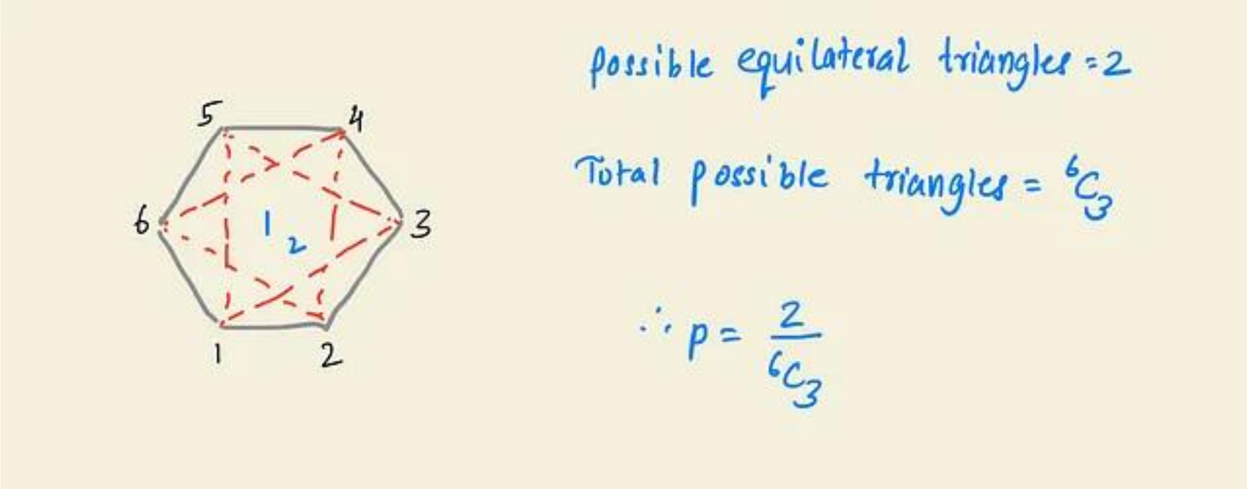
Top 14 questions of Probability for Data Science Interview

Basic Probability

Que 1) 3 vertices (corners) of a regular hexagon are randomly joined. What is the probability that an equilateral triangle is formed?

In a regular hexagon all sides and angle measurements are equal. And an equilateral triangle has all three sides equal.

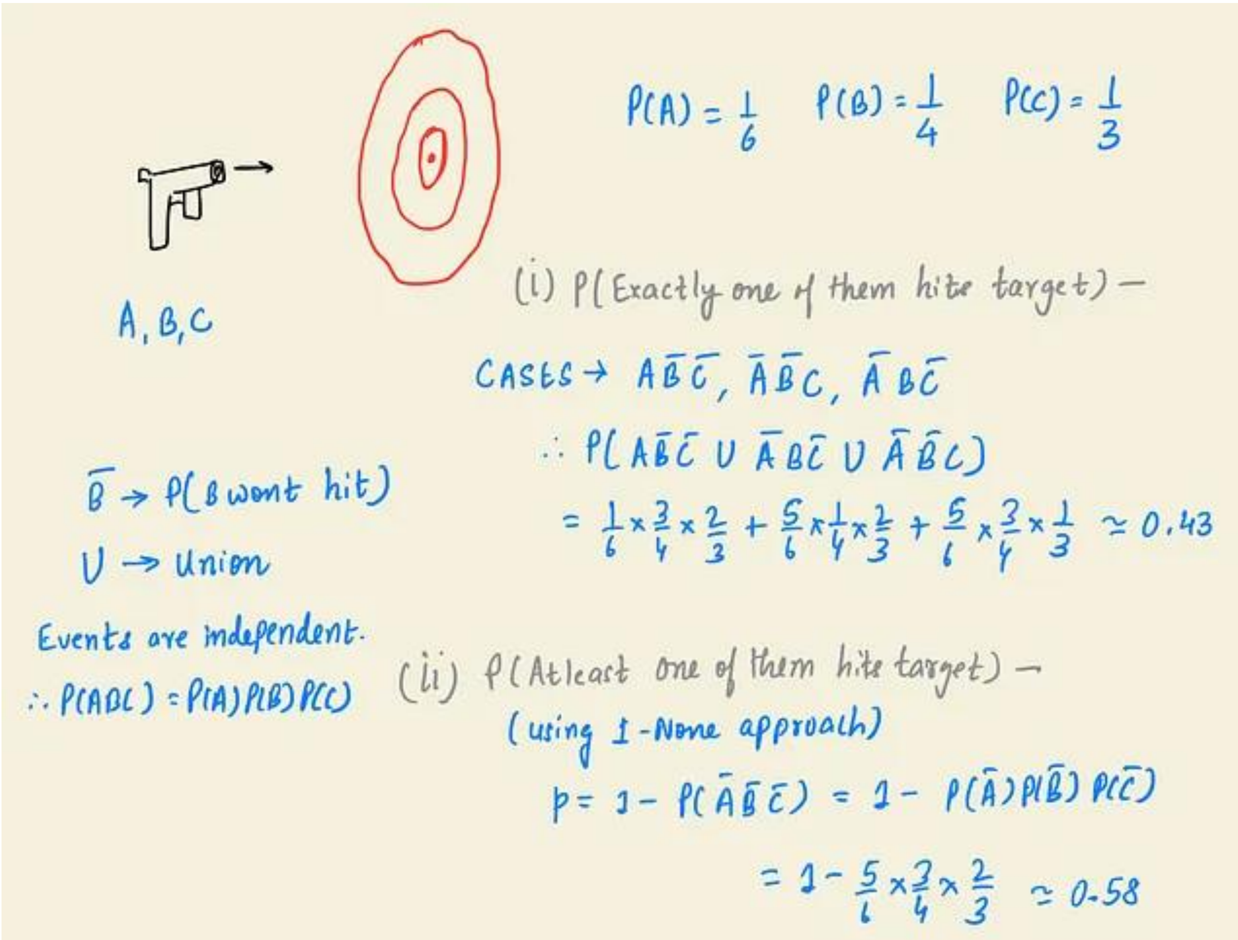


So, P = $2/(6!/((6-3)! * 3!))$ = 2/20 = 0.1

Que 2) 3 persons A, B, C independently fire at a target. What is the probability that
(i) Exactly one of them hits the target,
(ii) At least one of them hits the target?
Given: Probability of hitting the target. P(A) = 1/6, P(B) = 1/4, P(C) = 1/3.

(i) Probability that exactly one of them hits the target requires the other two not to hit the target. We can readily see three cases for this event to happen. Finally, we calculate the probability by taking a union of those cases.

(ii) Probability that at least one of them hits the target is solved by creating several cases and taking a union as we did in the earlier part. A much easier approach will be to calculate the negation of the same event and subtract it from 1. (Since the firings are independent P(ABC) becomes P(A)P(B)P(C)).



Que 3) The probability that a teacher takes a surprise test is 0.55. If a student remains absent for two days. What is the probability that he misses exactly one test, and at most one test?

(i) Similar to the previous question
(ii) Missing at most 1 test means missing either 0 test or 1 test
PS: this is similar to the Uber and Lyft problem.

$P(T_i)$ = Probability that teacher takes test on day i. $= 0.55$

Negation implies not taking the test.

$p(T_1 \cap \bar{T}_2) = P(T_1)\,P(\bar{T}_2)$

As the events are independent

(i) P(Missing Exactly 1 test) →

$$p = P(T_1 \cap \bar{T}_2 \ \cup \ \bar{T}_1 \cap T_2)$$
$$= P(T_1 \cap \bar{T}_2) + P(\bar{T}_1 \cap T_2)$$
$$= 0.55 \times 0.45 + 0.45 \times 0.55$$
$$p = 0.495$$

(ii) P(Miss atmost 1 test) = P(Miss no test) + P(miss 1 test)

$$p = 0.45 \times 0.45 + 0.495$$
$$p = 0.6975$$

Que 4) A box contains 2 defective pens and 3 working pens. Pens are tested one by one until both defective ones are discovered. What is the probability that the testing procedure comes to an end at the end of
(i) 2nd testing,
(ii) 3rd testing?

For the test to come to an end at the end of two checks, the first two starting pens need to be defective. For the test to come to an end at the end of three checks, we can create cases and take the union.

P(D) = probability that the pen is defective

(i) Testing procedure comes at the end of 2^{nd} test

$\Rightarrow$ P(starting 2 pens defective)

2 Defective (D)

3 Working (W)

$P(DD) = \frac{2}{5} \times \frac{1}{4}$

(ii) Testing procedure comes at the end of 3^{rd} test -

CASES - WDD, DWD, WWW (remaing 2 defective).

$= P(WDD) + P(DWD) + P(WWW)$ {Union}

$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \simeq 0.3$

Que 5) If there are 30 people in a room, what is the probability that everyone has different birthdays? Assume 365 possible birthdays in a year.

Total possible ways 30 people can have birthdays = $(365)^{30}$

1^{st} person: 365 ways to choose

2^{nd} person: 364 ways to choose

⋮

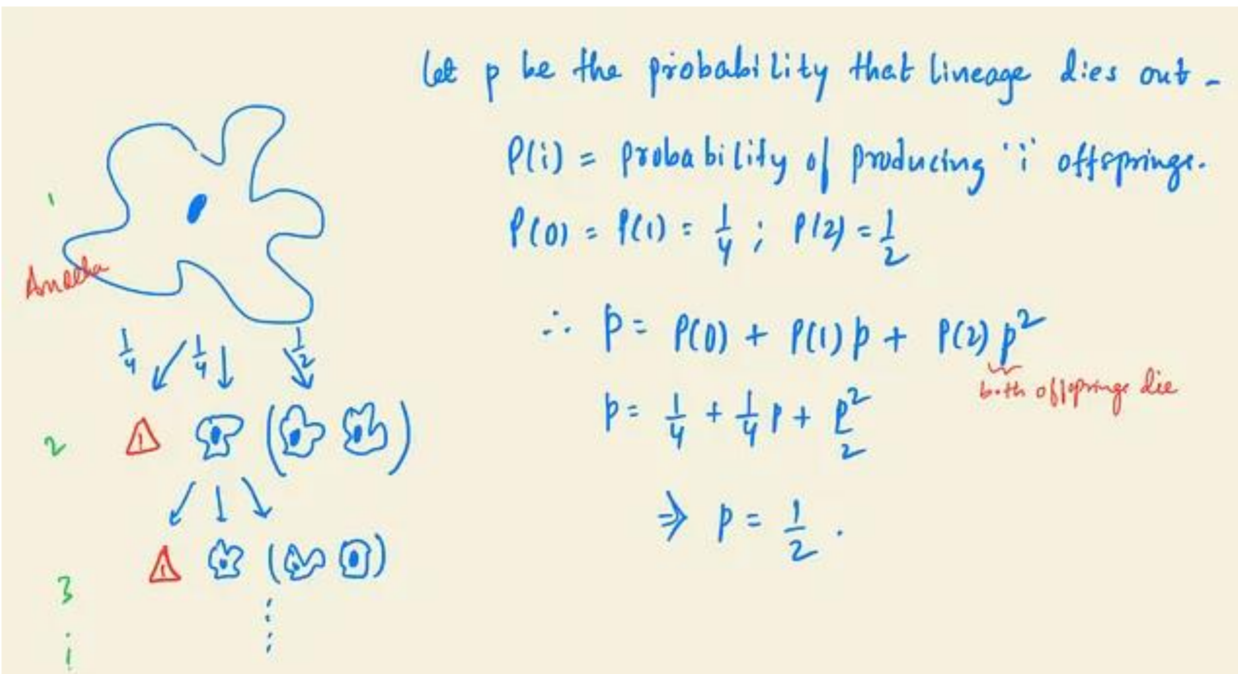
30^{th} person: 336 ways to choose -

$\therefore p = \frac{365 \cdot 364 \cdot \cdots \cdot 336}{(365)^{30}}$.

Algebraic Problems

Que 6) An amoeba has a 25%, 25%, and 50% chance of producing 0, 1, or 2 offspring, respectively. Each of the amoeba's descendants also has the same probabilities. What is the probability that the amoeba's lineage dies out?

For the amoeba lineage to die it needs to produce 0 offspring. If it produces 1 offspring then the child offspring must produce 0 offspring. Similarly for two children's offspring.



Que 7) The entries in a 2 x 2 matrix are integers that are independently chosen for each entry. The probability that the entry is odd is p. If the probability that the value of the determinant is even is 0.5, find p.

The probability that the determinant is odd/even can be computed by making cases for odd/even and then taking a sum of the probabilities of those cases.

Consider $M = \begin{pmatrix} a & d \\ b & c \end{pmatrix}_{2\times2}$, the determinant is given by:

$$|M| = ad - bc$$

given:
[p is the probability that the entry is odd]

$P(E) = 0.5 \Rightarrow P(O) = 0.5$ (probability that $|M|$ is odd).

for $|M|$ to be odd we have 2 cases:

(1) $ad \to$ odd, $bc \to$ even $\Rightarrow |M| \to$ odd

$\therefore p_1 = p^2(1-p^2)$

(2) $ad \to$ even, $bc \to$ odd $\Rightarrow |M| \to$ odd

$\therefore p_2 = p^2(1-p^2)$

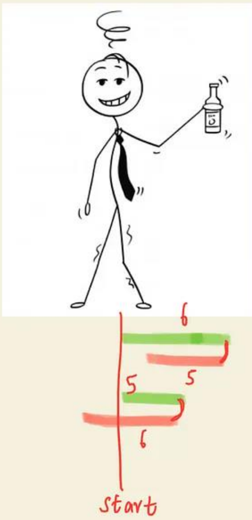
$\therefore 2p^2(1-p^2) = P(O) = 0.5$

$\Rightarrow p = \frac{1}{\sqrt{2}}$.

<u>Binomial Distribution</u>

Que 8) A drunker takes either a step forward or backward. The probability that he takes a forward step is 0.4. Find the probability that at the end of 11 steps he is 1 step away from the starting point?

Clearly, for the drunker to be 1 step away from start he can either take 5 steps forward (meaning 6 backward steps) hence ending 1 step behind the start OR he can take 6 forward (meaning 5 backward steps) hence ending 1 step front of the start. The final probability can be calculated by taking a union of the two events.



$P(F)$ = probability of taking forward step $= 0.4$

$\therefore P(B) = 0.6$
Backward step

$\therefore P(1 \text{ step away}) = P(5 \text{ steps forward}) + P(6 \text{ steps forward}) = p$

$$p = \binom{11}{5} 0.4^5\, 0.6^6 + \binom{11}{6} 0.4^6\, 0.6^5 = 0.367$$

Que 9) A coin is twice as likely to land head as a tail in a series of independent tosses. Find the probability that the 3rd head occurs on the 5th toss.

For the 3rd head to occur in the 5th toss, the earlier 2 heads can occur in any of the 4 tosses initially, which becomes a case of the binomial distribution.

$P(H) = \frac{2}{3}$ $\quad P(T) = \frac{1}{3}$

The probability of 2 heads (2 successes) in 4 trials is given by: $\binom{4}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = p_1$

Probability of a head in 5^{th} toss $= \frac{2}{3} = p_2$

$$\therefore P(3^{rd} \text{ head in } 5^{th} \text{ toss}) = p_1 \times p_2 = \binom{4}{2}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 \simeq 0.2$$

Law of Total Probability

Que 10) A rich lady has 4 compartments in her purse. The 1st compartment has 1 Rs (Rupees) and 2 Paise coins. 2nd has 2 Rs and 3 Paise coins. 3rd has 3 Rs and 4 Paise coins. 4th has 4 Rs and 6 Paise coins.
She selects a random compartment & draws a coin, what is the probability that the drawn coin is a rupee coin?

$P(R) = P(\text{Drawn coin is a Rupee coin})$.

$$P(R) = P(C_1)P(Rs|C_1) + P(C_2)P(Rs|C_2) + P(C_3)P(Rs|C_3) + P(C_4)P(Rs|C_4)$$

$$= \frac{1}{4} \times \frac{1}{3} + \frac{1}{4} \times \frac{2}{5} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{4}{10}$$

$$\approx 0.47$$

$P(C_i)$ = Probability that the coin is drawn from apartment i.

Bayes Theorem

Que 11) An HIV test is 99% accurate (both ways). Only 0.3% of the population is HIV +. What is the probability that a random person is HIV + given that the person tests +?

$P(\text{person is HIV+} \mid \text{person tests +}) = ?$

applying Bayes Theorem

$$p = \frac{p(\text{tests +} \mid \text{is HIV+})\, p(\text{is HIV+})}{p(\text{tests +} \mid \text{is HIV+})\, p(\text{is HIV+}) \times p(\text{tests +} \mid \text{is HIV−})\, p(\text{is HIV−})}$$

given: $p(\text{is HIV+}) = 0.3\%$

$\therefore\ p(\text{is HIV−}) = 99.7\%$

$$p = \frac{0.3\%\ 99\%}{0.3\%\ 99\% + 99.7\%\ 1\%} = 22.95\%$$

Que 12) A speaks truth in 70% of cases, B in 50% of cases. Find the probability that they will speak the same thing while describing a certain event?

$p(A) = 0.7$ {probability that A speaks truth}

$p(B) = 0.5$

Define: E_1: Both speak truth

E_2: Both speak lie

E_3: Both have same answer.

$p(E_1)$ is the probability that both A & B speak truth.

$\therefore\ p(E_3) = ??$

$= p(E_1)\, p(E_3/E_1) + P(E_2)\, P(E_3|E_2)$

$= 0.7 \times 0.5 \times 1 + 0.3 \times 0.5 \times 1$

$\simeq 0.5$.

Miscellaneous Card Problems

Que 13) Cards are dealt one by one from a pack of 52 well-shuffled cards. What is the probability that exactly 'k' cards are dealt before the 1st ace appears?

We are indirectly looking for the probability that the 1st ace appears in the (k+1)th card. () is the standard combination's notation.

4 Aces

Total ways to choose 'k' cards from 52 cards.

$$= \binom{52}{k}$$

Total ways to choose 'k' cards which are not Aces.

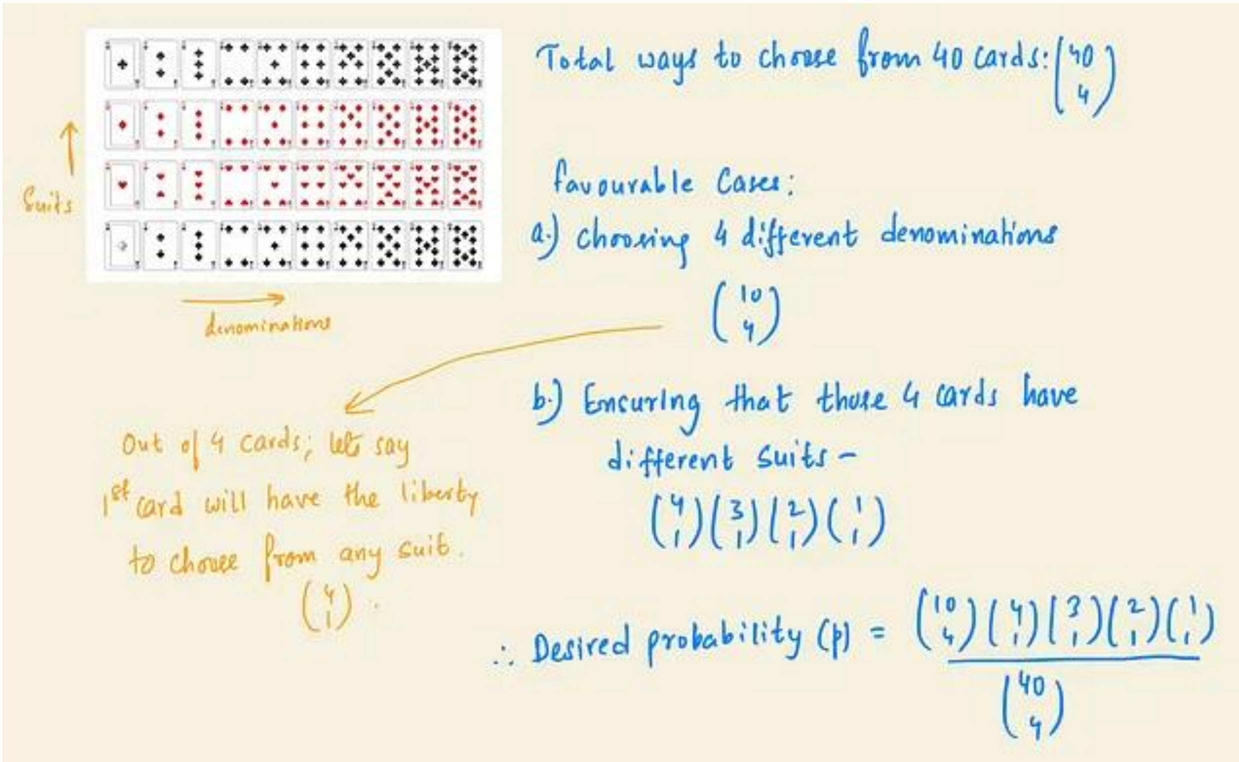
$$= \binom{48}{k}$$

Ways to choose an Ace $= \binom{4}{1}$; This choice will be made after 'k' cards are dealt !!

$$\therefore p = \frac{\binom{48}{k}}{\binom{52}{k}} \times \frac{\binom{4}{1}}{\binom{52-k}{1}}.$$

Que 14) All face cards are removed from a pack of 52 well-shuffled cards. From the remaining 40 cards, 4 cards are drawn randomly. What is the probability that 4 cards are from different suits and denominations?

Total suits = 4 (Spade, Hearts, Clubs, Diamonds); Total denominations = 13 (2, …, 10, A, J, Q, K).



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