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Roll. No.: 18ETCS002121

Course code: 19CSC311A

CSE - 'C' section.

Course Name : Graph Theory and Optimization.

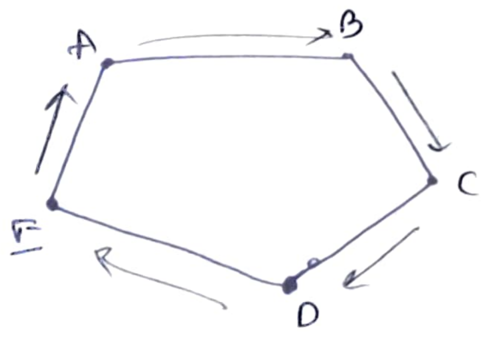
1.

(i) Hamiltonian graph.

a connected graph is called hamiltonian graph if it ~~follows~~ contains a hamiltonian cycle or hamitonian path.

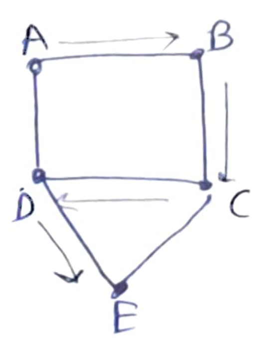
a hamitonian cycle in a graph must include all the vertices in the graph. It does not ~~mean~~ have to include all ~~the~~ the edges.

In the connected graph if a walk exist that visits every vertex of the graph exactly once without repeating the edges, that walk is called hamitoniaan. path.



hamitonian cycle.

=> A B C D E A



Hamiltonian path.

=> A BC DE

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1.

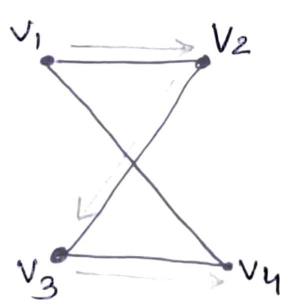
(ii) Euler graph

a graph is a euler graph if it ~~gottoas~~ contains euler path or euler circuit.

Euler path is a path where it can reach all the vertices but every edge is ~~a~~ used only once.

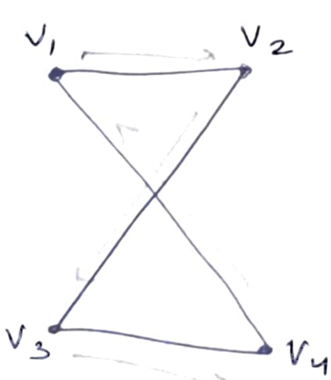
Euler circuit is a cycle where the starting and ending point is same.

eg.



euler path.

$V_1 \rightarrow V_2 \longrightarrow V_3 \longrightarrow V_4$



euther circuit

$V_1 \longrightarrow V_2 \longrightarrow V_3 \longrightarrow V_4 \longrightarrow V_1$

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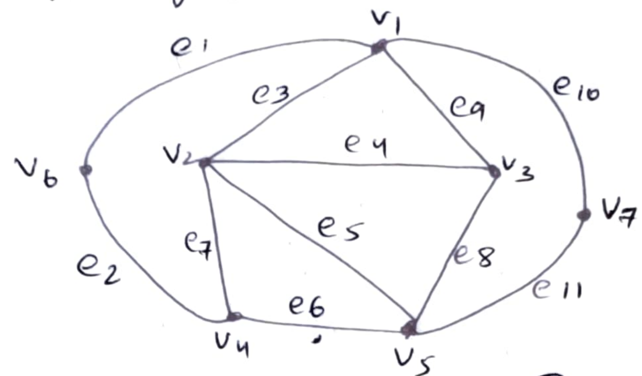
2.

a.) let the graph be G.



We know,

Euler's formula,

$$r = e - v + 2$$

where, r = no. of regions
e = no. of edges.
v = no. of vertices.

$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}\}$

$e = 11$

$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$

$v = 7$

$\therefore$, $r = 11 - 7 + 2$

$= 11 - 5 = 6$

no. of regions = 6.

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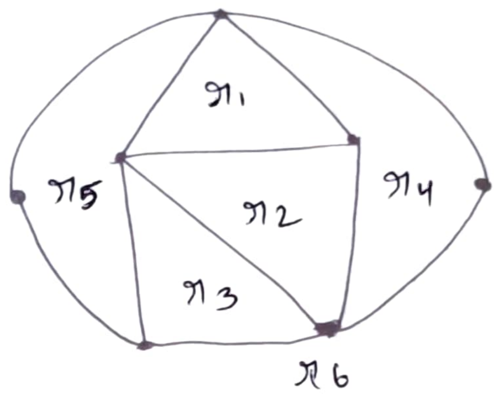
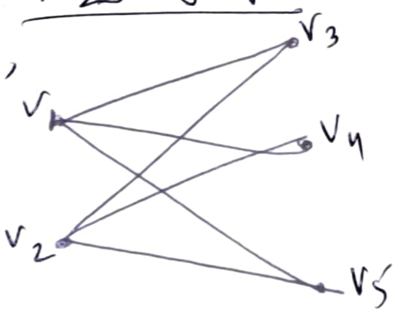


hence, there are 6 regions in graph G.
the graph has 5 finite regions, R_1, R_2, R_3, R_4, R_5 &
1 infinite region. R_6.

2.
b)

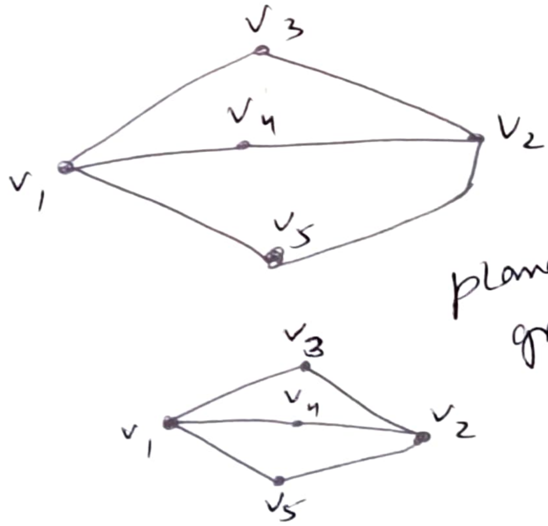
K_{23} graph

as,



complete bipartite graph

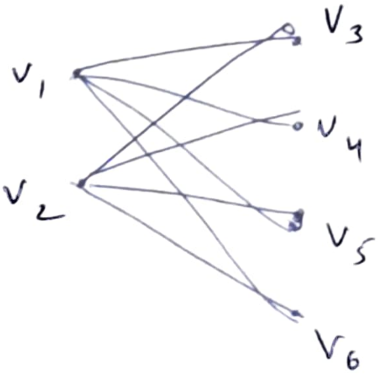
also can be drawn as



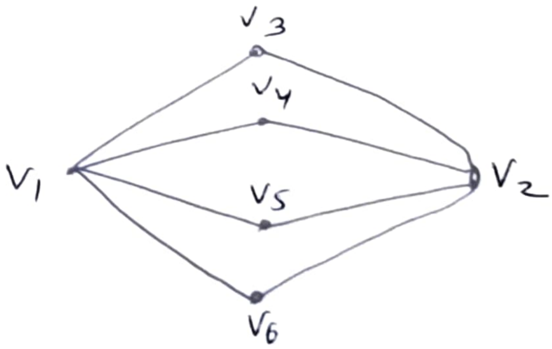
planar graph

K_{24} ~~graph~~ graph.

as complete bipartite graph.



also can be drawn as



planar graph.
where no edge is crossing each other.

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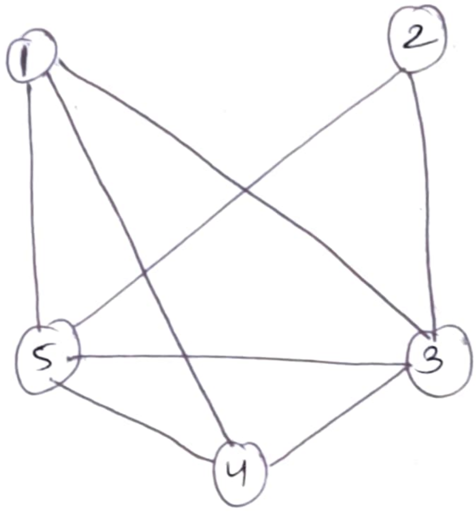
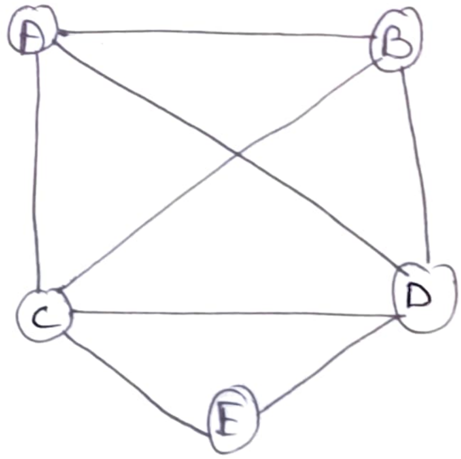
3. let the graph G_1 =



let the graph G_2 =



as we can see, V & E are equal.
mapping vertices from G_1 to G_2,

1 → A
5 → C
4 → B
3 → D
2 → E

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~~So~~, in G_1,

$d(1) = 3$
$d(2) = 2$
$d(3) = 4$
$d(4) = 3$
$d(5) = 4$

in G_2,

$d(A) = 3$
$d(B) = 3$
$d(C) = 4$
$d(D) = 4$
$d(E) = 2$

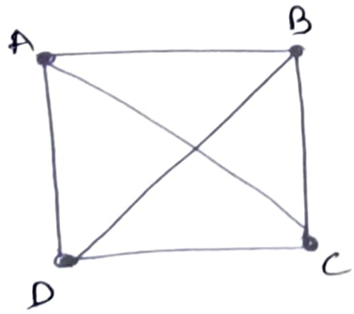
both G_1 & G_2 have 5 vertices & 8 edges.

hence, we can say, G_1 & G_2 are isomorphic.
as we know, two graphs are isomorphic when the mapping of their vertices is a bijection & their exists an edge b/w $f(u)$ & $f(v)$, iff there is an edge b/w $(u), (v)$.

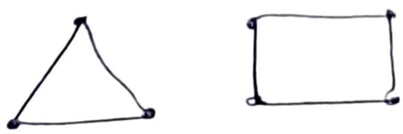
4.
=> **K - Regular graph.**

A graph is called regular graph if degree of each vertex is equal.
A graph is called k-regular if degree of each vertex in the graph is k.

eg.



3 - Regular graph



2 - regular graph.

1 - regular graph.

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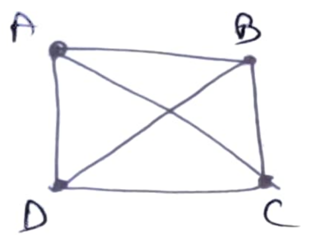
any simple k_n graph is $(k-1)$ regular
a complete graph with n vertices is called k_n graph:
In a complete graph of k vertices, each vertices is connected to all $(k-1)$ remaining vertices, so, degree of each vertex is $(k-1)$, Hence, the graph is $(k-1)$ regular. eg.



all the 4 vertices connected with 3 remaining vertices (all have degree 3)
Hence, it is a k_4 graph as well as 3-Regular graph.

5.

(i) G_2 has 16 edges and all vertices of degree 4

according to Handshake Lemma -

$$\sum d(v_i) = 2E$$

where E = Edges.
$d(v_i)$ = degree of v_i

$$\therefore \quad 4 \cdot V = 16 \times 2$$

$$V = 8 \text{ vertices.}$$

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(ii) G is regular with 15 edges.

we know, $E = \frac{N \times k}{2}$ where E = no. of edges
N = no. of vertices.

$$15 = \frac{N \times K}{2}$$

V, no. of vertices = $\frac{30}{k}$ where $k = 2, 3, 5 \ldots$

(iii) G has 10 edges with 2 vertices of degree 4 & all other vertics of degree 3.

according to Handshake Lemma -

$$\Sigma d(v_i) = 2E$$
$$= 2 \times 10$$

$$2 \times 4 + (v-2) \times 3 = 2 \times 10$$

$$8 + 3V - 6 = 20$$

$$3V + 2 = 20$$
$$V = 18/3$$

V = 6 vertices