Fuzzy Membership Functions – Numerical problems

1. If two fuzzy sets A and B are given with membership functions μA(x) = {0.2, 0.4, 0.8, 0.5, 0.1} μB(x) = {0.1, 0.3, 0.6, 0.3, 0.2} Then what is the value of complement of A∩B ?
   Explanation:- The fuzzy intersection of two fuzzy sets A and B on universe of discourse X: μA∩B(x) = min [μA(x), μB(x)] , where x∈X But here in the question, they are asking for complement of A intersection B and so the answer would be 1-min[A(x),B(x)].
   The minimum of 0.2 and 0.1 will be 0.1, and 1-0.1 will be 0.9
   The second value is min(0.4,0.3)=0.3 and 1-0.3=0.7
   The third value is min(0.8,0.6)=0.6 and 1-0.6=0.4
   The fourth value is min(0.5,0.3)=0.3 and 1-0.3=0.7
   The last value is min(0.1,0.2)=0.1 and 1-0.1=0.9
   The only option which has got the values 0.9,0.7,0.4,0.7 and 0.9, although the fourth value is given as 0.8 instead of 0.7 is **{0.9, 0.7, 0.4, 0.7, 0.9}**.

2. If A and B are two fuzzy sets with membership functions μA(x) = {0.2, 0.5, 0.6, 0.1, 0.9} μB(x) = {0.1, 0.5, 0.2, 0.7, 0.8} Then the value of μA ∩B is ----------

Intersection of two fuzzy sets

μA ∩B (x) = μA(x) ^ μB(x) = min(μA(x), μB(x))

μA(x) = {0.2, 0.5, 0.6, 0.1, 0.9}

μB(x) = {0.1, 0.5, 0.2, 0.7, 0.8}

μA ∩B={0.1,0.5,0.2,0.1,0.8}

3. If A and B are two fuzzy sets with membership functions μA(x) = {0.6, 0.5, 0.1, 0.7, 0.8} μB(x) = {0.9, 0.2, 0.6, 0.8, 0.5}, Then the value of μ Complement A∪B(x) will be

Union of two fuzzy sets

μAUB(x) = μA(x) V μB(x) = max(μA(x), μB(x))

μA(x) = {0.6, 0.5, 0.1, 0.7, 0.8}

μB(x) = {0.9, 0.2, 0.6, 0.8, 0.5}

μAUB(x) = {0.9,0.5,0.6,0.8,0.8}

Complement of μAUB(x)={0.1,0.5,0.4,0.2,0.2}

So, the correct answer is **{0.1, 0.5, 0.4, 0.2, 0.2}**

4. Consider a set of positive numbers

   X = {1,2,3,4,5,6,7}
   Fuzzy set = { 2 or so}

   A = {(1,0.5), (2,1), (3,0.2), (4,0), (5,0), (6,0), (7,0)}
   This is also written as
   A ={ $\frac{1}{0.5,}$, $\frac{2}{1}$, $\frac{3}{0.2}$, $\frac{4}{0}$, $\frac{5}{0}$, $\frac{6}{0}$, $\frac{7}{0}$}
5. Consider the age values of various people
   X = {1,20,15,35,78,80}
   Write a fuzzy set of “Young” people
   A = {(1,0),(20,1),(15,0.7),(35,0.4),(80,0)}

Fuzzy Inference System

Fuzzy Inference System is the key unit of a fuzzy logic system having decision making as its primary work. It uses the “IF…THEN” rules along with connectors “OR” or “AND” for drawing essential decision rules.

Characteristics of Fuzzy Inference System

Following are some characteristics of FIS −

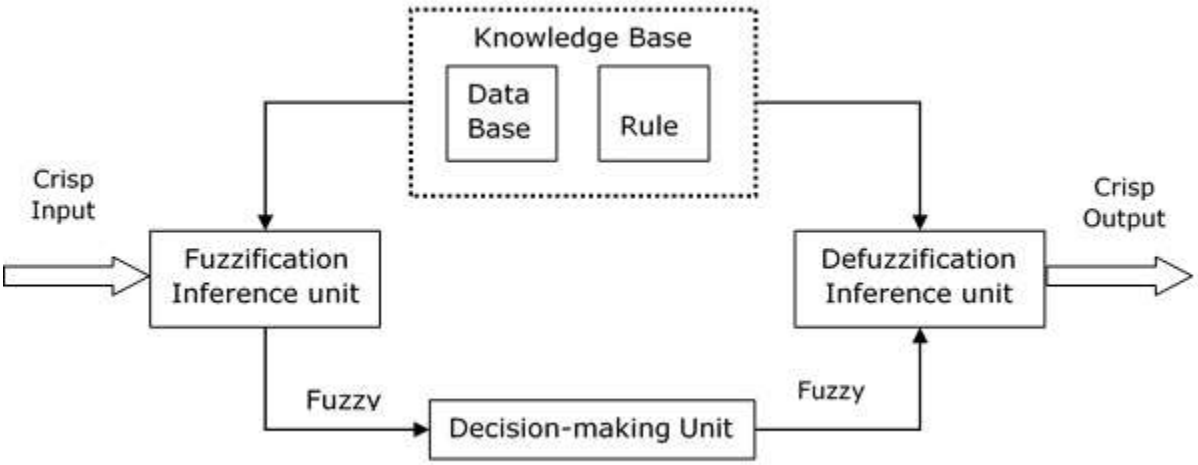
- The output from FIS is always a fuzzy set irrespective of its input which can be fuzzy or crisp.
- It is necessary to have fuzzy output when it is used as a controller.

- A defuzzification unit would be there with FIS to convert fuzzy variables into crisp variables.

Functional Blocks of FIS

The following five functional blocks will help you understand the construction of FIS −

- **Rule Base** − It contains fuzzy IF-THEN rules.
- **Database** − It defines the membership functions of fuzzy sets used in fuzzy rules.
- **Decision-making Unit** − It performs operation on rules.
- **Fuzzification Interface Unit** − It converts the crisp quantities into fuzzy quantities.
- **Defuzzification Interface Unit** − It converts the fuzzy quantities into crisp quantities. Following is a block diagram of fuzzy interference system.



Working of FIS

The working of the FIS consists of the following steps −

- A fuzzification unit supports the application of numerous fuzzification methods, and converts the crisp input into fuzzy input.
- A knowledge base - collection of rule base and database is formed upon the conversion of crisp input into fuzzy input.
- The defuzzification unit fuzzy input is finally converted into crisp output.

Methods of FIS

Let us now discuss the different methods of FIS. Following are the two important methods of FIS, having different consequent of fuzzy rules −

- Mamdani Fuzzy Inference System
- Takagi-Sugeno Fuzzy Model (TS Method)

Mamdani Fuzzy Inference System

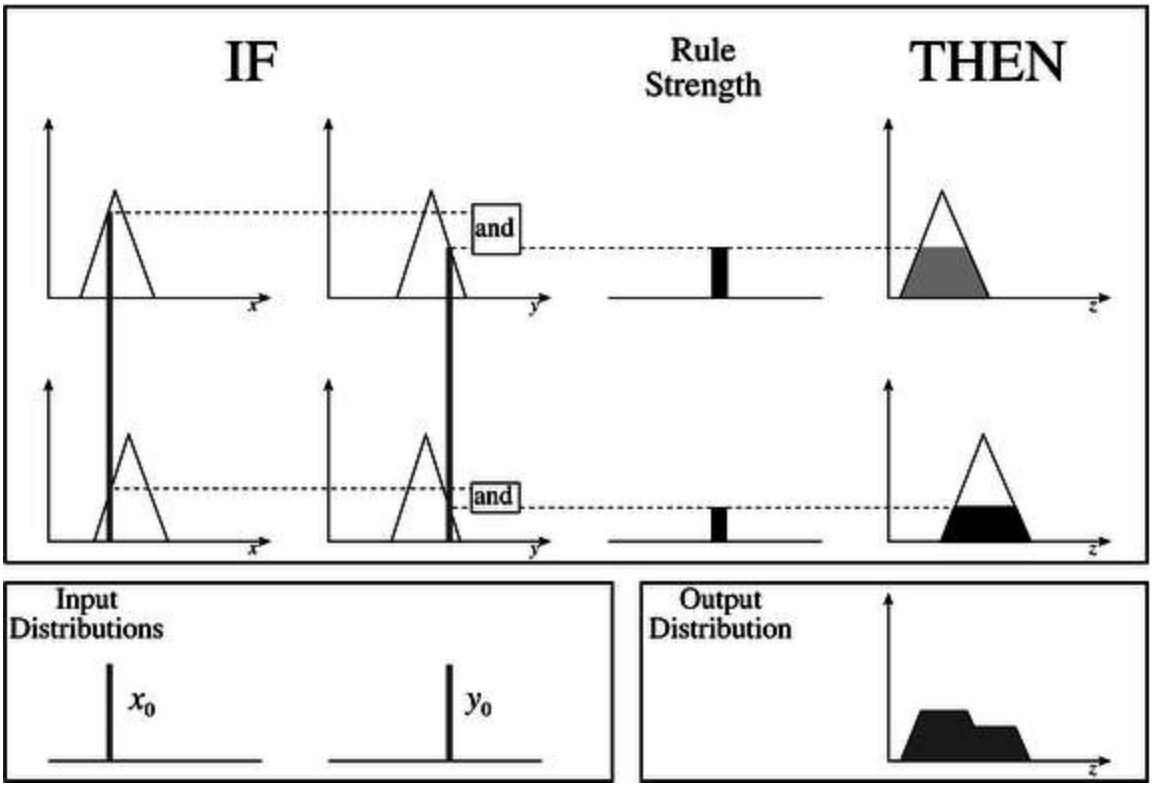
This system was proposed in 1975 by Ebhasim Mamdani. Basically, it was anticipated to control a steam engine and boiler combination by synthesizing a set of fuzzy rules obtained from people working on the system.

Steps for Computing the Output

Following steps need to be followed to compute the output from this FIS −

- **Step 1** − Set of fuzzy rules need to be determined in this step.
- **Step 2** − In this step, by using input membership function, the input would be made fuzzy.
- **Step 3** − Now establish the rule strength by combining the fuzzified inputs according to fuzzy rules.
- **Step 4** − In this step, determine the consequent of rule by combining the rule strength and the output membership function.
- **Step 5** − For getting output distribution combine all the consequents.
- **Step 6** − Finally, a defuzzified output distribution is obtained.

Following is a block diagram of Mamdani Fuzzy Interface System.



Takagi-Sugeno Fuzzy Model (TS Method)

This model was proposed by Takagi, Sugeno and Kang in 1985. Format of this rule is given as −

IF x is A and y is B THEN Z = f(x,y)

Here, *AB* are fuzzy sets in antecedents and *z = f(x,y)* is a crisp function in the consequent.

Fuzzy Inference Process

The fuzzy inference process under Takagi-Sugeno Fuzzy Model (TS Method) works in the following way −

- **Step 1: Fuzzifying the inputs** − Here, the inputs of the system are made fuzzy.
- **Step 2: Applying the fuzzy operator** − In this step, the fuzzy operators must be applied to get the output.

Rule Format of the Sugeno Form

The rule format of Sugeno form is given by −

if 7 = x and 9 = y then output is z = ax+by+c

Comparison between the two methods

Let us now understand the comparison between the Mamdani System and the Sugeno Model.

- **Output Membership Function** − The main difference between them is on the basis of output membership function. The Sugeno output membership functions are either linear or constant.
- **Aggregation and Defuzzification Procedure** − The difference between them also lies in the consequence of fuzzy rules and due to the same their aggregation and defuzzification procedure also differs.
- **Mathematical Rules** − More mathematical rules exist for the Sugeno rule than the Mamdani rule.
- **Adjustable Parameters** − The Sugeno controller has more adjustable parameters than the Mamdani controller.

We have studied in our previous chapters that Fuzzy Logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" logic. It deals with reasoning that is approximate rather than precise to solve problems in a way that more resembles human logic, hence database querying process by the two valued realization of Boolean algebra is not adequate.

Fuzzy Scenario of Relations on Databases

The Fuzzy Scenario of Relations on Databases can be understood with the help of the following example −

Example

Suppose we have a database having the records of persons who visited India. In simple database, we will have the entries made in the following way −

Name	Age	Citizen	Visited Country	Days Spent	Year of Visit
John Smith	35	U.S.	India	41	1999
John Smith	35	U.S.	Italy	72	1999

John Smith	35	U.S.	Japan	31	1999

Now, if anyone queries about the person who visited India and Japan in the year 99 and is the citizen of US, then the output will show two entries having the name of John Smith. This is simple query generating simple output.

But what if we want to know whether the person in the above query is young or not. According to the above result, the age of the person is 35 years. But can we assume the person to be young or not? Similarly, same thing can be applied on the other fields like days spent, year of visit, etc.

The solution of the above issues can be found with the help of Fuzzy Value sets as follows −

- FV(Age){ very young, young, somewhat old, old }
- FV(Days Spent){ barely few days, few days, quite a few days, many days }
- FV(Year of Visit){distant past, recent past, recent }
- Now if any query will have the fuzzy value then the result will also be fuzzy in nature.

Fuzzy Query System

A fuzzy query system is an interface to users to get information from the database using (quasi) natural language sentences. Many fuzzy query implementations have been proposed, resulting in slightly different languages. Although there are some variations according to the particularities of different implementations, the answer to a fuzzy query sentence is generally a list of records, ranked by the degree of matching.

In modeling natural language statements, quantified statements play an important role. It means that NL heavily depends on quantifying construction which often includes fuzzy concepts like “almost all”, “many”, etc. Following are a few examples of quantifying propositions −

- Every student passed the exam.
- Every sport car is expensive.
- Many students passed the exam.
- Many sports cars are expensive.

In the above examples, the quantifiers “Every” and “Many” are applied to the crisp restrictions “students” as well as crisp scope “(person who)passed the exam” and “cars” as well as crisp scope ”sports”.

Fuzzy Events, Fuzzy Means and Fuzzy Variances

With the help of an example, we can understand the above concepts. Let us assume that we are a shareholder of a company named ABC. And at present the company is selling each of its share for ₹40. There are three different companies whose business is similar to ABC but these are offering their shares at different rates - ₹100 a share, ₹85 a share and ₹60 a share respectively.

Now the probability distribution of this price takeover is as follows −

Price	₹100	₹85	₹60
Probability	0.3	0.5	0.2

Now, from the standard probability theory, the above distribution gives a mean of expected price as below −

$$100\times0.3+85\times0.5+60\times0.2=84.5100\times0.3+85\times0.5+60\times0.2=84.5$$

And, from the standard probability theory, the above distribution gives a variance of expected price as below −

$$(100-84.5)2\times0.3+(85-84.5)2\times0.5+(60-84.5)2\times0.2=124.825(100-84.5)2\times0.3+(85-84.5)2\times0.5+(60-84.5)2\times0.2=124.825$$

Suppose the degree of membership of 100 in this set is 0.7, that of 85 is 1, and the degree of membership is 0.5 for the value 60. These can be reflected in the following fuzzy set −

$$\{0.7100,185,0.560,\}\{0.7100,185,0.560,\}$$

The fuzzy set obtained in this manner is called a fuzzy event.

We want the probability of the fuzzy event for which our calculation gives −

$$0.7\times0.3+1\times0.5+0.5\times0.2=0.21+0.5+0.1=0.810.7\times0.3+1\times0.5+0.5\times0.2=0.21+0.5+0.1=0.81$$

Now, we need to calculate the fuzzy mean and the fuzzy variance, the calculation is as follows −

Fuzzy_mean $=(10.81)\times(100\times0.7\times0.3+85\times1\times0.5+60\times0.5\times0.2)=(10.81)\times(100\times0.7\times0.3+85\times1\times0.5+60\times0.5\times0.2)$

$=85.8=85.8$

Fuzzy_Variance $=7496.91-7361.91=135.27$