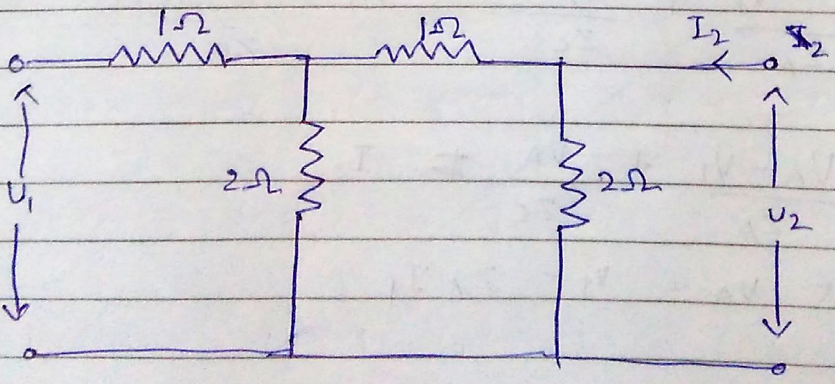
$$\therefore V_2 = \alpha Z_B I_1 + I_2 Z_B + Z_C I_1 + Z_C I_2$$

$$\therefore V_2 = (\alpha Z_B + Z_C) I_1 + I_2(Z_B + Z_C).$$

so, $Z_{21} = \alpha Z_B + Z_C$, $Z_{22} = Z_B + Z_C$.

→ so, $Z_{12} \neq Z_{21}$. So, It is not Reciprocal.

Que



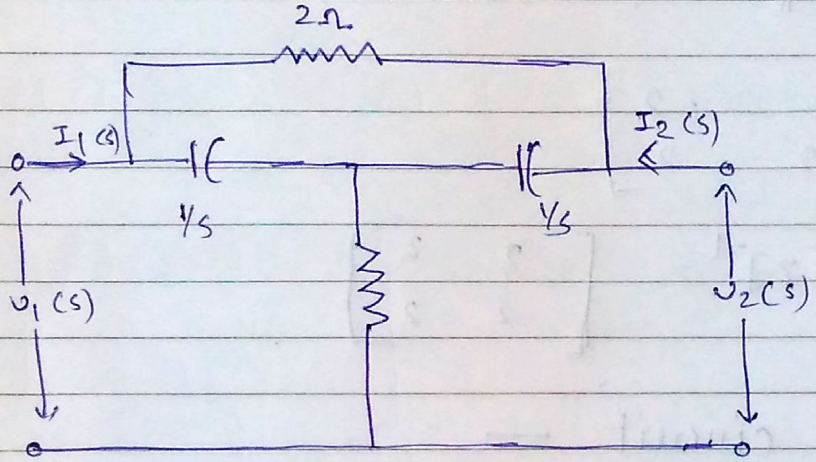
ABCD = ?

$$\begin{bmatrix} 4 & 5/2 \\ 2 & 3/2 \end{bmatrix}$$

$AD - BC = 1 \rightarrow$ Reciprocal.

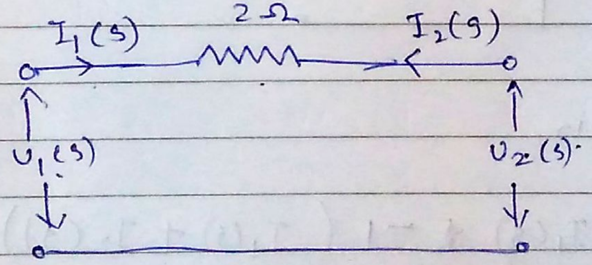
$A \neq D \rightarrow$ Not symmetrical.

Que



find Y Parameter.

→ for 1st circuit –



Case-1 Output port is shorted.

$V_1(s) + 2(I_2(s) - I_1(s)) = 0.$

$\therefore V_1(s) = 2I_1(s) - 2I_2(s)$

$Z_{11} = 2, \quad Z_{12} = -2.$

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Case-2 Input port is shorted. ($V_1 = 0$)

∴ $V_2(s) + 2(I_1(s) - I_2(s)) = 0$

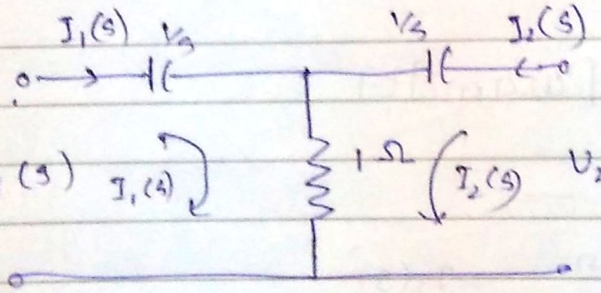
∴ $V_2(s) = 2I_2(s) - 2I_1(s)$

$Z_{21} = 2$, $Z_{22} = -2$.

$$Z = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\therefore [Y] = [Z]^{-1} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

→ for 2^{nd} circuit —



Mesh Analysis.

$V_1(s) - (1/s) I_1(s) - 1(I_1(s) + I_2(s)) = 0$

∴ $V_1(s) = \frac{1}{s} I_1(s) + I_1(s) + I_2(s)$

∴ $V_1(s) = \left(\frac{1}{s} + 1\right) I_1(s) + I_2(s)$

$Z_{11} = \frac{1}{s} + 1$, $Z_{12} = 1$.

$$V_2(s) - \left(\frac{1}{s}\right) I_2(s) - 1\left(I_1(s) + I_2(s)\right) = 0$$

$$\therefore V_2(s) = \frac{1}{s} I_2(s) + I_1(s) + I_2(s)$$

$$\therefore V_2(s) = \left(\frac{1}{s} + 1\right) I_2(s) + I_1(s)$$

$$\therefore \cancel{V_2(s)} \Rightarrow Z_{21} = 1, \quad Z_{22} = 1/s + 1.$$

$$[Z] = \begin{bmatrix} 1/s + 1 & 1 \\ 1 & 1/s + 1 \end{bmatrix}$$

$$\therefore [Y] = [Z]^{-1}$$