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7/1/2020

* Partial Differential Equation :-
An Equation involves one or more partial derivatives of dependent function of two or more variables is known as Partial Differential Equation.

		order	degree
Ex → ①	$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ ⇒ Linear	1	1
②	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$	2	1
③	$\frac{\partial^2 u}{\partial y^2} + u\frac{\partial u}{\partial x} + x^3 = 0$	2	1
④	$\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial^2 u}{\partial y^2} = 0$	2	1

non Line

* Order of PDE → is the order of highest order partial derivative which involves in a given equation

* Degree → A Degree of PDE is the degree of the highest order derivative in the equation ~~of~~ the after rationalization. (free from of fraction)
[Order & degree is always a positive integers]

* An Equation is said to be linear if degree of dependent variable and its derivatives is one and are not multiplied to each other otherwise A PDE is non linear

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(5) $\frac{\partial u}{\partial t} + k\frac{\partial^2 u}{\partial x^2} = 0$ linear

(6) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ Linear

(7) $u_x + u \cdot u_y = 0$ NON Linear

* ## Classification of first order partial differential Equation

[The first order Equation is said to linear only if it ~~sta~~ satisfies this if power is less th... o...

$P(x,y)p' + Q(x,y)q' = R(x,y)z' + S(x,y)$

(i) $xp + yq = z + x + y$

(ii) $\sin x p + \sin y q = \sin(x+y) z$

$Pp + Qq = R(x,y,z)$]

⇒ A first order PDE is of the form

$f(x, y, z, p, q) = 0$ —— (1)

eq (1) is said to be linear if it is linear in p, q, z order and are ~~not~~ multiplied x, y ⇒

i.e the given equation is of the form

$P(x,y)p + Q(x,y)q = R(x,y)z + S(x,y)$ —— (2)

Example

(1) $xp + yq = z + x + y$

(2) $\sin x p + \sin y q = \sin(x+y)z$

(3) $p + q = z + xy$

(4) $yx^2p + xy^2q = xyz + x^2y^3$

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⇒ A PDE $f(x,y,z,p,q)=0$ is known as Semilinear PDE if it is linear in p and q and coefficients of p and q are only functions of x and y i.e the given equation is of the form

$$P(x,y)p+Q(x,y)q=R(x,y,z) \quad \text{③}$$

Example

① $x^2yp+xy^2q=x^2y^2z^2$
② $xp+yq=xz^2$
③ $(x-y)p+(x+y)q=x^2+y^2+z^2$

than one. ⇒ A PDE $f(x,y,z,p,q)=0$ is known as quasilinear PDE if it is linear in p and q and coeffients of p and q are functions of x, y, and z i.e the given equation of the form

$$P(x,y,z)p+Q(x,y,z)q=R(x,y,z)$$

Example

① $(x-y)p+(y-z)q=z-x$
→ independent ② $x^2p+y^2zq=x^2y^2z$
③ $x^2zp+y^2zq=x^2y^2$
④ $(x^2-yz)p+(y^2-xz)q=(x^2+y^2)z$

⇒ A PDE $f(x,y,z,p,q)=0$ is known as non linear PDE if p, q and z in the given equation is of form.

$\cancel{P(x+y)p^n+Q(x+y)q^n=R(x+y)}$

all pq terms are multipled by each other.

$$P(x,y,z)p^n+Q(x,y,z)q^n=R(x,y,z)$$

if the degree and derivative of the dependent variable is ~~z~~ greater than 1 is called Non linear.

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⇒ A PDE which is not a linear, semi & quasilinear is known as NON-linear
Example
(1) $pq = 0$
(2) $xyp^2 + yq = 0$
(3) $\sin x p^2 + \cos y q^2 = z^2$

10/1/2020

Formation of PDE by eliminating arbitrary constants

Consider the function

$$f(x, y, z, a, b) = 0 \quad \text{--- (1)}$$

where a & b are arbitary constants
Diff (1) wrt x, we get

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial x} = 0 \quad \text{--- (2)}$$

Diff (1) wrt y we get,

$$\frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} = 0 \quad \text{--- (3)}$$

Eliminating arbitrary constants in (1) (2) & (3), we get the required partial differential equation.

Q1) $z = (x-a)^2 + (y-b)^2$
Soln → Given

$$z = (x-a)^2 + (y-b)^2 \quad \text{--- (1)}$$

diff given eq (1) wrt x. we get

$$\frac{\partial z}{\partial x} = 2(x-a).1$$

$$(x-a) = \frac{1}{2}\frac{\partial z}{\partial x} \quad \text{--- (2)}$$

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diff eq ① wrt y we get

$$\frac{\partial z}{\partial y} = 2(y-b)\cdot 1$$

$$(y-b) = \frac{1}{2}\frac{\partial z}{\partial y} \quad \text{—③}$$

Using ② & ③ in ① we get

$$z = \left(\frac{1}{2}\left(\frac{\partial z}{\partial x}\right)\right)^2 + \left(\frac{1}{2}\frac{\partial z}{\partial y}\right)^2$$

$$\boxed{4z = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$$ this is the required PDE

it is non linear.

Q2) $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Sol Given

$$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2} \quad \text{—①}$$

diff eq ① wrt x we get

$$2\frac{\partial z}{\partial x} = \frac{\cancel{2x}}{a^2}\;\frac{2x}{a^2} \quad \text{—②} \quad \frac{1}{a^2} = \frac{1}{x}\frac{\partial z}{\partial x}$$

diff eq ② wrt y we get

$$2\frac{\partial z}{\partial x} = \frac{2y}{b^2} \quad \text{—③} \quad \frac{1}{b^2} = \frac{1}{y}\frac{\partial z}{\partial y}$$

$$\boxed{2z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}}$$ required PDE linear

(3) Sol

$\log(az-1) = x + ay + b$

Given

$$\log(az-1) = x + ay + b \quad \text{—(1)}$$

diff eq (1) wrt x

$$\frac{1}{az-1}\, a\frac{\partial z}{\partial x} = 1 \quad \text{—(2)}$$

diff eq (1) wrt y

$$\frac{1}{az-1}\, \cancel{a}\frac{\partial z}{\partial y} = \cancel{a} \quad \text{—}\cancel{(3)}$$

$$\frac{1}{az-1}\, \frac{\partial z}{\partial y} = 1$$

$$\frac{\partial z}{\partial y} = (az-1)$$

$$az = \frac{\partial z}{\partial y} + 1$$

$$\boxed{a = \frac{1}{z}\left(\frac{\partial z}{\partial y} + 1\right)} \quad \text{—(3)}$$

Using (3) in (2) we get

$$\frac{1}{\frac{1}{z}\left(1+\frac{\partial z}{\partial y}\right)z - 1} \cdot \frac{\left(1+\frac{\partial z}{\partial y}\right)}{z}\frac{\partial z}{\partial x} = 1$$

$$\frac{\left(1+\frac{\partial z}{\partial y}\right)}{z\frac{\partial z}{\partial y}}\frac{\partial z}{\partial x} = 1$$

$$\boxed{\left(1+\frac{\partial z}{\partial y}\right)\frac{\partial z}{\partial x} = z\frac{\partial z}{\partial y}}$$

non linear PDE

Qu $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ — (1)

Sol diff eq (1) w.r.t x

$$\frac{2x}{a^2} + \frac{2z}{c^2}\frac{\partial z}{\partial x} = 0$$

$\div 2$ we get $\rightarrow$ $\frac{x}{a^2} = -\frac{z}{c^2}\frac{\partial z}{\partial y}$

$$\frac{x}{a^2} + \frac{z}{c^2}\frac{\partial z}{\partial x} = 0 \quad — (2)$$

diff eq (2) w.r.t x

$$\frac{1}{a^2} + \frac{1}{c^2}\left(\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial x} + z\frac{\partial^2 z}{\partial x^2}\right) = 0$$

$$\frac{1}{a^2} = -\frac{1}{c^2}\left(\left(\frac{\partial z}{\partial x}\right)^2 + z\frac{\partial^2 z}{\partial x^2}\right) \quad — (3)$$

diff eq (1) w.r.t y

$$\frac{2y}{b^2} + \frac{2z}{c^2}\frac{\partial z}{\partial y} = 0$$

$\div 2$ we get

$$\frac{y}{b^2} + \frac{1}{c^2} z\frac{\partial z}{\partial y} = 0 \qquad \frac{y}{b^2} = -\frac{1}{c^2} z\frac{\partial z}{\partial y} \quad — (4)$$

diff it again by y

$$\frac{1}{b^2} + \frac{1}{c^2}\left(\frac{\partial z}{\partial y}\frac{\partial z}{\partial y} + z\frac{\partial^2 z}{\partial y^2}\right) = 0$$

$$\frac{1}{b^2} = -\frac{1}{c^2}\left(\left(\frac{\partial z}{\partial y}\right)^2 + z\frac{\partial^2 z}{\partial y^2}\right) = 0 \quad — (5)$$

~~using (3) & (4) in (1)~~ we get

$$\cancel{-\frac{x^2}{c^2}\left(\left(\frac{\partial z}{\partial x}\right)^2 + z\frac{\partial^2 z}{\partial x^2}\right) - \frac{y^2}{c^2}\left(\left(\frac{\partial z}{\partial y}\right)^2 + z\frac{\partial^2 z}{\partial y^2}\right) + \frac{z^2}{c^2}} = 1$$

bine (2) & (4)

$$\frac{x}{a^2} = -\cancel{\frac{z}{c^2}}\frac{\partial z}{\partial x} \qquad \frac{b^2}{a^2} = \frac{y}{x}\frac{\partial z/\partial x}{\partial z/\partial y}$$

$$\frac{y}{b^2} = -\cancel{\frac{z}{c^2}}\frac{\partial z}{\partial y}$$

Divide ③ & ⑤

$$\frac{\cancel{1}/a^2}{1/b^2} = \frac{-\cancel{1/c^2}\left(\left(\frac{\partial z}{\partial x}\right)^2 + z\frac{\partial^2 z}{\partial x^2}\right)}{-\cancel{1/c^2}\left(\left(\frac{\partial z}{\partial y}\right)^2 + z\frac{\partial^2 z}{\partial y^2}\right)}$$

$$\frac{b^2}{a^2} = \frac{\left(\frac{\partial z}{\partial x}\right)^2 + z\frac{\partial^2 z}{\partial x^2}}{\left(\frac{\partial z}{\partial y}\right)^2 + z\frac{\partial^2 z}{\partial y^2}}$$

$$\boxed{\frac{y\frac{\partial z}{\partial x}}{x\frac{\partial z}{\partial y}} = \frac{\left(\frac{\partial z}{\partial x}\right)^2 + z\frac{\partial^2 z}{\partial x^2}}{\left(\frac{\partial z}{\partial y}\right)^2 + z\frac{\partial^2 z}{\partial y^2}}}$$

13/10/2020

* Formation of PDE by eliminating arbitrary function

(1) $z = f(x^2 - y^2)$ 1st order

$f = f(u)$
$u = u(x, y)$
$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$
f'

Given

$z = f(x^2 - y^2)$ — ①

diff ① wrt x we get

$\frac{\partial z}{\partial x} = f'(x^2-y^2)\,2x$

$f'(x^2-y^2) = \frac{1}{2x}\frac{\partial z}{\partial x}$ — ②

Diff ① wrt y we get

$\frac{\partial z}{\partial y} = f'(x^2-y^2)(-2y)$

$f'(x^2-y^2) = -\frac{1}{2y}\frac{\partial z}{\partial y}$ — ③

from (2) & (3)

$$\frac{1}{\cancel{2}x}\frac{\partial z}{\partial x} = -\frac{1}{\cancel{2}y}\frac{\partial z}{\partial y}$$

$$\boxed{y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0}$$ the required pde is linear

2) $z = \phi(x+ay) + \psi(x-ay)$ — 2nd order

Given

$z = \phi(x+ay) + \psi(x-ay)$ — (1)

diff (1) wrt x we get

$\frac{\partial z}{\partial x} = \phi'(x+ay)1 + \psi'(x-ay)1$ — (2)

$\cancel{\phi'(x+ay) = \frac{\partial z}{}}$

diff (1) wrt y we get

$\frac{\partial z}{\partial y} = \phi'(x+ay)a + \psi'(x-ay)(-a)$

$\frac{\partial z}{\partial y} = a(\phi'(x+ay) - \psi'(x-ay))$ — (3)

diff (2) with x we get

$\frac{\partial^2 z}{\partial x^2} = \phi''(x+ay) + \psi''(x-ay)$

diff (3) with y we get

$\frac{\partial^2 z}{\partial y^2} = a(\phi''(x+ay)a - \psi''(x-ay)(-a))$

$= a^2(\phi''(x+ay) + \psi''(x-ay))$

$\frac{\partial^2 z}{\partial y^2} = a^2\frac{\partial^2 z}{\partial x^2}$ it is the required PDE & is linear. and it is wave equation.

3) $lx + my + nz = \phi(x^2 + y^2 + z^2)$

Given

$$lx + my + nz = \phi(x^2 + y^2 + z^2) \quad \text{— 1st order PDE}$$

diff ① wrt x. we get

$$l + n\frac{\partial z}{\partial x} = \phi'(x^2 + y^2 + z^2)\left(2x + 2z\frac{\partial z}{\partial x}\right)$$

$$\phi'(x^2+y^2+z^2) = \frac{1}{2x}\left(l + n\frac{\partial z}{\partial x}\right) \quad ②$$

diff ① wrt y we get

$$m + n\frac{\partial z}{\partial y} = \phi'(x^2+y^2+z^2)\left(2y + 2z\frac{\partial z}{\partial y}\right)$$

$$\phi'(x^2+y^2+z^2) = \frac{1}{2y}\left(m + n\frac{\partial z}{\partial y}\right) \quad ③$$

from ② & ③

$$\frac{1}{2x}\left(l + n\frac{\partial z}{\partial x}\right) = \frac{1}{2y}\left(m + n\frac{\partial z}{\partial y}\right)$$

$$ly + ny\frac{\partial z}{\partial x} = mx + nx\frac{\partial z}{\partial y}$$

÷ ② & ③ we get

$$\frac{l + n\frac{\partial z}{\partial x}}{m + n\frac{\partial z}{\partial y}} = \frac{\cancel{\phi(x^2+y^2+z^2)}\,\cancel{2}\left(x + z\frac{\partial z}{\partial x}\right)}{\cancel{\phi(x^2+y^2+z^2)}\,\cancel{2}\left(y + z\frac{\partial z}{\partial y}\right)}$$

$$\left(1+x\frac{\partial z}{\partial x}\right)\left(y+z\frac{\partial z}{\partial y}\right)=\left(m+n\frac{\partial z}{\partial y}\right)\left(x+z\frac{\partial z}{\partial x}\right)$$

is required pde and is nonlinear

(4) $z = x\phi(y) + y\cdot\psi(x)$.

Given

$z = x\phi(y) + y\cdot\psi(x)$

diff (1) wrt x we get

$$\frac{\partial z}{\partial x} = \phi(y) + y\,\psi'(x)$$

~~again diff it with x we get~~

$$\cancel{\frac{\partial^2 z}{\partial x^2} = y\psi''(x)} \quad \text{— (2)}$$

~~diff (1) wrt y we get~~

$$\cancel{\frac{\partial z}{\partial y} = x\phi'(y) + \psi(x)}$$

~~diff again.~~

$$\cancel{\frac{\partial^2 z}{\partial y^2} = x\phi''(y) + 0} \quad \text{— (3)}$$

$$\psi'(x) = \frac{1}{y}\left[\frac{\partial z}{\partial x} - \phi(y)\right] \quad \text{— (2)}$$

diff (1) y we get

$$\frac{\partial z}{\partial y} = x\phi'(y) + \psi(x) \quad \text{— (3)}$$

$$\phi'(y) = \frac{1}{x}\left[\frac{\partial z}{\partial y} - \psi(x)\right] \quad \text{— (4)}$$

Diff (3) wrt x we get

$$\frac{\partial^2 z}{\partial x\partial y} = \phi'(y) + \psi'(x)$$

it is linear.

using (2) & (4) we get

$$\frac{\partial^2 z}{\partial x\partial y} = \frac{1}{x}\left(\frac{\partial z}{\partial y} - \psi(x)\right) + \frac{1}{y}\left(\frac{\partial z}{\partial x} - \phi(y)\right)$$

$$\frac{\partial^2 z}{\partial x\partial y} = \frac{1}{xy}\left[y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x} - \left(y\psi(x) + x\phi(y)\right)\right]$$

$$\therefore \boxed{xy\frac{\partial^2 z}{\partial x\partial y} = y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x}}$$

14/1/2020

5) $\phi(x+y+z,\ x^2+y^2-z^2)=0$

$u = x+y+z$ $\qquad\qquad \phi(u,v)$

$v = x^2+y^2-z^2$ $\qquad\qquad u = u(x,y,z)$

$\phi(u,v)=0$ —— (2)

Diff equation (2) wrt x we get

$$\frac{\partial\phi}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial\phi}{\partial v}\frac{\partial v}{\partial x}=0$$

$$\frac{\partial\phi}{\partial u}\left(1+\frac{\partial z}{\partial x}\right)+\frac{\partial\phi}{\partial v}\left(2x-2z\frac{\partial z}{\partial x}\right)=0$$

$$\frac{\frac{\partial\phi}{\partial u}}{\frac{\partial\phi}{\partial v}}=-\frac{\left(2x-2z\frac{\partial z}{\partial x}\right)}{\left(1+\frac{\partial z}{\partial x}\right)} \quad \text{—— (3)}$$

Diff equation (2) wrt y we get

$$\frac{\partial\phi}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial\phi}{\partial v}\frac{\partial v}{\partial y}=0$$

$$\frac{\partial\phi}{\partial u}\left(1+\frac{\partial z}{\partial y}\right)+\frac{\partial\phi}{\partial v}\left(2y-2z\frac{\partial z}{\partial y}\right)=0$$

$$\frac{\frac{\partial\phi}{\partial u}}{\frac{\partial\phi}{\partial v}}=-\frac{\left(2y-2z\frac{\partial z}{\partial y}\right)}{\left(1+\frac{\partial z}{\partial y}\right)} \quad \text{—— (4)}$$

From (3) & (4) we get

$$\frac{\left(2x-2z\frac{\partial z}{\partial x}\right)}{\left(1+\frac{\partial z}{\partial x}\right)}=\frac{\left(2y-2z\frac{\partial z}{\partial y}\right)}{\left(1+\frac{\partial z}{\partial y}\right)}$$

$$=\cancel{2}\left(2x-2z\frac{\partial z}{\partial x}\right)\left(1+\frac{\partial z}{\partial y}\right)=\cancel{2}\left(2y-2z\frac{\partial z}{\partial y}\right)\left(1+\frac{\partial z}{\partial x}\right)$$

Ans: $\left(x-z\frac{\partial z}{\partial x}\right)\left(1+\frac{\partial z}{\partial y}\right)=\left(y-z\frac{\partial z}{\partial y}\right)\left(1+\frac{\partial z}{\partial x}\right)$

$\Rightarrow x + x\frac{\partial z}{\partial y} - z\frac{\partial z}{\partial x} - \cancel{z\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}} =$

$y - z\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} - \cancel{z\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}}$

$\Rightarrow y\frac{\partial z}{\partial x} + z\frac{\partial z}{\partial x} - z\frac{\partial z}{\partial y} - x\frac{\partial z}{\partial y} = x - y$

$$\boxed{(y+z)\frac{\partial z}{\partial x} - (x+z)\frac{\partial z}{\partial y} = (x-y)}$$

is requied PDE and it is quasilinear.

* <u>Lagrange</u> <u>Equation</u> :- A quasilinear PDE of the form

$Pp + Qq = R$ is known as L.E

<u>The</u> <u>General</u> <u>theorm</u> :- the equation $Pp + Qq = R$ is

$\phi(u, v) = 0$ where u and v are arbitary functions and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ are independent solutions of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ where c_1 and c_2 are arbitary constants.

$u(x, y, z)$ and $v(x, y, z)$ are said to be independent if $\frac{u}{v}$ is not merely a constant.

* where C_1 and C_2 are arbitary constants and atleast one of u and v involves z.

* Lagrange Equation of type 1

It is of the form $Pp + Qq = R$ — (1)

Auxillary equation of eq (1) is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{dR} \quad \text{— (2)}$$

Eq (1) is said type 1, if any two fractions in eqn (2) is independent from the other variable [i.e, if we consider 1 & 2 fraction and these two are independent from z then (1) is of type 1]

Q1) $xp + yq = z$

Given

Sol

$$xp + yq = z \quad \text{— (1)}$$

There

$$P = x, \quad Q = y, \quad R = z$$

Auxillary equation is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} \quad \text{— (2)}$$

consider (1) & (2) fraction of eq (2)

$$\frac{dx}{x} = \frac{dy}{y}$$

$$\int \frac{dx}{x} = \int \frac{dy}{y} + c_1$$

$$\log x = \log y + c_1$$

$$\log x - \log y = c_1$$

$$\log \frac{x}{y} = c_1$$

$$\frac{x}{y} = e^{c_1} = k_1$$

$$\boxed{\frac{x}{y} = k_1} \quad \text{— (3)}$$

$$\boxed{u(x,y,z) = \frac{x}{y}}$$

Consider ① & ③ we get

$$\frac{dx}{x} = \frac{dz}{z}$$

$$\int \frac{dx}{x} = \int \frac{dz}{z} + c_2$$

$$\log x = \log z + c_2$$

$$\log x - \log z = c_2$$

$$\log \frac{x}{z} = c_2 = k_2$$

$$\frac{x}{z} = e^{c_2} = k_2$$

$$\boxed{\frac{x}{z} = k_2}$$

$$\boxed{u(x, y, z) = \frac{x}{z}}$$

The general solution of given eq is

$$\phi(u, v) = 0$$

$$\boxed{\phi\left(\frac{x}{y}, \frac{x}{z}\right) = 0}$$

independent

Q2) $\tan x\, p + \tan y\, q = \tan z$

Given

Sol $\tan x\, p + \tan y\, q = \tan z$ —— ①

Here

$P = \tan x$, $Q = \tan y$, $R = \tan z$

Auxillary equation is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z} \quad \text{—— ②}$$

consider ① & ② fraction & eq ②

$$\frac{dx}{\tan x} = \frac{dy}{\tan y}$$

$$\int \frac{dx}{\tan x} = \int \frac{dy}{\tan y} + \log c_1$$

$$\ln|\sin x| = \ln|\sin y| + \log c_1$$

$$\ln|\sin x| - \ln(\sin y) = \log c_1$$

$$\ln\left|\frac{\sin x}{\sin y}\right| = \log c_1$$

$$\frac{\sin x}{\sin y} = e^{c_1} = k_1$$

$$\boxed{\frac{\sin x}{\sin y} = k_1} = u(x,y,z)$$

Conside ② & ③

$$\boxed{\frac{\sin y}{\sin z} = k_2}$$

General sol $\phi(u,v) = 0$.

$$\phi\left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$$ //

17/1/2020

③ $y^2p - xyq = x(z-2y)$

sol

Given :-

$$y^2p - xyq = x(z-2y) \quad \text{—— ①}$$

$P = y^2$, $Q = -xy$, $R = x(z-2y)$

Auxillary equation is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)} \quad \text{—— ②}$$

Considering ① ② fractions of eq ②

$$\frac{dx}{y^2} = \frac{dy}{-xy}$$

taking ln b.s

$$\int \frac{dx}{y^2} = -\int \frac{dy}{xy}$$

$$\cancel{\frac{x}{y^2}} = \cancel{-\frac{\log y}{x}} + c_1$$

$\cancel{x^2 + y^2 \log y = c_1}$

$$-x\,dx = \frac{y^2\,dy}{y}$$

$$-x\,dx = y\,dy$$

taking integration b.s

$$\int -x\,dx = \int y\,dy$$

$$\frac{x^2}{2} = -\frac{y^2}{2} + C$$

$$\boxed{\frac{x^2}{2} + \frac{y^2}{2} = C_1} \quad \therefore \boxed{x^2 + y^2 = C_1}$$

$$\frac{dy}{-\cancel{x}y} = \frac{dz}{\cancel{x}(z-2y)}$$

$$-\frac{dy}{y} = \frac{dz}{(z-2y)}$$

~~taking integration b.s we get~~

$$\frac{dz}{dy} = -\frac{(z-2y)}{y}$$

$$\frac{dz}{dy} + \frac{1}{y} \cdot z = 2$$

$$\frac{dz}{dy} + \frac{z}{y} = 2$$

$$\boxed{\frac{dy}{dx} + Py = Q \quad I.F\, y = \int I.F\, Q\,dx + c}$$

$$I.F = e^{\int P\,dx}$$

$$I.F = e^{\int \frac{1}{y} dy} = e^{\log_e y} = y$$

$$y \cdot z = \int 2y\,dy + C_2$$

$$\boxed{u = \frac{x^2}{2} + \frac{y^2}{2}}$$

$$yz = y^2 + C_2$$

$$\boxed{v = yz - y^2}$$

General sol

$$\phi(x^2 + y^2,\; yz - y^2) = 0$$

(ii) $(x^2+2y^2)p - xyq = xz$

Sol Given

$(x^2+2y^2)p - xyq = xz$

Auxillary equation

$P = (x^2+2y^2)$, $Q = -xy$, $R = xz$

So

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{x^2+2y^2} = \frac{dy}{-xy} = \frac{dz}{xz}$$

Now

$$\frac{dx}{x^2+2y^2} = -\frac{dy}{\cancel{xy}\,xz}$$

$$\frac{dx}{dy} = -\frac{(x^2+2y^2)}{x\cancel{z}y}$$

$$\frac{dx}{dy} = -\frac{x^2}{xy} - \frac{2y^2}{xy}$$

$$\frac{dx}{dy} = -\frac{x}{y} - \frac{2y}{x}$$

$$\frac{dx}{dy} + \frac{x}{y} = -\frac{2y}{x}$$ ∴ it is in the form $\frac{dx}{dy} + Py = Q$. So

$IF = e^{\int P\,dy}$

$= e^{\int \frac{1}{y}dy} = e^{\log y} = y$

$$y \cdot x = \int y \cdot \left(\frac{-2y}{x}\right) dy + C$$

$$yx = \int -\frac{2y^2}{x}\,dy + C$$

$$yx = -\frac{2}{x}\int y^2\,dy + C$$

$$yx = -\frac{2}{x}\,\frac{y^3}{3} + C$$

$$\frac{dx}{dy} + \frac{x}{y} = \frac{-2y}{x}$$

$x^2 = t$

$2x\,dx = dt$

$x\,dx = \frac{1}{2} dt$

multiply x b.s

$$x\frac{dx}{dy} + \frac{x^2}{y} = -2y$$

$$\frac{1}{2}\frac{dt}{dy} + \frac{t}{y} = -2y \qquad \times^{m} \text{ by } 2,$$

$$\frac{dt}{dy} + \frac{2t}{y} = -4y$$

$$IF = e^{\int P\,dy} = e^{\int 2/y\,dy} = e^{2\log y} = y^2$$

$$y^2 \cdot t = -\int 4y^3\,dy + C_1$$

$$y^2 t = -\cancel{4}\cdot\frac{y^4}{\cancel{4}} + C_1$$

$$y^2 x^2 = -y^4 + C_1$$

$$\boxed{x^2y^2 + y^4 = C_1}$$

$$\frac{dy}{-\cancel{x}y} = \frac{dz}{\cancel{x}z}$$

$$+\int\frac{dy}{y} = -\int\frac{dz}{z}$$

$$+\log y = -\log z + \log c_2$$

$$\log y + \log z = \log c_2$$

$$\cancel{\log}\left[\frac{y}{z}\right] = \log c_2$$

$$\cancel{\log}\ \boxed{yz = c_2}$$

$$\phi(u, v) = 0$$

$$\phi(x^2y^2 + y^4, yz) = 0$$

Required soulution.

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20/11/2020

* Lagrange equation Type - 2

Lagrange equation is of the form
$$Pp + Qq = R \quad \text{—} (1)$$
A Es of (1) are given by
$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \quad \text{—} (2)$$
Suppose that one of the integral of equation two is known by using type 1 method & the other integral cannot be obtained by the type 1 method then one of the known integral is used to find another integral.

Q1) $p + 3q = 5z + \tan(y - 3x)$

Sol Given

$$p + 3q = 5z + \tan(y - 3x) \quad \text{—} (1)$$
$P = 1 \quad Q = 3 \quad R = 5z + \tan(y - 3x)$
$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$
$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y - 3x)} \quad \text{—} (2)$$
from (1) & (2) we get
$$\frac{dx}{1} = \frac{dy}{3}$$
integrating b.s
$$\int \frac{dx}{1} = \int \frac{dy}{3}$$
$$x = \frac{y}{3} + C$$
$$\boxed{c_1 = -3C}$$
$$3x - y = 3C$$
$$\boxed{y - 3x = c_1} \quad \text{—} (3)$$

Consider ① & ③ in eq ② we get

$$\frac{dx}{1} = \frac{dz}{5z + \tan(y-3x)}$$

using c_1 in this we get

$$\frac{dx}{1} = \frac{dz}{5z + \tan c_1}$$

On integrating we get

$$\int dx = \int \frac{dz}{5z + \tan c_1}$$

$$x = \frac{1}{5} \log|5z + \tan c_1| + c_2$$

$$5x - \log(5z + \tan(y-3x)) = c_2 5$$

$$5x - \log(5z + \tan(y-3x)) = c_2 \quad \text{--- ④}$$

from ③ & ④ we get

$$\phi(u, v) = 0$$

$$\phi(y-3x,\ 5x - \log(5z + \tan(y-3x))) = 0$$

(Q2) $z(z^2+xy)(px - qy) = x^4$

Sol Given

$$z(z^2+xy)(px - qy) = x^4 \quad \text{--- ①}$$

$$px - qy = \frac{x^4}{z(z^2+xy)}$$

$$P = x, \quad Q = -y, \quad R = \frac{x^4}{z(z^2+xy)}$$

$$\frac{dP}{x} = \frac{dQ}{-y} = \frac{dR}{\frac{x^4}{z^3+xyz}} \quad \text{--- ②}$$

Consdn ① & ② & ②

$$\frac{dx}{x} = -\frac{dy}{+y}$$

integrating

$$\int \frac{dx}{x} = -\int \frac{dy}{y}$$

$$\log x + \log y = \log c_1$$

$$\boxed{\log xy = \log c_1}$$

$$\boxed{xy = c_1}$$

① & ③ we get

$$\frac{dy}{y} = \frac{dx}{x^4}\left(z(x^2+xy)\right)$$

$$\frac{dx}{x} = + \frac{z(z^2+c_1)\,dz}{x^4}$$

$$\frac{dx}{x}x^4 = +z(z^2+c_1)\,dz$$

$$x^3\,dx = +z(z^2+c_1)\,dz$$

$$\frac{x^4}{4} = +(z^3+zc_1)\,dz$$

$$\frac{x^4}{4} = +\frac{z^4}{4} + \frac{z^2}{2}$$

$$\boxed{\frac{x^4}{4} - \frac{z^4}{4} - \frac{z^2 c}{2} = c_2}$$

$$\boxed{\frac{x^4}{4} - \frac{z^4}{4} - \frac{z^2 c_1}{2} = c_2}$$

$$\phi\left(\cancel{\log}\, xy,\ x^4 - z^4 - 2z^2c\right) = 0$$

$$\boxed{\phi\left(xy,\ x^4 - z^4 - 2z^2xy\right) = 0}$$

Q3 $xzp + yzq = xy$

$P = xz \quad q = yz \quad R = xy$

$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy}$$

① & ② we get

$$\frac{dx}{xz} = \frac{dy}{yz}$$

$$\int\frac{dx}{x} = \int\frac{dy}{y}$$

$$\log x = \log y + \log c_1$$

$$\log x - \log y = \log c_1$$

$$\log x/y = \log c_1$$

$$\boxed{c_1 = \frac{x}{y}}$$

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Consider (1) & (3) fraction

$$\frac{dx}{\cancel{xz}} = \frac{dz}{yx}$$

$$y\,dx = z\,dz$$

$$\frac{x}{c_1}\,dx = z\,dz$$

$$\frac{x^2}{2c_1} = \frac{z^2}{2} + c_2$$

$$\frac{x^2 y}{2x} - \frac{z^2}{2} = c_2 \qquad \frac{xy}{2} - \frac{z^2}{2} = c_2$$

$$\boxed{\frac{xy}{2} - \frac{z^2}{2} = c_2} \qquad xy - z^2 = 2c_2$$

$$\phi\left(x/y,\ \frac{xy}{2} - \frac{z^2}{2}\right) = 0$$

$$\boxed{\phi\left(\frac{x}{y},\ xy - z^2\right) = 0}$$ General Sol

Q4 $xyp + y^2 q = xyz + 2x^2$

$P = xy,\ Q = y^2,\ R = xyz \mp 2x^2$

$$\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{xyz \mp 2x^2} \quad \text{—— } (\Sigma)$$

(1) & (2) of (Σ) we get

$$\frac{dx}{xy} = \frac{dy}{y^2}$$

$$\frac{dx}{x} = \frac{dy}{y}$$

$$\int \frac{dx}{x} = \int \frac{dy}{y}$$

$$\log x = \log y + \log c_1$$

$$\log x - \log y = \log c_1$$

$$\log \frac{x}{y} = \log c_1$$

$$\boxed{\frac{x}{y} = c_1}$$

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1 & 3

$$\frac{dx}{xy} = \frac{dz}{xyz - 2x^2}$$

$$\frac{dx}{xy} = \frac{dz}{x(yz-2x)}$$

$$\frac{dx}{y} = \frac{dz}{yz-2x}$$

$$\frac{dx}{x/c_1} = \frac{dz}{xc_1 z - 2x}$$

$$\frac{dx}{x/c_1} = \frac{dz}{x(c_1 z - 2)}$$

$$\frac{dx}{c_1} = \frac{dz}{zc_1 - 2}$$

$$\Rightarrow \frac{x}{c_1} = \frac{1}{c_1}\log\left(\frac{z}{c_1} - 2\right)$$

$$\Rightarrow x = \log(z - 2c_1) - \log c_1$$

$$\Rightarrow x - \log(z - 2c_1) = c_2$$

$$\frac{dx}{x/c_1} = \frac{dz}{\frac{x}{c_1}z - 2x}$$

$$\frac{dx}{\frac{x}{c_1}} = \frac{dz}{\frac{x(z-2c_1)}{c_1}}$$

$$dx = \frac{dz}{z - 2c_1}$$

$$x = \log(z - 2c_1) + c_2$$

$$\phi\left(x/y,\ x - \log(z - 2c_1)\right) = 0 \quad //$$

required

21/1/2020

* Lagrange's equation :- Type-3

Lagrange equation is of the form

$$Pp + Qq = R \longrightarrow (1)$$

AE of eqn (1) are given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Let P_1, Q_1, R_1 be the fun of x, y, z then by a principle of algebra each fraction in equation 2 is equal to

$$\frac{P_1 dx + Q_1 dy + R_1 dz}{P_1 P + Q_1 Q + R_1 R} \quad \text{if } P_1 P + Q_1 Q + R_1 R = 0$$

then Numerator is also zero i.e
$P_1 dx + Q_1 dy + R_1 dz = 0$ which is integrated
to give $u(x,y,z) = C_1$.
To find another integral this procedure may be repeated with another set of P_1, Q_1, R_1 where P_1, Q_1, R_1 are multipliers

Q1) $x(y-z)p + y(z-x)q = z(x-y)$

Sol Given

$$x(y-z)p + y(z-x)q = z(x-y) \quad \text{--- (1)}$$

Here

~~Pdx~~ $P = x(y-z)$ $\quad Q = y(z-x)$

$R = z(x-y)$

AE's are given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)} \quad \text{--- (2)}$$

$x(y-z) \quad y(z-x) \quad z(x-y)$

$1(\cancel{xy} - \cancel{xz}) + 1(yz - \cancel{xy}) + 1(\cancel{zx} - \cancel{yz})$

Let choose 1,1,1 as multipliers Thus,

$$P_1P + Q_1Q + R_1R = x(y-z) + y(z-x) + z(x-y) = 0$$

$P_1 dx + Q_1 dy + R_1 dz = 0$

$dx + dy + dz = 0$ & integrate,

$$\boxed{x + y + z = C_1}$$

$yx + xy - z^2$

$2(xy)$

PTO

Let choose $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as multipliers

$$P_1P + Q_1Q + R_1R = \frac{x}{x}(y-z) + \frac{y}{y}(z-x) + \frac{z}{z}(x-$$

$$= 0$$

$$Pdx + Qdy + Rdz = 0$$

$$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

$$\log x + \log y + \log z = \log c_2$$

$$\log(xyz) = \log c_2$$

$$\boxed{xyz = c_2}$$

$$U(x,y,z) = x+y+z$$

$$V(x,y,z) = xyz$$

Thus, The requ. sol is

$$\phi(U,V) = 0$$

$$\phi(x+y+z, xyz) = 0$$

Q 2) $z(x+y)p + z(x-y) = x^2+y^2$

$P = z(x+y)$ $Q = z(x-y)$ $R = x^2+y^2$

AE's is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$$

Let us choose $-x, y, \& z$ as multipliers

$$P_1P + Q_1Q + R_1R = 0$$

$$-x(z(x+y)) + y(z(x-y)) + z(x^2+y^2) = 0$$

$$\cancel{-x^2z} - \cancel{xyz} + \cancel{xyz} - \cancel{zy^2} + \cancel{zx^2} + \cancel{zy^2}$$

$$P_1dx + Q_1dy + R_1dz = 0$$

$$-xdx + ydy + zdz = 0$$

$$-\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{c_1}{2}$$

$$\boxed{-x^2 + y^2 + z^2 = c_1}$$

Let choose $y, x, -z$ as

$P_1 dx + Q_1 dy + R_1 dz = 0$

$y\,dx + x\,dy - z\,dz = 0$

$d(xy) - z\,dz = 0$

on integration $[du - z\,du = 0]$

$xy - \frac{z^2}{2} = C_2$

$$\boxed{2xy - z^2 = 2C_2}$$

Thus, the general solⁿ of given eqn is

$\phi(-x^2 + y^2 + z^2,\ 2xy - z^2) = 0$

Q3) $(mz - ny)p + (nx - lz)q = ly - mx$

Sol= $P = (mz - ny)$ $\quad Q = (nx - lz)$

$R = ly - mx$

AE's is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{(mz-ny)} = \frac{dy}{(nx-lz)} = \frac{dz}{(ly-mx)}$$

Let us choose x, y, z as multipliers

$P_1 P + Q_1 Q + R_1 R = 0$

$x(mz - ny) + y(nx - lz) + z(ly - mx)$

$xmz - xny + nxy - lyz + zly - mxz = 0$

$\therefore$

$P_1 dx + Q_1 dy + R_1 dz = 0$

$x\,dx + y\,dy + z\,dz = 0$ on integrating

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{C_1}{2}$$

$\therefore$ $\boxed{x^2 + y^2 + z^2 = 2C_1}$

$v(x, y, z) = u(x^2 + y^2 + z^2)$

Let us choose $l\ m\ n$ are multiple

$P_1P + Q_1Q + R_1R = 0$

$l(mz - ny) + m(nx - lz) + n(ly - mx)$

$lmz - lny + mnx - mlz + nly - mxn = 0$

$\therefore$

$P_1 dx + Q_1 dy + R_1 dz = 0$

$l\,dx + m\,dy + n\,dz = 0$

on integration

$lx + my + nz = C_2$

$\therefore$

$v(x, y, z) = lx + my + nz$

So

Thus the required general sol is

$\phi(u, v) = 0$

$\phi(x^2 + y^2 + z^2,\ lx + my + nz) = 0$

Q4) $x(x^2 + 3y^2)p - y(3x^2 + y^2)q = 2z(y^2 - x^2)$

$P = x(x^2 + 3y^2) \quad Q = -y(3x^2 + y^2) \quad R = 2z(y^2 - x^2)$

AE's is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{x(x^2+3y^2)} = \frac{dy}{-y(3x^2+y^2)} = \frac{dz}{2z(y^2-x^2)}$$

Let us chose $\frac{1}{x}\ \frac{1}{y}\ \frac{1}{z}$ - multiple

$P_1P + Q_1Q + R_1R = 0$

$\frac{1}{x}x(x^2+3y^2) + \frac{1}{y}\,-y(3x^2+y^2) + \frac{1}{z}2z(y^2-x^2)$

$\frac{1}{x}(x^3 + 3xy^2) + \frac{1}{y}\,-(3x^2y + y^3) + \frac{1}{z}\,2zy^2 - 2zx^2$

$x^2 + 3y^2 - 3x^2 - y^2 + 2y^2 - 2x^2 = 0$

$-2x^2 + 2y^2 - 2y^2 + 2x^2 = 0$

$P_1 dx + Q_1 dy + R_1 dz = 0$

$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$

on integration we get

$$\log|x| - \log|y| - \log|z| = \log|c_1|$$

$$\log\left|\frac{xy}{z}\right| = \log|c_1|$$

$$\boxed{\frac{xy}{z} = c_1}$$

Consider ① & ② fraction

$$\frac{dx}{x(x^2+3y^2)} = \frac{dy}{-y(3x^2+y^2)}$$

$$(3x^2y + y^3)dx + (x^3+3xy^2)dy = 0$$

it is in the terms of $Mdx + Ndy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

$$\frac{\partial M}{\partial y} = 3x^2+3y^2 \qquad \frac{\partial N}{\partial x} = 3x^2+3y^2$$

Solution on is given by

$$\int M\,dx + \int [\text{terms of } N \text{ indepenent form } x]\,dy = C_1$$

$$\int(3x^2y+y^3)dx = C_2$$

$$\boxed{x^3y + xy^3 = C_2}$$

∴ The required Solution is

$$\phi(u, v) = 0$$

$$\phi\left(\frac{xy}{z}, x^3y+xy^3\right) = 0$$ //

27/1/2020

⇒ An expression of the type $f(x,y,z,a,b)$ is a Solution of PDE $\phi(x,y,z,p,q)=0$ where a & b are arbitary constant is said to be complete integral of first order PDE

⇒ Charpit's Method → Given PDE of first order and non linear p and q be $f(x,y,z,p,q)=0$

Charpits arbitary equations are

$$\frac{dx}{-f_p} = \frac{dy}{-f_q} = \frac{dz}{-pf_p - qf_q} = \frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z}$$

Procedure

Transform all the terms of the given equation to LHS and define entire expression by f write down Charpits Auxillary equations using value of f find f_x, f_y, f_z, f_p and f_q and put in AE After simplifying, Select two proper fractions so that resulting integral may come out to be the simplest relation involving p/q both. Two simplest relations is Solved along with given equation to determine p and q Put these values in $dz = p\,dx + q\,dy$ which on integration gives complete integral of the PDE

Q1 $z = px + qy + p^2 + q^2$

$px + qy + p^2 + q^2 - z = 0$

AE is

$$\frac{dx}{-f_p} = \frac{dy}{-f_q} = \frac{dz}{-pf_p - qf_q} = \frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z}$$

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$f_p = x + 2p$; $f_q = y + 2q$; $f_z = -1$; $f_y = q$;
$f_x = p$

$\therefore \frac{dx}{-(x+2p)} = \frac{dy}{-(y+2q)} = \frac{dz}{-p(x+2p)-q(y+2q)} = \frac{dp}{p-p} = \frac{dq}{q-q}$

$= \frac{dx}{-(x+2p)} = \frac{dy}{-(y+2q)} = \frac{dz}{-(px-2p^2+qy+2q^2)} = \frac{dp}{0} = \frac{dq}{0}$

$dp = 0 \Rightarrow p = a$;
$dq = 0 \Rightarrow q = b$
$\therefore \boxed{z = ax + by + a^2 + b^2}$

Q2 $q = 3p^2 \Rightarrow q - 3p^2 = 0$
AE
$\frac{dx}{-f_p} = \frac{dy}{-f_q} = \frac{dz}{pf_p - qf_q} = \frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z}$

$f_p = -6p$; $f_q = 1$; $f_z = 0$; $f_x = 0$; $f_y = 0$;
$\therefore$

$\frac{dx}{-6p} = \frac{dy}{-1} = \frac{dz}{6p-q} = \frac{dp}{0} = \frac{dq}{0}$

$b = 3a^2$

$dp = 0 \Rightarrow p = a$; $dq = 0 \Rightarrow q = b = 3a^2$
$dz = p\,dx + q\,dy \Rightarrow dz = a\,dx + 3a^2 dy$
$\boxed{z = ax + 3a^2 y + b}$

Q3 $px + qy = pq$
Sol
$px + qy - pq = 0$
$f_p = x - q$ $f_q = y - p$ $f_x = p$
$f_y = q$ $f_z = 0$
Charpits auxillary equation
$\frac{dx}{-(x-q)} = \frac{dy}{-(y-p)} = \frac{dz}{-p(x-q)-q(y-p)} = \frac{dp}{p} = \frac{dq}{q}$

Consider last 2 fraction

$\frac{dp}{p} = \frac{dq}{q} \Rightarrow \log p = \log q + \log a$

$p = aq$

using given we get

$aqx + qy = aq$

$(ax+y)q = aq^2$

$ax + y = aq$

wkt $\quad p = aq = ax+y$

$q = \frac{ax+y}{a}$

wkt

$dz = p\,dx + q\,dy$

$dz = (ax+y)\,dx + \left(\frac{ax+y}{a}\right)dy$

$a\,dz = a(ax+y)\,dx + (ax+y)\,dy$

$a\,dz = a^2x\,dx + ay\,dx + ax\,dy + y\,dy$

$az = \frac{a^2x^2}{2} + ayx + \frac{y^2}{2} + \cancel{axy}$

$ay\,dx + ax\,dy$

$\frac{y}{v}\,d\frac{(a-x)}{u} + \frac{ax}{u}\,\frac{dy}{va}$

$d(ax-y)$

$$\boxed{z = \frac{ax^2}{2} + yx + \frac{y^2}{2a} + \frac{b}{a}}$$

Q 4 $\quad p^2 + y^2q = y^2 - x^2$

$y^2 - x^2 - p^2 + y^2q = 0$

sol $\quad f_p = -2p \quad f_q = y^2 \quad f_x = -2x$

$f_y = 2y + 2yq \quad f_z = 0$

Chap A E

$\frac{dx}{2p} = \frac{dy}{-y^2} = \frac{dz}{2p^2 - qy^2} = \frac{dp}{2x} = \frac{dq}{2y+2yq}$

Consider ① & ④

$\frac{dx}{2p} = \frac{dp}{-2x}$ $\qquad \frac{x^2}{2} = \frac{-p^2}{2} + \frac{a^2}{2}$

$\Rightarrow \frac{x^2}{2} + \frac{p^2}{2} = \frac{a^2}{2}$

$p^2 = \sqrt{a^2 - x^2}$

Consider ② & ⑤

$\frac{dy}{y^2} = \frac{dq}{-2y(q+1)}$

$\log y = -\frac{1}{2}\log(q+1) + \log b$

$-2\log y = \log(q+1) - 2\log b$

$-2(\log y - \log b) = \log(q+1)$

$-2\log y/b = \log(q+1)$

$\left(\frac{y}{b}\right)^{-2} = q+1$

$q = \frac{b^2}{y^2} - 1$

$dz = p\,dx + q\,dy$

$dz = \sqrt{a^2-x^2}\,dx + \left(\frac{b^2}{y^2} - 1\right) dy$

$\therefore$

$z = \frac{x}{2}\sqrt{a^2-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{b^2}{-y} - y + c$

is the complete integral of the given equation

Q5 $2zx - px^2 - 2qxy + pq = 0$

$f_p = -x^2 + q \qquad f_q = -2xy + p$

$f_x = 2z - p2x - 2qy$

$f_y = -2qx \qquad f_z = 2x$

CAE is

$$\frac{dx}{x^2-q} = \frac{dy}{2xy-p} = \frac{dz}{x^2p-qp+2xyq-pq}$$

$$= \frac{dp}{-2qy+2z-p2x+2xp} = \frac{dq}{-2qx+2xq}$$

$dq = 0$

$q = a$

$2zx - px^2 - 2qxy + pq = 0$

$2zx - px^2 - 2axy + pa = 0$

$\therefore$

$$p = \frac{2zx + 2axy}{(a-x^2)} \qquad \boxed{q=a}$$

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$dz = p\,dx + q\,dy$

$$dz = \left(\frac{2zx - 2axy}{x^2-a}\right)dx + a\,dy$$

$(x^2-a)dz = (2zx - 2axy)dx + a(x^2-a)dy$

$(x^2-a)dz - 2zx\,dx = a(x^2-a)dy - 2axy\,dx$

$$\frac{(x^2-a)dz - 2zx\,dx}{(x^2-a)^2} \qquad d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$d(uv) = u\,dv + v\,du$

$$= \frac{(x^2-a)a\,dy - 2axy\,dx}{(x^2-a)^2} = d\left(\frac{ay}{x^2-a}\right)$$

on integrating we get

$$\frac{z}{x^2-a} = \frac{ay}{x^2-a} + b$$

$$\boxed{z = ay + b(x^2-a)}$$

$(x^2-a)dz = 2zx\,dx - 2axy\,dx + a(x^2+a)dy$

$(x^2-a)dz - 2zx\,dx = a(x^2-a)dy - 2axy\,dx$

$\cancel{(x^2-a)^2}$

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Q6). $z^2(p^2+q^2+1)=1$

$p^2z^2+q^2z^2+z^2=1$

Charpit's A E is

$$\frac{dx}{-f_p}=\frac{dy}{-f_q}=\frac{dz}{-pf_p-qf_q}=\frac{dp}{f_x+pf_z}=\frac{dq}{f_y+qf_z}$$

Here

$f_p=2pz^2 \quad f_q=2qz^2 \quad f_x=0 \quad f_y=0$

$f_z=2z(p^2+q^2+1)$

$$\frac{dx}{-2pz^2}=\frac{dy}{-2qz^2}=\frac{dz}{-}$$

x ——— x ——— x

* Classification of 2nd order PDE

Second order PDE is in the form
$AU_{xx}+BU_{xy}+CU_{yy}+DU_x+EU_y+FU=\phi(x,y)$ ①
where A, B, C, D, E, F are functions of independent variable x and y. Discriminent of eq ① is given by $\Delta=B^2-4AC$ and eq ① is classified as followed.

if $\Delta>0$ then the given eq ① is hyperbolic.

if $\Delta=0$ then the given eq ① is equal to Parabolic.

if $\Delta<0$ the given eq is elliptic

Q) $U_{xx}+3U_{yy}-2U_x+12U_y+5U=0$

Sol Here
$A=1,\ B=0,\ C=3,$

$\Delta=B^2-4AC$

$-4(1)(3)=-12<0$

∴ given eq is elliptic

2) $yU_{xx} - 4U_{xy} + 4xU_{yy} = 0$

Here $A = y$

$B = -4$

$C = 4x$

$\therefore \Delta = B^2 - 4AC$

$= (-4)^2 - 4(y)(4x)$

$\Delta = 16 - 16xy$

if $x = y = 1$, then $\Delta = 0$ the given eqn is Parabolic

If $16 - 16xy < 0$

$1 - xy < 0$

$xy > 1$

$\boxed{x > 1/y}$ then $\Delta < 0$ elliptic.

if $x < \frac{1}{y}$ $xy < 1$

$\therefore \Delta = B^2 - 4AC > 0$ then hyperbolic

Q3 $U_{xx} + U_{yy} = 0$ (4) $U_t = U_{xx}$

(5) $U_{tt} = U_{xx}$

Q3 $A = 1$, $B = 0$, $C = 1$ (4) $A = 1$, $B = 0$, $C = 0$

$\Delta = -4 < 0$ elliptic $\Delta = 0$; Parabolic

Q5) $U_{tt} - U_{xx}$ $A = 1$, $B = 0$, $C = -1$

$\Delta = B^2 - 4AC = 0 - 4 \cdot 1(-1)$

$= 4$

$\Delta > 0$

Thus the given eq is hyperbolic

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Q6) $U_{xx} + xU_{yy} = 0$ Here $B = 0$

$A = 1$

$C = x$

$\Delta = B^2 - 4AC$

$0 - 4(1)(x)$

$\Delta = -4x$

$\Delta < 0$ $x > 0$ the eq is hyperbolic

$\Delta = 0$ $x = 0$ " Parabolic

$\Delta > 0$ $x < 0$ " elliptic.

$\therefore$ This equation is Trichomic equal

Q7) $xyr - (x^2-y^2)s - xy + py - qx = 2(x^2-y^2)$

Here

$r = \frac{\partial^2 u}{\partial x^2}$, $s = \frac{\partial^2 u}{\partial x \partial y}$, $t = \frac{\partial^2 u}{\partial y^2}$

$p = \frac{\partial u}{\partial x}$, $q = \frac{\partial u}{\partial y}$

$A = xy$, $B = -(x^2-y^2)$, $C = -xy$

$B^2 - 4AC = (-(x^2-y^2))^2 - 4(xy)(\cancel{x}y)$

$= x^4 + y^4 - 2x^2y^2 + 4x^2y^2$

$= x^4 + y^4 + 2x^2y^2$

$(x^2+y^2)^2 > 0$

if $\forall (x,y)$ $x \neq 0, y \neq 0$. This the given equation is hyperbolic

4/2/2020

* Solution of 1-D heat equation by variable separable method

Let $u(x,t)$ denote temperature of a thin long rod of length l run's from $x = 0$ to $x = h$.

Assume that the ends of rod are insulated i.e heat neither enters nor leaves from the rod on all sides

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also ~~create~~ heat energy neigther c
nor distroyed
the temperature $u(x,t)$ satisfies
the heat equation

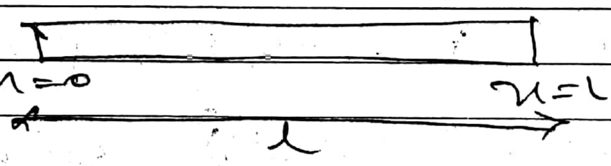
$$\boxed{\frac{\partial u}{\partial t} = \chi \frac{\partial^2 u}{\partial x^2}} \quad (1)$$

where $\chi = \frac{K}{\rho c_p}$ is thermal diffusivity

K = thermal conductivity
ρ = is density
c_p is sp. heat at const pre

Both the ends are kept at constant temperature. thus $\boxed{u(0,t) = u(l,t) = 0}$ — (2a)

Initial temperature of a rod at the position x is $\boxed{u(x,0) = f(x)}$ — (2b)



Sol:- Heat eqn is given by

$$\frac{\partial u}{\partial t} = \chi \frac{\partial^2 u}{\partial x^2} \quad (1)$$

subject to BCs: $u(0,t) = 0 = u(l,t)$ — (2a)

IC: $u(x,0) = f(x)$ — (2b)

variable separable solution of eq (1) is given by:

$$u(x,t) = X(x)T(t) \quad (3)$$

Using (2a) in (3) we get

$$u(0,t) = 0$$

$$X(0)T(t) = 0 \quad (4)$$

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$u(L,t) = 0$

$X(L)T(t) = 0$

$X(L) = 0 \longrightarrow (5)$

Using equation (3) in equation (1)

$X(x)T'(t) = \chi X''(x)T(t)$

$\div$ by $\chi X(x)T(t)$

$$\frac{\cancel{X(x)}T'(t)}{\chi \cancel{X(x)}T(t)} = \frac{\cancel{\chi} X''(x)\cancel{T(t)}}{\cancel{\chi} X(x)\cancel{T(t)}}$$

$$\boxed{\frac{T'(t)}{\chi T(t)} = \frac{X''(x)}{X(x)} = \lambda} \quad (6)$$

Case (1) If $\lambda = 0$

$$\frac{X''(x)}{X(x)} = 0$$

$X''(x) = 0$

integrate twice, we get

$X'(x) = a$

$X(x) = ax + b \quad (7)$

From eq (4) $X(0) = 0$

$a \cdot 0 + b = 0$

$\boxed{b = 0}$

From (5) $X(L) = 0$

$aL + b = 0$

$a \cdot L + 0 = 0$

$a \cdot L = 0$

$\boxed{a = 0}$

Thus

$X(x) = 0$

$u(x,t) = 0 \cdot T(t)$

$\boxed{u(x,t) = 0}$

so
$u(x,t) = 0$ is a trivial solⁿ

we neglect this case.

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Case 2 : λ is positive $(\lambda > 0)$, $\lambda = \mu^2$, $(\mu \neq 0)$

$$\frac{X''(x)}{X(x)} = \mu^2$$

$$X''(x) = \mu^2 X(x)$$

AE is given by

$$m^2 = \mu^2 \Rightarrow m = \pm\mu \quad (\mu \in \mathbb{R})$$

$$X(x) = c_1 e^{\mu x} + c_2 e^{-\mu x} \quad \text{——— (a)}$$

From eq (4) $X(0) = 0$

$$c_1 e^{\mu \cdot 0} + c_2 e^{-\mu \cdot 0} = X(0)$$

$$c_1 + c_2 = 0$$

~~Thus~~ $\boxed{c_2 = -c_1}$

$$X(x) = c_1 e^{\mu x} - c_2 e^{-\mu x}$$

From eq (5) $X(l) = 0$

$$X(l) = c_1 e^{\mu l} - c_2 e^{-\mu l}$$

$$0 = c_1 (e^{\mu l} - e^{-\mu l})$$

$$\boxed{c_1 = 0}$$

$$\boxed{c_2 = 0}$$

Thus equa (a) becomes $X(x) = 0$

$u(x, t) = 0$ it is a trivial sol

thus we neglet this case.

Case 3:- λ is negative $(\lambda < 0)$, $\lambda = -\mu^2$ $(\mu \neq 0)$

$$\frac{X''(x)}{X(x)} = -\mu^2$$

$$X''(x) = -\mu^2 X(x)$$

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AE is given by.

$m^2 = -\mu^2 \Rightarrow m = \pm \mu i$ $\quad e^{\alpha + \beta i} = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)$

$X(x) = C_1 \cos\mu x + C_2 \sin\mu x$ —— (10)

From eq (4) $X(0) = 0$

$X(0) = C_1 \cos\mu(0) + C_2 \sin\mu(0)$

$\boxed{C_1 = 0}$

from eq (5) $X(l) = 0$

$X(l) = C_1 \cos\mu(l) + C_2 \sin\mu(l)$

$0 = C_2 \sin\mu(l)$

$\cancel{\boxed{C_2 = 0}}$

~~Thus eq (10) become~~ $\cancel{X(x) = 0}$ /

$\cancel{u(x,t) = 0}$

$C_2 \neq 0$; $\sin\mu l = 0$

$\mu l = n\pi$, $n = 1, 2, 3 \ldots$ $\quad \therefore \lambda_n = -\frac{n^2\pi^2}{l^2}$

$\mu = \frac{n\pi}{l}$

Thus, $X_n(x) = C_2 \sin\left(\frac{n\pi x}{l}\right)$ —— (11)

$\frac{X''}{X} = \frac{T'}{XT} = \lambda \rightarrow$ (6)

From eq (6)

$\frac{T'}{XT} = \lambda$ $\quad T' = X\lambda T$ $\quad \Big| \; T' = \frac{dT}{dt}$

AE is $m = X\lambda$

Solution of eq (12) is given by

$T(t) = C_3 e^{X\lambda t}$

$\boxed{T_n(t) = C_3 e^{-\frac{Xn^2\pi^2}{l^2}t}}$ —— (13)

Therefore, the solution of given eqn is

$u(x,t) = \sum_{n=1}^{\infty} X_n(x) T_n(t)$

$$= \sum_{n=1}^{\infty} C_2 \sin\left(\frac{n\pi x}{L}\right) C_3 \cdot e^{-\frac{\lambda n^2\pi^2 t}{l^2}}$$

$$\boxed{u(x,t) = \sum_{n=1}^{\infty} A_n \cdot e^{-\frac{\lambda n^2\pi^2 t}{l^2}} \cdot \sin\left(\frac{n\pi x}{L}\right)}$$

where $A_n = C_2 C_3$

Given $u(x,0) = f(x)$

put $t=0$ in eq (14) we get

$$u(x,0) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right)$$

$$f(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right) \quad \text{—— (15)}$$

Eqn (15) is a half range fourier sine series where A_n is given by

$$\boxed{A_n = \frac{2}{l}\int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx} \quad \text{—— (16)}$$

Therefore, the general of (1) is given by eq (14) and A_n is given by eq (16)

Q. Solve $u_t = k u_{xx}$ subjected to

$u(0,t) = u(l,t) = 0$

$u(x,0) = 6\sin\left(\frac{\pi x}{l}\right)$

Ans Given $u_t = k u_{xx}$

$u(0,t) = u(l,t) = 0$

$u(x,0) = 6\sin\left(\frac{\pi x}{l}\right)$

The solution of heat eqn is given by

$$u(x,t) = \sum_{n=1}^{\infty} A_n e^{-\frac{k n^2\pi^2 t}{l^2}} \cdot \sin\left(\frac{n\pi x}{l}\right)$$

where

$$A_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$A_n = \frac{2}{l}\int_0^l 6\sin\left(\frac{\pi x}{l}\right)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$A_n = \begin{cases} \frac{\cancel{12}^{6}}{l}\cdot\frac{l}{2} & n=1 \\ 0 & n>1 \end{cases}$$

$$\left[\int_0^{2l} \sin mx \sin nx\, dx = \begin{cases} 0, & m\neq n \\ l, & m=n \end{cases}\right]$$

$$A_n = \begin{cases} 6 & n=1 \\ 0 & n>1 \end{cases}$$

Therefoure the solution is given by

$$u(x,t) = A_1 e^{-\frac{k\pi^2 t}{l^2}} \cdot \sin\left(\frac{\pi x}{l}\right)$$

$$\boxed{u(x,t) = 6e^{-\frac{k\pi^2 t}{l^2}} \sin\left(\frac{\pi x}{l}\right)}$$

• d) $u_t = k\,u_{xx}$

$u(0,t) = u(l,t) = 0$

$u(x,0) = 12\sin\left(\frac{9\pi x}{l}\right) - 7\sin\left(\frac{4\pi x}{l}\right)$

Ans: Given

The solution of heat eq is given by

$$u(x,t) = \sum_{n=1}^{\infty} A_n e^{-\frac{kn^2\pi^2}{l^2}t}\sin\left(\frac{n\pi x}{l}\right)$$

where

$$A_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

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$$A_n = \frac{2}{l}\int_0^l \left(12\sin\frac{9\pi x}{l} - 7\sin\left(\frac{4\pi x}{l}\right)\right)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$A_n = \begin{cases} \frac{-\cancel{14}\,7}{l}\frac{l}{\cancel{2}} & n=4 \\ \frac{\cancel{24}\,12}{l}\frac{l}{\cancel{2}} & n=9 \\ 0 & n\neq 4 \,\&\, n\neq 9 \end{cases}$$

Thus

$A_4 = -7$, $A_9 = 12$, $A_n = 0 \ \forall n, n\neq 4, n\neq 9$

$$\boxed{u(x,t) = 12e^{-\frac{81\pi^2 K}{l^2}t}\sin\left(\frac{9\pi x}{l}\right) + 7e^{-\frac{16\pi^2 K}{l^2}t}\sin\left(\frac{4\pi x}{l}\right)}$$

11/2/2020 solve by urs.

Q1 $u_t = u_{xx}$ Subjected to

$u(0,t) = u(1,t) = 0 \quad t>0$

$u(x,0) = x - x^2, \quad 0<x<1$

$A_n = \frac{4}{n^3\pi^3}(1-(-1)^n) \quad n = 1,2,3\ldots$

Q2 $u_t = u_{xx}$ $\quad u(x,0) = \begin{cases} x & , 0\leq x\leq 50 \\ 100-x & , 50\leq x<100 \end{cases}$

$u(0,t) = u(100,t) = 0$

$A_m = \frac{400}{(2m-1)^2\pi^2}(-1)^m, \quad m = 1,2,3\ldots$

$$u(x,t) = \sum_{m=0}^{\infty} \frac{400}{(2m-1)^2\pi^2} e^{-\frac{(2m-1)\pi}{100}t} \sin\left(\frac{(2m-1)\pi x}{100}\right)$$