Engineering Mathematics-3

Fourier Series

Tutorial and Assignment-1

Tutorial-1

1. Obtian a Fourier series expansion for

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 \le x < \pi, \end{cases} \qquad f(x+2\pi) = f(x).$$

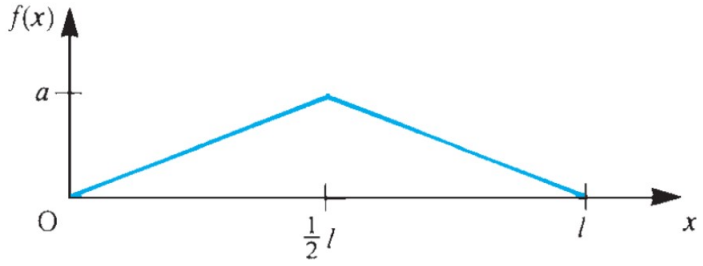
Hence deduce that

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} + \cdots .$$

2. Obtian a Fourier series expansion for $f(x) = |x|$, $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$.

3. Let $f(x) = x + \pi$, $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$. Then determine its Fourier series expansion for $f(x)$.

4. A tightly stretched flexible uniform string has its ends fixed at the points $x = 0$ and $x = \ell$. The midpoint of the string is displaced a distance a, as shown in the following Figure. If $f(x)$ denotes the displaced profile of the string, express $f(x)$ as a Fourier series expansion consisting only of sine terms.



Assignment-1

1. Obtain a Fourier series expansion of the function: $f(x) = (\pi - x)^2$, $0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$. Hence deduce that

$$\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots .$$

2. Determine Fourier series expansion of

$$f(x) = \begin{cases} 1 & -1 < x < 0 \\ x & 0 \le x < 1, \end{cases} \qquad f(x+2) = f(x).$$

Note: Submit assignment to the respective course leader on or before 5^{th} September 2019.