HOMEWORK 7

(1) Check that $\bar{d}$ is a metric.

(2) Let d and d' be two metrics on the set X. Let $\mathscr{T}$ and $\mathscr{T}'$ be the topologies they induce respectively. Then show that $\mathscr{T}'$ is finer than $\mathscr{T}$ if and only if for each $x \in X$ and each $\epsilon > 0$, there exists a $\delta > 0$ such that

$$B_{\delta,d'}(x) \subseteq B_{\epsilon,d}(x)$$

where the d and d' in the subscript denote the δ-balls or ϵ-balls in those metrics respectively.

(3) Show that the pointwise convergence topology on functions $\{f : \mathbb{R} \to \mathbb{R}\}$ is not metrizable.

(4) Topology (Munkres), Chapter 2, Section 20, Exercise (4).

(5) Topology (Munkres), Chapter 2, Section 20, Exercise (6).

(6) Topology (Munkres), Chapter 2, Section 20, Exercise (10).

(7) Topology (Munkres), Chapter 2, Section 20, Exercise (8).

(8) Topology (Munkres), Chapter 2, Section 20, Exercise (11).

(9) Topology (Munkres), Chapter 2, Section 21, Exercise (4).

(10) Topology (Munkres), Chapter 2, Section 21, Exercise (7).

(11) Topology (Munkres), Chapter 2, Section 21, Exercise (8).

(12) Topology (Munkres), Chapter 2, Section 21, Exercise (12).