# HOMEWORK 13

**Note:** $*$ marked problems might be slightly more difficult or interesting than the unmarked ones.

(1) Show that a locally compact Hausdorff space $X$ is dense as a subspace of its one-point compactification $Y$.

(2) Show that the construction of one-point compactification carried for the space $X = \mathbb{Q}$ produces $Y$ that is not Hausdorff.

(3) Show that a space $X$ is homeomorphic to an open subspace of a compact Hausdorff space if and only if $X$ is locally compact Hausdorff.

(4)$^*$ Let $f : X \to Y$ be a continuous map between $X$ and $Y$ both locally compact Hausdorff spaces. Then let $f^* : X^* \to Y^*$ denote the one-point extension of $f$ between one-point compactifications $X^*$ and $Y^*$ of $X$ and $Y$ respectively, which is defined as $f^*(\infty_X) = \infty_Y$, and $f^*|_X = f$. Show that $f^*$ is continuous if and only if $f$ is proper.

(5)$^*$ Let $f : X \to Y$ be a continuous map and let $X$ be a Hausdorff space and $Y$ be a locally compact Hausdorff space. Then show that $f$ is proper if and only if $f$ is closed and for each $y \in Y$ the set $f^{-1}(y)$ is compact.

(6) Let $f : X \to Y$ and $g : Y \to Z$ be continuous. Show that :

(1) If $f$ and $g$ are proper, then $g \circ f$ is also proper.
(2) If $Y$ is Hausdorff, and $g \circ f$ is proper, then $f$ is proper.
(3) If $f$ is onto, and $g \circ f$ is proper, then $g$ is proper.

(7)$^*$ Show that $S_\Omega$ is locally compact. Show that $\tilde{S}_\Omega$ is homeomorphic to its one-point compactification.

(8)$^*$ Topology (Munkres), Chapter 3, Section 29, Exercise (4).

(9) Topology (Munkres), Chapter 3, Section 29, Exercise (8).

(10) Topology (Munkres), Chapter 3, Section 29, Exercise (11).