CE 331A GEOINFORMATICS

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Criteria for selection of layout of triangles

1. Simple triangles should be preferably equilateral.
2. Braced quadrilaterals should be preferably approximate squares.
3. Centered polygons should be regular.
4. Arrangement should be such that the computations can be done through two or more independent routes.
5. Arrangement should be such that at least one route and preferably two routes form well-conditioned triangles.
6. No angle of the figure, opposite a known side should be small, whichever end of the series is used for computation.
7. Angles of simple triangles should not be less than 45°, and in the case of quadrilaterals, no angle should be less than 30°. In the case of centered polygons, no angle should be less than 40°.
8. Sides of the figures should be of comparable lengths. Very long lines and very short lines should be avoided.
9. Layout should be such that it requires least work to achieve maximum progress.
10. As far as possible, complex figures should not involve more than 12 conditions.

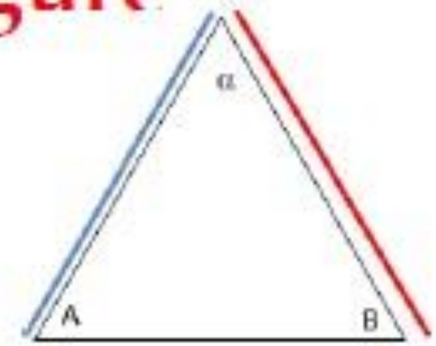
Strength of Figure

- Factor to be considered for triangles for which the computations can be maintained within a decided degree of accuracy.
- Plays also an important role in deciding layout of a triangulation system.
- U.S. Coast and Geodetic Surveys developed a method of evaluating strength of a triangulation figure. Based on the fact that
  - Computations in triangulation involve use of angles of triangle and length of one known side. The other two sides are computed by sine law.
  - For a given change in the angles, the sine of small angles change more rapidly than those of large angles. Hence, smaller angles less than 30° should not be used in computation of triangulation. If, due to unavoidable circumstances, angles less than 30° are used, then it must be ensured that this is not opposite the side whose length is required to be computed for carrying forward the triangulation series.
- Based on expression for square of probable error (L^2), that would occur in the sixth place of the logarithm of any side, if the computations were carried out from a known side through a single chain of triangles after the net has been adjusted for the side and angle conditions.

Formula for strength of figure

$$L^2 = \frac{4d^2}{3}\left(\frac{D-C}{D}\right)\sum\left(\delta A^2 + \delta A\delta B + \delta B^2\right) = \frac{4d^2}{3}R$$

$$R = \left(\frac{D-C}{D}\right)\sum\left(\delta A^2 + \delta A\delta B + \delta B^2\right)$$



d = Probable error of an observed direction in seconds

D = Number of directions observed (forward and or backward) excluding those along the known or fixed line

= Total number of directions observed

δA = Difference per second in sixth place of logarithm of the sine of distance angle A of each triangle in the chain used.

A & B = Distance angles A and B of a triangle, where

A: angle opposite to the known side

B: angle opposite to the side which is to be computed

Azimuth angle = Third angle not used in this formula is called azimuth angle (α)

R = Relative strength, contains terms affected by shape of figure

C = Number of angle and side conditions to be satisfied in the triangulation network

Number of conditions in triangulation

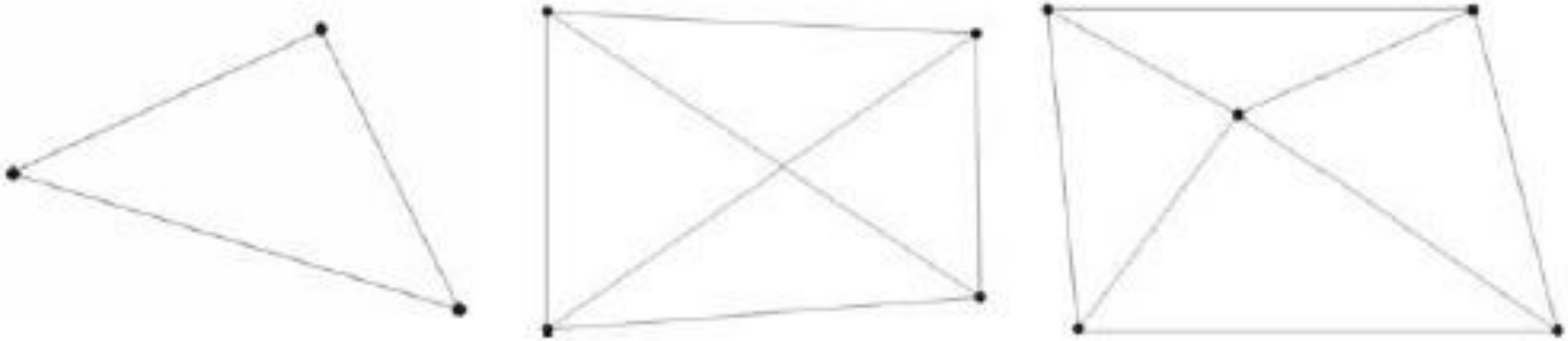
D	=	number of directions observed (forward and or backward) excluding those along the known or fixed line
C	=	number of angle and side conditions to be satisfied in the triangulation net
	=	(Number of angle + Number of side) conditions
	=	$(N' - S' + 1) + (N - 2S + 3)$
$N' - S' + 1$	=	number of angle conditions
$N - 2S + 3$	=	number of side conditions
N'	=	total number of lines observed in both direction
N	=	total number of lines including known side
S'	=	number of occupied stations
S	=	total number of stations

Factors affecting strength of figure

- Strength of figure depends upon:
  - number of directions observed
  - number of geometrical conditions imposed by the shape of the figure, together with the number of stations occupied in field.
  - size of angles used in computations
- Relative strength can be computed in terms of factor R
  - Lower the value of R, stronger the figure.

$$R = \left(\frac{D - C}{D} \right) \sum \left(\delta A^2 + \delta A \times \delta B + \delta B^2 \right)$$

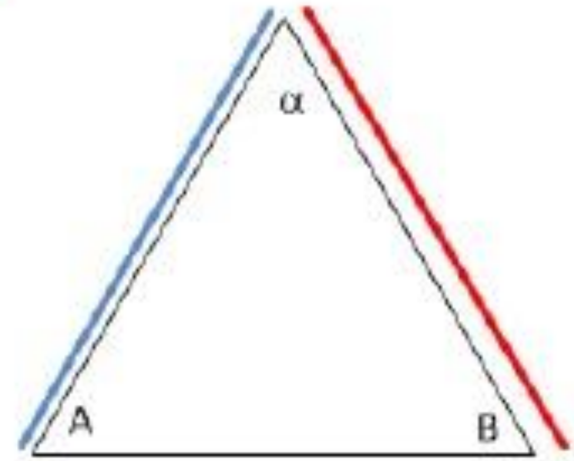
Examples for computations (D-C) / D for different figure



- Triangle
- Braced quadrilateral
- Four sided central point figure without diagonals
- Combination of different figures

1. Triangle

- $N' = 3$
- $N = 3$
- $S = 3$
- $S' = 3$
- $C = (3 - 3 + 1) + (3 - 2 \times 3 + 3) = 1$
- $D = 2 \times (3 - 1) = 4$
- $(D - C)/D = (4 - 1)/4 = 0.75$



N'	=	total number of lines observed in both direction
N	=	total number of lines including known side
S'	=	number of occupied stations
S	=	total number of stations
C	=	number of angle and side conditions to be satisfied in the triangulation net
	=	(Number of angle + Number of side) conditions
	=	$(N' - S' + 1) + (N - 2S + 3)$
D	=	number of directions observed (forward and or backward) excluding those along the known or fixed line
$N' - S' + 1$	=	number of angle conditions
$N - 2S + 3$	=	number of side conditions

Braced quadrilateral

$N' = 6$

$N = 6$

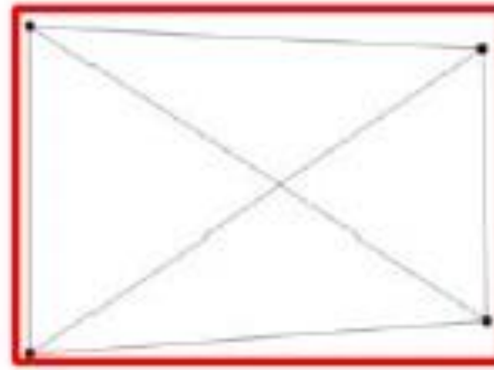
$S = 4$

$S' = 4$

$C = (6 - 4 + 1) + (6 - 2 \times 4 + 3) = 4$

$D = 2 \times (6 - 1) = 10$

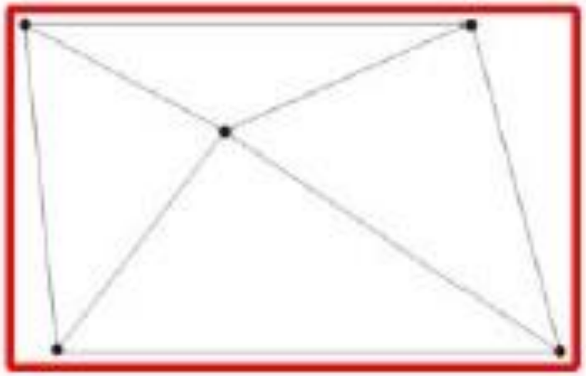
$(D - C)/D = (10 - 4)/10 = 0.6$



D	=	number of directions observed (forward and or backward) excluding those along the known or fixed line
C	=	number of angle and side conditions to be satisfied in the triangulation net
	=	(Number of angle + Number of side) conditions
	=	$(N' - S' + 1) + (N - 2S + 3)$
$N' - S' + 1$	=	number of angle conditions
$N - 2S + 3$	=	number of side conditions
N'	=	total number of lines observed in both direction
N	=	total number of lines including known side
S'	=	number of occupied stations

Four sided central point figure without diagonals

- $N' = 8$
- $N = 8$
- $S = 5$
- $S' = 5$
- $C = (8 - 5 + 1) + (8 - 2 \times 5 + 3) = 5$
- $D = 2 \times (8 - 1) = 14$
- $(D - C)/D = (14 - 5)/14 = 0.64$



N'	=	total number of lines observed in both direction
N	=	total number of lines including known side
S'	=	number of occupied stations
S	=	total number of stations
D	=	number of directions observed (forward and or backward) excluding those along the known or fixed line
C	=	number of angle and side conditions to be satisfied in the triangulation net
	=	(Number of angle + Number of side) conditions
	=	$(N' - S' + 1) + (N - 2S + 3)$
$N' - S' + 1$	=	number of angle conditions
$N - 2S + 3$	=	number of side conditions

Four-sided central-point figure with one diagonal

$N = 9$

$N' = 9$

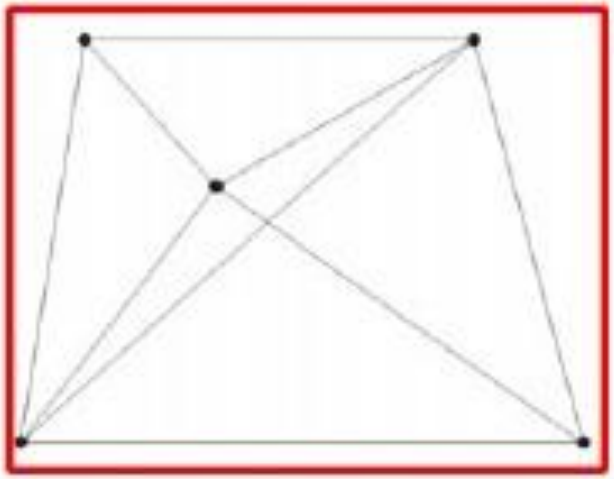
$S = 5$

$S' = 5$

$C = (9 - 5 + 1) + (9 - 2 \times 5 + 3) = 7$

$D = 2 \times (9 - 1) = 16$

$(D - C)/D = (16 - 7)/16 = 0.56$



N'	=	total number of lines observed in both direction
N	=	total number of lines including known side
S'	=	number of occupied stations
S	=	total number of stations
D	=	number of directions observed (forward and or backward) excluding those along the known or fixed line
C	=	number of angle and side conditions to be satisfied in the triangulation net
	=	(Number of angle + Number of side) conditions
	=	$(N' - S' + 1) + (N - 2S + 3)$
$N' - S' + 1$	=	number of angle conditions
$N - 2S + 3$	=	number of side conditions

Combination of figures

$N' = 19$

$N = 19$

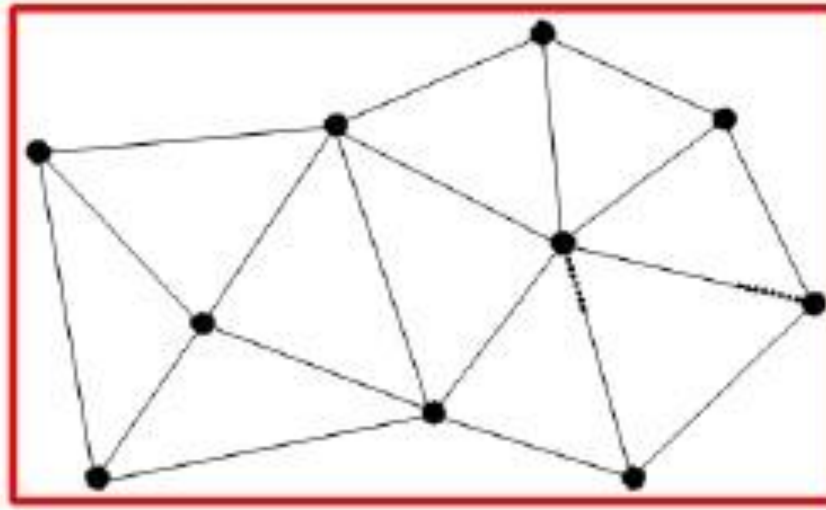
$S = 10$

$S' = 10$

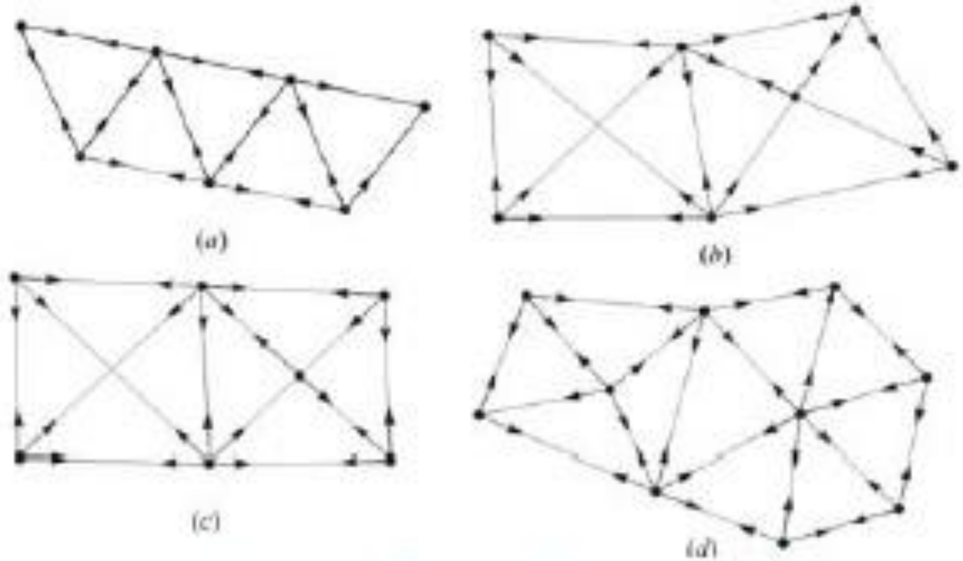
$C = (19 - 10 + 1) + (19 - 2 \times 10 + 3) = 12$

$D = 2 \times (19 - 1) = 36$

$(D - C)/D = (36 - 12)/36 = 0.67$



N'	=	total number of lines observed in both direction
N	=	total number of lines including known side
S'	=	number of occupied stations
S	=	total number of stations
D	=	number of directions observed (forward and or backward) excluding those along the known or fixed line
C	=	number of angle and side conditions to be satisfied in the triangulation net
	=	(Number of angle + Number of side) conditions
	=	$(N' - S' + 1) + (N - 2S + 3)$
$N' - S' + 1$	=	number of angle conditions
$N - 2S + 3$	=	number of side conditions



(a) (b) (c) (d)

Figure (a): Directions observed are shown with arrows

C = (N' – S' + 1) + (N – 2S+ 3)

N = total number of lines including known side= 11

N′ = total number of lines observed in both directions = 9

S = total number of stations = 7

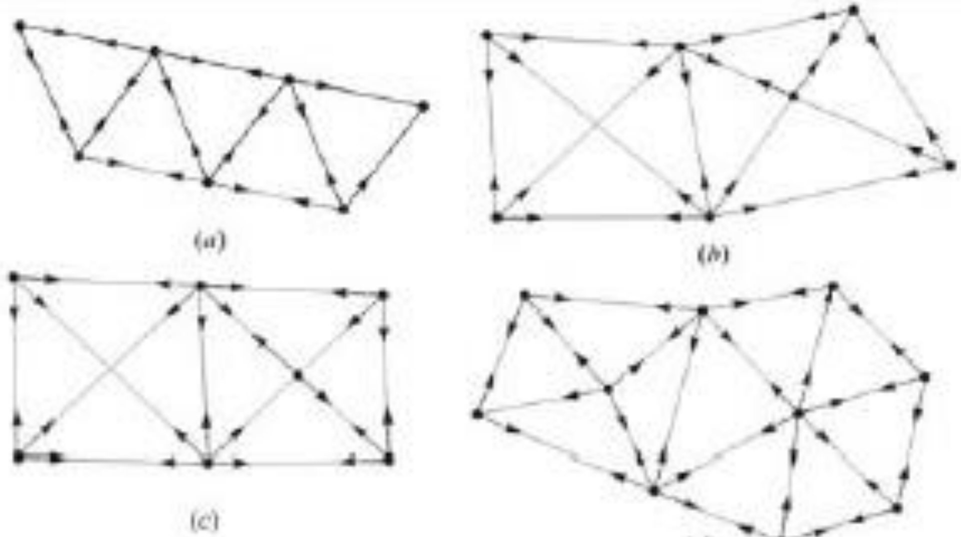
S′ = total number of stations occupied = 6

C = (9 – 6 + 1) + (11 – 2 × 7 + 3) = 4

D = total number of directions observed excluding the known side

= 2 × (N' – 1) + number of lines observed in one direction = 2 × (9 – 1) + 2 = 18

(D - C)/D = (18 - 4)/18 = 0.78



(a) (b) (c) (d)

Figure (b): Directions observed are shown with arrows

C = (N' – S' + 1) + (N – 2S+ 3)

N = total number of lines including known side = 13

N′ = total number of lines observed in both directions = 11

S = total number of stations = 7

S′ = total number of stations occupied = 7

C = (11 – 7 + 1) + (13 – 2 × 7 + 3) = 7

D = total number of directions observed excluding the known side

= 2 × (N' – 1) + number of lines observed in one direction = 2 × (11 – 1) + 2 = 22

(D - C)/D = (22 - 7)/22 = 0.68

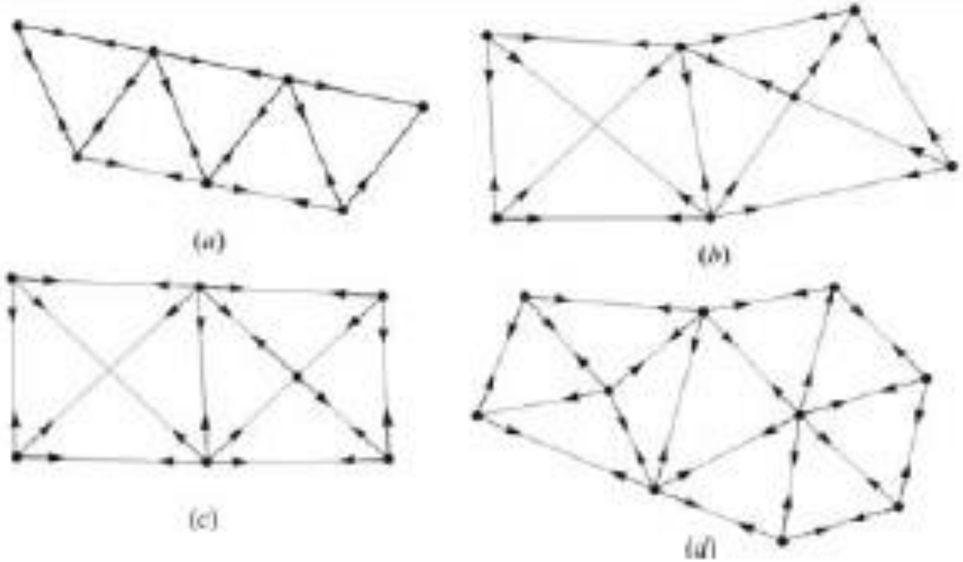


Figure (c): Directions observed are shown with arrows

$C = (N' - S' + 1) + (N - 2S + 3)$

N = total number of lines including known side = 13

N' = total number of lines observed in both directions = 11

S = total number of stations = 7

S' = total number of stations occupied = 7

$C = (11 - 7 + 1) + (13 - 2 \times 7 + 3) = 7$

D = total number of directions observed excluding the known side

$= 2 \times (N' - 1)$ + number of lines observed in one direction $= 2 \times (11 - 1) + 2 = 22$

$(D - C)/D = (22 - 7)/22 = 0.68$

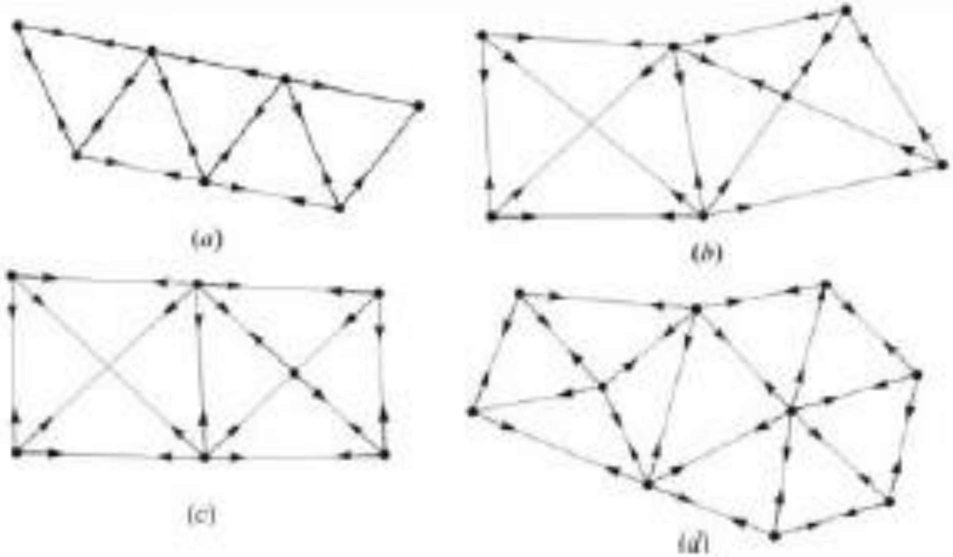


Figure (d): Directions observed are shown with arrows

$C = (N' - S' + 1) + (N - 2S + 3)$

N = total number of lines including known side = 19

N′ = total number of lines observed in both directions = 19

S = total number of stations = 10

S′ = total number of stations occupied = 10

$C = (19 - 10 + 1) + (19 - 2 \times 10 + 3) = 12$

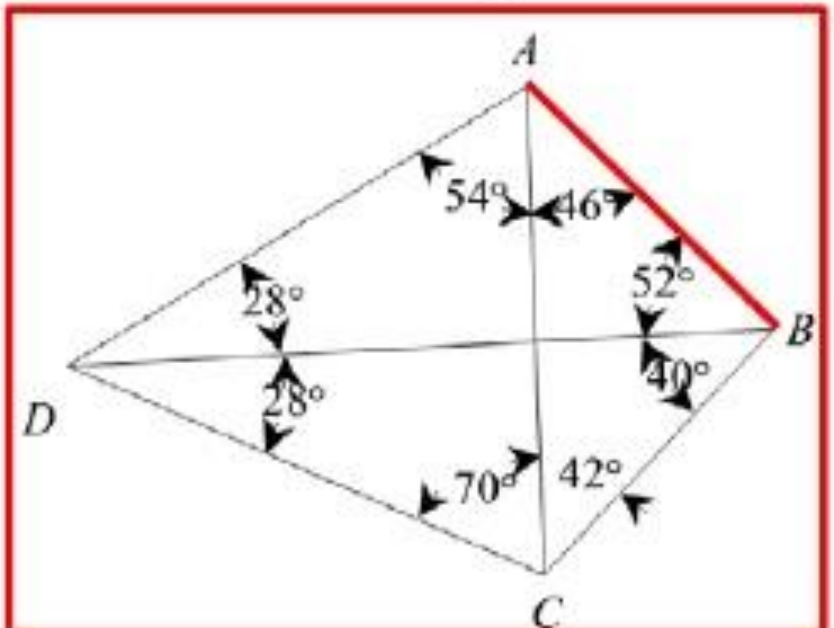
D = total number of directions observed excluding the known side

$= 2 \times (N' - 1) +$ number of lines observed in one direction $= 2 \times (19 - 1) + 0 = 36$

$(D - C)/D = (36 - 12)/36 = 0.67$

Strength of figure

Question: Compute strength of figure ABCD for all the routes by which length CD (to be computed) can be determined from the known side AB assuming that all stations have been occupied, and find the strongest route.



Four possible route:

Route-1: ΔABD and ΔBDC having common side BD

Route-2: ΔABD and ΔADC having common side AD

Route-3: ΔABC and ΔADC having common side AC

Route-4: ΔABC and ΔBCD having common side BC

Relative strength of figures can be computed quantitatively in terms of the factor R as given below:

$$R=\left(\frac{D-C}{D}\right)\sum\left(\delta A^2+\delta A\delta B+\delta B^2\right)$$

- By means of computed values of R alternative routes of computations can be compared, and hence, the best or strongest route can be selected.
- As strength of a figure is approximately equal to the strength of the strongest chain, the lowest value of R is a measure of the strongest route.
- For braced quadrilateral (D-C)/D is = 0.6 (computed earlier.