1

Assignment 10

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Abstract

This document explains the procedure of determining the rank of the given matrix.

Download latex-tikz codes from

https://github.com/subhasishsaikia22/EE5609-Matrix-theory

1 Problem

What is the rank of the following matrix?

$$\begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 2 & 2 & 2 & 2 \\
1 & 2 & 3 & 3 & 3 \\
1 & 2 & 3 & 4 & 4 \\
1 & 2 & 3 & 4 & 5
\end{pmatrix}$$

- 1) 2
- 2) 3
- 3) 4
- 4) 5

2 Explanation

Rank of a matrix	The rank of the given matrix is determine by reducing it to row reduced echelon form.	
	A matrix is in row echelon form if:	
	>all rows consisting of only zeroes are at the bottom.	
	>the leading coefficient of a nonzero row is always strictly	
	to the right of the leading coefficient of the row above it	

TABLE 1: Definition

3 Solution

Given Given
$$\begin{cases} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \xrightarrow{R_3 = R_3 - R_1} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

$$\begin{cases} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 3 & 3 \\ 0 & 1 & 2 & 3 & 4 \end{pmatrix} \xrightarrow{R_4 = R_4 - R_3} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 3 & 3 \\ 0 & 1 & 2 & 3 & 4 \end{pmatrix} \xrightarrow{R_3 = R_3 - R_4} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 = R_3 - R_4 - R_5} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 = R_3 - R_4 - R_5} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 = R_3 - R_4 - R_5} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 = R_3 - R_4 - R_5} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 = R_3 - R_4 - R_5} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 = R_3 - R_4 - R_5} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus the rank of the given matrix is 5

TABLE 2: RREF and Rank

4 Solution

Option	Solution	True/
		False
1	2	False
2	3	False
3	4	False
4	5	True

TABLE 3: correct option