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In[40]:= (* Define the prime counting function  $\pi(x)$  *)
primePiFunction[x_] := PrimePi[x]

(* Precompute  $\pi(x/e)$ ,  $\pi(x/e^2)$ , and  $\pi(x/e^3)$  to optimize the function kFunction *)
precomputePrimePi[x_] := Module[{e = E},
  {primePiFunction[x], primePiFunction[x/e],
   primePiFunction[x/(e^2)], primePiFunction[x/(e^3)]}
]

(* Define the function  $\mathcal{K}(x)$  using precomputed  $\pi$  values *)
kFunction[x_] := Module[{e = E, logX = Log[x],  $\pi$ Values},
   $\pi$ Values = precomputePrimePi[x];
   $\pi$ Values[[1]]^4 - (4 e x / logX)  $\pi$ Values[[2]]^3 +
    (6 e^2 x / logX^2)  $\pi$ Values[[3]]^2 - (4 e^3 x / logX^3)  $\pi$ Values[[4]]
]

(* Evaluate the function for  $x = 10^k$ , where  $4 \leq k \leq 7$  *)
resultsK1 = Table[{10^k, N[kFunction[10^k]]}, {k, 4, 7}]

Out[43]=
{{10 000, 6.7855  $\times 10^{11}$ }, {100 000, 2.85871  $\times 10^{15}$ },
 {1 000 000, 1.36574  $\times 10^{19}$ }, {10 000 000, 7.37684  $\times 10^{22}$ }}

In[45]:= (* Evaluate the function for  $x = 10^k$ , where  $8 \leq k \leq 11$  *)
resultsK2 = Table[{10^k, N[kFunction[10^k]]}, {k, 8, 11}]

Out[45]=
{{100 000 000, 4.2993  $\times 10^{26}$ }, {1 000 000 000, 2.6665  $\times 10^{30}$ },
 {10 000 000 000, 1.73943  $\times 10^{34}$ }, {100 000 000 000, 1.18212  $\times 10^{38}$ }}

In[47]:= (* Evaluate the function for  $x = 10^k$ , where  $12 \leq k \leq 14$  *)
resultsK3 = Table[{10^k, N[kFunction[10^k]]}, {k, 12, 14}]

Out[47]=
{{1 000 000 000 000, 8.31051  $\times 10^{41}$ },
 {10 000 000 000 000, 6.01092  $\times 10^{45}$ }, {100 000 000 000 000, 4.45417  $\times 10^{49}$ }}

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In[48]:= (* Evaluate the function for x = 10^k, where 15 ≤ k ≤ 16 *)
resultsK4 = Table[{10^k, N[kFunction[10^k]]}, {k, 15, 16}]
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Out[48]:= {{1 000 000 000 000 000 000, 3.37014 × 1053}, {10 000 000 000 000 000, 2.59663 × 1057}}
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In[49]:= (* Evaluate the function for x = 10^k, where k = 17 *)
resultK17 = {10^17, N[kFunction[10^17]]}
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Out[50]:= {100 000 000 000 000 000 000, 2.03278 × 1061}
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In[51]:=
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