Support Vector classifier indepth Maths

① Eqn of line, plane, hyperplane.

$y = mx + c$

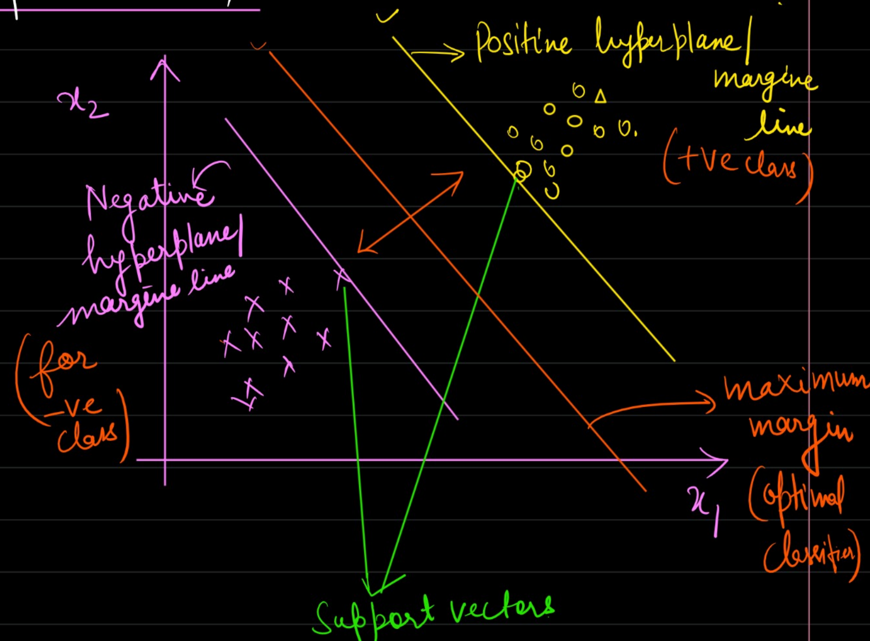
$y = \theta_0 + \theta_1 x_1$

$ax + by + c = 0$

$by = -ax - c$

$y = \boxed{\frac{-a}{b}}\, x - \boxed{\frac{c}{b}}$

$y = \underset{m}{m}x + \underline{c}$



in morethan 3d

$y = \theta_0 + \theta_1 x_1$

$y = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \dots \theta_n x_n \longrightarrow \underline{\text{hyperplane}}$

$y = b + w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots w_n x_n$

b - bias
w - weights

$y = b + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4$

$$W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$W^T = [w_1 \ w_2 \ w_3 \ w_4] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \Rightarrow w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + b$$

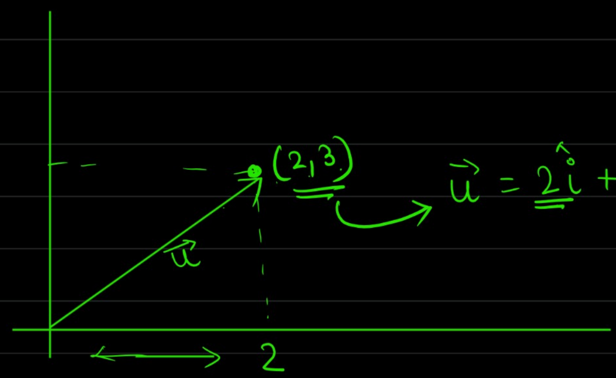
W^T X

$$\boxed{y = w^T x + b} \quad \left(y = mx + c \quad ax + by + c = 0 \right)$$

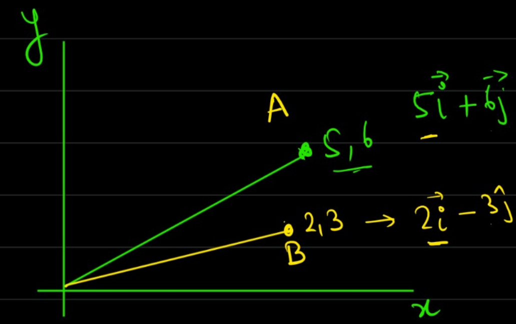
$$\Downarrow$$

$$w^T x + b = 0$$

2



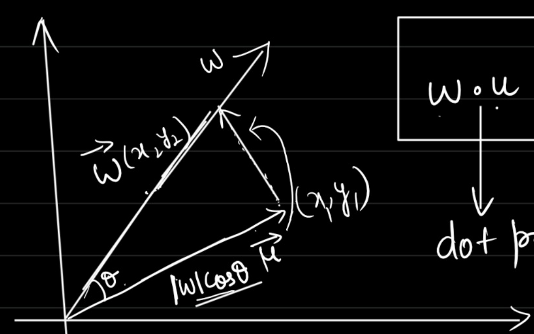
$\vec{u} = 2\hat{i} + 3\hat{j}$



3 Vector subtraction $\vec{A} - \vec{B}$

$$(5-2)\hat{i} + (6-3)\hat{j} = 3\hat{i} + 3\hat{j}$$

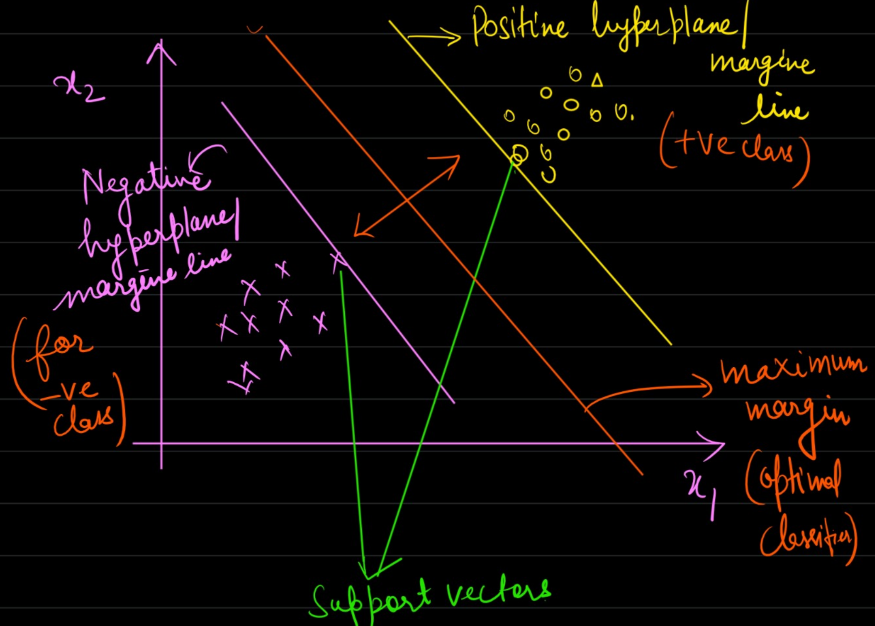
4 Dot product of vectors.



magnitude of w, magnitude of u

$$\boxed{w \cdot u = |w| \cos\theta \cdot |u|}$$

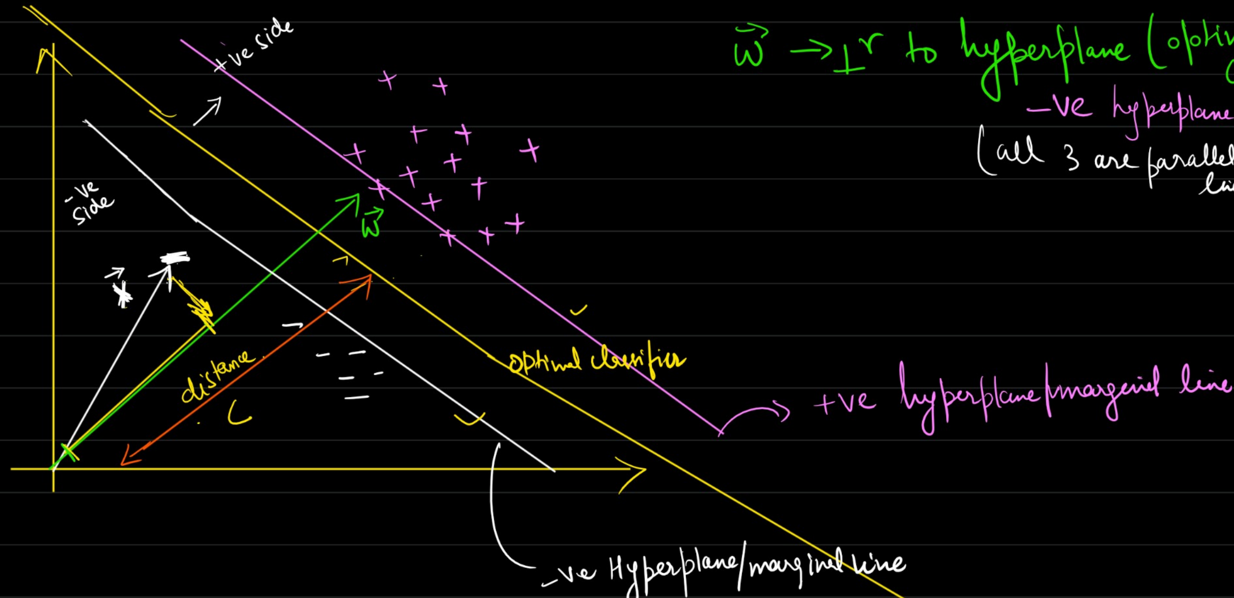
dot product means projection of $\vec{u}$ on $\vec{w}$



$\vec{w} \rightarrow \perp^r$ to hyperplane (optimal classifier)

−ve hyperplane / +ve hyperplane

(all 3 are parallel lines)

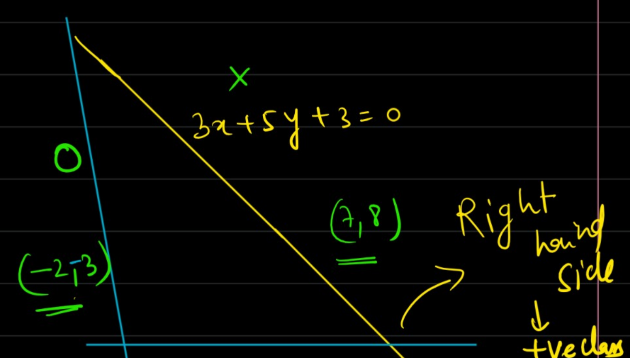


Project is dot product.

$\vec{X} \cdot \vec{w} = (c)$ distance (the point lies on decision boundary)

$\vec{X} \cdot \vec{w} > (c)$ distance (positive class)

$\vec{X} \cdot \vec{w} < (c)$ distance (negative class)

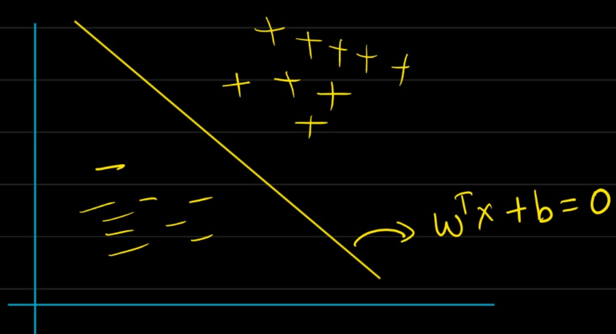


$(7,8) \Rightarrow 3\times7 + 5\times8 + 3 > 0$

$(-2,-3) \Rightarrow 3\times-2 + 5\times-3 + 3 < 0$

−ve side (left side)

−ve class.



$\vec{X} \cdot \vec{w} \geq C$

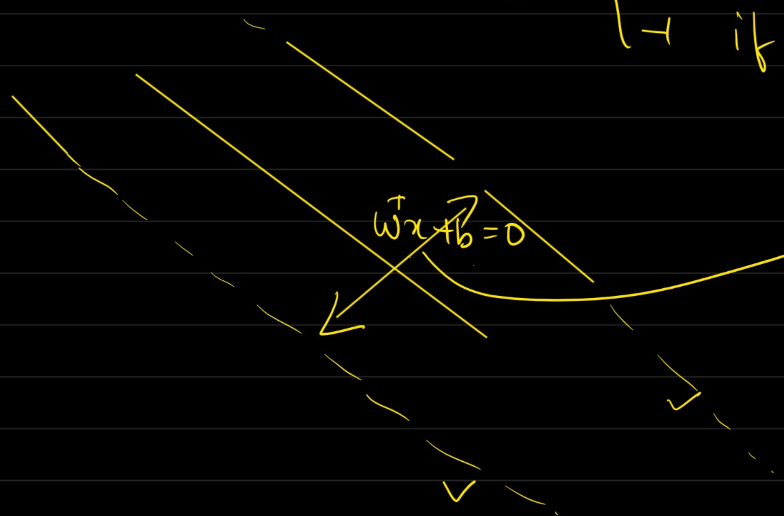
$\vec{X} \cdot \vec{w} - C \geq 0$

$-C$ as b

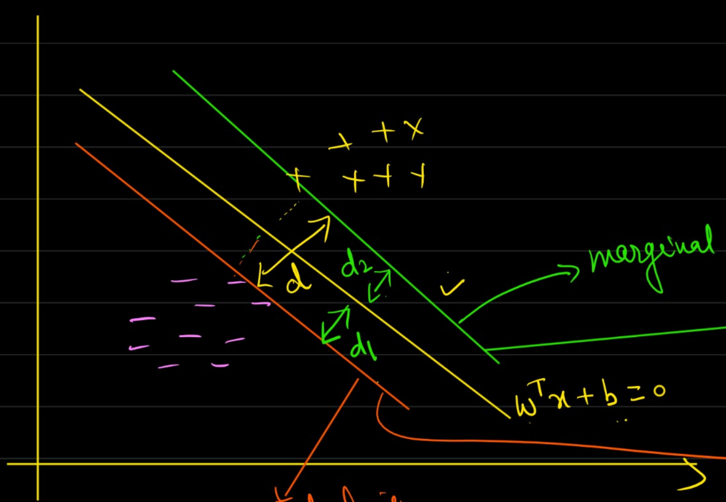
$\vec{X} \cdot \vec{w} + b \geq 0 \longrightarrow$ +ve point

$\vec{x} \cdot \vec{w} + b \geq 0$

$$y = \begin{cases} +1 & \text{if } \vec{x} \cdot \vec{w} + b \geq 0 \\ -1 & \text{if } \vec{x} \cdot \vec{w} + b < 0 \end{cases}$$



→ d is maximum



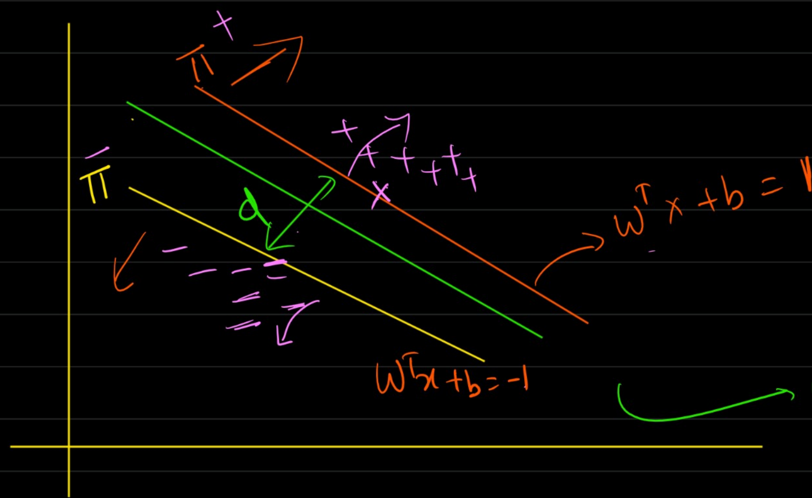
for positive class +1
for negative class −1

* Why equal (both 1)? → d_1 & d_2 should be equidistant (optimal line should pass through center of margin.)

* Why only 1? → It doesn't make a difference

Both will be same line ⟵ $\begin{cases} 2x + y = 1 \\ 2x + y = 2 \end{cases}$

→ even if we multiply the whole equation with some other number the line doesn't change



* We want calculate distance (d) such that no positive or negative point can cross the margin line

+1 for +ve class dps
$$\vec{w}^T \cdot \vec{x} + b \geq 1$$

-1 for -ve class datapoints
$$\vec{w}^T \cdot \vec{x} + b \leq -1$$

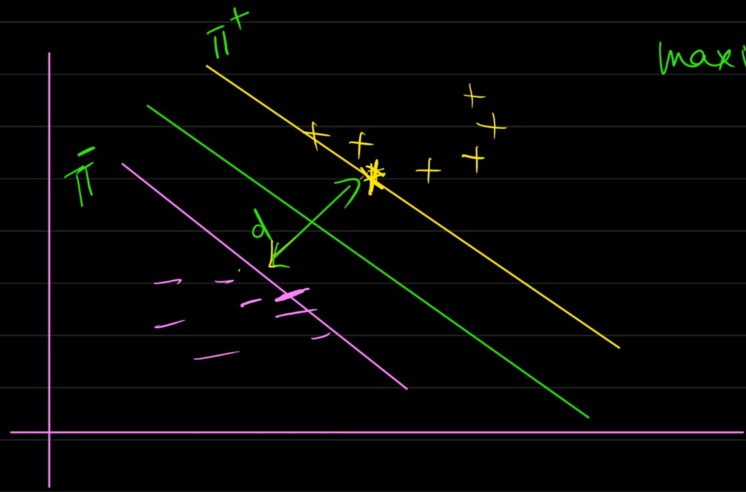
We want to maximize 'd' such that this constraint holds true

for +ve class → $\underset{+1}{y_i} (\underbrace{\vec{w}^T \cdot \vec{x}}_{+ve} + b) \geq 1$

for -ve class = $y_i (\vec{w}^T \vec{x} + b) \leq -1$

$= +1 (\vec{w}^T \cdot \vec{x} + b) \geq +1$

$= y_i (\vec{w}^T \cdot \vec{x}_i + b) \geq 1$



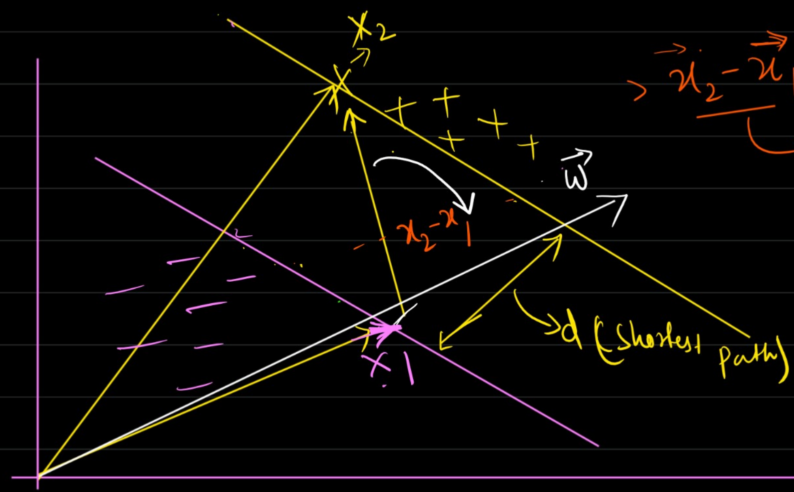
maximise d

$$y_i(\vec{w}^T\cdot\vec{x}+b) \geq 1$$

for support vector

$$y_i(\vec{w}^T\cdot\vec{x}+b) = 1$$

equality because Support Vectors falls on marginal hyperplane.



→ $\vec{x}_2 - \vec{x}_1$ → b/w vectors x_2, x_1

→ To get shortest distance we need a unit vector ⊥r to all marginal hyperplane

→ Projection of $\vec{x}_2 - \vec{x}_1$ on Unit Vector $\vec{w}$ to get d

$$d = (\vec{x}_2 - \vec{x}_1)\cdot\frac{\vec{W}}{\|W\|}$$

$$\frac{x_2\cdot\vec{w} - \vec{x}_1\cdot w}{\|W\|} \quad \text{— ①}$$

(x_1 & x_2 are support Vectors, they lie on marginal hyperplane)

Since $x_1 x_2$ are Support Vectors, it should follow

$$y_i(\vec{w}\cdot\vec{x}+b) = 1$$

for +ve class = $y_i = 1$ for x_1

$$1 \times (w^T x_1 + b) = 1$$

$$\vec{w}\vec{x}_1 = 1 - b \quad \text{— ②}$$

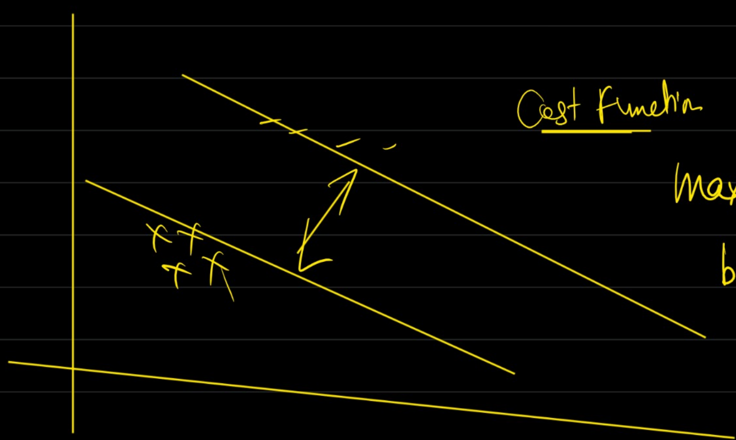
for -ve class $-1(w^T x_2 + b) = 1$

$$\vec{w}^T\cdot\vec{x}_2 = -b - 1 \quad \text{— ③}$$

Putting eqn ② & ③ in ①

$$\frac{(1-b)-(-b-1)}{\|W\|} = \frac{2}{\|W\|} = d$$

$$\underset{(w,b)}{\text{maximise}} \ \frac{2}{\|W\|} \quad \text{such that} \quad y_i\left(\vec{W}^T \cdot \vec{X} + b\right) \geq 1$$



Cost Function

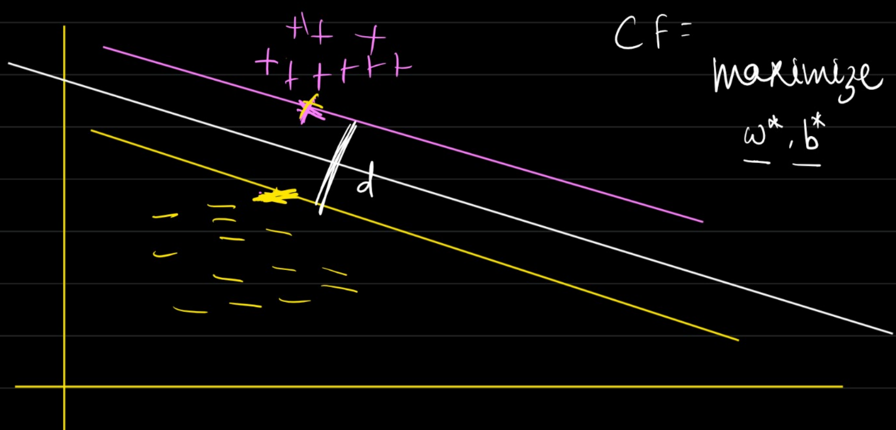
$$\text{maximise} \ \frac{2}{\|W\|}$$

by changing w, b (intercept / coeff)

* Modified Cost fn for Hardmargin SVC

$$\text{minimise} \ \frac{\|W\|}{2} \quad \text{by varying } w \ \& \ b$$

Constraint $y_i(w^T x_i + b) \geq 1$



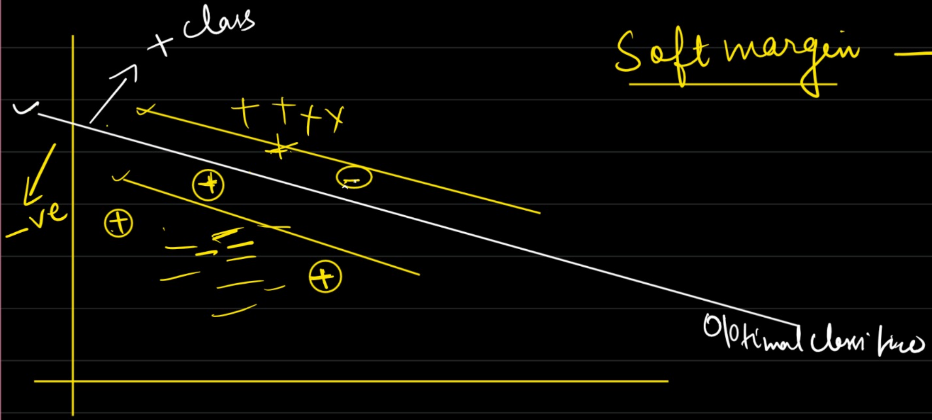
$$CF = \underset{w^*, b^*}{\text{maximize}} \ \frac{2}{\|W\|}$$

Constraint $y_i\,(w^T x_i + b) \geq 1$

$$\max f(x) \Leftrightarrow \min \frac{1}{f(x)}$$

$$CF = \min_{w^* b^*} \frac{\|w\|}{2}$$

$$y_i(w^T x_i + b) \geq 1$$



<u>Soft margin</u> — No of datapoint you want to sacrifice / error datapoint / misclassified datapoint because such distinguishable scenario is not possible, there will be some overlapping datapoints.

→ Error dp <<< correctly classified dp's

<u>CF for Soft margin SVC</u>

$$\underline{\min_{w^* b^*} \frac{\|W\|}{2}} + C \sum_{i=1}^{n} \xi_i$$

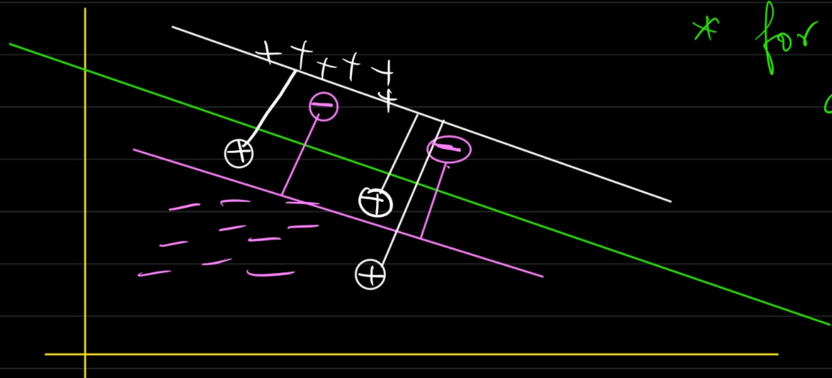
ξ → Zeta

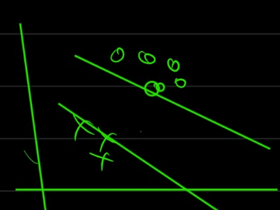
Such that $y_i(w^T x_i + b) \geq 1$

C ↓ no of misclassified dp's (hyperparameter).

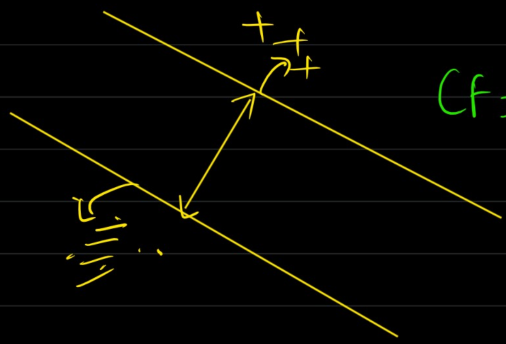
ξ → is the distance of all misclassified dp's to <u>correct marginal plane</u>.

* for all correctly classified dp's ⇒ hard margin $\xi = 0$





→ higher the d ⇒ distance b/w the hyperplanes of two Class, lower the error



Hinge loss

$$CF = \underbrace{\frac{\|W\|}{2}}_{\text{SVM error}} + \underbrace{C \sum_{i=1}^{n} \xi_i}_{\text{classification error}}$$

Such that $y_i(W^T x_i + b) \geq 1$

$$\boxed{C = \frac{1}{\lambda}}$$