Image Enchantement in Frequency Domain

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CONTENTS

- 2D DFT and Its Inverse,
- Filtering in the Frequency Domain,
- Correspondence between Filtering in the Spatial and Frequency Domains,
- Smoothing Frequency-Domain Filters,
  - Ideal Lowpass Filters,
  - Butterworth Lowpass Filters,
  - Gaussian Lowpass Filters,
  - Additional Examples of Lowpass Filtering,
- Sharpening Frequency Domain Filters,
  - Ideal Highpass Filters,
  - Butterworth Highpass Filters,
  - Gaussian Highpass Filters.

Introduction

- Image enhancement is basically improving the interpretability or perception of information in images for human viewers and providing `better input for other automated image processing techniques.
- The principal objective of image enhancement is to modify attributes of an image to make it more suitable for a given task and a specific observer.
- During this process, one or more attributes of the image are modified.

Image Enhancement

- There exist many techniques that can enhance a digital image without spoiling it.
- The enhancement methods can broadly be divided in to the following two categories:
  - Spatial Domain Methods
  - Frequency Domain Methods

Cont..

- Unfortunately, there is no general theory for determining what is ‘good’ image enhancement when it comes to human perception. If it looks good, it is good.

- However, when image enhancement techniques are used as pre-processing tools for other image processing techniques, then quantitative measures can determine which techniques are most appropriate.

Spatial Domain Enhancements

- In spatial domain techniques , we directly deal with the image pixels.
- The pixel values are manipulated to achieve desired enhancement.
- The value of a pixel with coordinates (x; y) in the enhanced image ‘ F’ is the result of performing some operation on the pixels in the neighborhood of (x; y) in the input image ‘ f’ .

Frequency Domain Enhancements

- In frequency domain methods, the image is 1^{st} transferred into frequency domain.
- It means that, the Fourier Transform of the image is computed 1^{st} .
- All the enhancement operations are performed on the Fourier transform of the image and then the Inverse Fourier transform is performed to get the resultant image.

Cont..

- These enhancement operations are performed in order to modify the image brightness, contrast or the distribution of the grey levels.
- As a consequence the pixel value (intensities) of the output image will be modified according to the transformation function applied on the input values.
- Image enhancement is applied in every field for example medical image analysis, analysis of images from satellites etc.

Cont..

- Image enhancement simply means, transforming an image F into image G using T. (Where T is the transformation function.
- The values of pixels in images F and G are denoted by r and s, respectively. As said, the pixel values r and s are related by the expression

$$s = T(r)$$

- Where T is a transformation that maps a pixel value r into a pixel value s.

Cont..

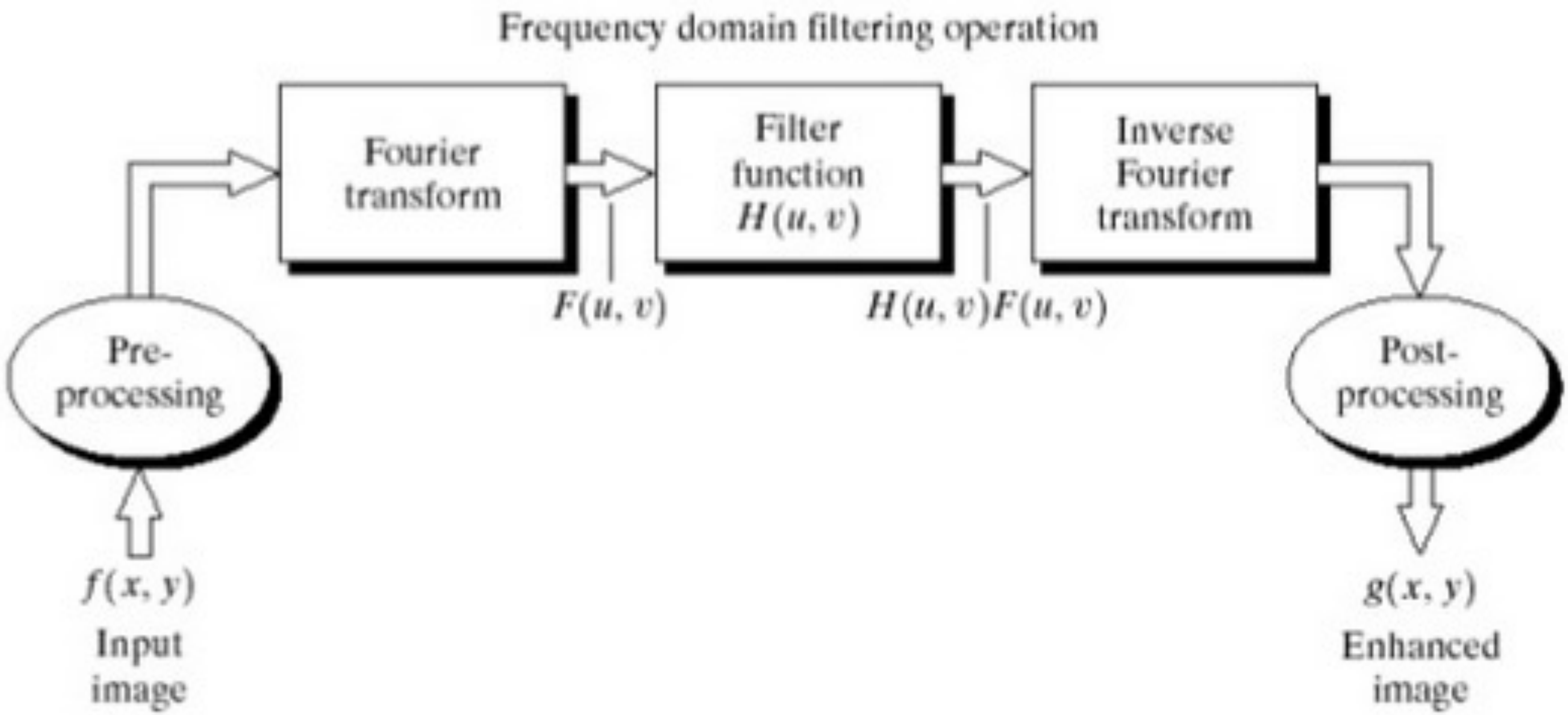


FIGURE 4.5 Basic steps for filtering in the frequency domain.

Cont..

- We can therefore directly design a transfer function and implement the enhancement in the frequency domain as follows.

$$\boxed{\underbrace{G(u,v)}_{\text{Enhanced Image}} = \overbrace{H(u,v)}^{\text{Transfer function}}\underbrace{F(u,v)}_{\text{Given Image}}}$$

Cont..

- The concept of filtering is easier to visualize in the frequency domain.

- Therefore, enhancement of image can be done in the frequency domain, based on its DFT.

2-D Discrete Fourier Transform and Its Inverse

DFT:

$$F(\mu,\nu) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(\mu x/M+\nu y/N)}$$

$\mu = 0,1,2,\ldots,M-1; \nu = 0,1,2,\ldots,N-1;$

$f(x,y)$ is a digital image of size M $\times$ N.

IDFT:

$$f(x,y) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(\mu,\nu)e^{j2\pi(\mu x/M+\nu y/N)}$$

Properties of the 2-D DFT

relationships between spatial and frequency intervals

Let ΔT and ΔZ denote the separations between samples, then the seperations between the corresponding discrete, frequency domain variables are given by

$$\Delta\mu = \frac{1}{M\Delta T}$$

and

$$\Delta\nu = \frac{1}{N\Delta Z}$$

Properties of the 2-D DFT

translation and rotation

$$f(x,y)e^{j2\pi(\mu_0 x/M+\nu_0 y/N)} \Leftrightarrow F(\mu-\mu_0,\nu-\nu_0)$$

and

$$f(x-x_0,y-y_0) \Leftrightarrow F(\mu,\nu)e^{-j2\pi(\mu x_0/M+\nu y_0/N)}$$

Using the polar coordinates

$x=r\cos\theta \quad y=r\sin\theta \quad \mu=\omega\cos\varphi \quad \nu=\omega\sin\varphi$

results in the following transform pair:

$$f(r,\theta+\theta_0) \Leftrightarrow F(\omega,\varphi+\theta_0)$$

Properties of the 2-D DFT

periodicity

$2-D$ Fourier transform and its inverse are infinitely periodic

$$F(\mu,\nu)=F(\mu+k_1M,\nu)=F(\mu,\nu+k_2N)=F(\mu+k_1M,\nu+k_2N)$$

$$f(x,y)=f(x+k_1M,y)=f(x,y+k_2N)=f(x+k_1M,y+k_2N)$$

$$f(x)e^{j2\pi(\mu_0 x/M)}\Leftrightarrow F(\mu-\mu_0)$$

$$\mu_0=M/2,\quad f(x)(-1)^x\Leftrightarrow F(\mu-M/2)$$

$$f(x,y)(-1)^{x+y}\Leftrightarrow F(\mu-M/2,\nu-N/2)$$

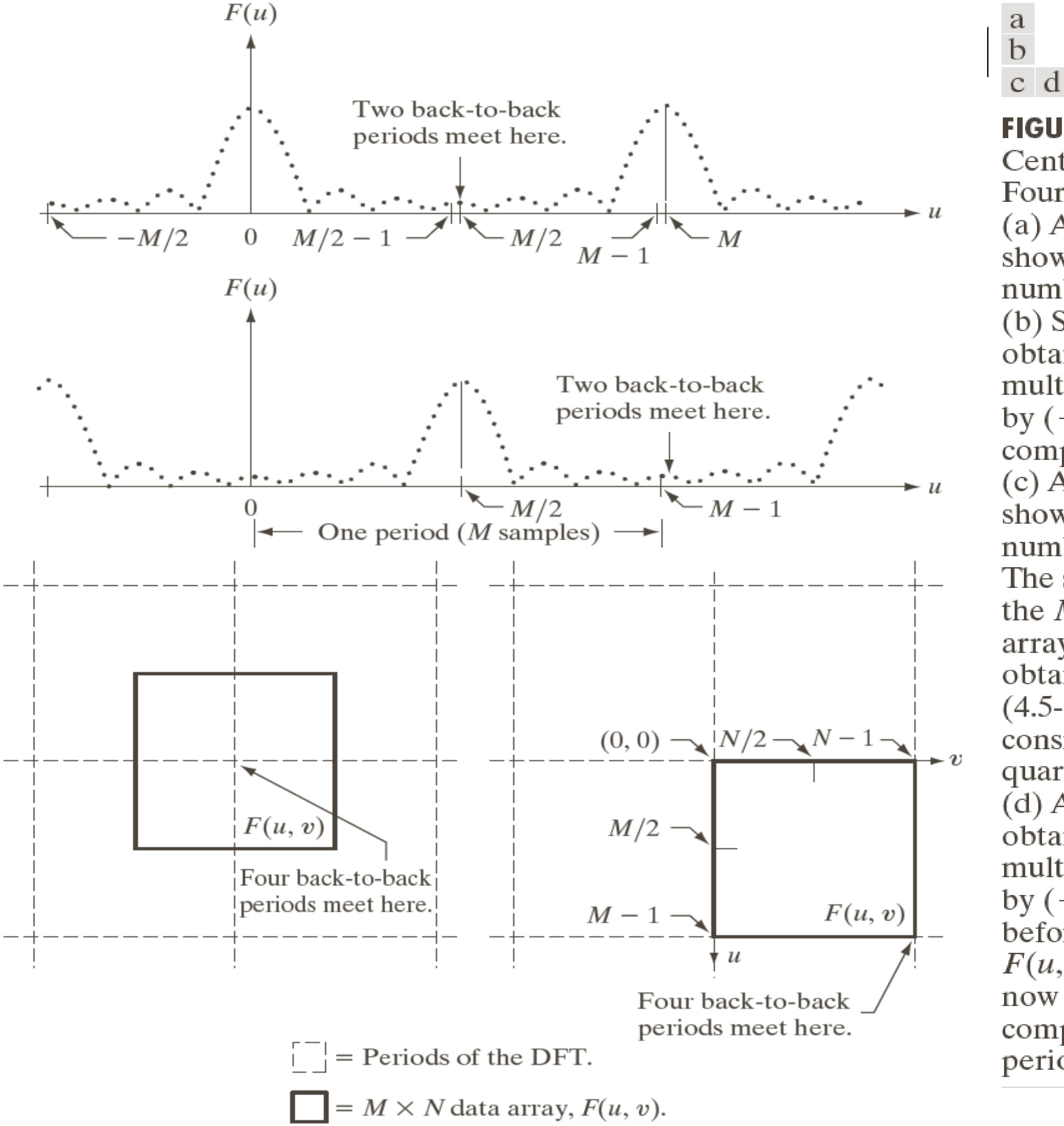


FIGURE 4.23 Centering the Fourier transform. (a) A 1-D DFT showing an infinite number of periods. (b) Shifted DFT obtained by multiplying $f(x)$ by $(-1)^x$ before computing $F(u)$. (c) A 2-D DFT showing an infinite number of periods. The solid area is the $M \times N$ data array, $F(u, v)$, obtained with Eq. (4.5-15). This array consists of four quarter periods. (d) A Shifted DFT obtained by multiplying $f(x, y)$ by $(-1)^{x+y}$ before computing $F(u, v)$. The data now contains one complete, centered period, as in (b).

Properties of the 2-D DFT

Symmetry

	Spatial Domain[†]		Frequency Domain[†]
1)	$f(x, y)$ real	$\Leftrightarrow$	$F^*(u, v) = F(-u, -v)$
2)	$f(x, y)$ imaginary	$\Leftrightarrow$	$F^*(-u, -v) = -F(u, v)$
3)	$f(x, y)$ real	$\Leftrightarrow$	$R(u, v)$ even; $I(u, v)$ odd
4)	$f(x, y)$ imaginary	$\Leftrightarrow$	$R(u, v)$ odd; $I(u, v)$ even
5)	$f(-x, -y)$ real	$\Leftrightarrow$	$F^*(u, v)$ complex
6)	$f(-x, -y)$ complex	$\Leftrightarrow$	$F(-u, -v)$ complex
7)	$f^*(x, y)$ complex	$\Leftrightarrow$	$F^*(-u - v)$ complex
8)	$f(x, y)$ real and even	$\Leftrightarrow$	$F(u, v)$ real and even
9)	$f(x, y)$ real and odd	$\Leftrightarrow$	$F(u, v)$ imaginary and odd
10)	$f(x, y)$ imaginary and even	$\Leftrightarrow$	$F(u, v)$ imaginary and even
11)	$f(x, y)$ imaginary and odd	$\Leftrightarrow$	$F(u, v)$ real and odd
12)	$f(x, y)$ complex and even	$\Leftrightarrow$	$F(u, v)$ complex and even
13)	$f(x, y)$ complex and odd	$\Leftrightarrow$	$F(u, v)$ complex and odd

[†]Recall that x, y, u, and v are *discrete* (integer) variables, with x and u in the range $[0, M - 1]$, and y, and v in the range $[0, N - 1]$. To say that a complex function is *even* means that its real *and* imaginary parts are even, and similarly for an odd complex function.

TABLE 4.1 Some symmetry properties of the 2-D DFT and its inverse. $R(u, v)$ and $I(u, v)$ are the real and imaginary parts of $F(u, v)$, respectively. The term *complex* indicates that a function has nonzero real and imaginary parts.

Properties of the 2-D DFT

Fourier Spectrum and Phase Angle

2-D DFT in polar form

$$F(u,v) = |F(u,v)| e^{j\phi(u,v)}$$

Fourier spectrum

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v) \right]^{1/2}$$

Power spectrum

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

Phase angle

$$\phi(\mathrm{u,v}) = \arctan\left[\frac{I(u,v)}{R(u,v)} \right]$$

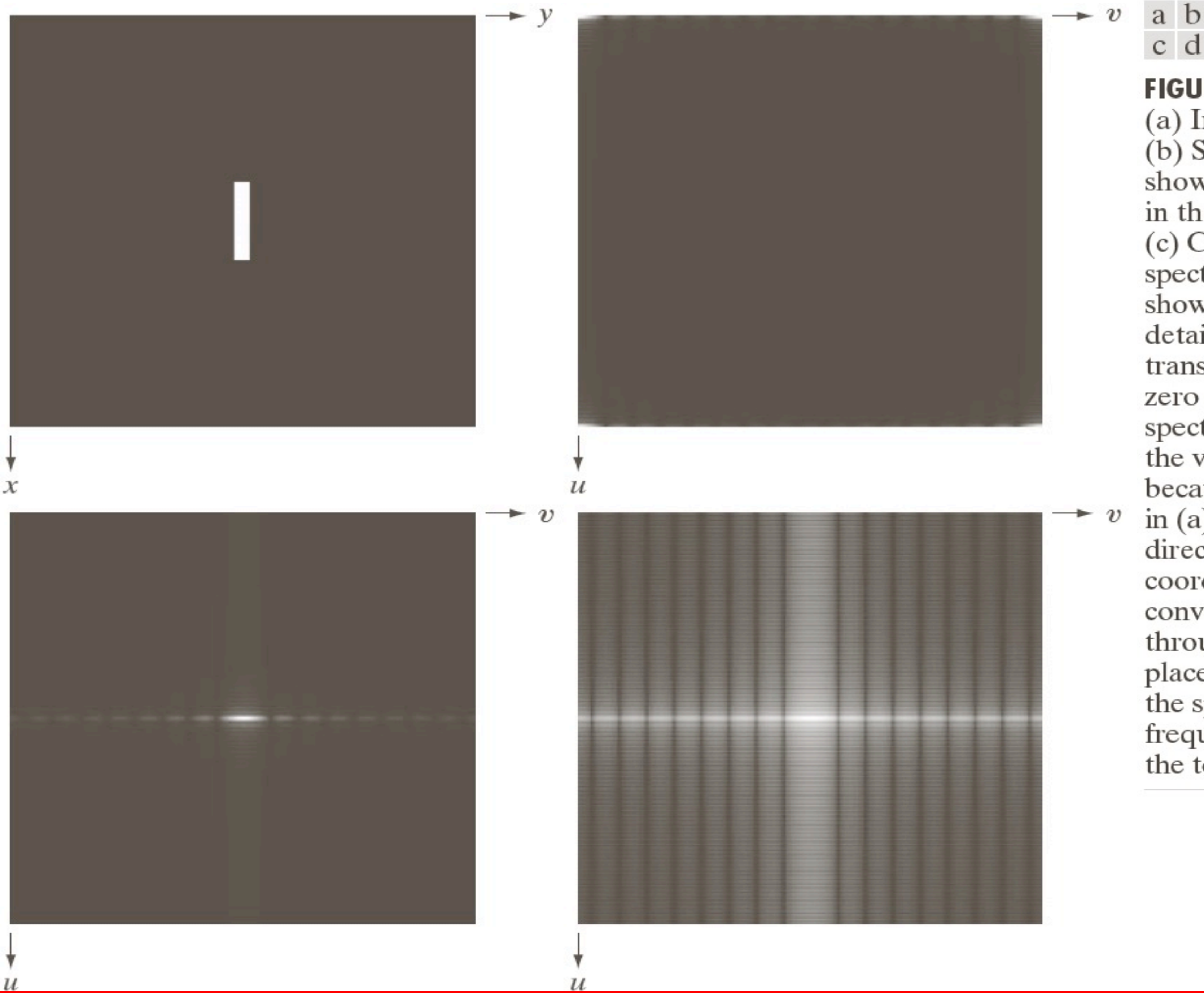
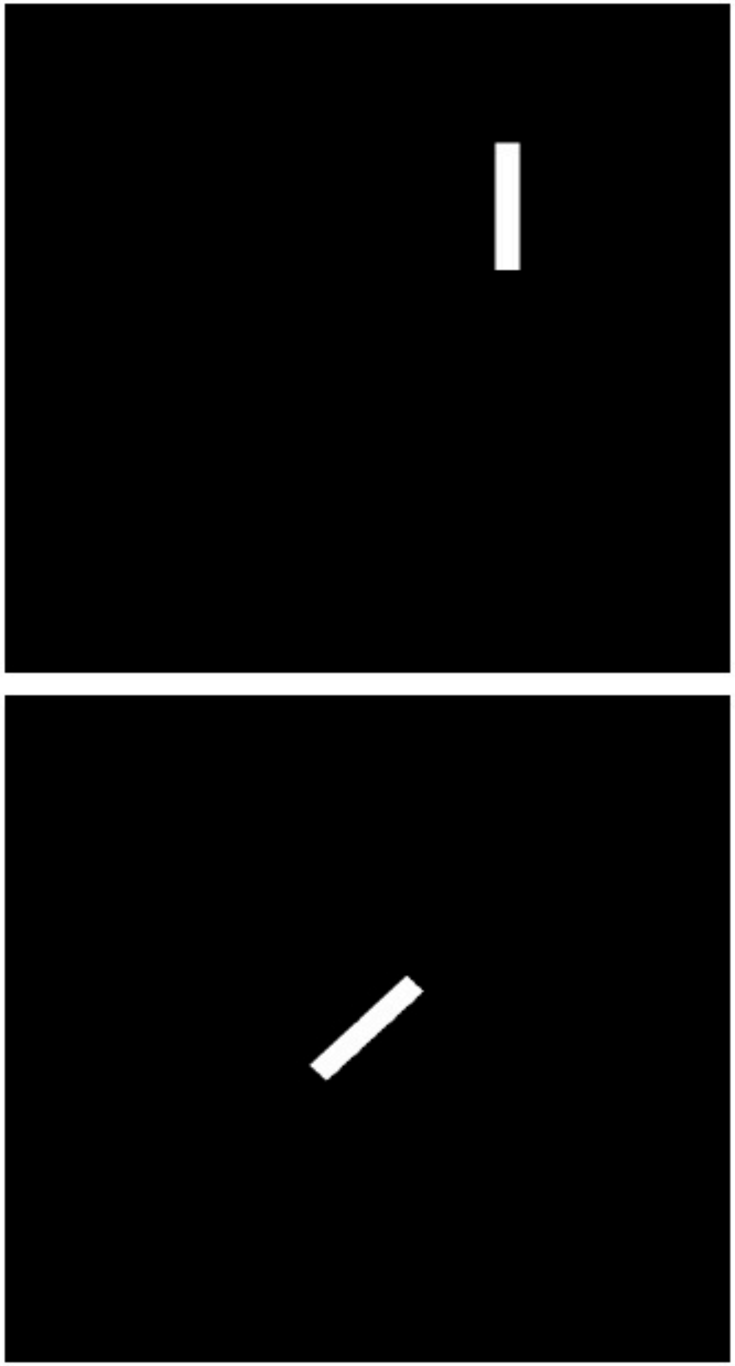


FIGURE 4.24
(a) Image. (b) Spectrum showing bright spots in the four corners. (c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.



a b
c d

FIGURE 4.25
(a) The rectangle in Fig. 4.24(a) translated, and (b) the corresponding spectrum. (c) Rotated rectangle, and (d) the corresponding spectrum. The spectrum corresponding to the translated rectangle is identical to the spectrum corresponding to the original image in Fig. 4.24(a).

Example: Phase Angles

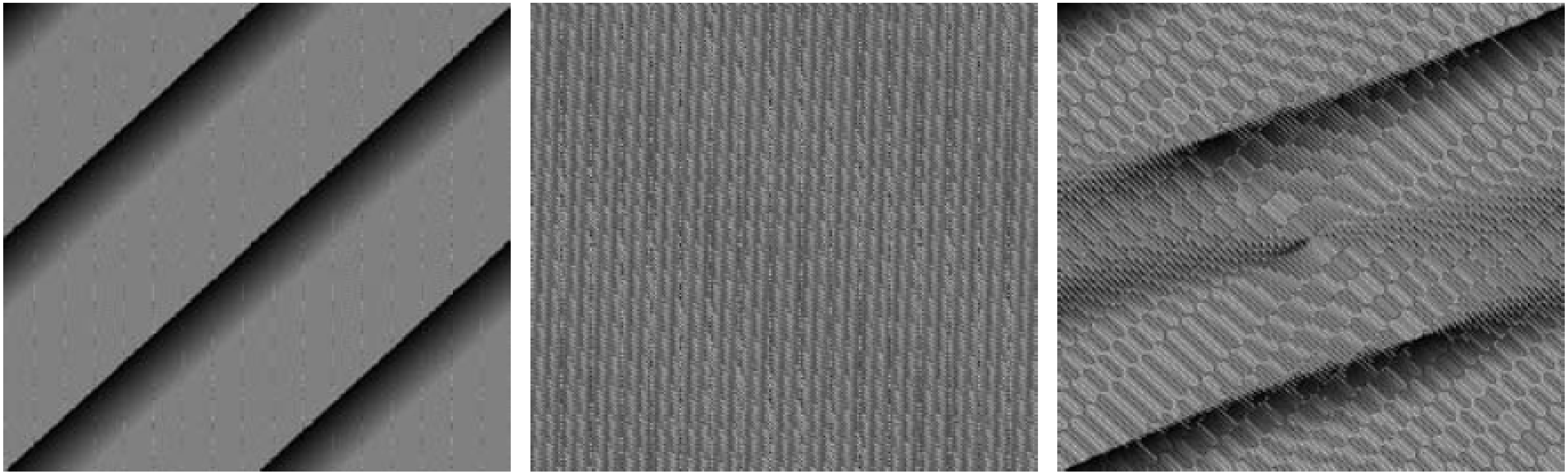


FIGURE 4.26 Phase angle array corresponding (a) to the image of the centered rectangle in Fig. 4.24(a), (b) to the translated image in Fig. 4.25(a), and (c) to the rotated image in Fig. 4.25(c).

Cont..

- Filtering can be divided in two categories namely
- Low pass filtering
- High Pass filtering

Cont..

- Edges and sharp transitions in gray values in an image contribute significantly to high-frequency content of its Fourier transform.
- Regions of relatively uniform gray values in an image contribute to low-frequency content of its Fourier transform.

Cont..

- Edges and sharp transitions in gray values in an image contribute significantly to high-frequency content of its Fourier transform.
- Regions of relatively uniform gray values in an image contribute to low-frequency content of its Fourier transform.

Cont..

- Hence, an image can be smoothed in the Frequency domain by attenuating the high-frequency content of its Fourier transform.

- This would be a low pass filter.

- For simplicity, we will consider only those filters that are real and symmetric.

  – Low pass (LP) filters – only pass the low frequencies, drop the high ones

  – High-pass (HP) filters – only pass the frequencies above a minimum value

Basics of filtering in the frequency domain

1. multiply the input image by $(-1)^{x+y}$ to center the transform to u = M/2 and v = N/2 (if M and N are even numbers, then the shifted coordinates will be integers)
2. computer F(u,v), the DFT of the image from (1)
3. multiply F(u,v) by a filter function H(u,v)
4. compute the inverse DFT of the result in (3)
5. obtain the real part of the result in (4)
6. multiply the result in (5) by $(-1)^{x+y}$ to cancel the multiplication of the input image.

Image Smoothing Using Filter Domain Filters: ILPF

Ideal Lowpass Filters (ILPF)

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

D_0 is a positive constant and $D(u,v)$ is the distance between a point (u,v) in the frequency domain and the center of the frequency rectangle

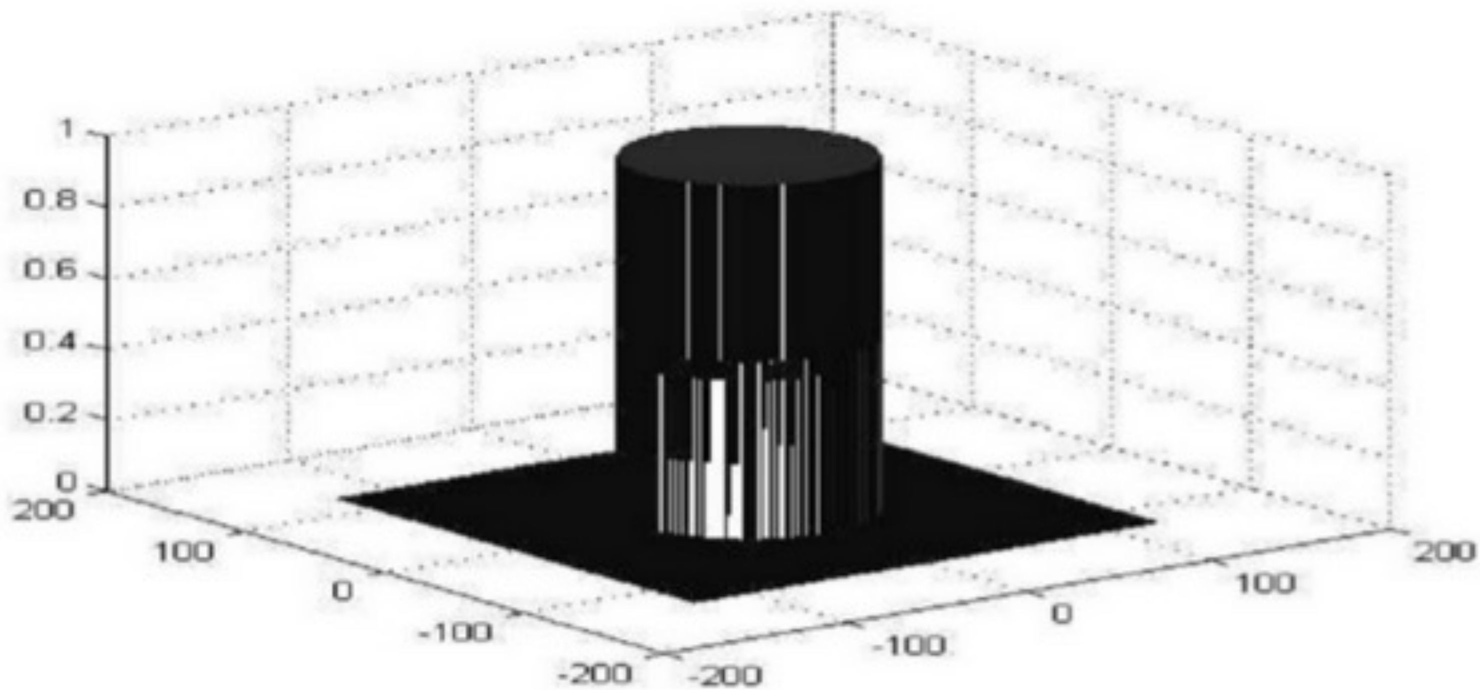
$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$$

Ideal Lowpass Filter

- An ideal low pass filter with cutoff frequency ' r_0' is given by following relation.

$$H(u,v)=\begin{cases}1, & \text{if } \sqrt{u^2+v^2} \le r_0 \\ 0, & \text{if } \sqrt{u^2+v^2} > r_0\end{cases}$$

Ideal Lowpass Filter



Ideal LPF with $r_0 = 57$

Example

Original Image

LPF image, $r_0 = 57$

Example

LPF image, $r_0 = 36$

LPF image, $r_0 = 26$

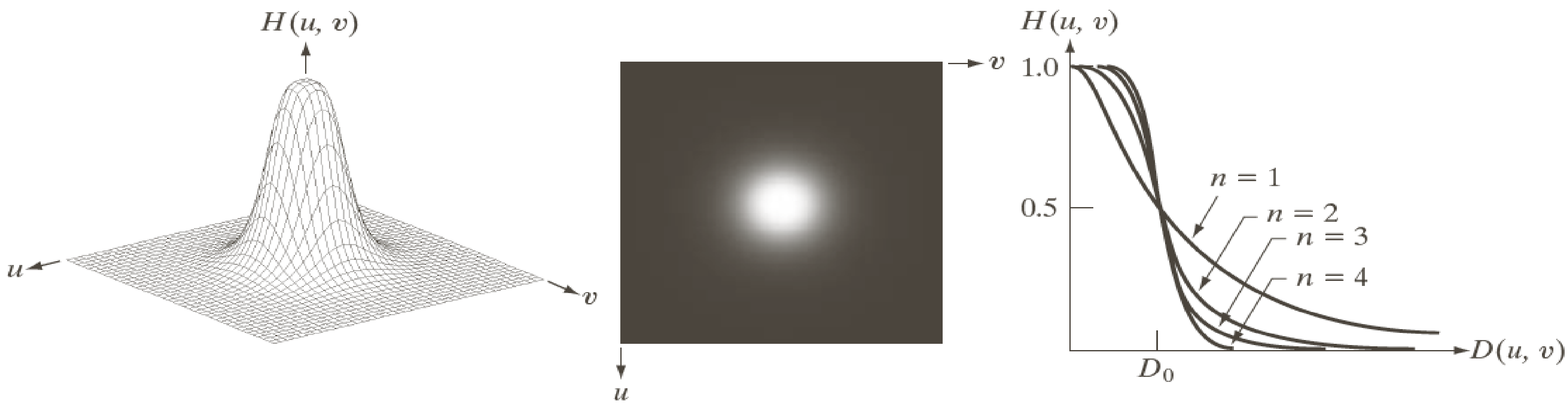
Ideal Lowpass Filter

- The cutoff-frequency of the ideal LPF determines the amount of frequency components passed by the filter.
- Smaller the value of r_0, more the number of image components eliminated by the filter.
- In general, the value of r_0 is chosen such that most components of interest are passed through, while most components not of interest are eliminated.

Image Smoothing Using Filter Domain Filters: BLPF

Butterworth Lowpass Filters (BLPF) of order n and with cutoff frequency D_0

$$H(u,v) = \frac{1}{1+\left[D(u,v)/D_0\right]^{2n}}$$



a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

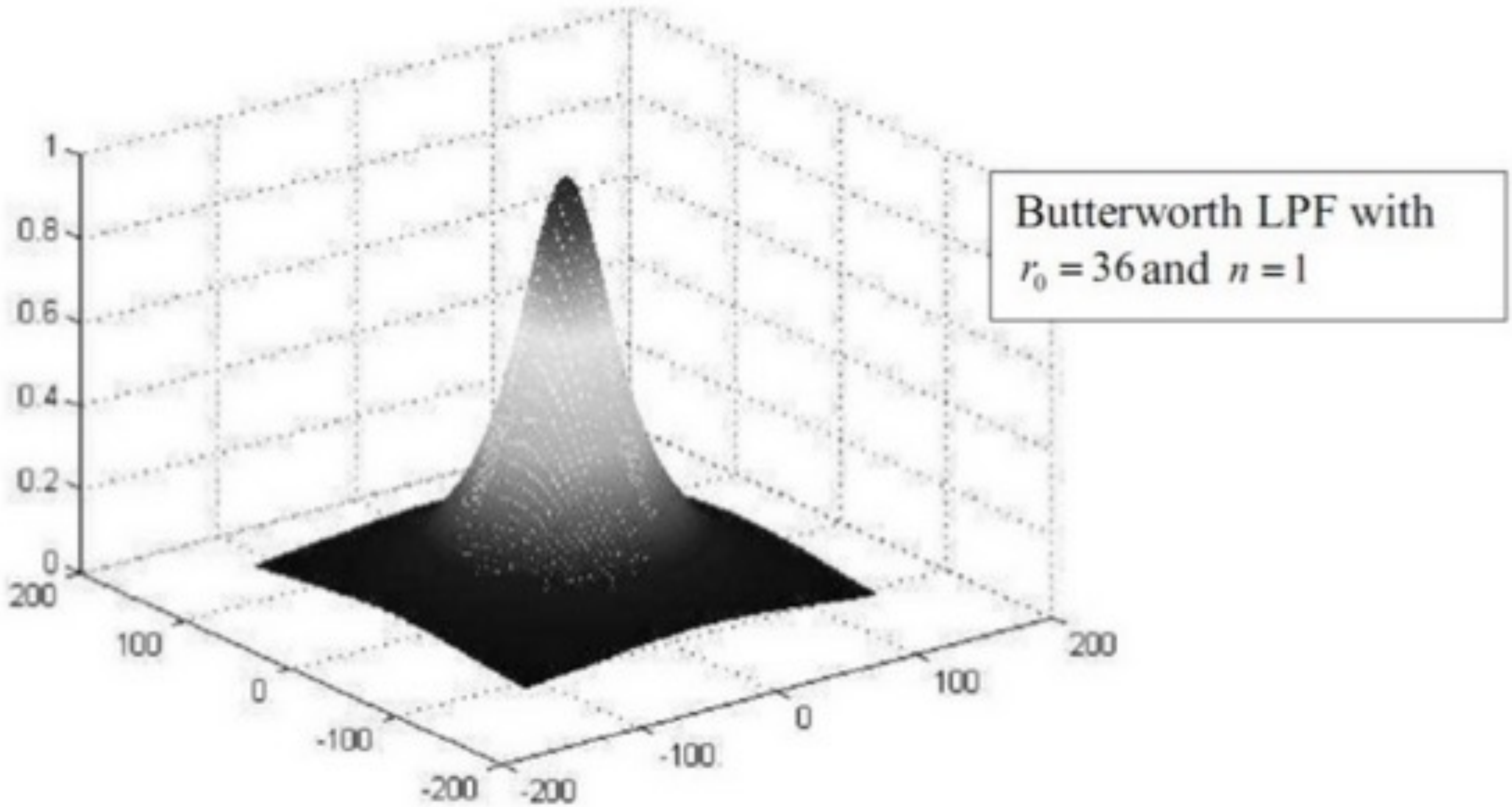
Butterworth Lowpass Filters

- A two-dimensional Butterworth low pass filter has transfer function:

$$H(u,v) = \frac{1}{1 + \left[\frac{\sqrt{u^2 + v^2}}{r_0}\right]^{2n}}$$

n : filter order, r_0 : cutoff-frequency

Example



1
0.8
0.6
0.4
0.2
0
200
100
0
-100
-200
-200
-100
0
100
200
Butterworth LPF with
$r_0 = 36$ and $n = 1$

Butterworth Lowpass Filters

- Frequency response does not have a sharp transition as in the ideal LPF.

- This is more appropriate for image smoothing than the ideal LPF, since this not introduce ringing.

Example

Original Image

LPF image, $r_0 = 18$

Example

LPF image, $r_0 = 13$

LPF image, $r_0 = 10$

Image Smoothing Using Filter Domain Filters: GLPF

Gaussian Lowpass Filters (GLPF) in two dimensions is given

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

By letting $\sigma = D_0$

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

Gaussian Lowpass Filters

- The form of a Gaussian low pass filter in 2- D is given by

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2} \qquad D(u,v) = \sqrt{u^2 + v^2}$$

  - Where D is the distance from origin in frequency plane

- The parameter σ measures the dispersion of the Gaussian curve. Larger the value of σ, larger the cutoff frequency and milder the filtering.

Image Sharpening Using Frequency Domain Filters

A highpass filter is obtained from a given lowpass filter using

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

A 2-D ideal highpass filter (IHPL) is defined as

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Highpass Filters

- Hence, an image can be smoothed in the Frequency domain by attenuating the low-frequency content of its Fourier transform.
- This would be a high pass filter.
- For simplicity, we will consider only those filters that are real and symmetric

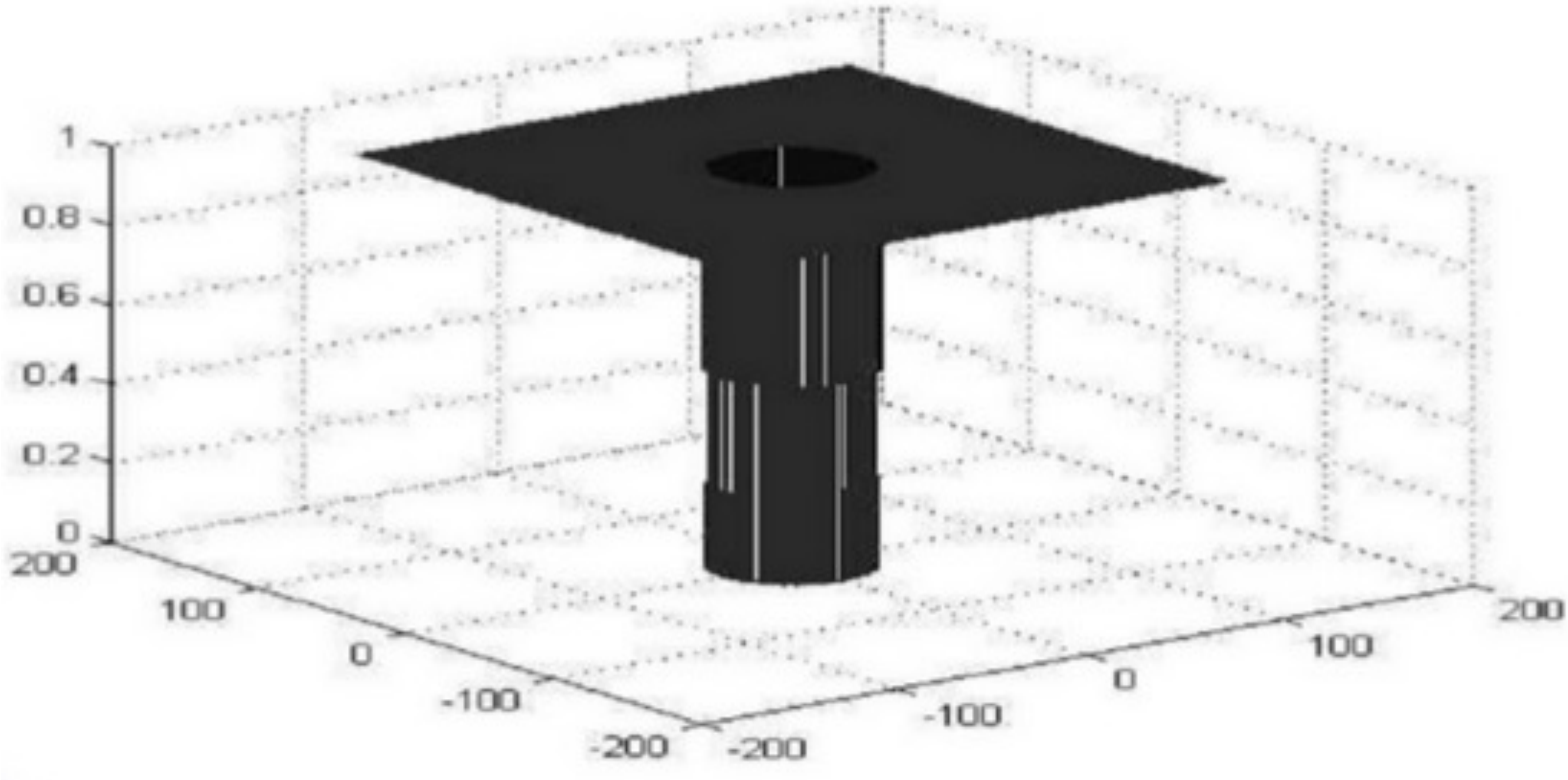
Ideal Highpass Filters

- An ideal high pass filter with cutoff frequency r_0.

$$H(u,v) = \begin{cases} 0, & \text{if } \sqrt{u^2 + v^2} \le r_0 \\ 1, & \text{if } \sqrt{u^2 + v^2} > r_0 \end{cases}$$

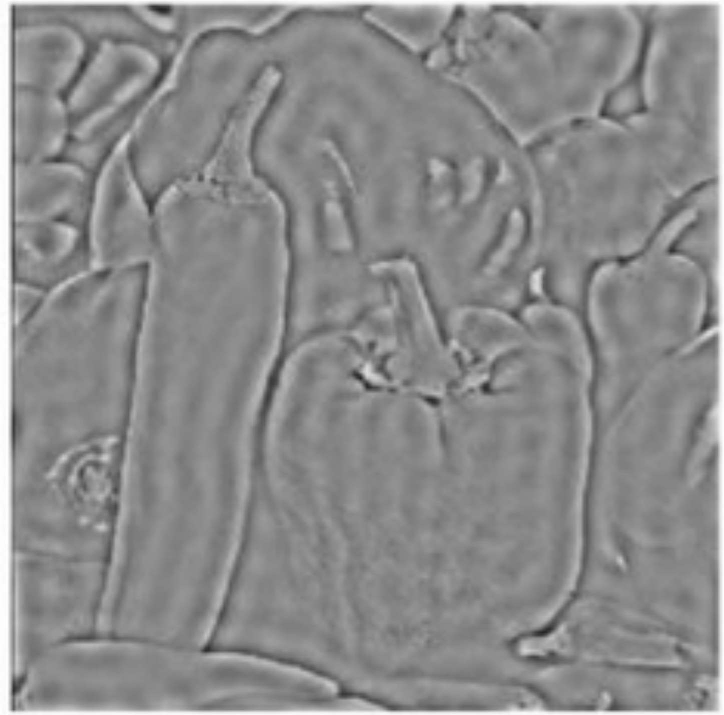
Example

- Ideal HPF with cutoff frequency r_0=36



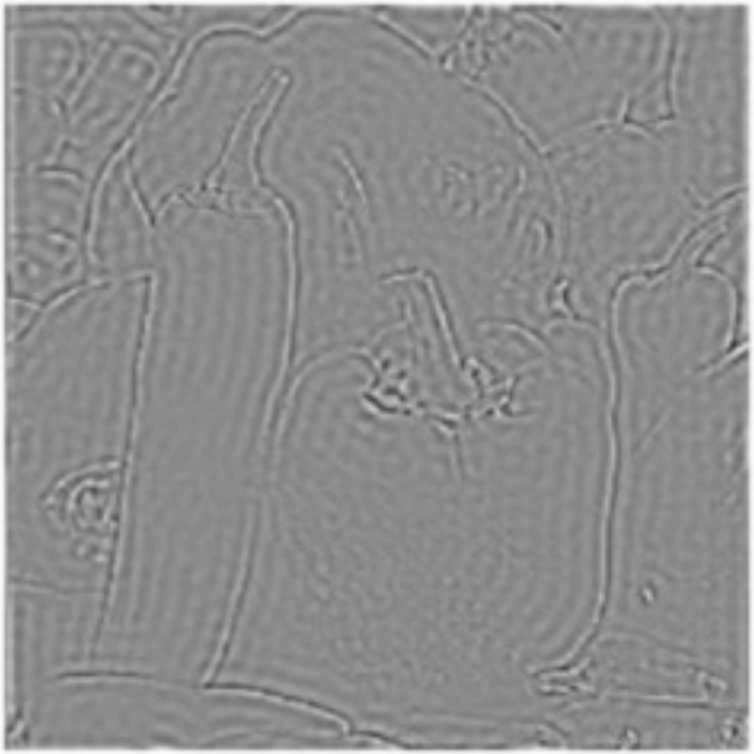
Example

Original Image

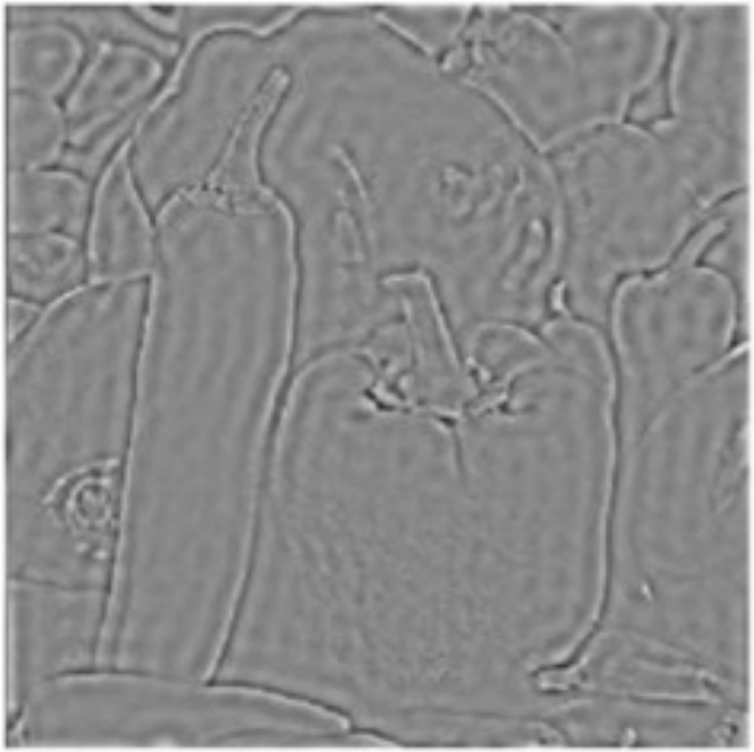


HPF image, $r_0 = 18$

Example



HPF image, $r_0 = 36$



HPF image, $r_0 = 26$

Butterworth Highpass Filters

A 2-D Butterworth highpass filter (BHPL) is defined as

$$H(u,v) = \frac{1}{1+\left[D_0 / D(u,v)\right]^{2n}}$$

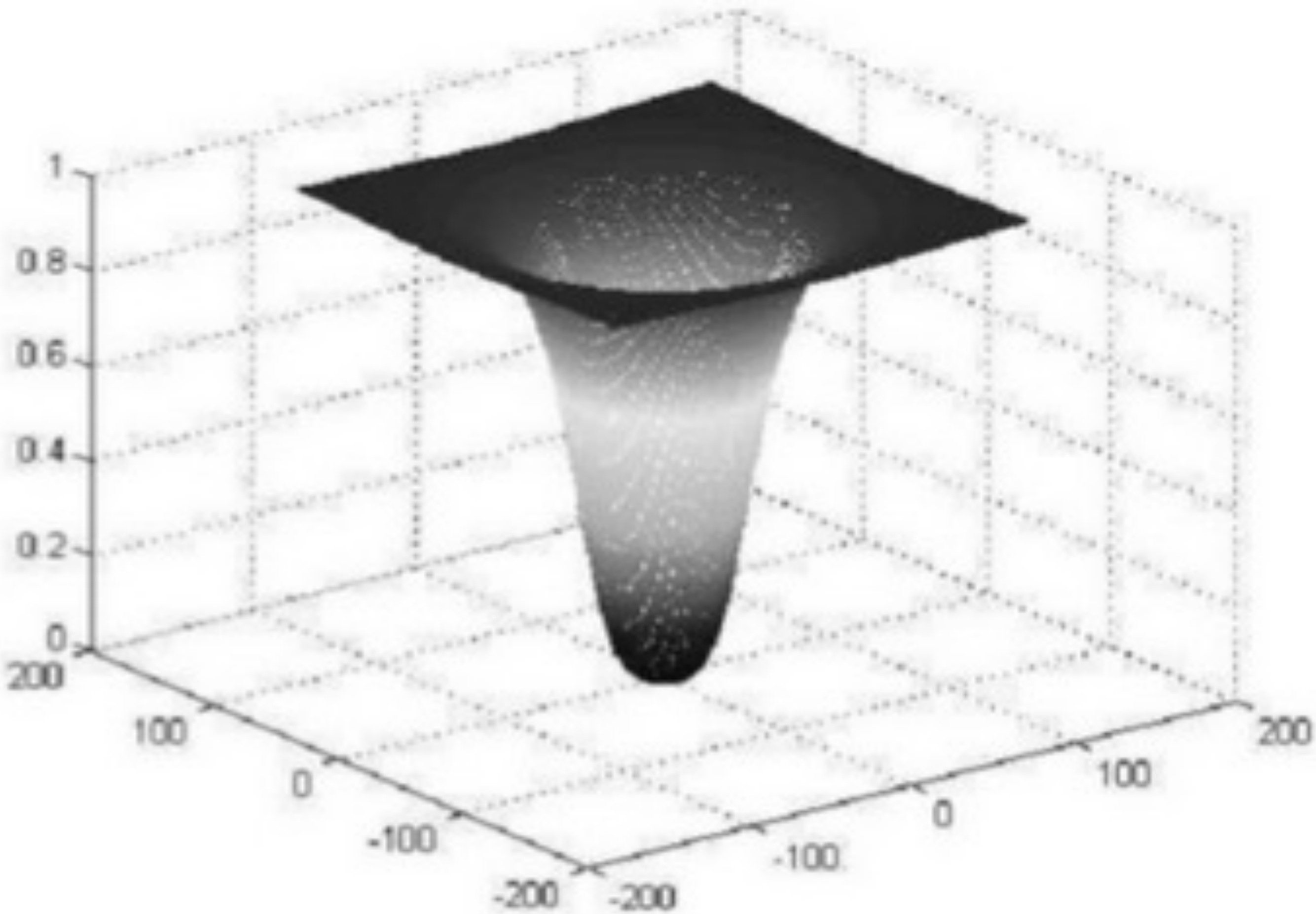
- A two-dimensional Butterworth high pass filter has transfer function:

$$H(u,v) = \frac{1}{1+\left[\dfrac{r_0}{\sqrt{u^2+v^2}}\right]^{2n}}$$

- n : filter order, r_0 : cutoff-frequency

Example

- Butterworth HPF with cutoff frequency r_0=47, order, n = 2

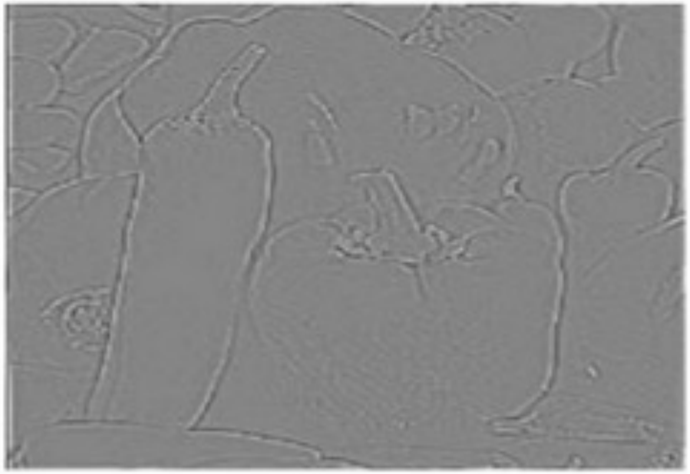


Butterworth Highpass Filters

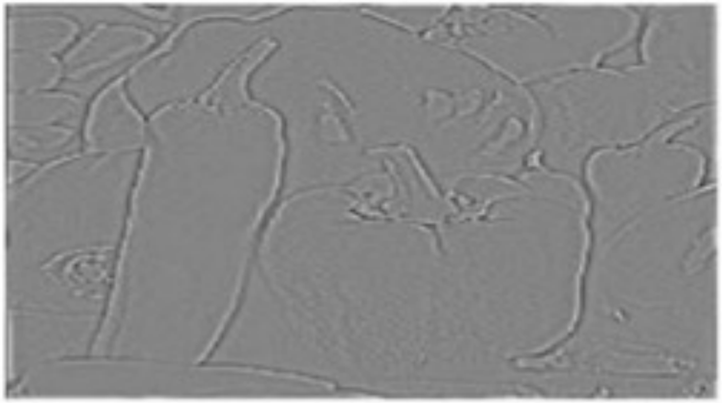
- Frequency response does not have a sharp transition as in the ideal HPF.
- This is more appropriate for image sharpening than the ideal HPF, since this not introduce ringing.
- Example of Butterworth high pass filtering is given on next slide.

Example

Original Image



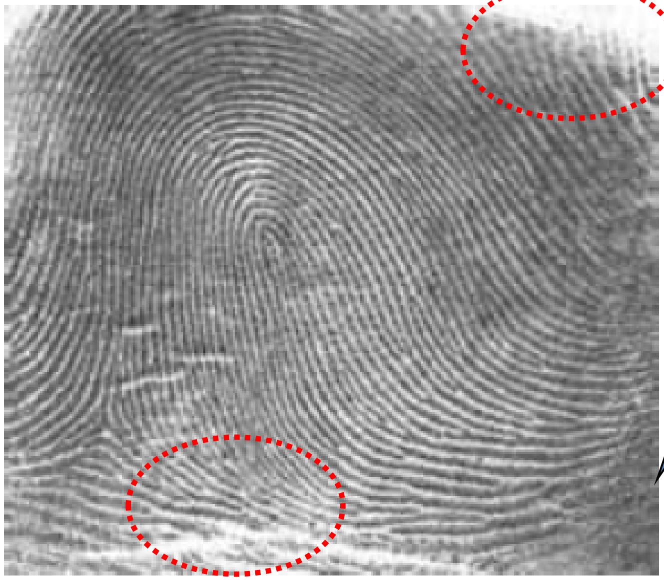
HPF image, $r_0 = 47$



HPF image, $r_0 = 36$

HPF image, $r_0 = 81$

Using Highpass Filtering and Threshold for Image Enhancement



BHPF
(order 4 with a cutoff frequency 50)

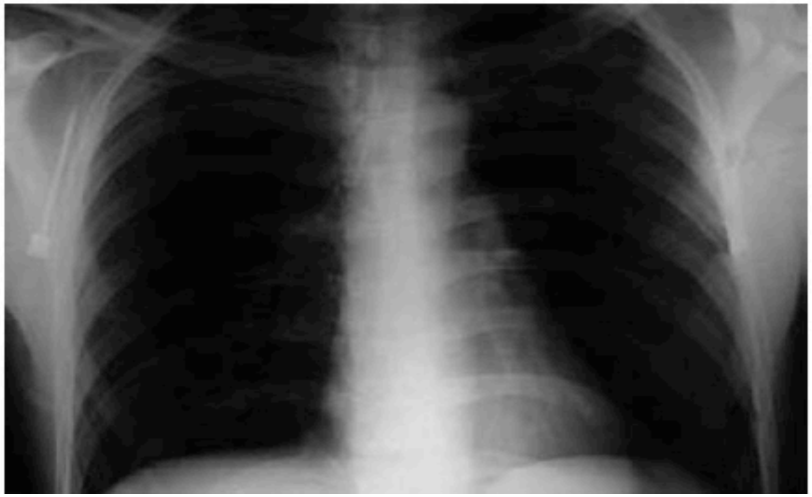
a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Gaussian Highpass Filters

A 2-D Gaussian highpass filter (GHPL) is defined as

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$



Gaussian Filter
D_0=40

High-Frequency-Emphasis Filtering
Gaussian Filter
K1=0.5, k2=0.75

a	b
c	d

FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

Filtering Results by IHPF

a b c

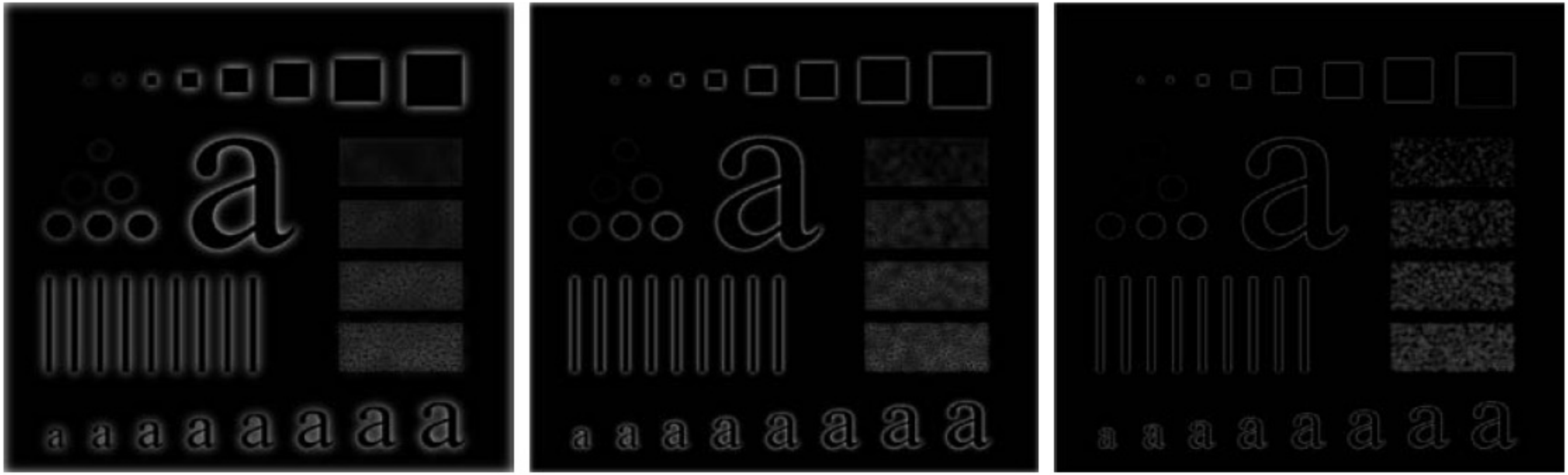
FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60,$ and 160.

Filtering Results by BHPF

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Filtering Results by GHPF



a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

Conclusion

- Image enhancement algorithms offer a wide variety of approaches for modifying images to achieve visually acceptable images.
- The choice of such techniques is a function of the specific task, image content, observer characteristics, and viewing conditions.