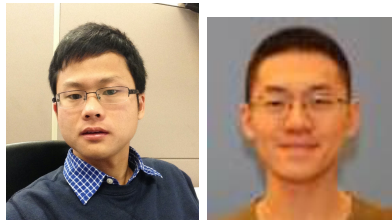# Implementing A Particle Collision Model:
## A Report for CS 6491 Fall 2015 Project 1

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# 1 Objective

To simulate the precise motion of a large number of balls in a cube in computer graphics.

# 2 Input

We are given the intial prositions and velocities of $n$ balls. We have the following assumptions: All balls are in the cube, have the same radius and mass, and are pairwise disjoint. Balls starts in constant velocity motion. All collisions are elastic.

# 3 Implementation

## 3.1 Key Classes and Methods

We implement new classes and methods for *balls* class, some of which are:

### 3.1.1 class Colli

A class used to store collision object and time. It has two variables: $p$ denotes the index of ball to collide, and $t$ is the collision time.

### 3.1.2 Colli[] colli

An array to store next collisions of all balls with balls or walls (where walls are denoted by negative integers).

### 3.1.3 void initPointsOnGrid()

A modified version of ball collisions to help debugging.

### 3.1.4  float calB2BTime()

Calculate the collision time between two balls. If two balls cannot collide in positive finite time, the return -1. We solved the quadratic equation with regard to $P_0$, $P_1$, $V_0$, $V_1$, $r$, $t$ to get the minimal collision time.

### 3.1.5  float calB2WTime()

Calculate the collision time between one ball and one wall. If they cannot collide in positive finite time, then return -1. We solve linear equation with regard to $P_0$, $V_0$, $r$, $R$, $t$ to get the minimal collision time.

### 3.1.6  Colli calOneBallColli()

Calculate the smallest collision time between ball i and other balls and walls. This function checks all possibilities between one ball and all the other balls, and all six walls. A minimum collision time is maintained along with the next collision object (a ball or a wall). The complexity of this function is O(n), where n is the number of balls.

### 3.1.7  void initCollision()

Initialize the collision array by calculating next collision of all balls. This complexity is $O(n^2)$. Luckily we only need to do it once in initialization phase.

### 3.1.8  void bounceTwoBalls()

Solve two balls' velocities after collision. Normal compatible components (along the line conneting these two balls) will exchange.

### 3.1.9  void updateB2BColli()

We calculate the velocities of two balls after collisions, then update the next collisions of these two balls. We also need to update balls that "WERE going to collide into these two balls", and balls that "ARE supposed to collide into these two balls". All the rest do not need to recalculate. In the first case, since the initial velocities of all balls are randomized, the expected number of collisions for a ball with other balls is $nP$, $n$ is the number of balls, and $P$ is the probability of collision, which is proportional to $r$, where $r$ is inversely proportional to $n$. Therefore, the number of collision is O(1). In the second case, it is clearly no more than two. Therefore, the complexity is $O(n)$.

### 3.1.10  void updateB2WColli()

We calculate the velocities of the ball after collision with a wall, then update the next collisions of this ball. We also need to update balls that "WERE going to collide into this ball", and the ball that "IS supposed to collide into this ball". All the rest do not need to recalculate. The complexity is $O(n)$.

### 3.1.11  void updateColli()

The main function to update the state whenever collision is about to happen. Update collision array using either one of the update functions.

## 3.2  General Picture

### 3.2.1  Initialization Phase: Before the graphic process begins

After adding balls to the scene, we call $initCollisions()$ to initialize collision array $colli[]$.

### 3.2.2 Update Phase: At each frame of the graphic process

We call updateColli() to update collision array if next collision is about to happen.

# 4 Complexity/Performance

$O(n^2)$ for pairwise comparison in the initialization phase, and $O(n)$ in the update phase.