

Question No. 3

For the given initial value problem

$$\frac{dy}{dx} = x^3 + y, y(0) = 1$$

- Write the MATLAB function to solve numerically using Runge Kutta fourth order method.
- Find the exact solution using MATLABs built-in function ***'dsolve'***.
- Plot the exact and numerical solution in the interval [0,1] choosing step size h=0.1 in the same figure.

Solution:

```
function [] = runge_kutta( f,yo,xo,h,xn )
x= xo:h:xn;
n = length(x);
y = zeros(1,n);
y(1) = yo;
for i = 2:n
    k1 = h*f(x(i-1),y(i-1));
    k2 = h*f(x(i-1)+h/2,y(i-1)+k1/2);
    k3 = h*f(x(i-1)+h/2,y(i-1)+k2/2);
    k4 = h*f(x(i-1)+h,y(i-1)+k3);
    y(i) = y(i-1) + 1/6*(k1+2*k2+2*k3+k4);
end
z = eval(dsolve('Dy=x^3+y','y(0)=1','x'));
fprintf("numerical solution: ");
disp(y)
fprintf("exact solution: ");
disp(z);
plot(x,y,'r',x,z,'k*')
end
```

```
Command Window
>> f = @(x,y) x^3+y;
>> xo = 0;
>> yo = 1;
>> h=0.1;
>> xn = 1;
>> runge_kutta( f,yo,xo,h,xn )
numerical solution:    1.0000    1.1052    1.2218    1.3520    1.4988    1.6660    1.8588    2.0833    2.3468    2.6582    3.0280
exact solution:       1.0000    1.1052    1.2218    1.3520    1.4988    1.6660    1.8588    2.0833    2.3468    2.6582    3.0280
fx >> |
```

Figure 1 assigning values and calling function in command window

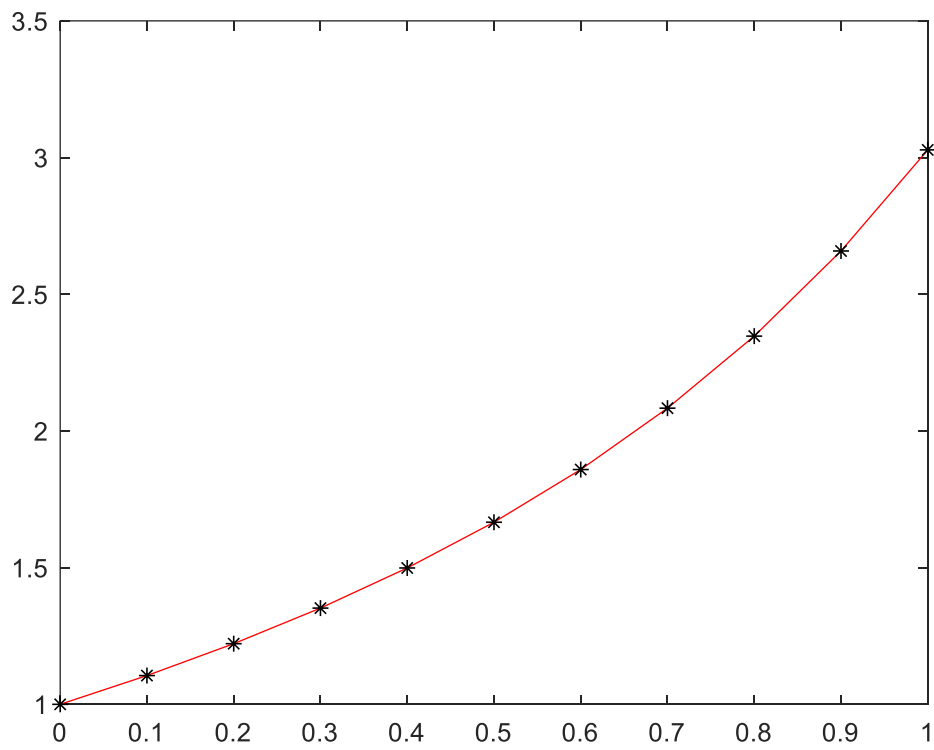


Figure 2 Graph output