In[72]:=
```
(* Define the prime counting function π(x) *)
primePiFunction[x_] := PrimePi[x]


(* Precompute π(x/k), π(x/(ek)) and log(x/k), log(x/(ek)) for k = 1,
2, 3, 4, 5 to optimize the function fFunction *)
precomputePrimePiAndLog[x_] := Module[{e = E, values1, values2, logs1, logs2},
 values1 = Table[primePiFunction[x / k], {k, 1, 5}];
 logs1 = Table[Log[x / k], {k, 1, 5}];
 values2 = Table[primePiFunction[x / (e * k)], {k, 1, 5}];
 logs2 = Table[Log[x / (e * k)], {k, 1, 5}];
 {values1, logs1, values2, logs2}
]


(* Define the function \mathcal{F}(x) using precomputed π and log values *)
fFunction[x_] := Module[{e = E, logX = Log[x], πValues, logValues},
 {πValues, logValues} = precomputePrimePiAndLog[x][[1 ;; 2]];
 {πValuesE, logValuesE} = precomputePrimePiAndLog[x][[3 ;; 4]];
 (Total[πValues / logValues]^2) - (e x / logX) Total[πValuesE / logValuesE]
]


(* Evaluate the function for x = 10^m, where 4 ≤ m ≤ 16 *)
(* Evaluate the function for x = 10^m, where 4 ≤ m ≤ 7 *)
resultsF1 = Table[{10^m, N[fFunction[10^m]]}, {m, 4, 7}]
```

Out[75]=
$$\{\{10\,000, -377\,275.\}, \{100\,000, -1.83018\times10^{7}\}, \{1\,000\,000, -1.02039\times10^{9}\}, \{10\,000\,000, -6.2567\times10^{10}\}\}$$

In[76]:=
```
(* Evaluate the function for x = 10^m, where 8 ≤ m ≤ 11 *)
resultsF2 = Table[{10^m, N[fFunction[10^m]]}, {m, 8, 11}]
```

Out[76]=
$$\{\{100\,000\,000, -4.11092\times10^{12}\}, \{1\,000\,000\,000, -2.84512\times10^{14}\}, \{10\,000\,000\,000, -2.05049\times10^{16}\}, \{100\,000\,000\,000, -1.5265\times10^{18}\}\}$$

In[77]:=
```
(* Evaluate the function for x = 10^m, where 12 ≤ m ≤ 14 *)
resultsF3 = Table[{10^m, N[fFunction[10^m]]}, {m, 12, 14}]
```

Out[77]=
$$\{\{1\,000\,000\,000\,000, -1.16701 \times 10^{20}\},\ \{10\,000\,000\,000\,000, -9.12168 \times 10^{21}\}, \{100\,000\,000\,000\,000, -7.26483 \times 10^{23}\}\}$$