

Assignment 2

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Abstract—This document explains the properties of a directional vector and how to find out if the given points are the vertices of a parallelogram, using directional vectors

Download all python codes from

<https://github.com/subhasishsaikia22/EE5609-Matrix-theory>

and latex-tikz codes from

<https://github.com/subhasishsaikia22/EE5609-Matrix-theory>

1 PROBLEM

Using directional vectors, show that the points

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 7 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (1.0.1)$$

are vertices of a parallelogram.

2 EXPLANATION

Two lines are parallel if their respective directional vectors are in the same ratio.

Let the points be denoted by:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{C} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{D} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (2.0.4)$$

The directional vector of \mathbf{AB} is

$$\begin{pmatrix} 2 - 5 \\ 1 - 4 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \quad (2.0.5)$$

The directional vector of \mathbf{BC} is

$$\begin{pmatrix} 5 - 4 \\ 4 - 7 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad (2.0.6)$$

The directional vector of \mathbf{CD} is

$$\begin{pmatrix} 4 - 1 \\ 7 - 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad (2.0.7)$$

The directional vector of \mathbf{AD} is

$$\begin{pmatrix} 2 - 1 \\ 1 - 4 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad (2.0.8)$$

The directional vector of \mathbf{AC} is

$$\begin{pmatrix} 2 - 4 \\ 1 - 7 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \end{pmatrix} \quad (2.0.9)$$

Since the directional vectors of \mathbf{AB} and \mathbf{CD} are in the same ratio, so \mathbf{AB} and \mathbf{CD} are parallel and also opposite to each other.

Similarly, the directional vectors of \mathbf{BC} and \mathbf{AD} are in the same ratio, hence \mathbf{BC} and \mathbf{AD} are parallel and opposite.

Since the two pairs of opposite sides are parallel, the given points are the vertices of the parallelogram.

Moreover the sum of the directional vectors of \mathbf{AB} and \mathbf{BC}

$$\begin{pmatrix} -3 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 + 1 \\ -3 - 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$$

Thus $\mathbf{AB} + \mathbf{BC} = \mathbf{AC}$, which satisfy parallelogram law of vector addition i.e vector sum of two adjacent side of a parallelogram is the diagonal vector of the parallelogram.

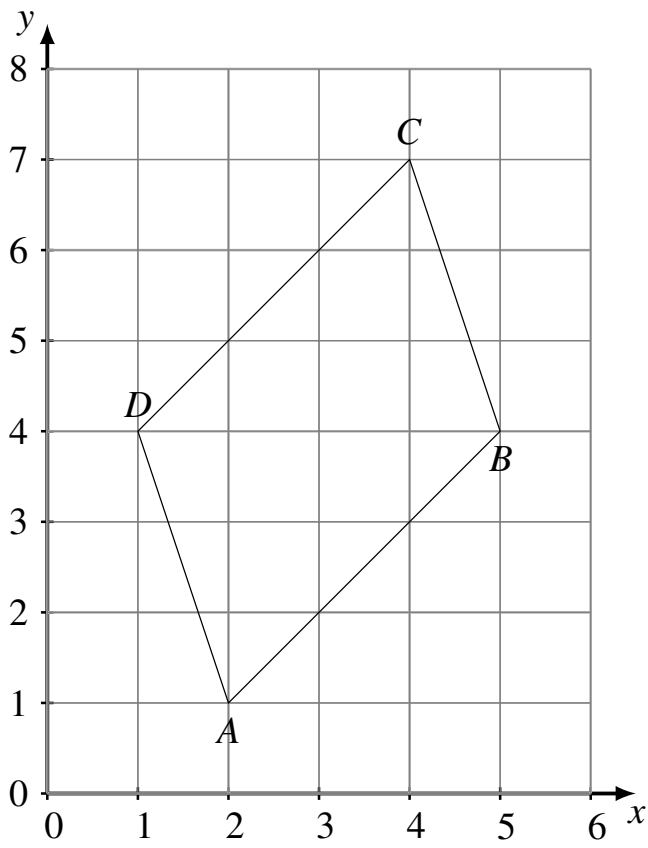


Fig. 1: This is the parallelogram with the given vertices