National University of Computer & Emerging Sciences, Karachi
Final Examination, Spring-2022, School of Computing
Monday, June 6, 2022,   Time: 12:00pm to 03:00pm

Course Code: MT-1006	Course Name: Differential Equations	
Instructor Name(s) :	Abdul Basit khan	M Nadeem Khan
	Urooj	Javeria Iftikhar
	Asma Masood	Jamil Usmani
Student Roll No:	Section No:	

Instruction:

1. Write all the answers on script according to the sequence given in the question paper. Credit will be awarded for correct content and clarity of presentation.

2. Scientific calculator is allowed. There are 8 questions and 8 pages.

Time: 180 minutes. Max Points : 100

Question 1: . . . . CLO 1. . . . . *10 points*

Using the appropriate substitution, find the solution to the given initial value problem.

$$y(\ln y - \ln x - 1)\, dx + x\, dy = 0, \; y(1) = e$$

Question 2: . . . . CLO 1. . . . . *10 points*

Attempt each part.

(i) $\boxed{5 \text{ points}}$ Evaluate the solution to the given non-exact differential equation.

$$xy\, dx + (2x^2 + 3y^2 - 20)\, dy = 0$$

(ii) $\boxed{5 \text{ points}}$ A 100-volt electromotive force is applied to an RC-series circuit in which the resistance is 200 ohms and the capacitance is 10^{-4} farad. Find the charge $q(t)$ on the capacitor if $q(0) = 0$ by using RC-series circuit model $R\dfrac{dq}{dt} + \dfrac{1}{C}q = E(t)$

Question 3: . . . . CLO 1. . . . . *10 points*

The number $N(t)$ of people in a community who are exposed to a particular advertisement is governed by the logistic equation $\dfrac{dN}{dt} = N(a - bN)$. Initially, $N(0) = 500$, and it is observed that $N(1) = 1000$. Solve for $N(t)$ if it is predicted that the limiting number of people in the community who will see the advertisement is $50,000$.

Question 4: . . . . CLO 2. . . . . *10 points*

Determine the general solution of the given differential equation

$$4y'' - 4y' - 5y = 1 - \sin^2 x$$

Question 5: ....CLO 2....................................... *10 points*

Compute the solution for each part.

(i) $\boxed{\text{5 points}}$ $x^2y'' + xy' - y = \ln x$

(ii) $\boxed{\text{5 points}}$ $y''' + y' = 0, y(\pi) = 0, y'(\pi) = 2, y''(\pi) = -1$

Question 6: ....CLO 2....................................... *10 points*

Solve the differential equation using the annihilator approach.

$$y'' + 3y' - 10y = x(e^{2x} + 1)$$

Question 7: ....CLO 3....................................... *20 points*

Attempt each part.

(i) $\boxed{\text{10 points}}$ Using the method of separation of variables to find the solution of

$$16\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

(ii) $\boxed{\text{10 points}}$ Solve the wave equation subject to the conditions

$$u(0,t) = 0, \qquad u(L,t) = 0, \qquad t > 0$$

$$u(x,0) = x(L-x), \qquad \frac{\partial u}{\partial t}\Big|_{t=0} = x(L-x), \qquad 0 < x < L.$$

Question 8: ....CLO 3....................................... *20 points*

Attempt each part.

(i) $\boxed{\text{10 points}}$ Expand the Fourier series of function on the given interval

$$f(x) = x + x^2, -\pi < x < \pi$$

(ii) $\boxed{\text{10 points}}$ Determine whether the function is even or odd and expand function in an appropriate cosine or sine series.

$$f(x) = \begin{cases} x + 5 & \text{if } -2 < x < 0, \\ -x + 5 & \text{if } 0 \le x < 2. \end{cases}$$