PYL204 Assignment 1

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1 Approach

- Generated **Lagrange Interpolation** polynomials for China and India using **Divided Difference** coefficients
- The polynomials and their derivatives were evaluated using **Horner's** method (modified to use with divided difference coefficients)
  - **Horner's** method was preferred due to its $O(N)$ running time, lower error and more stable value due to fewer number of computations required
  - In contrast, using **Neville's** method to evaluate the polynomial takes $O(N^2)$ running time
- To find when India's population crosses China is equivalent to finding the root of difference of polynomials that fit to China and India respectively
- I try to find an initial estimate of this root (upto 0.5 tolerance) through the **Bisection** method
- Using the above value, I tried to use **Newton's** method to get the root upto 10^{-9} tolerance in very few iterations

2 Code

- **background_functions.py** : Contains the **LagrangePolynomial** class and functions for **bisection** and **newton**
- **main1.py** : Finds the root of the difference of China and India's population using functions imported from **background_functions.py**
- **main2.py** : Finds where the **derivative** of the difference between China and India is 0 (finding the extremum point) using functions imported from **background_functions.py**

3 Instructions to run the code

1. **main1.py** and **main2.py** are to be executed.

2. Install the library requirements using **pip install - r "path to requirements.txt"** where the absolute path of requirements.txt is to be put inside the "" on the terminal. **requirements.txt** is present in the zip file submitted

3. Run main1.py or main2.py using **python -u "file/path"** in the terminal where **file/path** is the absolute path to the python file

4 Results and Discussion

- As per the polynomial fit, **India never crosses China's population**. Newton's method does not converge for any initial value. Bisection method also cannot be applied since the function always remains positive over the whole interval

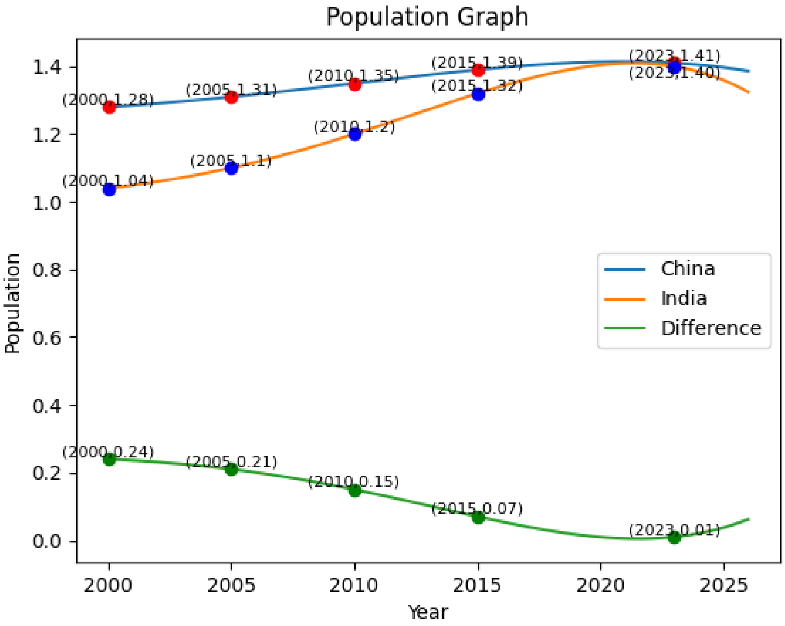
- Newton's method diverges as seen in the execution of main1.py even after 10 steps

- One can even verify this himself by further increasing **maxiter_newton** variable in main1.py

- We can also infer the same from the plots below

- We can calculate the extremum points of the **Difference** polynomial by calculating the roots of its derivative. This is put in main2.py file

  - **China_der** and **India_der** polynomials storing the derivatives of China and India are created
  - The root of their difference is calculated between **2000** and **2026** by getting an initial estimate using bisection method, then refining using newton's method
  - Even after checking all mini intervals in between 2000 and 2026, we only get one root = **2021.5236877539198** which can be seen on running main2.py
  - The value of the difference of the population at this root is **0.0046722764866684585** Clearly this value cannot be the maxima as the difference achieves much higher values than this at many instances (Like at x=2000, difference is 0.24 billions)
  - So, **2021.5236877539198** must be the minima of the difference. This rigorously confirms that there exists no root of the difference since the minimum value it attains is **0.0046722764866684585** which is positive


Population Graph
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Population
2000
2005
2010
2015
2020
2025
Year
China
India
Difference
(2000,1.28)
(2005,1.31)
(2010,1.35)
(2015,1.39)
(2023,1.41)
(2000,1.04)
(2005,1.1)
(2010,1.2)
(2015,1.32)
(2023,1.40)
(2000,0.24)
(2005,0.21)
(2010,0.15)
(2015,0.07)
(2023,0.01)