HS5004 Econometrics – Term Paper

A study on predicting resale price of used cars

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Abstract

As the production of cars go up each year, the sales of used cars also contribute a significant portion of income for the automobile industry. With multiple start-ups focusing on the used car segments with renting and subscription models, it becomes an important part of their business to calculate the price of the vehicle after it has been used for a certain period. This study aims to achieve the task by using the *Ordinary Least Square (OLS)* regression over multiple independent variables.

Motivation

The used car market is an ever-rising industry with rapidly growing market share and multiple companies like *Zoom Car*, *Car Dekho* etc. competing to make their mark in this segment. While these companies also have subscription models, those cars would eventually be sold at the end of a stipulated period.

Deciding the selling price of used cars and buyers ascertaining the worth of the posted prices on online sites can prove to be difficult due to various factors involved in finding the resale price. This study uses the OLS estimator to ascertain the retail value of the car on the date of sale.

Dataset

For this study, we use the dataset from *Car Dekho* dataset on Kaggle that is referenced at the end of this document. This dataset consists of 9 columns that are discussed below:

1. **Car_Name**: The name of the car that is under consideration. It is a text column with multiple values.
2. **Year**: This column represents the year in which the car was bought. It is an integer column of the form YYYY.
3. **Selling_Price**: This is the selling price of the car. This variable is our dependent variable, and we need to predict this variable using the OLS estimator.
4. **Present_Price**: This column tells the present price of the car i.e., the current price of the car when bought as the first-hand user.
5. **Kms_Driven**: This column tells the total distance that the car has travelled from the date of purchase of the car.
6. **Transmission**: This column tells if the car is a manual of automatic transmission.
7. **Owner**: It tells the number of owners that the car has previously had.

Dataset statistics

In this section, we will be seeing some important statistics about the raw dataset which will help us decide the pre-processing steps that has to be performed.

The dataset has a total of 301 rows and 9 columns. Various statistics about the 9 columns are listed in Table 1.

Table 1: Basic data statistics

#	Column	Non-null count	Data type
1	Car_Name	301	string
2	Year	301	integer
3	Selling_Price	301	float
4	Present_Price	301	float
5	Kms_Driven	301	int
6	Fuel_Type	301	Enum {Petrol | Diesel | CNG}
7	Seller_Type	301	Enum {Dealer | Individual}
8	Transmission	301	Enum {Manual | Automatic}
9	Owner	301	int

Table 2: An example row from the dataset

Column	Value
Car_Name	Bajaj Avenger 220
Year	2017
Selling_Price (in Lakhs)	0.90
Present_Price (in Lakhs)	0.950
Kms_Driven	1300
Fuel_Type	Petrol
Seller_Type	Individual
Transmission	Manual
Owner	0

Exploratory Data Analysis (EDA)

EDA is used in data science projects to get closer to certainty that the future results will be valid, correctly interpreted, and applicable to the desired problem statement. This is done by validating the raw data, checking for anomalies, and ensuring the dataset was collected without errors and noise.

Basic statistics

Table 3: Statistics of numerical columns in the dataset

	selling_price	present_price	kms_driven	age
count	301.000000	301.000000	301.000000	301.000000
mean	4.661296	7.628472	36947.205980	8.372093
std	5.082812	8.644115	38886.883882	2.891554
min	0.100000	0.320000	500.000000	4.000000
25%	0.900000	1.200000	15000.000000	6.000000
50%	3.600000	6.400000	32000.000000	8.000000
75%	6.000000	9.900000	48767.000000	10.000000
max	35.000000	92.600000	500000.000000	19.000000

The above statistics show a basic analysis for the range and the spread of the dataset. These values can be analysed and looking at the 75th percentile and max values, we can expect that the dataset has outliers which need to be removed.

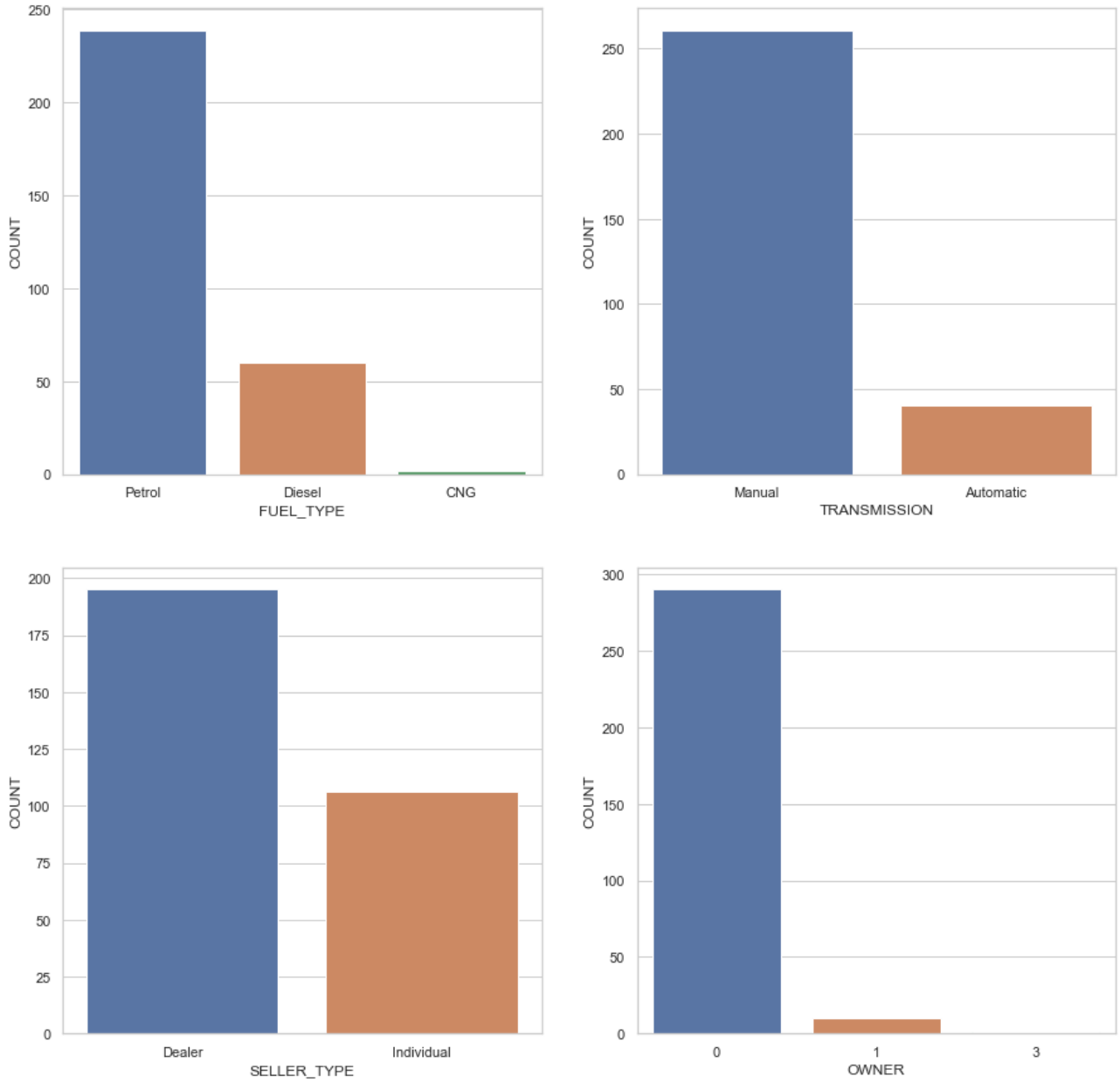
Univariate analysis

This dataset has five categorical variables, namely, *Car_Name, Fuel_Type, Seller_Type, Transmission* and *Owner.* Just by some simple inspection, it becomes very apparent that having *Car_Name* feature in the training set would lead the model to overfit since it can learn a one-one mapping from car name to the selling price, so I would not be analysing this variable further.

The first step involves plotting the bar-plots for all the categorical variables and deduce some conclusions from it. The below bar plots show the count of each categorical variable (on Y-axis) and the feature under observation (on X-axis).

Although the dataset is skewed, as we will be seeing in the bi-variate analysis, the *Present_Price* variable has a good linear relationship with the *Selling_Price,* so we can expect to get a good performance with the dataset.

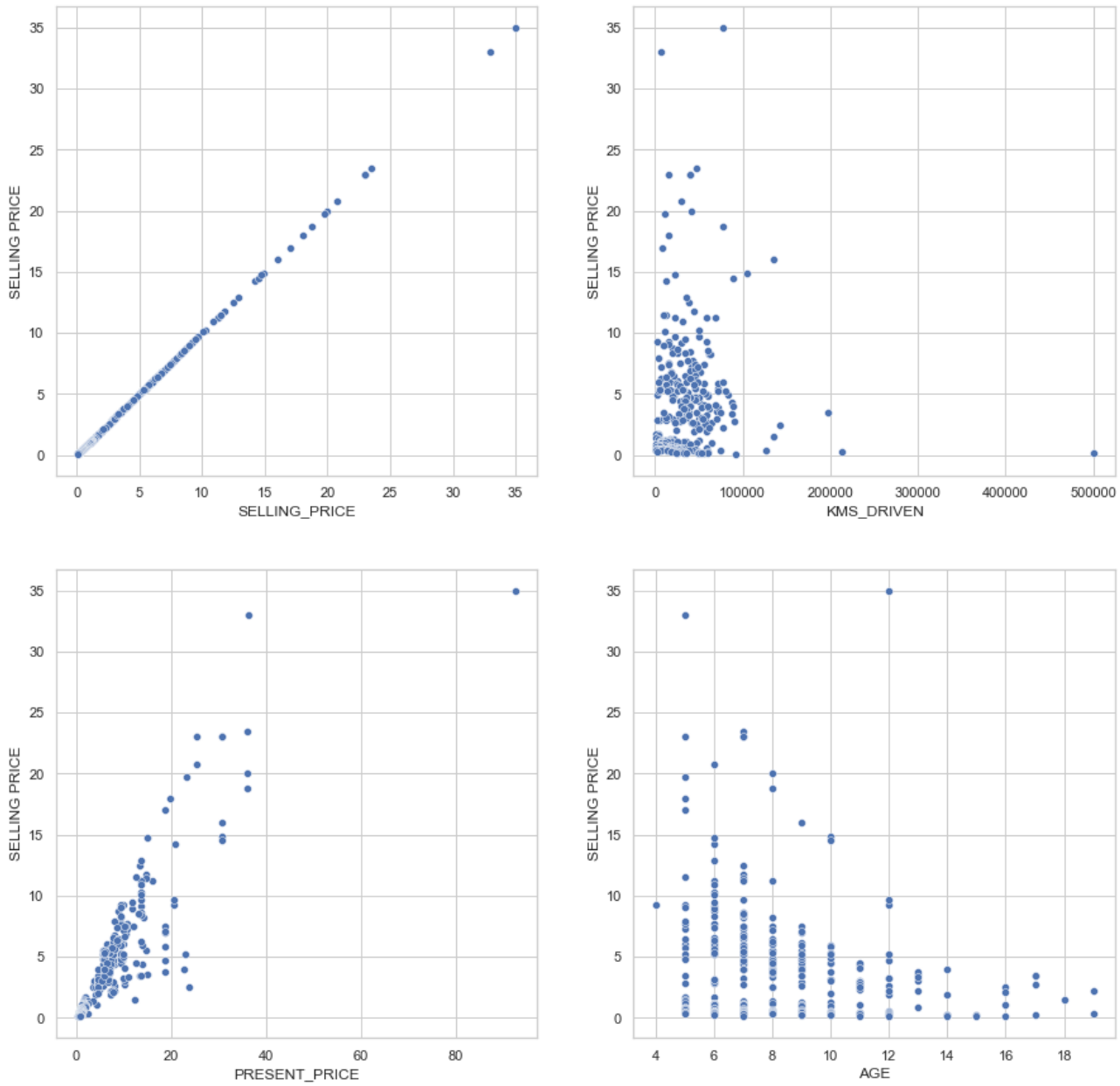
Figure 1: Blot plots between frequency and each categorical variable



Bivariate Analysis

In the previous section, we saw some bar-plots of each categorical variables, in this section we will be plotting the numerical variables against the *Selling_Price* to see what kind of relationships they exhibit. To perform this bivariate analysis, we will be seeing some scatter plots of the numerical variables.

Figure 2: Scatter plot between numerical features and the dependent variable



Note: The first image is plot with same features on both X and Y-axis, hence the straight line.

We can see that the *Present_Price* has a near-perfect linear relationship with the *Selling_Price*, this shows us that it will be influencing the *Selling_Price* to a large extent. Looking at the numerical plots, we can spot the outliers here too, so removing outliers will have to be considered as a necessary pre-processing step.

Possible pre-processing steps:

1. To make it uniform and to ease string comparisons (if any), I converted all the column names to lowercase, i.e., the column ***Owner*** becomes ***owner*** and the same applies to all other columns.
2. The ***Year*** column has values like 2014, 2015 etc., here it would make more sense to represent this column as the age of the vehicle as the price is influenced by the age of the vehicle. This can be done by subtracting the values from the current year (2022).
3. Since linear regression models take in numbers, it would be more appropriate to convert the enumerated columns (***Fuel_Type, Seller_Type, Transmission)*** to numbers.
   For ex: The ***Transmission*** column has two values, hence the value *Manual* can be assigned a value of 0 and the value *Automatic* can be assigned a value of 1.
4. Since ***Car_Name*** can be very different for each observation, it is not necessary to use it in the final training dataset, so this column can be dropped from the dataset.
5. From the analysis, we have seen that some columns have data outliers, and they will have to be removed from the dataset.

Gauss-Markov assumptions

To justify the usage of multiple linear regression, we need to satisfy the Gauss-Markov assumptions. These assumptions and the justification are given below:

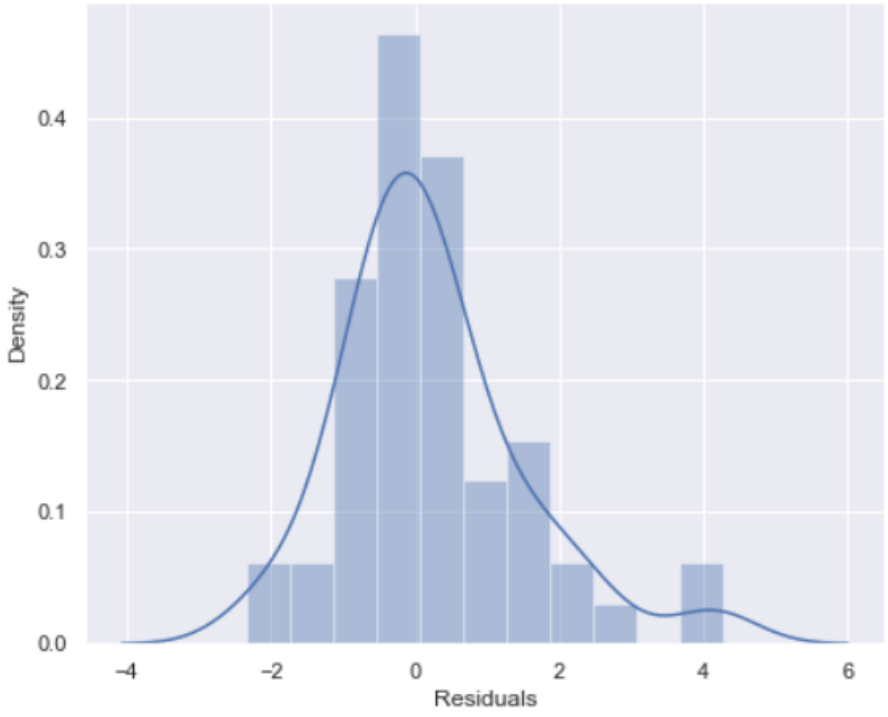
The model is linear in parameters and correctly specified

This assumption holds in our case since our model is linear in all variables in the dataset i.e., the model can be represented as $Y = \beta_1 + \beta_2 X_2 + \ldots + \beta_k X_k$

The disturbance has zero expectation

The below plot shows the plot of residuals vs their frequency. This plot shows that the residuals follow a normal distribution with 0 mean, hence this assumption is valid in our case.

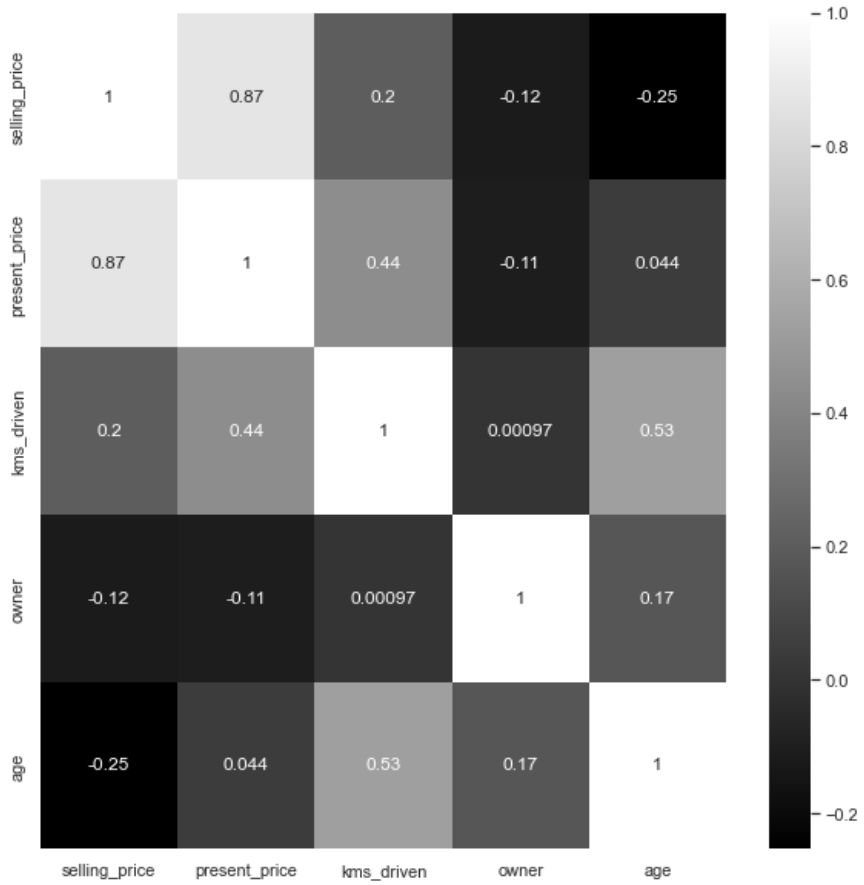
Figure 3: Plot of residuals obtained after training



No exist an exact linear relationship among the regressors (No multicollinearity)

This can be verified from the correlation plot. If two variables have a correlation factor of 1, then they are said to be in a perfect linear relation, but as we can see that none of the features are in perfect correlation with the *Selling_Price*.

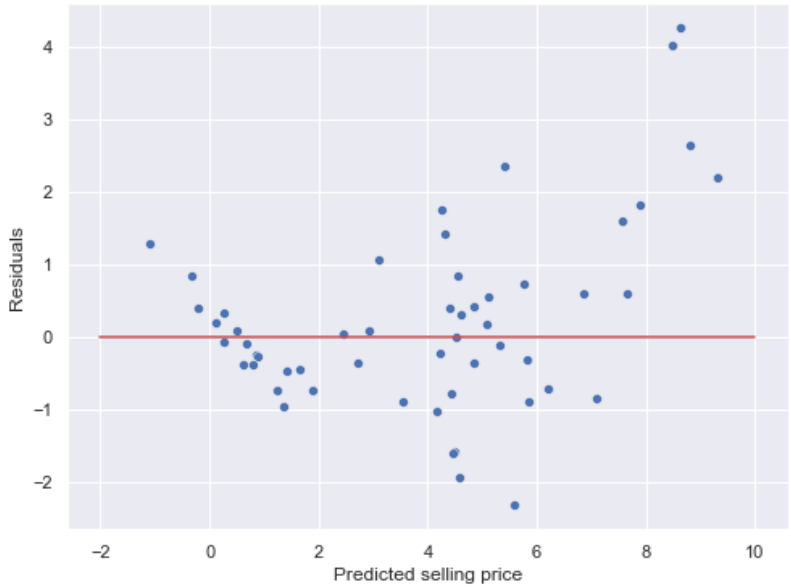
Figure 4: Correlation plot for all pairs of variables



The disturbance term is homoscedastic

This assumption states that our error terms have a constant variance (no heteroscedasticity). The figure below shows that the residuals are at a constant distance from the line except for some outliers, hence we are safe to conclude that our assumption of homoscedasticity holds.

Figure 5: Plot of residuals vs fitted values



The values of disturbance term have independent distributions

This dataset was published by the company *Car Dekho,* and we can assume that the data was randomly sampled based on the activities on their site, thus we satisfy this assumption. Also, from Figure 3, we can see that the residuals are randomly distributed along the horizontal line.

The disturbance term has a normal distribution

As we can see from Figure 3, the residuals or the disturbance term has a normal distribution with mean 0, hence this assumption holds.

We have shown that all the six assumptions of the Gauss-Markov theorem hold true in our case, hence the use of a multiple linear regression or the OLS regression model is justified.

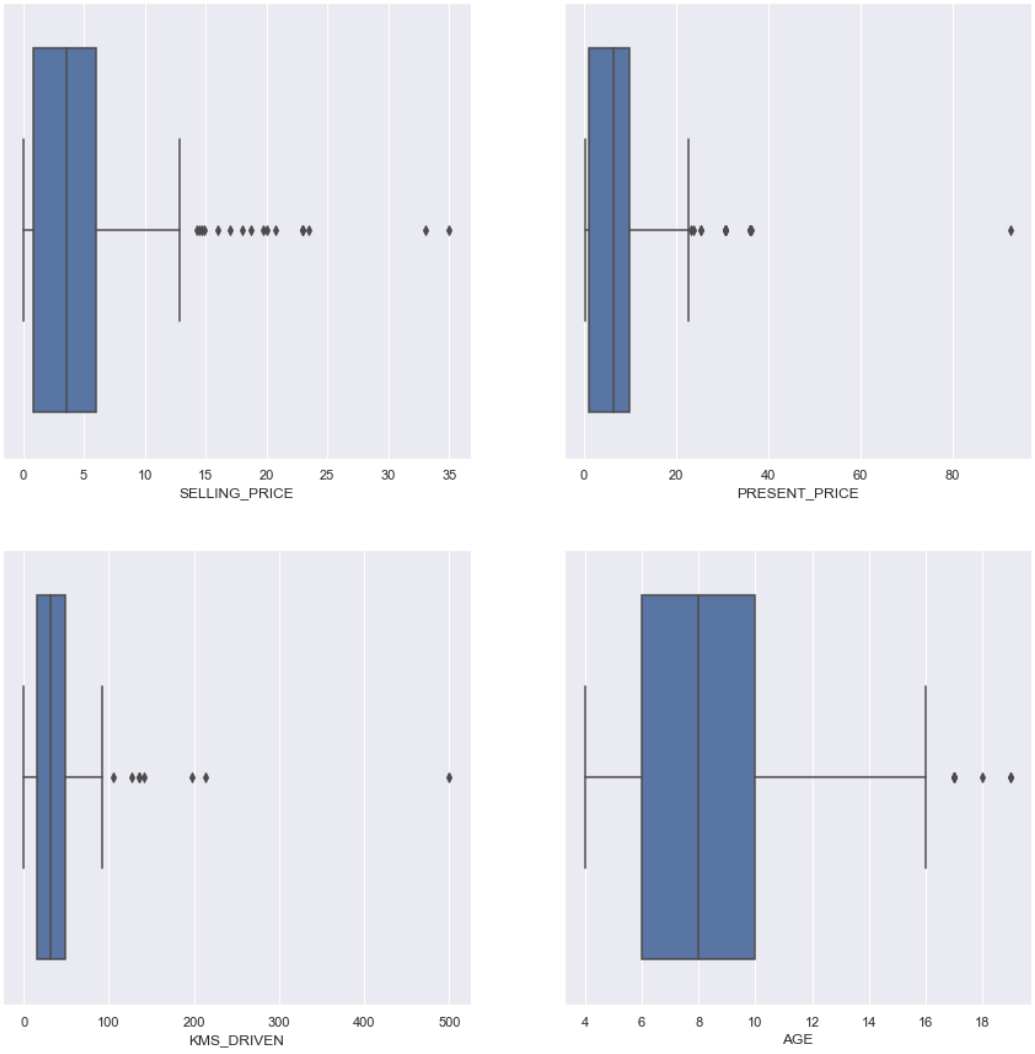
Processing dataset

Before any training task, it is wise to process the dataset by removing outliners, reducing multicollinearity, and performing other suitable steps depending on the dataset.

Removing data outliers

Outliers are those points in the dataset that are at the extremes with a very low density and usually indicate an erratic measurement, noise, or other unwanted factors. Such outliers disturb the accuracy of the model and lead to unreliable results.

Figure 6: Plot of numerical data before removing the outliers



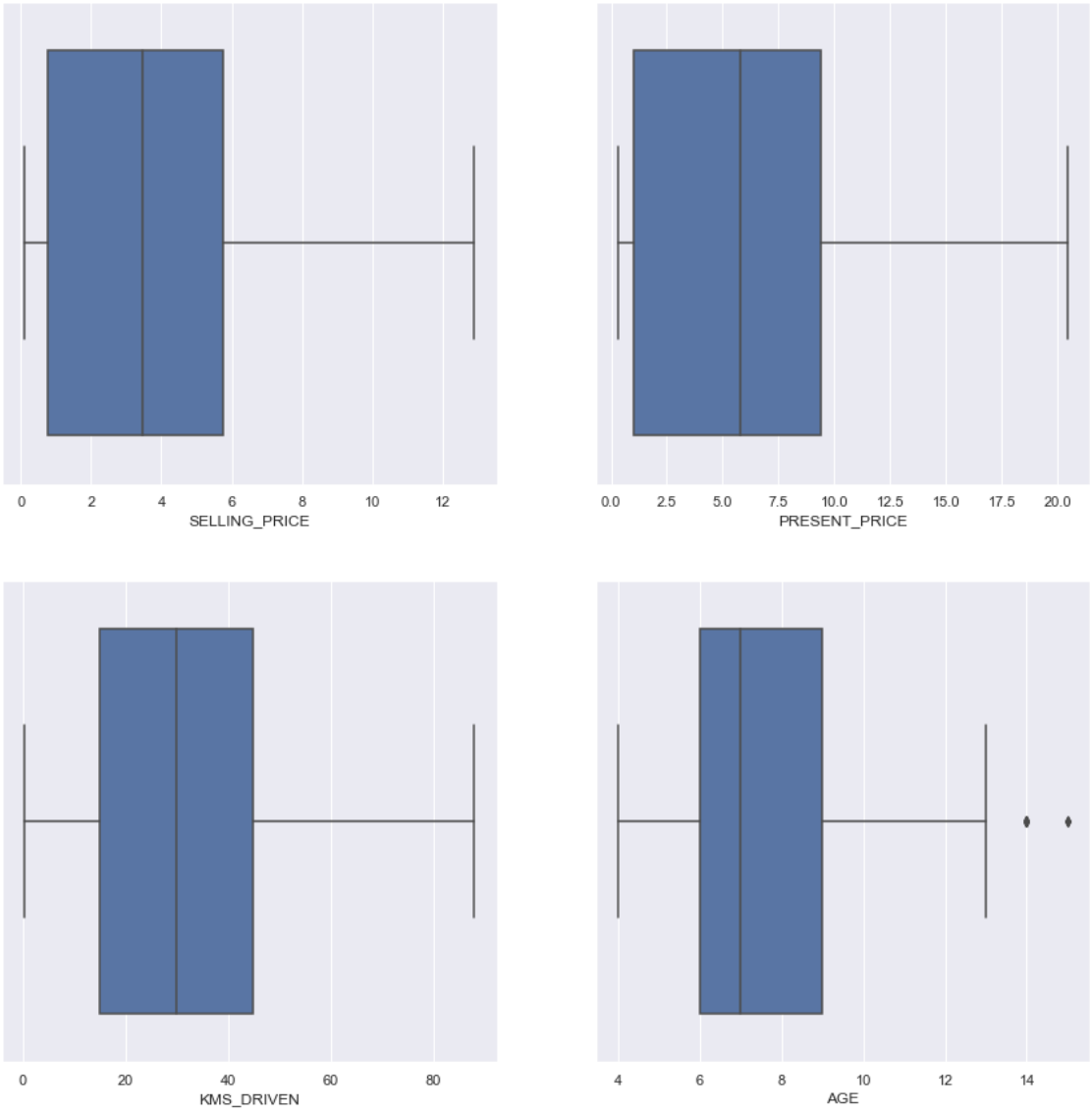
As we can see from Figure 6, there are some points that lie outside 1.5 x Inter Quartile range (IQR), we remove those points from the dataset to reduce noise.

The algorithm to remove outliers is as follows:

1. Find first quartile i.e., the 25th percentile (Q1).
2. Find the third quartile i.e., the 75th percentile (Q3).
3. The IQR is defined as Q3 – Q1.
4. Now, we remove all the points outside the range [Q1 – 1.5 * IQR, Q3 + 1.5 * IQR].

We can see from Figure 7 that almost all the outliers are removed from the desired columns. Thus, in this step, we have reduced unwanted discrepancies in the dataset by removing outliers.

Figure 7: Plots of numerical data after dropping the outliers



Dropping variables with high Variance Inflation Factor (VIF)

Variance inflation factor (VIF) is a measure of the amount of multicollinearity in a set of multiple regression variables. Mathematically, the VIF for a regression model variable is equal to the ratio of the overall model variance to the variance of a model that includes only that single independent variable.

We assume a threshold of 10 for VIF and drop columns with higher threshold until all columns are below the threshold.

The algorithm is as follows:

1. Find VIF of all independent variables.
2. Drop the variable with highest VIF greater than the threshold and repeat from step 1.

Table 4: VIF of independent variables and possible reasons for high correlation

	VIF Factor	features
0	6.975323	present_price
1	7.058405	kms_driven
2	19.552141	age
3	6.034425	fuel_type_Diesel
4	22.501061	fuel_type_Petrol
5	4.133359	seller_type_Individual
6	11.667031	transmission_Manual
7	1.088076	owner_1

Here, we can see that *fuel_type_Petrol* has the highest VIF of 22.5 and it is also above the desired threshold, so we will be removing this variable from the training set.
This is because *fuel_type_Petrol* is related to *fuel_type_Diesel* since vehicles are usually either of petrol or diesel.

	VIF Factor	features
0	5.309685	present_price
1	6.884100	kms_driven
2	14.846372	age
3	1.563838	fuel_type_Diesel
4	3.358380	seller_type_Individual
5	8.290393	transmission_Manual
6	1.082781	owner_1

After removing *fuel_type_Petrol*, the age variable still has a VIF of 14.8 which is also above the threshold.
This might be because *kms_driven* and *age* might be in a strong linear relationship.

	VIF Factor	features
0	4.724197	present_price
1	4.332166	kms_driven
2	1.486752	fuel_type_Diesel
3	2.590337	seller_type_Individual
4	6.717228	transmission_Manual
5	1.057800	owner_1

After remove both the variables, we can see that are variables are below the required threshold. We can be sure that we have eliminated any possible multicollinearity effects on the dataset.

Results

We will be using the OLS estimator to perform a linear regression on the dataset. The dependent variable in the dataset is the *Selling_Price* and the independent variables are show in the last image of Table 4.

Figure 8: OLS Regression results

OLS Regression Results

Dep. Variable:	selling_price	R-squared:	0.848
Model:	OLS	Adj. R-squared:	0.843
Method:	Least Squares	F-statistic:	192.8
Date:	Thu, 05 May 2022	Prob (F-statistic):	4.39e-82
Time:	10:39:51	Log-Likelihood:	-338.37
No. Observations:	215	AIC:	690.7
Df Residuals:	208	BIC:	714.3
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	2.9297	0.391	7.486	0.000	2.158	3.701
present_price	0.4356	0.028	15.799	0.000	0.381	0.490
kms_driven	-0.0425	0.005	-9.006	0.000	-0.052	-0.033
fuel_type_Diesel	1.3670	0.243	5.615	0.000	0.887	1.847
seller_type_Individual	-1.7390	0.266	-6.526	0.000	-2.264	-1.214
transmission_Manual	-0.0500	0.305	-0.164	0.870	-0.651	0.551
owner_1	-0.1273	0.434	-0.294	0.769	-0.982	0.727

Omnibus:	17.417	Durbin-Watson:	1.990
Prob(Omnibus):	0.000	Jarque-Bera (JB):	54.466
Skew:	0.147	Prob(JB):	1.49e-12
Kurtosis:	5.448	Cond. No.	220.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

From the above image, we can see that the R-squared is valued at 0.85 which is very close to the adjusted R-squared. This says that our model is not overfitting. We can further confirm this by checking the R-squared score on the train and test data respectively.

The below figure confirms our assumption that our model is not overfitting the dataset. We can also see the cross-validation results.

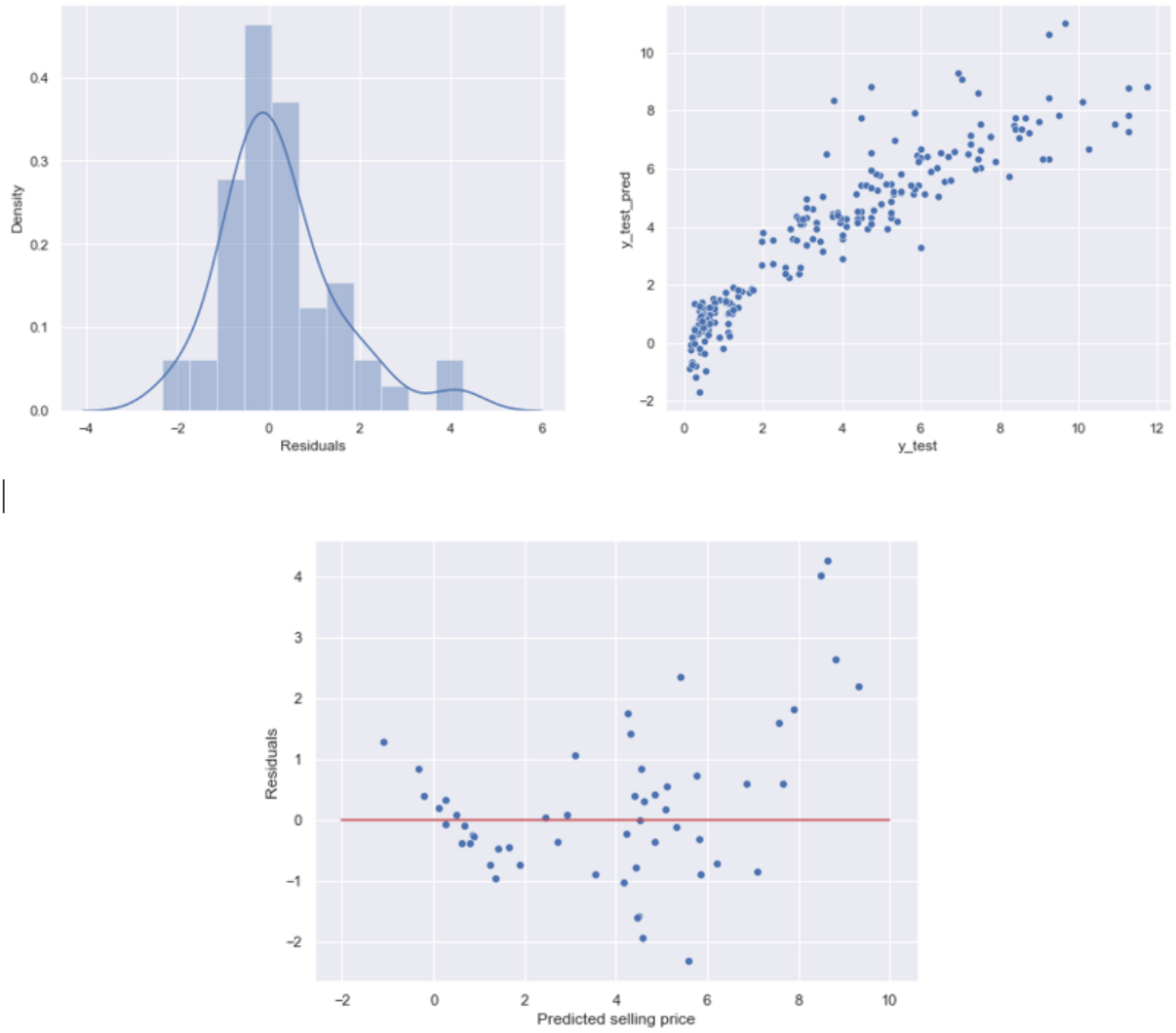
Figure 9: R-squared score on train and test data

```
from sklearn.metrics import r2_score
from sklearn.model_selection import cross_val_score

r2_train = r2_score(y_train, y_pred_train)
r2_test = r2_score(y_test, y_pred_test)
print("R2 score on train set: {}".format(round(r2_train, 2)))
print("R2 score on test set: {}".format(round(r2_test, 2)))

cv_train = cross_val_score(SMWrapper(sm.OLS), X_train, y_train, cv=5)
cv_train_mean = cv_train.mean()
print("Mean CV score on train set: {}".format(round(cv_train_mean, 2)))
```

```
R2 score on train set: 0.85
R2 score on test set: 0.85
Mean CV score on train set: 0.84
```

Plots

Figure 10: Plots obtained on the test dataset



1. The first plot (top-left) is the residual density plot. We can see that the residuals follow the normal distribution.
2. The second plot (top-right) shows the plot between predicted and actual values in the test dataset. Since there is a near linear relationship and there is not much deviation, we can infer that the model predicts *Selling_Price* with a very good accuracy.
3. The third plot (bottom) shows the plot between the residuals and predicted *Selling_Price*, we can see from the plot that the residuals are at a constant distance from the red line except for some outliers, thus we can conclude that residuals are randomly distributed, and the variance of error terms are equal.

Interpretation

All inferences will be drawn from Figure 8.

Practical significance

We have obtained an R-squared score of 0.84 on the dataset which is good for the size of data that we have. The *fuel_type_Diesel* has a positive value, which says that a diesel car usually adds to the *Selling_Price* and this is intuitive since petrol cars are more affordable. The negative coefficient of *kms_driven* is justified by the fact that cars with higher usage tend to be cheaper.

Statistical significance

From the referenced figure, we can see the lack of statistical significance for the number of owners and the transmission type of the car. This is because cars are usually priced based on the distance travelled and transmission type is merely a preference for individuals.

The high t-statistic of *Present_Price* indicates that it has a strong significance in deciding the selling price which is also expected since current market value of a car is always taken into consideration to decide the resale value.

Conclusion

Our study of analysing and predicting resale value of cars shows a promising model with good scores on test dataset. The t-tests and p-values complement the intuitive expectations on the resale value.

References

1. Used car details dataset by Car Dekho
2. Scikit-Learn Linear Regression
3. Correlation plots using seaborn
4. Removing outliers