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Faculty : FET
Department : CSE
Sem : 4th

Date : 4-06-2020
Course code : 19MHB211A
Course Title : Engg. Maths-4

question 1.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial^2 x}, \quad 0 < x < l, \quad t > 0$$

$$U(0,t) = u(l,t) = 0, \quad t > 0$$

$$u(x,0) = \mu(l-x) = f(x), \quad 0 < x < l$$

$$\frac{\partial u}{\partial t}(x,0) = 0 = g(x) \quad 0 < x < l$$

we know,

$$U_n(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{l}\right)\left[A_n \cos\left(\frac{n\pi ct}{l}\right) + B_n \sin\left(\frac{n\pi ct}{l}\right)\right]$$

where,

$$A_n = \frac{2}{l}\int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right)$$

$$B_n = \frac{2}{n\pi l}\int_0^l g(x) \sin\left(\frac{n\pi x}{l}\right)$$

given,

$$g(x) = 0$$

$$\Rightarrow B_n = 0$$

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$$A_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{2}{l}\int_0^l \mu(l-x)\sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{2}{l}\left[\frac{\mu(l-x)\left(-\cos\left(\frac{n\pi x}{l}\right)\right)}{\frac{n\pi}{l}} - \frac{(-\mu)\left(-\sin\left(\frac{n\pi x}{l}\right)\right)}{\frac{n^2\pi^2}{l^2}}\right]_0^l$$

$$= \frac{2}{l}\left[0-0-\left[\frac{\mu(l)(-1)}{\frac{n\pi}{l}} - 0\right]\right]$$

$$= \frac{2}{\cancel{l}}\left[\frac{\mu . l^{\cancel{2}}}{n\pi}\right) = \frac{2\mu l}{n\pi}$$

$$\boxed{A_n = \frac{2\mu l}{n\pi}}$$

$$\boxed{U_n = \sum_{n=1}^{\infty} \frac{2\mu l}{n\pi}\cos\left(\frac{n\pi C t}{l}\right)\sin\left(\frac{n\pi x}{l}\right)}$$

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question-2

x	y	xy	x^2
1	910	910	1
2	680	1360	4
3	520	1560	9
4	450	1800	16
5	370	1850	25
6	380	2280	36
7	340	2380	49
28	3650	12140	140

$\Sigma x = 28$

$\Sigma y = 3650$

$\Sigma(xy) = 12140$

$\Sigma(x^2) = 140$

$(\Sigma x)^2 = 784$

W/k $y = a + bx$

where,

$$a = \frac{(\Sigma y)(\Sigma(x^2)) - (\Sigma x)(\Sigma xy)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$b = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$a = \frac{(3650 \times 140) - (28 \times 12140)}{7(140) - 784} = 872.857$$

$$b = \frac{7(12140) - (28 \times 3650)}{7(140) - 784} = -82.143$$

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$$\therefore,\ \boxed{y = 872.857 - 82.143x}$$

is the estimated linear regression eqn.

Emission (y) in the year 2018:

$2010 - x = 1$
$2018 - x = 9$

$$\therefore,\ y = 872.857 - 82.143(9)$$

$$\boxed{y = 82.1428}$$

$\therefore$, 82.1428 units of NO is estimated to be emitted in the year 2018.

question 3

let X, Y, Z be the events that X, Y, Z become the director respectively.

let B be the event that the Bonus Scheme is introduced

given,

$P(X) = 30\% = 0.3$

$P(Y) = 40\% = 0.4$

$P(Z) = 30\% = 0.3$

$P(B/X) = 0.5$

$P(B/Y) = 0.3$

$P(B/Z) = 0.8$

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w/k

$$P(Z/B) = \frac{P(Z)\,P(B/Z)}{P(X)P(B/X)+P(Y)P(B/Y)+P(Z)\,P(Z/B)}$$

$$= \frac{0.8 \times \cancel{0.3}}{(0.3\times \cancel{0.3}) + (0.4 \times \cancel{0.3}) + (\cancel{0.3}\times 0.8)}$$

$$= \frac{0.8}{1.7}$$

$$P(Z/B) = 0.4705$$

∴ the probability of Z becoming the director given the bonus scheme is introduced is

0.4705.

question - 4

let D be the event that the bulb is defective

given,

$$P(\bar{D}) = \frac{9}{10} = 0.9$$

$$P(D) = 1 - P(\bar{D}) = \frac{1}{10} = 0.1$$

$n=12$, let $P(X=x_i)$ be the probability of x_i defective bulbs.

a) $P(X=3) = {}^{12}C_3\,(0.1)^3\,(0.9)^3$

[∵, By Binomial diff. formula]

$$\boxed{P(X=3) = 0.085}$$

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b) $P(X \geq 2)$

W/k

$P(X=0) + P(X=1) + P(X=2) + \dots P(X=n) = 1$

$\Rightarrow P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$= 1 - ({}^{12}C_0 (0.1)^0 (0.9)^{12}) - ({}^{12}C_1 (0.1)^1 (0.9)^{11})$

$= 1 - 0.2824 - 0.3765$

$= 0.3409$

∴ Probability of atleast 2 defective bulbs is $\underline{0.3409}$.

c) $P(X=0) = {}^{12}C_0 (0.1)^0 (0.9)^{12}$

$= 0.2824$

∴ Probability of getting no defective bulb is $\underline{0.2824}$.

<u>question 5</u>

given,

$$\oint_C \frac{e^z}{z(z-2)(z-3)} dz \quad \text{where } C: |z| = 2.5$$

By Cauchy's integral theorem,

$$\oint_C \frac{f(z)}{(z-a)} dz = 2\pi i f(a), \text{ where } a \text{ is a pole.}$$

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$$I = \oint_C \frac{e^z}{z(z-2)(z-3)}\,dz$$

$$\frac{1}{z(z-2)(z-3)} = \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z-3}$$

$$1 = A(z-2)(z-3) + B(z-3)(z) + C(z-2)(z)$$

put $z=2$

$$\boxed{B = -1/2}$$

put $z=0$

$$\boxed{A = 1/6}$$

put $z=3$

$$\boxed{C = 1/3}$$

$$I = \oint_C \left(\frac{1}{6(z-0)} - \frac{1}{2(z-2)} + \frac{1}{3(z-3)}\right) e^z\,dz$$

By Cauchy Integral theorem.

$$= 2\pi i\left(\frac{e^0}{6} - \frac{e^z}{2}\right) \quad [\because |z| = 2.5]$$

$$= \pi i\left(1/3 - e^z\right)$$

$$\boxed{\therefore I = \frac{\pi i(1-3e^z)}{3}}$$

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question 6

$$U_{xx} + U_{yy} = 0$$

$$U(0,y) = U(1,y) = 0, \quad l = 1$$

$$u(x,0) = 0$$

$$u(x,a) = u(x,4) = x = f(x), \quad a = 4$$

wkt,

$$U(x,y) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right) \sinh\left(\frac{n\pi y}{l}\right)$$

where,

$$A_n = \frac{2}{l \sinh\left(\frac{n\pi a}{l}\right)} \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

$$A_n = \frac{2}{1 \cdot \sinh(4n\pi)} \int_0^1 x \sin(n\pi x)\, dx$$

$$= \frac{2}{\sinh(4n\pi)} \left[\frac{x(-\cos(\pi n x))}{n\pi} - \frac{(1)(-\sin(n\pi x))}{n^2\pi^2}\right]_0^1$$

$$= \frac{2}{\sinh(4n\pi)} \left[\frac{(1)(-(-1)^n)}{n\pi} - 0 - (0-0)\right]$$

$$= \frac{2}{\sinh(4n\pi)} \left[\frac{-(-1)^n}{n\pi}\right] = \frac{(-2)(-1)^n}{(n\pi)\sinh(4n\pi)}$$

$$\boxed{U(x,y) = \sum_{n=1}^{\infty} \frac{-2}{n\pi} \times \frac{(-1)^n}{\sinh(4n\pi)} \sin(n\pi x) \sinh(n\pi y)}$$

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