HOMEWORK 26

Note: $*$ marked problems might be slightly more difficult or interesting than the unmarked ones.

(1) Let X be the subspace of $\mathbb{R}^2$ consisting of the horizontal segment $[0,1] \times \{0\}$ together with all the vertical segments $\{r\} \times [0, 1-r]$ for r a rational number r in the interval $[0,1]$. Show that X deformation retracts to any point $(a, 0)$ for $a \in [0,1]$, but not to any other point.

(2) Show that a homotopy equivalence $f : X \to Y$ induces a bijection between the set of path components of X and the set of path components of Y, and that f restricts to a homotopy equivalence from each path component of X to the corresponding path component of Y.

(3) Show that any two deformation retractions r_t^0 and r_t^1 of a space X onto a subspace A can be joined by a continuous family of deformations retractions $r_t^s, 0 \leq s \leq 1$, of X onto A, where continuity means that the map $X \times I \times I \to X$ sending (x, s, t) to $r_t^s(x)$ is continuous.

(4) Let $X \subset \mathbb{R}^m$ be the union of convex open sets $X_1, X_2, \ldots, X_n$ such that $X_i \cap X_j \cap X_k \neq \varnothing$ for all i, j, k. Show that X is simply connected.

(5) Show that there does not exist a covering map from $\mathbb{RP}^2$ to the torus.

(6) Topology (Munkres), Chapter 9, Section 59, Exercise (4).

(7) Topology (Munkres), Chapter 9, Section 60, Exercise (5).