Session : Fundamentals of Optimization

Course Title: Computational Intelligence
Course Code: 19CSE422A

Course Leader:
Dr. Vaishali R. Kulkarni
Assistant Professor, Department of Computer Science and Engineering
Faculty of Engineering and Technology
Ramaiah University of Applied Sciences, Bengaluru
Email: vaishali.cs.et@msruas.ac.in
Tel: +91-804-906-5555 Ext:2325
Website: www.msruas.ac.in/staff/fet_cse#Vaishali

Objectives of this Session

I wish to:

1. Introduce and define global optimization

Objectives of this Session

I wish to:

1. Introduce and define global optimization
2. Give examples of various types of optimization

Objectives of this Session

I wish to:

1. Introduce and define global optimization
2. Give examples of various types of optimization
3. Introduce combinatorial optimization

Objectives of this Session

I wish to:

1. Introduce and define global optimization
2. Give examples of various types of optimization
3. Introduce combinatorial optimization
4. Give examples of combinatorial optimization

Objectives of this Session

I wish to:

1. Introduce and define global optimization
2. Give examples of various types of optimization
3. Introduce combinatorial optimization
4. Give examples of combinatorial optimization
5. Discuss pros and cons of deterministic approaches to optimization

Objectives of this Session

I wish to:

1. Introduce and define global optimization
2. Give examples of various types of optimization
3. Introduce combinatorial optimization
4. Give examples of combinatorial optimization
5. Discuss pros and cons of deterministic approaches to optimization
6. Introduce heuristic approaches to optimization and

Objectives of this Session

I wish to:

1. Introduce and define global optimization
2. Give examples of various types of optimization
3. Introduce combinatorial optimization
4. Give examples of combinatorial optimization
5. Discuss pros and cons of deterministic approaches to optimization
6. Introduce heuristic approaches to optimization and
7. Discuss bio-inspired CI approaches to optimization

Intended Outcomes of this Session

At the end of this session, the student will be able to:

1. Formulate optimization mathematically

Intended Outcomes of this Session

At the end of this session, the student will be able to:

1. Formulate optimization mathematically
2. Exemplify various types of optimization

Intended Outcomes of this Session

At the end of this session, the student will be able to:

1. Formulate optimization mathematically
2. Exemplify various types of optimization
3. List the deterministic approaches to optimization

Intended Outcomes of this Session

At the end of this session, the student will be able to:

1. Formulate optimization mathematically
2. Exemplify various types of optimization
3. List the deterministic approaches to optimization
4. Discuss relative advantages of deterministic approaches to optimization and

Intended Outcomes of this Session

At the end of this session, the student will be able to:

1. Formulate optimization mathematically
2. Exemplify various types of optimization
3. List the deterministic approaches to optimization
4. Discuss relative advantages of deterministic approaches to optimization and
5. Summarize bio-inspired CI approaches to optimization

Optimization: A Day-to-Day Example 1

Optimization: A Day-to-Day Example 1

1. Ingredients: Water, Tea-powder, Sugar and Milk

Optimization: A Day-to-Day Example 1

1. Ingredients: Water, Tea-powder, Sugar and Milk
2. Assumption: Taste of the tea depends only on ingredients

Optimization: A Day-to-Day Example 1

1. Ingredients: Water, Tea-powder, Sugar and Milk
2. Assumption: Taste of the tea depends only on ingredients
3. **How much of each ingredient do we use per cup of tea?**

Optimization: A Day-to-Day Example 1

1. Ingredients: Water, Tea-powder, Sugar and Milk
2. Assumption: Taste of the tea depends only on ingredients
3. **How much of each ingredient do we use per cup of tea?**
4. Desired Outcome: Excellent taste!

Optimization: Day-to-Day Example 2

Optimization: Day-to-Day Example 2

- Two knobs: H and C

Optimization: Day-to-Day Example 2

- Two knobs: H and C
- Knob settings θ_h and θ_c

Optimization: Day-to-Day Example 2

- Two knobs: H and C
- Knob settings θ_h and θ_c
- **What should be the right settings of θ_h and θ_c?**

Optimization: Day-to-Day Example 2

- Two knobs: H and C
- Knob settings θ_h and θ_c
- **What should be the right settings of θ_h and θ_c?**
- Desired Outcome: Very comfortable shower!

Optimization

- In mathematics, computer science and economics, optimization refers to **choosing the best element** from a set of available alternatives

Optimization

- In mathematics, computer science and economics, optimization refers to **choosing the best element** from a set of available alternatives
- The objective of optimization is to maximize(or minimize) a **real function** by systematically choosing the values of variables from an allowed set

Optimization

- In mathematics, computer science and economics, optimization refers to **choosing the best element** from a set of available alternatives
- The objective of optimization is to maximize(or minimize) a **real function** by systematically choosing the values of variables from an allowed set
- Mathematically, an optimization problem can be represented as:
  Given: a function $f : A \subseteq \mathbb{R}^N \to \mathbb{R}$
  Sought: an element $\mathbf{x}^*$ in A such that
  $f(\mathbf{x}^*) \geq f(\mathbf{x})\ \forall \mathbf{x} \in\ A$ and $f(\mathbf{x}^*) \neq \infty$

Optimization

- In mathematics, computer science and economics, optimization refers to **choosing the best element** from a set of available alternatives
- The objective of optimization is to maximize(or minimize) a **real function** by systematically choosing the values of variables from an allowed set
- Mathematically, an optimization problem can be represented as:
  Given: a function $f : A \subseteq \mathbb{R}^N \to \mathbb{R}$
  Sought: an element $\mathbf{x}^*$ in A such that
  $f(\mathbf{x}^*) \geq f(\mathbf{x})\ \forall \mathbf{x} \in\ A$ and $f(\mathbf{x}^*) \neq \infty$
- The function f is called the **objective function**

Optimization

- In mathematics, computer science and economics, optimization refers to **choosing the best element** from a set of available alternatives
- The objective of optimization is to maximize(or minimize) a **real function** by systematically choosing the values of variables from an allowed set
- Mathematically, an optimization problem can be represented as:
  Given: a function $f : A \subseteq \mathbb{R}^N \rightarrow \mathbb{R}$
  Sought: an element $\mathbf{x}^*$ in A such that
  $f(\mathbf{x}^*) \geq f(\mathbf{x}) \; \forall \mathbf{x} \in A$ and $f(\mathbf{x}^*) \neq \infty$
- The function f is called the **objective function**
- It is commonly referred to as the **cost function**, health function or **fitness function**

Example 3: Design Optimization

- An engineer wants to design a cylindrical can to hold 200 ml of a soft drink

Example 3: Design Optimization

- An engineer wants to design a cylindrical can to hold 200 ml of a soft drink
- The can will be made from aluminum sheet

Example 3: Design Optimization

- An engineer wants to design a cylindrical can to hold 200 ml of a soft drink
- The can will be made from aluminum sheet
- Radius r and height h are the optimization variables

Example 3: Design Optimization

- An engineer wants to design a cylindrical can to hold 200 ml of a soft drink
- The can will be made from aluminum sheet
- Radius r and height h are the optimization variables
- What are the values of r^* and h^* that result in the minimum surface area?

Example 3: Design Optimization

- An engineer wants to design a cylindrical can to hold 200 ml of a soft drink
- The can will be made from aluminum sheet
- Radius r and height h are the optimization variables
- What are the values of r^* and h^* that result in the minimum surface area?
- This is a two-dimensional problem

Example 3: Design Optimization

- An engineer wants to design a cylindrical can to hold 200 ml of a soft drink
- The can will be made from aluminum sheet
- Radius r and height h are the optimization variables
- What are the values of r^* and h^* that result in the minimum surface area?
- This is a two-dimensional problem
- The objective function to minimize is $A = 2\pi rh + 2(\pi r^2)$

Example 3: Design Optimization

- An engineer wants to design a cylindrical can to hold 200 ml of a soft drink
- The can will be made from aluminum sheet
- Radius r and height h are the optimization variables
- What are the values of r^* and h^* that result in the minimum surface area?
- This is a two-dimensional problem
- The objective function to minimize is $A = 2\pi rh + 2(\pi r^2)$
- This is a relatively easy problem

Example 4: Time Management

- A student has registered for N courses $C_i,\ i = 1, 2, \cdots, N$

Example 4: Time Management

- A student has registered for N courses C_i, $i = 1, 2, \cdots, N$
- The number of hours she spends studying course i is H_i

Example 4: Time Management

- A student has registered for N courses C_i, $i = 1, 2, \cdots, N$
- The number of hours she spends studying course i is H_i
- She expects to get M_i marks in course i

Example 4: Time Management

- A student has registered for N courses C_i, $i = 1, 2, \cdots, N$
- The number of hours she spends studying course i is H_i
- She expects to get M_i marks in course i
- The objective is to earn as many total marks as possible, $\sum_{i=1}^{N} M_i$

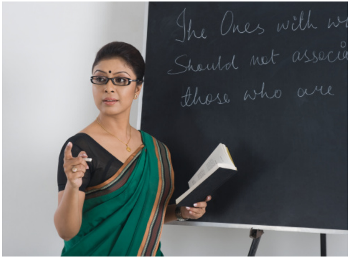
Example 4: Time Management

- A student has registered for N courses $C_i,\ i = 1, 2, \cdots, N$
- The number of hours she spends studying course i is H_i
- She expects to get M_i marks in course i
- The objective is to earn as many total marks as possible, $\sum_{i=1}^{N} M_i$
- What are the best values of $H_i^*,\ i = 1, 2, \cdots, N$?

Example 4: Time Management

- A student has registered for N courses $C_i,\ i = 1, 2, \cdots, N$
- The number of hours she spends studying course i is H_i
- She expects to get M_i marks in course i
- The objective is to earn as many total marks as possible, $\sum_{i=1}^{N} M_i$
- What are the best values of $H_i^*,\ i = 1, 2, \cdots, N$?
- Discrete optimization!

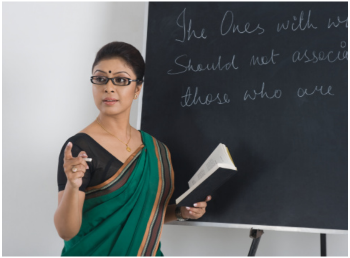
Example 5: HR Optimization



- A department seeks to appoint N faculty members from four cadres:

L	Lecturer
AP	Assistant Professor
ASP	Associate Professor
P	Professor

Example 5: HR Optimization

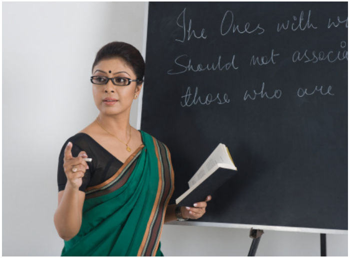


- A department seeks to appoint N faculty members from four cadres:

L	Lecturer
AP	Assistant Professor
ASP	Associate Professor
P	Professor

- The number of faculty members in these cadres is N_L, N_{AP}, N_{ASP} and N_P, respectively

Example 5: HR Optimization

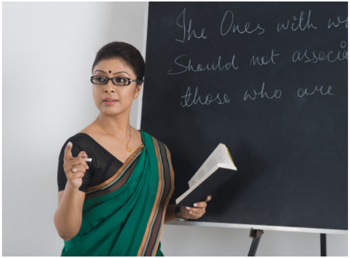


- A department seeks to appoint N faculty members from four cadres:

L	Lecturer
AP	Assistant Professor
ASP	Associate Professor
P	Professor

- The number of faculty members in these cadres is N_L, N_{AP}, N_{ASP} and N_P, respectively
- The objective is to pay as **low salary** as possible, but to get the **best possible quality**

Example 5: HR Optimization

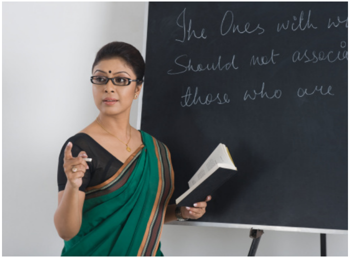


- A department seeks to appoint N faculty members from four cadres:

L	Lecturer
AP	Assistant Professor
ASP	Associate Professor
P	Professor

- The number of faculty members in these cadres is N_L, N_{AP}, N_{ASP} and N_P, respectively
- The objective is to pay as **low salary** as possible, but to get the **best possible quality**
- What are the best numbers N_L^*, N_{AP}^*, N_{ASP}^* and N_P^*?

Example 5: HR Optimization



- A department seeks to appoint N faculty members from four cadres:

L	Lecturer
AP	Assistant Professor
ASP	Associate Professor
P	Professor

- The number of faculty members in these cadres is N_L, N_{AP}, N_{ASP} and N_P, respectively
- The objective is to pay as **low salary** as possible, but to get the **best possible quality**
- What are the best numbers N_L^*, N_{AP}^*, N_{ASP}^* and N_P^*?
- Multi-objective integer optimization!

Example 6: Rastrigin Function

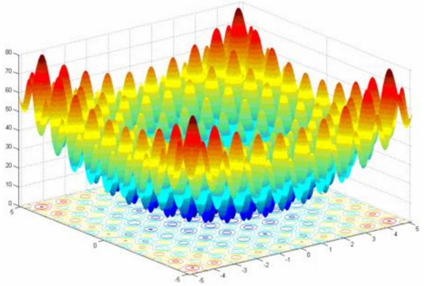
- Let $\mathbf{x} = \{x_1, x_2, \ldots, x_N\}, x_i \in [-5.12, 5.12]$
- $f(\mathbf{x}) = 10N + \sum_{i=1}^{N} [x_i^2 - 10\cos(2\pi x_i)]$

Example 6: Rastrigin Function

- Let $\mathbf{x} = \{x_1, x_2, \ldots, x_N\}, x_i \in [-5.12, 5.12]$
- $f(\mathbf{x}) = 10N + \sum_{i=1}^{N} [x_i^2 - 10\cos(2\pi x_i)]$
- The objective is to find the point $\mathbf{x}^*$ such that $f(\mathbf{x}^*)$ is minimum

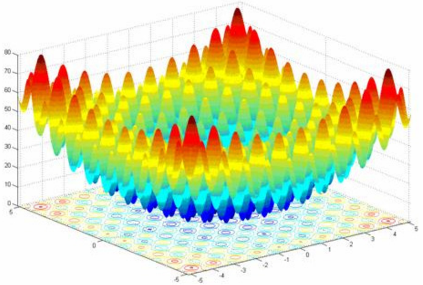
Example 6: Rastrigin Function

- Let $\mathbf{x} = \{x_1, x_2, \ldots, x_N\}, x_i \in [-5.12, 5.12]$
- $f(\mathbf{x}) = 10N + \sum_{i=1}^{N} [x_i^2 - 10\cos(2\pi x_i)]$
- The objective is to find the point $\mathbf{x}^*$ such that $f(\mathbf{x}^*)$ is minimum
- The fitness landscape is as shown:



Example 6: Rastrigin Function

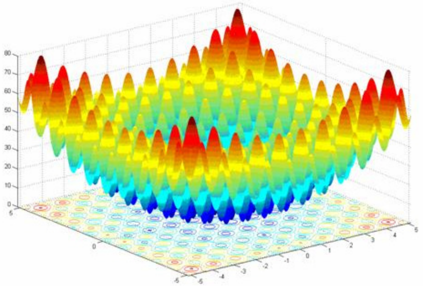
- Let $\mathbf{x} = \{x_1, x_2, \ldots, x_N\}, x_i \in [-5.12, 5.12]$
- $f(\mathbf{x}) = 10N + \sum_{i=1}^{N} [x_i^2 - 10\cos(2\pi x_i)]$
- The objective is to find the point $\mathbf{x}^*$ such that $f(\mathbf{x}^*)$ is minimum
- The fitness landscape is as shown:



- Solution is $x_i^* = 0, i = 1, 2, \cdots, N$

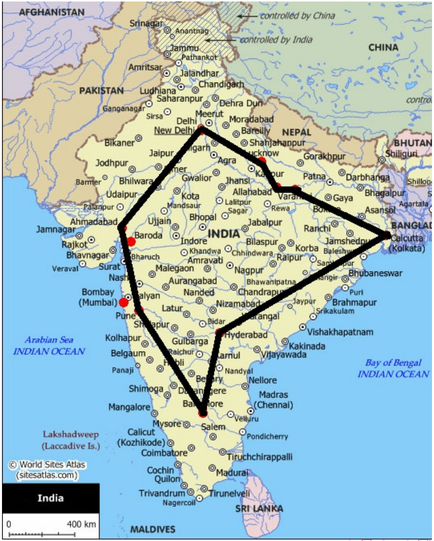
Example 6: Rastrigin Function

- Let $\mathbf{x} = \{x_1, x_2, \ldots, x_N\}, x_i \in [-5.12, 5.12]$
- $f(\mathbf{x}) = 10N + \sum_{i=1}^{N} [x_i^2 - 10\cos(2\pi x_i)]$
- The objective is to find the point $\mathbf{x}^*$ such that $f(\mathbf{x}^*)$ is minimum
- The fitness landscape is as shown:



- Solution is $x_i^* = 0, i = 1, 2, \cdots, N$
- Multimodal landscape. Global optimization!

Combinatorial Optimization: Example 1

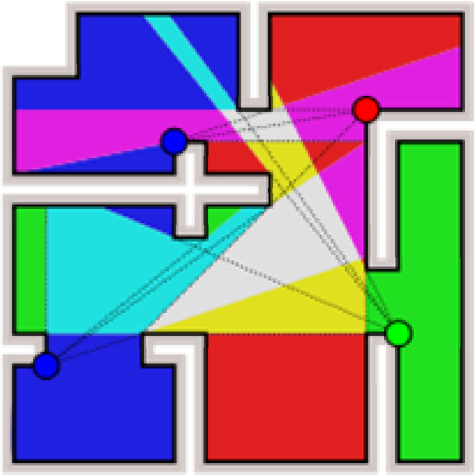


The Traveling salesman problem

Combinatorial Optimization: Example 2

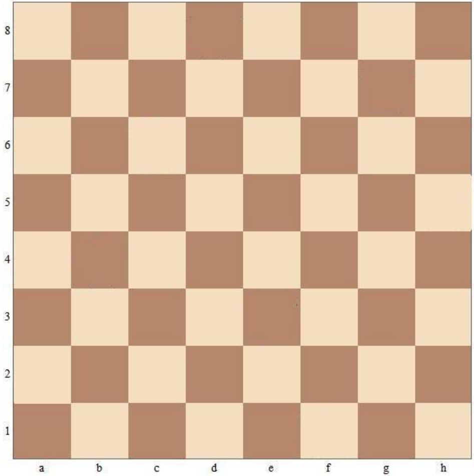
The Knapsack problem

Combinatorial Optimization: Example 3



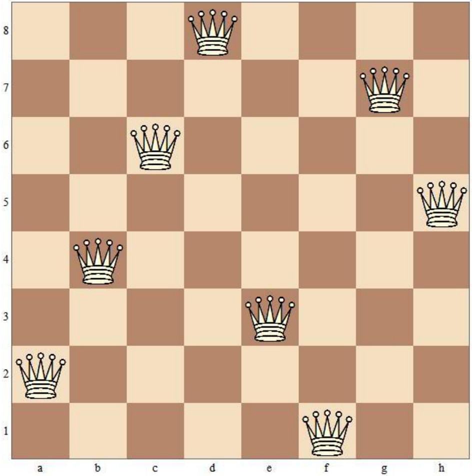
The Art gallery problem

Combinatorial Optimization: Example 4



The 8-Queens Problem

Combinatorial Optimization: Example 4



A Solution to the 8-Queens Problem

Combinatorial Optimization: Example 5

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

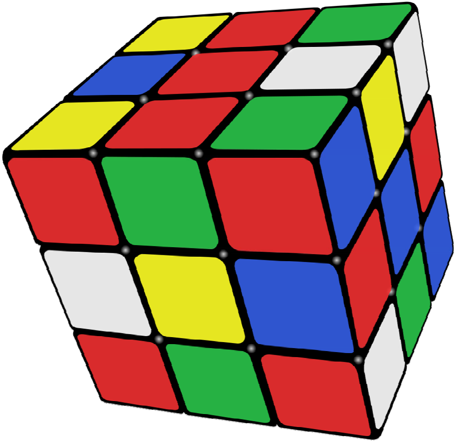
A Sudoku Puzzle

Combinatorial Optimization: Example 5

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

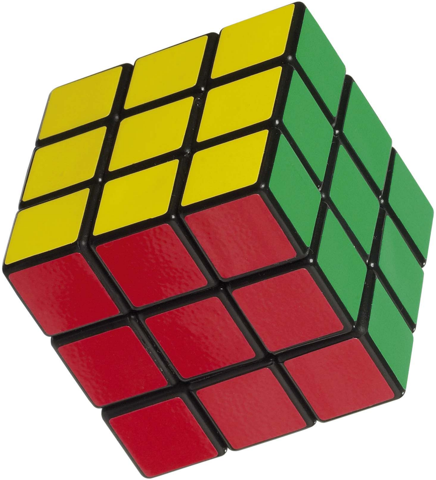
A Solved Sudoku Puzzle

Combinatorial Optimization: Example 6



The Rubik's Cube Puzzle

Combinatorial Optimization: Example 6



Solved Robik's Cube Puzzle

Real-World Problems

- Search spaces are huge

Real-World Problems

- Search spaces are huge
- The model of the problem is difficult to design

Real-World Problems

- Search spaces are huge
- The model of the problem is difficult to design
- The quality of solutions varies with time

Real-World Problems

- Search spaces are huge
- The model of the problem is difficult to design
- The quality of solutions varies with time
- Search spaces are highly constrained

Real-World Problems

- Search spaces are huge
- The model of the problem is difficult to design
- The quality of solutions varies with time
- Search spaces are highly constrained
- Objective functions may be non-differentiable, non-linear, noisy, or may have many local minima

Real-World Problems

- Search spaces are huge
- The model of the problem is difficult to design
- The quality of solutions varies with time
- Search spaces are highly constrained
- Objective functions may be non-differentiable, non-linear, noisy, or may have many local minima

Deterministic Approaches to Optimization

- Linear Programming
- Nonlinear Programming
- Dynamic Programming
- Multi-Criteria Decision Making

Advantages

- ✓ They are the first option to solve optimization problems

Advantages

- ✓ They are the first option to solve optimization problems
- ✓ If there is a match between the features of the problem and the conditions required by the method, the results are very competitive

Advantages

- ✓ They are the first option to solve optimization problems
- ✓ If there is a match between the features of the problem and the conditions required by the method, the results are very competitive
- ✓ Under certain conditions, the computational cost can be low

Advantages

- ✓ They are the first option to solve optimization problems
- ✓ If there is a match between the features of the problem and the conditions required by the method, the results are very competitive
- ✓ Under certain conditions, the computational cost can be low
- ✓ They produce the same results always

Disadvantages

✗ In some problems, these techniques are either difficult to apply or they may take considerable time to reach to an acceptable solution

Disadvantages

- ✗ In some problems, these techniques are either difficult to apply or they may take considerable time to reach to an acceptable solution
- ✗ These techniques suffer from the **curse-of-dimensionality**

Disadvantages

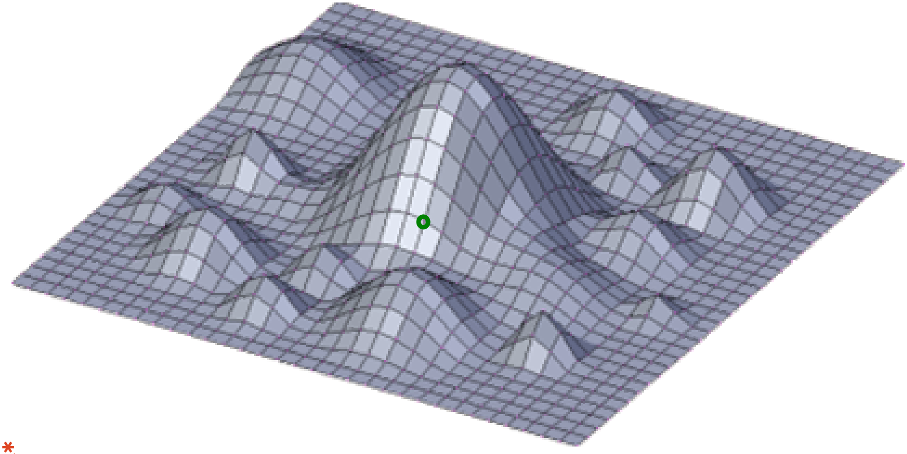
- ✗ In some problems, these techniques are either difficult to apply or they may take considerable time to reach to an acceptable solution
- ✗ These techniques suffer from the **curse-of-dimensionality**
- ✗ The application of these methods may require a transformation of the original model of the problem

Disadvantages

- ✗ In some problems, these techniques are either difficult to apply or they may take considerable time to reach to an acceptable solution
- ✗ These techniques suffer from the **curse-of-dimensionality**
- ✗ The application of these methods may require a transformation of the original model of the problem
- ✗ Some methods are difficult to use

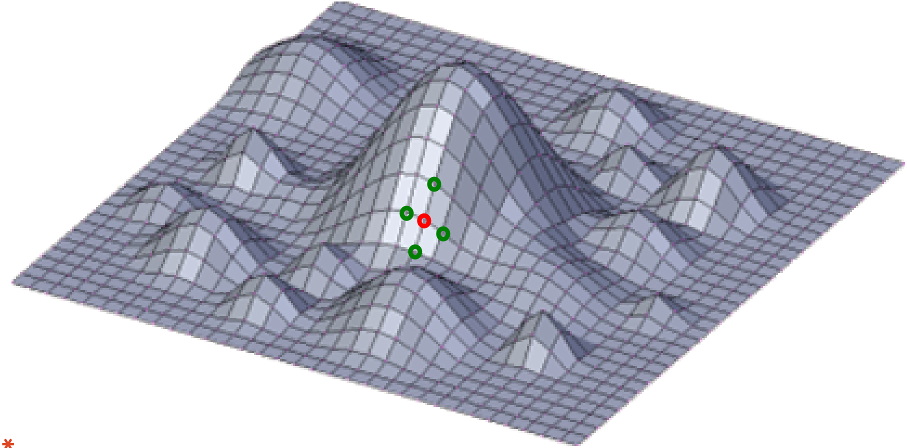
Heuristic Search: Hill Climbing

Start with a random solution.



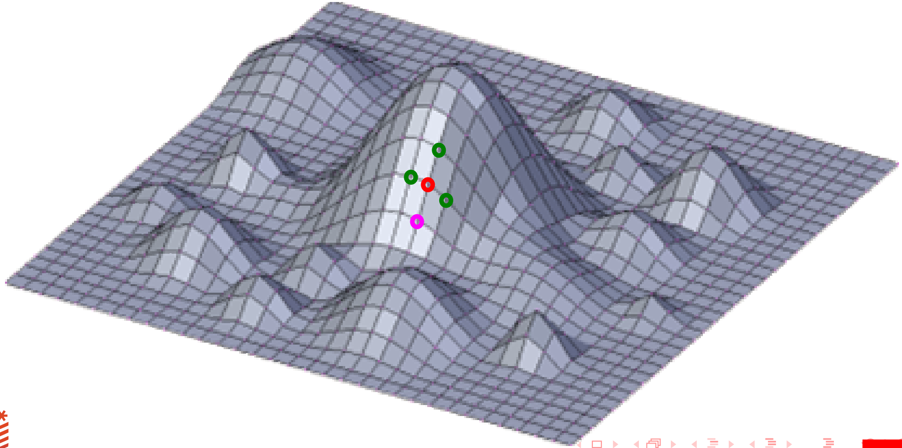
Heuristic Search: Hill Climbing

Take incremental steps in each dimension.
Choose the best position as the solution.



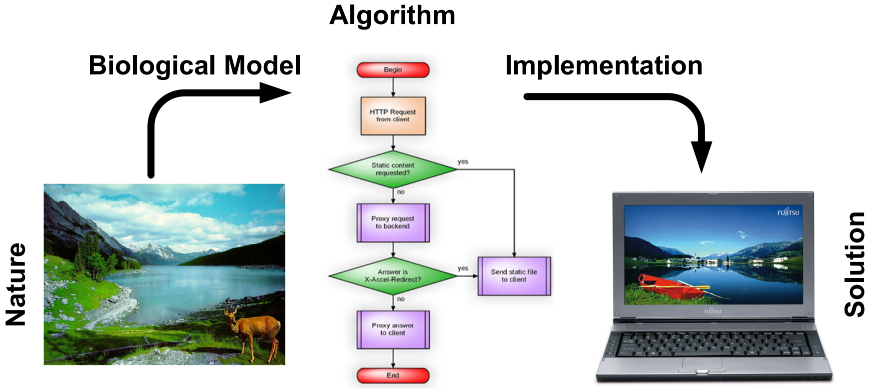
Heuristic Search: Hill Climbing

Repeat this for a large number of steps, or until no more improvement takes place



Bio-Inspired Optimization

Bio-inspired optimization is the group of optimization algorithms whose behaviors are based on **biological phenomena**



Why Biology?

- Biology is a source of **adaptive mechanisms** where intelligent behavior emerges in changing and complex environments
- These mechanisms are capable of:
  1. Learning
  2. Generalizing
  3. Abstracting
  4. Discovering
  5. Associating
- These are studied under the title CI
- The five dominant paradigms of CI are:
  1. Artificial Immune Systems
  2. Artificial Neural Networks
  3. Evolutionary Computing
  4. Fuzzy Logic
  5. Swarm Intelligence

Session Summary

1. Global optimization: Choosing the best values of independent variables in such a way that the value of a dependent variable is minimum (or maximum)

Session Summary

1. Global optimization: Choosing the best values of independent variables in such a way that the value of a dependent variable is minimum (or maximum)

2. Combinatorial optimization consists of finding an optimal object from a finite set of objects

Session Summary

1. Global optimization: Choosing the best values of independent variables in such a way that the value of a dependent variable is minimum (or maximum)

2. Combinatorial optimization consists of finding an optimal object from a finite set of objects

3. Deterministic approaches to optimization include linear programming, integer programming, nonlinear programming, dynamic programming, etc

Session Summary

1. Global optimization: Choosing the best values of independent variables in such a way that the value of a dependent variable is minimum (or maximum)

2. Combinatorial optimization consists of finding an optimal object from a finite set of objects

3. Deterministic approaches to optimization include linear programming, integer programming, nonlinear programming, dynamic programming, etc

4. Curse of dimensionality is a major shortcoming of the deterministic methods

Session Summary

1. Global optimization: Choosing the best values of independent variables in such a way that the value of a dependent variable is minimum (or maximum)

2. Combinatorial optimization consists of finding an optimal object from a finite set of objects

3. Deterministic approaches to optimization include linear programming, integer programming, nonlinear programming, dynamic programming, etc

4. Curse of dimensionality is a major shortcoming of the deterministic methods

5. This has motivated the development of many biologically inspired metaheuristics for solving optimization problems

Session Summary

1. Global optimization: Choosing the best values of independent variables in such a way that the value of a dependent variable is minimum (or maximum)

2. Combinatorial optimization consists of finding an optimal object from a finite set of objects

3. Deterministic approaches to optimization include linear programming, integer programming, nonlinear programming, dynamic programming, etc

4. Curse of dimensionality is a major shortcoming of the deterministic methods

5. This has motivated the development of many biologically inspired metaheuristics for solving optimization problems

Any Questions?

Thank You