

## Assignment - 4

Sol. 1.

a) Probability distribution  $\sum P(x_i) = 1$   
finding  $K$ .

$$\frac{2K-3}{10} + \frac{K-2}{10} + \frac{K-1}{10} + \frac{K+1}{10} = 1$$

$$2K-3 + K-2 + K-1 + K+1 = 10$$

$$5K-5 = 10$$

$$5K = 15$$

$$\boxed{K=3}$$

i)  $P(-3 < X < 4)$

$$\Rightarrow P(-1) + P(2)$$

$$\Rightarrow \frac{K-2}{10} + \frac{K-1}{10} = \frac{2K-3}{10} = \frac{2(3)-3}{10}$$

$$= \frac{3}{10} = \underline{\underline{0.3}}$$

ii)  $P(X \leq 2)$

$$\Rightarrow P(-3) + P(-1) + P(2)$$

$$\Rightarrow \frac{2K-3}{10} + \frac{K-2}{10} + \frac{K-1}{10} = \frac{4K-6}{10}$$

$$\Rightarrow \frac{4(3)-6}{10} = \frac{6}{10} = \underline{\underline{0.6}}$$

Soln 2.

$$U(x, y) = e^x \cos(y) + xy$$

$$\frac{\partial U}{\partial x} = \cos y e^x + y$$

$$\frac{\partial^2 U}{\partial x^2} = e^x \cos y$$

$$\frac{\partial U}{\partial y} = -\sin y e^x + x$$

$$\frac{\partial^2 U}{\partial y^2} = -\cos y e^x$$

we know,

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

$\therefore U$  is harmonic

let  $V(x, y)$  be the imaginary part,

$$f'(z) = \frac{\partial U}{\partial x} - i \frac{\partial V}{\partial y}$$

$$f'(z) = (e^x \cos y + y) - i(-\sin y e^x + x)$$

$$\Rightarrow e^x \cos y + i \sin y e^x + (y - ix)$$

$$\Rightarrow e^{(x+iy)} + (y - ix)$$

By Milne-Thomson method,

Replace  $x$  by  $z$  and  $y$  by  $0$ .

$$f'(z) = e^z - iz$$

Integrating w.r.t  $z$ , we get

$$f(z) = \int (e^z - iz) dz$$

$$\boxed{f(z) = e^z - i \frac{z^2}{2} + C}$$