RAMAIAH UNIVERSITY OF APPLIED SCIENCES

Faculty of Mathematical & Physical Sciences
Model Question paper – B. Tech.

Department	**:** Mathematics
Programme	**:** B. Tech.
Semester/Batch	**:** 3rd /2018
Course Code	**:** 19MHB201A
Course Title	**:** Engineering Mathematics - 3

Model Question paper MATLAB

INSTRUCTIONS TO STUDENTS:
1. Answer all questions.
2. Missing data may be appropriately assumed.
3. Indicate the question numbers clearly against your answers.

Maximum Duration: 2 Hour **Maximum Marks: 50**

Question No. 1 **(6+4=10 Marks)**

a. Use MATLAB built-in function to determine the Laplace transform of

$$f(t) = \begin{cases} t^2 & 0 \le t < 3, \\ 2t - 1 & 3 \le t < 5\,, \\ 9 & t \ge 5. \end{cases}$$

b. Use MATLAB built-in function to determine the inverse Laplace transform of

$$F(s) = \frac{1}{s^2(s^2+1)}.$$

Question No. 2 **(5+5=10 Marks)**

a. Use MATLAB built-in function to determine the Fourier transform of the function

$$f(t) = \begin{cases} 2 & |t| \le 1 \\ 0 & |t| > 1. \end{cases}$$

b. Plot the vector field $\mathbf{F}(x, y, z) = (3x^2)\mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$ in the interval $-3 \le x \le 3,\ -3 < y \le 3$ and $-3 \le z \le 3$.

Question No. 3: **(5+5=10 Marks)**

a. Use MATLAB built-in function to obtain quadratic fit of the given data and hence plot the given data and obtained quadratic curve in the same graph:

x	1.5	2.0	2.5	3.0	3.5	4.0
y	1.3	6.7	12.0	25.7	35.4	44.1

b. Plot the periodic function $f(x) = x^2, \forall x \in (-2,2), f(x+4) = f(x)$ in the interval $(-6, 6)$.

Question No. 4 **(10 Marks)**

In a machine the displacement of y of a point is given for a certain angle θ as follows:

θ^0	30	60	90	120	150	180	210	240	270	300	330	360
y	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

a. Write a MATLAB program to compute the Fourier series expansion of the displacement y, up to third harmonic.

b. Plot the data point and the Fourier series expansion in the same graph, and record the output.

Question No. 5: [Manual calculation] **(10 Marks)**

Using the method of least squares, fit a relation of the form $y = ae^{bx}$ to the following data:

x	2	3	4	5	6
y	144	172.8	207.5	250	298

Hence predict the value of y when $x = 5.4$.

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Solution manual

1a. Use MATLAB built-in function to determine the Laplace transform of

$$\boldsymbol{f(t)} = \begin{cases} \boldsymbol{t^2} & \boldsymbol{0 \le t < 3}, \\ \boldsymbol{2t-1} & \boldsymbol{3 \le t < 5}, \\ \boldsymbol{9} & \boldsymbol{t \ge 5}. \end{cases}$$

Soln.:- Express given $f(t)$ in terms of Heaviside function.

$$f(t) = t^2 + (2t - 1 - t^2)H(t-3) + \big(9 - (2t-1)\big)H(t-5)$$

MATLAB code:

```
syms t s
f = t^2+(2*t-1-t^2)*heaviside(t-3)+(9-(2*t-1))*heaviside(t-5);
F = laplace(f,t,s);
F = simplify(F)
disp(F)
```

Output:

```
-(2*exp(-3*s) + 4*s*exp(-3*s) + 2*s*exp(-5*s) + 4*s^2*exp(-
3*s) - 2)/s^3
```

Exercise:

Determine Laplace transform for the following:

1. $f(t) = \begin{cases} t-1, & 0 \le t < 4 \\ 2t, & t \ge 4 \end{cases}$
2. $f(t) = e^{3t}\cos(2t) + t^2\sin(3t)$
3. $f(t) = t^3\cos(3t)\,H(t-3)$

b. Use MATLAB built-in function to determine the inverse Laplace transform of

$$\boldsymbol{F(s) = \frac{1}{s^2(s^2+1)}}.$$

MATLAB code:

```
syms s t
F = 1/(s^2*(s^2+1));
f = ilaplace(F,s,t);
f = simplify(f);
disp(f)
```

Output:

```
t - sin(t)
```

Exercise:

Determine inverse Laplace transform for the following:

1. $F(s) = \frac{(s+1)e^{-s}}{s(s^2+1)^2}$
2. $F(s) = \frac{s+3}{(s-3)^2(s+4)}$

3. $F(s) = \frac{s^2+2}{s^2+2s+9}$

2a. Use MATLAB built-in function to determine the Fourier transform of the function

$$\boldsymbol{f(t) = \begin{cases} 2 & |t| \le 1 \\ 0 & |t| > 1. \end{cases}}$$

Soln.:- Express given $f(t)$ in terms of Heaviside function.

$$f(t) = 2\big(H(t+1) - H(t-1)\big)$$

MATLAB code:

```
syms t w
f = 2*(heaviside(t+1)-heaviside(t-1));
F = fourier(f,t,w);
F = simplify(F);
disp(F)
```

Output:
```
(4*sin(w))/w
```

Exercise:
Determine Fourier transform for the following:

1. $f(t) = e^{-t}H(t)$
2. $f(t) = \begin{cases} 2\sin t & |t| \le \pi \\ 0 & |t| > \pi \end{cases}$
3. $f(t) = \cos(4t)\,\big(H(t+2) - H(t-2)\big)$

b. Plot the vector field $\mathbf{F}(x,y,z) = (3x^2)\mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$ in the interval $-3 \le x \le 3$, $-3 \le y \le 3, -3 \le z \le 3$.

Soln.:- Note the components of $\mathbf{F}$ are $f_1 = 3x^2$, $f_2 = 2xz - y$ and $f_3 = z$

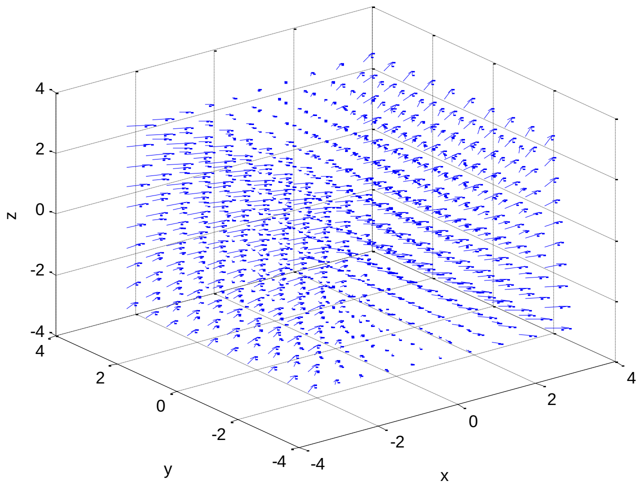
Now use MATLAB editor and write a script file to perform following steps
Step 1: create set of x, y and z values using `linspace`
Step 2: generate mesh grid points for x, y and z using `meshgrid(x,y,z)`
Step 3: Enter given f_1, f_2 and f_3
Step 4: use built-in function `quiver3(x,y,z,f1,f2,f3)`
Step 5: label x, y, z axes
Step 6: Copy and paste graph

MATLAB code:

```
[x,y,z] = meshgrid(-3:1:3);
f1 = 3*x.^2;
f2 = 2*x.*z-y;
f3 = z;
quiver3(x,y,z,f1,f2,f3)
xlabel('x')
ylabel('y')
zlabel('z')
```

OUTPUT

```
>> vector_plot
```



Practice Problems

1. $\mathbf{F}(x,y,z) = -y\mathbf{i} + x\mathbf{j}$ in the interval $-1 \le x \le 1,\ -1 \le y \le 1.$
2. $\mathbf{F}(x,y,z) = x\mathbf{i} + ye^{z}\mathbf{j} + z^2\mathbf{k}$ in the interval $-1 \le x \le 1,\ -1 \le y \le 1, -1 \le z \le 1.$
3. $\mathbf{F}(x,y,z) = \cos x\,\mathbf{i} + \sin y\,\mathbf{j} + (z-y)\mathbf{k}$ in the interval $-3 \le x \le 3, -3 < y \le 3,$ $-3 \le z \le 3.$
4. $\mathbf{F}(x,y,z) = (3x+2y)\mathbf{i} + x^2y^3\mathbf{j} + 2xe^{y}\mathbf{k}$ in the interval $-4 \le x \le 4,\ -4 < y \le 4,$ $-4 \le z \le 4.$
5. $\mathbf{F}(x,y,z) = xy\mathbf{i} + y^2z\mathbf{j}$, in the interval $-1 \le x \le 1, -1 \le y \le 1$

3a. Use MATLAB built-in function to obtain quadratic fit of the following data and hence plot the given data and obtained curve in same graph:

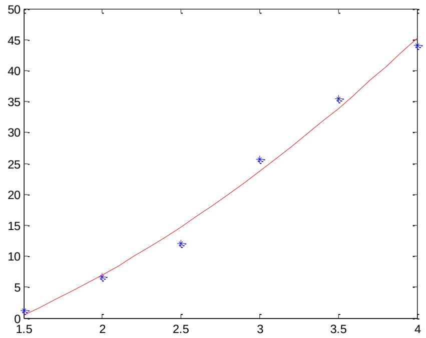
x	1.5	2.0	2.5	3.0	3.5	4.0
y	1.3	6.7	12.0	25.7	35.4	44.1

Solution.
MATLAB code:

```
x = [1.5 2 2.5 3 3.5 4];
y = [1.3 6.7 12 25.7 35.4 44.1];
p = polyfit(x,y,2);
syms t
yp = p(1)*t^2+p(2)*t+p(3);
fprintf('Quaratic fit for the given data is: y = ')
disp(vpa(yp,4))
plot(x,y,'*')
hold on
t=min(x):0.1:max(x);
z=eval(yp);
plot(t,z,'r')
```

Output:

Quaratic fit for the given data is: y = 2.436*t^2 + 4.535*t - 11.8



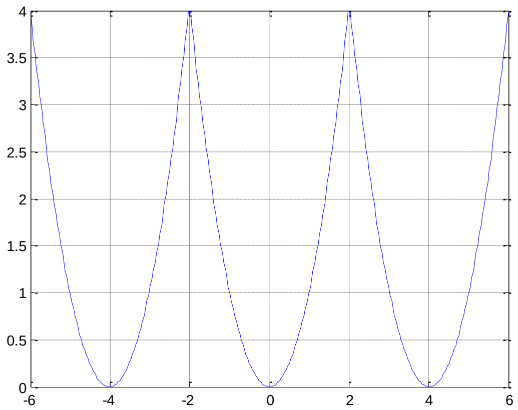
b. Plot the periodic function $f(x) = x^2, \forall x \in (-2, 2), f(x+4) = f(x)$ in the interval $(-6, 6)$.

Solution:

MATLAB code:

```
f=@(x) x.^2;
x=linspace(-2,2,100);
fx = repmat(f(x),1,3);                    %%%%%  3=(6-(-6))/4
x1 = linspace(-6,6,length(fx));
plot(x1,fx,'b')
grid on
```

Output:



Other example:

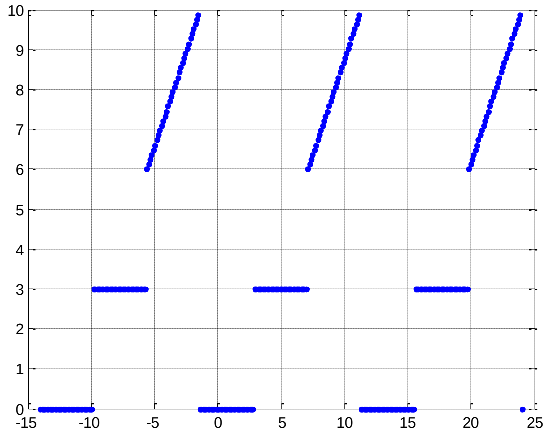
Plot the following periodic function in the interval $[-14, 24]$

$$f(x) = \begin{cases} 0 & -2 \le x \le 2 \\ 3 & 2 < x < 6 \\ x & 6 \le x < 10 \end{cases} , \qquad f(x+12) = f(x)$$

MATLAB code:

```
f= @(x) 0.*(-2<=x & x<2)+3.*(2<=x & x<6)+x.*(6<=x & x<10);
x=linspace(-2,10,100);
fx = repmat(f(x),1,3);                    %%%%%  3=(24-(-14))/(10-(-
2))
x1 = linspace(-14,24,length(fx));
plot(x1,fx,'.b')
grid on
```

Output:



Exercise:

1. Plot the periodic function:
   $f(x) = x^2 + 1 \qquad 0 \le x \le 2,\ f(x+2) = f(x)$ in the interval $[0,10]$.
2. Plot the periodic function:
   $$f(x) = \begin{cases} \frac{3x}{4}, & 0 \le x \le 4 \\ 7 - x, & 4 \le x \le 10 \\ -3, & 10 \le x \le 13 \end{cases},\ f(x+13) = f(x) \text{ in the interval } [-26, 26]$$
3. Plot the periodic function in the interval $[-2\pi, 4\pi]$
   $$f(x) = \begin{cases} 2\sin x & 0 \le x < \pi \\ x & \pi \le x \le 2\pi \end{cases}, \qquad f(x+2\pi) = f(x).$$
4. Plot the periodic function $f(x) = e^{3x}$, $\ 0 \le x \le 2$ and $f(x+2) = f(x)$ in the interval $[0, 8]$.
5. Plot the periodic function $f(x) = \begin{cases} 1, & 0 \le x < 1 \\ x, & 1 \le x \le 2 \end{cases}$ and $f(x+2) = f(x)$ in the interval $[0, 6]$.

4. In a machine the displacement of y of a point is given for a certain angle θ as follows:

θ^0	30	60	90	120	150	180	210	240	270	300	330	360
y	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

c. Write a MATLAB program to compute the Fourier series expansion of the displacement y, up to third harmonic.

d. Plot the data point and the Fourier series expansion in the same graph, and record the output.

Solution:

MATLAB PROGRAM (Includes Plotting commands):

```
x=[30 60 90 120 150 180 210 240 270 300 330 360];
x=(pi/180)*x;% only if theta is in terms of degree, not
required otherwise
y=[2.34 3.01 3.68 4.15 3.69 2.20 0.83 0.51 0.88 1.09 1.19
1.64];
syms t
```

```
T=2*pi; % Period, change according to question
w=2*pi/T;
h=3; % number of hormonics, change according to question
a0=2*mean(y);
HS=a0/2;
for i=1:h
    a(i)=2*mean(y.*cos(i*w*x));
    b(i)=2*mean(y.*sin(i*w*x));
    HS=HS+a(i)*cos(i*w*t)+b(i)*sin(i*w*t);
end
HS=vpa(HS,3);
disp(HS)
%Plotting
t=linspace(min(x),max(x),100);
y1=eval(HS);
plot(x,y,'*',t,y1,'r')
```

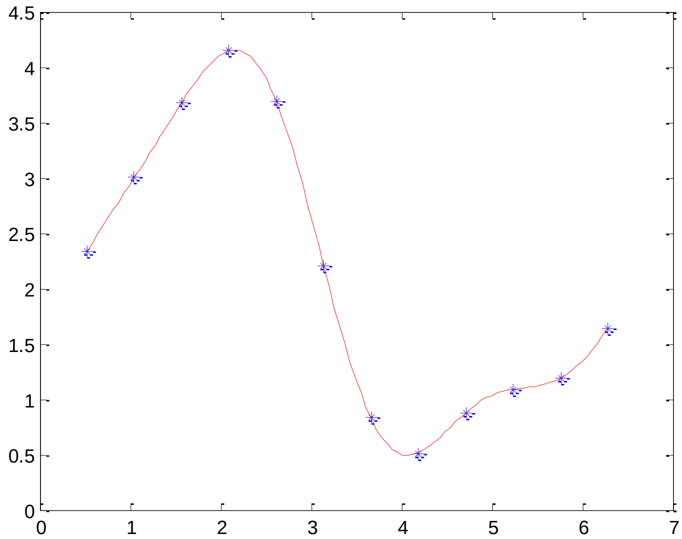
Above script is saved with a filename : HarmonicAnalysis.m

Output:

>> HarmonicAnalysis

4.2*cos(36.0*t) - 1.31e-14*sin(36.0*t) + 4.2*cos(12.0*t) + 4.2*cos(24.0*t) - 5.7e-15*sin(12.0*t) - 1.14e-14*sin(24.0*t) + 2.1

Plot:



Practice Problems:

1. The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel.

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$
$f(x)$	0	9.2	14.4	17.8	17.3	11.7

a. Write a MATLAB program to compute the Fourier series expansion of the displacement y, up to 20th harmonic.

b. Plot the data point and the Fourier series expansion in the same graph, and record the output.

2. In a machine the displacement of y of a given point is given for a certain angle θ as follows.

θ^o	0	30	60	90	120	150	180	210	240	270	300	330
y	7.9	8.0	7.2	5.6	3.6	1.7	0.5	0.2	0.9	2.5	4.7	6.8

a. Write a MATLAB program to compute the Fourier series expansion of the displacement y, up to 10th harmonic.

b. Plot the data point and the Fourier series expansion in the same graph, and record the output.

3. For the given data

x	0	1	2	3	4	5
$f(x)$	4	8	15	7	6	2

a. Write a MATLAB program to compute the Fourier series expansion of the displacement y, up to 25th harmonic.

b. Plot the data point and the Fourier series expansion in the same graph, and record the output.

4. For the given data

x	0	2	4	6	8	10	12
$f(x)$	9.0	18.2	24.4	27.8	27.5	22.0	9.0

a. Write a MATLAB program to compute the Fourier series expansion of the displacement y, up to 10th harmonic.

b. Plot the data point and the Fourier series expansion in the same graph, and record the output.

5. Using the method of least squares, fit a relation of the form $y = ae^{bx}$ to the following data:

x	**2**	**3**	**4**	**5**	**6**
y	**144**	**172.8**	**207.5**	**250**	**298**

Hence predict the value of y when $x = 5.4$.

Soln: To fit a curve

$$y = ae^{bx} \dots \dots (1)$$

By taking log to both sides of the above equation,

$$log\, y = log\, a + bx \dots \dots (2)$$

Put,

$$Y := log\, y, \quad A = log\, a \dots \dots \dots (3)$$

With this (2) implies

$$Y = A + bx \dots\dots (4)$$

Now fit a curve (4) for the data:

x	2	3	4	5	6
$Y \coloneqq log\, y$	4.9698	5.1521	5.3351	5.5215	5.6971

We use the method of least squares to find the best fit. Using this method we obtain two normal equations to find A and b in (4).

Normal equations for $Y = A + bx$ are:

$$\sum_i Y_i = An + b \sum_i x_i$$
$$\sum_i Y_i x_i = A \sum_i x_i + b \sum_i x_i^2$$

Or

$$\begin{pmatrix} \sum Y \\ \sum Yx \end{pmatrix} = \begin{pmatrix} n & \sum x \\ \sum x & \sum x^2 \end{pmatrix} \begin{pmatrix} A \\ b \end{pmatrix} \dots\dots (5)$$

Where

$\sum x = sum(x) = 20$

$\sum x^2 = sum(x.* x) = 20$

$\sum Y = sum(Y) = 26.6756$

$\sum Yx = sum(x.* Y) = 108.5264$, (Find using MATLAB)

$n = 5$ (number of terms)

Substituting in (5), we get

$$\begin{pmatrix} 26.6756 \\ 108.5264 \end{pmatrix} = \begin{pmatrix} 5 & 20 \\ 20 & 90 \end{pmatrix} \begin{pmatrix} A \\ b \end{pmatrix} \dots\dots (5)$$

If $N = \begin{pmatrix} 26.6756 \\ 108.5264 \end{pmatrix}$ and $M = \begin{pmatrix} 5 & 20 \\ 20 & 90 \end{pmatrix}$, then

$\begin{pmatrix} A \\ b \end{pmatrix} = M \backslash N = \begin{pmatrix} 4.6055 \\ 0.1824 \end{pmatrix}$ (find using MATLAB)

But,

$$A = log\, a \Rightarrow a = e^A = \exp(4.6055) = 100.0330.$$

Hence, required curve is

$$y = 100.033 e^{0.1824x}.$$

Exercise:

1. Find a least square cure $y = ax^b$ to the following data:

X	1	2	3	4	5
Y	0.5	2	4.5	8	12.5

2. Using a method of least square fit a relation of the form $y = ab^x$ to the following data:

X	2	4	6	8	10
Y	25	38	56	84	122

3. Fit a parabola $y = a + bx + cx^2$ to the following data:

X	2	4	6	8	10
Y	3.07	12.85	31.47	57.38	91.29