



National University of Computer & Emerging Sciences, Karachi
FAST, School of Computing,
Fall 2022



Final

29th December, 2022, 8:30 am – 11:30pm

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| Course Code: MT 1006 | Course Name: Differential Equations |
| Instructor Names: Ms. Amber Shaikh and Ms. Alishba Tariq | |
| Student Roll No: | Section : |

Instructions:

- Solve all questions and return the question paper.
- Read each question completely before answering it.
- This paper consists of 3 pages and 7 Questions.

Time: 180 minutes.

Total Marks:100

Qno1 –

CLO 3

10 Marks

Short Questions.

I) What is the dimension of given partial differential equation.

$$\frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial t^2} = 0$$

II) Write a nonlinear and non-homogeneous partial differential equation.

III) What is the Fundamental period of $\cos 2x + \sin 3x$.

IV) Identify the given PDE as elliptic, hyperbolic or parabolic PDE.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$$

V) Can the given PDE be solved by variable separable method. (Yes/ NO)

$$\alpha^2 \frac{\partial^2 u}{\partial x^2} - g - \frac{\partial^2 u}{\partial t^2} = 0, \text{ where } g \text{ and } \alpha \text{ are constants.}$$

VI) Write the condition of two functions $f_1(x)$ and $f_2(x)$ to be orthogonal on the interval $[a, b]$.

VII) Write the order of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$.

VIII) Period of $\sin 5x$ is given by.

IX) Draw the graph of function given in Question no 2.

X) Identify the nonlinear term in given PDE.

$$\frac{\partial^2 u}{\partial x^2} + xy \frac{\partial^2 u}{\partial x \partial y} + u \frac{\partial^2 u}{\partial y^2} = 0$$

Qno2 –**CLO 3****15 Marks**

Find Fourier Series of

$$f(x) = \begin{cases} 0, & -\pi/2 < x < 0 \\ \cos x, & 0 \leq x < \pi/2 \end{cases}$$

Qno3 –**CLO 3****15 Marks**

Find any 3 Solutions of given partial differential equation.

$$\frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$

Qno4 –**CLO 3****(10+5) Marks**

a) Find Maxima, Minima or Saddle point of given bivariate function.

$$f(x, y) = x^4 + y^4 - 4xy + 1$$

b) Find $\left. \frac{dz}{dt} \right|_{t=\pi}$ of given function by using chain rule.

$$z = \sqrt{x^2 + y^2 + 1}, \quad x = \ln t, \quad y = \cos t$$

Qno5-**CLO 3****(10+5) Marks**

a) Prove that given set of function is orthogonal.

$$\sin \left\{ \frac{n\pi x}{p} \right\}, n = 1, 2, 3, \dots; [0, p]$$

b) Find value of c for which given solution satisfy heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

$$u = e^{-t} \sin x$$

Qno6-**CLO 2****(12+3)Marks**

a) Find solution of given higher order differential equation.

$$y'' - 5y' + 4y = 8e^x + 5$$

b) Suppose $m_1 = 3, m_2 = -5$ and $m_3 = 1$ are roots of multiplicity one, two and three, respectively, of an auxiliary equation. Write down the general solution and the corresponding linear DE if it is,

I) an equation with constant coefficients

II) a Cauchy Euler equation

Solve the given first order differential equation

a) $\frac{dy}{dx} + y = f(x), y(0) = 0$ where $f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$

b) $(e^{2y} - y \cos xy)dx + (2xe^{2y} - x \cos xy + 2y)dy = 0$

END