Image Restoration

Delivered by

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Introduction to Image Restoration

At the end of this session, student will be able to

- Describe the process of image restoration
- Identify the noise models used for degrading an image
- Implement image restoration using standard software

Session Topics

- Image Restoration
- A Model of the Image Degradation/Restoration Process
- Noise Models
  - Spatial and Frequency Properties of Noise
  - Some Important Noise Probability Density Functions
  - Periodic Noise
- Restoration in Presence of Noise only Spatial Filter
  - Mean Filters
    - Arithmetic Mean Filter
    - Geometric Mean Filter
    - Harmonic Mean Filter
    - Contra-harmonic Mean Filter

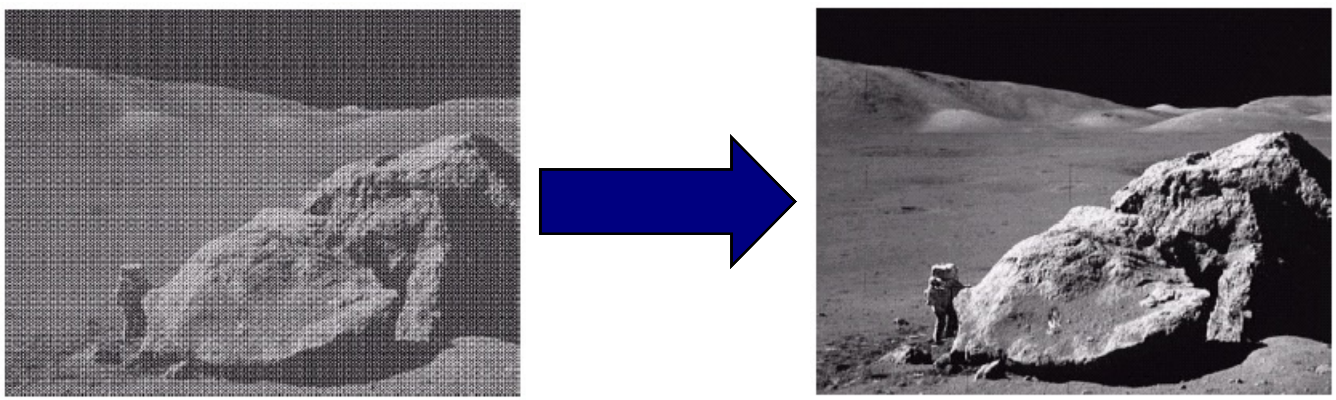
Cont..

- Order-Statistics Filters
  - Median Filter
  - Max and Min Filter
  - Midpoint Filter
  - Alpha trimmed Mean Filter
- Adaptive Filters
  - Adaptive median filter

- Periodic Noise Reduction by Frequency Domain Filtering
  - Band Reject Filter
  - Band Pass Filter

What is Image Restoration?

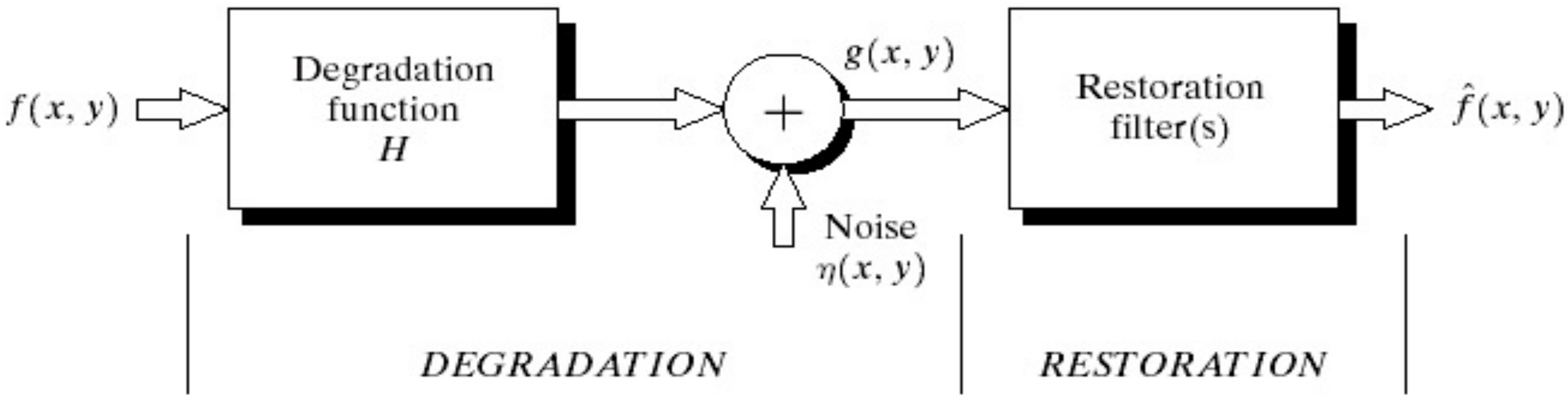
- Image restoration attempts to restore images that have been degraded
  - Identify the degradation process and attempt to reverse it
  - Similar to image enhancement, but more objective



- Model the degradation and applying the inverse process in order to recover the original image.

Concept of Image Restoration

- *Image restoration is to restore a degraded image back to the original image while image enhancement is to manipulate the image so that it is suitable for a specific application.*



Degradation model:

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

where *h(x,y)* is a system that causes image distortion and $\eta(x,y)$ is noise.

Noise Model

- We can consider a noisy image to be modelled as follows:

$$g(x, y) = f(x, y) + \eta(x, y)$$

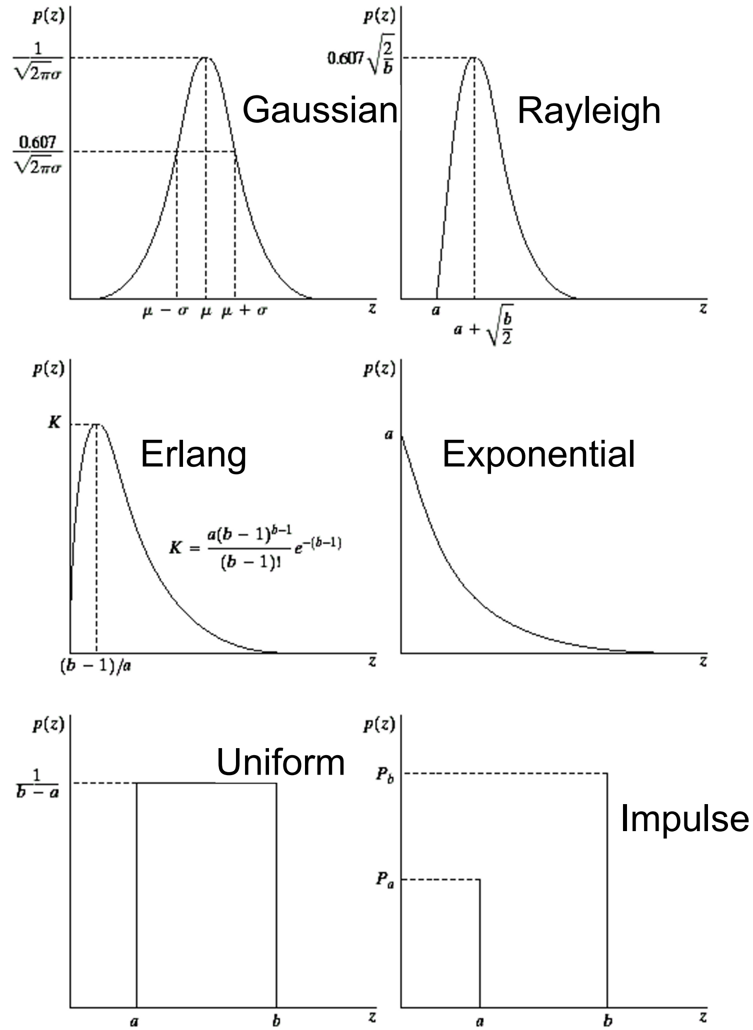
- where $f(x, y)$ is the original image pixel, $\eta(x, y)$ is the noise term and $g(x, y)$ is the resulting noisy pixel

- If we can estimate the noise model we can figure out how to restore the image

- Noise cannot be predicted but can be approximately described in statistical way using the probability density function (PDF)

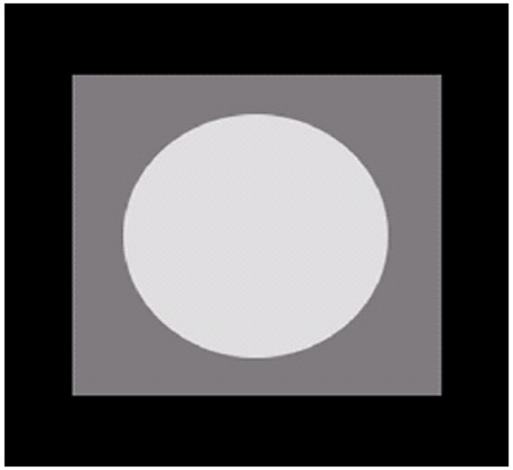
Cont..

- There are many different models for the image noise term $\eta(x, y)$:
  - Gaussian
    - Most common model
  - Rayleigh
  - Erlang (Gamma)
  - Exponential
  - Uniform
  - Impulse
    - *Salt and pepper* noise



Noise Example

- The test pattern to the right is ideal for demonstrating the addition of noise
- The following slides will show the result of adding noise based on various models to this image



Image

Histogram

Gaussian Noise

The PDF of Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

where, z represents intensity

$\bar{z}$ is the mean (average) value of z

σ is the standard deviation

Cont..

The PDF of Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$

- 70% of its values will be in the range

$$\left[(\mu-\sigma),(\mu+\sigma)\right]$$

- 95% of its values will be in the range

$$\left[(\mu-2\sigma),(\mu+2\sigma)\right]$$

Rayleigh Noise

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = a + \sqrt{\pi b / 4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

Erlang (Gamma) Noise

The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = b / a$$

$$\sigma^2 = b / a^2$$

Exponential Noise

The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = 1/a$$

$$\sigma^2 = 1/a^2$$

Uniform Noise

The PDF of uniform noise is given by

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{for a} \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = (a+b)/2$$

$$\sigma^2 = (b-a)^2/12$$

Impulse (Salt-and-Pepper) Noise

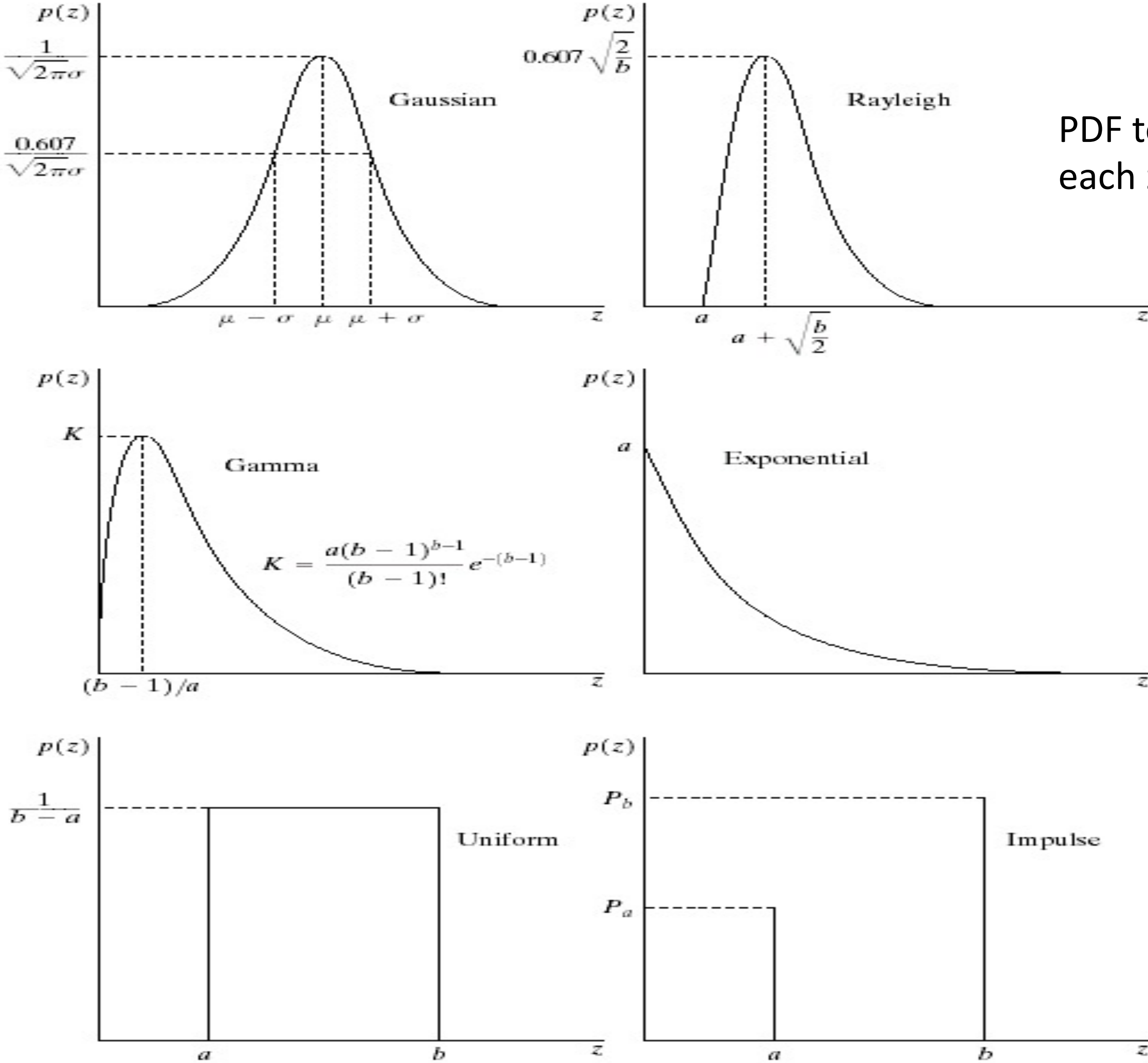
The PDF of (bipolar) impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

if $b > a$, gray-level b will appear as a light dot, while level a will appear like a dark dot.

If either P_a or P_b is zero, the impulse noise is called *unipolar*

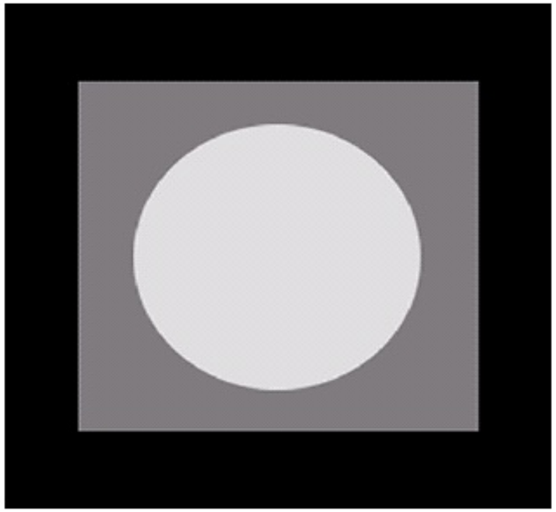
PDF: Statistical Way to Describe Noise



PDF tells how much each z value occurs.

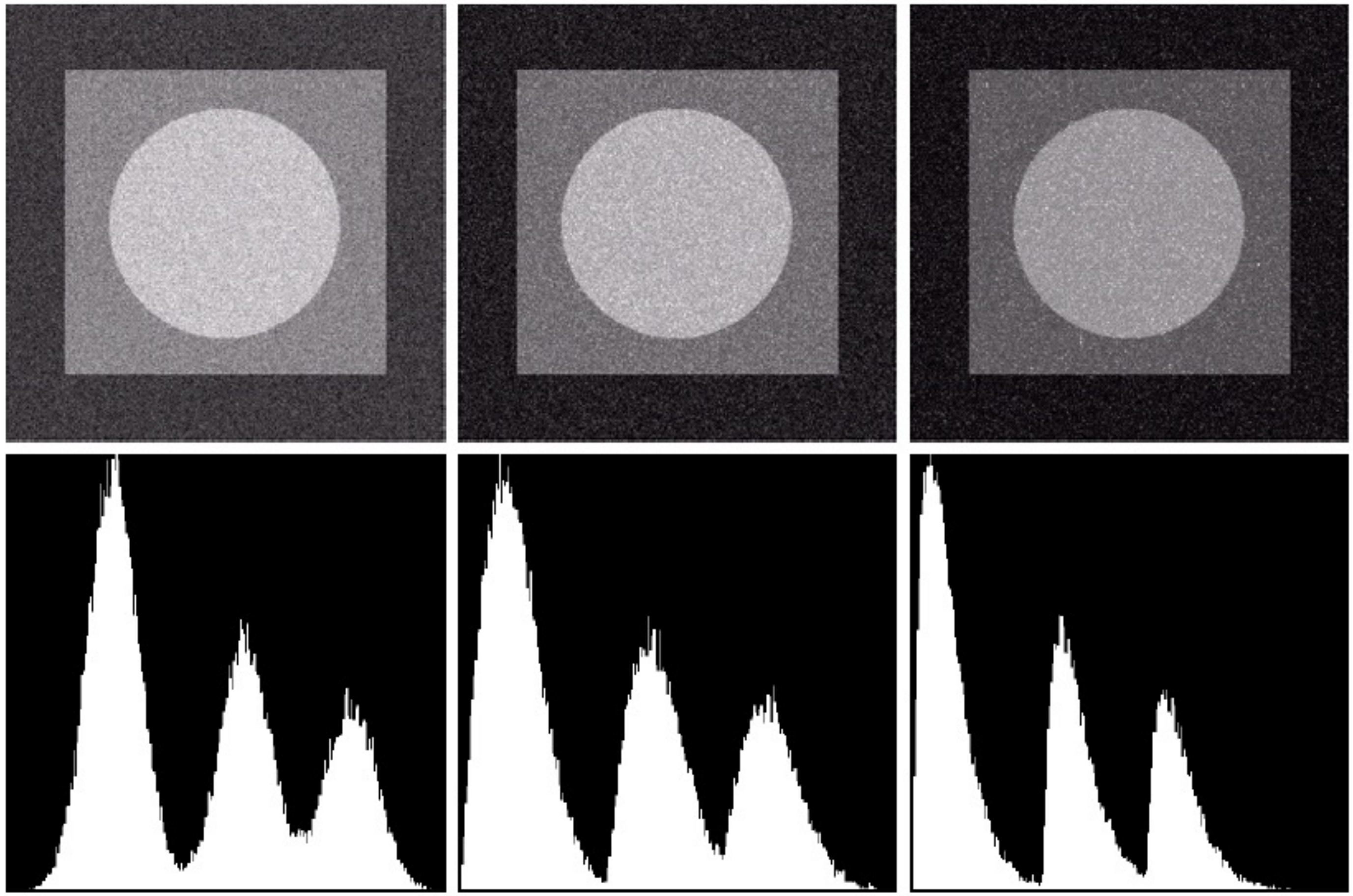
Image Degradation with Additive Noise

$$g(x,y) = f(x,y) + \eta(x,y)$$



Original image

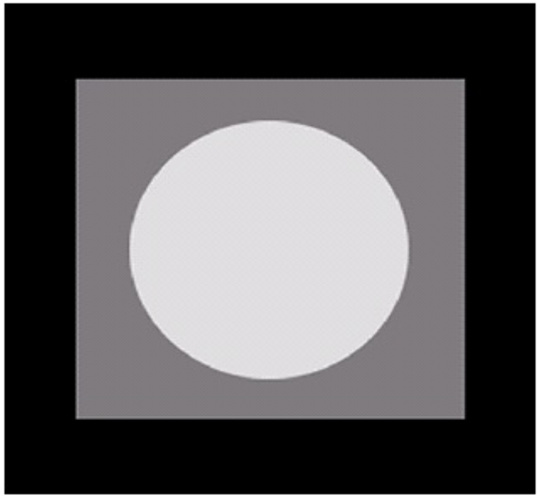
Degraded images



Histogram

Gaussian Rayleigh Erlang

Cont…

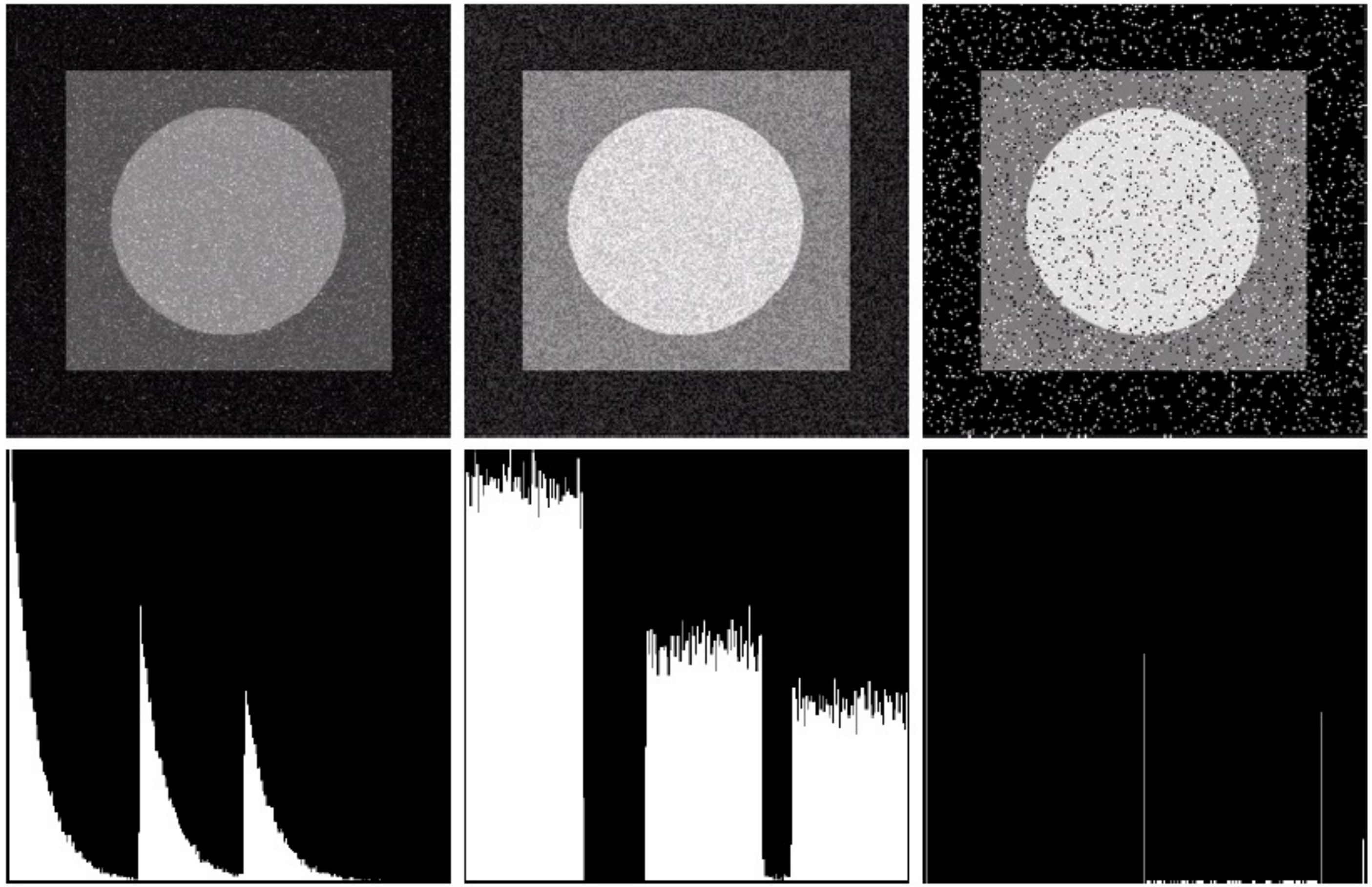


Original image

$$g(x,y) = f(x,y) + \eta(x,y)$$

Degraded images

Histogram



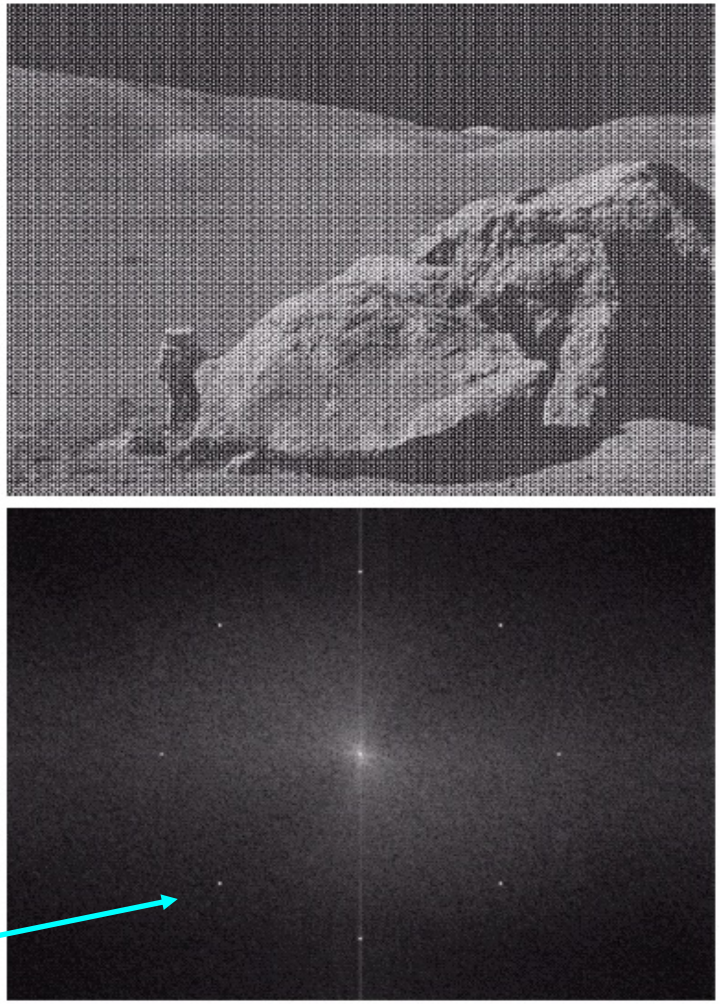
Exponential Uniform Impulse

Periodic Noise

- Periodic noise in an image arises typically from electrical or electromechanical interference during image acquisition.
- It is a type of spatially dependent noise
- Periodic noise can be reduced significantly via frequency domain filtering

Periodic Noise

- Typically arises due to electrical or electromagnetic interference.
- Gives rise to regular noise patterns in an image.
- Frequency domain techniques in the Fourier domain are most effective at removing periodic noise.



Periodic noise looks like dots In the frequency domain

Restoration in the Presence of Noise Only – Spatial Filtering

Noise model without degradation

$$g(x, y) = f(x, y) + \eta(x, y)$$

and

$$G(u,v) = F(u,v) + N(u,v)$$

Filtering to Remove Noise

- We can use spatial filters of different kinds to remove different kinds of noise
- The *arithmetic mean* filter is a very simple one and is calculated as follows:

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

This is implemented as the simple smoothing filter

It blurs the image and reduces the noise.

Other Means

- There are different kinds of mean filters all of which exhibit slightly different behaviour:
  - Arithmetic Mean Filter
  - Geometric Mean
  - Harmonic Mean
  - Contra-harmonic Mean

Cont..

- **Arithmetic Mean:**

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

m x n = size of moving window;

S_{xy}= set of coordinates in m x n centered at x, y;

Simplest median filter.

Calculate Avg value of the corrupted image $g(x,y)$ in S_{xy} the area.

Implemented using Conv Mask and all coefficients have value 1/mn.

Cont..

- **Geometric Mean:**

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Achieves similar smoothing to the arithmetic mean, but tends to lose less image detail.

Cont..

- **Harmonic Mean:**

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise, but fails for pepper noise.

Also does well for other kinds of noise such as Gaussian noise.

Cont..

- **Contra-harmonic Mean:**

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q is the *order* of the filter.

Positive values of Q eliminate pepper noise.

Negative values of Q eliminate salt noise.

It cannot eliminate both simultaneously.

It reduces to AM filter if $Q = 0$ and HM if $Q = -1$

Geometric Mean Filter: Example

Original image

Image corrupted by AWGN

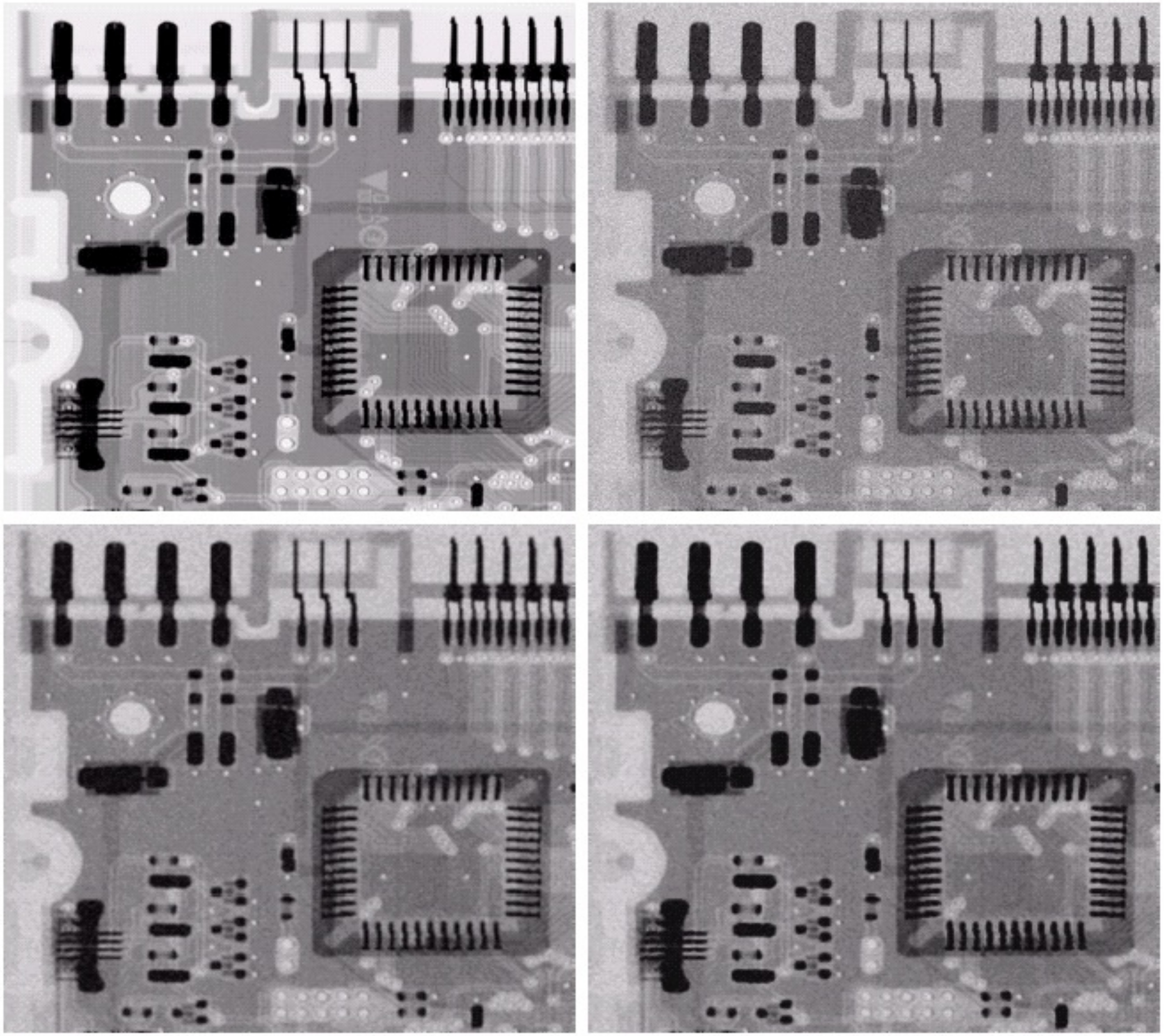


Image obtained using a 3x3 arithmetic mean filter

Image obtained using a 3x3 geometric mean filter

AWGN: Additive White Gaussian Noise

Contra-harmonic Filters: Example

Image corrupted by pepper noise with prob. = 0.1

Image corrupted by salt noise with prob. = 0.1

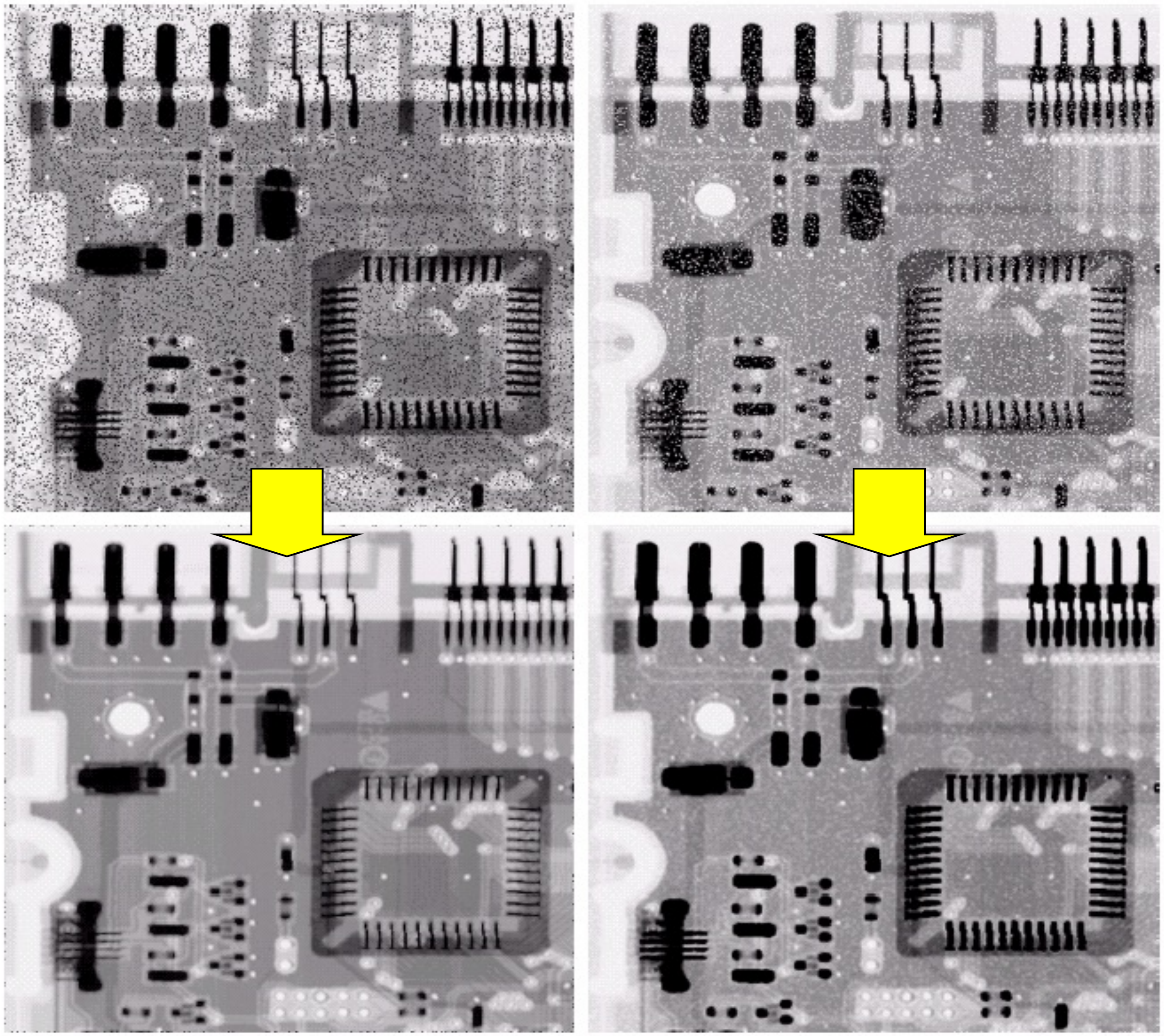


Image obtained using a 3x3 contra-harmonic mean filter With Q = 1.5

Image obtained using a 3x3 contra-harmonic mean filter With Q=-1.5

Contra-harmonic Filters: Incorrect Use Example

Image corrupted by pepper noise with prob. = 0.1

Image corrupted by salt noise with prob. = 0.1

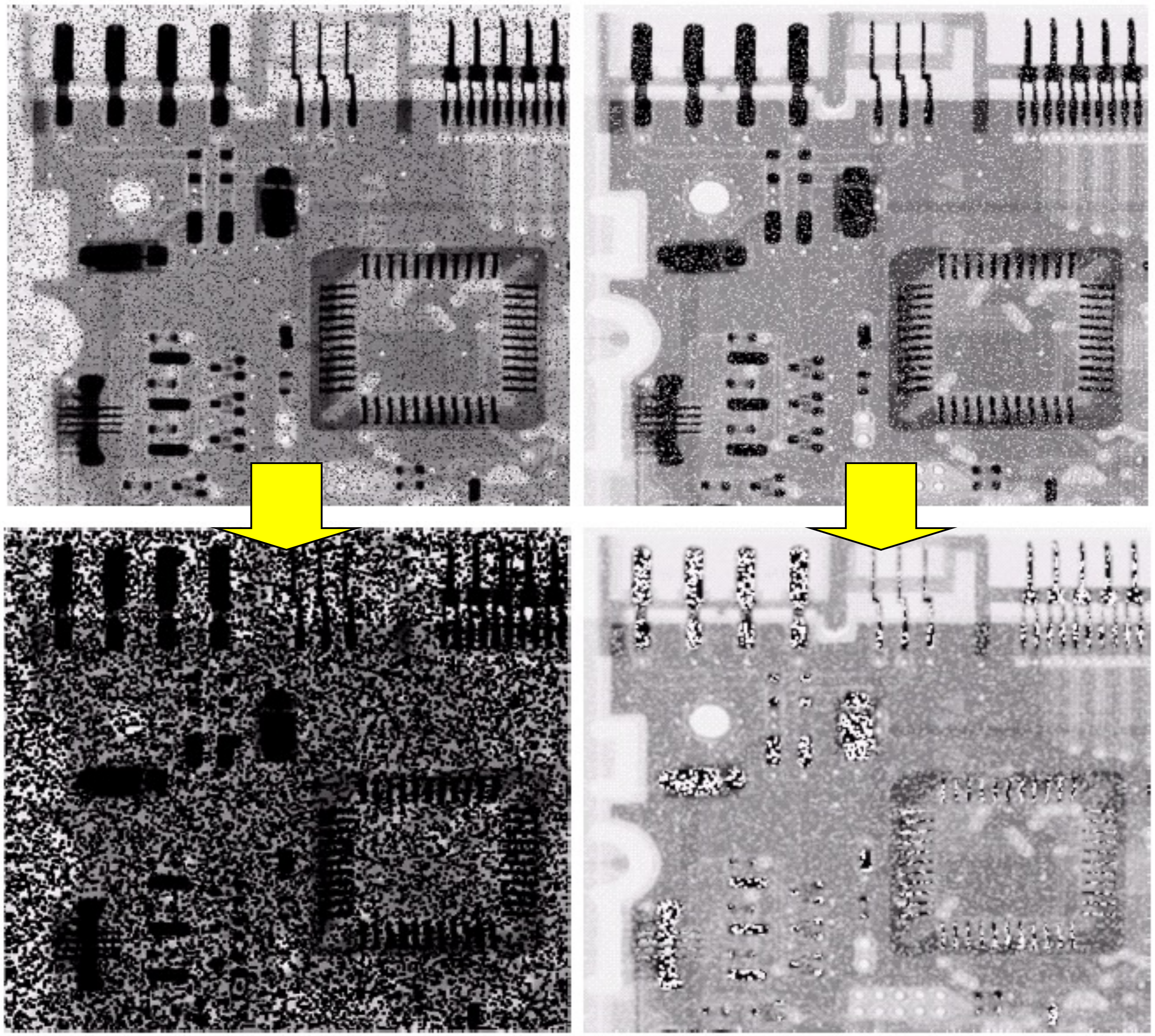
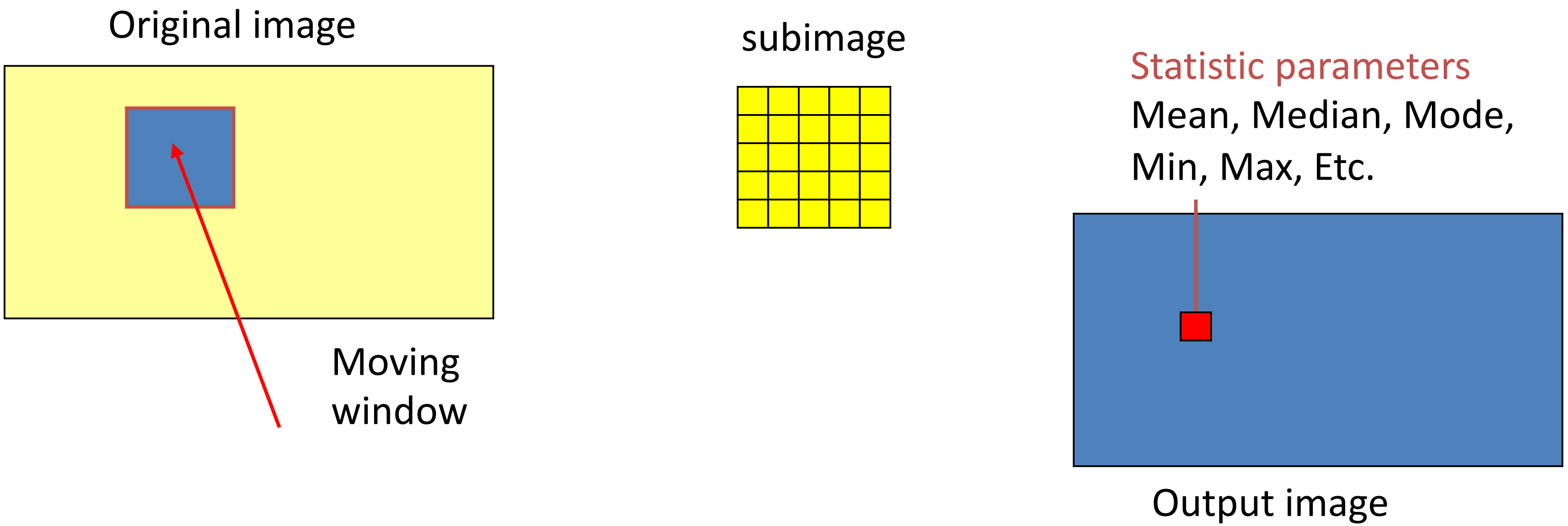


Image obtained using a 3x3 contra-harmonic mean filter With Q=-1.5

Image obtained using a 3x3 contra-harmonic mean filter With Q=1.5

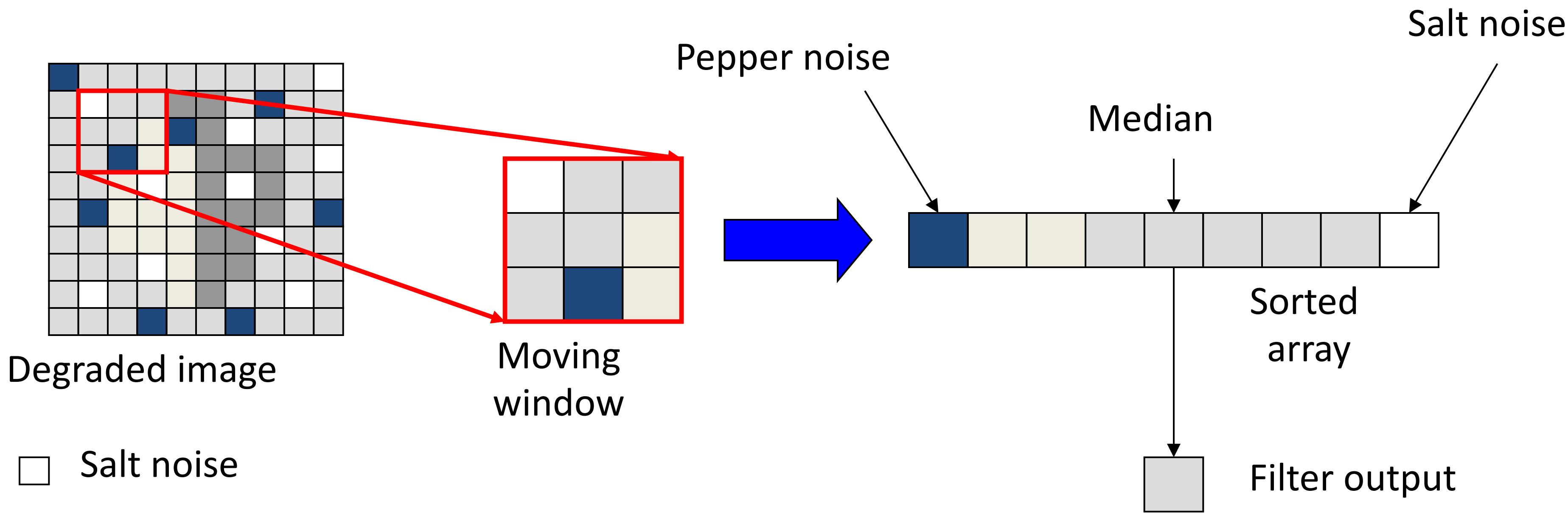
Order-Statistic Filters: Revisit



- Spatial filters based on ordering the pixel values that make up the neighbourhood defined by the filter support.
- Useful spatial filters include
  - Median filter
  - Max and min filter
  - Midpoint filter
  - Alpha trimmed mean filter

Median Filter : How it works

- A median filter is good for removing impulse, isolated noise



Salt noise

Pepper noise

Normally, impulse noise has high magnitude and is isolated. When we sort pixels in the moving window, noise pixels are usually at the ends of the array.

Therefore, it's rare that the noise pixel will be a median value.

Median Filter

Median Filter:

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

Excellent at noise removal, without the smoothing effects that can occur with other smoothing filters.

Particularly good when salt and pepper noise is present.

Max and Min Filter

Max Filter:

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Min Filter:

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Max filter is good for dark or pepper noise and Min filter is good for bright or salt noise.

Midpoint Filter

Midpoint Filter:

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Good for random Gaussian and uniform noise.

Alpha-Trimmed Mean Filter

Alpha-Trimmed Mean Filter:

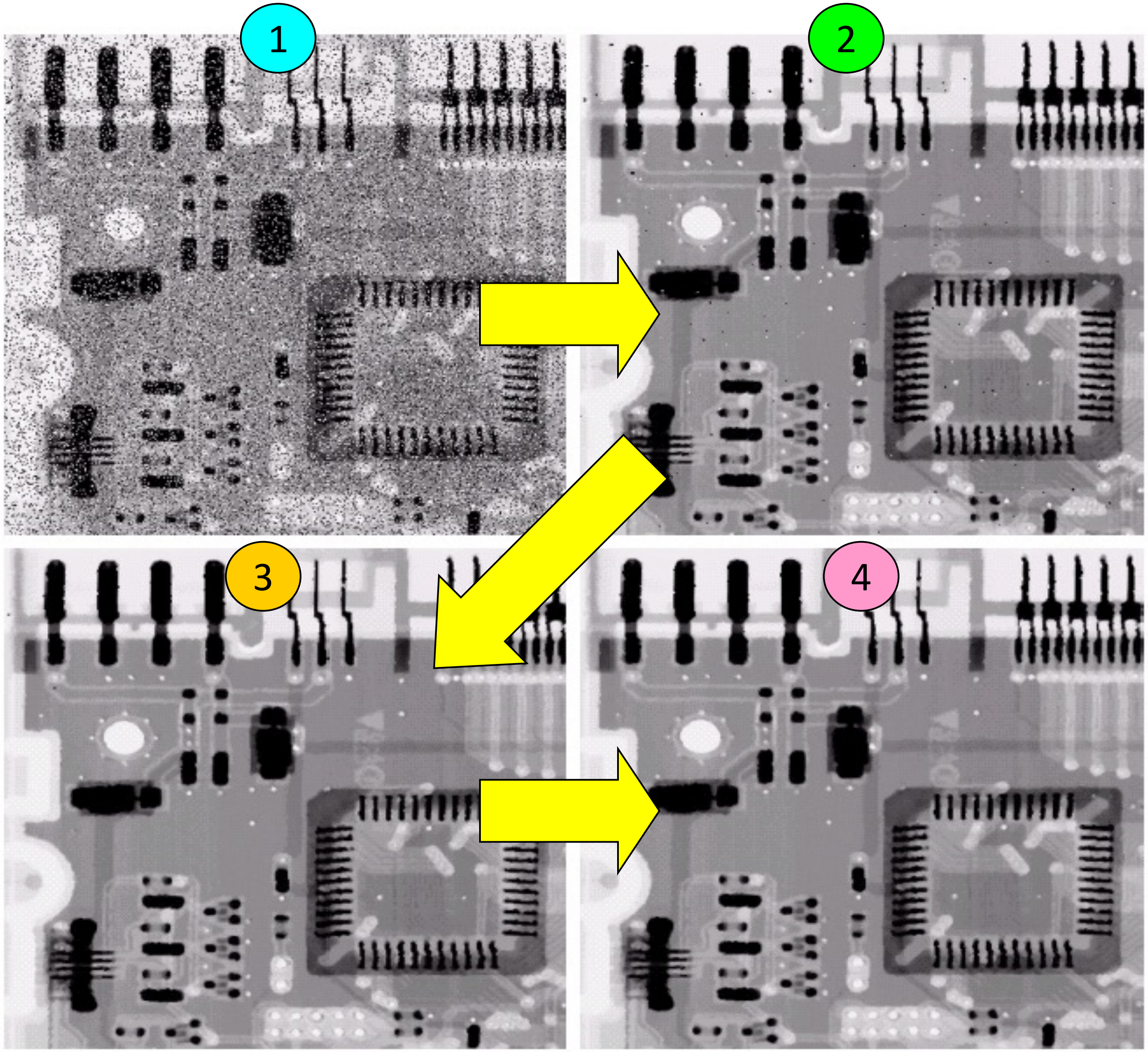
$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

We can delete the $d/2$ lowest and $d/2$ highest grey levels $g(s,t)$. So $g_r(s, t)$ represents the remaining $mn - d$ pixels.

This filter is useful in situations involving multiple types of noise such as a combination of salt-and-pepper and Gaussian noise.

Median Filter : Example

Image corrupted by salt-and-pepper noise with $p_a = p_b = 0.1$



Images obtained by using a 3x3 median filter

Max and Min Filters: Example

Image corrupted by pepper noise with prob. = 0.1

Image corrupted by salt noise with prob. = 0.1

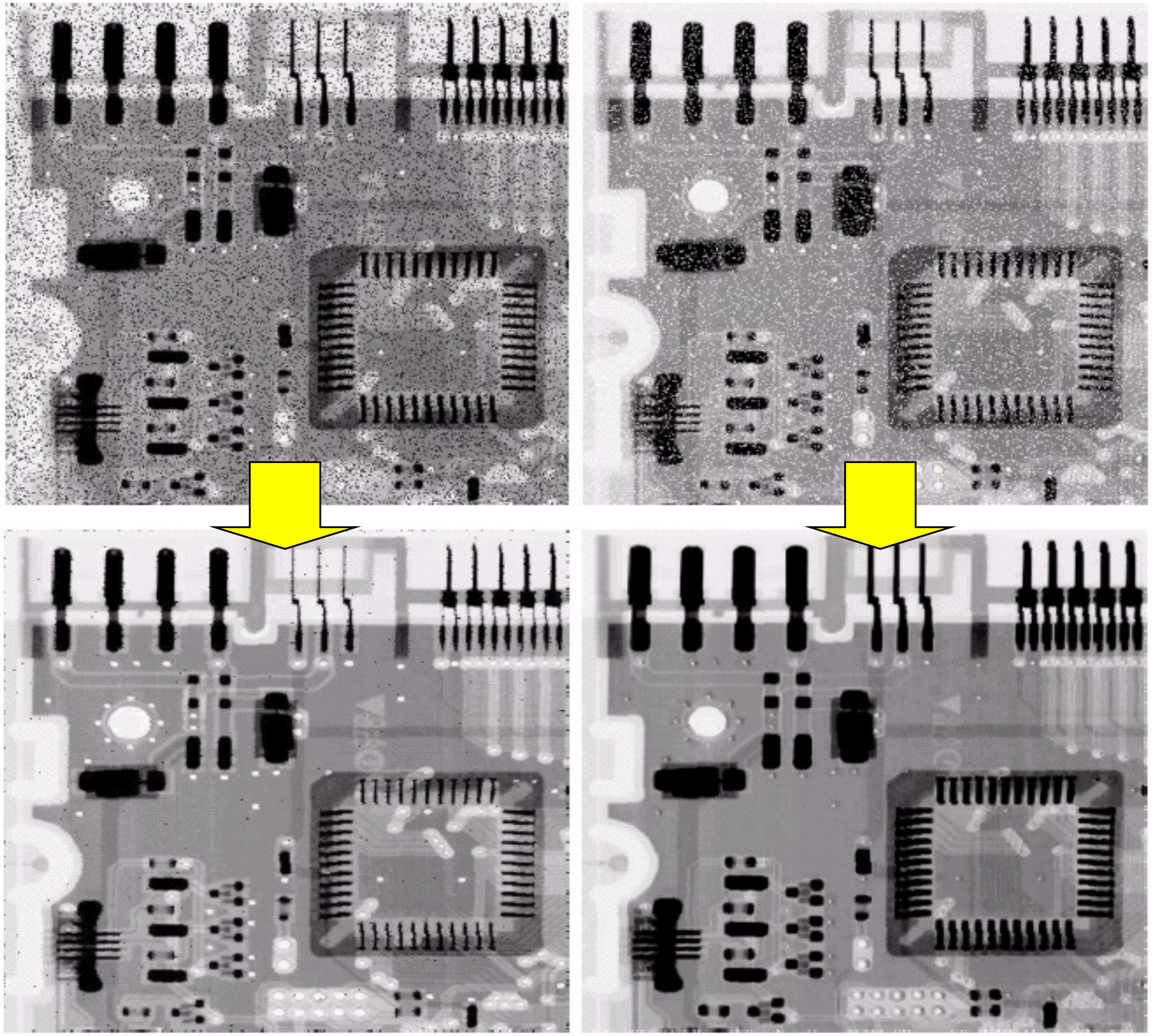


Image obtained using a 3x3 max filter

Image obtained using a 3x3 min filter

Alpha-trimmed Mean Filter: Example

Image corrupted by additive uniform noise

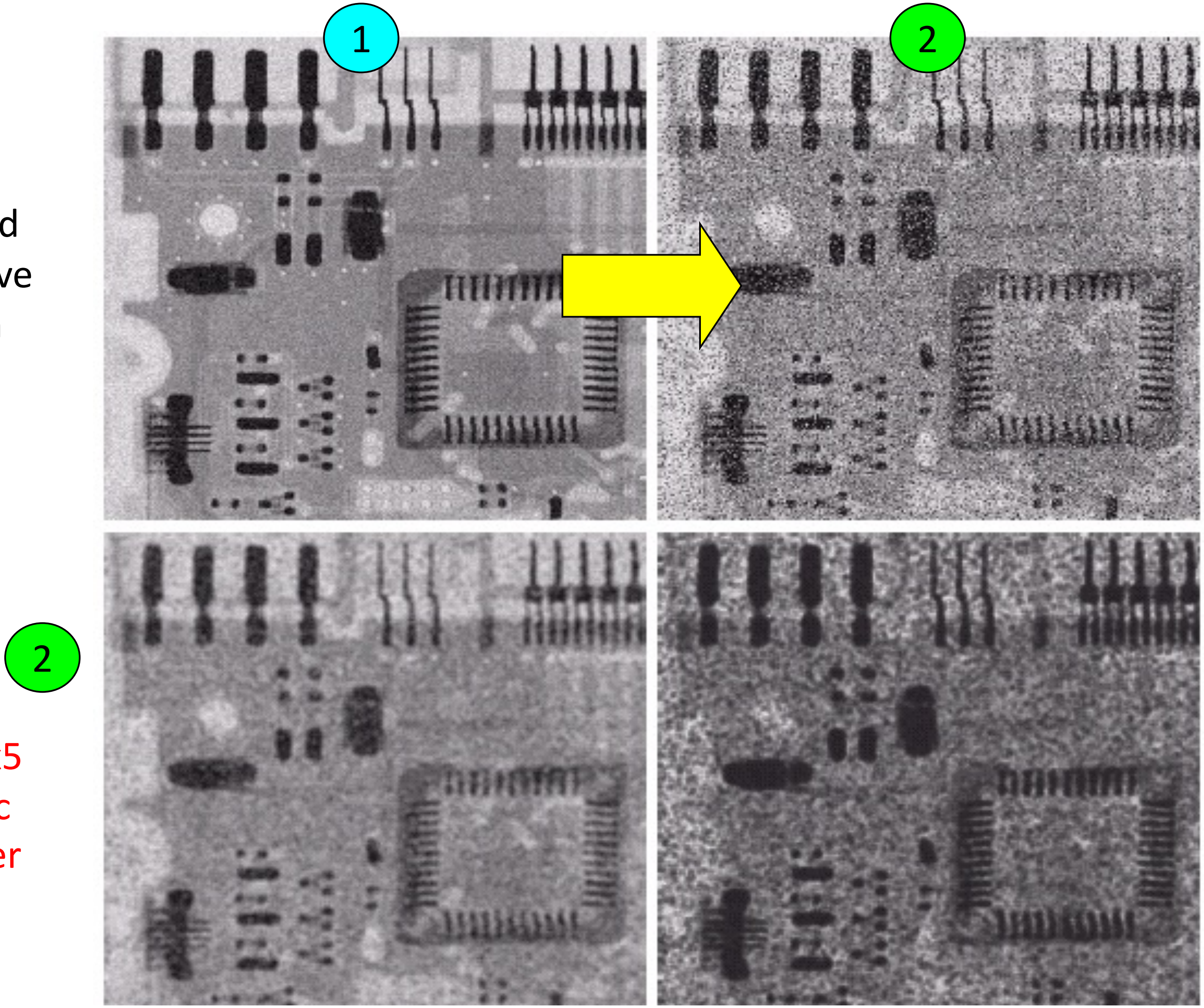


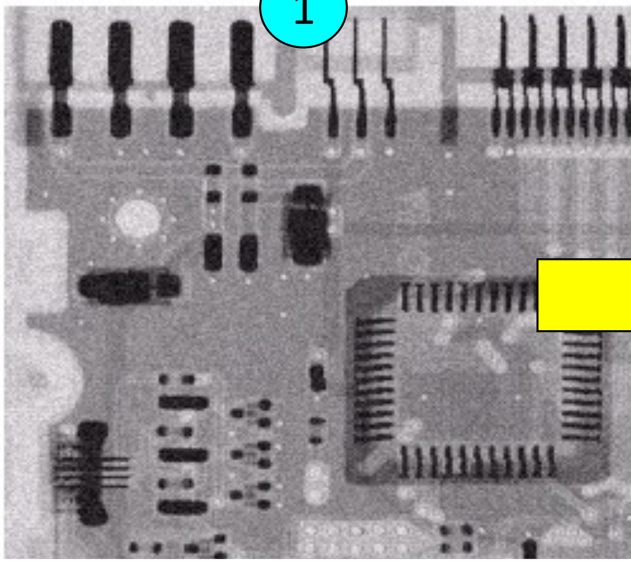
Image additionally corrupted by additive salt-and-pepper noise

Image obtained using a 5x5 arithmetic mean filter

Image obtained using a 5x5 geometric mean filter

Cont…

Image corrupted by additive uniform noise



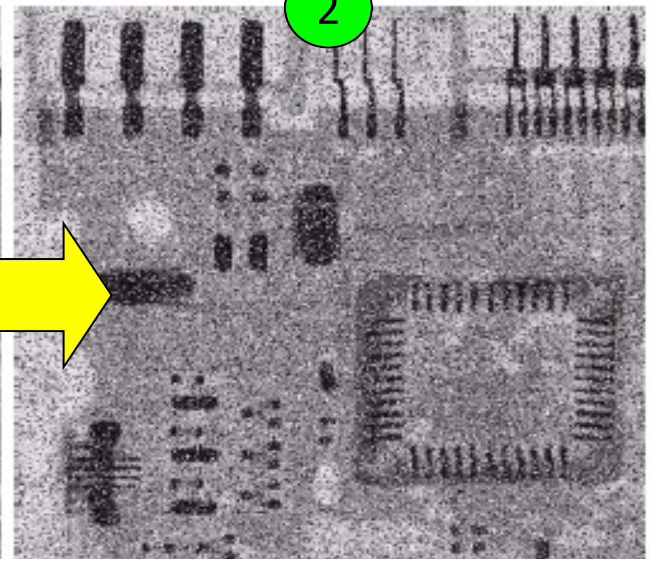
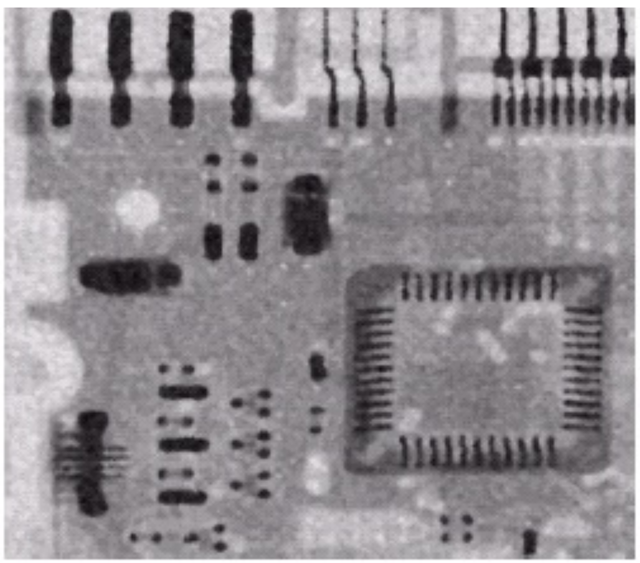


Image additionally corrupted by additive salt-and-pepper noise

Image obtained using a 5x5 median filter

2



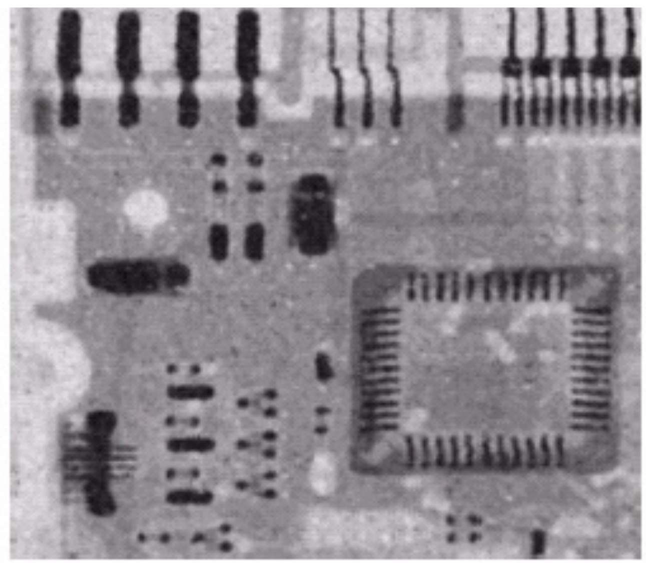
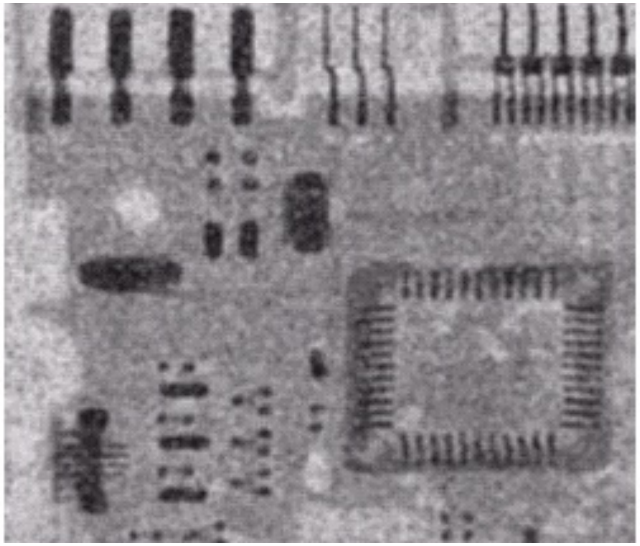


Image obtained using a 5x5 alpha-trimmed mean filter with *d* = 5

2

Cont…

Image obtained using a 5x5 arithmetic mean filter



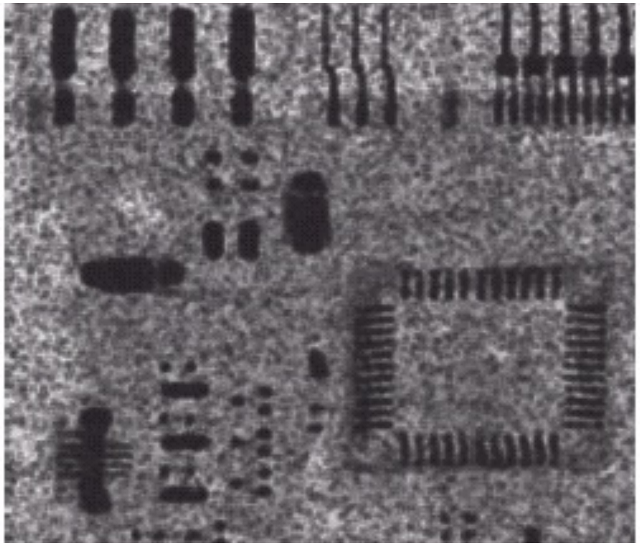
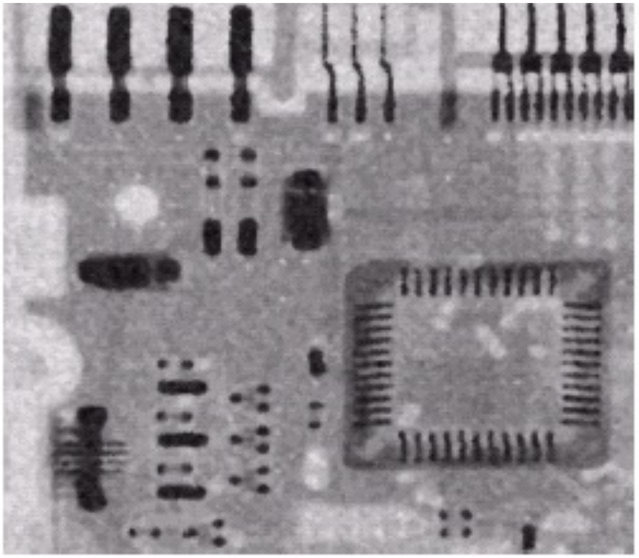


Image obtained using a 5x5 geometric mean filter

Image obtained using a 5x5 median filter



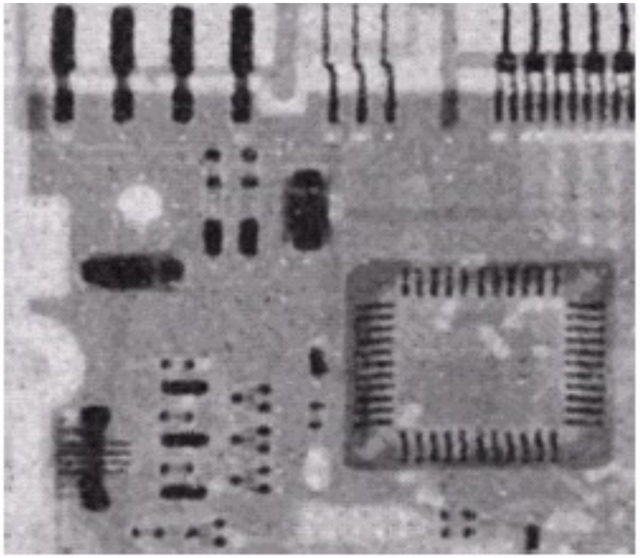


Image obtained using a 5x5 alpha-trimmed mean filter with d = 5

Adaptive Filter

General concept:

- Filter behavior depends on statistical characteristics of local areas inside *m* x *n* moving window
- More complex but superior performance compared with "fixed" filters or discussed filters

Statistical characteristics:

Local mean:

$$m_L = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Noise variance:

$$\sigma_\eta^2$$

Local variance:

$$\sigma_L^2 = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} (g(s,t) - m_L)^2$$

Cont..

S_{xy}: local region

The response of the filter at the center point (x,y) of S_{xy} is based on four quantities:

(a) $g(x, y)$, the value of the noisy image at (x, y);

(b) σ_η^2, the variance of the noise corrupting $f(x, y)$ to form $g(x, y)$;

(c) m_L, the local mean of the pixels in S_{xy};

(d) σ_L^2, the local variance of the pixels in S_{xy}.

Cont..

The behavior of the filter:

(a) if σ_η^2 is zero, the filter should return simply the value of $g(x, y)$.

(b) if the local variance is high relative to σ_η^2, the filter should return a value close to $g(x, y)$;

(c) if the two variances are equal, the filter returns the arithmetic mean value of the pixels in S_{xy}.

Cont..

An adaptive expression for obtaining $\hat{f}(x, y)$ based on the assumptions:

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L\right]$$

Adaptive, Local Noise Reduction Filter

Purpose: want to preserve edges

Concept:

1. If σ_η^2 is zero, → No noise
   the filter should return $g(x,y)$ because $g(x,y) = f(x,y)$

2. If σ_L^2 is high relative to σ_η^2, → Edges (should be preserved),
   the filter should return the value close to $g(x,y)$

3. If $\sigma_L^2 = \sigma_\eta^2$, → Areas inside objects
   the filter should return the arithmetic mean value m_L

Formula:

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left(g(x,y) - m_L\right)$$

Adaptive Noise Reduction Filter: Example

Image corrupted by additive Gaussian noise with zero mean and σ^2=1000

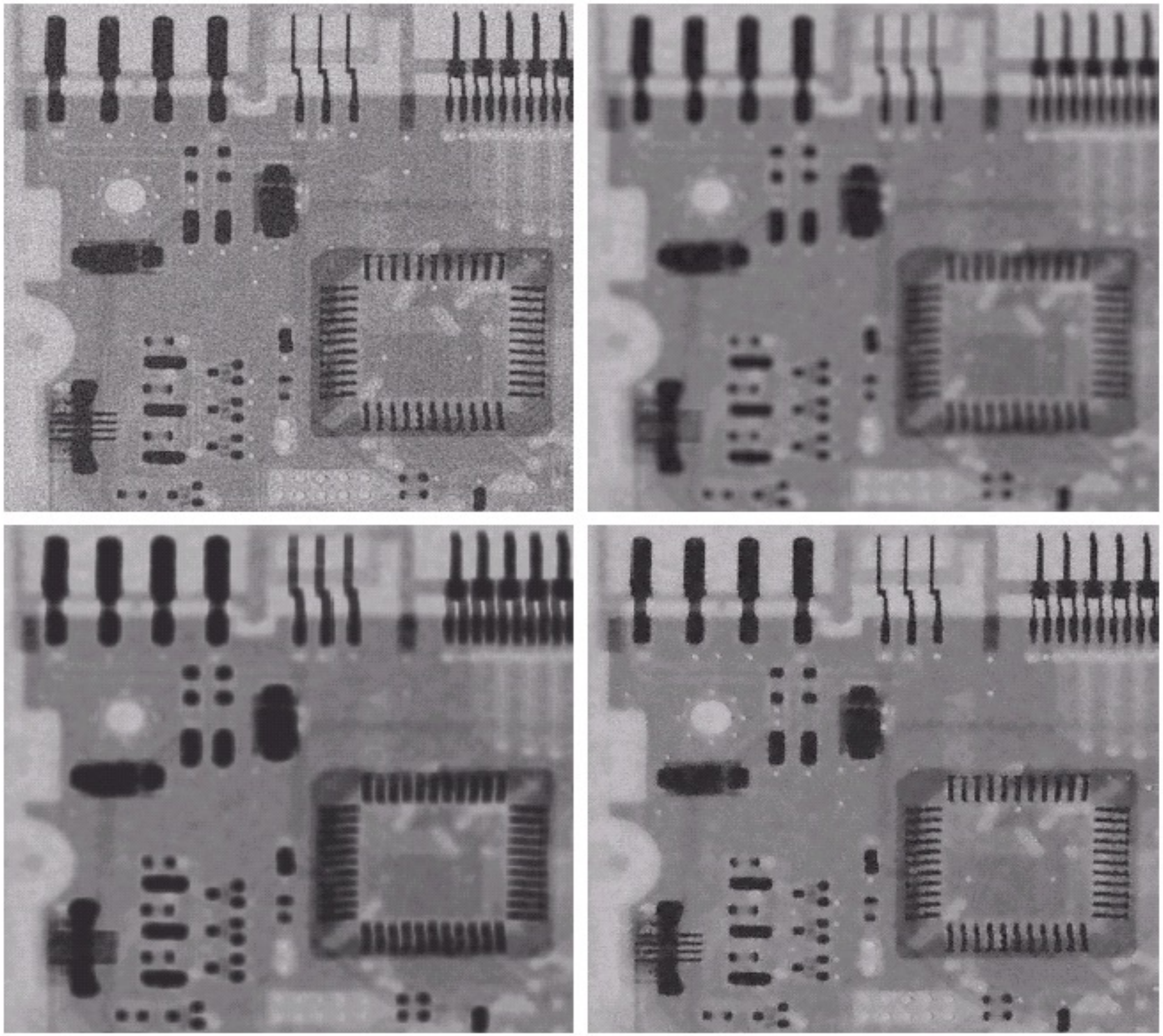


Image obtained using a 7x7 arithmetic mean filter

Image obtained using a 7x7 geometric mean filter

Image obtained using a 7x7 adaptive noise reduction filter

Cont..

The notation:

$$z_{\text{min}} = \text{minimum intensity value in } S_{xy}$$

$$z_{\text{max}} = \text{maximum intensity value in } S_{xy}$$

$$z_{\text{med}} = \text{median intensity value in } S_{xy}$$

$$z_{xy} = \text{ intensity value at coordinates } (x, y)$$

$$S_{\text{max}} = \text{maximum allowed size of } S_{xy}$$

Adaptive Median Filters

Purpose: want to remove impulse noise while preserving edges

The adaptive median-filtering works in two stages:

Stage A:

$$A1 = z_{\text{med}} - z_{\min}; \quad A2 = z_{\text{med}} - z_{\max}$$

if A1>0 and A2<0, go to stage B

Else increase the window size

if window size $\leq S_{\max}$, repeat stage A; Else output z_{med}

Stage B:

$$B1 = z_{xy} - z_{\min}; \quad B2 = z_{xy} - z_{\max}$$

if B1>0 and B2<0, output z_{xy}; Else output z_{med}

Cont..

The adaptive median-filtering works in two stages:

Stage A:

$A1 = z_{\text{med}} - z_{\min}; \quad A2 = z_{\text{med}} - z$

if A1>0 and A2<0, go to stage B

Else increase the window size

if window size $\leq S_{\max}$, repeat stage A; Else output z_{med}

The median filter output is an impulse or not

Stage B:

$B1 = z_{xy} - z_{\min}; \quad B2 = z_{xy} - z$

if B1>0 and B2<0, output z_{xy}; Else output z_{med}

The processed point is an impulse or not

Adaptive Median Filter: Example

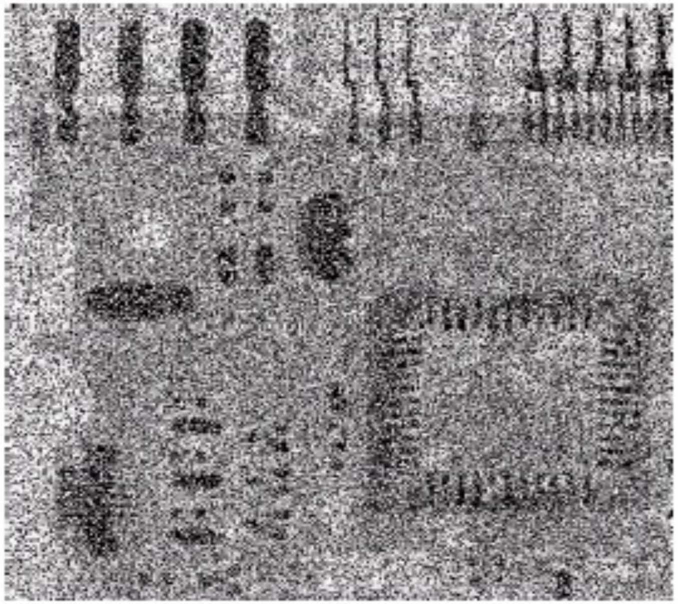


Image corrupted by salt-and-pepper noise with $p_a = p_b = 0.25$

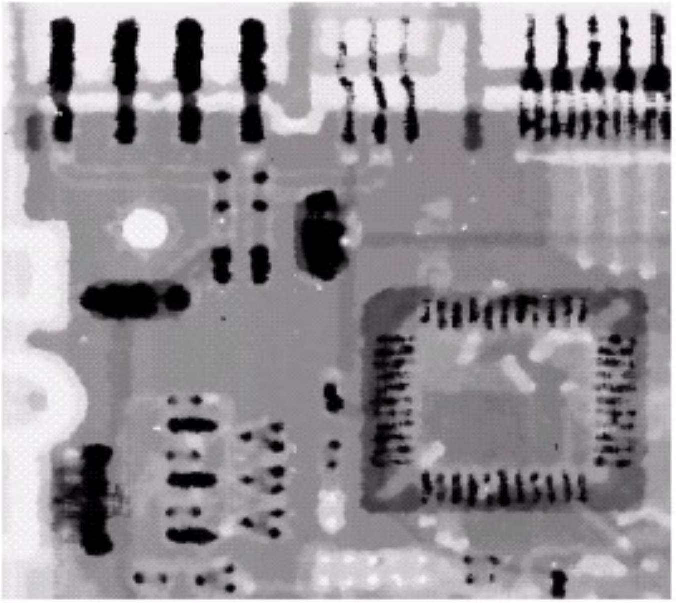


Image obtained using a 7x7 median filter

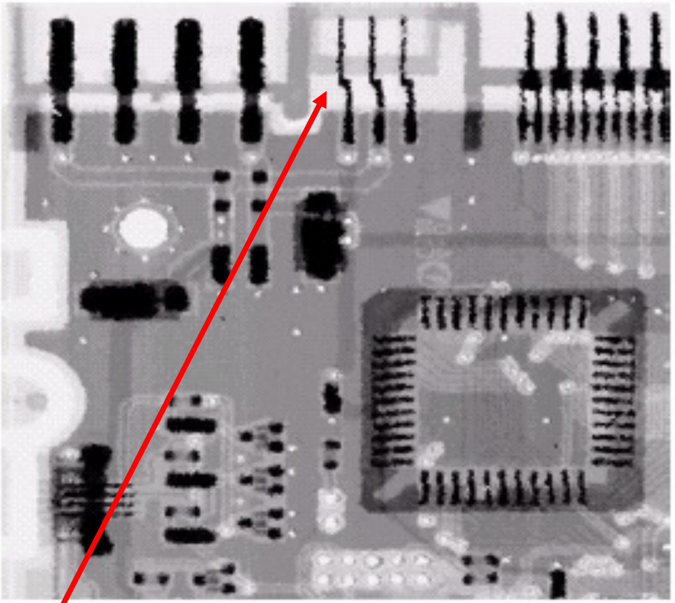
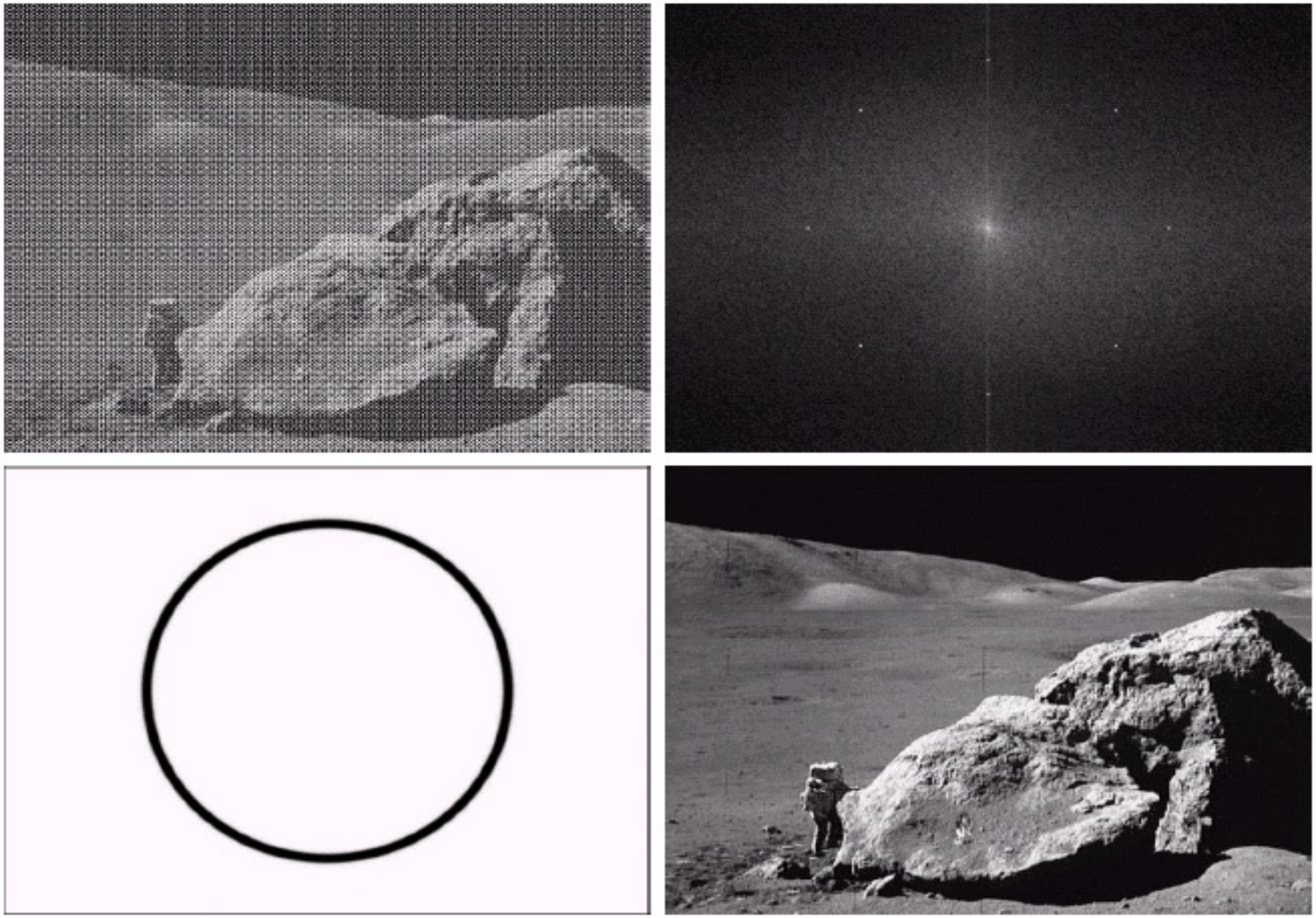


Image obtained using an adaptive median filter with $S_{max} = 7$

More small details are preserved

Periodic Noise Reduction by Freq. Domain Filtering

Degraded image DFT



Band reject filter Restored image

Periodic noise can be reduced by setting frequency components corresponding to noise to zero.

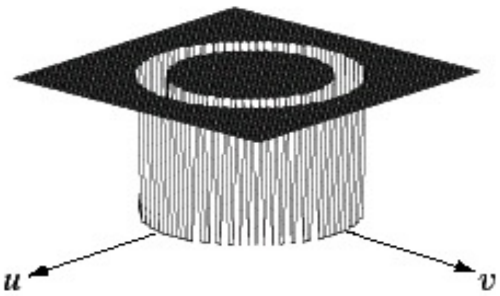
Band Reject Filters

- Removing periodic noise form an image involves removing a particular range of frequencies from that image.
- *Band reject* filters can be used for this purpose
- An ideal band reject filter is given as follows:

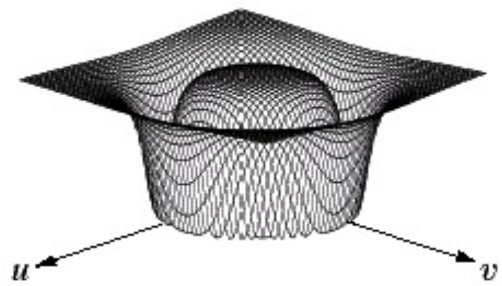
$$H(u,v) = \begin{cases} 1 & if \; D(u,v) < D_0 - \dfrac{W}{2} \\ 0 & if \; D_0 - \dfrac{W}{2} \le D(u,v) \le D_0 + \dfrac{W}{2} \\ 1 & if \; D(u,v) > D_0 + \dfrac{W}{2} \end{cases}$$

Cont..

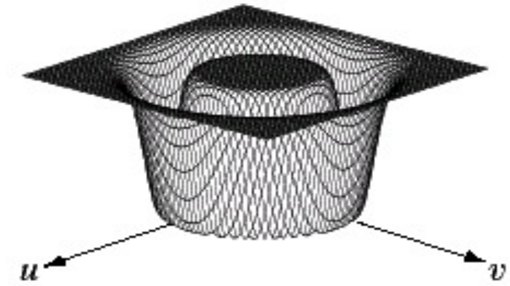
The ideal band reject filter is shown below, along with Butterworth and Gaussian versions of the filter



Ideal Band Reject Filter



Butterworth Band Reject Filter (of order 1)

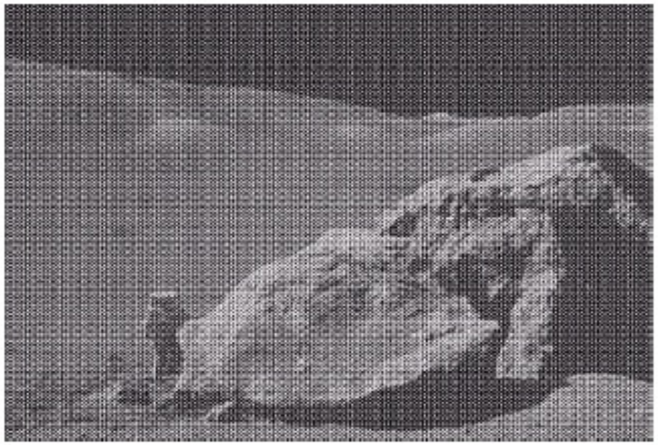


Gaussian Band Reject Filter

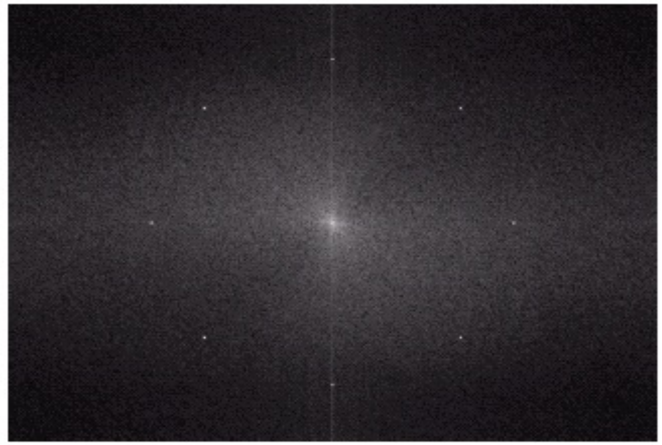
Use to eliminate frequency components in some bands

Band Reject Filter : Example

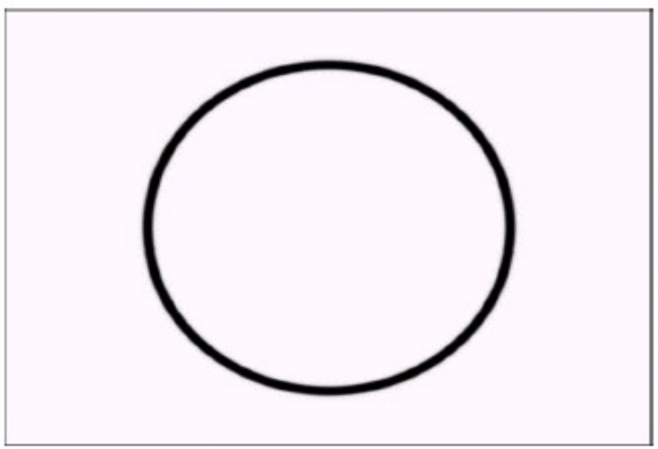
Image corrupted by sinusoidal noise



Fourier spectrum of corrupted image



Periodic noise can be reduced by setting frequency components corresponding to noise to zero.

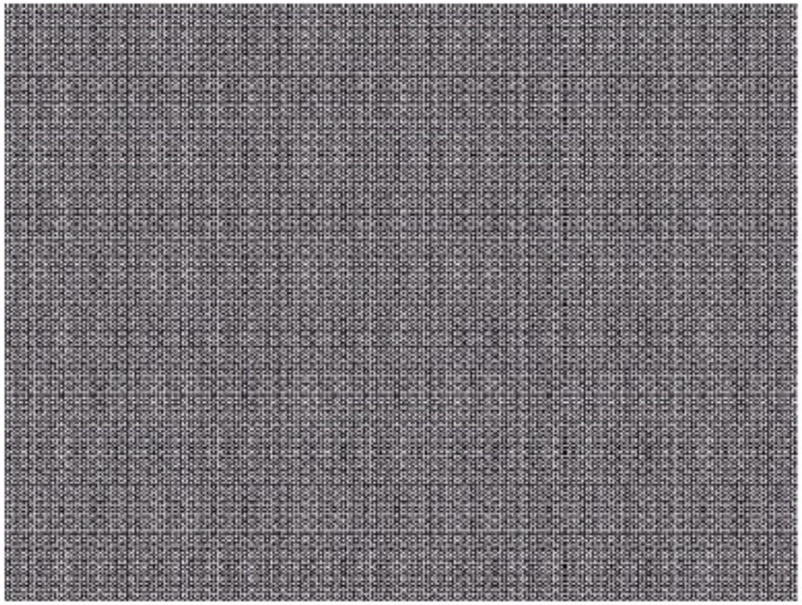


Butterworth band reject filter

Filtered image

Band Pass Filter : Example

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



$$H_{bp}(u, v) = 1 - H_{br}(u, v)$$

Problems

- Find the pixel value to replace the central pixel in the image given below by considering following filters

a. Mean

b. Median

c. Geometric

d. Harmonic Mean

e. Contra-harmonic Mean

f. Alpha trimmed with d=4

1	4	7
5	5	4
7	2	2

Cont..

- Mean

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

1	4	7
5	**5**	4
7	2	2

Cont..

- Mean

1	4	7
5	5	4
7	2	2

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

$$= \frac{1}{9}[1 + 4 + 7 + 5 + 5 + 4 + 7 + 2 + 2]$$

$$= \frac{1}{9}[37] = 4.11$$

Cont..

- Mean

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

1	4	7
5	**5**	4
7	2	2

$$= \frac{1}{9}[1+4+7+5+5+4+7+2+2]$$

$$= \frac{1}{9}[37] = 4.11$$

1	4	7
5	**4.11**	4
7	2	2

Cont..

- Median

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

1	**4**	**7**
5	**5**	**4**
7	**2**	**2**

Cont..

- Median

1	4	7
5	**5**	4
7	2	2

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

$$= median[1, 4, 7, 5, 5, 4, 7, 2, 2]$$

$$= median[1, 2, 2, 4, 4, 5, 5, 7, 7]$$

$$= \ 4$$

Cont..

- Median

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

1	4	7
5	5	4
7	2	2

$$= median[1,4,7,5,5,4,7,2,2]$$

$$= median[1,2,2,4,4,5,5,7,7]$$

$$= 4$$

1	4	7
5	4	4
7	2	2

Cont..

- Geometric Mean

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

1	4	7
5	5	4
7	2	2

Cont..

- Geometric Mean

$$\hat{f}(x,y)=\left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$$

1	4	7
5	5	4
7	2	2

$$= [1+4+7+5+5+4+7+2+2]^{\frac{1}{9}}$$

$$= [37]^{\frac{1}{9}} = 1.49$$

Cont..

- Geometric Mean

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

1	4	7
5	5	4
7	2	2

$$= [1 * 4 * 7 * 5 * 5 * 4 * 7 * 2 * 2]^{\frac{1}{9}}$$

$$= [78{,}400]^{\frac{1}{9}} = 3.49$$

1	4	7
5	3.49	4
7	2	2

Cont..

- Harmonic Mean

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

1	4	7
5	5	4
7	2	2

Cont..

- Harmonic Mean

1	4	7
5	5	4
7	2	2

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

$$= \frac{9}{[1+\frac{1}{4}+\frac{1}{7}+\frac{1}{5}+\frac{1}{5}+\frac{1}{4}+\frac{1}{7}+\frac{1}{2}+\frac{1}{2}]}$$

$$= \frac{9}{[\frac{140+2(35)+2(20)+2(28)+2(70)}{140}]} = \frac{9}{[\frac{140+70+40+56+140}{140}]}$$

$$= \frac{9}{[\frac{446}{140}]} = \frac{9(140)}{446} = \frac{1260}{446} = 2.8$$

Cont..

- Harmonic Mean

1	4	7
5	5	4
7	2	2

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

$$= \frac{9}{[1 + \frac{1}{4} + \frac{1}{7} + \frac{1}{5} + \frac{1}{5} + \frac{1}{4} + \frac{1}{7} + \frac{1}{2} + \frac{1}{2}]}$$

$$= \frac{9}{[\frac{140+2(35)+2(20)+2(28)+2(70)}{140}]} = \frac{9}{[\frac{140+70+40+56+140}{140}]}$$

$$= \frac{9}{[\frac{446}{140}]} = \frac{9(140)}{446} = \frac{1260}{446} = 2.8$$

1	4	7
5	2.8	4
7	2	2

Cont..

- Contra-harmonic Mean

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

1	4	7
5	5	4
7	2	2

Cont..

- Alpha trimmed with d=4

1	4	7
5	**5**	4
7	2	2

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

- Steps:-
  1. Place a window over element;
  2. Pick up elements;
  3. Order elements;
  4. Discard elements at the beginning and at the end of the got ordered set;
  5. Take an average — sum up the remaining elements and divide the sum by their number.

Cont..

1	4	7
5	**5**	4
7	2	2

- Alpha trimmed with d=4

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

$$= \frac{1}{9-4}[1+4+7+5+5+4+7+2+2]$$

$$= \frac{1}{5}[1+2+2+4+4+5+5+7+7]$$ Order elements

$$= \frac{1}{5}[2+4+4+5+5]$$ Discard elements

$$= \frac{1}{5}[20] = 4$$

Cont..

- Alpha trimmed with d=4

1	4	7
5	5	4
7	2	2

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

$$=\frac{1}{9-4}[1+4+7+5+5+4+7+2+2]$$

$$=\frac{1}{5}[1+2+2+4+4+5+5+7+7]$$ Order elements

$$=\frac{1}{5}[2+4+4+5+5]$$ Discard elements

$$=\frac{1}{5}[20]=4$$

1	4	7
5	4	4
7	2	2

Summary

- Image restoration restores a degraded image back to its original form
- Degradation model has to either known or estimated apriori to restoration
- Noise or blurring are two types of degradation
- Filters can be used in either spatial or frequency domain to restore an image

- Some/all of the images used in the slides are from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2^{nd} Edition.