HOMEWORK 1

(1) Prove using the epsilon-delta definition of continuity and/or the concept of limits, that the function g from $[0,1] \cup [2,3]$ to $\mathbb{R}$ defined by $g([0,1]) = 0$ and $g([2,3]) = 1$ is a continuous function.

(2) Show explicitly that (x, ∞) is an open set in $(\mathbb{R}, \mathscr{T}_{\mathbb{R}})$ for any real number x. Prove explicitly that $[a, b)$ is not open in $(\mathbb{R}, \mathscr{T}_{\mathbb{R}})$.

(3) When X is an infinite set, show that the finite complement topology $\mathscr{T}_f$ is strictly coarser than the discrete topology $\mathscr{T}_{\text{disc}}$, i.e. $\mathscr{T}_f \subsetneq \mathscr{T}_{\text{disc}}$.

(4) What can you say about the standard topology $\mathscr{T}_{\mathbb{R}}$ on $\mathbb{R}$ and the ray+ topology $\mathscr{T}_{\text{ray+}}$ on $\mathbb{R}$? Is one of them coarser or finer than the other? Justify your assertion.

(5) Under what assumptions on the set X is the finite complement topology $(X, \mathscr{T}_f)$ equal to the countable complement topology $(X, \mathscr{T}_c)$?

(6) Topology (Munkres), Chapter 1, Section 7, Exercise 4.

(7) Topology (Munkres), Chapter 1, Section 7, Exercise 6.

(8) Topology (Munkres), Chapter 1, Section 10, Exercise 6.

(9) Topology (Munkres), Chapter 2, Section 13, Exercise 6. (Read the definition of $\mathbb{R}_K$ from Section 13 for this problem.)