Assignment 4

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***Abstract*—This document explains the properties of inscribed circle and escribed circle and how to find out the centre of the corresponding circles given the equation of the three sides of the triangle.**

Download all python codes from

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https://github.com/subhasishsaikia22/EE5609−Matrix−theory
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and latex-tikz codes from

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https://github.com/subhasishsaikia22/EE5609−Matrix−theory
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1 Problem

The sides**AB**, **BC**, **CA**of a triangle have equations

$$\begin{pmatrix} 4 & -3 \end{pmatrix} x = 12 \tag{1.0.1}$$

$$\begin{pmatrix} 3 & 4 \end{pmatrix} x = 24 \tag{1.0.2}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} x = 2 \tag{1.0.3}$$

Find the coordinates of the centres of the inscribed circle and of the escribed circle opposite to the vertex **A**.

2 Explanation

If the three vertices are located at

$$\mathbf{A} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{y}_1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \mathbf{x}_2 \\ \mathbf{y}_2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \mathbf{x}_3 \\ \mathbf{y}_3 \end{pmatrix} \tag{2.0.1}$$

and the sides opposite these vertices have corresponding lengths **a**, **b** and **c** then:
The coordinates of the centre of the Inscribed circle

$$\mathbf{O}\begin{pmatrix} \mathbf{x}_0 \\ \mathbf{y}_0 \end{pmatrix} = \begin{pmatrix} \frac{a\mathbf{x}_1 + b\mathbf{x}_2 + c\mathbf{x}_3}{a+b+c} \\ \frac{a\mathbf{y}_1 + b\mathbf{y}_2 + c\mathbf{y}_3}{a+b+c} \end{pmatrix} \tag{2.0.2}$$

And the coordinates of the centre of the Escribed circle opposite to the vertex **A**

$$\mathbf{P}\begin{pmatrix} \mathbf{p}_0 \\ \mathbf{q}_0 \end{pmatrix} = \begin{pmatrix} \frac{b\mathbf{x}_2 + c\mathbf{x}_3 - a\mathbf{x}_1}{b+c-a} \\ \frac{b\mathbf{y}_2 + c\mathbf{y}_3 - a\mathbf{y}_1}{b+c-a} \end{pmatrix} \tag{2.0.3}$$

The vertices **A** is the intersection of the sides **AB** and **CA**.Thus **A** is obtained from

$$\begin{pmatrix} 4 & -3 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} 12 \\ 2 \end{pmatrix} \tag{2.0.4}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 4 & -3 & 12 \\ 0 & 1 & 2 \end{pmatrix} \xleftrightarrow{R_1 = R_1 + 3R_2} \begin{pmatrix} 4 & 0 & 18 \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_1 = \frac{1}{4}R_1} \begin{pmatrix} 1 & 0 & \frac{9}{2} \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.6}$$

$$\implies \mathbf{A}\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{y}_1 \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ 2 \end{pmatrix} \tag{2.0.7}$$

The vertices **B** is the intersection of the sides **AB** and **BC**.Thus **B** is obtained from

$$\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix} x = \begin{pmatrix} 12 \\ 24 \end{pmatrix} \tag{2.0.8}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 4 & -3 & 12 \\ 3 & 4 & 24 \end{pmatrix} \xleftrightarrow{R_2 = 4R_2 - 3R_1} \begin{pmatrix} 4 & -3 & 12 \\ 0 & 25 & 60 \end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow{R_2 = \frac{1}{25}R_2} \begin{pmatrix} 4 & -3 & 12 \\ 0 & 1 & \frac{12}{5} \end{pmatrix} \xleftrightarrow{R_1 = R_1 + 3R_2} \begin{pmatrix} 4 & 0 & \frac{96}{5} \\ 0 & 1 & \frac{12}{5} \end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow{R_1 = \frac{1}{4}R_1} \begin{pmatrix} 1 & 0 & \frac{24}{5} \\ 0 & 1 & \frac{12}{5} \end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{B}\begin{pmatrix} \mathbf{x}_2 \\ \mathbf{y}_2 \end{pmatrix} = \begin{pmatrix} \frac{24}{5} \\ \frac{12}{5} \end{pmatrix} \tag{2.0.12}$$

The vertices **C** is the intersection of the sides **BC** and**CA**.Thus **C** is obtained from

$$\begin{pmatrix} 3 & 4 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} 24 \\ 2 \end{pmatrix} \tag{2.0.13}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 3 & 4 & 24 \\ 0 & 1 & 2 \end{pmatrix} \xleftrightarrow{R_1 = R_1 - 4R_2} \begin{pmatrix} 3 & 0 & 16 \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.14}$$

$$\xleftrightarrow{R_1 = \frac{1}{3}R_1} \begin{pmatrix} 1 & 0 & \frac{16}{3} \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.15}$$

$$\implies \mathbf{C}\begin{pmatrix} \mathbf{x}_3 \\ \mathbf{y}_3 \end{pmatrix} = \begin{pmatrix} \frac{16}{3} \\ 2 \end{pmatrix} \tag{2.0.16}$$

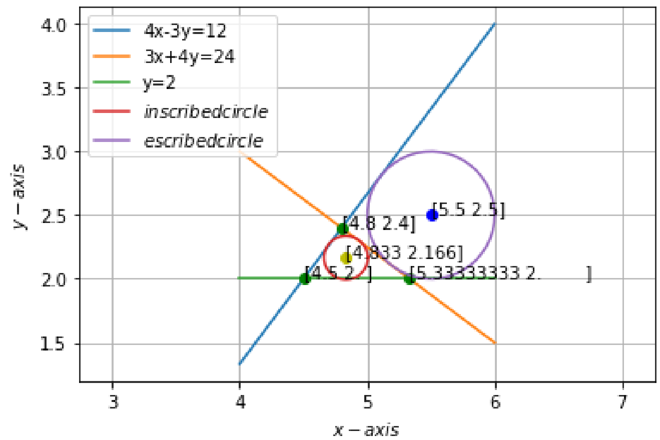

Fig. 1: This is the 2D diagram of the triangle , the inscribed circle and the escribed circle opposite to vertex **A**

The length of the side **AB**

$$\mathbf{c} = \sqrt{(\mathbf{x}_2 - \mathbf{x}_1)^2 + (\mathbf{y}_2 - \mathbf{y}_1)^2} = 0.5 \tag{2.0.17}$$

The length of the side **BC**

$$\mathbf{a} = \sqrt{(\mathbf{x}_3 - \mathbf{x}_2)^2 + (\mathbf{y}_3 - \mathbf{y}_2)^2} = 0.666 \tag{2.0.18}$$

The length of the side **CA**

$$\mathbf{b} = \sqrt{(\mathbf{x}_1 - \mathbf{x}_3)^2 + (\mathbf{y}_1 - \mathbf{y}_3)^2} = 0.833 \tag{2.0.19}$$

Therefore the coordinates of the centre of the Inscribed circle

$$\mathbf{O}\begin{pmatrix}\mathbf{x}_0\\ \mathbf{y}_0\end{pmatrix} = \begin{pmatrix}\frac{0.666\frac{9}{2}+0.833\frac{24}{5}+.5\frac{16}{3}}{.666+.833+.5}\\ \frac{0.666(2)+0.833\frac{12}{5}+.5(2)}{.666+.833+0.5}\end{pmatrix} = \begin{pmatrix}4.833\\ 2.166\end{pmatrix} \tag{2.0.20}$$

And the coordinates of the centre of the Escribed circle opposite to the vertex **A**

$$\mathbf{P}\begin{pmatrix}\mathbf{p}_0\\ \mathbf{q}_0\end{pmatrix} = \begin{pmatrix}\frac{0.833\frac{24}{5}+.5\frac{16}{3}-0.666\frac{9}{2}}{.833+0.5-.666}\\ \frac{0.833\frac{12}{5}+.5(2)-0.666(2)}{.833+0.5-.666}\end{pmatrix} = \begin{pmatrix}5.499\\ 2.499\end{pmatrix} \tag{2.0.21}$$