Engineering Mathematics-3

Laplace Transform

Tutorial and Assignment-3

Tutorial-3

1. Express the following piecewise-continuous causal functions in-terms of unit step functions, and hence determine the Laplace Transform

$$\text{i) } f(t) = \begin{cases} t & 0 \le t < 2, \\ 0 & t \ge 2. \end{cases} \qquad \text{ii) } f(t) = \begin{cases} 2 & 0 \le t < 3, \\ -2 & t \ge 3. \end{cases} \qquad \text{iii) } f(t) = \begin{cases} 2t^2 & 0 \le t < 3, \\ t+4 & 3 \le t < 5, \\ 9 & t \ge 5. \end{cases}$$

$$\text{iv) } f(t) = \begin{cases} \sin t & 0 \le t < 2\pi, \\ 0 & t \ge 2\pi. \end{cases} \qquad \text{v) } f(t) = \begin{cases} 0 & 0 \le t < 1, \\ t^2 & t \ge 1. \end{cases}$$

2. Obtain the inverse Laplace transforms of the following:

$$\text{i) } \frac{4s}{4s^2+1} \qquad \text{ii) } \frac{s+1}{s^2-4s} \qquad \text{iii) } \frac{s}{s^2+2s-3} \qquad \text{iv) } \left(\frac{2}{s} - \frac{1}{s^3}\right)^3$$

3. Solve the following initial value problem using Laplace transform.

   (a) $y'' - 4y' + 4y = t^3 e^{2t}, \quad y(0) = 0,\ y'(0) = 0.$

   (b) $y'' - 5y' + 6y = \sin t\ H(t - 2\pi), \quad y(0) = 1,\ y'(0) = 0.$

   (c) $y'' + 4y = f(t), \quad y(0) = 0,\ y'(0) = -1, \quad \text{where} \quad f(t) = \begin{cases} 1 & 0 \le t < 1, \\ 0 & t \ge 1. \end{cases}$

   (d) $y'' - 2y' = 1 + \delta(t-2), \quad y(0) = 0,\ y'(0) = 1.$

4. Using the convolution theorem, determine the following inverse Laplace transforms.

$$\text{i) } \mathcal{L}^{-1}\left\{\frac{1}{s(s+3)^3}\right\} \qquad \text{ii) } \mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2(s+3)^2}\right\} \qquad \text{iii) } \mathcal{L}^{-1}\left\{\frac{1}{s^2(s+4)}\right\}$$

Assignment-3

Marks-15

1. Express in terms of Heaviside unit step functions the following piecewise-continuous-functions. Obtain the Laplace transfrom of the function.

$$f(t) = \begin{cases} 3t^2 & 0 < t \le 4, \\ 2t - 3 & 4 < t < 6, \\ 5 & t > 6. \end{cases}$$

2. Obtain the inverse Laplace transforms of the following:

i) $\dfrac{2s^2+4s+9}{(s+2)(s^2+3s+3)}$ ii) $\dfrac{(1+e^{-2s})^2}{s+2}$

3. Use Laplace transform to solve the given initial value problem

$$\frac{d^2y}{dt^2}+y=f(t), \qquad y(0)=0, \quad \frac{dy}{dt}|_{t=0}=1$$

where

$$f(t)=\begin{cases} 0 & 0\le t\le \pi, \\ 1 & \pi\le t<2\pi, \\ 0 & t\ge 2\pi. \end{cases}$$

Note: Submit assignment to the respective course leader on or before 25 October 2019.