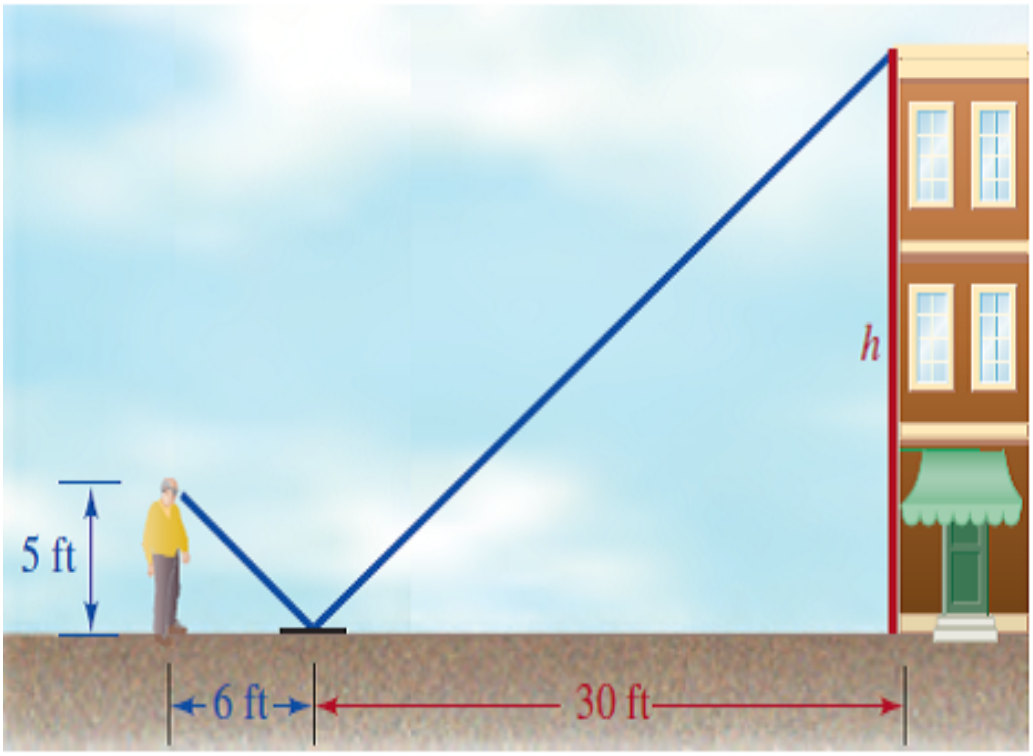
Basics of trigonometry

Nikola Pavličević

2019/12/27

There are some trigonometry basics presented here alongside some proofs. At first let us consider example in figure 1 (chegg 2019). Here the idea is to find height of a building. It can be done using triangle similarity and trigonometry. Triangle similarity is given first and trigonometry after. This is given through ratio $\frac{5}{h} = \frac{6}{30}$, $\frac{5}{h} = \frac{1}{5}$, $\frac{25}{h} = 1$, $h = 25$.

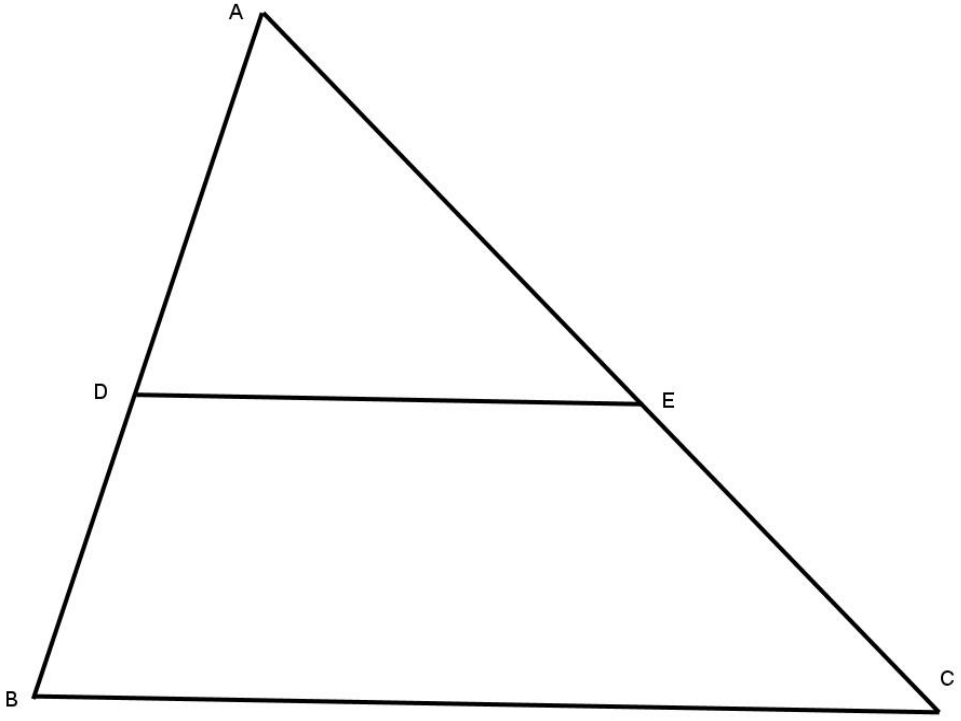
Figure 1: Check height of a building



Next the idea is to establish actual ratios. This is given next and figure 2 (natureofmathematics 2019) can be checked for more details.

$$DE||BC \text{ lines DE and BC are parallel} \tag{1}$$
$$D = B \text{ angles at D and B are same} \tag{2}$$
$$E = C \text{ angles at E and C are same} \tag{3}$$
$$ADE \text{ similar to } ABC \tag{4}$$
$$\frac{AB}{AD} = \frac{AC}{AE} \text{ because angles are same} \tag{5}$$
$$\frac{AD + DB}{AD} = \frac{AE + EC}{AE} \tag{6}$$
$$AE * AD + AE * DB = AE * AD + EC * AD \tag{7}$$
$$\frac{AD}{DB} = \frac{AE}{EC} \tag{8}$$

Figure 2: Check similar triangles



Next let us look at height of a building using trigonometric identities. Basic identities are given for right angle triangle and these are cosine (adjacent side over hypotenuse), sine (opposite side over hypotenuse) and tangent (opposite side over adjacent side). In current example $r^2 = 5^2 + 6^2 = 25 + 36 = 61$, $r = 7.81$, $\sin A = \frac{5}{7.81} = 0.64$, $A = 40$ degrees. Next $\tan A = \tan 40 = \frac{h}{30}$, $0.84 * 30 = 25.2 = h$ (almost same as before).

Next let us express trigonometric identities using symbols. This can be seen in figure 3 (a circle of radius r and a right angle triangle, vcemethods12 2019). Here $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$ and $\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$. Case is even simpler for unit circle (and right angle triangle having hypotenuse of length one, figure 4, stackexchange 2019). Here $\sin\theta = y$ and $\cos\theta = x$.

Figure 3: A circle of radius r

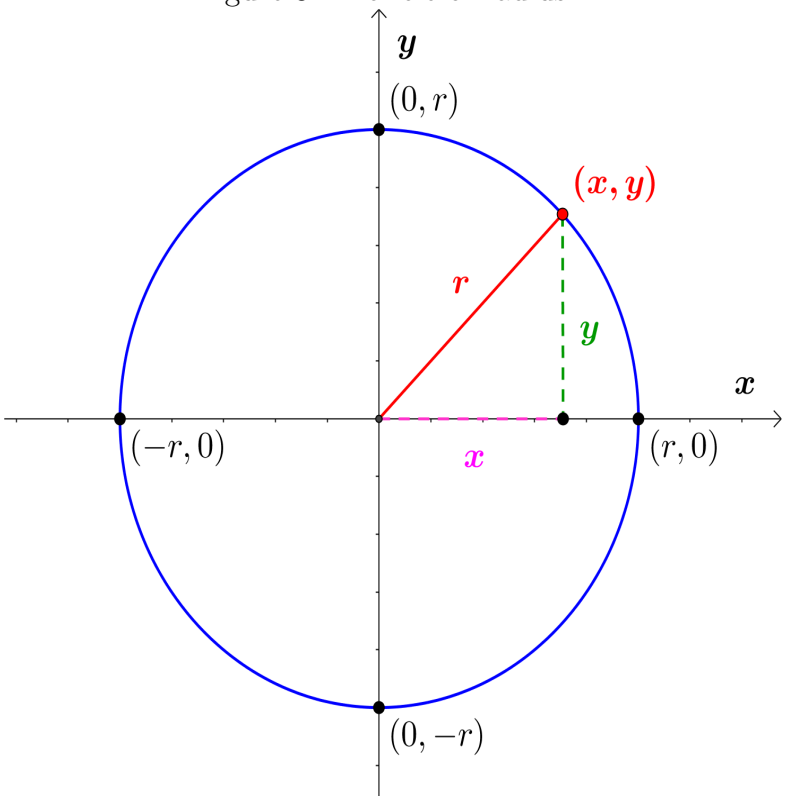
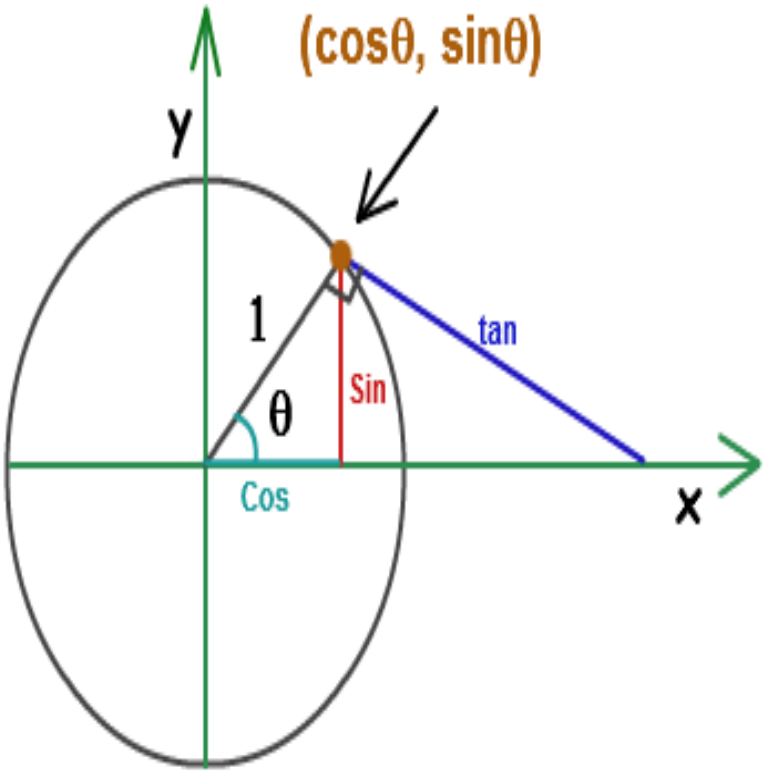


Figure 4: A unit circle



References

chegg (2019). *Check height of a building using triangle similarity*. Online. URL: https://www.chegg.com/homework-help/use-similar-triangles-solve-problem-see-example-9-height-bui-chapter-13.8-problem-98ss-solution-9781133169123-exc.

natureofmathematics (2019). *Triangle similarity*. Online. URL: https://natureofmathematics.wordpress.com/lecture-notes/thales/.

stackexchange (2019). *A unit circle*. Online. URL: https://math.stackexchange.com/questions/2673771/how-to-visualise-positive-and-negative-tangents/2674255#2674255.

vcemethods12 (2019). *A circle of radius r*. Online. URL: https://sites.google.com/site/vcemethods12/6---power-functions/6e---circles-and-semicircles.