In[66]:=
```
(* Define the prime counting function π(x) *)
primePiFunction[x_] := PrimePi[x]


(* Precompute π(x), π(x/e), and π(x/e^2) to optimize the function hNFunction *)
precomputePrimePi[x_] := Module[{e = E},
 {primePiFunction[x], primePiFunction[x / e], primePiFunction[x / (e ^ 2)]}
]


(* Define the function \mathcal{H}_n(x) for n=2 *)
h2Function[x_] := Module[{e = E, logX = Log[x], πValues},
 πValues = precomputePrimePi[x];
 (πValues[[1]] ^ 9) + (3 e ^ 2 x / logX ^ 2) (πValues[[3]] ^ 7) - (3 e x / logX) (πValues[[2]] ^ 8)
]


(* Define the function \mathcal{H}_n(x) for n=3 *)
h3Function[x_] := Module[{e = E, logX = Log[x], πValues},
 πValues = precomputePrimePi[x];
 (πValues[[1]] ^ 27) + (3 e ^ 2 x / logX ^ 2) (πValues[[3]] ^ 25) - (3 e x / logX) (πValues[[2]] ^ 26)
]


(* Evaluate the function for x = 10^m, where 4 ≤ m ≤ 15 for both n=2 and n=3 *)
resultsH2 = Table[{10 ^ m, N[h2Function[10 ^ m]]}, {m, 4, 15}]
resultsH3 = Table[{10 ^ m, N[h3Function[10 ^ m]]}, {m, 4, 15}]
```

Out[70]=
$$\{\{10\,000,\ 6.35373\times10^{27}\},\ \{100\,000,\ 6.83559\times10^{35}\},\ \{1\,000\,000,\ 1.12614\times10^{44}\},$$
$$\{10\,000\,000,\ 2.5176\times10^{52}\},\ \{100\,000\,000,\ 6.966\times10^{60}\},\ \{1\,000\,000\,000,\ 2.26314\times10^{69}\},$$
$$\{10\,000\,000\,000,\ 8.33495\times10^{77}\},\ \{100\,000\,000\,000,\ 3.39336\times10^{86}\},$$
$$\{1\,000\,000\,000\,000,\ 1.49953\times10^{95}\},\ \{10\,000\,000\,000\,000,\ 7.09417\times10^{103}\},$$
$$\{100\,000\,000\,000\,000,\ 3.55538\times10^{112}\},\ \{1\,000\,000\,000\,000\,000,\ 1.87205\times10^{121}\}\}$$

Out[71]=

$$\{\{10\,000,\ 2.61759\times10^{83}\},\ \{100\,000,\ 3.24749\times10^{107}\},\ \{1\,000\,000,\ 1.44938\times10^{132}\},$$
$$\{10\,000\,000,\ 1.61718\times10^{157}\},\ \{100\,000\,000,\ 3.4227\times10^{182}\},\ \{1\,000\,000\,000,\ 1.17297\times10^{208}\},$$
$$\{10\,000\,000\,000,\ 5.85681\times10^{233}\},\ \{100\,000\,000\,000,\ 3.95082\times10^{259}\},$$
$$\{1\,000\,000\,000\,000,\ 3.40831\times10^{285}\},\ \{10\,000\,000\,000\,000,\ 3.608074552069927\times10^{311}\},$$
$$\{100\,000\,000\,000\,000,\ 4.540919459707700\times10^{337}\},$$
$$\{1\,000\,000\,000\,000\,000,\ 6.627717169602305\times10^{363}\}\}$$