CELEBAL COE ASSIGNMENT

WEEK-4

Problem 1: Apply Bayes Theorem to solve a medical diagnosis problem given conditional probabilities:

$P(\text{Disease}) = 0.01$

$P(\text{No Disease}) = 0.99$

$P(\text{Positive Test} \mid \text{Disease}) = 0.95$

$P(\text{Positive Test} \mid \text{No Disease}) = 0.1$

We have to find the probability that a person has the ~~highest~~ disease given that they tested positive $P(\text{Disease} \mid \text{Positive Test})$.

Solution: Using Bayes Theorem:

$$P(\text{Disease} \mid \text{Positive Test}) = \frac{P(\text{Positive Test} \mid \text{Disease}) \cdot P(\text{Disease})}{P(\text{Positive Test})}$$

where,

$$\begin{aligned} P(\text{Positive Test}) &= P(\text{Positive Test} \mid \text{Disease}) \cdot P(\text{Disease}) + P(\text{Positive Test} \mid \text{No Disease}) \cdot P(\text{No Disease}) \\ &= (0.95 * 0.01) + (0.1 * 0.99) \\ &= 0.0095 + 0.099 \\ &= 0.1085 \end{aligned}$$

Now applying Bayes' Theorem:

$$\begin{aligned} P(\text{Disease} \mid \text{Positive Test}) &= \frac{0.95 * 0.01}{0.1085} \\ &= \frac{0.0095}{0.1085} \\ &= 0.0876 \end{aligned}$$

So, $P(\text{Disease} \mid \text{Positive Test}) \approx 0.0876$

Problem 2: Find the eigen values and corresponding eigen vectors of the given matrix.

$$A = \begin{pmatrix} 2 & 1 \\ 4 & 5 \end{pmatrix}$$

Solution: $\det(A - \lambda I) = 0$ (characteristic equation)

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 \\ 4 & 5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(5-\lambda) - 4 = 0$$

$$\Rightarrow \lambda^2 - 7\lambda + 6 = 0$$

$$\Rightarrow (\lambda - 1)(\lambda - 6) = 0$$

$$\Rightarrow \lambda = 1, 6$$

Eigen values of A is 1 and 6.

$\lambda_1 = 1 \rightarrow$

$$\begin{pmatrix} 2-\lambda & 1 \\ 4 & 5-\lambda \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 4 & 4 & 0 \end{array}\right) \xrightarrow{R_2 \to R_2 - 4R_1}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$x_1 + x_2 = 0$$

$$\Rightarrow x_1 = -x_2$$

let, $x_2 = 1$

$x_1 = -1$

$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ is eigen vector corresponding to $\lambda_1 = 1$

$\lambda_2 = 6 \rightarrow$

$$\begin{pmatrix} 2-\lambda & 1 \\ 4 & 5-\lambda \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -4 & 1 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\Rightarrow \left(\begin{array}{cc|c} -4 & 1 & 0 \\ 4 & -1 & 0 \end{array}\right) \xrightarrow{R_2 \to R_1 + R_2}$$

$$\left(\begin{array}{cc|c} -4 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$-4x_1 + x_2 = 0$$

or, $x_1 = \frac{1}{4} x_2$

let, $x_2 = 1$

$x_1 = 1/4$

$\begin{pmatrix} 1/4 \\ 1 \end{pmatrix}$ is eigen vector corresponding to $\lambda_2 = 6$

Problem 3: Calculate determinant of 3×3 matrix and find the inverse if possible.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$$

Solution:

$$\det(A) = 1(0-24) - 2(0-20) + 3(0-5)$$
$$= -24 + 40 - 15$$
$$= 1$$

Since, $\det(A) \neq 0$, inverse matrix is possible.

$$\mathrm{Adj}(A) = \begin{bmatrix} \begin{pmatrix} 1 & 4 \\ 6 & 0 \end{pmatrix} & -\begin{pmatrix} 0 & 4 \\ 5 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ 5 & 6 \end{pmatrix} \\ -\begin{pmatrix} 2 & 3 \\ 6 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 3 \\ 5 & 0 \end{pmatrix} & -\begin{pmatrix} 1 & 2 \\ 5 & 6 \end{pmatrix} \\ \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} & -\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix} & \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} -24 & 20 & -5 \\ 18 & -15 & 4 \\ 5 & -4 & 1 \end{bmatrix}^T = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{\det(A)} \cdot \mathrm{adj}(A)$$

$$\text{or, } A^{-1} = \frac{1}{1} \cdot \begin{pmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{pmatrix}$$

$$\text{or, } A^{-1} = \begin{pmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{pmatrix}$$

Problem 4: Describe the properties and applications of Normal distribution and calculate probabilities using its properties.

Solution: The Normal distribution $N(\mu, \sigma^2)$ has the following properties:

(a) Symmetric around the mean μ.

(b) Bell shaped curve

(c) Mean, median and mode are equal.

(d) Approximately 95% of the data falls within one standard deviation σ of the mean μ.

Applications:

(a) Used in statistical inference.

(b) Used in hypothesis testing

(c) Used in finance for asset return

(d) Used in quality control for process variability.

Finding the probability that a randomly selected observation from a normal distribution with $\mu = 50$ and $\sigma = 10$ falls between 40 and 60.

Ans → Step 1: Standardizing the variables

$$Z1 = (40-50)/10 = -1$$

$$Z2 = (60-50)/10 = 1$$

Step 2: Using standard normal distribution table to find the probabilities

$$P(Z \leq 1) = 0.8413$$

$$P(Z \leq -1) = 0.1587$$

Step 3: Calculating probability for the range

$$P(40 \leq X \leq 60) = P(Z \leq 1) - P(Z \leq -1) = 0.8413 - 0.1587 = 0.6826$$