HOMEWORK 3

(1) Are the rationals $\mathbb{Q}$ discrete in $\mathbb{R}$?

(2) Let X be a topological space, let Y be a subspace of X, and let A be a subset of Y. Then show that the subspace topology A inherits from Y is equal to the subspace topology it inherits from X.

(3) Let $(X, \mathscr{T}_X)$ and $(Y, \mathscr{T}_Y)$ be topological spaces with $\mathcal{B}_X$ and $\mathcal{B}_Y$ bases on X and Y respectively, such that they generate the respective topologies on X and Y. Then show that

$$\mathcal{B} := \{U \times V : U \in \mathcal{B}_X, V \in \mathcal{B}_Y\}$$

is a basis for the product topology on $X \times Y$.

(4) Check the gluing property for the basis defined in class for the order topology.

(5) Let X be a linearly ordered set with $\mathscr{T}_X$ being the order topology, and let Y be a convex subset of X. Then show that the order topology on Y obtained by restriction of the order relation $<$ is the same as the subspace topology on Y inherited from X.

(6) Show that a subset of S_Ω is countable if and only if it is bounded.

(7) Show that S_Ω is not discrete in its order topology.

(8) Topology (Munkres), Chapter 2, Section 16, Exercise (5).

(9) Topology (Munkres), Chapter 2, Section 16, Exercise (7).