National University of Computer & Emerging Sciences, Karachi
Spring-2017EE-Department
Final Exam
23rd May 2017, 9:00 am – 12 pm

Course Code:MT203	Course Name: Differential Equations
Instructor Name :	Muhammad Jamil usmani
Student Roll No:	Section No:

INSTRUCTIONS:

SUBJECTIVE PART – B [65 Marks]

1. Solve all the questions . Solve the question in order.
2. Part- B consist of two pages and you are allowed **100** minutes for this Part.
3. Read each question completely before answering it. There are **5 questions and 2 pages**.
4. In order to receive full credit, you must show your all necessary work with justification.

Q1 Solve the given 1st order differential equation (any two) **[10]**

a) $(3x^2y + e^y)dx + (x^3 + xe^y - 2y)dy = 0$

b) $\frac{dy}{dx} = \frac{y^2 + yx}{x^2}$

c) $x^2\frac{dy}{dx} - 2xy = 3y^4$

[10]

Q2 A small metal bar, whose initial temperature was 24° C, is dropped into a large container of boiling water.(use the fact that the boiling temperature of water is 100°C)

a) How long will it take the bar to reach 90° C if it is known that its temperature increases 2° in 1 second?

b) How long will it take the bar to reach 98° C?

[20]

Q3 Solve higher order differential equation using any appropriate method (any two)

a) $y'' + y' - 6y = 2x$

b) $y'' - 3y' = 8e^{3x} + 4sinx$

c) $y'' - 2y' + y = \frac{e^x}{1+x^2}$

Q4 (a) Find the Laplace transform of the given function: **[20]**

I. $f(t) = 4t^2 - 5\sin 3t + e^{4t} - t\cosh 3t$

II. $f(t) = (1 - e^t + 3e^{-4t})cos5t$

(b) Find the inverse Laplace transform. $F(S) = \frac{1}{4s+1} + \frac{2s-6}{s^2+9}$

(c) Use the Laplace Transform to solve the given initial value problem.(any one)

I. $y'' + 9y = \cos 3t\,,\quad y(0) = 2\,, y'(0) = 5$

II. $y'' + 5y' + 4y = 0\,, y(0) = 1\,, y'(0) = -1$

[05]

Q5 When a delta impulse force of 1 is applied to a certain spring -mass function initially at rest at time $t = 1$, the equation of motion of the mass is given by

$$y'' + 2y' = \delta(t-1)\,, y(0) = 0\ \,, y'(0) = 1$$

Find $y(t)$ using Laplace

TABLE 7.1 Brief Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}\,,\quad s > 0$
e^{at}	$\frac{1}{s-a}\,,\quad s > a$
$t^n\,,\quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}\,,\quad s > 0$
$\sin bt$	$\frac{b}{s^2+b^2}\,,\quad s > 0$
$\cos bt$	$\frac{s}{s^2+b^2}\,,\quad s > 0$
$e^{at}t^n\,,\quad n = 1, 2, \ldots$	$\frac{n!}{(s-a)^{n+1}}\,,\quad s > a$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}\,,\quad s > a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}\,,\quad s > a$