Session: Ant Colony Optimization

Course Title: Computational Intelligence
Course Code: 19CSE422A

Course Leader: Dr. Vaishali R. Kulkarni

Assistant Professor, Department of Computer Science and Engineering Faculty of Engineering and Technology
Ramaiah University of Applied Sciences, Bengaluru
Email: vaishali.cs.et@msruas.ac.in

Tel: +91-804-906-5555 Ext:2325 Website: www.msruas.ac.in/staff/fet_cse#Vaishali





Objectives of this Session

I wish to:

- 1. Highlight some interesting facts about biological ants
- 2. Introduce the intelligent social foraging behavior of ants
- 3. Discuss the random exploration and stigmergy-based foraging
- 4. Discuss the simple ant colony optimization algorithm (SACO)
- 5. Discuss the pros and cons of the SACO algorithm
- 6. Discuss the ant system (AS) algorithm
- 7. Introduce the variants of ant colony optimization
- 8. Discuss the influence of the various parameters on the effectiveness of algorithm

Intended Outcomes of this Session

At the end of this session, the student will be able to:

- 1. Summarize the interesting facts about ants in nature
- 2. Discuss the stigmergy-based foraging behavior of ants
- 3. Develop code for ant decision process
- 4. Develop code for pheromone depositing and evaporation
- 5. Develop the simple ant colony optimization (ACO) algorithm
- 6. Develop the ant systems algorithm
- 7. Distinguish between the numerous variants of ACO
- 8. Choose good values of parameters of ACO and
- 9. List the applications of ACO



Recommended Resources for this Session

- 1. Engelbrecht, A. P. (2007). *Computational intelligence: An introduction*. Chichester, England, John Wiley & Sons.
- 2. Konar, A. (2005). *Computational Intelligence: Principles, Techniques and Applications*. Secaucus, NJ, USA, Springer-Verlag New York, Inc.

Ants and their Colonies





 Ants have six legs, each with three joints. The legs are very strong so they can run very quickly



- Ants have six legs, each with three joints. The legs are very strong so they can run very quickly
- Ants can lift 20 times their own body weight



- Ants have six legs, each with three joints. The legs are very strong so they can run very quickly
- Ants can lift 20 times their own body weight
- An ant brain has about 250,000 brain cells (humans have 10^{10})

- Ants have six legs, each with three joints. The legs are very strong so they can run very quickly
- Ants can lift 20 times their own body weight
- An ant brain has about 250,000 brain cells (humans have 10^{10})
- The average life expectancy of an ant is 45-60 days



- Ants have six legs, each with three joints. The legs are very strong so they can run very quickly
- Ants can lift 20 times their own body weight
- An ant brain has about 250,000 brain cells (humans have 10¹⁰)
- The average life expectancy of an ant is 45-60 days
- Ants use their antennae for touch and sense of smell



- Ants have six legs, each with three joints. The legs are very strong so they can run very quickly
- Ants can lift 20 times their own body weight
- An ant brain has about 250,000 brain cells (humans have 10^{10})
- The average life expectancy of an ant is 45-60 days
- Ants use their antennae for touch and sense of smell
- Each ant has a pair of large, strong jaws. The jaws open and shut sideways like a pair of scissors



- Ants have six legs, each with three joints. The legs are very strong so they can run very quickly
- Ants can lift 20 times their own body weight
- An ant brain has about 250,000 brain cells (humans have 10¹⁰)
- The average life expectancy of an ant is 45-60 days
- Ants use their antennae for touch and sense of smell
- Each ant has a pair of large, strong jaws. The jaws open and shut sideways like a pair of scissors
- Adult ants cannot chew and swallow solid food. Instead, they swallow the juice which they squeeze from pieces of food





- Ants have six legs, each with three joints. The legs are very strong so they can run very quickly
- Ants can lift 20 times their own body weight
- An ant brain has about 250,000 brain cells (humans have 10¹⁰)
- The average life expectancy of an ant is 45-60 days
- Ants use their antennae for touch and sense of smell
- Each ant has a pair of large, strong jaws. The jaws open and shut sideways like a pair of scissors
- Adult ants cannot chew and swallow solid food. Instead, they swallow the juice which they squeeze from pieces of food
- The ant has two compound eyes, each of which has many smaller eyes



 The abdomen of the ant contains two stomachs. One stomach holds the food for itself and second is for food to be shared with other ants



- The abdomen of the ant contains two stomachs. One stomach holds the food for itself and second is for food to be shared with other ants
- Like all insects, the outside of their body is covered with a hard armour this is called the exoskeleton



- The abdomen of the ant contains two stomachs. One stomach holds the food for itself and second is for food to be shared with other ants
- Like all insects, the outside of their body is covered with a hard armour this is called the exoskeleton
- Ants have four distinct growing stages, the egg, larva, pupa and the adult



- The abdomen of the ant contains two stomachs. One stomach holds the food for itself and second is for food to be shared with other ants
- Like all insects, the outside of their body is covered with a hard armour this is called the exoskeleton
- Ants have four distinct growing stages, the egg, larva, pupa and the adult
- Biologists classify ants as a special group of wasps



- The abdomen of the ant contains two stomachs. One stomach holds the food for itself and second is for food to be shared with other ants
- Like all insects, the outside of their body is covered with a hard armour this is called the exoskeleton
- Ants have four distinct growing stages, the egg, larva, pupa and the adult
- Biologists classify ants as a special group of wasps
- There are over 10000 known species of ants. Each ant colony has at least one or more queens



- The abdomen of the ant contains two stomachs. One stomach holds the food for itself and second is for food to be shared with other ants
- Like all insects, the outside of their body is covered with a hard armour this is called the exoskeleton
- Ants have four distinct growing stages, the egg, larva, pupa and the adult
- Biologists classify ants as a special group of wasps
- There are over 10000 known species of ants. Each ant colony has at least one or more queens
- The job of the queen is to lay eggs which the worker ants look after. Worker ants are sterile, they look for food, look after the young, and defend the nest from unwanted visitors



• Ants are clean and tidy insects. Some worker ants are given the job of moving rubbish from the nest to a special dump!



- Ants are clean and tidy insects. Some worker ants are given the job of moving rubbish from the nest to a special dump!
- Each colony of ants has its own smell. Thus, intruders can be recognized immediately



- Ants are clean and tidy insects. Some worker ants are given the job of moving rubbish from the nest to a special dump!
- Each colony of ants has its own smell. Thus, intruders can be recognized immediately
- Many ants, such as the common Red, have a sting which they use to defend their nest

- Ants are clean and tidy insects. Some worker ants are given the job of moving rubbish from the nest to a special dump!
- Each colony of ants has its own smell. Thus, intruders can be recognized immediately
- Many ants, such as the common Red, have a sting which they use to defend their nest
- The common Black Ants and Wood Ants have no sting, but they can squirt formic acid. Some birds put ants in their feathers because the formic acid gets rid of parasites



- Ants are clean and tidy insects. Some worker ants are given the job of moving rubbish from the nest to a special dump!
- Each colony of ants has its own smell. Thus, intruders can be recognized immediately
- Many ants, such as the common Red, have a sting which they
 use to defend their nest
- The common Black Ants and Wood Ants have no sting, but they can squirt formic acid. Some birds put ants in their feathers because the formic acid gets rid of parasites
- The Slave-Maker Ant raids the nests of other ants and steals their pupae. When they hatch, they work as slaves within the colony



• At night, the worker ants move the eggs and larvae deep into the nest to protect them from the cold. During the day, they move the eggs and larvae to the top of the nest for warmth



- At night, the worker ants move the eggs and larvae deep into the nest to protect them from the cold. During the day, they move the eggs and larvae to the top of the nest for warmth
- If a worker ant finds a source of food, it leaves a trail of scent so that the other ants can find the food too



- At night, the worker ants move the eggs and larvae deep into the nest to protect them from the cold. During the day, they move the eggs and larvae to the top of the nest for warmth
- If a worker ant finds a source of food, it leaves a trail of scent so that the other ants can find the food too
- Army Ants are nomadic and they are always moving. They carry their larvae and their eggs with them in a long column

- At night, the worker ants move the eggs and larvae deep into the nest to protect them from the cold. During the day, they move the eggs and larvae to the top of the nest for warmth
- If a worker ant finds a source of food, it leaves a trail of scent so that the other ants can find the food too
- Army Ants are nomadic and they are always moving. They carry their larvae and their eggs with them in a long column
- The Leaf Cutter Ants are farmers. They cut out pieces of leaves which they take back to their nests. They chew them into a pulp and a special fungus grows on it

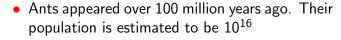




Ants appeared over 100 million years ago. Their population is estimated to be 10^{16}

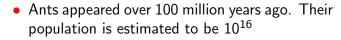






Ants are social insects. They live in colonies of 30 to millions

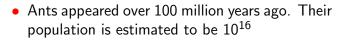




Ants are social insects. They live in colonies of 30 to millions

Ant foraging, division of labor, cemetery organization and brood care, and construction of nests have been studied extensively





Ants are social insects. They live in colonies of 30 to millions

 Ant foraging, division of labor, cemetery organization and brood care, and construction of nests have been studied extensively

 Most research efforts are concentrated on developing algorithms of foraging behavior





• Ants appeared over 100 million years ago. Their population is estimated to be 10^{16}

Ants are social insects. They live in colonies of 30 to millions

 Ant foraging, division of labor, cemetery organization and brood care, and construction of nests have been studied extensively

 Most research efforts are concentrated on developing algorithms of foraging behavior

Models have been developed for division of labor, cooperative support, self-assembly, and cemetery organization as well



Ant Colony Optimization



Ants possess enormous ability to find the shortest path between their nest and a food source

Ant Colony Optimization



- Ants possess enormous ability to find the shortest path between their nest and a food source
- This observation inspired Marco Dorigo (1992) to develop the first algorithmic model of the ant foraging behavior

Ant Colony Optimization



- Ants possess enormous ability to find the shortest path between their nest and a food source
- This observation inspired Marco Dorigo (1992) to develop the first algorithmic model of the ant foraging behavior
- Algorithms developed based on studies of ant foraging are collectively called instances of the ant colony optimization (ACO) meta-heuristic

Ant Colony Optimization



- Ants possess enormous ability to find the shortest path between their nest and a food source
- This observation inspired Marco Dorigo (1992) to develop the first algorithmic model of the ant foraging behavior
- Algorithms developed based on studies of ant foraging are collectively called instances of the ant colony optimization (ACO) meta-heuristic
- Since ACO models the foraging behavior of ants, we start from just there

Ants exhibit a random activity pattern in the search for food.
 When food is located, activity patterns get more organized and more and more ants follow the same path



- Ants exhibit a random activity pattern in the search for food.
 When food is located, activity patterns get more organized and more and more ants follow the same path
- This emergent behavior is a result of a recruitment mechanism whereby ants that have located a food source influence other ants towards the food source



- Ants exhibit a random activity pattern in the search for food.
 When food is located, activity patterns get more organized and more and more ants follow the same path
- This emergent behavior is a result of a recruitment mechanism whereby ants that have located a food source influence other ants towards the food source
- The recruitment mechanism differs across species, and can either be in the form of direct contact, or indirect communication



- Ants exhibit a random activity pattern in the search for food.
 When food is located, activity patterns get more organized and more and more ants follow the same path
- This emergent behavior is a result of a recruitment mechanism whereby ants that have located a food source influence other ants towards the food source
- The recruitment mechanism differs across species, and can either be in the form of direct contact, or indirect communication
- Most ant species use the latter form of recruitment in which the communication is via pheromone trails



- Ants exhibit a random activity pattern in the search for food.
 When food is located, activity patterns get more organized and more and more ants follow the same path
- This emergent behavior is a result of a recruitment mechanism whereby ants that have located a food source influence other ants towards the food source
- The recruitment mechanism differs across species, and can either be in the form of direct contact, or indirect communication
- Most ant species use the latter form of recruitment in which the communication is via pheromone trails
- When an ant locates a food source, it carries food to the nest while laying pheromone along the trail



 Forager ants decide which path to follow based on the pheromone concentrations on the different paths



- Forager ants decide which path to follow based on the pheromone concentrations on the different paths
- Paths with a larger pheromone concentration have a higher probability of being selected



- Forager ants decide which path to follow based on the pheromone concentrations on the different paths
- Paths with a larger pheromone concentration have a higher probability of being selected
- As more ants follow a specific trail, the desirability of that path is reinforced by more pheromone being deposited by the foragers, which attracts more ants to follow that path

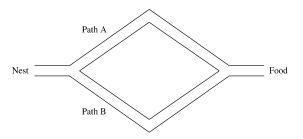


- Forager ants decide which path to follow based on the pheromone concentrations on the different paths
- Paths with a larger pheromone concentration have a higher probability of being selected
- As more ants follow a specific trail, the desirability of that path is reinforced by more pheromone being deposited by the foragers, which attracts more ants to follow that path
- The collective behavior that results is a form of autocatalytic behavior, where positive feedback about a food path causes that path to be followed by more and more ants

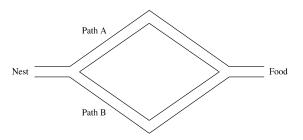


- Forager ants decide which path to follow based on the pheromone concentrations on the different paths
- Paths with a larger pheromone concentration have a higher probability of being selected
- As more ants follow a specific trail, the desirability of that path is reinforced by more pheromone being deposited by the foragers, which attracts more ants to follow that path
- The collective behavior that results is a form of autocatalytic behavior, where positive feedback about a food path causes that path to be followed by more and more ants
- The indirect communication where ants modify their environment by laying of pheromones to influence the behavior of other ants is referred to as stigmergy



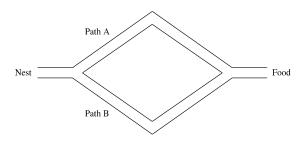


 An ant nest is separated from the food source by a bridge having two equally long branches free of any pheromones



- An ant nest is separated from the food source by a bridge having two equally long branches free of any pheromones
- After a finite time period, one of the branches is selected and most of the ants follow the path even though both branches are equally long





- An ant nest is separated from the food source by a bridge having two equally long branches free of any pheromones
- After a finite time period, one of the branches is selected and most of the ants follow the path even though both branches are equally long
- The selection of that branch is due to random fluctuations that caused higher concentrations on that path

• Let us assume that each ant deposits the same amount of pheromone and that pheromone does not evaporate



- Let us assume that each ant deposits the same amount of pheromone and that pheromone does not evaporate
- Let $n_A(t)$ and $n_B(t)$ denote the number of ants on paths A and B respectively at time step t



- Let us assume that each ant deposits the same amount of pheromone and that pheromone does not evaporate
- Let $n_A(t)$ and $n_B(t)$ denote the number of ants on paths A and B respectively at time step t
- The probability of the next ant to choose path A at time step t+1 is given as:

$$P_A(t+1) = \frac{(c+n_A(t))^{\alpha}}{(c+n_A(t))^{\alpha} + (c+n_B(t))^{\alpha}} = 1 - P_B(t+1)$$

- Let us assume that each ant deposits the same amount of pheromone and that pheromone does not evaporate
- Let $n_A(t)$ and $n_B(t)$ denote the number of ants on paths A and B respectively at time step t
- The probability of the next ant to choose path A at time step t+1 is given as:

$$P_A(t+1) = \frac{(c+n_A(t))^{\alpha}}{(c+n_A(t))^{\alpha} + (c+n_B(t))^{\alpha}} = 1 - P_B(t+1)$$

ullet Here c quantifies the degree of attraction of an unexplored branch and lpha is the bias to using pheromone deposits in the decision process



- Let us assume that each ant deposits the same amount of pheromone and that pheromone does not evaporate
- Let $n_A(t)$ and $n_B(t)$ denote the number of ants on paths A and B respectively at time step t
- The probability of the next ant to choose path A at time step t+1 is given as:

$$P_A(t+1) = \frac{(c+n_A(t))^{\alpha}}{(c+n_A(t))^{\alpha} + (c+n_B(t))^{\alpha}} = 1 - P_B(t+1)$$

- ullet Here c quantifies the degree of attraction of an unexplored branch and lpha is the bias to using pheromone deposits in the decision process
- The larger the value of α , the higher the probability that the next ant follows the path with a higher pheromone concentration $\alpha \approx 2$ and $c \approx 20$ matches to the observed behavior

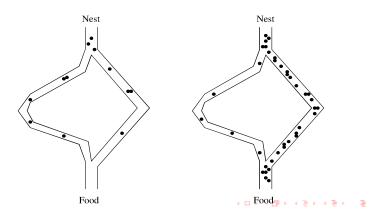
Extended Binary Bridge Experiment

• The decision rule of an ant at the binary bridge is: If $U(0,1) \le P_A(t+1)$ then follow path A; otherwise, follow path B



Extended Binary Bridge Experiment

- The decision rule of an ant at the binary bridge is: If $U(0,1) \le P_A(t+1)$ then follow path A; otherwise, follow path B
- Consider the bridge that has unequal paths





• Stigmergy refers to a form of indirect communication mediated by modifications of the environment. Sign-based stigmergy facilitates communication via a signaling mechanism via chemical compounds deposited by ants



- Stigmergy refers to a form of indirect communication mediated by modifications of the environment. Sign-based stigmergy facilitates communication via a signaling mechanism via chemical compounds deposited by ants
- Each ant drops an amount of pheromone as it moves from food source to the nest

- Stigmergy refers to a form of indirect communication mediated by modifications of the environment. Sign-based stigmergy facilitates communication via a signaling mechanism via chemical compounds deposited by ants
- Each ant drops an amount of pheromone as it moves from food source to the nest
- Future ants choose their paths based on the amount of pheromone. Higher pheromone concentration means higher chance that the path is chosen



- Stigmergy refers to a form of indirect communication mediated by modifications of the environment. Sign-based stigmergy facilitates communication via a signaling mechanism via chemical compounds deposited by ants
- Each ant drops an amount of pheromone as it moves from food source to the nest
- Future ants choose their paths based on the amount of pheromone. Higher pheromone concentration means higher chance that the path is chosen
- Over time, shorter paths have stronger pheromone concentrations since they are chosen by ants most often



- Stigmergy refers to a form of indirect communication mediated by modifications of the environment. Sign-based stigmergy facilitates communication via a signaling mechanism via chemical compounds deposited by ants
- Each ant drops an amount of pheromone as it moves from food source to the nest
- Future ants choose their paths based on the amount of pheromone. Higher pheromone concentration means higher chance that the path is chosen
- Over time, shorter paths have stronger pheromone concentrations since they are chosen by ants most often
- Pheromone evaporates over time. The pheromone concentrations on the longer paths decrease more quickly than on the shorter paths

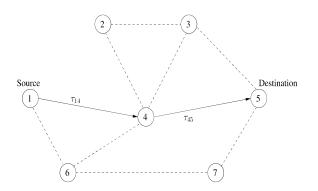
Artificial Ant Decision Process

Algorithm 7.1. Artificial Ant Decision Process

```
    Let r ~ U(0,1);
    for Each potential path A do
    Determine P<sub>A</sub>;
    if r < P<sub>A</sub> then
    Follow path A;
    Break;
    end if
```



Simple Ant Colony Optimization (SACO)



• SACO provides solution to the problem of finding the shortest path between two nodes on a graph G = (V, E). V is the set of nodes (vertexes) and E is a matrix representing the connections (edges) between nodes



• The graph has $n_G = |V|$ nodes



- The graph has $n_G = |V|$ nodes
- The length L^k of the path constructed by ant k is the number of hops in the path from the origin to the destination



- The graph has $n_G = |V|$ nodes
- The length L^k of the path constructed by ant k is the number of hops in the path from the origin to the destination
- ullet Each edge (i,j) has a pheromone concentration au_{ij}

- The graph has $n_G = |V|$ nodes
- The length L^k of the path constructed by ant k is the number of hops in the path from the origin to the destination
- ullet Each edge (i,j) has a pheromone concentration au_{ij}
- In SACO, each edge is assigned a small random initial pheromone $\tau_{ii}(0)$



- The graph has $n_G = |V|$ nodes
- The length L^k of the path constructed by ant k is the number of hops in the path from the origin to the destination
- ullet Each edge (i,j) has a pheromone concentration au_{ij}
- In SACO, each edge is assigned a small random initial pheromone $au_{ij}(0)$
- A number of ants $k=1,\ldots,n_k$, are placed on the source node. At each node, each ant executes a decision policy to determine the next link of the path



- The graph has $n_G = |V|$ nodes
- The length L^k of the path constructed by ant k is the number of hops in the path from the origin to the destination
- Each edge (i,j) has a pheromone concentration au_{ij}
- ullet In SACO, each edge is assigned a small random initial pheromone $au_{ij}(0)$
- A number of ants $k=1,\ldots,n_k$, are placed on the source node. At each node, each ant executes a decision policy to determine the next link of the path
- Thus each ant incrementally and iteratively constructs a path to the destination



• If ant k is currently located at node i, it selects the next node $j \in \mathcal{N}_i^k$ based on the transition probability

$$p_{ij}^{k}(t) = \begin{cases} \frac{\tau_{ij}^{\alpha}(t)}{\sum\limits_{j \in \mathcal{N}_{i}^{k}} \tau_{ij}^{\alpha}(t)} & \text{if } j \in \mathcal{N}_{i}^{k} \\ 0 & \text{if } j \notin \mathcal{N}_{i}^{k} \end{cases}$$

• If ant k is currently located at node i, it selects the next node $j \in \mathcal{N}_i^k$ based on the transition probability

$$p_{ij}^{k}(t) = \begin{cases} \frac{\tau_{ij}^{\alpha}(t)}{\sum\limits_{j \in \mathcal{N}_{i}^{k}} \tau_{ij}^{\alpha}(t)} & \text{if } j \in \mathcal{N}_{i}^{k} \\ 0 & \text{if } j \notin \mathcal{N}_{i}^{k} \end{cases}$$

• Here, $j \in \mathcal{N}_i^k$ is the set of feasible nodes connected to node i, with respect to ant k. If, for any node i and ant k, $\mathcal{N}_i^k = \emptyset$, then the predecessor to node i is included in \mathcal{N}_i^k



• If ant k is currently located at node i, it selects the next node $j \in \mathcal{N}_i^k$ based on the transition probability

$$p_{ij}^{k}(t) = \begin{cases} \frac{\tau_{ij}^{\alpha}(t)}{\sum_{j \in \mathcal{N}_{i}^{k}} \tau_{ij}^{\alpha}(t)} & \text{if } j \in \mathcal{N}_{i}^{k} \\ 0 & \text{if } j \notin \mathcal{N}_{i}^{k} \end{cases}$$

- Here, $j \in \mathcal{N}_i^k$ is the set of feasible nodes connected to node i, with respect to ant k. If, for any node i and ant k, $\mathcal{N}_i^k = \emptyset$, then the predecessor to node i is included in \mathcal{N}_i^k
- Loops, if any, are removed once the destination node has been reached



• If ant k is currently located at node i, it selects the next node $j \in \mathcal{N}_i^k$ based on the transition probability

$$p_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}^{\alpha}(t)}{\sum\limits_{j \in \mathcal{N}_i^k} \tau_{ij}^{\alpha}(t)} & \text{if } j \in \mathcal{N}_i^k \\ 0 & \text{if } j \notin \mathcal{N}_i^k \end{cases}$$

- Here, $j \in \mathcal{N}_i^k$ is the set of feasible nodes connected to node i, with respect to ant k. If, for any node i and ant k, $\mathcal{N}_i^k = \emptyset$, then the predecessor to node i is included in \mathcal{N}_i^k
- Loops, if any, are removed once the destination node has been reached
- ullet Here, lpha is a positive constant used to amplify the influence of pheromone concentrations



• After all ants have constructed a complete path from the origin node to the destination node, and all loops have been removed, each ant retraces its path to the source node and deposits a pheromone amount $\Delta \tau^k_{ij}(t) \propto \frac{1}{L^k(t)}$



- After all ants have constructed a complete path from the origin node to the destination node, and all loops have been removed, each ant retraces its path to the source node and deposits a pheromone amount $\Delta \tau_{ij}^k(t) \propto \frac{1}{L^k(t)}$
- ullet This means, $au_{ij}(t+1) = au_{ij}(t) + \sum_{k=1}^{n_k} \Delta au_{ij}^k(t)$



- After all ants have constructed a complete path from the origin node to the destination node, and all loops have been removed, each ant retraces its path to the source node and deposits a pheromone amount $\Delta \tau^k_{ij}(t) \propto \frac{1}{L^k(t)}$
- ullet This means, $au_{ij}(t+1) = au_{ij}(t) + \sum_{k=1}^{n_k} \Delta au_{ij}^k(t)$
- Here, $\Delta \tau_{ij}^k(t)$ is the quality of the constructed path (its length). Any other measure of quality can be used

- After all ants have constructed a complete path from the origin node to the destination node, and all loops have been removed, each ant retraces its path to the source node and deposits a pheromone amount $\Delta \tau^k_{ij}(t) \propto \frac{1}{L^k(t)}$
- ullet This means, $au_{ij}(t+1) = au_{ij}(t) + \sum_{k=1}^{n_k} \Delta au_{ij}^k(t)$
- Here, $\Delta \tau_{ij}^k(t)$ is the quality of the constructed path (its length). Any other measure of quality can be used
- In implicit evaluation of paths, all ants deposit the same amount of pheromone. In explicit, pheromone amounts are proportional to some quality measure of constructed solutions

- After all ants have constructed a complete path from the origin node to the destination node, and all loops have been removed, each ant retraces its path to the source node and deposits a pheromone amount $\Delta \tau^k_{ij}(t) \propto \frac{1}{L^k(t)}$
- ullet This means, $au_{ij}(t+1) = au_{ij}(t) + \sum_{k=1}^{n_k} \Delta au_{ij}^k(t)$
- Here, $\Delta \tau_{ij}^k(t)$ is the quality of the constructed path (its length). Any other measure of quality can be used
- In implicit evaluation of paths, all ants deposit the same amount of pheromone. In explicit, pheromone amounts are proportional to some quality measure of constructed solutions
- SACO terminates when a maximum number of iterations, n_t , has been exceeded, an acceptable solution has been found, or all ants follow the same path



Simple ACO Algorithm

Algorithm 17.2 Simple ACO Algorithm

```
Initialize \tau_{ij}(0) to small random values;
Let t = 0:
Place n_k ants on the origin node:
repeat
   for each ant k = 1, ..., n_k do
      //Construct a path x^k(t);
      x^k(t) = \emptyset:
      repeat
          Select next node based on the probability defined in equation (17.2);
          Add link (i, j) to path x^k(t);
      until destination node has been reached;
      Remove all loops from x^k(t);
      Calculate the path length f(x^k(t));
   end
   for each link (i, j) of the graph do
      //pheromone evaporation;
      Reduce the pheromone, \tau_{ij}(t), using equation (17.5);
   end
   for each ant k = 1, ..., n_k do
      for each link (i, j) of x^k(t) do
          \Delta \tau^k = \frac{1}{f(x^k(t))};
          Update \tau_{ij} using equation (17.4);
       end
   end
   t = t + 1;
until stopping condition is true;
Return the path x^k(t) with smallest f(x^k(t)) as the solution:
```



 The initial experiments on the binary bridge problem found that ants rapidly converge to a solution, and that little time is spent exploring alternative paths



- The initial experiments on the binary bridge problem found that ants rapidly converge to a solution, and that little time is spent exploring alternative paths
- To persuade ants to explore, and to prevent premature convergence, pheromone intensities on links are allowed to "evaporate" at each iteration of the algorithm

- The initial experiments on the binary bridge problem found that ants rapidly converge to a solution, and that little time is spent exploring alternative paths
- To persuade ants to explore, and to prevent premature convergence, pheromone intensities on links are allowed to "evaporate" at each iteration of the algorithm
- For each link (i,j) let $\tau_{ij}(t) \leftarrow (1-\rho)\tau_{ij}(t)$, with $\rho \in [0,1]$

- The initial experiments on the binary bridge problem found that ants rapidly converge to a solution, and that little time is spent exploring alternative paths
- To persuade ants to explore, and to prevent premature convergence, pheromone intensities on links are allowed to "evaporate" at each iteration of the algorithm
- For each link (i,j) let $au_{ij}(t) \leftarrow (1ho) au_{ij}(t)$, with $ho \in [0,1]$
- The constant ρ specifies the rate at which pheromones evaporate, causing ants to forget previous decisions



- The initial experiments on the binary bridge problem found that ants rapidly converge to a solution, and that little time is spent exploring alternative paths
- To persuade ants to explore, and to prevent premature convergence, pheromone intensities on links are allowed to "evaporate" at each iteration of the algorithm
- For each link (i,j) let $\tau_{ij}(t) \leftarrow (1-\rho)\tau_{ij}(t)$, with $\rho \in [0,1]$
- The constant ρ specifies the rate at which pheromones evaporate, causing ants to forget previous decisions
- ullet For large values of ho, pheromone evaporates rapidly, while for small values it evaporates slower



- The initial experiments on the binary bridge problem found that ants rapidly converge to a solution, and that little time is spent exploring alternative paths
- To persuade ants to explore, and to prevent premature convergence, pheromone intensities on links are allowed to "evaporate" at each iteration of the algorithm
- For each link (i,j) let $au_{ij}(t) \leftarrow (1-\rho) au_{ij}(t)$, with $\rho \in [0,1]$
- The constant ρ specifies the rate at which pheromones evaporate, causing ants to forget previous decisions
- For large values of ρ , pheromone evaporates rapidly, while for small values it evaporates slower
- The more pheromones evaporate, the more random the search becomes, facilitating better exploration. For $\rho=1$, the search is completely random



 SACO works well for very simple graphs. The shortest path is selected most often



- SACO works well for very simple graphs. The shortest path is selected most often
- For larger graphs, performance deteriorates. The algorithm becomes less stable and more sensitive to parameter choices



- SACO works well for very simple graphs. The shortest path is selected most often
- For larger graphs, performance deteriorates. The algorithm becomes less stable and more sensitive to parameter choices
- Convergence to the shortest path is good for a small number of ants, while too many ants cause non-convergent behavior

- SACO works well for very simple graphs. The shortest path is selected most often
- For larger graphs, performance deteriorates. The algorithm becomes less stable and more sensitive to parameter choices
- Convergence to the shortest path is good for a small number of ants, while too many ants cause non-convergent behavior
- Evaporation becomes more important for more complex graphs. If $\rho=0$, the algorithm does not converge. If a large ρ is used, the algorithm often converged to sub-optimal solutions

- SACO works well for very simple graphs. The shortest path is selected most often
- For larger graphs, performance deteriorates. The algorithm becomes less stable and more sensitive to parameter choices
- Convergence to the shortest path is good for a small number of ants, while too many ants cause non-convergent behavior
- Evaporation becomes more important for more complex graphs. If $\rho=0$, the algorithm does not converge. If a large ρ is used, the algorithm often converged to sub-optimal solutions
- For smaller α , the algorithm generally converges to the shortest path. For complex problems, large values of α result in worse convergence behavior



• AS include heuristic information to the transition probability p_{ij}^k , and adds a memory capability



- AS include heuristic information to the transition probability p_{ij}^k , and adds a memory capability
- The probability of moving from node i to node j is given as

$$p_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)}{\sum\limits_{u \in \mathcal{N}_i^k(t)} \tau_{iu}^{\alpha}(t)\eta_{iu}^{\beta}(t)} & \text{if } j \in \mathcal{N}_i^k(t) \\ 0 & \text{if } j \notin \mathcal{N}_i^k(t) \end{cases}$$

• τ_{ij} represents the a *posteriori* effectiveness of the move from node i to node j, as expressed in the pheromone intensity



- AS include heuristic information to the transition probability p_{ij}^k , and adds a memory capability
- The probability of moving from node i to node j is given as

$$p_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)}{\sum\limits_{u \in \mathcal{N}_i^k(t)} \tau_{iu}^{\alpha}(t)\eta_{iu}^{\beta}(t)} & \text{if } j \in \mathcal{N}_i^k(t) \\ 0 & \text{if } j \notin \mathcal{N}_i^k(t) \end{cases}$$

- τ_{ij} represents the a *posteriori* effectiveness of the move from node i to node j, as expressed in the pheromone intensity
- η_{ij} represents the *a priori* effectiveness (attractiveness) of the move from *i* to *j*, computed using some heuristic



- AS include heuristic information to the transition probability p_{ij}^k , and adds a memory capability
- The probability of moving from node i to node j is given as

The probability of moving from node
$$j$$
 to node j
$$p_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}^\alpha(t)\eta_{ij}^\beta(t)}{\sum\limits_{u \in \mathscr{N}_i^k(t)} \tau_{iu}^\alpha(t)\eta_{iu}^\beta(t)} & \text{if } j \in \mathscr{N}_i^k(t) \\ 0 & \text{if } j \notin \mathscr{N}_i^k(t) \end{cases}$$

- τ_{ij} represents the a *posteriori* effectiveness of the move from node i to node j, as expressed in the pheromone intensity
- η_{ij} represents the *a priori* effectiveness (attractiveness) of the move from *i* to *j*, computed using some heuristic
- The pheromone concentrations τ_{ij} indicate how profitable it has been to make a move from i to j (memory of previous best moves)



• The transition probability in AS is a balance between pheromone intensity (history of previous successful moves), τ_{ij} , and heuristic information (desirability of the move), η_{ij}



- The transition probability in AS is a balance between pheromone intensity (history of previous successful moves), τ_{ij} , and heuristic information (desirability of the move), η_{ii}
- This balances the trade-off between exploration and exploitation.
 The search process favors actions that it has found in the past and which proved to be effective, thereby exploiting knowledge obtained about the search space

- The transition probability in AS is a balance between pheromone intensity (history of previous successful moves), τ_{ij} , and heuristic information (desirability of the move), η_{ii}
- This balances the trade-off between exploration and exploitation.
 The search process favors actions that it has found in the past and which proved to be effective, thereby exploiting knowledge obtained about the search space
- In order to discover such actions, the search has to investigate previously unseen actions, thereby exploring the search space



- The transition probability in AS is a balance between pheromone intensity (history of previous successful moves), τ_{ij} , and heuristic information (desirability of the move), η_{ii}
- This balances the trade-off between exploration and exploitation.
 The search process favors actions that it has found in the past and which proved to be effective, thereby exploiting knowledge obtained about the search space
- In order to discover such actions, the search has to investigate previously unseen actions, thereby exploring the search space
- Balance between exploration and exploitation is achieved through proper selection α and β . If $\alpha=0$, no pheromone information is used, thus previous search experience is neglected



- The transition probability in AS is a balance between pheromone intensity (history of previous successful moves), τ_{ij} , and heuristic information (desirability of the move), η_{ij}
- This balances the trade-off between exploration and exploitation.
 The search process favors actions that it has found in the past and which proved to be effective, thereby exploiting knowledge obtained about the search space
- In order to discover such actions, the search has to investigate previously unseen actions, thereby exploring the search space
- Balance between exploration and exploitation is achieved through proper selection α and β . If $\alpha=0$, no pheromone information is used, thus previous search experience is neglected
- If $\beta = 0$, the attractiveness (or potential benefit) of moves is neglected and the search algorithm is similar to SACO \sim

• The heuristic information adds a bias towards the most attractive solutions. Therefore, it is problem-dependent



- The heuristic information adds a bias towards the most attractive solutions. Therefore, it is problem-dependent
- \mathcal{N}_i^k is the set of feasible nodes for ant k on node i. This may include only the immediate neighbors of node i



- The heuristic information adds a bias towards the most attractive solutions. Therefore, it is problem-dependent
- \mathcal{N}_i^k is the set of feasible nodes for ant k on node i. This may include only the immediate neighbors of node i
- Alternatively, to prevent loops, \mathcal{N}_i^k may include all nodes not yet visited by ant k



- The heuristic information adds a bias towards the most attractive solutions. Therefore, it is problem-dependent
- \mathcal{N}_i^k is the set of feasible nodes for ant k on node i. This may include only the immediate neighbors of node i
- Alternatively, to prevent loops, \mathcal{N}_i^k may include all nodes not yet visited by ant k
- For this purpose, a tabu list is usually maintained for each ant.
 As an ant visits a new node, that node is added to the ants tabu list



- The heuristic information adds a bias towards the most attractive solutions. Therefore, it is problem-dependent
- \mathcal{N}_i^k is the set of feasible nodes for ant k on node i. This may include only the immediate neighbors of node i
- Alternatively, to prevent loops, \mathcal{N}_i^k may include all nodes not yet visited by ant k
- For this purpose, a tabu list is usually maintained for each ant.
 As an ant visits a new node, that node is added to the ants tabu list
- Nodes in the tabu list are removed from \mathcal{N}_i^k ensuring that no node is visited more than once



1. After completion of a path by each ant, the pheromone on each link is updated as $\tau_{ij}(t+1) = \tau_{ij}(t) + \Delta \tau_{ij}(t)$, with

$$\Delta au_{ij}(t) = \sum_{k=1}^{n_k} \Delta au_{ij}^k(t)$$



1. After completion of a path by each ant, the pheromone on each link is updated as $\tau_{ij}(t+1) = \tau_{ij}(t) + \Delta \tau_{ij}(t)$, with

$$\Delta au_{ij}(t) = \sum_{k=1}^{n_k} \Delta au_{ij}^k(t)$$

2. Here $\Delta au^k_{ij}(t)$ is the amount of pheromone deposited by ant k on link (i,j) at time step t

1. After completion of a path by each ant, the pheromone on each link is updated as $\tau_{ij}(t+1) = \tau_{ij}(t) + \Delta \tau_{ij}(t)$, with

$$\Delta au_{ij}(t) = \sum_{k=1}^{n_k} \Delta au_{ij}^k(t)$$

- 2. Here $\Delta \tau_{ij}^k(t)$ is the amount of pheromone deposited by ant k on link (i,j) at time step t
- 3. Three variants of AS exist which differ in the way $\Delta \tau_{ij}^k(t)$ is calculated (assuming a minimization problem)

• Ant-cycle AS:

$$\Delta \tau_{ij}^k(t) = \begin{cases} \frac{Q}{f(x_k(t))} & \text{if link } (i,j) \text{ occurs in path } x^k(t) \\ 0 & \text{otherwise} \end{cases}$$

• Pheromone deposits are inversely proportional to the quality, $f(x_k(t))$, of the complete path constructed by the ant. Global information is used to update pheromone. Q is a positive constant

• Ant-cycle AS:

$$\Delta \tau_{ij}^k(t) = \begin{cases} \frac{Q}{f(x_k(t))} & \text{if link } (i,j) \text{ occurs in path } x^k(t) \\ 0 & \text{otherwise} \end{cases}$$

- Pheromone deposits are inversely proportional to the quality, $f(x_k(t))$, of the complete path constructed by the ant. Global information is used to update pheromone. Q is a positive constant
- Ant-density AS:

$$\Delta au_{ij}^k(t) = egin{cases} Q & ext{if link } (i,j) ext{ occurs in path } x^k(t) \ 0 & ext{otherwise} \end{cases}$$



• Ant-cycle AS:

$$\Delta \tau_{ij}^k(t) = \begin{cases} \frac{Q}{f(x_k(t))} & \text{if link } (i,j) \text{ occurs in path } x^k(t) \\ 0 & \text{otherwise} \end{cases}$$

• Pheromone deposits are inversely proportional to the quality, $f(x_k(t))$, of the complete path constructed by the ant. Global information is used to update pheromone. Q is a positive constant

• Ant-density AS:

$$\Delta \tau_{ij}^k(t) = \begin{cases} Q & \text{if link } (i,j) \text{ occurs in path } x^k(t) \\ 0 & \text{otherwise} \end{cases}$$

• Each ant deposits the same amount of pheromone. Total pheromone is proportional to the number of ants that followed link (i,j)



• Ant-quantity AS:

$$\Delta au_{ij}^k(t) = egin{cases} rac{Q}{d_{ij}} & ext{if link } (i,j) ext{ occurs in path } x^k(t) \\ 0 & ext{otherwise} \end{cases}$$

• Ant-quantity AS:

$$\Delta au_{ij}^k(t) = egin{cases} rac{Q}{d_{ij}} & ext{if link } (i,j) ext{ occurs in path } x^k(t) \\ 0 & ext{otherwise} \end{cases}$$

• Only local information d_{ij} is used to update pheromone concentrations. Lower cost links are made more desirable

• Ant-quantity AS:

$$\Delta \tau_{ij}^k(t) = \begin{cases} \frac{Q}{d_{ij}} & \text{if link } (i,j) \text{ occurs in path } x^k(t) \\ 0 & \text{otherwise} \end{cases}$$

- Only local information d_{ij} is used to update pheromone concentrations. Lower cost links are made more desirable
- If d_{ij} represents the distance between links, then ant-quantity AS prefers selection of the shortest links

AS Algorithm

Algorithm 17.3 Ant System Algorithm

```
t = 0:
Initialize all parameters, i.e. \alpha, \beta, \rho, Q, n_k, \tau_0;
Place all ants, k = 1, \ldots, n_k;
for each link (i, j) do
   \tau_{ij}(t) \sim U(0, \tau_0);
end
repeat
   for each ant k = 1, ..., n_k do
       x^k(t) = \emptyset;
       repeat
            From current node i, select next node j with probability as defined in
           equation (17.6);
            x^{k}(t) = x^{k}(t) \cup \{(i, j)\};
        until full path has been constructed;
       Compute f(x^k(t));
    end
   for each link (i, j) do
        Apply evaporation using equation (17.5);
       Calculate \Delta \tau_{ij}(t) using equation (17.10);
        Update pheromone using equation (17.4);
   end
   for each link (i, j) do
       \tau_{ij}(t+1) = \tau_{ij}(t);
   end
   t = t + 1;
until stopping condition is true:
                                                                4 D F 4 A F F 4 B F
Return x^k(t): f(x^k(t)) = \min_{t'=1}^k f(x^k(t))
```



- Ant Colony System: Differs from AS in these aspects
 - ▶ A different transition rule is used
 - ▶ A different pheromone update rule is defined
 - ▶ Local pheromone updates are introduced
 - ► Candidate lists are used to favor specific nodes



- Ant Colony System: Differs from AS in these aspects
 - ▶ A different transition rule is used
 - ▶ A different pheromone update rule is defined
 - ▶ Local pheromone updates are introduced
 - Candidate lists are used to favor specific nodes

• Max-Min Ant System:

- ▶ Address the premature stagnation problem of AS
- ▶ Pheromone intensities are restricted within given intervals
- Only the best ant may reinforce pheromones
- ▶ Initial pheromones are set to the max allowed value
- pheromone smoothing mechanism is used



- Ant Colony System: Differs from AS in these aspects
 - ▶ A different transition rule is used
 - ▶ A different pheromone update rule is defined
 - ► Local pheromone updates are introduced
 - Candidate lists are used to favor specific nodes
- Max-Min Ant System:
 - ▶ Address the premature stagnation problem of AS
 - ▶ Pheromone intensities are restricted within given intervals
 - ▶ Only the best ant may reinforce pheromones
 - ▶ Initial pheromones are set to the max allowed value
 - pheromone smoothing mechanism is used
- Ant-Q: The pheromone notion is dropped to be replaced by Ant-Q value. The goal is to learn AQ-values such that the discovery of good solutions is favored in probability



• Fast Ant System:

- ▶ Originally developed to solve the quadratic assignment problem
- ▶ Uses only one ant
- ► A different pheromone update rule is applied which does not make use of any evaporation



• Fast Ant System:

- ▶ Originally developed to solve the quadratic assignment problem
- ▶ Uses only one ant
- ► A different pheromone update rule is applied which does not make use of any evaporation

Antabu:

- ► Includes a local search using tabu search to refine solutions constructed by each iteration of AS
- ➤ The global update rule is changed such that each ant's pheromone deposit on each link of its constructed path is proportional to the quality of the path



- AS-rank: Modification of AS to:
 - ► Allow only the best ant update pheromone concentrations on the links of the global-best path
 - use elitist ants
 - ► Let ants update pheromone on the basis of a ranking of the ants



- AS-rank: Modification of AS to:
 - ► Allow only the best ant update pheromone concentrations on the links of the global-best path
 - use elitist ants
 - ► Let ants update pheromone on the basis of a ranking of the ants
- ANTS: ANTS differs from AS in:
 - ▶ The transition probability calculation
 - ► The global update rule, and (3) the approach to avoid stagnation



• The number of ants n_k : An obvious influence of n_k relates to computational complexity. The more ants used, the more paths have to be constructed, and the more pheromone deposits calculated



- The number of ants n_k : An obvious influence of n_k relates to computational complexity. The more ants used, the more paths have to be constructed, and the more pheromone deposits calculated
- The fewer ants used, the less the exploration ability of the algorithm, and consequently the less information about the search space is available to all ants

- The number of ants n_k : An obvious influence of n_k relates to computational complexity. The more ants used, the more paths have to be constructed, and the more pheromone deposits calculated
- The fewer ants used, the less the exploration ability of the algorithm, and consequently the less information about the search space is available to all ants
- Small values of n_k may then cause sub-optimal solutions to be found, or early stagnation



- The number of ants n_k : An obvious influence of n_k relates to computational complexity. The more ants used, the more paths have to be constructed, and the more pheromone deposits calculated
- The fewer ants used, the less the exploration ability of the algorithm, and consequently the less information about the search space is available to all ants
- Small values of n_k may then cause sub-optimal solutions to be found, or early stagnation
- Too many ants are not necessarily beneficial. With large values of n_k , it may take significantly longer for pheromone intensities on good links to increase to higher levels





• The maximum number of iterations n_t : If n_t is too small, ants may not have enough time to explore and to settle on a single path



- The maximum number of iterations n_t: If n_t is too small, ants may not have enough time to explore and to settle on a single path
- If n_t is too large, unnecessary computations may be done



- The maximum number of iterations n_t: If n_t is too small, ants may not have enough time to explore and to settle on a single path
- If n_t is too large, unnecessary computations may be done
- Initial pheromone τ_0 : During the initialization step, all pheromones are either initialized to a constant value τ_0 or to random values in the range $[0, \tau_0]$



- The maximum number of iterations n_t : If n_t is too small, ants may not have enough time to explore and to settle on a single path
- If n_t is too large, unnecessary computations may be done
- **Initial pheromone** τ_0 : During the initialization step, all pheromones are either initialized to a constant value τ_0 or to random values in the range $[0, \tau_0]$
- In the case of random values τ_0 is selected to be a small positive value. If a large value is selected, and random values are selected from the uniform distribution, then pheromone concentrations may differ significantly



- The maximum number of iterations n_t : If n_t is too small, ants may not have enough time to explore and to settle on a single path
- If n_t is too large, unnecessary computations may be done
- **Initial pheromone** τ_0 : During the initialization step, all pheromones are either initialized to a constant value τ_0 or to random values in the range $[0, \tau_0]$
- In the case of random values τ_0 is selected to be a small positive value. If a large value is selected, and random values are selected from the uniform distribution, then pheromone concentrations may differ significantly
- This may cause a bias towards the links with large initial concentrations, with links that have small pheromone values being neglected as components of the final solution



Applications of Ant Colony Optimization

- Assignment problems
- Bioinformatics
- Data clustering
- Robotics
- Routing
- Scheduling
- Sequential ordering problem
- Set covering
- Shortest common super-sequence
- Text mining



1. Algorithms developed based on studies of ant foraging are collectively called the ACO meta-heuristic



- 1. Algorithms developed based on studies of ant foraging are collectively called the ACO meta-heuristic
- 2. An ant carries food to the nest while laying pheromone along the trail. Forager ants decide which path to follow based on the pheromone concentrations on the different paths



- 1. Algorithms developed based on studies of ant foraging are collectively called the ACO meta-heuristic
- 2. An ant carries food to the nest while laying pheromone along the trail. Forager ants decide which path to follow based on the pheromone concentrations on the different paths
- 3. As more ants follow a specific trail, that path is reinforced by more pheromone which attracts more ants to follow



- 1. Algorithms developed based on studies of ant foraging are collectively called the ACO meta-heuristic
- 2. An ant carries food to the nest while laying pheromone along the trail. Forager ants decide which path to follow based on the pheromone concentrations on the different paths
- 3. As more ants follow a specific trail, that path is reinforced by more pheromone which attracts more ants to follow
- 4. This stigmergy based behavior has resulted in SACO and AS metaheuristics





- 1. Algorithms developed based on studies of ant foraging are collectively called the ACO meta-heuristic
- 2. An ant carries food to the nest while laying pheromone along the trail. Forager ants decide which path to follow based on the pheromone concentrations on the different paths
- 3. As more ants follow a specific trail, that path is reinforced by more pheromone which attracts more ants to follow
- 4. This stigmergy based behavior has resulted in SACO and AS metaheuristics
- 5. Numerous variants exist



- 1. Algorithms developed based on studies of ant foraging are collectively called the ACO meta-heuristic
- 2. An ant carries food to the nest while laying pheromone along the trail. Forager ants decide which path to follow based on the pheromone concentrations on the different paths
- 3. As more ants follow a specific trail, that path is reinforced by more pheromone which attracts more ants to follow
- This stigmergy based behavior has resulted in SACO and AS metaheuristics
- 5. Numerous variants exist
- 6. Important algorithmic parameters: Number of ants, maximum number of iterations, initial pheromone, evaporation rate, etc

Any Questions?







Thank You

