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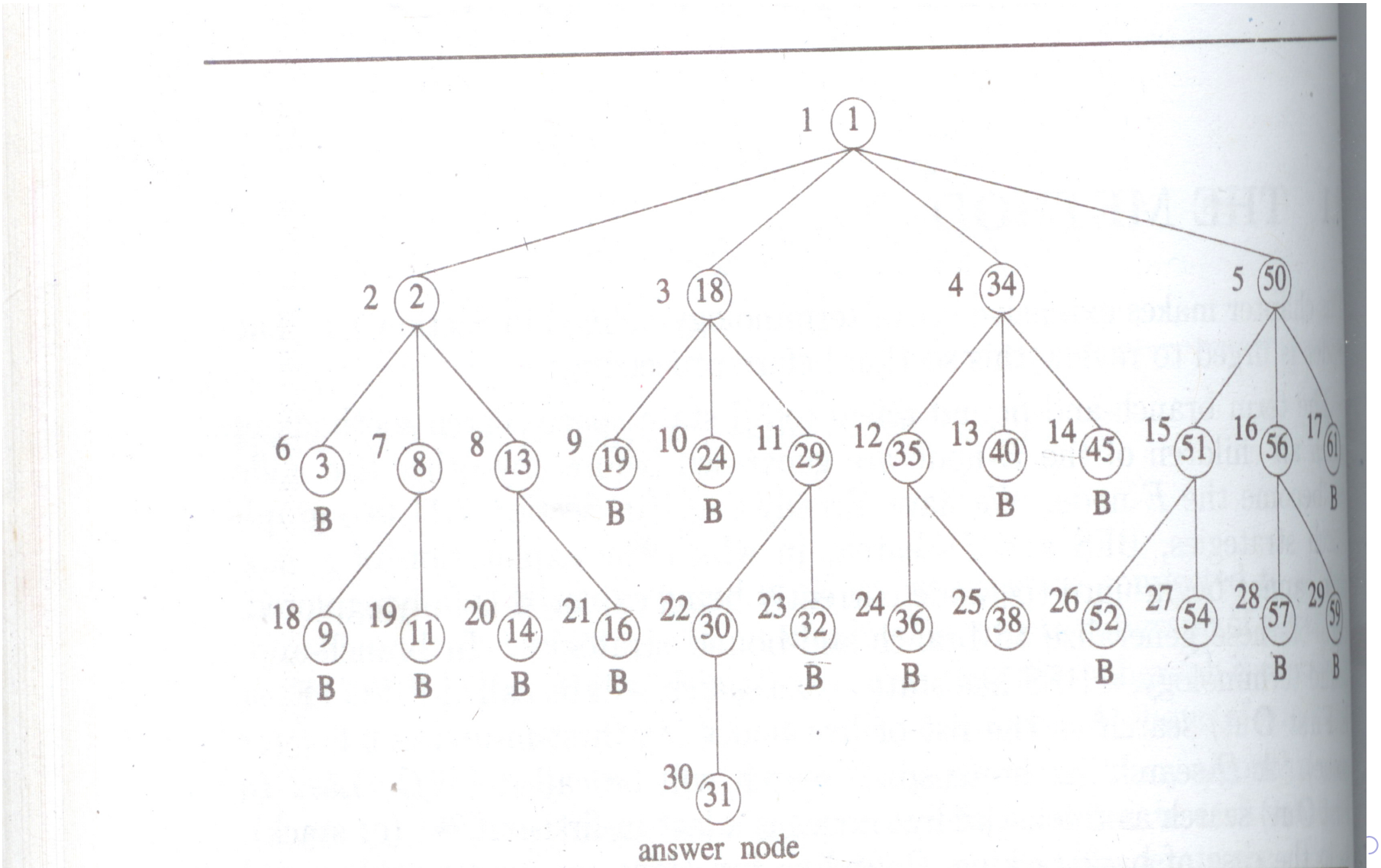
- bounding functions are used to avoid the generation of subtree that do not contain an answer node

4-queen problem

- portion of 4-queens state space tree generated by FIFO branch-and-bound



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- node 18 becomes the next E node and nodes 19, 24, and 29 are generated
- nodes 19 and 24 are killed and node 29 is added to queue of live nodes and the E node is node 34

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- the effort already expended in reaching the live nodes cannot be reduced and concerned is to minimize the additional effort to find an answer node

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- desirable not to overbias the search algorithm in favor of deep probes

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- LC search coupled with bounding functions is called an LC branch-and-bound search

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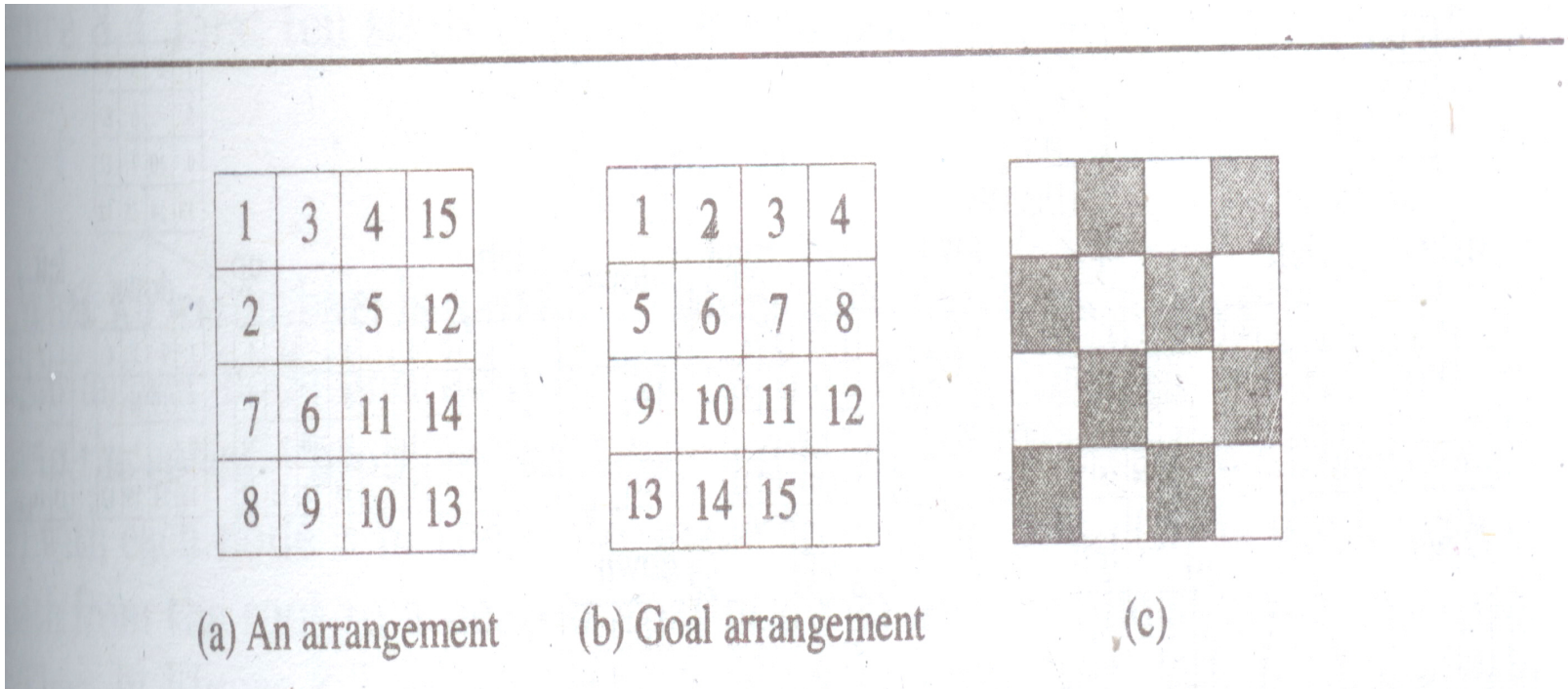

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- solution - search the state space for the goal state and use the path from the initial state to the goal state

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- given figure position 16 is the empty spot and $position(16)$ will denote the position of the empty spot

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- children of each node x in this tree represent states reachable from state x by one legal move

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- begin with the root as E node and generate a child node with $c()$ value the same as root

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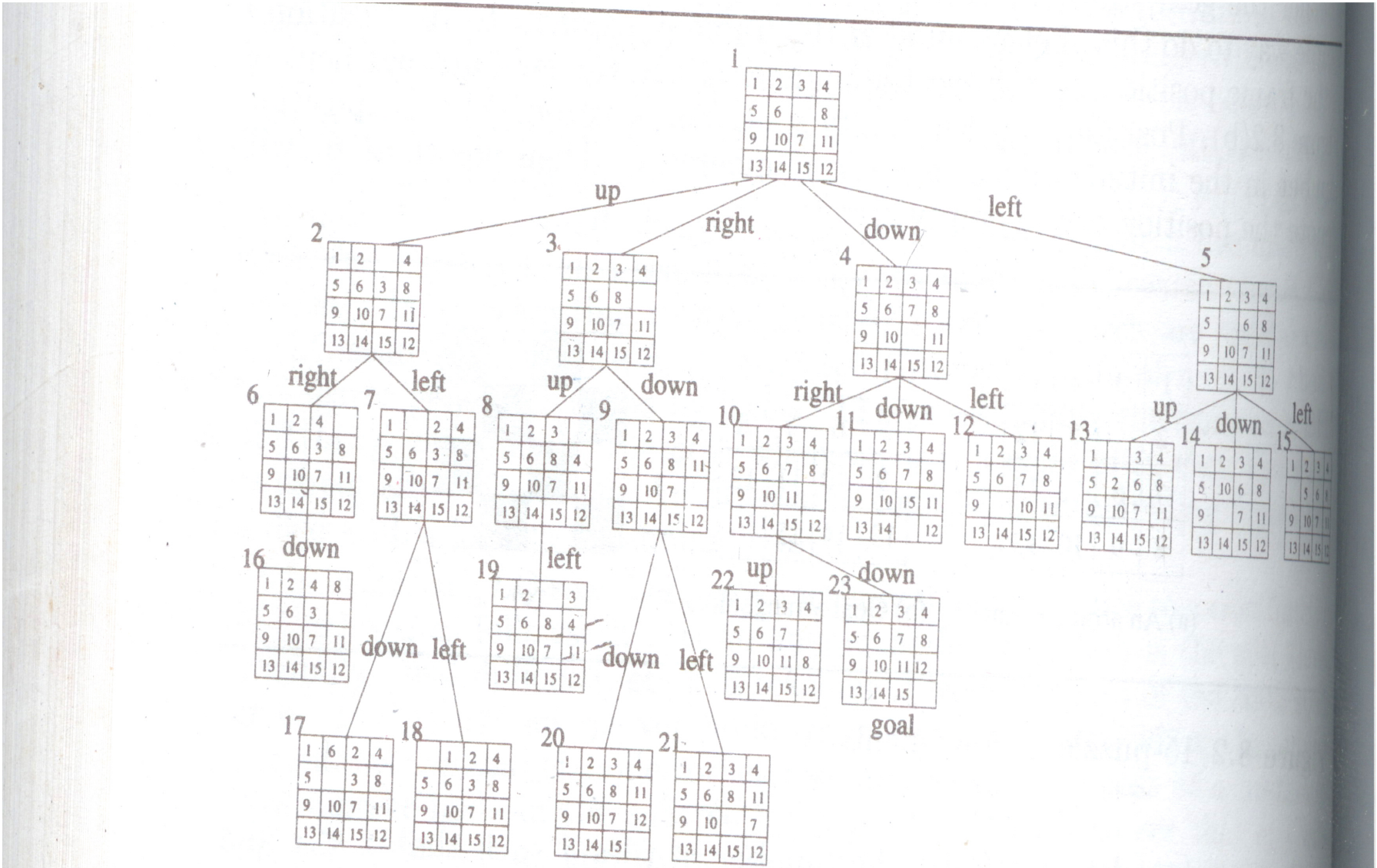

Edges are labeled according to the direction in which the empty space moves

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- here, nodes to become E nodes are nodes on the path from the root to a nearest goal node
- impractical as not possible to computer $c(\cdot)$ specified here

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13	14	15	7

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- suitable choice for $\hat{c}()$, LC search more selective than any of the other search methods

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13. } until (false)

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- starting value for *upper* can be set to ∞ and each time a new answer node is found the value of *upper* can be updated

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- cost $c(\cdot)$ is such that the solution node with least $c(\cdot)$ corresponds to a shortest tour in G

TSP problem

- one choice for $c(\cdot)$ is

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$$c(A) = \begin{cases} \text{length of tour defined by the path from the root to } A, \\ \qquad \text{if } A \text{ is a leaf} \\ \text{cost of a minimum-cost leaf in the subtree } A, \\ \qquad \text{if } A \text{ is not a leaf} \end{cases}$$

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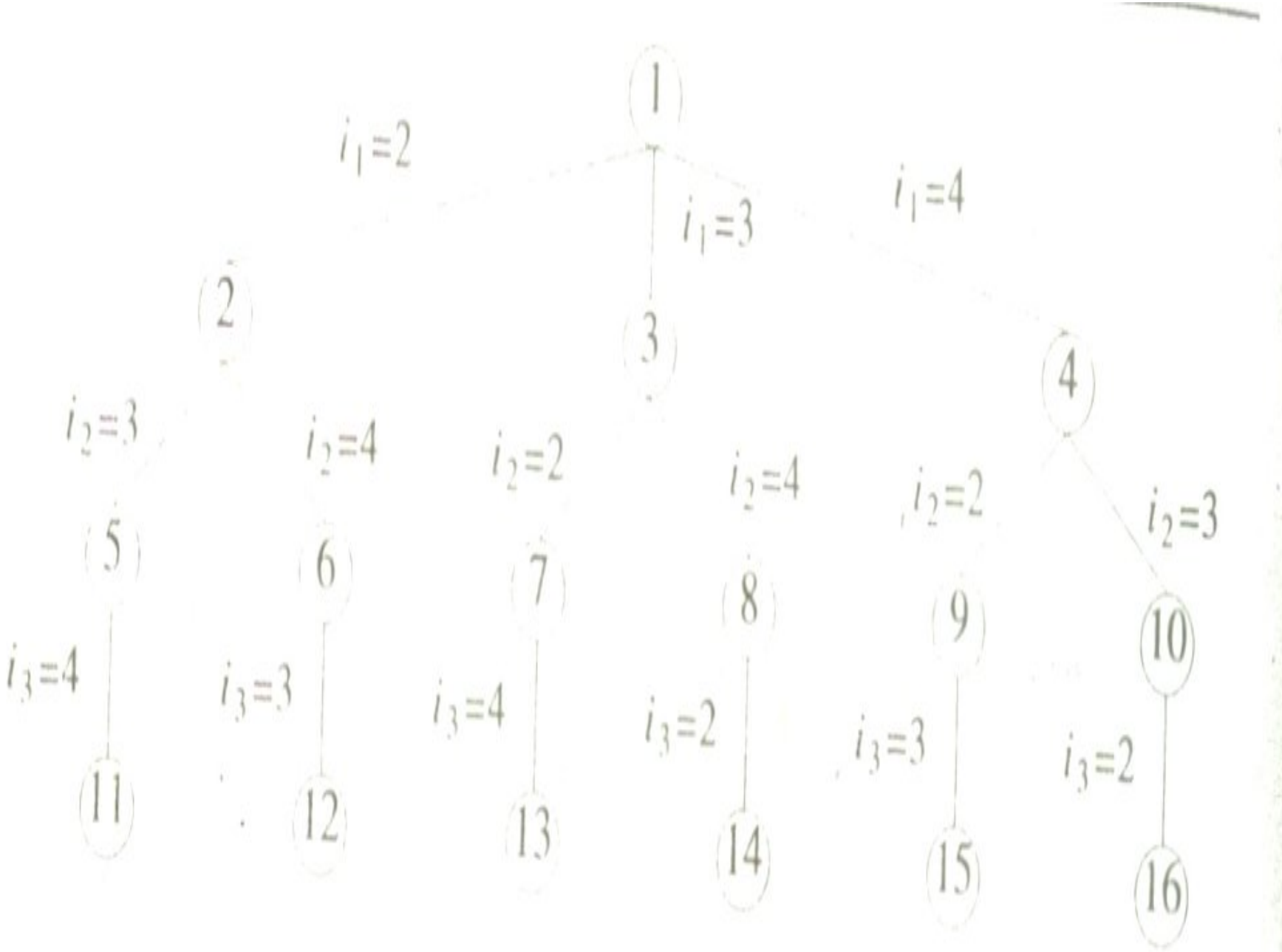
- a matrix is reduced iff every row and column is reduced

TSP problem


1
$i_1=2$
$i_1=3$
$i_1=4$
2
3
4
$i_2=3$
$i_2=4$
$i_2=2$
$i_2=4$
$i_2=2$
$i_2=3$
5
6
7
8
9
10
$i_3=4$
$i_3=3$
$i_3=4$
$i_3=2$
$i_3=3$
$i_3=2$
11
12
13
14
15
16

TSP problem

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- all tours in the original graph have a length at least 25

TSP problem

$$\begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \quad \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

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  3. reduce all rows and columns in the resulting matrix except for rows and columns containing only ∞

TSP problem

- let resulting matrix B

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- if r is the total amount subtracted in step 3 then $\hat{c}(S) = \hat{c}(R) + A(i, j) + r$

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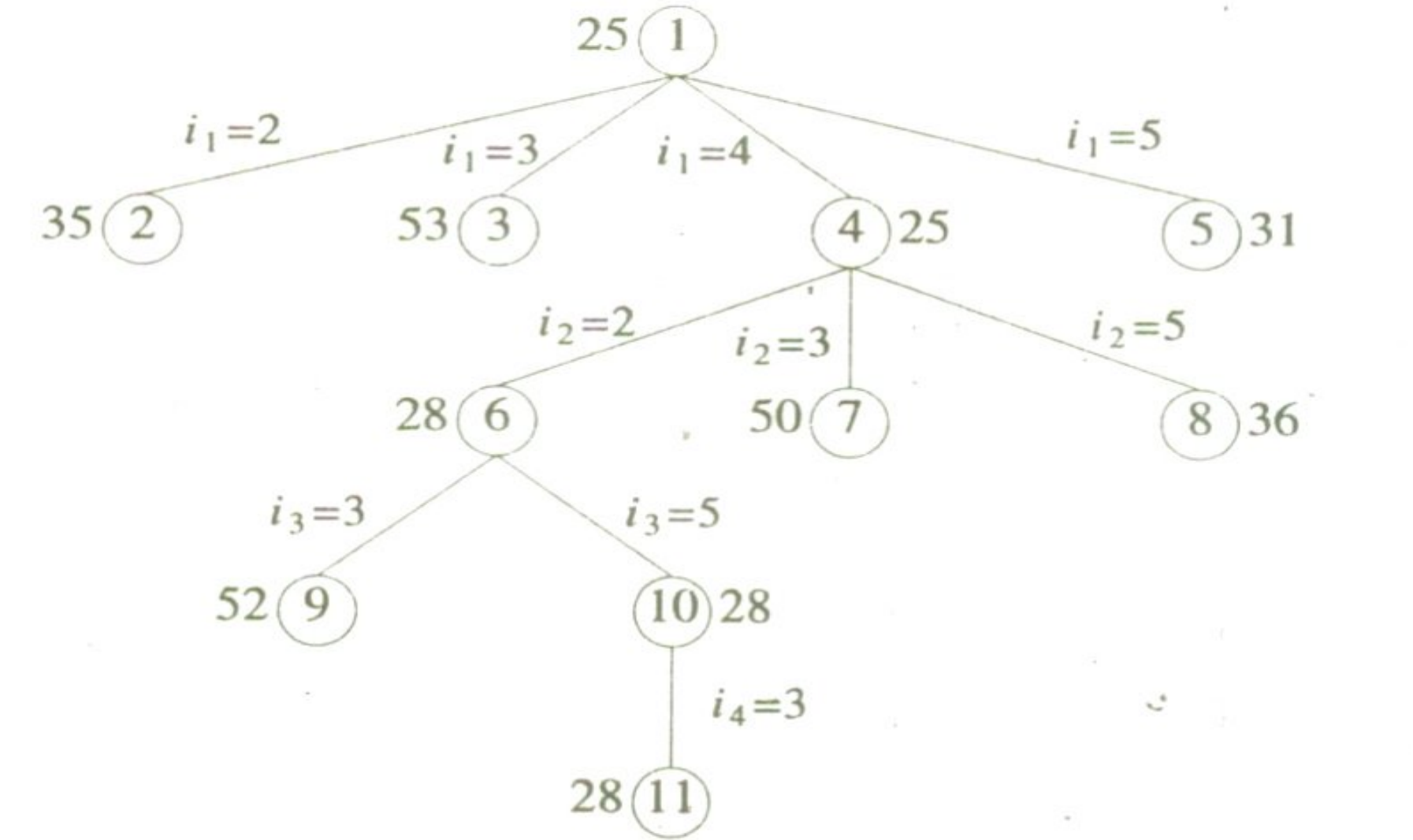
- for the upper bound function u use $u(R) = \infty$ for all nodes R
- use LCBB algorithm for the problem instance defined by initial matrix
- start with initial reduced matrix and $upper = \infty$
- starting with root node as E node, nodes 2, 3, 4, and 5 are generated

TSP problem


25 1
$i_1=2$
$i_1=3$
$i_1=4$
$i_1=5$
35 2
53 3
4 25
5 31
$i_2=2$
$i_2=3$
$i_2=5$
28 6
50 7
8 36
$i_3=3$
$i_3=5$
52 9
10 28
$i_4=3$
28 11

TSP problem

- reduced matrices corresponding to different nodes

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$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \qquad \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

(a) Path 1,2; for node 2 (b) Path 1,3; node 3

TSP problem

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$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

(a) Path 1,2; for node 2

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

(b) Path 1,3; node 3

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

(c) Path 1,4; for node 4

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

(b) Path 1,5; node 5

TSP problem

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \quad \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

(e) Path 1,4,2; for node 6 (f) Path 1,4,3; node 7

TSP problem

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

(e) Path 1,4,2; for node 6

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

(f) Path 1,4,3; node 7

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

(g) Path 1,4,5; for node 8

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

(h) Path 1,4,2,3; node 9

TSP problem

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

(i) Path 1,4,2,5; for node 10

TSP problem

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

(i) Path 1,4,2,5; for node 10

- for (b) all entries in row 1 and column 3 to ∞

TSP problem

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

(i) Path 1,4,2,5; for node 10

- for (b) all entries in row 1 and column 3 to ∞
- setting the element (3,1) to ∞, reducing column 1 by subtracting by 11

TSP problem

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

(i) Path 1,4,2,5; for node 10

- for (b) all entries in row 1 and column 3 to ∞
- setting the element (3,1) to ∞, reducing column 1 by subtracting by 11
- $\hat{c}$ for node 3 is 25 + 17 (the cost of edge $\langle 1, 3 \rangle$ in the reduced matrix) + 11 = 53

TSP problem

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

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- $\hat{c}$ for node 3 is $25 + 17$ (the cost of edge $\langle 1, 3 \rangle$ in the reduced matrix) $+ 11 = 53$
- calculate $\hat{c}$ values for other nodes similarly
- value of *upper* is unchanged and node 4 becomes the next E node

TSP problem

- children 6, 7, and 8 are generated, live nodes 2, 3, 5, 6, 7, and 8

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- tour length for this node $\hat{c}(11) = 28$ and *upper* is updated to 28
- next E node, node 5 having $\hat{c}(5) = 31 > upper$
- LCBB terminates with 1, 4, 2, 5, 3, 1 as the shortest length tour