HOMEWORK 12

(1) Show that S_Ω is not compact.

(2) Show that Heine-Borel theorem is not necessarily true for a general metric space X.

(3) Does there exist a metric d on $\mathbb{R}^n$ such that it induces the standard topology but Heine-Borel Theorem is not true in it?

(4) Show that the function $x \to d(x, A)$ is continuous.

(5) Let X be a nonempty compact Hausdorff space. If X has no isolated point, i.e. no point x such that $\{x\}$ is open, then show that X is uncountable.

(6) Complete the proof that if X is metrizable, it is compact if and only if sequentially compact if and only if limit point compact.

(7) Let X be a Hausdorff space. Then show that X is locally compact if and only if for each point $x \in X$ and each neighborhood U of x there is a neighborhood V of x such that its closure Cl V is compact and is contained in U :

$$x \in V \subseteq \text{Cl } V \subseteq U.$$

(8) Let X be locally compact Hausdorff. If $A \subseteq X$ is an open or closed subspace, then show that A is locally compact.

(9) Topology (Munkres), Chapter 3, Section 27, Exercise (2).

(10) Topology (Munkres), Chapter 3, Section 27, Exercise (4).

(11) Topology (Munkres), Chapter 3, Section 27, Exercise (6).

(12) Topology (Munkres), Chapter 3, Section 28, Exercise (2).

(13) Topology (Munkres), Chapter 3, Section 28, Exercise (6). Does the result still hold if X is not assumed to be compact?

(14) Topology (Munkres), Chapter 3, Section 29, Exercise (1).

(15) Topology (Munkres), Chapter 3, Section 29, Exercise (3).