DE MT1006- SOLUTION MID-II

Date: 31-03-2023

Q1(a)

$$x^2y'' - xy' + 2y = 0; \qquad y_1(x) = x\sin(lnx)$$

Dividing by x^2 we have

Identifying $P(x) = -1/x$ we have

$$y_2 = x\sin(\ln x)\int \frac{e^{-\int -dx/x}}{x^2\sin^2(\ln x)}\,dx = x\sin(\ln x)\int \frac{x}{x^2\sin^2(\ln x)}\,dx$$

$$= x\sin(\ln x)\int \frac{\csc^2(\ln x)}{x}\,dx = [x\sin(\ln x)][-\cot(\ln x)] = -x\cos(\ln x).$$

A second solution is $y_2 = x\cos(\ln x)$.

Q1(b)

I. $6x^2y'' + 5xy' - y = 0$

The auxiliary equation is $6m^2 - m - 1 = 0$ so that

$$y = c_1x^{1/2} + c_2x^{-1/3}.$$

II. $y''' + 3y'' + 3y' + y = 0$

The auxiliary equation is

$m^3 + 3m^2 + 3m + 1 = 0$ we obtain $m_1 = -1$, $m_2 = -1$, and $m_3 = -1$

$$y = c_1e^{-x} + c_2xe^{-x} + c_3x^2e^{-x}.$$

III. $\dfrac{d^3x}{dt^3} - \dfrac{d^2x}{dt^2} - 4x = 0$

The auxiliary equation is

$m^3 - m^2 - 4 = 0$ we obtain $m_1 = 2$ and $m_2 = -1/2 \pm \sqrt{7}\,i/2$ so that

$$x = c_1e^{2t} + e^{-t/2}[c_2\cos(\sqrt{7}\,t/2) + c_3\sin(\sqrt{7}\,t/2)].$$

Assistant Prof: Jamilusmani

Q2(a)

$$y'' + 2y' + 5y = 6sin2x + 7cos2x$$

The auxiliary Eqn

$$m^2 + 2m + 5 = 0$$
$$m = -1 \pm 2i$$
$$y_c = e^{-x}[C_1 \sin 2x + C_2 \cos 2x]$$

$Now \quad y_p = A\cos 2x + B\sin 2x$

$$y_p' = -2A\sin 2x + 2B\cos 2x$$
$$y''\text{p} = -4A\cos 2x - 4B\sin 2x$$
$$-4A\cos 2x - 4B\sin 2x + 2(-2A\sin 2x + 2B\cos 2x) +$$
$$5(A\cos 2x + B\sin 2x) = 6\sin 2x + 7\cos 2x$$
$$-4A\cos 2x + 4B\cos 2x + 5A\cos 2x - 4B\sin 2x - 4A\sin 2x$$
$$+5B\sin 2x = 6\sin 2x + 7\cos 2x$$
$$\cos 2x[-4A + 4B + 5A] = 7\cos 2x$$
$$compare\ A + 4B = 7 - - - (1)$$
$$\sin 2x[-4B - 4A + 5B] = 6\sin 2x$$
$$compare\ \ B - 4A = 6\ \ - - - (2)$$

solving equation (1) and (2) $using\ calculator$

$\boxed{B = 2}$

,$\boxed{A = -1}$

$$y_p = -\cos 2x + 2\sin 2x$$
$$y = y_c + y_p = e^{-x}[c_1 \sin 2x + c_2 \cos 2x] - \cos 2x + 2\sin 2x$$

Q2(b)

$$y'' + y = tanx\ secx$$

The auxiliary equation is $m^2 + 1 = 0$, so $y_c = c_1 \cos x + c_2 \sin x$ and

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

Identifying $f(x) = \sec x \tan x$ we obtain

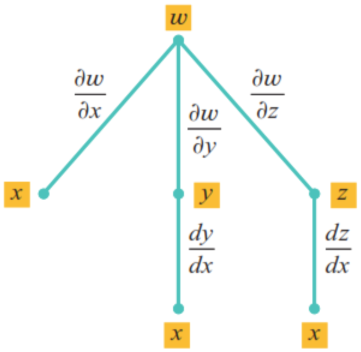
$$u_1' = -\sin x(\sec x \tan x) = -\tan^2 x = 1 - \sec^2 x$$
$$u_2' = \cos x(\sec x \tan x) = \tan x.$$

Then $u_1 = x - \tan x$, $u_2 = -\ln|\cos x|$, and

$$y = c_1 \cos x + c_2 \sin x + x\cos x - \sin x - \sin x \ln|\cos x|$$
$$= c_1 \cos x + c_3 \sin x + x\cos x - \sin x \ln|\cos x|.$$

Q3(a)

$$\boxed{w = xy + yz\,, \qquad y = sinx \quad , \qquad z = e^x}$$



$$\boxed{\frac{dw}{dx} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y}\frac{dy}{dx} + \frac{\partial w}{\partial z}\frac{dz}{dx}}$$

$$\begin{aligned}\frac{dw}{dx} &= y + (x + z)\cos x + ye^x \\ &= \sin x + (x + e^x)\cos x + e^x \sin x\end{aligned}$$

Q3(b)

$$g(x, y) = \frac{1}{3}x^3 + y^2 + 2xy - 6x - 3y + 4$$

Solution

step 1 , we first calculate $g_x(x, y)$ and $g_y(x, y)$, then set each of them equal to zero:

$$\begin{aligned} g_x(x, y) &= x^2 + 2y - 6 \\ g_y(x, y) &= 2y + 2x - 3. \end{aligned}$$

Setting them equal to zero yields the system of equations

$$\begin{aligned} x^2 + 2y - 6 &= 0 \\ 2y + 2x - 3 &= 0 \end{aligned}$$

To solve this system, first solve the second equation for y. This gives $y = \frac{3-2x}{2}$. Substituting this into the first equation gives

$$\begin{aligned} x^2 + 3 - 2x - 6 &= 0 \\ x^2 - 2x - 3 &= 0 \\ (x - 3)(x + 1) &= 0 \end{aligned}$$

Therefore, $x = -1$ or $x = 3$. Substituting these values into the equation $y = \frac{3-2x}{2}$

the critical points are $\left(-1, \frac{5}{2}\right)$ and $\left(3, -\frac{3}{2}\right)$.

Step 2 involves calculating the second partial derivatives of g :

$$\begin{aligned} g_{xx}(x, y) &= 2x \\ g_{xy}(x, y) &= 2 \\ g_{yy}(x, y) &= 2. \end{aligned}$$

Then, we find a general formula for D :

$$
\begin{aligned}
D &= g_{xx}(x_0, y_0) g_{yy}(x_0, y_0) - \left(g_{xy}(x_0, y_0)\right)^2 \\
&= (2x_0)(2) - 2^2 \\
&= 4x_0 - 4.
\end{aligned}
$$

Next, we substitute each critical point in D

$$
D\left(-1, \frac{5}{2}\right) = (2(-1))(2) - (2)^2 = -4 - 4 = -8
$$
$$
D\left(3, -\frac{3}{2}\right) = (2(3))(2) - (2)^2 = 12 - 4 = 8.
$$

step 3, we note that, applying the Second Derivative Test for Functions of Two Variables, $\left(-1, \frac{5}{2}\right)$ is a saddle point.

point $\left(3, -\frac{3}{2}\right)$ corresponds to a local minimum

ALL THE BEST

Assistant Prof: Jamilusmani