

Assignment - 2.

question 1.

x	1	2	4	8	16
$P(x)$	0.06	0.13	0.32	0.38	0.11

probability distribution of amount of memory (GB)
in a purchased flash drive.

$$\begin{aligned} \text{a) } P(x \leq 4) &= P(x = 1, 2, 4) \\ &= P(x_1 = 1) + P(x_2 = 2) + P(x_3 = 4) \\ &= 0.06 + 0.13 + 0.32 \\ &= 0.51 \end{aligned}$$

$$\begin{aligned} \text{b) } P(x > 8) &= P(x = 16) \\ &= P(x_5 = 16) \\ &= 0.11 \end{aligned}$$

$$\begin{aligned} \text{c) } P(2 < x \leq 8) &= P(x = 4, 8) \\ &= P(x_3 = 4) + P(x_4 = 8) \\ &= 0.32 + 0.38 \\ &= 0.7 \end{aligned}$$

question. 2

the ~~proper~~ probability density function of a continuous random variable x is given by -

$$f(x) = \begin{cases} cx+3 & ; -3 \leq x \leq -2 \\ 3-x & ; 2 \leq x \leq 3 \\ 0 & ; \text{else where.} \end{cases}$$

a) value of c ,

by using the properties of probability density function, we know,

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

hence,

$$\Rightarrow \int_{-3}^{-2} (cx+3) dx + \int_2^3 (3-x) dx = 1$$

$$\Rightarrow \left(\frac{cx^2}{2} + 3x \right) \Big|_{-3}^{-2} + \left(3x - \frac{x^2}{2} \right) \Big|_2^3 = 1$$

$$\Rightarrow \frac{4c}{2} - \frac{9c}{2} + 3(-2+3) + 3(3-2) - \frac{1}{2}(9-4) = 1$$

$$= -\frac{5c}{2} + 3 + 3 - \frac{5}{2} = 1$$

$$\frac{-5c-5}{2} = -5$$

$$\frac{c+1}{2} = 1$$

$$\boxed{c=1}$$

\therefore , the value of $c=1$.

$$\text{now, } f(x) = \begin{cases} x+3 & -3 \leq x \leq -2 \\ 3-x & -2 \leq x \leq 3 \\ 0 & \text{elsewhere.} \end{cases}$$

b) Expectation and variance.

$$\text{Expectation, } E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

$$E(X) = \int_{-3}^{-2} x(x+3) dx + \int_{-2}^3 x(3-x) dx$$

$$= \int_{-3}^{-2} (x^2 + 3x) dx + \int_{-2}^3 (3x - x^2) dx$$

$$= \left(\frac{x^3}{3} + \frac{3x^2}{2} \right)_{-3}^{-2} + \left(\frac{3x^2}{2} - \frac{x^3}{3} \right)_{-2}^3$$

$$= \frac{1}{3} (-8 + 27) + \frac{3}{2} (4 - 9) + \frac{3}{2} (9 - 4) - \frac{1}{3} (27 - 8)$$

$$= \frac{1}{3} (27 - 8) - \frac{1}{2} (27 - 8) + \frac{3}{2} (5) + \frac{3}{2} (-5)$$

$$\therefore, \boxed{E(X) = 0}$$

$$E(X^2) = \int_{-3}^{-2} x^2(x+3) dx + \int_{-2}^3 x^2(3-x) dx$$

$$= \left(\frac{x^4}{4} + \frac{3x^3}{3} \right)_{-3}^{-2} + \left(\frac{3x^3}{3} - \frac{x^4}{4} \right)_{-2}^3$$

$$= \frac{1}{4} (16 - 81) + (-8 + 27) + (27 - 8) - \frac{1}{4} (81 - 16)$$

$$= \frac{1}{2} (16 - 81) + 2(27 - 8)$$

$$\Rightarrow \frac{1}{2} (-65) + 2(19) \\ = -\frac{32}{2} + 38 = \frac{11}{2} = 5.5$$

$$\therefore, \boxed{E(X^2) = 5.5}$$

and, as we know,

$$\begin{aligned} \text{variance of } X, \text{ var}(X) &= E(X^2) - (E(X))^2 \\ &= 5.5 - (0)^2 \\ &= 5.5 \end{aligned}$$

$$\therefore, \boxed{\text{var}(X) = 5.5}$$

therefore,

Expectation and variance are

$$E(X) = 0 \quad \text{and} \quad \text{var}(X) = 5.5.$$