National University of Computer and Emerging Sciences, Karachi

FAST School of Computing

Midterm 1 Examination, Fall 2022

September 28, 2022, 08:30 am – 09:30 am

Course Code: CS1005	Course Name: Discrete Structures
Instructor Names: Mr. Shoaib Raza, Ms. Bakhtawer, Ms. Safia, Ms. Fizza Aqeel, Mr. Fahad Hussain and Mr. Sudais	
Student Roll No:	Section No:

Instructions:

- Return the question paper along with the answer script. Read each question completely before answering it. There are **3 questions and 2 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Answering all the questions in given sequence of the question paper.

Total Time: 60 minutes **Maximum Points**: 24

Question # 1 (Propositional Logic and Rules of Inference) **[CLO-3 C3]**

(a) Let P, Q, and R be the propositions. [2 Points]

P: Niagara Falls is in New York. Q: New York City is the capital state of United State.

R: New York City will have more snow in 2050.

Write these propositions using P, Q, and R and logical connectives (including negations):

(i) If Niagara Falls is in New York, New York City will not have more snow in 2050.

(ii) Neither Niagara Falls is in New York nor will New York City have more snow in 2050.

(iii) It is not the case that New York City is not the state capital of the United States.

(iv) New York City will not have more snow in 2050 only if New York City is not the state capital of the United States.

(b) Using the truth table, prove or disprove that the contrapositive of statement (i) in part (a) is equivalent to the converse of its inverse. [2 Points]

(c) Using the premises (statements) from part (a), apply rules of inference to obtain conclusion(s). [2 Points]

(d) Using laws of Logic, determine if the following statement is a tautology, contradiction or a contingency. [2 Points]

$$\big((P \vee Q) \wedge (P \rightarrow R)\big) \rightarrow (Q \vee R)$$

Question # 2 (Predicates and Quantifiers) **[CLO-2 C2]**

(a) Let F(x, y) means "x + y = 1", where 'x' and 'y' are integers. Determine the truth value of the following statement. [2 Points]

(i) $\forall x \exists y\, F(x, y)$

(ii) $\exists x \forall y\, F(x, y)$

(b) Translate each of the following statements into logical expressions using predicates, quantifiers, and logical connectives where C(x) is "x is a comedian" and F(x) is "x is funny" and the domain consists of all people. [2 Points]

(i) All comedians are funny.

(ii) Some comedians are funny.

(c) Translate each of the following statements into English where P(x) is "x is a professor,", Q(x) is "x is ignorant,", and R(x) is "x is vain," and the domain consists of all people. [2 Points]

(i) $\neg\exists x\, (P(x) \wedge Q(x))$

(ii) $\forall x\, (Q(x) \rightarrow R(x))$

Question # 3 (Set Theory and Functions) **[CLO-2 C2]**

(a) Out of 40 students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes. Using a Venn diagram, determine how many students are in either class and how many are in neither of the classes? [2 Points]

(b) Using Set identities, prove or disprove that $\overline{\overline{A} \cap \overline{B}} \cup B = \bar{A} \cup B$ [2 Points]

(c) Suppose f: Z→Z where f (m, n) = $x^3 + 1$. Determine whether the function is an onto (surjective) and/or a one-to-one (injective) or both (bijective). [2 Points]

(d) Given f(x) = x^3 + 18 and g(x) = 4x + 1, find (f o g) (x). [2 Points]

(e) Prove or disprove the statement $\lceil -x \rceil = -\lfloor x \rfloor$ for real number x. [2 Points]

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ALL THE BEST