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Assignment # 3

22K-4316 3F-BCS

1)

a) Quotient = 2, Remainder = 5

b) Quotient = -11, Remainder = 10

c) Quotient = 34, Remainder = 7

d) Quotient = 77, Remainder = 0

e) Quotient = 0, Remainder = 10

f) Quotient = 0, Remainder = 3

g) Quotient = -1, Remainder = 2

h) Quotient = 4, Remainder = 0

Question 2

a) i) a = -111, m = 99

a div m = -2, a mod m = 87

ii) a = -9999, m = 101

a div m = -99, a mod m = 0

iii) a = 10299, m = 999

a div m = 10, a mod m = 309

iv) a = 123456, m = 1001

a div m = 123, a mod m = 333

b)

(i) 80

~~$5 \equiv 5 \pmod{17}$, $80 \equiv 12 \pmod{17}$~~
~~80 is not congruent to 5 modulo 17~~

$80 \equiv 5 \pmod{17} \Rightarrow 80 - 5 = 75$

17 does not divide 75 so ~~75~~ 80 is not congruent to 5 modulo 17.

(ii) 103

$103 \equiv 5 \pmod{17} \Rightarrow 103 - 5 = 98$

98 ~~103~~ is not divided by 17, so 103 is not congruent to 5 modulo 17.

(iii) −29

$-29 \equiv 5 \pmod{17} \Rightarrow -29 - 5 = -34$

−34 is ~~not~~ divided by 17, so −29 is ~~not~~ congruent to 5 modulo 17.

(iv) −122

$-122 \equiv 5 \pmod{17} \Rightarrow -122 - 5 = -127$

−127 is not divided by 17 so,

−122 is not congruent to 5 modulo 17

Q3)

(i) 11, 15, 19	(ii) 14, 15, 21	(iii) 12, 17, 31, 37
gcd(11,15) = 1	gcd(14,15) = 1	gcd(12,17) = 1
gcd(15,19) = 1	gcd(15,21) = 3	gcd(17,31) = 1
gcd(11,19) = 1	gcd(14,21) = 7	gcd(31,37) = 1
		gcd(12,37) = 1
11, 15, 19 are pairwise relatively prime	14, 15, 21 are not pairwise relatively prime	12, 17, 31, 37 are pairwise relatively prime

(iv) 7, 8, 9, 11 → gcd(7,8) = 1, gcd(8,9) = 1, gcd(7,11) = 1, gcd(9,11) = 1 → 7, 8, 9, 11 are pairwise relatively prime.

(b)

(i) $88 = 2\times2\times2\times11 = 2^3\times11$

(ii) $126 = 2\times3\times3\times7 = 2\times3^2\times7$

(iii) $729 = 3\times\cancel{81}\times3\times\cancel{27}3\times\cancel{9}\times3\times3\times\cancel{3}^{3} = 3^6$

(iv) $1001 = \cancel{3\times}\, 7\times\cancel{13}\,11\times13\ \cancel{= 7\times11\times13}$

(v) $1111 = 11\times101$

(vi) $909 = 3\times\cancel{103}\,3\times101 = 3^2\times101$

Question 4

(144, 89)

$144 = 89\cdot1+55$

$89 = 55\cdot1+34$

$55 = 34\cdot1+21$

$34 = 21\cdot1+13$

$21 = 13\cdot1+8$

$13 = 8\cdot1+5$

$8 = 5\cdot1+3$

$5 = 3\cdot1+2$

$3 = 2\cdot1+1$

$2 = 1\cdot2+0$

Now working backwards:

$1 = 3 - 2\cdot 1$ — (i) | $5 = 13 - 8\cdot 1$ — (iv)

$2 = 5 - 3\cdot 1$ — (ii) | $8 = 21 - 13\cdot 1$ — (v)

$3 = 8 - 5\cdot 1$ — (iii) | $13 = 34 - 21\cdot 1$ — (vi)

$21 = 55 - 34\cdot 1$ — (vii)

$34 = 89 - 55\cdot 1$ — (viii)

$55 = 144 - 89\cdot 1$ — (ix)

Substituting (ii) eq into (i) yields:-

$\therefore$ (i) $\Rightarrow 1 = 3 - (5 - 3\cdot 1)\cdot 1 = 2\cdot 3 - 5\cdot 1$

substituting (iii)

$\Rightarrow 2(8 - 5\cdot 1) - 5\cdot 1 = 8\cdot 2 - 3\cdot 5$

Substituting (iv)

$\Rightarrow 8\cdot 2 - 3(13 - 8\cdot 1) = 5\cdot 8 - 3\cdot 13$

Substituting (v)

$\Rightarrow 5\cdot(21 - 13\cdot 1) - 3\cdot 13 = 5\cdot 21 - 8\cdot 13$

Substituting (vi)

$\Rightarrow 5\cdot 21 - 8(34 - 21\cdot 1) = 13\cdot 21 - 8\cdot 34$

Substituting (vii)

$\Rightarrow 13\cdot(55 - 34\cdot 1) - 8\cdot 34 = 13\cdot 55 - 21\cdot 34$

Substituting (viii)

$\Rightarrow 13\cdot 55 - 21\cdot(89 - 55\cdot 1) = 34\cdot 55 - 21\cdot 89$

Substituting (ix)

$\Rightarrow 34\cdot(144 - 89\cdot 1) - 21\cdot 89 \Rightarrow 1 = 34\cdot 144 - 55\cdot 89$

Assignment

$(1001, 100001)$

$\Rightarrow 100001 = 1001 \times 99 + 902$ ---(i)

$\Rightarrow 1001 = 902 \cdot 1 + 99$ ---(ii)

$\Rightarrow 902 = 99 \cdot 9 + 11$ ---(iii)

$\Rightarrow 99 = 11 \cdot 9 + 0$ ---(iv)

~~Substituting~~ working backwards

$\Rightarrow 11 = 902 - 99 \cdot 9$ — (i)

$\Rightarrow 99 = 1001 - 902 \cdot 1$ — (ii)

$\Rightarrow 902 = 100001 - 1001 \cdot 99$ — (iii)

Substituting (ii) in (i)

$\therefore$ (i) $\Rightarrow 11 = 902 - (1001 - 902 \cdot 1) \cdot 9$

$= 10 \cdot 902 - 9 \cdot 1001$

Substituting (iii)

$\Rightarrow 11 = 10 \cdot (100001 - 1001 \cdot 99) - 9 \cdot 1001$

$999 \cdot 1001$

$\Rightarrow 11 = 10 \cdot 100001 - \cancel{\ldots 1001} \cancel{+08 \cdot 1001}$

Q5

a) $55x \equiv 34 \pmod{89}$

$\gcd(55, 89) = 1$

$89 = 55 \cdot 1 + 34$

$55 = 34 \cdot 1 + 21$

$21 = 13 \cdot 1 + 8$

$13 = 8 \cdot 1 + 5$

$8 = 5 \cdot 1 + 3$

$3 = 2 \cdot 1 + 1 \Rightarrow 2 = 1 \cdot 2 + 0$

$1 = 3 - 2 \cdot 1$ — (i)

$2 = 5 - 3 \cdot 1$ — (ii)

$3 = 8 - 5 \cdot 1$ — (iii)

$5 = 13 - 8 \cdot 1$ — (iv)

$8 = 21 - 13 \cdot 1$ — (v)

$13 = 34 - 21 \cdot 1$ — (vi)

$21 = 55 - 34 \cdot 1$ — (vii)

$34 = 89 - 55 \cdot 1$ — (viii)

~~$55 = 144 - 89 \cdot 1$~~

Substituting eq-(ii) into eq-(i) yields:-

$\therefore$ (i) $\Rightarrow 1 = 3 - (5 - 3\cdot 1)\cdot 1 = 2\cdot 3 - 5\cdot 1$

Substituting (iii)

$\Rightarrow 2(8 - 5\cdot 1) - 5\cdot 1 = 8\cdot 2 - 3\cdot 5$

Substituting (iv)

$\Rightarrow 8\cdot 2 - 3(13 - 8\cdot 1) = 5\cdot 8 - 3\cdot 13$

Substituting (v)

$\Rightarrow 5(21 - 13\cdot 1) - 3\cdot 13 = 5\cdot 21 - 8\cdot 13$

Substituting (vi)

$\Rightarrow 5\cdot 21 - 8(34 - 21\cdot 1) = 13\cdot 21 - 8\cdot 34$

Substituting (vii)

$\Rightarrow 13(55 - 34\cdot 1) - 8\cdot 34 = 13\cdot 55 - 21\cdot 34$

Substituting (viii)

$\Rightarrow 13\cdot 55 - 21\cdot(89 - 55\cdot 1) = 34\cdot 55 - 21\cdot 89$

$\Rightarrow 1 = 34\cdot 55 - 21\cdot 89$

$\Rightarrow \bar{a} = 34$

Now, according to formula:-

$\cancel{x \equiv \bar{a}}$ $\therefore x \equiv \bar{a}b \pmod{m}$

$\therefore x \equiv 34\cdot 34 \pmod{89}$

$\therefore x \equiv 1156 \pmod{89} = 88$

$89x \equiv 2 \pmod{232}$

$\gcd(89, 232) = 1$

$232 = 89\cdot 2 + 54$

$89 = 54\cdot 1 + 35$

$54 = 35\cdot 1 + 19$

$35 = 19\cdot 1 + 16$

$19 = 16\cdot 1 + 3$

$16 = 3\cdot 5 + 1$

$3 = 1\cdot 3 + 0$

$1 = 16 - 3\cdot 5$

$3 = 19 - 16\cdot 1$

$16 = 35 - 19\cdot 1$

$19 = 54 - 35\cdot 1$

$35 = 89 - 54\cdot 1$

$54 = 232 - 89\cdot 2$

$1 = 16 - 5(3)$

$1 = 16 - 5(19 - 1\cdot 16)$

$1 = 35 - 1\cdot 19 - 5\cdot 19 + 5\cdot 16$

$1 = 35 - 6\cdot 19 + 5(35 - 1\cdot 19)$

$1 = 35 - 6\cdot 19 + 5\cdot 35 - 5\cdot 19$

$1 = \cancel{6(5}\,6\cdot 35 - 11\cdot 19$

$1 = 6(89 - 1\cdot 54) - 11(54 - 1(89 - 1\cdot 54))$

$1 = 6\cdot 89 - 6\cdot 54 - 11\cdot 54 + 11\cdot 89 - 11\cdot 54$

$1 = -2\overset{8}{7}\cdot 54 + 17\cdot 89$

$1 = -2\overset{8}{7}(232 - 89\cdot 2) + 17\cdot 89$

$1 = 73\cdot 89 - 28\cdot 232 \quad \Rightarrow \bar{a} = 73$

CamScanner

$\therefore x = \bar{a}b \pmod{m}$

$\therefore x = 146 \pmod{23}$

$\therefore x = 146$

CamScanner

Q 11)

a)

(i) $h(034567981) = (034567981) \bmod 97$

$= 91$

(ii) $h(183211232) = (183211232) \bmod 97$

$= 57$

(iii) $h(220195744) = (220195744) \bmod 97$

$= 21$

(iv) $h(987255335) = (987255335) \bmod 97$

$= 5$

(b)

(i) $h(104578690) = (104578690) \bmod 101$

$= 58$

(ii) $h(432222187) = (432222187) \bmod 101$

$= 60$

(iii) $h(372201919) = (372201919) \bmod 101$

$= 52$

(iv) $h(501338753) = (501338753) \bmod 101$

Question 12

$x_{n+1} = (4x_n+1) \bmod 7 \quad , x_0 = 3$

$\therefore x_1^{+1} = \{4(3)+1\} \bmod 7 \Rightarrow x_1 = \cancel{5}\,6$

$\therefore x_2+1 = \{4(\overset{6}{\cancel{8}})+1\} \bmod 7 = \cancel{-1}\,4$

$\therefore x_3^{+1} = \{4(\overset{4}{\cancel{7}})+1\} \bmod 7 \Rightarrow x_3 = 3$

$\therefore x_4+1 = \{4(3)+1\} \bmod 7 \Rightarrow x_4 = \cancel{5}\,6$

$\therefore x_5+1 = \{4(\overset{6}{\cancel{8}})+1\} \bmod 7 \Rightarrow x_5 = \cancel{-1}\,4$

$\therefore x_6+1 = \{(4)(\overset{4}{\cancel{7}})+1\} \bmod 7 \Rightarrow x_6 = 3$

$\therefore x_7+1 = \{4(3)+1\} \bmod 7 \Rightarrow x_7 = \cancel{8}\,6$

$\therefore x_8+1 = \{4(\overset{6}{\cancel{8}})+1\} \bmod 7 \Rightarrow x_8 = \cancel{-1}\,4$

$\therefore x_9+1 = \{4(\overset{4}{\cancel{1}})+1\} \bmod 7 \Rightarrow x_9 = 3$

$\therefore x_{10}+1 = \{4(3)+1\} \bmod 7 \Rightarrow x_{10} = \cancel{8}\,6$

$\therefore x_{11}+1 = \{4(\overset{6}{\cancel{8}})+1\} \bmod 7 \Rightarrow x_{11} = \cancel{-1}\,4$

$\therefore x_{12}+1 = \{4(\overset{4}{\cancel{7}})+1\} \bmod 7 \Rightarrow x_{12} = 3$

$\therefore x_{13}+1 = \{4(3)+1\} \bmod 7 \Rightarrow x_{13} = \cancel{8}\,6$

$\therefore x_{14}+1 = \{4(\overset{6}{\cancel{8}})+1\} \bmod 7 \Rightarrow x_{14} = \cancel{-1}\,4$

$\therefore x_{15}+1 = \{4(\overset{4}{\cancel{7}})+1\} \bmod 7 \Rightarrow x_{15} = 3$

$\therefore x_{16}+1 = \{4(3)+1\} \bmod 7 \Rightarrow x_{16} = \cancel{8}\,6$

$\therefore x_{17}+1 = \{4(\overset{6}{\cancel{8}})+1\} \bmod 7 \Rightarrow x_{17} = \cancel{-1}\,4$

$\therefore x_{18}+1 = \{4(\overset{4}{\cancel{7}})+1\} \bmod 7 \Rightarrow x_{18} = 3$

$\therefore x_{19}+1 = \{4(3)+1\} \bmod 7 \Rightarrow x_{19} = \cancel{8}\,6$

$\therefore x_{20}+1 = \{4(\overset{6}{\cancel{8}})+1\} \bmod 7 \Rightarrow x_{20} = \cancel{-1}\,4$

Sequence: 3, 1, 3, 1 ..... Q 13

(a)

(i) 7 3 ~~7~~ 2 3 2 ~~7~~ 1 8 4 4 3 4

$3x_1 + x_2 + 3x_3 + x_4 + 3x_5 + x_6 + 3x_7 + x_8 + 3x_9 + x_{10} +$
$3x_{11} + x_{12} \equiv 0 \pmod{10}$

$\Rightarrow 3(7) + 3 + 3(2) + 3 + 3(2) + 1 + 3(8) + 4 + 3(4)$
$+ 3 + 3(4) + x_{12} \equiv 0 \pmod{10}$

$\Rightarrow 21 + 3 + 6 + 3 + 6 + 1 + 24 + 4 + 12 + 3 + 12 + x_{12} \equiv 0 \pmod{10}$

$\Rightarrow 95 + x_{12} \equiv 0 \pmod{10}$

$\therefore x_{12} \equiv 0 \pmod{10} \Rightarrow$ So, the check digit is 5

(ii) 6 3 6 2 3 9 9 1 3 4 6

$\Rightarrow 3(6) + 3 + 3(6) + 2 + 3(3) + 9 + 3(9) + 1 + 3(3) + 4 +$
$3(6) + x_{12} \equiv 0 \pmod{10}$

$\Rightarrow 118 + x_{12} \equiv 0 \pmod{10} \Rightarrow x_{12} \equiv 0 \pmod{10}$

So, the check digit is 2.

(i) 036000291452

$3(0)+3+3(6)+0+3(0)+0+3(2)+9+3(1)+4+3(5)+2 \equiv 0 \pmod{10}$

$\Rightarrow 60 \equiv 0 \equiv 0 \pmod{10}$

Hence, given string of 12 digits is a valid UPC code

(ii) 012345678903

$3(0)+1+3(2)+3+3(4)+5+3(6)+7+3(8)+9+3(0)+3 \equiv 0 \pmod{10}$

$\Rightarrow 88 \equiv 8 \not\equiv 0 \pmod{10}$

Hence, given string of 12 digits is not a valid UPC code.

Q14

(a) ISBN: 0-07-119881

$x_{10} \equiv \sum_{i=1}^{9} i x_i \ \cancel{(\bmod 10)} \pmod{11}$

$x_{10} \equiv 1(0)+2(0)+3(7)+4(1)+5(1)+6(9)+7(8)+8(8)+9(1) \pmod{11}$

$x_{10} \equiv 213 \equiv 4 \pmod{11} \Rightarrow x_{10} = 4$

So, 4 is the check digit.

(b) ISBN:- 0-321-500Q1-8

~~$x_{10} = \sum_{i=1}^{9} i x_i \pmod{11}$~~

~~$x_{10} =$~~ $1(0)+2(3)+3(2)+4(1)+5(5)+6(0)+$

$\Rightarrow x_{10} = 7(0)+8(Q)+9(1)$ ~~$+10(8)$~~ $\bmod 11$

~~$x_{10} \Rightarrow 130+8Q \equiv 0 \pmod{11}$~~

~~$\Rightarrow \frac{130}{11} = -8Q \quad 11k \Rightarrow 130+8Q = 11k$~~

$\Rightarrow x_{10} =$ ~~$80+$~~ $8Q+50 \pmod{11}$

$\Rightarrow 8 = 8Q+50 \pmod{11}$

$\Rightarrow 8 = 8Q+6 \pmod{11}$

Subtracting 6 on both sides

$\Rightarrow 2 = 8Q \pmod{11}$

Mul. by 7 mod 11

$\Rightarrow 2 \cdot 7 \bmod 11 = 8 \cdot 7\, Q \pmod{11}$

$\Rightarrow 14 \bmod 11 = 56 Q \bmod 11$

$\Rightarrow 3 = Q \bmod 11$

$\therefore Q = 3$

Since Q is to be a digit b/w 0 to 9.

Question 28

Let $n=7$ then 2^n-1

$\Rightarrow 2^7-1=127$

$\because$ 127 is a prime number and also $n=7$ is greater than 5, the above statement holds true

So, there is an integer, $n>5$ such that 2^n-1 is prime.

(b)

Suppose that p divides a and (a+1) also, then:

$a=pr$ — (i) and $(a+1)=ps$ — (ii)

Subtracting both equations eq (ii) and (i)

$(a+1)-a=ps-pr$

$1=p(s-r)$

This shows that p divides 1 means $(s-r)$ is an integer which is not possible

Thus, our assumption is wrong and p can only divide a and ~~or~~ not a+1

Question 29

$\sqrt{(a+b)} = \sqrt{a} + \sqrt{b}$

Sq. on both sides

$\Rightarrow a+b = (\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$

$\therefore 2\sqrt{ab} = 0 \Rightarrow ab = 0$

Therefore, either a ~~or~~ or b must be zero or both $a=0$ and $b=0$

Now to satisfy the given condition

Let's take $b=0$ and 5

R.H.S:

$\sqrt{5+0} = \sqrt{5}$

L.H.S:-

$\sqrt{5} + \sqrt{0} = \sqrt{5}$ $\Rightarrow$ L.H.S = R.H.S

Proved!

So, given statement is true

(b)

Let's make contrapositive of given statement

if $x \leq 1$ and $x \geq -1$ then $|x| \leq 1$

$x \leq 1$ and $x \geq -1$

$-1 \leq x \leq 1$ which is true for $|x| \leq 1$

Equivalently, if $|x| > 1$ then $x > 1$ or $x < -1$ is also true

Question 30

(a) Let prime number be 19
$\Rightarrow n+2 = 19+2 = \cancel{19}\ 21$ which is not

(b) ~~Let the~~ Suppose the set of prime numbers is finite, Then:-

$p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7, \ldots\ldots p_n$

Consider the integer

$N = p_1 \cdot p_2 \cdot p_3 \text{———} p_n + 1$

Then $N > 1$, as any integer greater than 1 is divided by some prime number p

$\therefore p \mid N$

Also, since p is prime, p must equal one of the prime numbers.

$p_1, p_2, p_3, \ldots\ldots p_n$.

Thus

$p \mid (p_1, p_2, p_3, \ldots\ldots p_n)$

But then

$p \nmid (p_1, p_2, p_3, \text{———}, p_n + 1)$

So,

$p \nmid N$

Thus $p \mid N$ and $p \nmid N$, which is a contradiction.

Hence, the supposition is false and given statement is true.

Q31

Suppose n and m are odd and m+n is not even
Now, $m = 2k+1$, $n = 2l+1$
$\Rightarrow m+n = (2k+1)+(2l+1)$
$= 2k+2l+2$
$= 2(k+l+1) = 2x \Rightarrow$ where $(x = l+k+1)$
which is even and it is contradicting the supposition ~~the~~ that m+n ~~is~~ is odd. Hence, given statement is true.

(b)

By contraposition:-
Suppose that for all integers m and n, ~~then~~ if one is even and one is odd, then m+n is odd
$m = 2k$ (even), $n = 2l+1$ (odd)
$\Rightarrow m+n = 2k+2l+1$
$= 2(k+l)+1$
$= 2x+1$ $(x = k+l)$
which is true
Similarly, if we take m as odd and n as even then m+n will also be odd
* Equivalently, given statement will ~~b~~ also be true as its contrapositive is true

... and n are both odd.

Question 31

(a) Suppose that $6-7\sqrt{2}$ is rational.

$$6-7\sqrt{2} = \frac{m}{n} \text{ (with no common factors)}$$

$$\Rightarrow -7\sqrt{2} = \frac{m}{n} - 6 \Rightarrow \sqrt{2} = -\left(\frac{m-6n}{7n}\right)$$

$$\Rightarrow \sqrt{2} = \frac{-m+6n}{7n}$$

On right side, the equation is a ratio of two integers and $\sqrt{2}$ is on the left side. However, this is a contradiction because $\sqrt{2}$ is irrational. But since, we have assumed that $6-7\sqrt{2}$ is rational. Since assuming that $6-7\sqrt{2}$ is rational leads to a contradiction, our initial assumption must be false.

Therefore, we conclude that $6-7\sqrt{2}$ is irrational.

(b)

Suppose that $\sqrt{2}+\sqrt{3}$ is rational

$\sqrt{2}+\sqrt{3} = \frac{a}{b}$ (with no common factors) $\therefore b \neq 0$

$\Rightarrow \sqrt{2} = \frac{a}{b} - \sqrt{3}$ → sq. on both sides

$\Rightarrow 2 = \left(\frac{a}{b} - \sqrt{3}\right)^2 \Rightarrow 2 = \frac{a^2}{b^2} - \frac{2\sqrt{3}\,a}{b} + 3$

$\Rightarrow \cancel{4}\, 2-3 = \frac{a^2}{b^2} - \frac{2\sqrt{3}\,a}{b} \Rightarrow -1 = \frac{a^2}{b^2} - \frac{2\sqrt{3}\,a}{b}$

$\Rightarrow -1 = \frac{a^2 - 2\sqrt{3}\,ab}{b^2} \Rightarrow \frac{-b^2 - a^2}{-2ab} = \sqrt{3}$

$\Rightarrow \frac{b^2+a^2}{2ab} = \sqrt{3}$

The right side is a rational number but leads to a contradiction as we know that $\sqrt{3}$ is irrational.

Therefore, our initial assumption that $\sqrt{2}+\sqrt{3}$ is rational must be false and we conclude that $\sqrt{2}+\sqrt{3}$ is irrational.

Question 33

Let $P(n) = 1^2+2^2+3^2+\ldots\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

1. $P(1)$ is true

$1^2 = \frac{1(1+1)(2\times1+1)}{6} = \frac{\cancel{6}\,1}{\cancel{6}\,1}$

$1=1$ $\therefore$ L.H.S = R.H.S

(b)

Now, Suppose $P(k)$ is true for, some integers $k \geq 1$

$\Rightarrow 1^2+2^2+3^2+\ldots\ldots+k^2 = \frac{k(k+1)(2k+1)}{6}$ — (i)

To prove $P(k+1)$ is true:-

$\Rightarrow 1^2+2^2+3^2+\ldots\ldots+(k+1)^2 = \frac{(k+1)(k+2)(2k+2+1)}{6}$

$\cancel{= \frac{(k+1)(k+2)(2k+3)}{6}}$ $\cancel{=(k+}$

$\Rightarrow 1^2+2^2+3^2+\ldots\ldots+(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$ — (ii)

Consider L.H.S of (ii)

$\Rightarrow 1^2+2^2+3^2+\ldots\ldots+(k+1)^2 = 1^2+2^2+3^2+\ldots+k^2+(k+1)^2$

$\Rightarrow \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$\Rightarrow (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$

$\Rightarrow (k+1)\left[\frac{\cancel{k}\cancel{2k}\,2k^2+\cancel{k}+6k+6}{6}\right]$

$\Rightarrow (k+1)\left[\frac{2k^2+7k+6}{6}\right] = \frac{(k+1)(2k^2+4k+3k+6)}{6}$

$\Rightarrow \frac{(k+1)(2k+3)(k+2)}{6} \rightarrow L.H.S$

(2) $\rightarrow L.H.S = R.H.S$

Hence by principle of mathematical induction the given result is true for all integers n. ~~greater or equal to 1~~

Part (b)

Let $P(n) = 1+2+2^2+ \ldots\ldots + 2^n = 2^{n+1} - 1$

1. $P(0)$ is true

$1 = 2^{0+1} - 1$

$1 = 1 \quad \therefore L.H.S = R.H.S$

(2) Now, suppose $P(k)$ is true for:- some integers $k \geq 0$

$\Rightarrow 1+2+2^2+\ldots\ldots+2^k = 2^{k+1} - 1$ — (i) prove P(k+1) is true:-

$1+2+2^2+ \cdots\cdots + 2^{k+1} = \cancel{2^k} 2^{k+2} - 1$ — (ii)

Taking L.H.S

$\Rightarrow 1+2+2^2+ \text{-----} + 2^{k+1} = 1+2+2^2+ \cdots + 2^k + 2^{k+1}$

From (i)

$\Rightarrow 2^{k+1} - 1 + 2^{k+1} = \cancel{2^{k+1+k+1} - 1 - 2\,2}$

$\Rightarrow 2 \cdot 2^{k+1} - 1 = 2^{k+2} - 1$

From (2)

L.H.S = R.H.S

Hence, by principle of mathematical induction the given result is true for all integers $n \geq 0$

Part (c)

Let, $P(n) = 1^3+2^3+3^3+ \cdots + n^3 = \frac{1}{4}n^2(n+1)^2$

1. P(1) is true

$1^3 = \frac{1}{4}(1)^2(1+1)^2 \Rightarrow 1 = 1 \Rightarrow$ L.H.S = R.H.S

(2) Now, suppose P(k) is true for some integers n.

$\Rightarrow 1^3+2^3+3^3+ \cdots + k^3 = \frac{1}{4}k^2(k+1)^2$ — (i)

To prove P(k+1) is true:-

$\Rightarrow 1^3+2^3+3^3+ \cdots + (k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2$ — (ii)

Taking L.H.S of (ii)

$\Rightarrow 1^3+2^3+3^3+ \ldots +(k+1)^3 = 1^3+2^3+3^3+\ldots+k^3+(k+1)^3$

From (i)

$\Rightarrow \frac{1}{4}k^2(k+1)^2+(k+1)^3 \Rightarrow (k+1)^2\left\{\frac{1}{4}k^2+k+1\right\}$

$\Rightarrow (k+1)^2\left\{\frac{\cancel{1}k^2+4k+4}{\cancel{4}\ 4}\right\} \to$ Mul. by 4

$\Rightarrow \cancel{(k+1)^2(k^2+16k+16) \Rightarrow (k+1)^2}$

$\Rightarrow \frac{1}{4}(k+1)^2(k^2+4k+4) \Rightarrow \frac{1}{4}(k+1)^2(k+2)^2$

From (ii)

$\therefore$ L.H.S = R.H.S

Hence, by principle of mathematical induction the given result is true for all integers n.

Question 34

a) Combination:-

(i) Team Formation:- Used in sports, business or academic team formation to cal. the ways to choose ~~individuals~~ individuals without considering order.

(ii) Lottery systems:- Employed in designing lottery systems to calculate the no. of ways a set of numbers can be drawn without order consideration.

Permutation:-

(i) Password Generation:- Applied in password generation algorithms to enhance security by varying order of characters.

(ii) Seating Arrangements:- Utilized in event planning for optimizing seating arrangements and managing logistics efficiently.

(c) Binomial Theorem:-

1. Probability Distribution:- Used in probability theory for calculating the probability of a specific no. of successes in repeated independent trials.

2) ~~Fina~~ Finance and Investments:- Applied in finance, especially in option pricing models for calculating the future value of financial instruments under different scenarios.

(d) Proof Methods:-

(i) Cryptography:- Employed in cryptography for proving the security of encryption algorithms against various attacks.

② Software Verification:-
Used in computer science for formally proving the correctness of software, reducing bugs and vulnerabilities.

e) Mathematical Induction:-

1) Algorithm correctness:-
Used to prove the correctness of algorithms by demonstrating their validity for a base case and in subsequent steps.

2) Summation Formulas:-
Applied in proving summation formulas, such as the sum of natural numbers, with applications in mathematics and engineering.