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MATHS ASSIGNMENT - 3

Question No. 1(ILO 4)

Approximate the solution to the following heat equation up to second time level using Schmidth method with $h = \frac{1}{4}$ and k $= \frac{1}{2}$

$$\frac{\delta u}{\delta t} = \frac{1}{16}\frac{\delta^2 u}{\delta x^2}$$

subjected to

$$u(0,t) = u(1,t) = 0, t > 0,$$
$$u(x,0) = 2\,sin(2\pi x), 0 \leq x \leq 1.$$

Solution 1: Given**,**

$$\frac{\delta u}{\delta t} = \frac{1}{16}\frac{\delta^2 u}{\delta x^2}$$

$$Boundary\ Conditions : u(0,t) = u(1,t) = 0, t > 0$$

$$Initital\ Condition : u(x,0) = 2\,sin(2\pi x), 0 \leq x \leq 1.$$

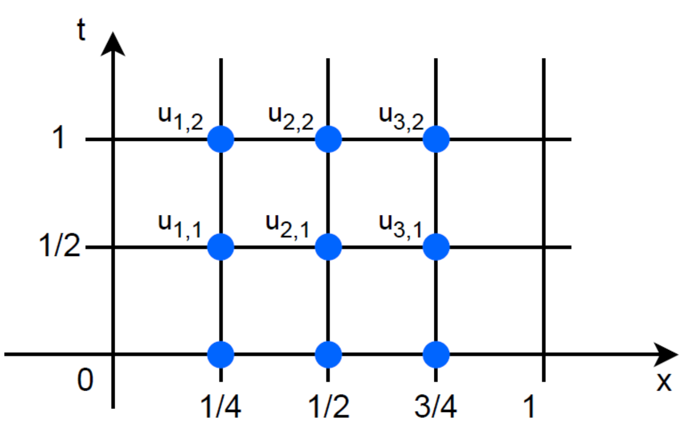
Also,

$$h = \frac{1}{4}, k = \frac{1}{2}$$

$$\boldsymbol{\lambda} = \frac{KC^2}{h^2} = \frac{\frac{1}{2} \times \frac{1}{16}}{\frac{1}{16}} = \boldsymbol{\frac{1}{2}}$$

From initial Condition,

$$u\left(\frac{1}{4}, 0\right) = 2\sin\frac{\pi}{2} = 2$$
$$u\left(\frac{1}{2}, 0\right) = 2\sin\pi = 0$$
$$u\left(\frac{3}{4}, 0\right) = 2\sin\frac{3\pi}{2} = -2$$



According to Schimidth Method,

$$u_{i,j+1} = \frac{(u_{i+1}, j + u_{i-1}, j)}{2}$$

For first Time level $j = 0$:

$$i = 1 : \ u_{1,1} = \frac{1}{2}(u_{0,0} + u_{2,0}) = 0.5 \times (0 + 0) = 0$$
$$i = 2 : \ u_{2,1} = \frac{1}{2}(u_{1,0} + u_{3,0}) = 0.5 \times (2 - 2) = 0$$
$$i = 3 : \ u_{3,1} = \frac{1}{2}(u_{2,0} + u_{4,0}) = 0.5 \times (0 + 0) = 0$$

For Second time level $j = 1$:

$$i = 1 : \ u_{1,2} = \frac{1}{2}(u_{0,1} + u_{2,1}) = 0.5 \times (0 + 0) = 0$$
$$i = 2 : \ u_{2,2} = \frac{1}{2}(u_{1,1} + u_{3,1}) = 0.5 \times (0 + 0) = 0$$
$$i = 3 : \ u_{3,2} = \frac{1}{2}(u_{2,1} + u_{4,1}) = 0.5 \times (0 + 0) = 0$$

Question No. 2(ILO 4)

A random sample of income from the users of a telecom company showed

Number of users(x) (in millions)	26	29	32	34	36	37	40
Income(y) (in billions)	48	68	66	69	76	67	84

(a). Obtain the correlation coefficient and comment on the type of relation between the variables.

(b). Determine the regression line between the variables.

Solution 2:

x	y	xy	x^2	y^2
26	48	1248	676	2304
29	68	1972	841	4624
32	66	2112	1024	4356
34	69	2346	1156	4761
36	76	2736	1296	5776
37	67	2479	1369	4489
40	84	3360	1600	7056

$$\sum x = 234 , \sum y = 478 , n = 7$$

$$\sum xy = 16253 , \sum x^2 = 7962 , \sum y^2 = 33366$$

a)

Substituting the data in below equation:

$$correlation\ coefficient\ (r) = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{\left((n\sum x^2) - (\sum x)^2\right) \times \left((n\sum y^2) - (\sum y)^2\right)}}$$

$$r = \frac{7(16253) - (234)(478)}{\sqrt{\left((7 * 7962) - 7962^2\right) * \left((7 * 33366) - 33366^2\right)}}$$

$$r = \frac{113771 - 11852}{\sqrt{978 * 5078}}$$

$$r = \frac{1919}{2228.51} = \mathbf{0.861}$$

$$\boldsymbol{Correlation\ coefficient\ (r)\ =\ 0.861}$$

Therefore, there is a string correlation between the two variables.

b)

$$Regression\ Line : Y = a + bx$$

$$where\ , a\ is\ y - intercept,$$
$$b\ is\ slope\ of\ the\ line$$

$$we\ know\ b = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{(n\sum x^2) \times (n\sum y^2)}}$$

Substituting the data,

$$\boldsymbol{b} = \frac{1919}{978} = \mathbf{1.962}$$

$$we\ know\ , a = \overline{y} - b\overline{x}$$

here,

$$\overline{y} = \frac{\sum y}{n} = \frac{478}{7} = 68.28$$

$$\overline{x} = \frac{\sum x}{n} = \frac{234}{7} = 33.42$$

$$a = 68.28 - 1.962 \times (33.42)$$

$$= 68.28 - 65.58$$

$$\boldsymbol{a = 2.7}$$

Substitution in values of a and b in equation, we get

$$\boldsymbol{Y = 1.962x + 2.7}$$

Hence, this is the required line equation.

Question No. 3(ILO 5)

Two groups are competing for the position on the board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Solution 3:

Given data:

Let A and B be two groups.

$$Probability\ that\ A\ wins, P(A) = 0.6$$

$$Probability\ that\ B\ wins, P(B) = 0.4$$

Let C be the event of introducing new product new product introduced after A wins, $P(C|A) = 0.7$

New product introduced after B wins, $P(C|B) = 0.3$

$$P(C|A) = \frac{P(A \cap C)}{P(A)} = 0.7$$

$$P(A \cap C) = P(A) \times 0.7$$

$$P(A \cap C) = 0.6 \times 0.7 = 0.42$$

$$P(C|B) = \frac{P(B \cap C)}{P(B)} = 0.3$$

$$= P(B) \times 0.3$$

$$P(B \cup C) = 0.4 \times 0.3 = 0.12$$

Since both are independent events applying total probability,

$$P(C) = P(A \cap C) + P(B \cup C)$$

$$P(C) = 0.42 + 0.12 = 0.54$$

$$P(B|C) = \frac{P(B \cap C)}{P(C)}$$

$$\boldsymbol{P(B|C) = \frac{0.12}{0.54} = 0.22}$$

Therefore, the probability that the new product introduced was by the second group is $\mathbf{0.22}$.