Assignment 1 LA

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Question 1

a) $A = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Augmented Matrix:-

$$AI = \left[\begin{array}{ccc:ccc} 1 & 1 & 4 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

b) Given that:

$A^{-1} = B$

$$AI = \left[\begin{array}{ccc:ccc} 1 & 1 & 4 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 - 3R_1$

$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$

$$AI = \left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & -6 & -2 & 1 & 0 \\ 0 & -1 & -11 & -3 & 0 & 1 \end{array}\right]$$

$R_3 + R_2 \ , R_1 - R_2$

$$AI = \left[\begin{array}{ccc|ccc} 1 & 0 & 10 & 3 & -1 & 0 \\ 0 & 1 & -6 & -2 & 1 & 0 \\ 0 & 0 & -17 & -5 & 1 & 1 \end{array}\right]$$

Mul. by $-\frac{1}{17}$ in R_3

$$AI = \left[\begin{array}{ccc|ccc} 1 & 0 & 10 & 3 & -1 & 0 \\ 0 & 1 & -6 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5/17 & -1/17 & -1/17 \end{array}\right]$$

$R_2 + 6R_3 \ , R_1 - 10R_3$

$$AI = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/17 & -7/17 & 10/17 \\ 0 & 1 & 0 & -4/17 & 11/17 & -6/17 \\ 0 & 0 & 1 & 5/17 & -1/17 & -1/17 \end{array}\right]$$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_5 = \begin{bmatrix} -1/17 & 0 & 0 \\ 0 & -1/17 & 0 \\ 0 & 0 & -1/17 \end{bmatrix}$$

$$E_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1/17 & -7/17 & 10/17 \\ -4/17 & 11/17 & -6/17 \\ 5/17 & -1/17 & -1/17 \end{bmatrix}$$

$$E_7 = \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e

$B = E_7 E_6 E_5 E_4 E_3 E_2 E_1 I_3$

c) $Ax = b$, $b = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

$x = A^{-1} b$

$$x = \begin{bmatrix} 1/17 & -7/17 & 10/17 \\ -4/17 & 11/17 & -6/17 \\ 5/17 & -1/17 & -1/17 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

$$x = \begin{bmatrix} -1/17 - 21/17 + 10/17 \\ 4/17 + 33/17 - 6/17 \\ -5/17 - 3/17 - 1/17 \end{bmatrix}$$

$$x = \begin{bmatrix} -12/17 \\ 31/17 \\ -9/17 \end{bmatrix}$$

Q2) Find a degree three polynomial $p(n) = an^3+bn^2+cn+d$ such that

$$p(n) = \sum_{k=1}^{n} k^2$$

Given that:-

$$\left[\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 & 5 \\ 27 & 9 & 3 & 1 & 14 \\ 64 & 16 & 4 & 1 & 30 \end{array}\right]$$

$R_2-8R_1,\ R_3-27R_1,\ R_4-64R_1$

$$\left[\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 1 \\ 0 & -4 & -6 & -7 & -3 \\ 0 & -18 & -24 & -26 & -13 \\ 0 & -48 & -60 & -63 & -34 \end{array}\right]$$

Mul. R_2 by -4

$$\left[\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3/2 & 7/4 & 3/4 \\ 0 & -18 & -24 & -26 & -13 \\ 0 & -48 & -60 & -63 & -34 \end{array}\right]$$

$R_3+18R_2,\ R_4+48R_2$

$$\left[\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3/2 & 7/4 & 3/4 \\ 0 & 0 & 3 & 11/2 & 1/2 \\ 0 & 0 & 12 & 21 & 2 \end{array}\right]$$

Mul. R_3 by $\frac{1}{3}$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3/2 & 7/4 & 3/4 \\ 0 & 0 & 1 & 11/6 & 1/6 \\ 0 & 0 & 12 & 21 & 2 \end{array}\right]$$

$R_4 - 12R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3/2 & 7/4 & 3/4 \\ 0 & 0 & 1 & 11/6 & 1/6 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right]$$

Mul. R_4 by -1

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3/2 & 7/4 & 3/4 \\ 0 & 0 & 1 & 11/6 & 1/6 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$R_3 - \frac{11}{6}R_4$, $R_2 - \frac{7}{4}R_4$, $R_1 - R_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 3/2 & 0 & 3/4 \\ 0 & 0 & 1 & 0 & 1/6 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$R_2 - \frac{3}{2}R_3$ $-R_1 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 1/6 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 - R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 0 & 1/2 \\ 0 & 0 & \cancel{0}1 & 0 & 1/6 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$a = 1/3$, $b = 1/2$, $c = 1/6$, $d = 0$

$$P(n) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n + 0$$

Q3) Given that:

$ax_1 + bx_2 + cx_3 = 0$

writing it as

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

suppose if $a \neq 0$ ~~and divide x_1 by a~~

then it is obvious that x_2 and x_3 are free variables, suppose $x_2 = s, x_3 = t$

then $x_1 = \frac{-bs-ct}{a}$

Since we have two free variables, the solution set is the same form R_3

$$\begin{bmatrix} \frac{-bs-ct}{a} \\ s \\ t \end{bmatrix}$$

If each of a, b, c are zero then there are no constraints and the solution set is whole R_3.