



FAST – National University of Computer & Emerging Sciences
EE Department (Karachi Campus)

Course Code: MT204	Differential Equations	Date: 22/02/2015
Time Allowed: 03 Hour	FINAL Spring 2015	Max. Marks: 70
Instructions:		
<ul style="list-style-type: none">Solve all the question and return the question paper.All the answers must be solved according to the sequence given in the question paper.		

Question 1 **Estimated Time: 20 minutes** **Marks (10)**

- a) Form the differential equation $y = C_1 e^{2x} + C_2 x e^{2x}$, where C_1 and C_2 are arbitrary constants.
b) Consider $y'' - 4y' + 4y = 0$, use Reduction of order to find $y_2(x)$ if $y_1(x) = e^{2x}$
and verify that $y_1(x)$ and $y_2(x)$ are linearly independent.

Question 2 **Estimated Time: 30 minutes** **Marks (15)**

Identify then solve the following 1st order differential equation

- I. $x \frac{dy}{dx} = y + \sqrt{x^2 - y^2}$
II. $\frac{dy}{dx} = \frac{2+y e^{xy}}{2y-x e^{xy}}$
III. $x \frac{dy}{dx} = (1+x)y + xy^2$

Question 3 **Estimated Time: 30 minutes** **Marks (15)**

Find the general solution of the following 2nd order differential equation

- i. $y'' - 2y' + y = \frac{e^x}{1+x^2}$
ii. $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$
iii. $y'' + y = 4x + 10\sin x, y(\pi) = 0, y'(\pi) = 2$

Question 4 **Estimated Time: 30 minutes** **Marks (7+8)**

- a) Evaluate i) $\mathcal{L}^{-1} \left\{ \frac{3s-6}{9s^2-36} + \frac{2}{s^2} \right\}$ ii) $\mathcal{L} \{ t e^{4t} \}$

- b) Use the Laplace Transform to solve the given initial value problem.

$$y'' - 4y = 24 \cos 2t, y(0) = 3, y'(0) = 4$$

Question 5**Estimated Time: 20 minutes****Marks (10)**

a) Solve the given initial / boundary value problems.

$$y''' + 12y'' + 36y' = 0; y(0) = 0, y'(0) = 1, y''(0) = -7$$

b) Solve the given Cauchy –Euler equation.

i. $x^2y'' + xy' + y = 0, y(1) = 1, y'(1) = 2$

ii. $xy'' + 2y' = 0; y(1) = 0, y'(1) = 2$

Question 6**Estimated Time: 30 minutes****Marks (15)**a) Show that using definition of laplace (**any one**)

i. $\mathcal{L}(\sin at) = \frac{a}{s^2 + a^2}$

ii. $\mathcal{L}(t^2) = \frac{2}{s^3}$

b) Evaluate Laplace of given function

i. $f(t) = t \cos 2t$

ii. $f(t) = \begin{cases} t, & \text{if } 0 \leq t < 1 \\ 1, & \text{if } t \geq 1 \end{cases}$

Question 7**Estimated Time: 40 minutes****Marks (10)**Let $f(x)$ be a function of period 2π such that

$$f(x) = \begin{cases} 1, & -\pi < x < 0 \\ 0, & 0 < x < \pi. \end{cases}$$

a) Sketch a graph of $f(x)$ in the interval $-2\pi < x < 2\pi$ b) Show that the Fourier series for $f(x)$ in the interval $-\pi < x < \pi$ is

$$\frac{1}{2} - \frac{2}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right]$$

c) By giving an appropriate value to x , show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

WISH YOU ALL THE BEST