Assignment 5

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***Abstract*—This document explains the properties of tangent to a circle and how to find the equation of the tangent to the circle at a given point.**

Download all python codes from

https://github.com/subhasishsaikia22/EE5609−Matrix−theory

and latex-tikz codes from

https://github.com/subhasishsaikia22/EE5609−Matrix−theory

1 Problem

Find the equation of the tangent to the following curve at the points stated:

$$\mathbf{x}^{\mathbf{T}}\mathbf{x} = 25, \begin{pmatrix}3\\4\end{pmatrix} \tag{1.0.1}$$

2 Explanation

The given equation of the curve:

$$\mathbf{x}^{\mathbf{T}}\mathbf{x} = 25 \tag{2.0.1}$$

The general equation of a second degree can be expressed as:

$$\mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f = 0 \tag{2.0.2}$$

Comparing (2.0.2) with (2.0.1):

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = \begin{pmatrix}0\\0\end{pmatrix} and f = -25 \tag{2.0.3}$$

For $\mathbf{V} = \mathbf{I}$, (2.0.2) represents a circle. $\mathbf{c}$ represent the center , r the radius and $\mathbf{q}$ the point of contact of the tangent to the circle .
The center and radius is given by:

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.4}$$

$$r = \sqrt{\mathbf{u}^{\mathbf{T}}\mathbf{u} - f} = \sqrt{0 - (-25)} = 5 \tag{2.0.5}$$

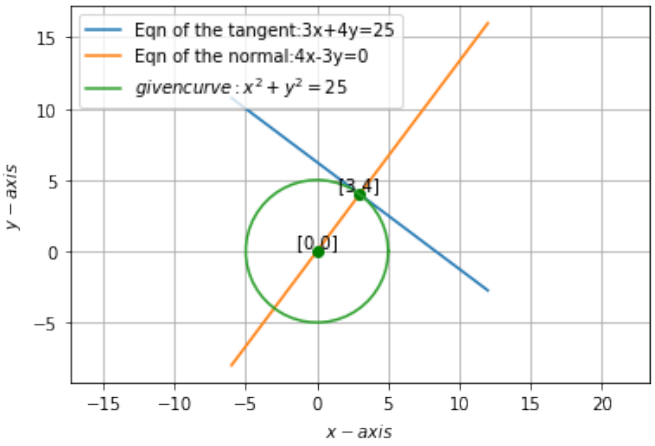

Fig. 1: This is the 2D diagram of the given curve $\mathbf{x}^{\mathbf{T}}\mathbf{x} = 25$ and the tangent to it at $\begin{pmatrix}3\\4\end{pmatrix}$

The given point of contact

$$\mathbf{q} = \begin{pmatrix}3\\4\end{pmatrix} \tag{2.0.6}$$

The direction vector of the line joining the point $\mathbf{q}$ and the center $\mathbf{c}$ is:

$$\mathbf{n} = \mathbf{q} - \mathbf{c} \tag{2.0.7}$$

$$\implies \mathbf{n} = \mathbf{q} + \mathbf{u} = \begin{pmatrix}3\\4\end{pmatrix} \tag{2.0.8}$$

The vector $\mathbf{n}$ is normal to the tangent of the circle, drawn at $\mathbf{q}$
The equation of the tangent is

$$\mathbf{n}^{\mathbf{T}}\left(\mathbf{x} - \mathbf{q}\right) = 0 \tag{2.0.9}$$

$$\mathbf{n}^{\mathbf{T}}\mathbf{x} = c \tag{2.0.10}$$

$$\tag{2.0.11}$$

where

$$c = \mathbf{n}^{\mathbf{T}}\mathbf{q} \tag{2.0.12}$$

$$\implies c = \begin{pmatrix}3 & 4\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix} = 25 \tag{2.0.13}$$

Thus the equation of the tangent to the curve at $\mathbf{q}$ is

$$\mathbf{n}^{\mathbf{T}}\mathbf{x} = 25 \tag{2.0.14}$$

$$\implies \begin{pmatrix}3 & 4\end{pmatrix}\mathbf{x} = 25 \tag{2.0.15}$$