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The Improved Algorithm of Edge Detection Based on Mathematics Morphology

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Abstract

In morphology, the structural elements of different sizes and shapes have different abilities in maintaining image detail and anti-noise, an adaptive algorithm of edge detection based on multi-structure and multi-scale element is presented in this paper. First, the existing morphological operators for edge detection are improved. Then, the weight coefficients of different scales and different shapes are determined respectively by using the method of morphological difference and calculating the information entropy of the edge-detected. Last, fuse together the edges to achieve the image edge. For the image with noise, the algorithm can filter out noise effectively, and the edge is continuous, smooth, clear outline. The experiment results show that the algorithm has good effect in edge detection, strong anti-noise capacity, and the objective evaluation and visual effect are good, too.

Keywords: *image processing, edge detection, mathematical morphology, integration*

1. Introduction

Image edge is one of the most basic characteristic of the image; it not only contains a large amount of information, but also is an important basis about the image analysis and image segmentation. Therefore, image edge has a wide application prospect [1-3].

Edge detection is widely used in image segmentation, image matching, feature extraction, and other fields. It is also an important research direction of computer vision. There are many kinds of algorithms for image edge detection. The traditional algorithms for edge detection have Laplace operator, Roberts operator, Sobel operator, Canny operator and so on [4], the main function of these operators is high-pass filter, each of them has different characteristics and pertinence. But the traditional algorithms involve in orientation, so anti-noise performance is generally poor, it is difficult to detect the edge of the complex image [5].

The basic idea of mathematics morphology is to use the structural element with a certain form to measure and to extract the corresponding shape in the image. Mathematics morphology can achieve the purposes of the image analysis and target recognition [6]. It has unique advantages in image edge detection and could make the accuracy of edge detection and noise immunity to reach a certain balance [7]. After 1985, mathematics morphology became a tool in the image processing, and shows a great application prospect increasingly [8-9]. In order to improve the performance of morphological edge detection, the scholars mainly researched in the following three aspects during nearly thirty years. First, study the operators of the morphological edge detection. Most of them use the complex type anti-noise edge detection operator [10-13]. For example, literatures
[11, 12] put forward the improved operators about edge detection, but their results either

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cannot detect the details of the edge well, or cannot filter out the noise well. Next, study how to select the structural element. Someone may use the multi-structure structural element, and someone may use multi-scale structural element [13, 14]. But select a single type of structural element cannot detect the details of the image edge well. For example, literature [13] only select single-scale structural element, the image edge is not complete. Last, study the image fusion. General calculate weight coefficients according to the inverse proportion of image difference or according to the direct proportion of information entropy [15, 16]. But when using the structural elements with different orientations to detect the image, the effect of edge fusion is not very ideal.

The above studies have different defects respectively. In order to improve the anti-noise ability of the morphological edge detection and to make the edge with continuous, complete, without jagged, this paper proposes an adaptive multi-structure and multi-scale edge detection algorithm by improving the existing operators. The experimental results show that this algorithm is greater than the literatures [5, 11-13].

2. The Basic Principle

2.1. The Basic Principle of Morphology

Mathematics morphology is a non-linear image processing based on set theory, now, it is applied to image analysis, pattern recognition, etc widely. The basic operations of mathematics morphology are: dilation, corrosion, opening and closing. $f(x,y)$ represents the input grey-scale image, $s(x,y)$ represents a structural element, and D_f and D_s represent the field of function $f(x,y)$ and $s(x,y)$ respectively. For the gray-scale morphology, the definition of dilation, erosion, opening and closing are shown as follows:

Definition 1 Gray-scale dilation operation is

$$(f \oplus s)(x,y) = \max\left\{ f(x-x', y-y') + s(x',y') \mid x-x', y-y' \in D_f, x',y' \in D_s \right\}$$

Definition 2 Gray-scale erosion operation is

$$(f \Theta s)(x,y) = \min\left\{ f(x+x', y+y') - s(x',y') \mid x+x', y+y' \in D_f, x',y' \in D_s \right\}$$

Definition 3 Gray-scale opening operation is

$$f \circ s = (f \Theta s) \oplus s$$

Gray-scale opening operation is erosion followed by dilation, it can smooth the contour of image and remove small extrudes.

Definition 4 Gray-scale closing operation is

$$f \bullet s = (f \oplus s) \Theta s$$

Gray-scale closing operation is dilation followed by erosion, it can smooth the image outline and recover the holes and cracks.

2.2. The Existing Operators based on Morphology

The edge detection based on morphology is to do the dilation and erosion operation by using structural element, then to establish certain mathematical operations and to get the relevant operator of image edge detection. $G(x,y)$ Represents the function of the image edge, $f(x,y)$ represents the input image, $s(x,y)$ and represents a structural element.

According to the basic operation of morphology, there are some existing edge detection operators based on morphology.

1) Three kinds of classical edge detection operators

The dilation operator of edge detection is

$$G_d^1 = f(x,y) \oplus s(x,y) - f(x,y) \tag{1}$$

The erosion operator of edge detection is

$$G_e^1 = f(x,y) - f(x,y) \Theta s(x,y) \tag{2}$$

The operator of morphological gradient is

$$G_{de}^1 = f(x,y) \circ s(x,y) - f(x,y) \Theta s(x,y) \tag{3}$$

The above three operators implement simply and run fast, but they are sensitive to noise particularly, the detection results is fuzzy and lost the edge information either.

2) The edge detection operators which were put forward in literature [11] are listed as follows:

$$G_{\min}^2 = \min\left\{G_d^1, G_e^1, G_{de}^1\right\} \tag{4}$$

$$G_{\max}^2 = \max\left\{G_d^1, G_e^1, G_{de}^1\right\} \tag{5}$$

$$G_{dec}^2 = G_{\max}^2 - G_{\min}^2 \tag{6}$$

The operators reduce the degree of fuzziness of the detection results, but anti-noise ability is poor.

3) The edge detection operators which were put forward in literature [12] are listed as follows:

$$G_d^3 = f(x,y) \circ s(x,y) \bullet s(x,y) \oplus s(x,y) - f(x,y) \circ s(x,y) \bullet s(x,y) \tag{7}$$

$$G_e^3 = f(x,y) \circ s(x,y) \bullet s(x,y) - f(x,y) \circ s(x,y) \bullet s(x,y) \Theta s(x,y) \tag{8}$$

$$G_{de}^3 = f(x,y) \circ s(x,y) \bullet s(x,y) \oplus s(x,y) - f(x,y) \circ s(x,y) \bullet s(x,y) \Theta s(x,y) \tag{9}$$

The operators are suitable for the image with the small noise, but they cannot filter out the noise well when detecting the images with the big noise.

4) Literature [5] combines the alternating sequential filter (ASF) with the dilation operator of edge detection, which can filter out the noise effectively, but the detected edge is jagged and discontinuous.

For the above operators have different defects respectively, in order to overcome these shortcomings, it is necessary to improve the existing operators.

3. The Improved Algorithm

3.1. The Composite Filter Smooth Image with Multiple Structural Elements

The close operation of morphology can fill small space, connect to the nearby objects and smooth the boundary. The open operation of morphology removes the small bulges, separates objects in the fine and smooths outline of images. And as the effect of the morphology for image processing is not only related to morphological transformation way, but also related to the selection of structural elements. Literature [13] combines morphological transformation mode with structure elements to constitute a multi-structure elements composite filter. Namely:

$$F = \frac{1}{2}\left[(f \circ B_1 \bullet B_2) + (f \bullet B_1 \circ B_2)\right] \tag{10}$$

Where $B_1(x,y) = [1,0,1;0,1,0;1,0,1]$ and $B_2(x,y) = [1,1,1;1,1,1;1,1,1]$ are structural elements, $f(x,y)$ represents the input image.

3.2. Improved Operator of Morphology in Edge Detection

When using the operators of classical edge detection or the operators is put forward in the literatures [5, 11, 12] for edge detection, the detection results are either not filter out noise extremely, or the edge appear fuzzy, discontinuous and jagged. In order to overcome these shortcomings, it is necessary to improve the existing operators.

Close operation of morphology can fill gaps, connect to the nearby objects and smooth the boundary, so which can ensure the edges more continuous, complete, and no jagged. Also we know that the advantages of morphology algorithm are not only rest with the flexible and changeable of morphology transformation way, but also rest with the selection of structural element. Therefore, this paper combines close operation of morphology with structure element, and proposes the improved operators as follows:

$$G_d^5(i)_m = F \bullet b_i^m \oplus b_i^m - F \bullet b_i^m \tag{11}$$

$$G_e^5(i)_m = F \bullet b_i^m - F \circ b_i^m \Theta b_i^m \tag{12}$$

$$G_{de}^5(i)_m = F \bullet b_i^m \oplus b_i^m - F \circ b_i^m \Theta b_i^m \tag{13}$$

Where F is gave by formula (10). $b_i^m(x,y)$ Represents structural element, $i=1,2,3,4$ represent directions of structural elements respectively, $m=1,2$ represent scales of structural elements with 3×3 and 5×5 respectively. They are as follows:

$$b_1^1 = [0,0,0;1,1,1;0,0,0]\ ;\ b_2^1 = [0,0,1;0,1,0;1,0,0]\ ;$$

$$b_3^1 = [0,1,0;0,1,0;0,1,0]\ ;\ b_4^1 = [1,0,0;0,1,0;0,0,1]\ ;$$

$$b_1^2 = [0,0,0,0,0;0,0,0,0,0;1,1,1,1,1;0,0,0,0,0;0,0,0,0,0]\ ;$$

$$b_2^2 = [0,0,0,0,1;0,0,0,1,0;0,0,1,0,0;0,1,0,0,0;1,0,0,0,0]\ ;$$

$$b_3^2 = [0,0,1,0,0;0,0,1,0,0;0,0,1,0,0;0,0,1,0,0;0,0,1,0,0]\ ;$$

$$b_4^2 = [1,0,0,0,0;0,1,0,0,0;0,0,1,0,0;0,0,0,1,0;0,0,0,0,1].$$

Calculating the minimum of $G_d^5(i)_m$ and $G_e^5(i)_m$, we will get the edge details. Namely:

$$G_{\min}^5(i)_m = \min\left\{G_d^5(i)_m, G_e^5(i)_m\right\} \tag{14}$$

Furthermore, we obtain the improved operation:

$$G^5(i)_m = G_{de}^5(i)_m + G_{\min}^5(i)_m \tag{15}$$

3.3. The New Edge Detection Algorithm

When using the structural elements of different orientations to detect the edge, this paper presents a new rule about fusion. When using the structural elements of different scales to detect the edges, this paper uses the information entropy method to fuse together the edges.

3.3.1. The Weight Coefficient of New Edge Detection Algorithm: Because a structural element is only sensitive to the edge with the same direction and geometric structure, the edges would be smoothed out with different directions and geometric structure. Therefore, in this paper, we propose a new rule about the fusion. We need to calculate the difference of the four directions in each pixel first. Then if the directions have the biggest difference (such as the horizontal direction) explain that the edge will incline to the vertical direction. Select the structural elements which have the same directions with the template of difference (the structure element in the horizontal).

Might as well set n_i^m (i =1-4, $m=1,2$) as the number of structural elements in different orientation, the weight coefficient is:

$$\beta_i^m = n_i^m \Big/ \sum_{l=1}^{4} n_l^m \tag{16}$$

$$\begin{bmatrix} & & \\ -1 & 0 & 1 \\ & & \end{bmatrix} \qquad \begin{bmatrix} & & -1 \\ & 0 & \\ 1 & & \end{bmatrix} \qquad \begin{bmatrix} & -1 & \\ & 0 & \\ & 1 & \end{bmatrix} \qquad \begin{bmatrix} -1 & & \\ & 0 & \\ & & 1 \end{bmatrix}$$

Difference Template in 0° Direction Difference Template; in 45° Direction Difference Template; in 90° Direction Difference Template; in 135° Direction

Figure 1. Difference Template

When using the structural elements of different scales to detect the edges, this paper uses the information entropy method to calculate the weight coefficient.

1) Calculate the information entropy of the edge image, and the calculation method of information entropy is

$$H_m = -\sum_{k=0}^{255} p_k^m \log_2 p_k^m \tag{17}$$

In which p_k^m ($m=1,2$) is the probability of the image grey-value which is *k*.

2) Calculate the weight coefficient of the edge image:

$$\alpha_m = H_m \Big/ \sum_{l=1}^{2} H_l \tag{18}$$

3.3.2. Fusion Processing: Structure element is the basic element of mathematics morphology; the structural elements may also affect edge detection results with different scales and shapes. For example, Literature [13] uses single scale structure element, the edge of detection is not so complete.

After the analysis and comparison, this paper adopts the omni-directional multi-scale structural elements for edge detection. According to the improved operation (15), this paper puts forward a new algorithm for edge detection:

$$\begin{cases} E = \sum_{m=1}^{2} \alpha_m E_m \\ E_m = \sum_{i=1}^{4} \beta_i^m G^5\left(i\right)_m \end{cases} \tag{19}$$

Among them, E_m ($m=1,2$)is the image edge that is obtained by using structural elements with different scales. And α_m and β_i^m are weight coefficients. $G^5\left(i\right)_m$ is the image edge that is obtained by using structural elements with different orientations in the same scale.

First, we put the structural elements of 3×3 size of different orientations ($b_1^1, b_2^1, b_3^1, b_4^1$) into formula(15) respectively, fuse together the detection results to get the image edge E_1. Then, we use the structural elements of 5×5 size of different orientations ($b_1^2, b_2^2, b_3^2, b_4^2$) for edge detection, fuse together the detected edges to get the image edge E_2. Last, fuse together the two edges (E_1 and E_2) to get the final edge.

3.4. Improving Algorithm Process

In this paper, the main steps to the improved algorithm are shown as follows:

Step1. Set structural elements ($b_1^1, b_2^1, b_3^1, b_4^1$) and ($b_1^2, b_2^2, b_3^2, b_4^2$) with the structure of 3×3 and 5×5, and their directions are $0^\circ, 45^\circ, 90^\circ, 135^\circ$, see section 3.2.

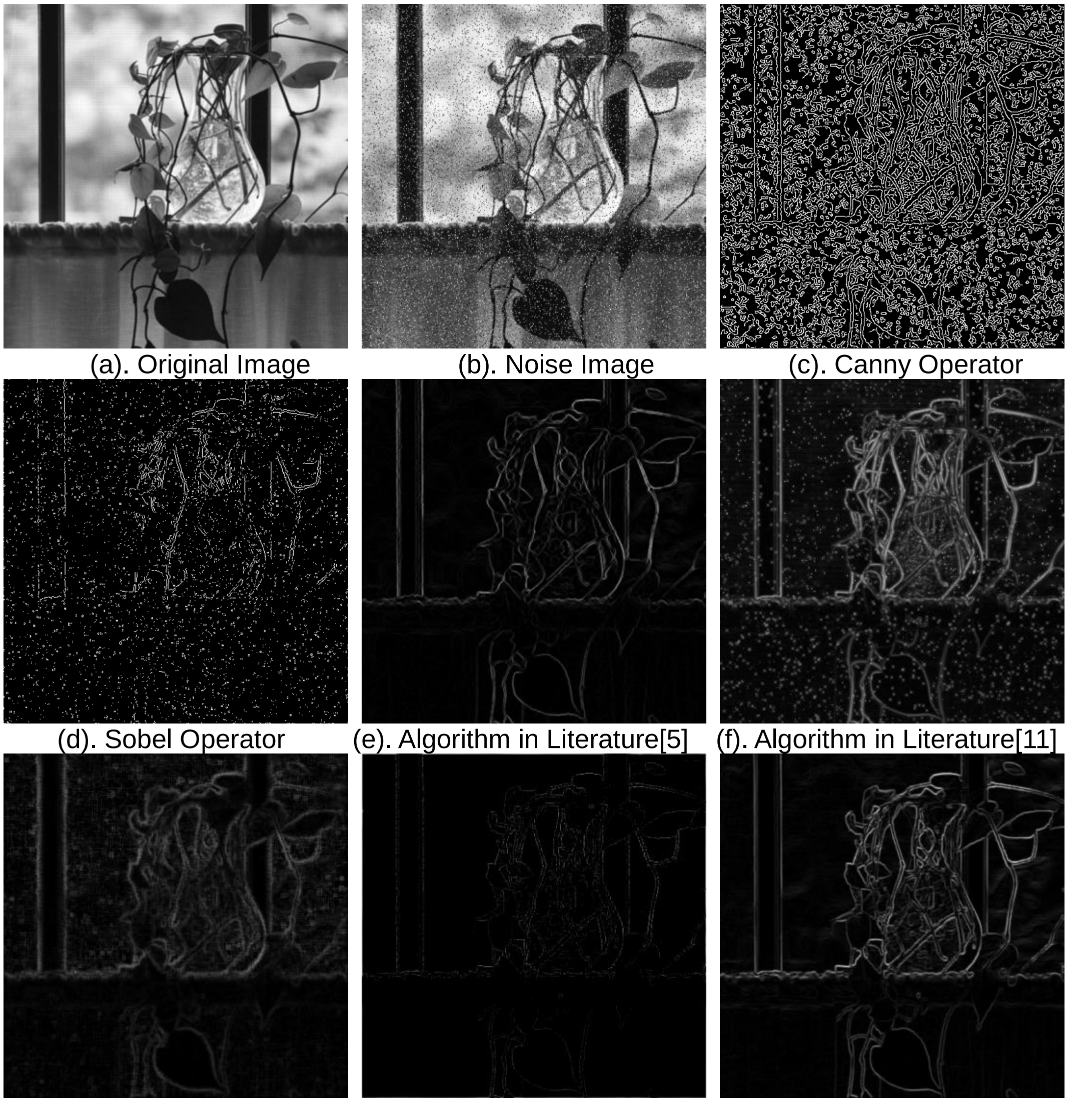
Step2. Put the four structural elements ($b_1^1, b_2^1, b_3^1, b_4^1$) which are 3×3 with different directions into the formula (15), then calculate the corresponding weight coefficients β_i^m （i =1,2,3,4, $m=1,2$) by the method of morphological difference, see formula(16). Last multiplying respectively the detection results $G^5(i)_m$ (i =1,2,3,4, $m=1,2$) by weight coefficients β_i^m (i =1,2,3,4, $m=1,2$), we can obtain the image edge E_1.

Step3. Put the other four structural elements ($b_1^2, b_2^2, b_3^2, b_4^2$) which are 5×5 with different directions and to go on the same step as Step2, we can obtain the image edge E_2.

Step4. Calculate the weight coefficient β_i^m (i =1-4, $m=1,2$) of each scale by the information entropy (see formula (18)), and get the final edge E by formula (19).

4. The Results and Analysis of Simulation

In order to detect the performance of the algorithm in this paper, first, we add 'salt and pepper' noise with density of 0.1 into the images of Flower, Bottle, Man, Watch and Lena. Then we use the traditional operators of edge detection (Sobel and Canny), the algorithms in literatures [5, 11-13] and the algorithm in this paper to detect the edge of the noise images. The detection results are shown in Figures 2-6. Through the analysis of simulation results we see that Canny operator and Sobel operator are sensitive to noise. The algorithms have certain ability of inhibition for noise in literatures [5] and [13], but the detection results appear jagged edges and miss the edge information. For example, the edge of the Bottle’s neck is discontinuous, the edges of the Man's hand and coat buttons are missed, and the edge of the top of the hat of Lena is missed, *etc.*, The algorithms can make the edges continuous in literatures [11, 12], but the noises are not completely filtered out. The improved algorithm in this paper can filter out the noise and retain the edge continuous, complete and no jagged.

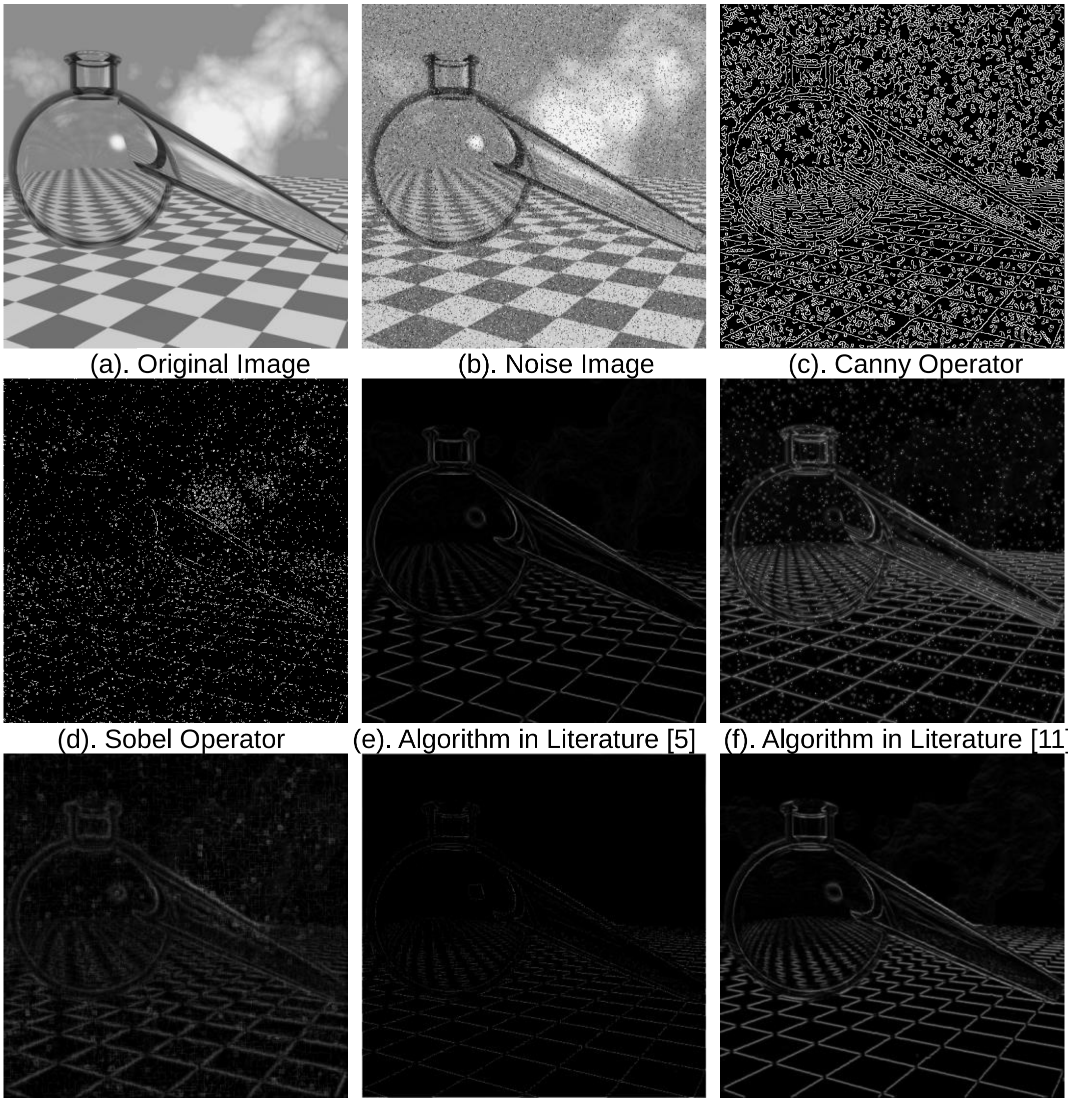


(a). Original Image (b). Noise Image (c). Canny Operator

(d). Sobel Operator (e). Algorithm in Literature[5] (f). Algorithm in Literature[11]

(g). Algorithm in Literature[12] (h). Algorithm in Literature[13] (i). Algorithm in This Paper

Figure 2. Edge Detection Results of Flower Image by using Various Algorithms

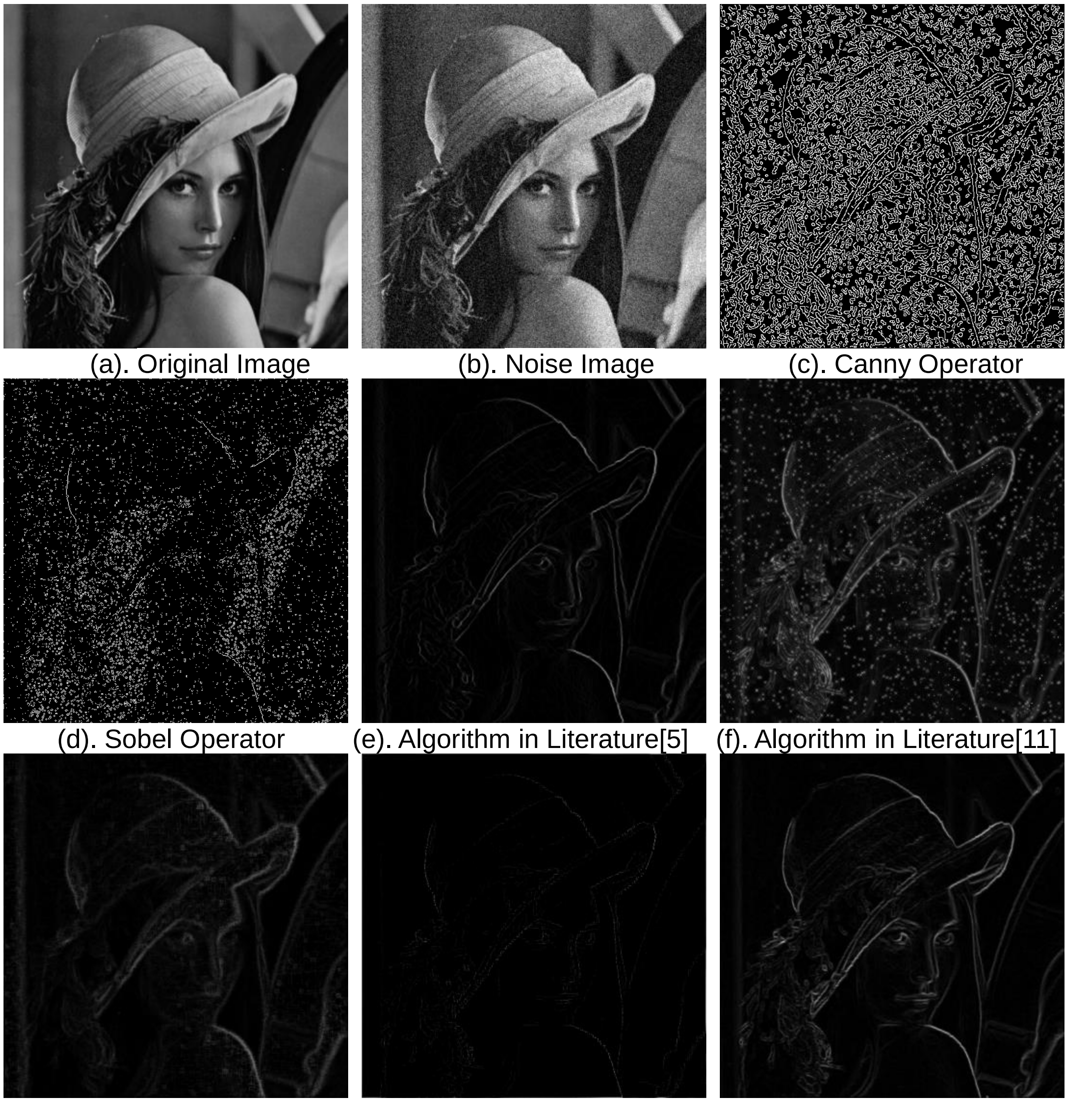


(a). Original Image (b). Noise Image (c). Canny Operator

(d). Sobel Operator (e). Algorithm in Literature [5] (f). Algorithm in Literature [11]

(g). Algorithm in Literature [12] (h). Algorithm in Literature [13] (i). Algorithm in This Paper

Figure 3. Edge Detection Results of Bottle Image by using Various Algorithms

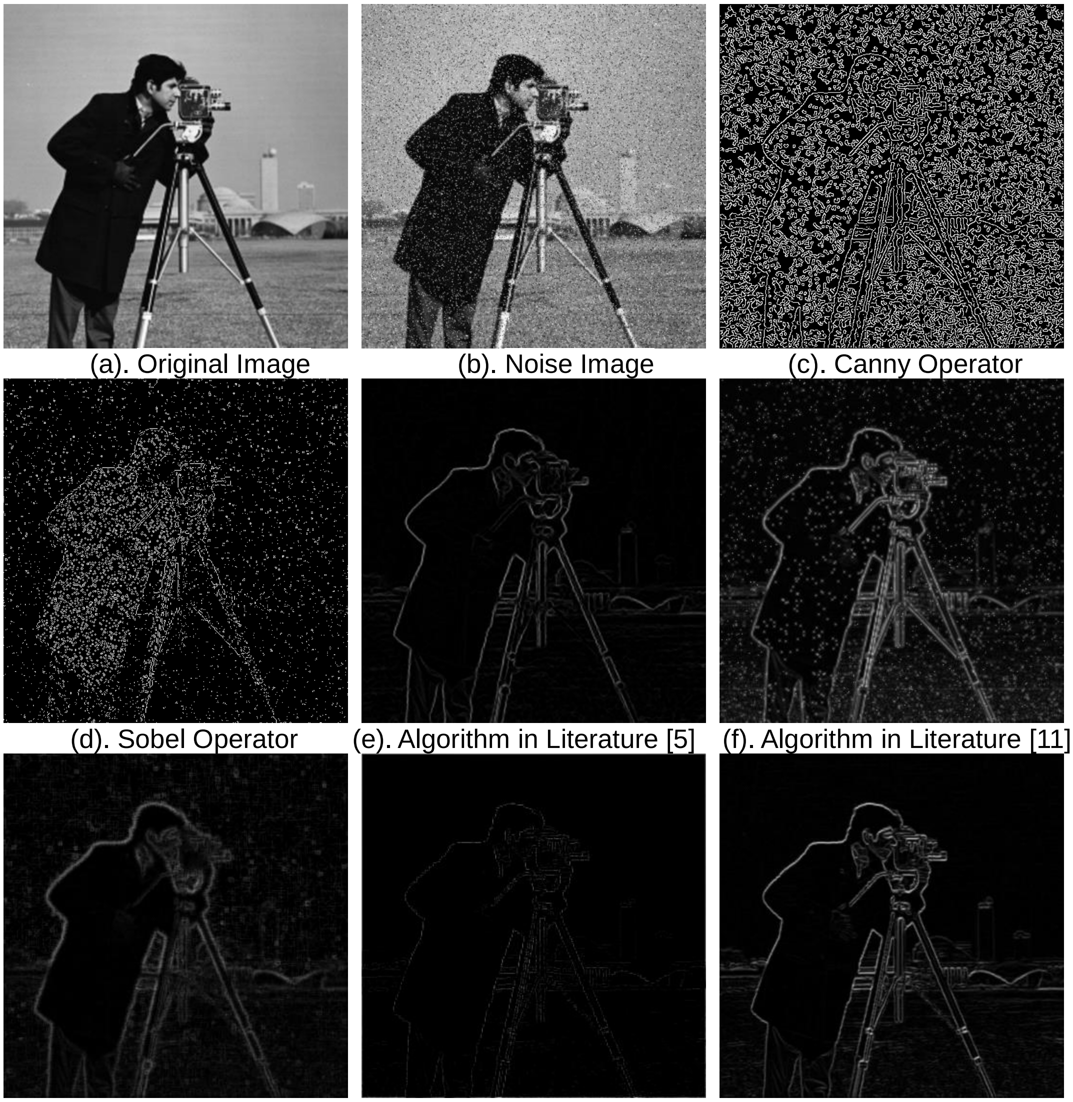


(a). Original Image (b). Noise Image (c). Canny Operator

(d). Sobel Operator (e). Algorithm in Literature[5] (f). Algorithm in Literature[11]

(g). Algorithm in Literature [12] (h). Algorithm in Literature [13] (i). Algorithm in This Paper

Figure 4. Edge Detection Results of Lena Image by using Various Algorithms

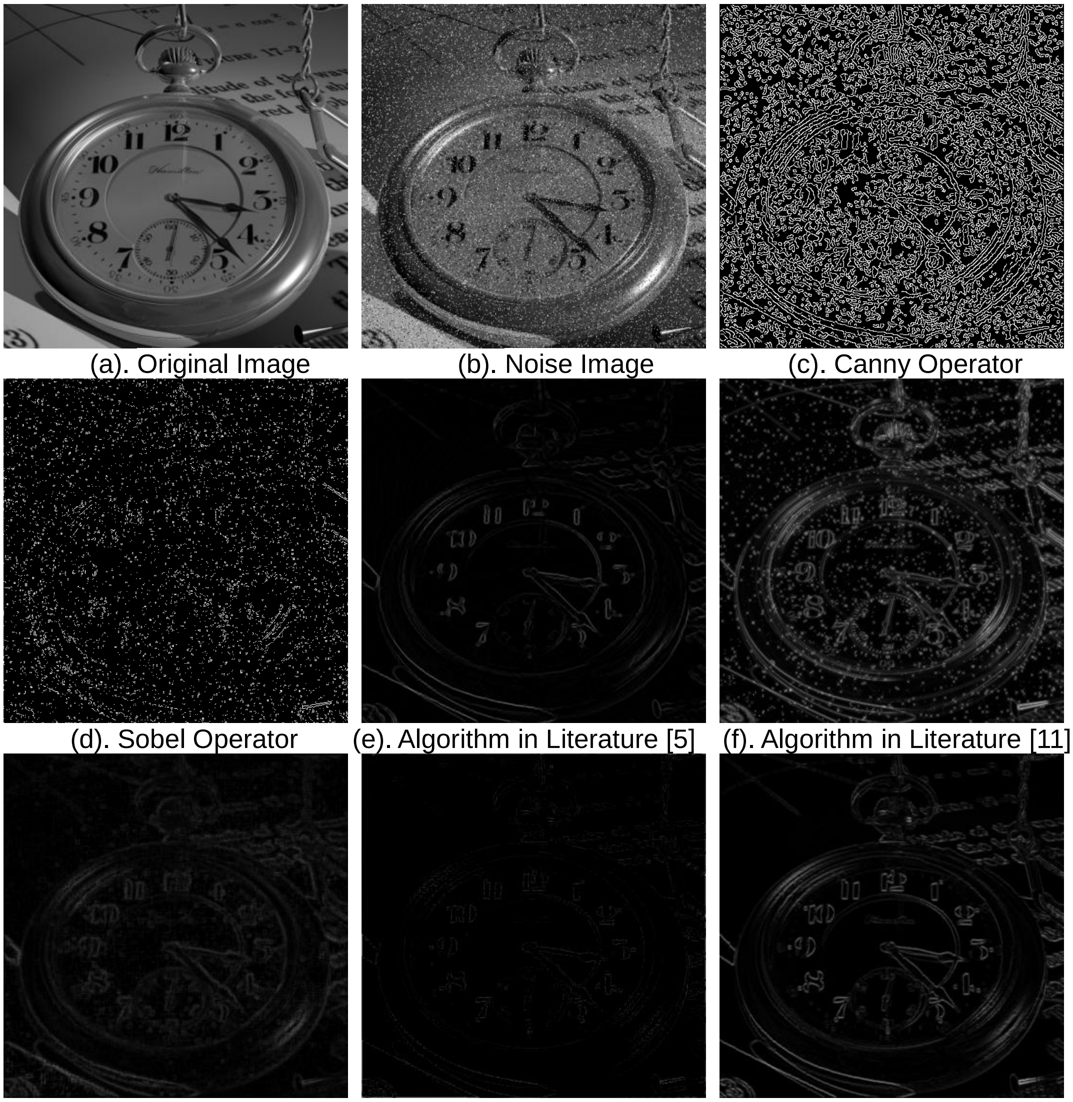


(a). Original Image (b). Noise Image (c). Canny Operator

(d). Sobel Operator (e). Algorithm in Literature [5] (f). Algorithm in Literature [11]

(g). Algorithm in Literature [12] (h). Algorithm in Literature [13] (i). Algorithm in This Paper

Figure 5. Edge Detection Results of Man Image by using Various Algorithms



(a). Original Image (b). Noise Image (c). Canny Operator

(d). Sobel Operator (e). Algorithm in Literature [5] (f). Algorithm in Literature [11]

(g). Algorithm in Literature [12] (h). Algorithm in Literature [13] (i). Algorithm in This Paper

Figure 6. Edge Detection Results of Watch Image by using Various Algorithms

In order to evaluate the effect of improved algorithm more objectively and effectively in this paper, we use the MSE (Mean Square Error) and PSNR (Peak Signal to Noise Ratio) as evaluation standards. The results are shown in Table 1.

Table 1. The Comparison of MSE and PSNR in Five Algorithms

		Algorithm of Literature [5]	Algorithm of Literature [11]	Algorithm of Literature [12]	Algorithm of Literature [13]	Algorithm in This Paper
MSE	Flower	17726	15983	15602	18342	15492
	Bottle	24142	22098	21998	24677	21975
	Lena	7054	6204	6199	7359	6165
	Man	17323	16031	16005	17296	15898
	Watch	5954	5638	5563	6415	5542
PSNR	Flower	5.6448	6.2008	6.2011	5.4963	6.2297
	Bottle	4.6108	4.7069	4.6993	4.2079	4.7095
	Lena	9.6433	10.1967	10.1977	9.5622	10.2309
	Man	5.7447	6.0989	6.0993	5.7315	6.1175
	Watch	10.6316	10.6559	10.6775	9.9599	10.7089

Through the data in Table 1 it can be found that with the same concentration of noise, the improved algorithm in this paper is better than the algorithms in literatures [5, 11-13] in these two indicators of MSE and PSNR. Although the dates of literatures [11, 12] are similar to the dates of this paper, the algorithms cannot filter out noise completely in literatures [11, 12].

In conclusion, the algorithm has a very good effect in visual and objective evaluation aspects in this paper.

5. Conclusions

Edge detection is one of the important research topics in image processing and computer vision, and morphology is a more effective method in image processing. In morphology, the structural elements of different sizes and shapes have different abilities in maintaining image detail and anti-noise. This paper proposes a new algorithm of edge detection based on morphology by improving the existing operators and also proposes a new method about fusion. The experimental results show that the algorithm can effectively restrain the influence of various noises on the edge detection, can detect the detailed information of image edges, and the detected edges are continuous and complete. The algorithm in this paper is superior to the algorithms in the literatures [5, 11-13], thus it is more conducive to image analysis and processing. But the theory of mathematics morphology is very rich, and a better method needs to be further research.

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