TYPES OF SETS

27.1 INTRODUCTION

In this chapter, we shall be finding whether :

(i) a given set has a countable number of elements or not,

(ii) a given set is empty or not,

(iii) two given sets have identical elements or not,

(iv) two given sets have an equal number of elements or not,

(v) two given sets have some elements in common or not, etc.

27.2 FINITE SET

A set is said to be a finite set if it has a limited number of elements,

i.e. the number of elements in it can be counted.

For example :

1. P = { Natural numbers less than 50 }

= { 1, 2, 3, ..., 49 }

2. Q = { x : x is a student in your school }

3. R = { Whole numbers between 5 and 45 } and so on.

> Set P is finite as it has a countable number of elements.

27.3 INFINITE SET

A set is said to be infinite if it has an unlimited number of elements

i.e. the number of elements of such a set cannot be counted.

For example :

1. Q = Set of whole numbers = { 0, 1, 2, 3, ... }

2. R = { Stars in the sky }

3. A = { x : x is a natural number greater than 32 }

= { 33, 34, 35, 36, ... } and so on.

> Each of these sets has an uncountable number of elements

Such sets are expressed in the Roster Form by writing a few elements and then putting some dots to show that the elements continue till infinity.

27.4 THE EMPTY SET or THE NULL SET

> The empty set is a finite set.

It is a set that has no elements.

The empty set is expressed by a pair of curly braces with no element written inside them, *i.e.* { } represents an empty set.

The empty set (the null set) is also represented by the Greek letter $\varnothing$, spelt as phi. The symbols { } and $\varnothing$ represent the same set, *i.e.* { } = $\varnothing$.

Examples :

(i)   Let A   =   { Triangles with four sides }

         then A   = $\varnothing$, i.e. A = { }   *There is no triangle with four sides.*

(ii)   Let B   =   { Natural numbers less than 1 }

         then B   =   { }, i.e. B = $\varnothing$ *There is no natural number less than 1.*

27.5   EQUAL SETS

Two sets are said to be equal if the elements of the two sets are the same, i.e. the elements of the two sets are identical. The symbol used for equality of sets is the usual sign " = ", i.e. "equal to".

For example :

        Let A   =   { 1, 2, 3, 4 }

        and B   =   { Natural numbers less than 5 }. Then, set A is equal to set B.

And so we write : Set A = Set B, or simply A = B

27.6   EQUIVALENT SETS

Two sets are said to be equivalent if the number of elements in both the sets are equal. The elements may be the the same or they may be different, but each set must contain the same number of elements.

For example :

         Consider A   =   { x, y, z }

            and B   =   { Patna, Calcutta, Delhi }

Here, A and B are equivalent sets because they have an equal number of elements (both A and B have three elements).

> 1. *Equal sets are equivalent but the converse is not always true.*
> 2. *Two empty sets are always equal.*
> 3. *Two infinite sets are always equivalent.*

27.7   DISJOINT SETS

If two given sets have no element in common, they are disjoint sets.

For example :

(i)   Let A   =   Set of students of Class X

        and B   =   Set of students of Class XII

Since, no student can be common to the two classes, **sets A and B are disjoint.**

(ii)   Let P   =   { a, b, c, d } and Q = { 1, 2, 3, 4, 5 }

Clearly, sets P and Q have no element in common; therefore, **sets P and Q are disjoint sets.**

27.8 OVERLAPPING SETS

If two given sets have at least one element in common, they are said to be overlapping sets.

> Overlapping sets are also known as joint sets.

For example :

(i) If set A = { 5, 6, 7, 8, 9, 10 } and set B = { 4, 6, 8, 10, 12 }, **sets A and B are overlapping** as they have elements 6, 8 and 10 in common.

(ii) Let P = Set of students of Class X

and Q = Set of students of Class X in Sophia School.

Clearly, the students of Class X in Sophia School are common to the two given sets; therefore **sets P and Q are overlapping sets.**

─────────────── EXERCISE 27 ───────────────

1. State whether the given set is **infinite** or **finite** :

  (i) { 3, 5, 7, ... }

  (ii) { 1, 2, 3, 4 }

  (iii) { ..., –3, –2, –1, 0, 1, 2 }

  (iv) { 20, 30, 40, 50, ..., 200 }

2. Which of the following sets is **empty** ?

  (i) Set of counting numbers between 5 and 6.

  (ii) Set of odd numbers between 7 and 19.

  (iii) Set of odd numbers between 7 and 9.

  (iv) Set of even numbers that are not divisible by 2.

  (v) { 0 }.

3. State which pair of sets given below are **equal sets** and which are **equivalent** :

  (i) { 3, 5, 7 } and { 5, 3, 7 }

  (ii) { 8, 6, 10, 12 } and { 3, 2, 4, 6 }

  (iii) { 7, 7, 2, 1, 2 } and { 1, 2, 7 }

  (iv) { 2, 4, 6, 8, 10 } and { a, b, d, e, m }

4. State which of the following are **finite** sets and which are **infinite** :

  (i) Set of integers   (ii) { Multiples of 5 }

  (iii) { Fractions between 1 and 2 }   (iv) { Number of people in India }

  (v) Set of trees in the world   (vi) Set of leaves on a tree

  (vii) Set of children in all the schools of Delhi

  (viii) { ...., – 4, – 2, 0, 2, 4, 6, 8 }   (ix) { – 12, – 9, – 6, – 3, 0, 3, 6, ... }

  (x) { Number of points in a line segment 4 cm long }.

5. State whether or not the following sets are **empty** :

    (i) { Prime numbers divisible by 2 }   (ii) { Negative natural numbers }

    (iii) { Women with height 5 metre }   (iv) { Integers less than 5 }

    (v) { Prime numbers between 17 and 23 }

    (vi) Set of even numbers not divisible by 2

    (vii) Set of multiples of 3 that are more than 9 and less than 15.

6. State if the given pairs of sets are **equal sets** or **equivalent sets** :

    (i) { Natural numbers less than five } and { Letters of the word 'BOAT' }.

    (ii) { 2, 4, 6, 8, 10 } and { even natural numbers less than 12 }.

    (iii) { 1, 3, 5, 7, ... } and set of odd natural numbers.

    (iv) { Letters of the word MEMBER } and { Letters of the word 'REMEMBER' }.

    (v) { Negative natural numbers } and { 50th day of a month }

    (vi) { Even natural numbers } and { Odd natural numbers }.

7. State whether the following are **finite** or **infinite sets** :

    (i) { 2, 4, 6, 8, ..., 800 }

    (ii) { ...., – 5, – 4, – 3, – 2 }

    (iii) { $x : x$ is an integer between – 60 and 60 }

    (iv) { No. of electrical appliances working in your house }

    (v) { $x : x$ is a whole number greater than 20 }

    (vi) { $x : x$ is a whole number less than 20 }.

8. For each statement given below, write **True** or **False** :

    (i) { ...., – 8, – 4, 0, 4, 8 } is a finite set.

    (ii) { – 32, – 28, – 24, – 20, ..., 0, 4, 8, 16 } is an infinite set.

    (iii) { $x : x$ is a natural number less than 1 } is the empty set.

    (iv) { Whole numbers between 15 and 16 } = { Natural numbers between 5 and 6 }.

    (v) { Odd numbers divisible by 2 } is the empty set.

    (vi) { Even natural numbers divisible by 3 } is the empty set.

    (vii) { $x : x$ is positive and $x < 0$ } is the empty set.

    (viii) { ...., –5, –3, –1, 1, 3, 5, ... } is a finite set.

9. State, giving reasons, which of the following pairs of sets are **disjoint sets** and which are **overlapping sets** :

    (i) A = { Girls with ages below 15 years } and

       B = { Girls with ages above 15 years }

    (ii) C = { Boys with ages above 20 years } and

       D = { Boys with ages above 27 years }

    (iii) A = { Natural numbers between 35 and 60 } and

       B = { Natural numbers between 50 and 80 }

(iv) P = { Students of Class IX studying in I.C.S.E. Board } and

    Q = { Students of Class IX }

(v) A = { Natural numbers that are multiples of 3 and less than 30 } and

    B = { Natural numbers divisible by 4 and lying between 20 and 45 }

(vi) P = { Letters in the word 'ALLAHABAD' } and

    Q = { Letters in the word 'MUSSOORIE' }

—————————— **Revision Exercise (Chapter 27)** ——————————

1. Give three examples of finite sets from your surroundings.

2. Give two examples of infinite sets from your surroundings.

3. Give three examples of empty sets.

4. Classify each of the following sets as empty set, finite set or infinite set :

    (i) { x : x is a prime number between 13 and 17 }

    (ii) { Lines drawn on the page of your note books }

    (iii) { Whole numbers less than 30 }

    (iv) { Whole numbers more than 30 }

    (v) { Integers between 35 and 36 }

    (vi) { Integers below 35 or above 36 }

    (vii) Set of fractions between 1 and 2

    (viii) { Men with four legs }

    (ix) Set of schools in India.

    (x) Set of all even prime numbers greater than 2.

5. Find which of the following pairs of sets are **disjoint sets** and which of them are **overlapping**

    (i) N = { Natural numbers greater than 15 } and

    W = { Whole numbers less than 20 }

    (ii) A = { Integers between −5 and 10 } and

    B = { Whole numbers less than 5 }

    (iii) B = { Girls in your class } and

    G = { Girls in your school }

    (iv) N = { Natural numbers between 10 and 20 } and

    I = { Integers between 5 and 15 }.

6. Which of the following sets are empty ?

    (i) { x : x is a whole number and $x^2 < 12$}

    (ii) { x : x is an integer and $3x - 2 = 0$}

    (iii) { x : x is an even prime number between 10 and 20}

    (iv) { x : x is a natural number less than 1}

    (v) { x : x is a natural number and $37 \leq x < 49$}

    (vi) { x : x is a natural number and $49 < x \leq 37$}