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(1)

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Subject: Graph Theory and Optimization.

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CSE - C Section.

TT-2

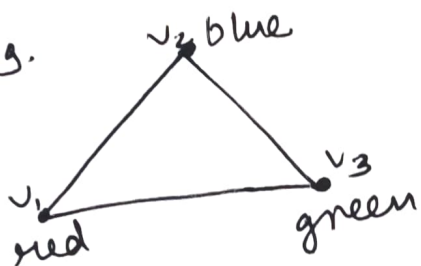
x

1.

a) Chromatic number

A graph which is coloured using the minimum of colours such a graph is called properly coloured graph. and the number of colours in a properly coloured graph is called chromatic number.

e.g.



3-chromatic graph.

chromatic no. = 3

b) Tree

A graph is said to be a tree if it is connected and has no cycle.

eg.



1.

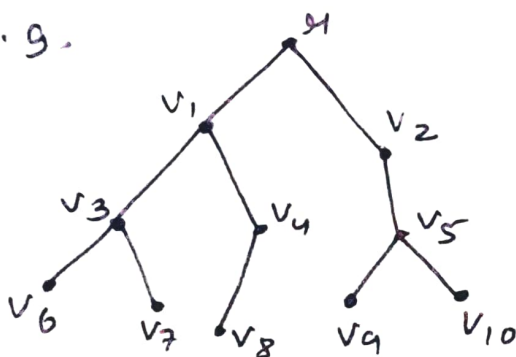
c) Internal vertices of a tree

Q

In a rooted tree a vertex whose out-degree is 0 is called a leaf.

~~a leaf~~ a vertex which is not a leaf is called an internal vertex of the tree.

e.g.



~~Q~~ In above graph,

except $v_6, v_7, v_8, v_9, v_{10}$, all other vertices are internal vertices.

because. $v_6, v_7, v_8, v_9, v_{10}$ are ~~leaf~~ leaves.

Internal vertices are $v_3, v_4, v_5, v_1, v_2, r$.

d) Spanning tree

let G be a connected graph

A subgraph T of G is called a spanning tree of G , if

- (i) T is a tree, and
- (ii) T contains all vertices of G .

e.g. a connected graph can have more than one spanning tree.



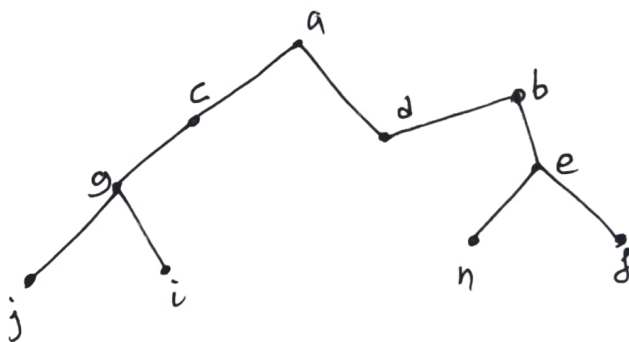
spanning tree of K_3 graph are-



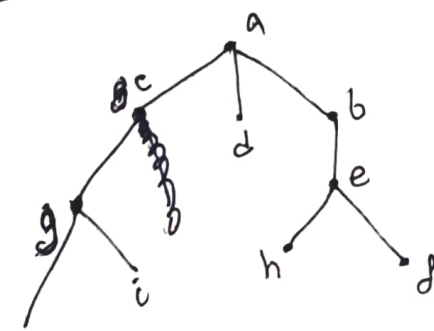
2.

DFS spanning tree.

$a \rightarrow c \rightarrow g \rightarrow j \rightarrow (\text{backtrace}) \rightarrow g \rightarrow i \rightarrow (\text{backtrace}) \rightarrow g \rightarrow$
 $(\text{backtrace}) \rightarrow c \rightarrow (\text{backtrace}) \rightarrow a \rightarrow d \rightarrow b \rightarrow$
 $\rightarrow e \rightarrow (\text{backtrace}) \rightarrow b \rightarrow (\text{backtrace}) \rightarrow d \rightarrow$
 $(\text{backtrace}) \rightarrow a.$



BFS spanning tree.



$a \rightarrow c \rightarrow d \rightarrow b \rightarrow g \rightarrow e \rightarrow j \rightarrow i \rightarrow h \rightarrow f$

5.

Courses are 681, 782, 792, 846, 857, 904, 909, 933

681 - 782, 792, 846, 857, 904, 909, 933.

782 - 681, 792, 857, 904, 909, 933

792 - 681, 782, 933

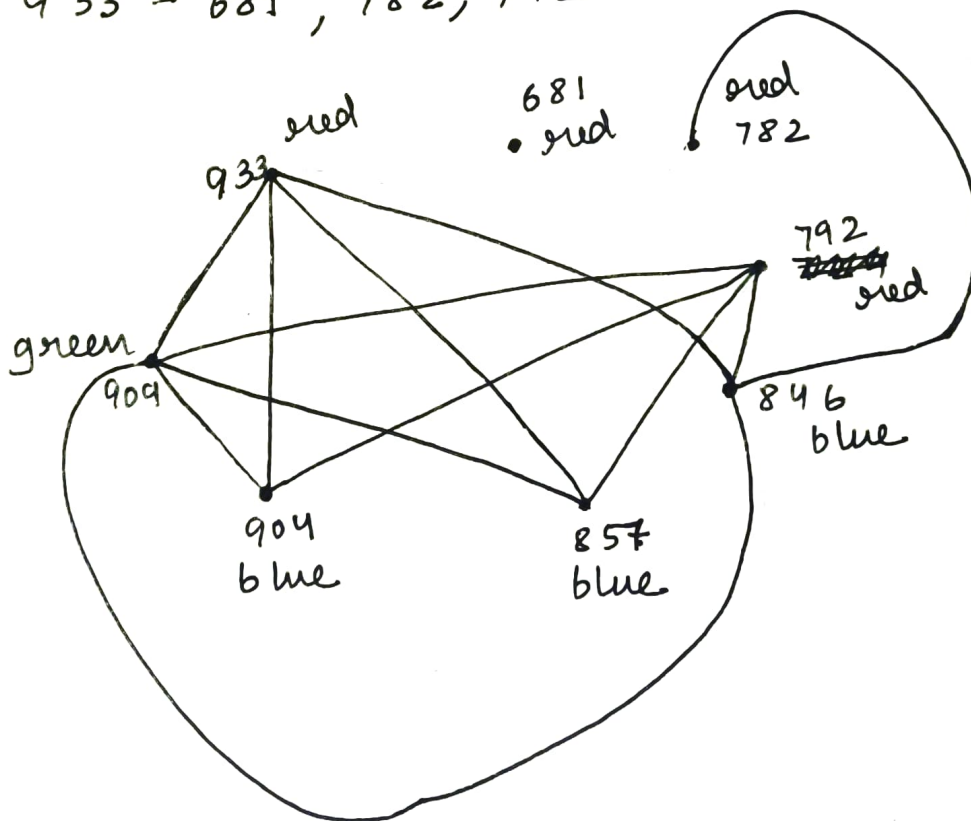
846 - 681, 857, 904,

857 - 681, 782, 846, 904

904 - 681, 782, 846, 857

909 - 681, 782

933 - 681, 782, 792



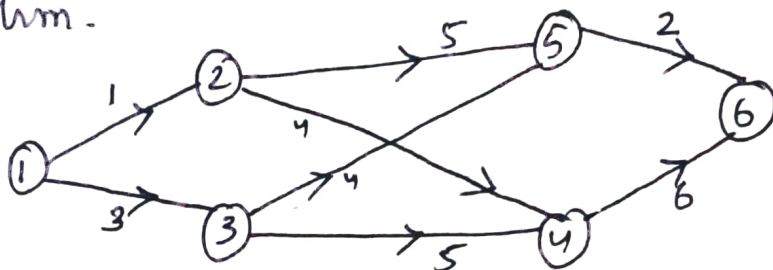
hence, ~~Red~~ Red \rightarrow 681, 782, 792, 933

Blue \rightarrow 846, 857, 904

Green \rightarrow 909

4.

To find the shortest path in the given weighted directed graph, we can use Dijkstra's algorithm.



given, the starting vertex is 1.

Let us consider the initial distance from 1 to all the vertices

	2	3	4	5	6	selected vertex
1		3	∞	∞	∞	2
<u>1</u>		3	5	6	∞	3
<u>1</u>		<u>3</u>	5	6	∞	4
<u>1</u>		<u>3</u>	5 5	6	11	5
<u>1</u>		<u>3</u>	<u>5</u>	<u>6</u>	<u>8</u>	6

on each selected value v , we check if the distance to the v + the cost to the next node is less than the distance taken to ~~the~~ reach the vertex directly from start node s .

if $d(u) > d(v) + c(v, u)$
then, update $d(u)$

∴, the shortest path from 1 are -

$$1 \rightarrow 2 \quad \text{cost} = 1$$

$$1 \rightarrow 3 \quad \text{cost} = 3$$

$$1 \rightarrow 2 \rightarrow 5 \quad \text{cost} = 6$$

$$1 \rightarrow 2 \rightarrow 4 \quad \text{cost} = 5$$

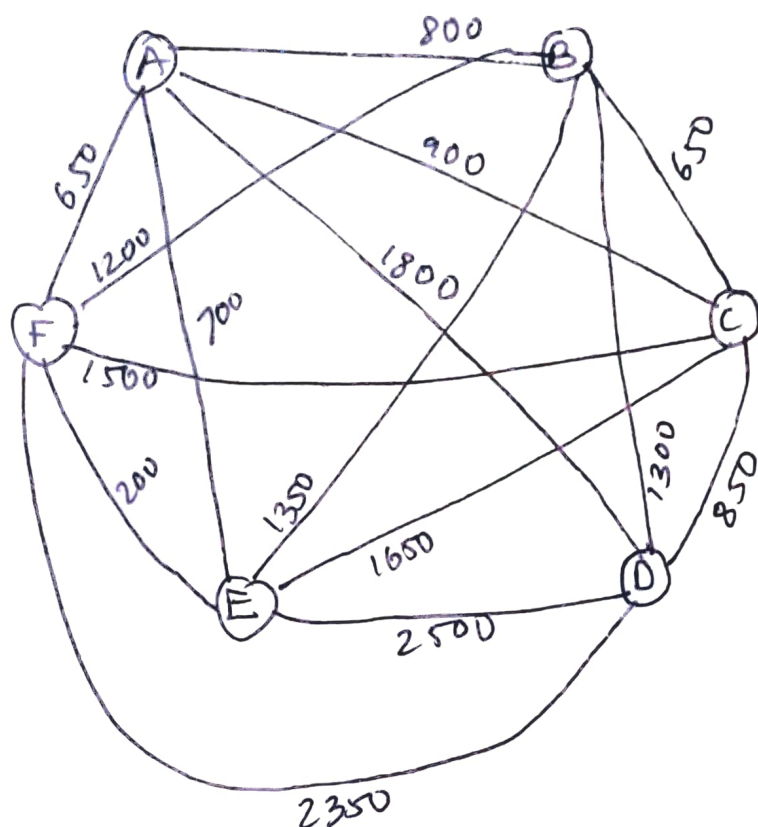
$$1 \rightarrow 2 \rightarrow 5 \rightarrow 6 \quad \text{cost} = 8$$

3. given, the distance b/w all the cities
It is a undirected graph.

To find shortest path covering all the cities

i.e minimum spanning tree.

we can use Prim's algorithm to find the
min. span tree because this is a dense
graph.



Start A,

AB \rightarrow 800

AC \rightarrow 900

AD \rightarrow 1800

AE \rightarrow 700

AF \rightarrow 650 \leftarrow

we choose A \rightarrow F
because it has minimum
cost.

Start A F

FE = 200

AE = 700

we choose
FE, because it has min cost

Start AFE

AB = 800

FB = 1200

EB = 1350

~~we~~ we choose, AB
because it has min cost.

Start A F E B,

$$BC = 650$$

$$AC = 900$$

$$FC = 1500$$

$$EC = 1650$$

~~we choose~~ we choose, BC, because it has min cost

Start, A F E B C

$$AD = 1800$$

$$CD = 850$$

$$FD = 2350$$

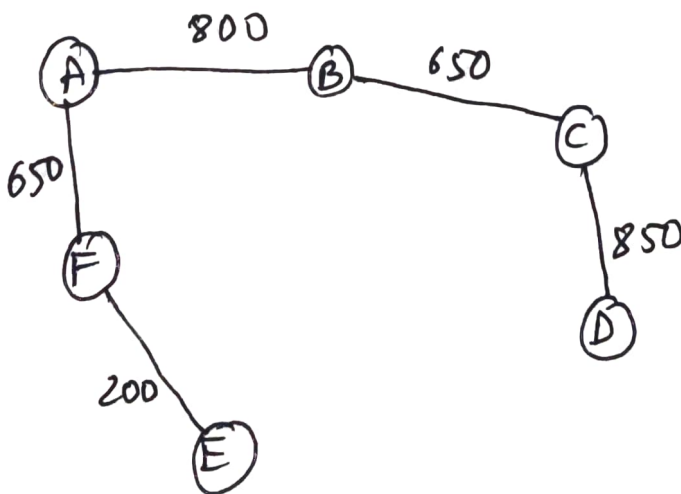
$$ED = 2500$$

$$BD = 1350$$

we choose CD, because, it has min cost.

hence, cost of shortest path is

$$200 + 650 + 800 + 650 + 850 = \underline{\underline{3150}}$$



is the required min spanning tree.