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17-5-2021

Roll. No.: 18ETCS002121

Course Code: 19CSC311A

CSE - 'C' section.

Course Name: Graph Theory and Optimization.

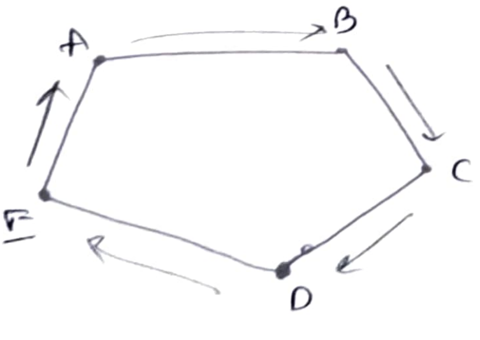
1.

(i) Hamiltonian graph.

a connected graph is called hamiltonian graph if it ~~Follows~~ contains a hamiltonian cycle or hamitonian path.

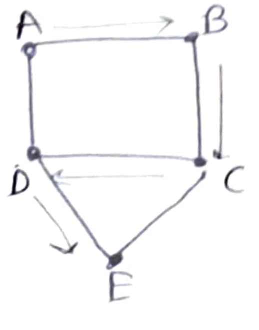
a hamitonian cycle in a graph must include all the vertices in the graph. It does not ~~mean~~ have to include all the the edges.

In the connected graph if a walk exist that visits every vertex of the graph exactly once without repeating the edges, that walk is called hamitoniaan path.



hamitonian cycle.

=> A B C D E A



Hamiltonian path.

=> A BC DE

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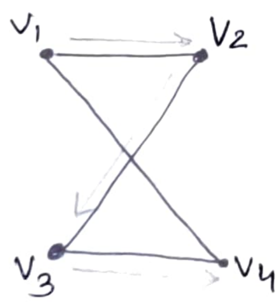
1. (ii) Euler graph

a graph is a euler graph if it ~~[illegible]~~ contains euler path or euler circuit.

Euler path is a path where it can reach all the vertices but every edge is ~~[illegible]~~ used only once.

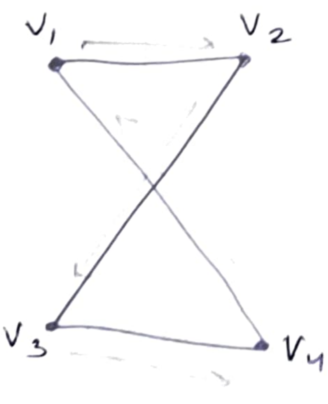
Euler circuit is a cycle where the starting and ending point is same.

eg.



euler path.

$V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4$



euther circuit

$V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4 \rightarrow V_1$

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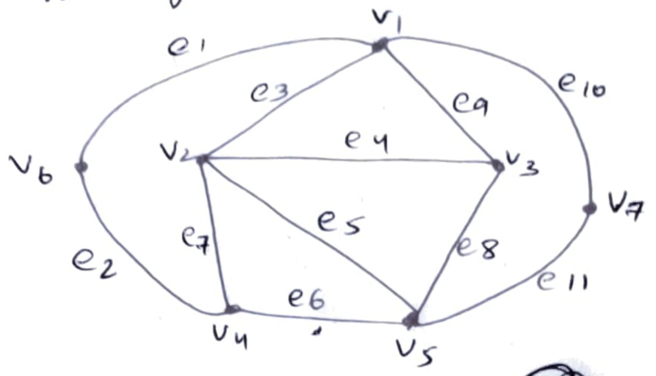
2.

a.). let the graph be G.



We know,

Euler's formula,

$$r = e - v + 2$$

where, r = no. of regions
e = no. of edges.
v = no. of vertices.

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}\}$$

$$e = 11$$

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$$

$$v = 7$$

$\therefore$,

$$r = 11 - 7 + 2$$

$$= 11 - 5 = 6$$

no. of regions = 6.

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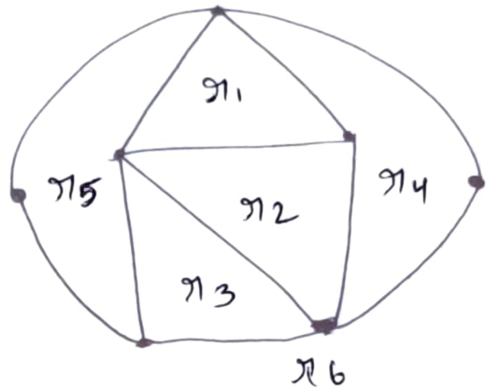


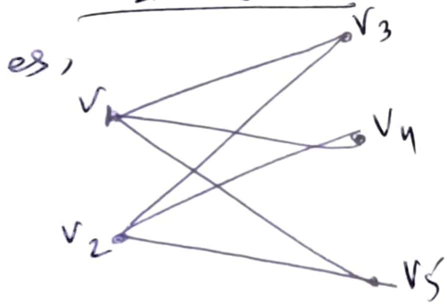
hence, there are 6 regions in graph G. the graph has 5 finite regions, R_1, R_2, R_3, R_4, R_5 & 1 infinite region. R_6.

2.

b)

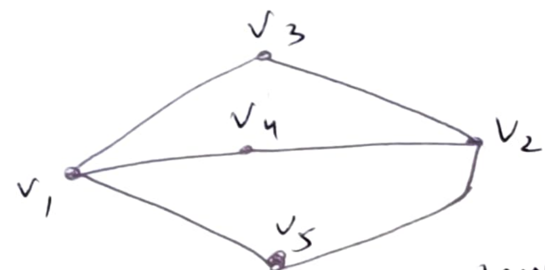
K_{23} graph

as,

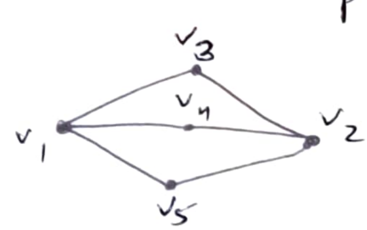


complete bipartite graph

also can be drawn as

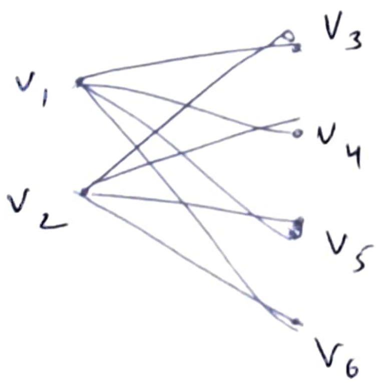


planar graph.

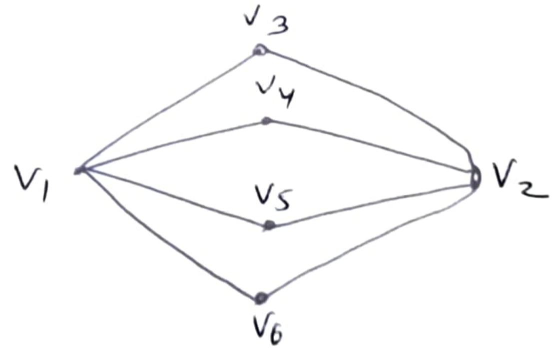


K_{24} ~~graph~~ graph.

as complete bipartite graph.



also can be drawn as



planar graph. where no edge is crossing each other.

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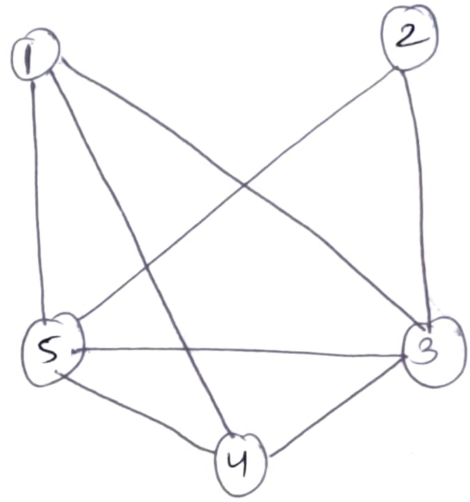
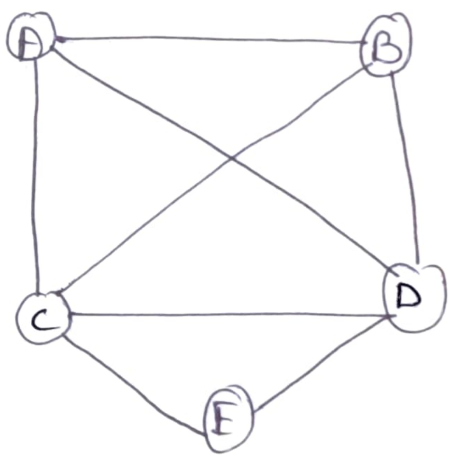
3. let the graph G_1 =



let the graph G_2 =



as we can see, V & E are equal.
mapping vertices from G_1 to G_2,

1 $\longrightarrow$ A
5 $\longrightarrow$ C
4 $\longrightarrow$ B
3 $\longrightarrow$ D
2 $\longrightarrow$ E

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~~Also~~, in G_1,

$d(1) = 3$
$d(2) = 2$
$d(3) = 4$
$d(4) = 3$
$d(5) = 4$

in G_2,

$d(A) = 3$
$d(B) = 3$
$d(C) = 4$
$d(D) = 4$
$d(E) = 2$

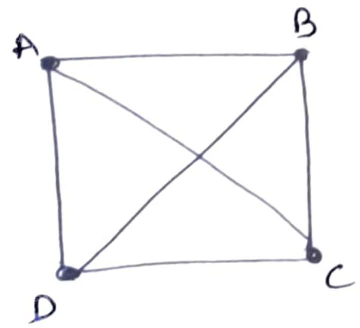
both G_1 & G_2 have 5 vertices & 8 edges.

hence, we can say, G_1 & G_2 are isomorphic.
as we know, two graphs are isomorphic when the mapping of their vertices is a bijection & their exists an edge b/w $f(u)$ & $f(v)$, iff there is an edge b/w $(u), (v)$.

4.
⇒ K - Regular graph.

A graph is called regular graph if degree of each vertex is equal.
A graph is called k-regular if degree of each vertex in the graph is k.

eg.



3 - Regular graph

2 - regular graph.

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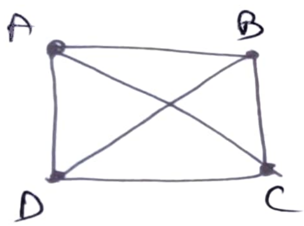
any simple kn graph is (k-1) regular
a complete graph with n vertices is called kn graph :
In a complete graph of k vertices, each vertices is connected to all (k-1) remaining vertices, so, degree of each vertex is (k-1), hence, the graph is (k-1) regular. eg.



all the 4 vertices connected with 3 remaing vertices (all have degree 3)
hence, i

5.
(i) G has 16 edges and all vertices of degree 4.

according to Handshake Lemma -

$$\Sigma d(v_i) = 2E$$ where E = Edges.
d(v_i) = degree of v_i

$\therefore$, $4 \cdot V = 16 \times 2$

$V = 8$ vertices.

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(ii) G is regular with 15 edges.

we know, $E = \frac{N \times k}{2}$ when E = no. of edges, N = no. of vertices.

$15 = \frac{N \times K}{2}$

V, no. of vertices = $\frac{30}{k}$ when $k = 2, 3, 5 \ldots$

(iii) G has 10 edges with 2 vertices of degree 4 & all other vertics of degree 3.

according to Handshake Lemma -

$\sum d(v_i) = 2E$

$= 2 \times 10$

$2 \times 4 + (v-2) \times 3 = 2 \times 10$

$8 + 3V - 6 = 20$

$3V + 2 = 20$

$V = 18/3$

$\underline{V = 6 \text{ vertices}}$