

# RBF Network

```
In [1]: import numpy as np
import seaborn as sns
import pandas as pd
import matplotlib.pyplot as plt
from matplotlib.pyplot import figure
from itertools import repeat
import random
from IPython.display import Image
```

## Utilities used for manipulation of data/matrices in the task

```
In [2]: # Sampling of data points according to the given rule
def createSampleFeatures():
    xi = []
    xj = []
    for x in range(0, 21):
        xi.extend(repeat((-2 + (0.2*x)), 21))
        xj.extend(repeat((-2 + (0.2*x)), 21))
    random.shuffle(xi)
    random.shuffle(xj)
    x1 = np.array(xi)
    x2 = np.array(xj)
    return np.vstack((x1, x2)).T
```

```
In [3]: # Returns Labels of each input according to the given rule
def calculateSampleTargetsGivenFeatures(data):
    y = []
    for feature in data:
        f = feature[0]**2 + feature[1]**2
        if f<=1:
            y.append(1)
        if f>1:
            y.append(-1)
    return np.array(y)
```

```
In [4]: # Returns g values of the of all nodes in the hidden layer, non linear transformation
def calculateGaussianFunctionForAllNodes(inputMatrix, centres, spreadParam):
    g = []
    for centroid in centres:
        g.append(calculateGaussianFunctionForOneNode(inputMatrix,
                                                    centroid.reshape(centroid.shape
                                                                    spreadParam))
    return np.array(g)
```

```
In [5]: # Returns g values of the of one node in the hidden layer, non linear transformation
def calculateGaussianFunctionForOneNode(inputMatrix, centroid, spreadParam):
    input = inputMatrix.transpose()
    diff = input - centroid
    diffSquare = diff ** 2
    sumDiffSquare = np.sum(diffSquare, axis=0) * (-0.5/(spreadParam**2))
```

```
expTerm = np.exp(sumDiffSquare).transpose()
return expTerm
```

In [6]: `Image(filename='img/q3_3.png')`

Out[6]:

$$W = G^+ D,$$

- where  $G^+$  denotes the pseudo-inverse matrix of  $G$ , which can be defined as

$$G^+ = (G^T G)^{-1} G^T$$

In [7]: `# Returns weights between hidden and output layer using inverse matrix calculation,`  
`def calculateWOfOutputLayer(gMatrix, oMatrix):`  
 `ggt = np.dot(gMatrix, gMatrix.transpose())`  
 `ggt_i = np.linalg.pinv(ggt)`  
 `gg_i_g = np.dot(ggt_i, gMatrix)`  
 `gg_i_g_o = np.dot(gg_i_g, oMatrix)`  
 `#print('ggt', ggt)`  
 `#print('gg_i', gg_i_g)`  
 `#print('ggt * gg_i', np.dot(ggt_i, ggt))`  
 `#print('gg_i_g', gg_i_g.shape)`  
 `return gg_i_g_o.transpose()`

In [8]: `Image(filename='img/q3_4.png')`

Out[8]:

$$o_j(x) = \sum_{i=1}^n w_{ij} g_i(x), \quad j = 1, \dots, r.$$

In [9]: `# W(t)G Returns the predicted output give weights and g values, above image states t`  
`def calculateOutput(weights, g):`  
 `return np.dot(weights, g).transpose()`

In [10]: `# Displays scatter plot of generated sample`  
`def showScatterPlot(features, outputs):`  
 `outputs = outputs.reshape(outputs.shape[0], -1)`  
 `mergedFeaturesOutputData = np.hstack((features, outputs))`  
 `df = pd.DataFrame(mergedFeaturesOutputData, columns = ['X1', 'X2', 'Y'])`  
 `sns.pairplot(df, hue='Y', height=5, aspect=1)`

In [11]: `# Returns total error of all data samples given target(t) and output(0)`  
`def errorCumulative(target, output):`  
 `# sum of (-0.5 * (t-0)^2) of all outputs`

```

error = np.sum(np.sum((0.5) * ((target - output) * (target - output)),
                      dtype=np.float64, axis=1), dtype=np.float64, axis=0)

return error

```

```

In [12]: # Generates n unique samples from a given dataset
def generateRandomNSamples(dataset, n):
    row_idx = np.array(random.sample(range(0, dataset.shape[0]), n))
    return dataset[row_idx]

```

```

In [13]: # predicts the output given sample of inputs and parameters of a trained network
def predict(inputMatrix, targets, centers, spreadParam, weights):
    g = calculateGaussianFunctionForAllNodes(inputMatrix=inputMatrix,
                                             centres=centers,
                                             spreadParam=spreadParam)
    predictedOutput = calculateOutput(weights=weights, g=g)
    return showPerformanceOfModel(target = targets, predicted = predictedOutput)

```

```

In [14]: # Returns Labels of prediction value 1 or -1
def generateLabelsOfPrediction(predictions):
    labels = []
    for prediction in predictions:
        if prediction > 1:
            labels.append([1])
        else:
            labels.append([-1])
    return np.array(labels)

```

```

In [15]: # Returns accuracy given target(t) and output(O)
def calculateAccuracy(t, O):
    correctPreds = np.sum((t == O).all(1))
    accuracy = (correctPreds / (t.shape[0]))
    return accuracy

```

```

In [16]: # Displays performance metrics of the model on give dataset
def showPerformanceOfModel(target, predicted):
    accuracy = calculateAccuracy(trainY,
                                generateLabelsOfPrediction(predicted))*100
    print('Accuracy of model', accuracy)
    print('Cummulative mean square error',
          (errorCumulative(target=testY, output=predicted)/target.shape[0]))
    return accuracy

```

```

In [17]: # Plot model performance
def plotModelPerformance(trainAccuracies, testAccuracies, spreadParams):
    plt.figure(figsize=(8,8))
    plt.plot(spreadParams, trainAccuracies, label = "Training data")
    plt.plot(spreadParams, testAccuracies, label = "Test data")
    plt.legend()
    plt.xlabel("Spread params")
    plt.ylabel("Accuracy")
    plt.show()

```

Two features x1 and x2. Given below is the data generation policy for training process.

```
In [18]: Image(filename='img/q3_1.png')
```

```
Out[18]:
```

$$f(x_1, x_2) = \begin{cases} +1 & \text{if } x_1^2 + x_2^2 \leq 1 \\ -1 & \text{if } x_1^2 + x_2^2 > 1 \end{cases}$$

over region  $-2 < x_1 < 2$  and  $-2 < x_2 < 2$

As a training set, use 441 randomly sampled data points defined as

$$x = (x_i, x_j)$$

Where

$$x_i = -2 + 0.2 i \quad i = 0, 1, \dots, 20$$

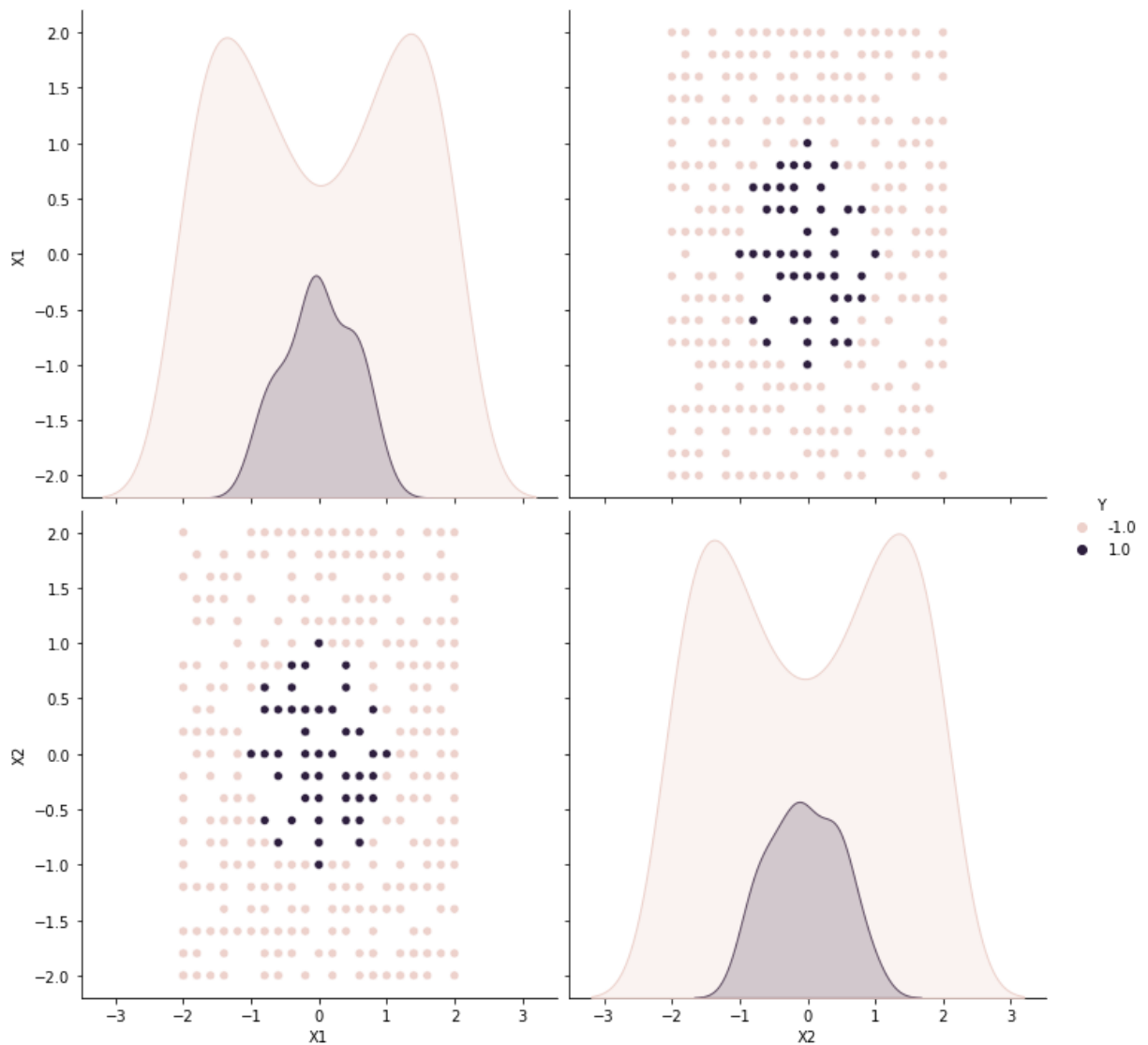
$$x_j = -2 + 0.2 j \quad j = 0, 1, \dots, 20$$

## Creating data samples

```
In [19]: globalFeatures = createSampleFeatures()
         globalOutputs = calculateSampleTargetsGivenFeatures(globalFeatures)
```

## Visualizing 441 randomly sampled datapoints

```
In [20]: showScatterPlot(globalFeatures, globalOutputs)
```



## Splitting Train and Test sets (80% Train, 20% Test)

```
In [21]: trainX = globalFeatures[:352, :]
trainY = globalOutputs[:352]
trainY = trainY.reshape(trainY.shape[0], -1)

testX = globalFeatures[89:, :]
testY = globalOutputs[89:]
testY = testY.reshape(testY.shape[0], -1)
```

## Calculating G values of hidden layer

### Train Method for RBF network

```
In [22]: def trainRBFNetwork(spreadParam, centres):
hiddenG = calculateGaussianFunctionForAllNodes(trainX,
                                                centres=centres,
                                                spreadParam=spreadParam)
#print('Hidden G values after non linear transformation', hiddenG.shape)
w = calculateWOfOutputLayer(hiddenG, trainY)
#print('Final weights calculated using matrix inversion technique', w)
return w
```

## Performance analysis

[1]

## 1. Using Gaussian kernel functions

## 2. Spread parameter is same for all kernel functions

## 3. Considering all the data points as the centers of the RB functions

### Case study: Using spread param = 0.5

In [24]:

```
width = 0.2
trainAccuracies = []
testAccuracies = []
spreadParams = []
networkWeights = trainRBFNetwork(spreadParam=width, centres = trainX)
print('Performance report on training set \n -----')
trainAccuracies.append(
    predict(trainX, trainY, centers=trainX, spreadParam=width,
            weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=trainX,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 95.45454545454545

Cummulative mean square error 0.5284090909090943

.....

Performance report on test set

-----

Accuracy of model 78.4090909090909

Cummulative mean square error 0.00906728865769017

### Case study: Using spread param = 0.7

In [25]:

```
width = 0.7
networkWeights = trainRBFNetwork(spreadParam=width, centres = trainX)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=trainX,
                              spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=trainX,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 91.76136363636364

Cummulative mean square error 0.4879289649454822

.....

Performance report on test set

-----

Accuracy of model 80.68181818181817

Cummulative mean square error 0.03956360599067274

### Case study: Using spread param = 0.9

In [26]:

```
width = 0.9
networkWeights = trainRBFNetwork(spreadParam=width, centres = trainX)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=trainX,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=trainX,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----  
 Accuracy of model 91.76136363636364  
 Cumulative mean square error 0.4775680197733197

.....  
 Performance report on test set

-----  
 Accuracy of model 80.11363636363636  
 Cumulative mean square error 0.042921065678687344

### Case study: Using spread param = 1

In [27]:

```
width = 1
networkWeights = trainRBFNetwork(spreadParam=width, centres = trainX)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=trainX,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=trainX,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----  
 Accuracy of model 91.47727272727273  
 Cumulative mean square error 0.47833386712012377

.....  
 Performance report on test set

-----  
 Accuracy of model 80.11363636363636  
 Cumulative mean square error 0.04521893512093031

### Case study: Using spread param = 5

In [28]:

```
width = 5
networkWeights = trainRBFNetwork(spreadParam=width, centres = trainX)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=trainX,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=trainX,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----  
 Accuracy of model 85.22727272727273  
 Cumulative mean square error 0.42003809407355663

.....  
 Performance report on test set

Accuracy of model 85.22727272727273  
 Cumulative mean square error 0.1027068927411142

## Case study: Using spread param = 10

In [29]:

```
width = 10
networkWeights = trainRBFNetwork(spreadParam=width, centres = trainX)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=trainX,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=trainX,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 85.22727272727273  
 Cumulative mean square error 0.35832159234349065

.....

Performance report on test set

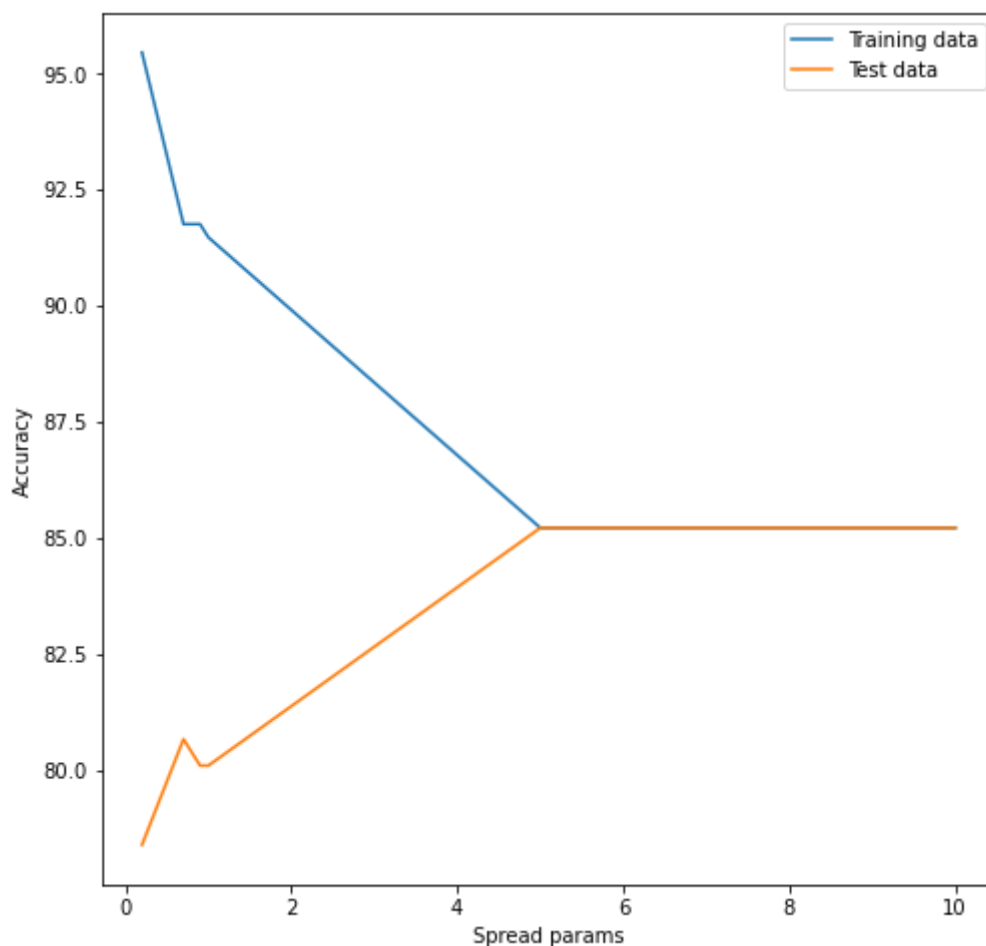
-----

Accuracy of model 85.22727272727273  
 Cumulative mean square error 0.17819241437964617

## Behaviour of model with increase of spread param in train and test sets

In [30]:

```
plotModelPerformance(trainAccuracies=trainAccuracies,
                    testAccuracies=testAccuracies,
                    spreadParams=spreadParams)
```





## [2a]

## 1. Using Gaussian kernel functions

## 2. Spread parameter is same for all kernel functions

## 3. Considering 150 randomly picked data points as the centers of the RB functions

```
In [32]: clusterCenters = generateRandomNSamples(trainX, n=150) # generating 150 random samples
```

## Case study: Using spread param = 0.1

```
In [33]: width = 0.1
trainAccuracies = []
testAccuracies = []
spreadParams = []
networkWeights = trainRBFNetwork(spreadParam=width,
                                  centres = clusterCenters)

print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters, spreadParam=width,
                               weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 90.625

Cummulative mean square error 0.4751365874958686

.....

Performance report on test set

-----

Accuracy of model 81.5340909090909

Cummulative mean square error 0.181101393874726

## Case study: Using spread param = 0.3

```
In [34]: width = 0.3
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 90.9090909090909

Cummulative mean square error 0.4837486533525857

.....

Performance report on test set

-----

Accuracy of model 81.5340909090909

Cummulative mean square error 0.034222911492273786

## Case study: Using spread param = 0.5

In [35]:

```
width = 0.5
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 91.47727272727273

Cummulative mean square error 0.4853165227286292

.....

Performance report on test set

-----

Accuracy of model 80.68181818181817

Cummulative mean square error 0.034802669623049017

## Case study: Using spread param = 0.8

In [36]:

```
width = 0.8
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 91.76136363636364

Cummulative mean square error 0.47717614652717927

.....

Performance report on test set

-----

Accuracy of model 80.11363636363636

Cummulative mean square error 0.04233647398007102

## Case study: Using spread param = 1

In [37]:

```
width = 1
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 92.32954545454545

Cummulative mean square error 0.4760350816330556

.....

Performance report on test set

```
-----
Accuracy of model 79.26136363636364
Cumulative mean square error 0.046126328724813766
```

### Case study: Using spread param = 2

In [38]:

```
width = 2
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

```
-----
Accuracy of model 89.77272727272727
Cumulative mean square error 0.45532388369676435
```

```
.....
Performance report on test set
```

```
-----
Accuracy of model 81.5340909090909
Cumulative mean square error 0.06547447078312532
```

### Case study: Using spread param = 5

In [39]:

```
width = 5
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

```
-----
Accuracy of model 85.22727272727273
Cumulative mean square error 0.4198934306737813
```

```
.....
Performance report on test set
```

```
-----
Accuracy of model 85.22727272727273
Cumulative mean square error 0.10268214871575684
```

### Case study: Using spread param = 10

In [40]:

```
width = 10
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

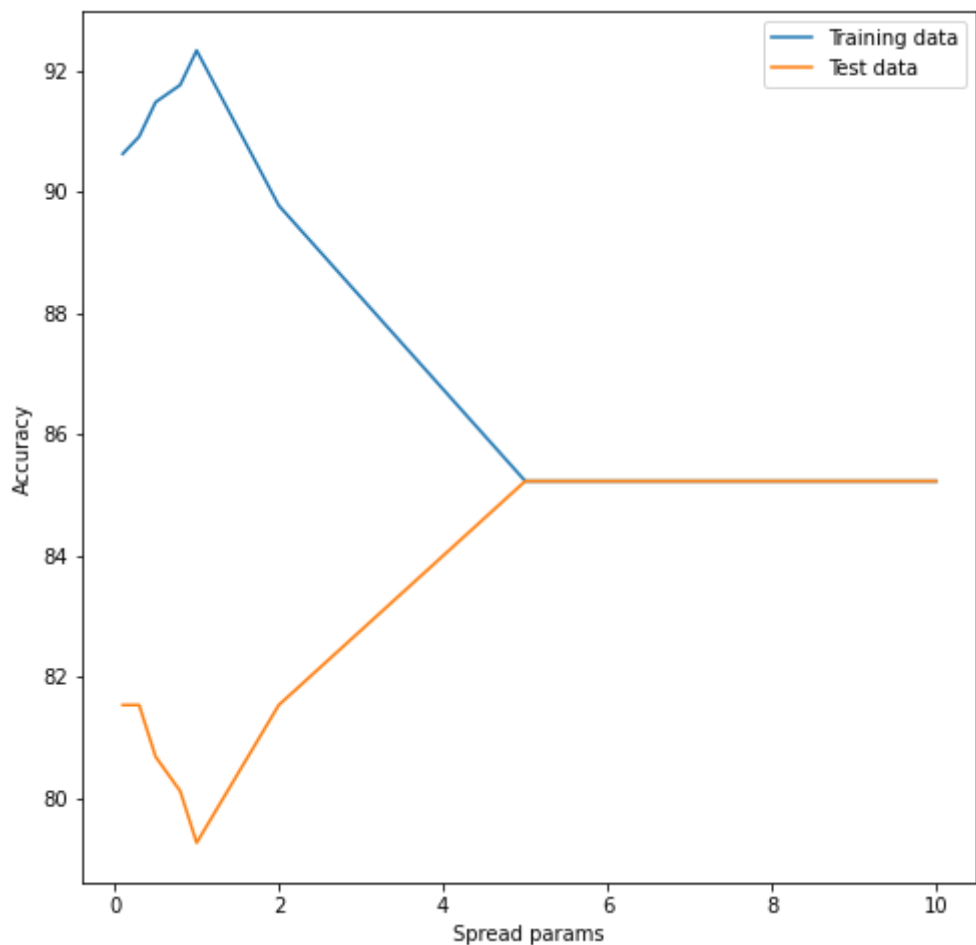
-----  
Accuracy of model 85.22727272727273  
Cumulative mean square error 0.3583156561192663

.....  
Performance report on test set

-----  
Accuracy of model 85.22727272727273  
Cumulative mean square error 0.17819300769123253

## Behaviour of model with increase of spread param in train and test sets

```
In [41]: plotModelPerformance(trainAccuracies=trainAccuracies,  
                             testAccuracies=testAccuracies,  
                             spreadParams=spreadParams)
```



[2b]

1. Using Gaussian kernel functions
2. Spread parameter is same for all kernel functions
3. Considering 150 data points as the centers from K-means clustering algorithm

```
In [43]: from sklearn.cluster import KMeans
```

```
In [44]: kmeans = KMeans(n_clusters=150, random_state=0).fit(trainX)
         clusterCenters = kmeans.cluster_centers_
```

### Case study: Using spread param = 0.1

```
In [45]: width = 0.1
         trainAccuracies = []
         testAccuracies = []
         spreadParams = []
         networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
         print('Performance report on training set \n -----')
         trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                                         spreadParam=width, weights=networkWeights))
         print('\n.....\nPerformance report on test set \n -----')
         testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                                       spreadParam=width, weights=networkWeights))
         spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 92.89772727272727

Cumulative mean square error 0.500661650224841

.....

Performance report on test set

-----

Accuracy of model 80.96590909090909

Cumulative mean square error 0.08278602445802938

### Case study: Using spread param = 0.3

```
In [46]: width = 0.3
         networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
         print('Performance report on training set \n -----')
         trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                                         spreadParam=width, weights=networkWeights))
         print('\n.....\nPerformance report on test set \n -----')
         testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                                       spreadParam=width, weights=networkWeights))
         spreadParams.append(width)
```

Performance report on training set

-----

Accuracy of model 90.625

Cumulative mean square error 0.49786142900410774

.....

Performance report on test set

-----

Accuracy of model 81.81818181818183

Cumulative mean square error 0.027355780334170122

### Case study: Using spread param = 0.5

```
In [47]: width = 0.5
         networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
         print('Performance report on training set \n -----')
         trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                                         spreadParam=width, weights=networkWeights))
         print('\n.....\nPerformance report on test set \n -----')
         testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                                       spreadParam=width, weights=networkWeights))
         spreadParams.append(width)
```

```

Performance report on training set
-----
Accuracy of model 91.19318181818183
Cumulative mean square error 0.4925999714513436

.....
Performance report on test set
-----
Accuracy of model 80.68181818181817
Cumulative mean square error 0.030450815840545268

```

### Case study: Using spread param = 1

In [48]:

```

width = 1
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)

```

```

Performance report on training set
-----
Accuracy of model 92.32954545454545
Cumulative mean square error 0.4764772848778543

.....
Performance report on test set
-----
Accuracy of model 79.82954545454545
Cumulative mean square error 0.04565299163418663

```

### Case study: Using spread param = 5

In [49]:

```

width = 5
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                               spreadParam=width, weights=networkWeights))
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                              spreadParam=width, weights=networkWeights))
spreadParams.append(width)

```

```

Performance report on training set
-----
Accuracy of model 85.22727272727273
Cumulative mean square error 0.42000175738907797

.....
Performance report on test set
-----
Accuracy of model 85.22727272727273
Cumulative mean square error 0.1027067887097561

```

### Case study: Using spread param = 10

In [50]:

```

width = 10
networkWeights = trainRBFNetwork(spreadParam=width, centres = clusterCenters)
print('Performance report on training set \n -----')
trainAccuracies.append(predict(trainX, trainY, centers=clusterCenters,
                               spreadParam=width, weights=networkWeights))

```

```
print('\n.....\nPerformance report on test set \n -----')
testAccuracies.append(predict(testX, testY, centers=clusterCenters,
                             spreadParam=width, weights=networkWeights))
spreadParams.append(width)
```

Performance report on training set

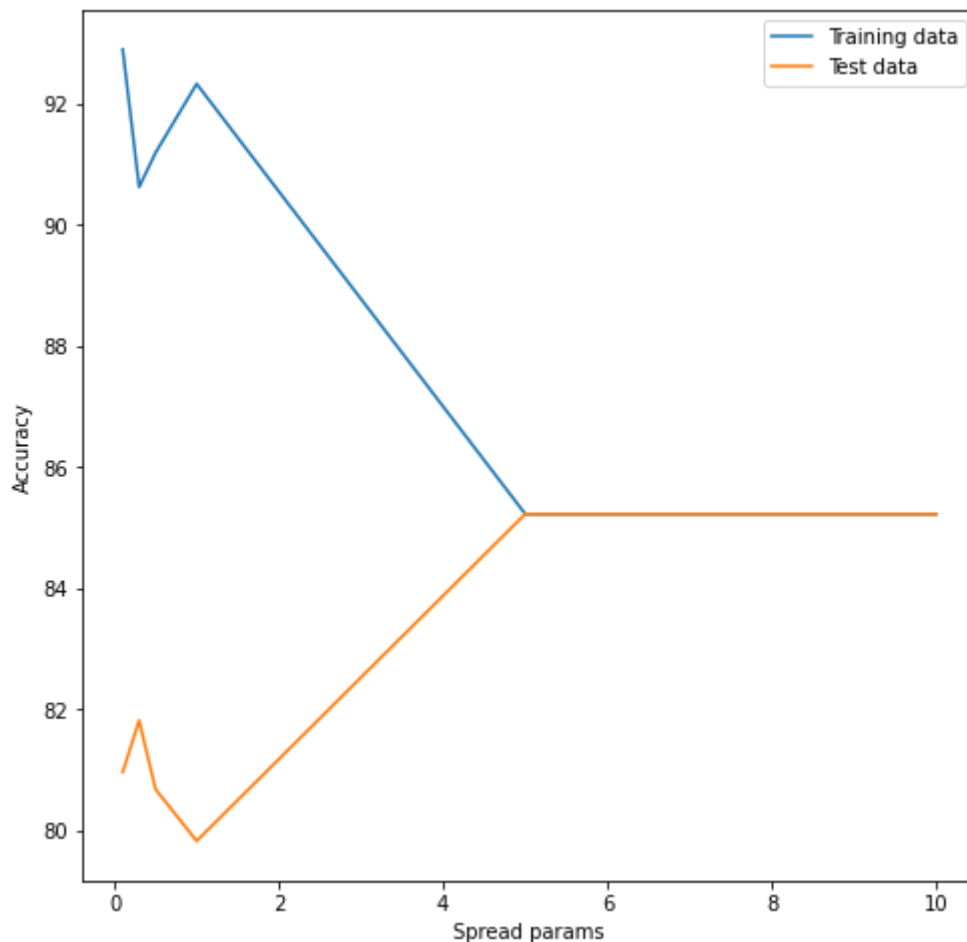
-----  
 Accuracy of model 85.22727272727273  
 Cumulative mean square error 0.358326371657712

.....  
 Performance report on test set

-----  
 Accuracy of model 85.22727272727273  
 Cumulative mean square error 0.17819308800385728

## Behaviour of model with increase of spread param in train and test sets

```
In [51]: plotModelPerformance(trainAccuracies=trainAccuracies,
                             testAccuracies=testAccuracies,
                             spreadParams=spreadParams)
```



## Takeaway

1. When the spread parameter is less in the range of 0-4,

the model fails to give a proper interpolation for the unseen data,

hence giving better accuracy for train set over test set.

2. A spread value above 5 gives a better result