## HOMEWORK 2

(1) Show that the two definitions $\mathscr{T}_{\mathcal{B}}$ and $\mathscr{T}'_{\mathcal{B}}$ of topologies generated by a basis $\mathcal{B}$ are equivalent to each other.

(2) Complete the proof that the basis $\mathcal{C}$ generates the same topology as $\mathscr{T}_X$ under given conditions. (We have checked in the notes that this is actually a basis.)

(3) Let $X$ be the set of all functions $f(x)$ from $[0,1]$ to $[0,1]$. Given any subset $A \subseteq [0,1]$, define

$$B_A := \{f(x) : f(x) = 0 \text{ for all } x \in A\}.$$

This is a subset of $X$ for each $A$. Then show that $\mathcal{B} := \{B_A : A \subseteq [0,1]\}$ is a basis on $X$.

(4)($a$) Let $d, r \in \mathbb{Z}$ with $d \neq 0$. Define $B_{d,r} := \{dn + r : n \in \mathbb{Z}\}$. Show that the collection

$$\mathcal{B} := \{B_{d,r}\}_{d,r}$$

is a basis on $\mathbb{Z}$.

($b$) Show that no nonempty finite set can be open in the topology generated by $\mathcal{B}$.

($c$) Show that $\mathbb{Z} \setminus B_{d,r}$ is open for any basic open set $B_{d,r}$.

($d$) Show that for any distinct integers $a$ and $b$, there are disjoint open sets $U$ and $V$ such that $a \in U$ and $b \in V$.

(5) For a prime $p$, define $S_p := \{n \in \mathbb{N} : p|n\}$. Show that

$$\mathcal{S} := \{S_p : p \text{ prime}\} \cup \{1\}$$

is a subbasis on $\mathbb{N}$. How do the open sets in the topology generated by $\mathcal{S}$ look like?

(6) Topology (Munkres), Chapter 2, Section 13, Exercise (7).

(7) Topology (Munkres), Chapter 2, Section 13, Exercise (8).