Assignment 11

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Abstract

This document explains the transformation of vector space $\mathbb{P}_n$

Download latex-tikz codes from

https://github.com/subhasishsaikia22/EE5609−Matrix−theory

1 Problem

Consider the vector space $\mathbb{P}_n$ of real polynomials in x of degree less than or equal to n. Define $\mathbf{T} : \mathbb{P}_2 \to \mathbb{P}_3$ by $\mathbf{T}(f(x)) = \int_0^x f(t)\,dt + f'(x)$.Then the matrix representation of $\mathbf{T}$ with respect to the bases $\left\{1, x, x^2\right\}$ and $\left\{1, x, x^2, x^3\right\}$ is

1) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \frac{1}{2} & 0 \\ 0 & 2 & 0 & \frac{1}{3} \end{pmatrix}$

2) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$

3) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{3} \end{pmatrix}$

4) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \frac{1}{2} \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$

2 Solution

Given	$$\mathbf{T}(f(x)) = \int_0^x f(t)\,dt + f'(x) \quad (2.0.1)$$ The basis vector of domain space $\left\{1, x, x^2\right\}$. The basis vector of co-domain space $\left\{1, x, x^2, x^3\right\}$. For the Transformation, we first find the images of the basis vector of domain space with respect to T, which is then expressed as the linear combination of basis vector

	of codomain space basis.
solution	Base vector 1: $$f(x) = 1 \implies f'(x) = 0 \quad (2.0.2)$$ $$\mathbf{T}(1) = \int_0^x 1\, dt + 0 = [t]_0^x = x \quad (2.0.3)$$ $$= 0 + 1.x + 0.x^2 + 0.x^3 \quad (2.0.4)$$ $$\mathbf{T}(1) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.5)$$ Base vector 2: $$f(x) = x \implies f'(x) = 1 \quad (2.0.6)$$ $$\mathbf{T}(x) = \int_0^x t\, dt + 1 = \left[\frac{t^2}{2}\right]_0^x + 1 = \frac{x^2}{2} + 1 \quad (2.0.7)$$ $$= 1 + 0.x + \frac{1}{2}.x^2 + 0.x^3 \quad (2.0.8)$$ $$\mathbf{T}(x) = \begin{pmatrix} 1 \\ 0 \\ \frac{1}{2} \\ 0 \end{pmatrix} \quad (2.0.9)$$ Base vector 3: $$f(x) = x^2 \implies f'(x) = 2x \quad (2.0.10)$$ $$\mathbf{T}(x^2) = \int_0^x t^2\, dt + 2t = \left[\frac{t^3}{3}\right]_0^x + [2t]_0^x = \frac{x^3}{3} + 2x \quad (2.0.11)$$ $$= 0 + 2.x + 0.x^2 + \frac{1}{3}.x^3 \quad (2.0.12)$$ $$\mathbf{T}(x^2) = \begin{pmatrix} 0 \\ 2 \\ 0 \\ \frac{1}{3} \end{pmatrix} \quad (2.0.13)$$ Therefore $$\mathbf{T} = \begin{pmatrix} \mathbf{T}(1) & \mathbf{T}(x) & \mathbf{T}(x^2) \end{pmatrix} \quad (2.0.14)$$ $$= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix} \quad (2.0.15)$$

TABLE 2: Solution

3 OPTION

Option	**Solution**	**True/ False**
1	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \frac{1}{2} & 0 \\ 0 & 2 & 0 & \frac{1}{3} \end{pmatrix}$	False
2	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$	True
3	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{3} \end{pmatrix}$	False
4	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \frac{1}{2} \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$	False

TABLE 3: correct option