In[14]:=
```
(* Define the prime counting function π(x) *)
primePiFunction[x_] := PrimePi[x]


(* Precompute π(x/e) and π(x/e^2) to optimize the function hFunction *)
precomputePrimePi[x_] := Module[{e = E},
 {primePiFunction[x], primePiFunction[x / e], primePiFunction[x / (e ^ 2)]}
]


(* Define the function \mathcal{H}(x) using precomputed π values *)
hFunction[x_] := Module[{e = E, logX = Log[x], πValues},
 πValues = precomputePrimePi[x];
 πValues[[1]] ^ 3 - (3 e x / logX) πValues[[2]] ^ 2 + (3 e ^ 2 x / logX ^ 2) πValues[[3]]
]


(* Evaluate the function for x = 10^k, where 4 ≤ k ≤ 7 *)
results1 = Table[{10 ^ k, N[hFunction[10 ^ k]]}, {k, 4, 7}]
```

Out[17]=
$$\{\{10\,000, -4.82295\times10^{8}\}, \{100\,000, -1.95356\times10^{11}\}, \{1\,000\,000, -9.74267\times10^{13}\}, \{10\,000\,000, -5.37332\times10^{16}\}\}$$

Out[12]=
$$\{\{10\,000, -4.82295\times10^{8}\}, \{100\,000, -1.95356\times10^{11}\}, \{1\,000\,000, -9.74267\times10^{13}\}, \{10\,000\,000, -5.37332\times10^{16}\}\}$$

In[21]:=
```
(* Evaluate the function for x = 10^k, where 8 ≤ k ≤ 11 *)
results2 = Table[{10 ^ k, N[hFunction[10 ^ k]]}, {k, 8, 11}]
```

Out[13]=
$$\{\{100\,000\,000, -3.27769\times10^{19}\}, \{1\,000\,000\,000, -2.1425\times10^{22}\}, \{10\,000\,000\,000, -1.47382\times10^{25}\}, \{100\,000\,000\,000, -1.05557\times10^{28}\}\}$$

In[19]:=
```
(* Evaluate the function for x = 10^k, where 12 ≤ k ≤ 14 *)
results3 = Table[{10 ^ k, N[hFunction[10 ^ k]]}, {k, 12, 14}]
```

Out[19]=
$$\{\{1\,000\,000\,000\,000, -7.81095\times10^{30}\}, \{10\,000\,000\,000\,000, -5.93755\times10^{33}\}, \{100\,000\,000\,000\,000, -4.61633\times10^{36}\}\}$$

In[18]:=
```
(* Evaluate the function for x = 10^k, where 15 ≤ k ≤ 16 *)
results5 = Table[{10^k, N[hFunction[10^k]]}, {k, 15, 16}]
```

Out[18]=
$$\{\{1\,000\,000\,000\,000\,000, -3.65848 \times 10^{39}\}, \{10\,000\,000\,000\,000\,000, -2.9475 \times 10^{42}\}\}$$

In[20]:=
```
(* Evaluate the function for x = 10^k, where 17 ≤ k ≤ 18 *)
results6 = Table[{10^k, N[hFunction[10^k]]}, {k, 17, 18}]
```

Out[20]=
$$\{\{100\,000\,000\,000\,000\,000, -2.40891 \times 10^{45}\}, \{1\,000\,000\,000\,000\,000\,000, -1.99359 \times 10^{48}\}\}$$