

```
In[54]:= (* Define the prime counting function  $\pi(x)$  *)
primePiFunction[x_] := PrimePi[x]
```

```
(* Precompute  $\pi(x/k)$ ,  $\pi(x/(ek))$ , and  $\pi(x/(e^2k))$  for  $k = 1, 2, 3, 4, 5$  to optimize the function n3Function *)
precomputePrimePi[x_] := Module[{e = E, values1, values2, values3},
  values1 = Table[primePiFunction[x/k], {k, 1, 5}];
  values2 = Table[primePiFunction[x/(e*k)], {k, 1, 5}];
  values3 = Table[primePiFunction[x/(e^2*k)], {k, 1, 5}];
  {values1, values2, values3}
]
```

```
(* Define the function  $\mathcal{N}_3(x)$  using precomputed  $\pi$  values *)
n3Function[x_] := Module[{e = E, logX = Log[x],  $\pi$ Values1,  $\pi$ Values2,  $\pi$ Values3},
  { $\pi$ Values1,  $\pi$ Values2,  $\pi$ Values3} = precomputePrimePi[x];
  (Total[ $\pi$ Values1]^3) - (e x / logX) (Total[ $\pi$ Values2]^3) + (Total[ $\pi$ Values3]^3)
]
```

```
(* Evaluate the function for  $x = 10^m$ , where  $4 \leq m \leq 15$  *)
resultsN3 = Table[{10^m, N[n3Function[10^m]]}, {m, 4, 15}]
```

Out[61]=

```
{{10 000, -6.20482  $\times 10^{12}$ }, {100 000, -2.05389  $\times 10^{16}$ }, {1 000 000, -8.54031  $\times 10^{19}$ },
{10 000 000, -4.14692  $\times 10^{23}$ }, {100 000 000, -2.25025  $\times 10^{27}$ }, {1 000 000 000, -1.32491  $\times 10^{31}$ },
{10 000 000 000, -8.30409  $\times 10^{34}$ }, {100 000 000 000, -5.46741  $\times 10^{38}$ },
{1 000 000 000 000, -3.746  $\times 10^{42}$ }, {10 000 000 000 000, -2.65231  $\times 10^{46}$ },
{100 000 000 000 000, -1.93044  $\times 10^{50}$ }, {1 000 000 000 000 000, -1.43841  $\times 10^{54}$ }}
```