HOMEWORK 14

Note: $*$ marked problems might be slightly more difficult or interesting than the unmarked ones.

(1) Show that any subspace of a regular space X is regular. Show that any product of regular spaces X_α is regular.

(2) Show that if X be a T_1 space, then X is normal if and only if for each closed subset $A \subset X$ and neighborhood W of x there is a neighborhood U containing A such that $A \subseteq U \subseteq \text{Cl } U \subseteq W$. (Hint: Mimic what we did in class for a similar result for regular spaces.)

(3) Show that a closed subspace of a normal space is normal.

(4)* Show that the space $\mathbb{R}_\ell$ is first countable, separable, and Lindelöf, but it is not second countable.

(5) Show that a finite product of two Lindelöf spaces need not be Lindelöf. (Hint : Look at $\mathbb{R}_\ell^2$.)

(6) Show that every compact metrizable space is second countable.

(7) Topology (Munkres), Chapter 4, Section 30, Exercise (5).

(8)* Topology (Munkres), Chapter 4, Section 30, Exercise (6).

(9) Topology (Munkres), Chapter 4, Section 31, Exercise (3).

(10)* Topology (Munkres), Chapter 4, Section 31, Exercise (6).

(11) Topology (Munkres), Chapter 4, Section 32, Exercise (3).