Engineering Mathematics-4

19MHB211A

Tutorial and Assignment-2

Tutorial-2

1. Use Charpits method to determine the completeintegral of the following equations:

   1.1 $pxy + pq + qy = yz$.

   1.2 $p^2x + q^2y = z$.

   1.3 $px + qy = pq$.

   1.4 $z^2(p^2 + q^2 + 1) = 1$.

   1.5 $p^2 - y^2q = y^2 - x^2$.

   1.6 $z = px + qy + p^2 + q^2$.

2. Obtain the solution of each of the following intial and boundary value problems:

   2.1 $u_t = 4u_{xx}, 0 < x < l, t > 0$, subject to

   $$\text{IC: } u(x,0) = x^2(1-x), 0 \leq x \leq \pi, \text{and}$$
   $$\text{BC: } u(0,t) = u(l,t) = 0, t > 0.$$

   2.2 $u_t = 4u_{xx}, 0 < x < \pi, t > 0$, subject to

   $$\text{IC: } u(x,0) = \sin^2 x, 0 \leq x \leq \pi, \text{and}$$
   $$\text{BC: } u(0,t) = u(\pi,t) = 0, t > 0.$$

   2.3 $u_t = u_{xx},\ 0 \leq x \leq 2, t \geq 0$, subject to

   $$\text{IC: } u(x,0) = x, 0 < x < 2, \text{and}$$
   $$\text{BC: } u(0,t) = u(2,t) = 0, t > 0.$$

   2.4 $u_t = u_{xx},\ 0 \leq x \leq 1, t \geq 0$, subject to

   $$\text{IC: } u(x,0) = 3\sin \pi x, 0 \leq x \leq 1, \text{and}$$
   $$\text{BC: } u(0,t) = u(1,t) = 0, t > 0.$$

3. Determine the temperature distribution in a rod of length l whose ends are kept at zero temperature and the intial temperature is $x(l-x)$.

4. Solve the heat equation
$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 \leq x \leq l, t \geq 0$$
subject to the boundary conditions $u(0,t) = u(l,t) = 0$, $\forall t > 0$ and intial condition $u(x,0) = f(x), 0 < x < l$, where
$$f(x) = \begin{cases} kx & \text{for } 0 < x \leq \frac{l}{2} \\ k(l-x) & \text{for } \frac{l}{2} \leq x \leq l \end{cases}$$

5. Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 \leq x \leq l, t \geq 0$$

subject to the boundary conditions $u(0,t) = u(l,t) = 0$, $\forall t > 0$ and intial conditions $u(x,0) = f(x)$, $\frac{\partial u(x,0)}{\partial t} = g(x)$, $0 < x < l$, where

a. $f(x) = 0$, $g(x) = \sin^2 x$ for $0 < x < 2$.

b. $f(x) = 0$, $g(x) = \begin{cases} x^2 & \text{for } 0 < x \leq 1 \\ kx & \text{for } 1 < x \leq 3 \end{cases}$.

c. $f(x) = x^2$, $g(x) = x$ for $0 < x < 1$.

Assignment-2

Question No. 1(ILO 4)

Use Charpit's method to determine the complete integral of given function:

$$p(p^2 + 1) + (b - z)q = 0$$

(5 marks)

Question No. 2(ILO 4)

Determine the temperature distribution in a bar of length π whose end points are held fixed at zero temperature. The intial temperature distribution is

$$u(x,t) = \begin{cases} x^2 & \text{for } 0 < x \leq \pi \\ \pi - x & \text{for } \frac{\pi}{2} < x \leq \pi \end{cases}$$

(5 marks)

Question No. 3(ILO 5)

For the given intial value problem

$$\frac{dy}{dx} = x^3 + y, y(0) = 1.$$

(5 marks)

a. Write the MATLAB function to solve numerically using Runge Kutta fourth order method.

b. Find the exact solution using MATLABs built-in function '*dsolve*'.

c. Plot the exact and numerical solution in the interval [0,1] choosing step size h=0.1 in the same figure.

Note: Submit assignment to the respective course leader on or before 15^{th} February 2020.