UNIT – 7	CHAPTER 26

UNIT – 7 SET THEORY

CHAPTER 26 IDEA OF A SET

26.1 INTRODUCTION

In our day-to-day life we often speak or hear about different types of collections.

Such as :
1. A collection of stamps.
2. A collection of toys.
3. A collection of books, etc.

Each collection is well defined

In the same way, we have different types of groups made for different activities.

Such as :
1. A group of boys playing hockey.
2. A group of girls playing badminton.
3. A group of students going for picnic, etc.

Each group is well defined

In mathematics, a collection of particular things or a group of particular objects is called a **set.**

26.2 DEFINITION OF A SET

A set is a collection of well-defined objects.

Meaning of "WELL-DEFINED" :

Well-defined means, it must be absolutely clear which object belongs to the set and which does not.

For example :

(i) **A collection of "Lovely Flowers"** is not a set *because the objects* (flowers) *to be included are* **not well-defined.**
   Reason : The word "Lovely" is a relative term. What may be lovely to one person may not be so to another person.

(ii) **A collection of "Red Flowers"** is a set because every red flower will be included in this set, *i.e.* **the objects of the set are well-defined.**

(iii) **A group of "Young Players"** is not a set, as the range of the age for young players is not given. *i.e.* it cannot be decided which player is to be considered young, *i.e.* **the objects are not well-defined.**

(iv) **A group of "Players with ages between 14 years and 18 years"** is a set, because the range of age of the players is given and so that it can be easily decided which players are to be included and which are to be excluded.

Hence, *the objects are well-defined.*

Example 1 :

State, giving reason, whether or not the following objects form a set :
(i) All the problems in this book that are difficult to solve.

(ii) All the problems in this book that are difficult for Mohit to solve.

(iii) All objects heavier than 28 kg.

Solution :

(i) The given **objects do not form a set.** (Ans.)

Reason : Some problems may be difficult for one person but not so difficult for others. Hence the given **objects are not well-defined.**

Thus, the given objects do **not** form **a set.**

(ii) The given **objects form a set.** (Ans.)

Reason : It can be easily found which problems are difficult to solve for Mohit and which are not.

(iii) The given **objects form a set.** (Ans.)

Reason : Every object can be compared with certainty in relation to weight of 28 kg. Thus, it is very easy to select objects which are heavier than 28 kg, *i.e.* the **objects are well-defined,** so they **form a set.**

The members (objects) of each of the following collections form a set :

(i) students in a class room,

(ii) books in your school bag,

(iii) counting numbers between 10 and 20,

(iv) the students of your class, that are taller than you   and so on.

─────────────── EXERCISE 26(A) ───────────────

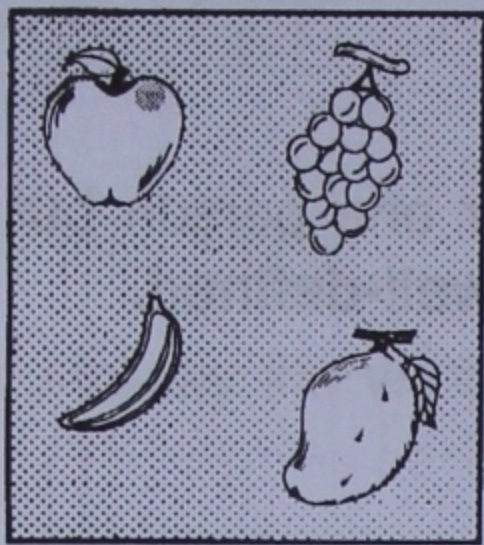1. State whether or not the following elements form a set; if not, give reason :

(i) All the easy problems in your text book.

(ii) All the three sided figures.

(iii) The first five counting numbers.

(iv) All the tall boys of your class.

(v) The last three days of a week.

(vi) All triangles that are difficult to draw.

(vii) The first three letters of the English alphabet.

(viii) All tasty fruits.

(ix) All the clever boys of class 6.

(x) All the good schools in Delhi.

(xi) All the girls in your class whose heights are less than your height.

(xii) All the boys in your class whose heights are more than your height.

(xiii) All the problems in your Mathematics book that are difficult for Amit.

26.3 **WAYS OF REPRESENTING SETS**
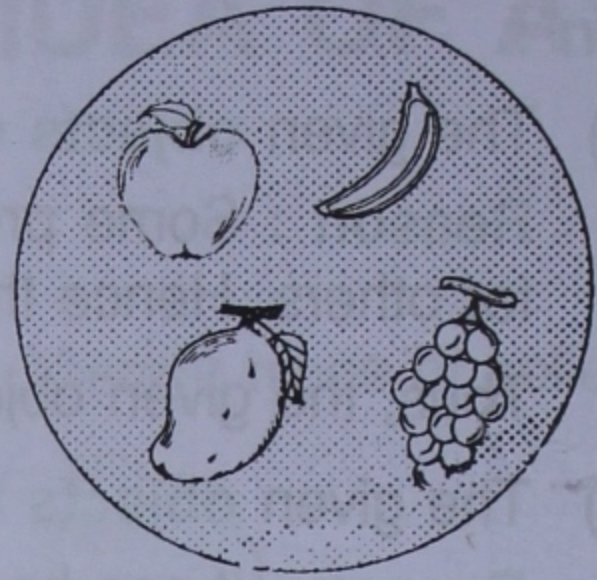
A set or a collection of well-defined objects may be represented by pictures drawn inside closed figures, like a circle, a rectangle, a square, etc.
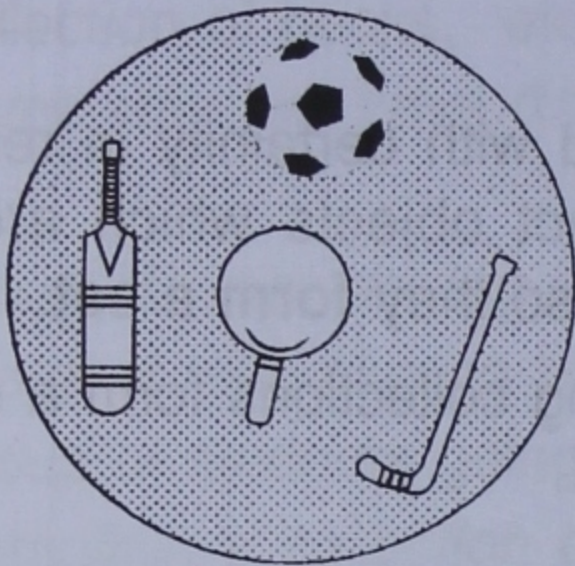
For example :

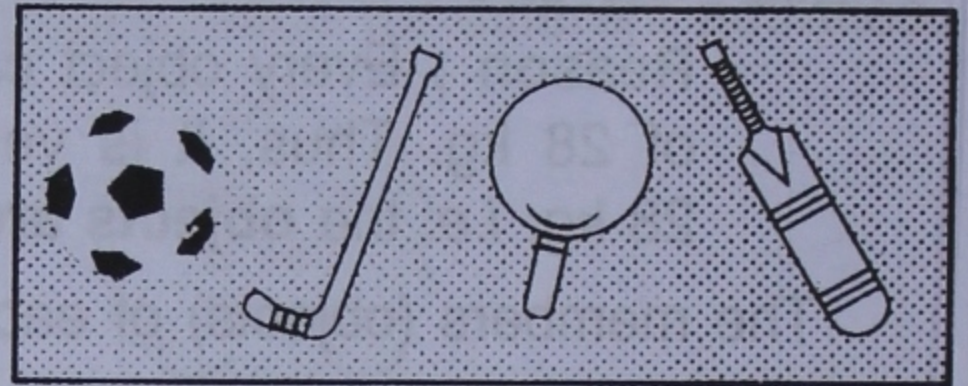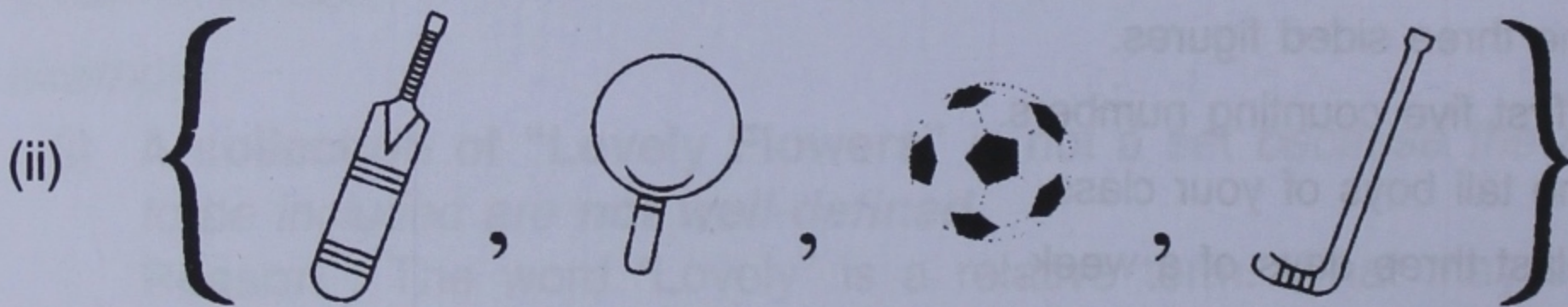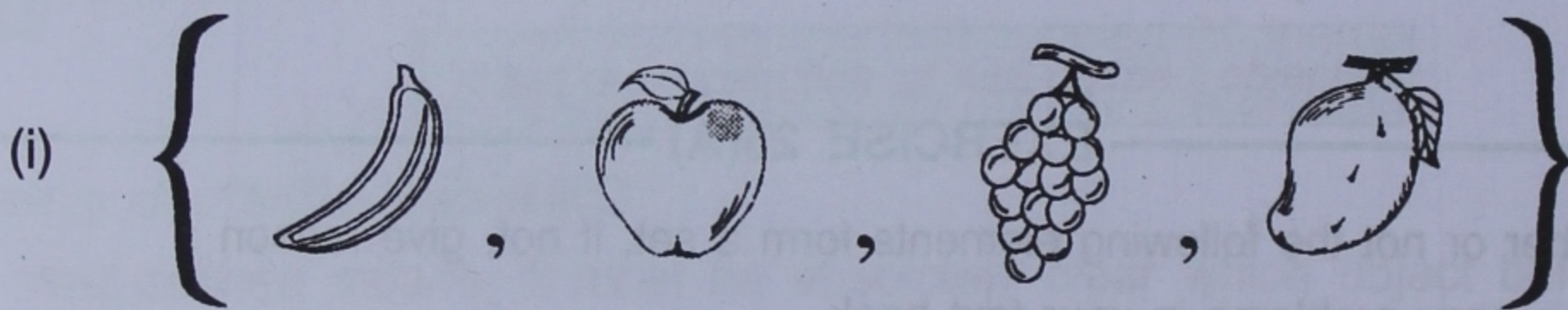(i) A set of some fruit can be shown as :

or

(ii) A set of some sports goods can be shown as :

or

Instead of showing (drawing) these pictures inside a circle, a rectangle, etc., we can also draw them within a pair of curly braces (brackets).

(i)

(ii)

The two sets shown above by pictures can also be expressed by writing their names as shown below :

(i) { banana, apple, grapes, mango }.

(ii) { cricket bat, tennis racket, football, hockey }.

It is convenient and time-saving to write (express) sets in this way.

Thus, the elements of a set can be expressed through pictures or by writing their names inside a pair of curly braces.

| 26.4 | ELEMENTS (or MEMBERS) OF A SET

The objects used to form a set are called its **elements** or its **members**.

Generally, the elements of a set are written inside a pair of curly (middle) braces and separated by commas. *The name of the set is always written in capital letters.*

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For example :

$$A = \{ p, q, r, s, t \}$$

Here, 'A' is the name of the set, whose elements (members) are p, q, r, s and t.

1. The pair of curly braces { } denotes a set.
2. The Greek letter **Epsilon** '∈' is used for the words "belongs to", "is an element of", etc.

   ∴ *p* ∈ *A will be read as "p belongs to set A" or as "p is an element of set A".*

   In the same way; q ∈ *A, r* ∈ *A, s* ∈ *A and t* ∈ *A.*

   The symbol '∉' stands for "does not belong to" or "is not an element of".

   ∴ *a* ∉ *A will be read as "a does not belong to set A" or as "a is not an element of set A".*

   In the same way b ∉ *A, c* ∉ *A and so on.*

Example 2 :

If set A = { 3, 6, 9, 12, 15, 18 }, state whether the following statements are **true** or **false** :

(i) 8 ∈ A   (ii) 12 ∉ A   (iii) 15 ∈ A

(iv) 9, 12 ∈ A   (v) 12, 14, 15 ∈ A

Solution :

(i) **False** : as element 8 does not belong to the given set A.   (Ans.)

(ii) **False** : as element 12 belongs to the given set A.   (Ans.)

(iii) **True** : as element 15 belongs to the given set A.   (Ans.)

(iv) **True** : as elements 9 and 12 both belong to the given set A.   (Ans.)

(v) **False** : as element 14 does not belong to the given set A.   (Ans.)

26.5   SOME PROPERTIES OF A SET

1. *A change in order of writing the elements does not make any change in the set.*

For example :

Set { 5, 6, 7, 8 } is same as set { 6, 8, 7, 5 }

i.e.   { 5, 6, 7, 8 } = { 6, 8, 7, 5 }

Similarly, { b, d, n, y, z } = { y, z, b, d, n } = { z, d, y, b, n }   and so on.

2. *If one or many elements of a set are repeated, the set remains the same.*

For example :

The set { a, b, d, a, c, b } is the same as the set { a, b, c, d }

i.e.   { a, b, d, a, c, b } = { a, b, c, d }

In general, *the elements of a set are not repeated*. Thus,

(i) if Q is the set of letters of the word **book**; then Q = { b, o, k }.

There are two **o**s in the word **book** but in the set it is written only once.

(ii) if R = { letters of the word 'PUPPET' }; then R = { p, u, e, t }.

> **Review :**
> 1. *The elements of a set are written inside a pair of curly braces and separated by commas.*
> 2. *The set is represented by a capital letter.*
> 3. *If the elements of a set are alphabets, these elements are written in small letters.*
> 4. *The elements of a set may be written in any order.*
> 5. *No element of a set must be repeated.*

Example 3 :

(i)   Write the set of letters used in the word 'MEERUT'.

(ii) Write the set of vowels used in the word "MUSSOORIE".

(iii) Write the set of consonants used in the word "AMRITSAR".

Solution :

(i)   **Required set = { *m, e, r, u, t* }**   (Ans.)

(ii) **Required set = { *u, o, i, e* }**   (Ans.)

(iii) **Required set = { *m, r, t, s* }**   (Ans.)

> Each element is written only once

Example 4 :

For each statement given below, state whether it is **true** or **false** :

(i)   { 5, 5, 5, ... } = { 5 }   (ii) { 3, 4, 7 } = { 4, 3, 7 }

(iii) { 6 – 2, 5 – 2, 4 – 2, 3 – 2 } = { 4, 3, 2, 1 }

Solution :

(i)   **True** : as repetition of elements does not change a set.   (Ans.)

(ii) **True** : as the change in order of writing the elements
       does not change a set   (Ans.)

(iii) **True.**   (Ans.)

─────────────── **EXERCISE 26(B)** ───────────────

1. If set A = { 2, 3, 4, 5, 6 }, state which of the following statements are **true** and which are **false** :

(i) $2 \in A$ (ii) $5, 6 \in A$ (iii) $3, 4, 7 \in A$ (iv) $2, 8 \in A$

2. If set B = { 4, 6, 8, 10, 12, 14 }, state which of the following statement is **correct** and which is **wrong** :

(i) $5 \in B$ (ii) $12 \in B$ (iii) $14 \in B$ (iv) $9 \in B$

(v)   B is the set of even numbers between 2 and 16.

(vi) 4, 6 and 10 are the members of the set B.

Also write the wrong statements correctly.

3. State whether **true** or **false** :

(i)   Sets { 4, 9, 6, 2 } and { 6, 2, 4, 9 } are not the same.

(ii) Sets { 0, 1, 3, 9, 4 } and { 4, 0, 1, 3, 9 } are the same.

(iii) Sets { 5, 4 } and { 5, 4, 4, 5 } are not the same.

(iv) Sets { 8, 3 } and { 3, 3, 8 } are the same.

(v)   Collection of vowels used in the word 'ALLAHABAD' forms a set.

(vi) If P is the set of letters in the word 'ROOP'; then P = { p, o, r }

(vii) If M is the set of letters used in the word 'MUMBAI', then :

M = { m, u, b, a, i }

4. Write the set containing :
    (i) the first five counting numbers.   (ii) the three types of angles.
    (iii) the three types of triangles.   (iv) the members of your family.
    (v) the first six consonants of the English alphabet.
    (vi) the first four vowels of the English alphabet.
    (vii) the names of any three Prime Ministers of India.

5.   (a) Write the members (elements) of each set given below :
      (i) { 3, 8, 5, 15, 12, 7 }   (ii) { c, m, n, o, s }

   (b) Write the sets whose elements are :
      (i) 2, 4, 8, 16, 64 and 128   (ii) 3, 5, 15, 45, 75 and 90

6.   (i) Write the set of letters used in the word 'BHOPAL'.
   (ii) Write the set of vowels used in the word 'BENGAL'.
   (iii) Write the set of consonants used in the word 'HONG-KONG'.

26.6 NOTATION (REPRESENTATION) OF A SET

For representing a set, the following methods are commonly used :
   (i) *Description Method*   (ii) *Roster or Tabular Method*
   (iii) *Rule or Set-builder Method*

1. Description Method :

In this method, a well-defined description of the elements of a set is made. Sometimes, the discription of elements is enclosed within the curly brackets.

For example :
   (i) A set of cricket players with ages between 20 years and 28 years.
     = {cricket players with ages between 20 years and 28 years}
   (ii) A set of numbers greater than 40 and smaller than 75.
     = {numbers greater than 40 and smaller than 75} and so on

2. Roster or Tabular Method :

In this method, the elements (members) of the set under consideration are written inside a pair of curly braces and are separated by commas.

For example :
   (i) If P is the set of the last four months of the year.   [Description Method]
     then, P = {September, October, November, December}.   [Roster Method]
   (ii) If Q is the set of counting numbers less than 6.   [Description Method]
     then, Q = { 1, 2, 3, 4, 5 }   [Roster Method]

3. Rule or Set-Builder Method :

In this method, *the actual elements of a set are not listed; rather a brief rule or statement or formula is written inside a pair of curly braces.*

For example :

If A is the set of counting numbers greater than 12,

Set A is written in set-builder form as :

A = { x : x is a counting number greater than 12 }

or

A = { x | x is a counting number greater than 12 }

This will be read as "A is the set of elements x such that x is a counting number greater than 12".

The symbol ':' or 'l' placed between two *xs* stands for "such that".

Thus, if B is the set of *x* such that *x* is a factor of 6 then

B = { x : x is a factor of 6 } or B = { x | x is a factor of 6 }

26.7 | SOME IMPORTANT SETS :

N = Natural Numbers

= Set of all counting numbers starting from 1   [Description Method]

= Set of numbers 1, 2, 3, 4, ...

= { 1, 2, 3, 4, ... }   [Roster Method]

= { x : x is a counting number starting from 1 }   [Set-Builder Method]

W = Whole Numbers

= Set containing zero and all the natural numbers.

= { 0, 1, 2, 3, 4, 5, 6, ... }

Z or I = Integers

= Set containing the negatives of the natural numbers, zero and the natural numbers.

= { ..., –3, –2, –1, 0, 1, 2, 3, 4, ... }

E = Even Natural Numbers

= Set of natural numbers that are divisible by 2.

= { 2, 4, 6, 8, ... }

O = Odd Natural Numbers

= Set of natural numbers that are not divisible by 2.

= { 1, 3, 5, 7, 9, ... }

Almost every set can be expressed in all the three methods (forms) discussed above.

For example :

(i)   If A = Set of odd natural numbers less than 15   [Description Form]

A = { 1, 3, 5, 7, 9, 11, 13 }   [Roster Form]

and A = { x : x is an odd natural number less than 15 }   [Set-Builder Form]

(ii)   If B = { x : x is a letter in the word 'MEERUT' }   [Set-Builder Form]

B = Set of letters used in the word 'MEERUT'   [Description Form]

and B = { m, e, r, u, t }   [Roster Form]

(iii)   If P = { 5, 10, 15, 20, ... }   [Roster Form]

P = { x : x is a natural number divisible by 5 }   [Set-Builder Form]

and, P = {natural numbers divisible by 5}   [Description Form]

EXERCISE 26(C)

1. Write each of the following sets in the **Roster Form** :
   - (i)   The set of five numbers each of which is divisible by 3.
   - (ii) The set of integers between – 4 and 4.
   - (iii) { x : x is a letter in the word 'SCHOOL' }
   - (iv) { x : x is an odd natural number between 10 and 20 }
   - (v)   { Vowels used in the word 'AMERICA' }
   - (vi) { Consonants used in the word 'MADRAS' }

2. Write each given set in the **Roster Form** :
   - (i)   All prime numbers between 1 and 20.
   - (ii)   The squares of the first four natural numbers.
   - (iii) Even numbers between 1 and 9.
   - (iv)   The first eight letters of the English alphabet.
   - (v)   The letters of the word 'BASKET'.
   - (vi)   Four cities of India whose names start with the letter J.
   - (vii) Any four closed geometrical figures.
   - (viii) Vowels used in the word 'MONDAY'.
   - (ix)   Single digit numbers that are perfect squares as well.

3. Write each given set in the **Set-Builder Form** :
   - (i)   { 2, 4, 6, 8, 10 }   (ii) { 2, 3, 5, 7, 11 }
   - (iii) { January, June, July }   (iv) { a, e, i, o, u }
   - (v)   { Tuesday, Thursday }   (vi) { 1, 4, 9, 16, 25 }
   - (vii) { 5, 10, 15, 20, 25, 30 }

4. Write each of the following sets in **Roster** (tabular) **Form** and also in **Set-Builder Form** :
   - (i)   Set of all natural numbers that can divide 24 completely.
   - (ii) Set of odd numbers between 20 and 35.
   - (iii) Set of letters used in the word 'CALCUTTA'.
   - (iv) Set of names of the first five months of a year.
   - (v)   Set of all two-digit numbers that are perfect squares as well.

5. Write, in **Roster Form,** the set of :
   - (i)   the first four odd natural numbers each divisible by 5.
   - (ii) the counting numbers between 15 and 35; each of which is divisible by 6.
   - (iii) the names of the last three days of a week.
   - (iv) the names of the last four months of a year.

Revision Exercise (Chapter 26)

1. State which of the following collections form a set :
   - (i) collection of odd natural numbers less than 25 and divisible by 3.
   - (ii) collection of the six most beautiful girls of your school.

(iii) collection of any three colours of a rainbow.

(iv) collection of all the rich people of Delhi.

(v) collection of all the integers between –10 and 5.

2. Write each of the following sets in tabular form :

   (i) { whole numbers between –6 and 6 }

   (ii) collection of consonants used in the word "UTTAR PRADESH".

   (iii) collection of all two-digit numbers the sum of whose digits is 8.

   (iv) { prime factors of 240 }.

3. Write each of the following sets in roster form as well as in set builder form :

   (i) set of factors of 48.

   (ii) set of integers between –3 and 8.

   (iii) set of integers from –3 to 8.

4. Write each of the following sets in tabular form as well as in discriptive form :

   (i) { x : x is a natural number and x < 10 }

   (ii) { x : x is a whole number as –3 < x < 4 }

   (iii) { x : x is a whole number, x > 30 and x is divisible by 3 }

   (iv) { x : x is a letter in the word RESONANCE }

   (v) { x : x is a whole number and x is divisible by 5 }

   (vi) { x : x is a prime number that lies between 10 and 30 }

5. Write down the **elements** of :

   (i) set A, if set A contains the squares of the first five whole numbers.

   (ii) set B, if set B contains the cubes of first three even natural numbers.

   (iii) set C, if set C contains whole numbers between 20 and 40; all divisible by 4.

   (iv) set D, containing natural numbers between 15 and 40; each divisible by 3 or by 4.

   (v) set E, containing natural numbers between 15 and 40; each divisible by 3 and 4.

6. If M is the set of the names of months with less than 31 days, write the set M in roster form.

7. If P is the set of factors of 18, express P in tabular form.

8. Represent each of the following sets in tabular form :

   (i) Set of even natural numbers between 10 and 24.

   (ii) Set of even natural numbers from 10 to 24.

   (iii) Set of letters in the word PINEAPPLE.

   (iv) Set of the prime factors of 40.

   (v) Set of all two-digit numbers divisible by 12.