

Naïve Baye's Algorithm (only for classification problem)

- ① Probability
- ② Baye's theorem.

* Probability \Rightarrow Probability is how likely something is to happen.

$$P = \frac{\text{No of favourable outcomes}}{\text{Total no of outcomes}}$$

$$P(H) = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

→ Independent Events → Two events are independent if the outcome of one event does not affect the outcome of other event.

○ ○ ○	3 yellow balls
○ ○	2 green balls

$$P(Y) = \frac{3}{3+2} = \frac{3}{5}$$

$$\Rightarrow P(G) = \frac{2}{2+3} = \frac{2}{5}$$

* After picking the ball for first time, you are returning the ball to the bag and due to this the probabilities remained the same for second selection.

* Dependent Events

Two events are dependent if the outcome of one event affects the outcome of the other event.

○ ○ ○
○ ○

What is the probability of picking a yellow ball and then a green ball?

$\boxed{\text{Step-1}} \quad p \text{ of picking a yellow ball} \Rightarrow P(Y) = \frac{3}{5} \rightarrow \text{Event 1}$
 $\boxed{\text{Step-2}} \quad p \text{ of picking green ball when yellow ball has been already picked} \Rightarrow P(G|Y) = \frac{2}{4} \rightarrow \text{Event 2}$

$P(G|Y) \Rightarrow \text{Probability of Green given Yellow}$

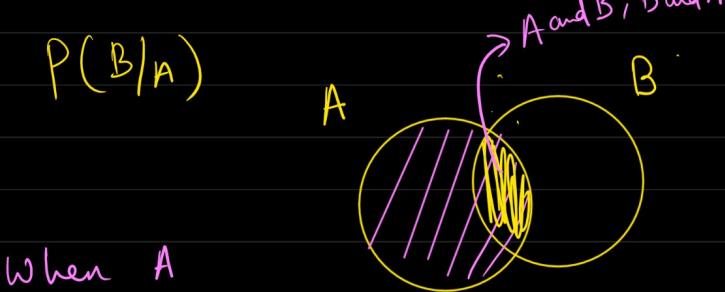
$P(Y \text{ and } G) = \text{Combine both the steps / events.}$

$$= P(Y) * P(G|Y) = \frac{3}{5} * \frac{2}{4} \Rightarrow \frac{3}{10}$$

↑ 1st event ↑ 2nd event.

$$P(A \text{ and } B) = P(\underline{A}) * P(\underline{B/A}) \rightarrow \text{conditional probability.}$$

$P(B/A)$ = Probability of event B,
 given A event has
 already occurred $\underline{=}$



When A has already occurred, the share of B will be the overlapping part.

$$P = \frac{\text{Favorable outcome}}{\text{Total outcomes}} = \frac{P(A \text{ and } B)}{P(A)} = P(B/A)$$

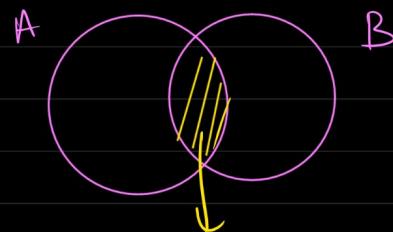
$$P(A \text{ and } B) = P(B/A) * P(A)$$

* Bayes' theorem

$$P(A \text{ and } B) = P(B \text{ and } A)$$

$$P(B/A) \cdot P(A) = P(A/B) \cdot P(B)$$

$$\boxed{P(A/B) = \frac{P(A) * P(B/A)}{P(B)}} \quad \boxed{\Rightarrow \text{Bayes' theorem.}}$$



A and B or

B and A

A n B or
B n A

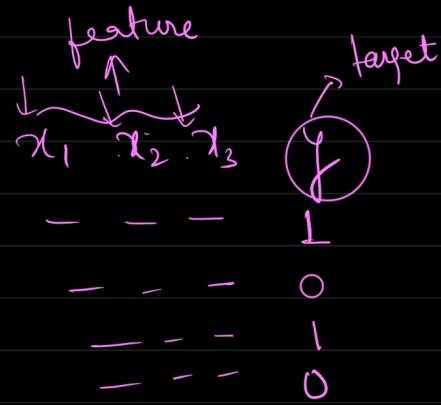
$P(A|B) \Rightarrow$ Prob. of Event A given B has occurred

$P(A) \Rightarrow$ Prob. of Event A

$P(B) \Rightarrow$ Prob. of Event B

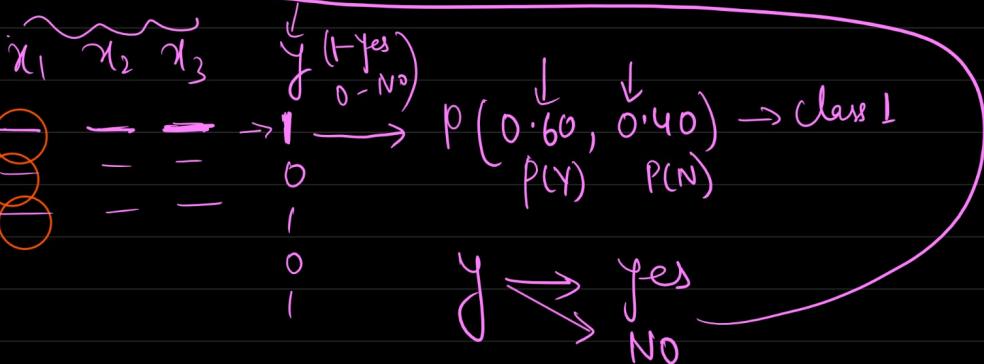
$P(B/A) \Rightarrow$ Prob. of Event B given A has occurred.

$$\boxed{P(A|B) = \frac{P(A) * P(B/A)}{P(B)}}$$



$$P(Y/x_1, x_2, x_3) = \frac{P(Y) * P(x_1, x_2, x_3 | Y)}{P(x_1, x_2, x_3)}$$

$$\boxed{P(Y/x_1, x_2, x_3) = \frac{P(Y) * P(x_1|Y) * P(x_2|Y) * P(x_3|Y)}{P(x_1) * P(x_2) * P(x_3)}}$$



Y has only two possibility \leftarrow Yes
No

$$\hookrightarrow P(\text{Yes} | (x_1, x_2, x_3)) = \frac{P(\text{Yes}) * P(x_1 | \text{yes}) * P(x_2 | \text{yes}) * P(x_3 | \text{yes})}{P(x_1) * P(x_2) * P(x_3)} = 0.70$$

$$\hookrightarrow P(\text{No} | (x_1, x_2, x_3)) = \frac{P(\text{No}) * P(x_1 | \text{No}) * P(x_2 | \text{No}) * P(x_3 | \text{No})}{P(x_1) * P(x_2) * P(x_3)} = 0.30$$

$P(x_1) * P(x_2) * P(x_3) \rightarrow \text{constant}$

$$\begin{aligned}\text{predicted class} &= \max \text{ class of Prob.} \\ &= (0.70, 0.30) \\ &\quad \downarrow \\ &\quad \perp\end{aligned}$$

* For multiclass classification

$$P(C_k | X) = \frac{P(X | C_k) * P(C_k)}{P(X)}$$

$$\left\{ \begin{array}{l} P(C_1 | x_1, x_2, x_3) = P(C_1) * \frac{P(x_1 | C_1) * P(x_2 | C_1) * P(x_3 | C_1)}{P(x_1) * P(x_2) * P(x_3) \rightarrow \text{const}} \\ P(C_2 | x_1, x_2, x_3) = () \\ P(C_3 | x_1, x_2, x_3) \Rightarrow P(C_3) * \frac{P(x_1 | C_3) * P(x_2 | C_3) * P(x_3 | C_3)}{\text{const.}} \end{array} \right.$$

maximum probability will be predicted class.

Outlook	Temperature	Humidity	Wind	Play
Sunny	Hot	High	Weak	No
Sunny	Hot	High	Strong	No
Overcast	Hot	High	Weak	Yes
Rain	Mild	High	Weak	Yes
Rain	Cool	Normal	Weak	Yes
Rain	Cool	Normal	Strong	No
Overcast	Cool	Normal	Strong	Yes
Sunny	Mild	High	Weak	No
Sunny	Cool	Normal	Weak	Yes
Rain	Mild	Normal	Weak	Yes
Sunny	Mild	Normal	Strong	Yes
Overcast	Mild	High	Strong	Yes
Overcast	Hot	Normal	Weak	Yes
Rain	Mild	High	Strong	No

$$P(Y|X_1, X_2, X_3) = P(Y) * \frac{P(X_1|Y) * P(X_2|Y) * P(X_3|Y)}{P(X_1) * P(X_2) * P(X_3)}$$

$$= \frac{P(N|X_1, X_2, X_3)}{P(X_1) * P(X_2) * P(X_3)}$$

$$P(N|X_1, X_2, X_3) = P(N) * \frac{P(X_1|N) * P(X_2|N) * P(X_3|N)}{P(X_1) * P(X_2) * P(X_3)}$$

const.

Outlook	Yes	No	P(E y)	P(E No)
Sunny	2	3	2/9	3/5
Overcast	4	0	4/9	0
Rain	3	2	3/9	2/5
Total	9	5	1	1

Temperature

	Yes	No	P(E y)	P(E No)
Hot	2	2	2/9	2/5
Mild	4	2	4/9	2/5
Cool	3	1	3/9	1/5
Total	9	5	1	1

$$P(\text{play}(Y|N))$$

$$\begin{array}{ccccc} & \text{Yes} & & 9 & P(\text{Yes}) = 9/14 \\ & \text{No} & & 5 & P(\text{No}) = 5/14 \end{array}$$

Test day (Sunny, Hot) → output

$$P(Y|X_1, X_2, X_3) = P(Y) * \frac{P(X_1|Y) * P(X_2|Y) * P(X_3|Y)}{P(X_1) * P(X_2) * P(X_3)}$$

$$\Rightarrow P(\text{Yes}/\text{Sunny, Hot}) =$$

$$P(\text{Yes}) * P(\text{Sunny}| \text{Yes}) * P(\text{Hot}| \text{Yes})$$

$$P(N|X_1, X_2, X_3) = P(N) * \frac{P(X_1|N) * P(X_2|N) * P(X_3|N)}{P(X_1) * P(X_2) * P(X_3)}$$

$$\Rightarrow \frac{9}{14} * \frac{2}{9} * \frac{2}{9} = \frac{2}{63} = 0.031$$

$$* P(\text{No}/\text{Sunny, Hot}) = P(\text{No}) * P(\text{Sunny}| \text{No}) * P(\text{Hot}| \text{No})$$

$$= \frac{5}{14} * \frac{3}{5} * \frac{2}{5} = \frac{3}{35} = 0.085$$

* Normalise the prob. to get in percentage form:-

$$P(Y \mid \text{Sunny, Hot}) = \frac{0.03}{0.03 + 0.085} = 0.27 \Rightarrow 27\%$$

$$P(\text{No Sun, Hot}) = \frac{0.085}{0.031 + 0.085} = 0.73 = 73\%$$

Test data — Sunny Temp hot opp
73°. ~~1~~ → $P(\text{No} | \text{Sunny}, \text{Hot})$
27°.

The person will not go out to play

* Why Naive Bayes ?

Bayes' theorem, so Bayes'

Why Naive? \Rightarrow It assumes that the features are independent of each other; not dependent on other factors. This is why it is called Naive.

→ No Multicollinearity.
↳ collinear

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)}$$

↑ likelihood

\Downarrow

Posterior Probability
(Post A)
or
updated B

\Downarrow

evidence

* Advantage

- based on probability
- faster Algorithm
- Works well for less training data
- Performance is good

* disadvantage

- It assumes that the features are independent

Application → Text Classification, Sentiment Analysis, Recommendation System.