

Intensity Transform and Spatial Filtering

Delivered by

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Image Enhancement

- **Objective:** To process an image so that the result is more suitable than the original image for a specific application.
- The word specifics is important, because it establishes at the outset; the techniques we will discuss here are very much problem oriented.
- Method that is useful for enhancing **X-ray images** may not necessarily be the best approach for enhancing pictures of **Mars transmitted by a space probe**.
- Regardless of the method used, however, image enhancement is one of the most interesting and visually appealing areas of image processing.



Intensity Transform and Spatial Filtering

- **Image enhancement** is the process that improves the quality of the image for a specific application. It fall into two broad categories:
 - **spatial domain** methods and
 - **frequency domain** methods
- **Image enhancement methods:**
 - 1. Spatial domain methods (image plane)**
It refers the image plane itself; Techniques are based on direct manipulation of pixels in the image.
 - 2. Frequency domain methods**
Techniques are based on modifying the Fourier transform of the image.
 - 3. Combinations methods:**
There are some enhancement techniques based on various combinations of methods from the two first categories.



Cont..

- **Spatial domain** refers to the aggregate of pixels composing an image.
- Spatial domain methods are **procedures** that operate directly on pixels (spatial domain) can be denoted by the expression:

$$g(x,y) = T[f(x,y)]$$

Where: $f(x,y)$ is the **input** image, $g(x,y)$ is the processed (**output**) image;

- **Mask/Filter:** neighborhood of a point (x, y) can be used, by using a **square/ rectangular** (commonly used) or **circular sub image** area centre at (x, y) as shown in Fig 3.1.
- The **center of the sub image** is moved from pixel to pixel starting at the top left of the corner



Cont..

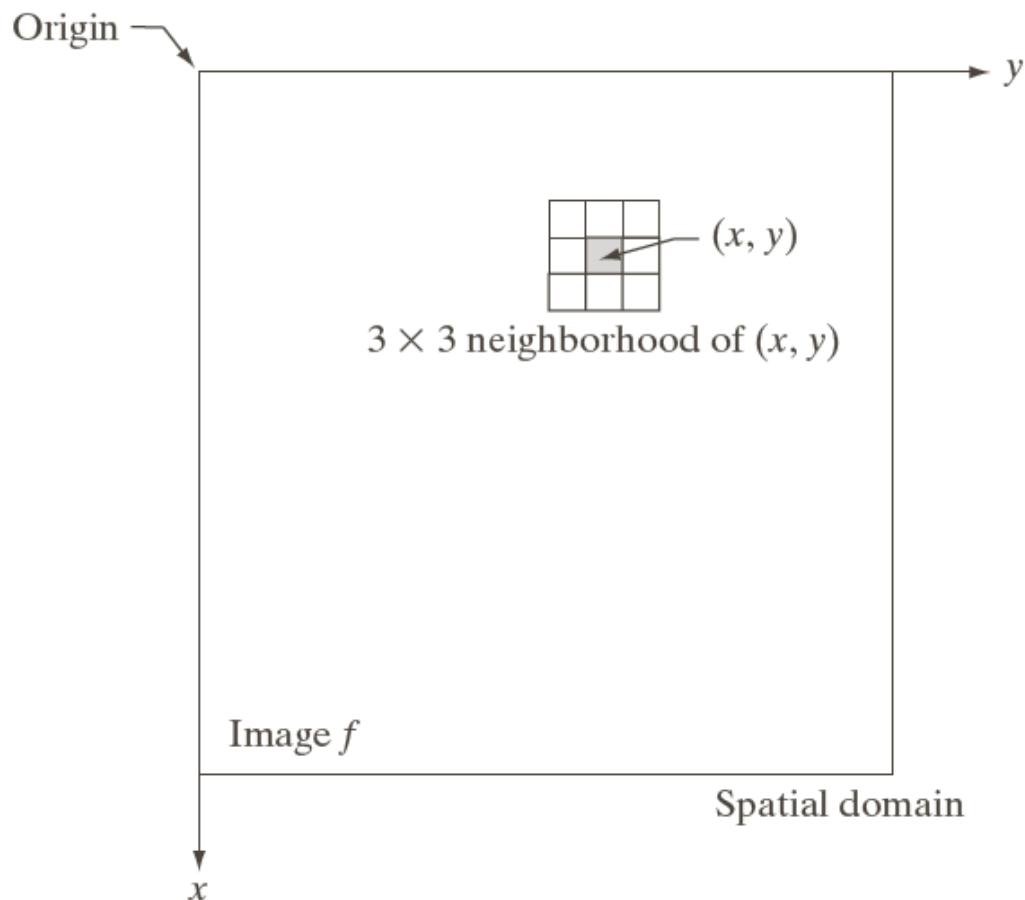


FIGURE 3.1
A 3×3 neighborhood about a point (x, y) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

T-is an operator on $f(x,y)$ defined over some neighborhood of (x,y) ;
T can operate on a set of I/P images, **example** performing the pixel-by-pixel sum of K images for noise reduction.

Contrast Stretching and Thresholding

- **Intensity Transformation Functions**

- **Point processing:**

- **Neighborhood:** **1*1 pixel** (that is, a single pixel). **g** depends on only the value of **f** at **(x,y)**.
 - And **T** becomes an **intensity transformation function** (*gray-level or mapping function*) of the form:

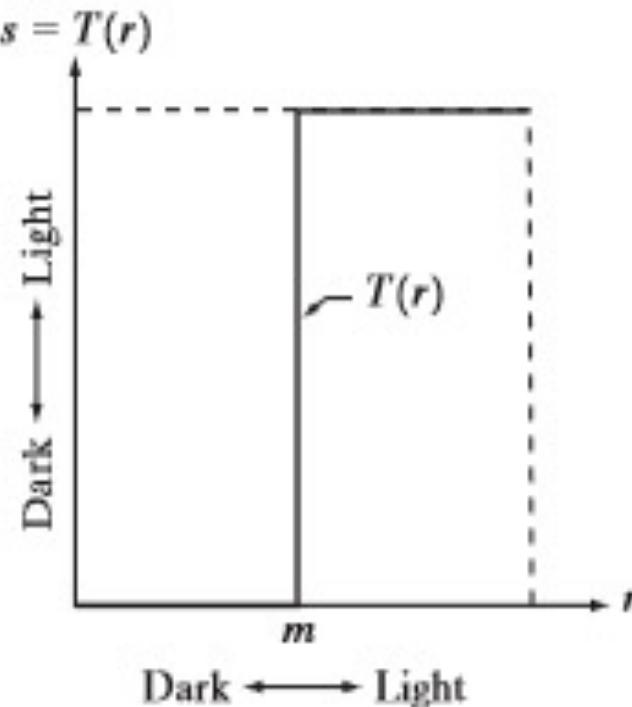
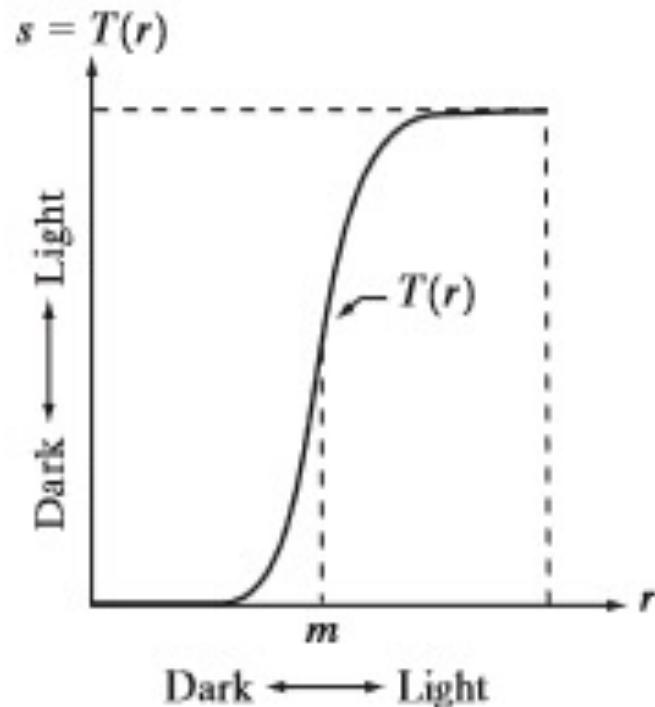
$$s = T(r)$$

Where: **r** = gray level of **f (x,y)** , **s** = gray level of **g(x,y)**

- **Example**, if $T(r)$ has form as in Fig.3.2(a), then T produce an image of higher contrast than the original by **darkening** the levels below **m** and **brightening** the levels above **m** in the original image. This technique is known as **contrast stretching**, the values of **r** below **m** are **compressed** by the transformation function into a **narrow range** of **s**, toward **black**.



Cont..



a b

FIGURE 3.2
Intensity transformation functions.
(a) Contrast-stretching function.
(b) Thresholding function.

Opposite effect takes place for values of r above m . Case shown in Fig.3.2(b), $T(r)$ produces a **two-level (binary)** image. A mapping of this form is called a **thresholding function**. Because enhancement at any point in an image depends only on the gray level at that point, techniques in this category often are referred to as point processing.



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- Intensity transformation example:
 1. **Contrast stretching:** (*Normalization*) If $T(r)$ has the form as shown in figure, the effect of applying the transformation to every pixel of f to generate the corresponding pixels in g would:
 - Produce *higher contrast than the original by*:
 - *Darkening the levels below m in the original image.*
 - *Brightening the levels above m in the original image.*

Contrast stretching: simple image enhancement technique that improves the contrast in an image by '*stretching*' the range of intensity values it contains to span a desired range of values; typically, it uses a *linear scaling function*

2. **Thresholding:**
 - Produce a *two-level (binary) image, a mapping of this form called thresholding function.*



Some Intensity Transform Function

- Procedures that operate directly on pixels (spatial domain) can be denoted by the expression:
- **Some Basic Intensity Transformation Functions:**
- Three basic types of functions used frequently for image enhancement:
 - *Linear functions: negative and identity transformations.*
 - *Logarithmic: log and inverse log transformations.*
 - *Power-law functions (n_{th} power and n_{th} root transformations, Gamma correction)*
 - *Piecewise linear transformations*



Some Intensity Transform Function

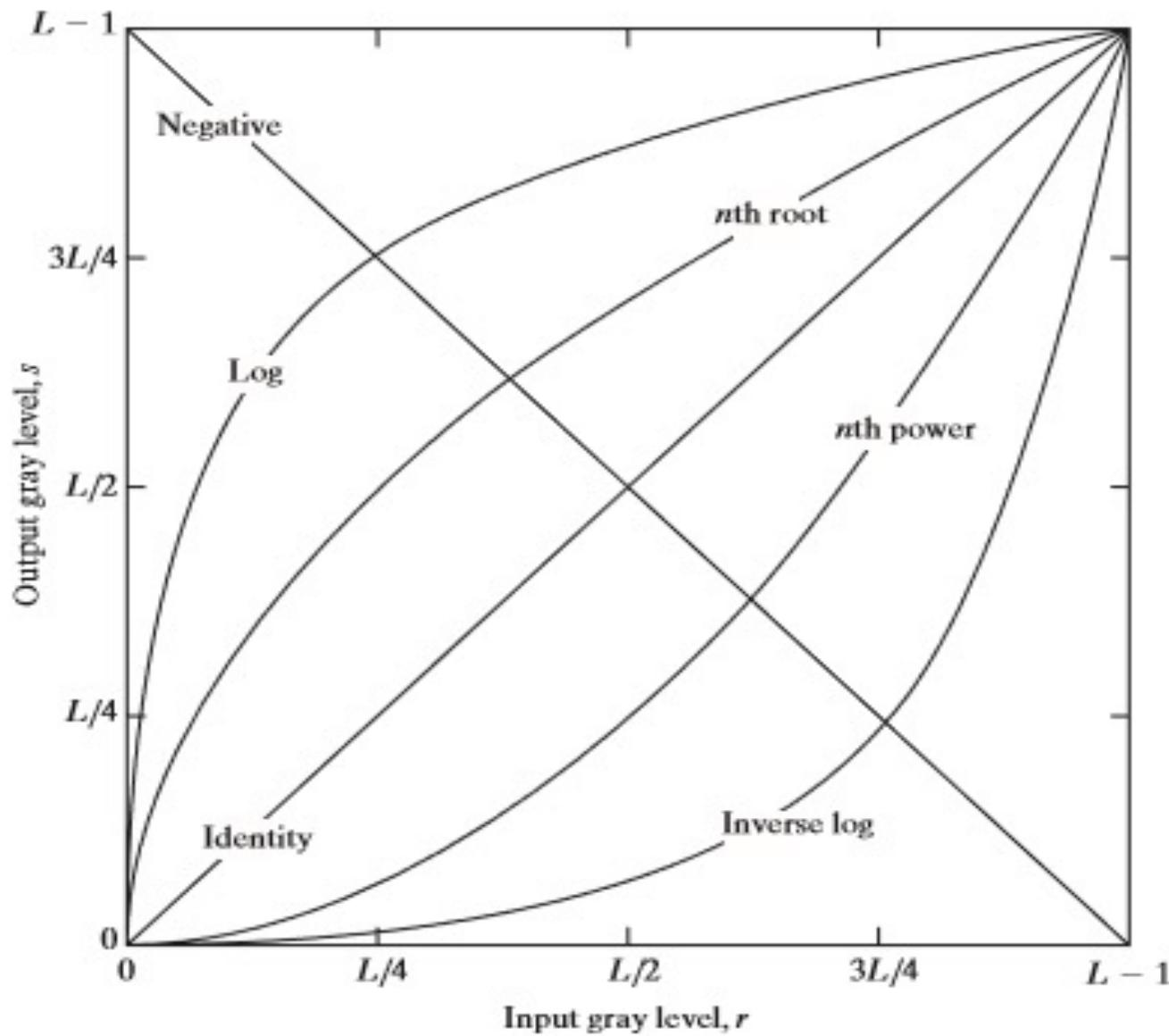


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Cont..

1. *Identity function:*

- Output intensities to input intensities.
- Is included in the graph only for completeness.

2. *Image negatives:*

- The negative of an image with gray level in the range **[0, L-1]**, where **L = 2ⁿ = 2⁸, n=1,2,...** is obtained by using the negative transformation's expression:

$$S = L-1 - r //$$

- That reverses the intensity levels of an input image.
- The negative transformation is suitable for enhancing white or gray detailed embedded in dark regions of an image, especially when the black area are dominant in size.



Algorithm

INPUT : Gray scale Image input($M \times N$)

OUTPUT: Negative gray scale image output($M \times N$)

STEPS :

1. Read the Input Gray scale Image ($M \times N$)

– `//I=imread('1.jpg'); figure, imshow(I); [m, n]=size(I);`

2. Calculate negative of the input image by applying the equation, $s = L - 1 - r$

`for (i=0; i<M; i++)`

`for (j=0; j<N; j++)`

`output[i][j] = L - 1 - input[i][j] ;`

`//I2[i][j]=255-I[i][j];`

`//figure, imshow(I2);`

Where **r** is the **input pixel**

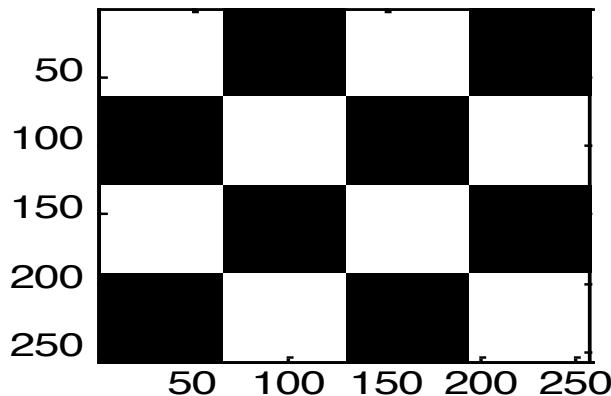
s is the **output pixel**

L is the **gray scale level**

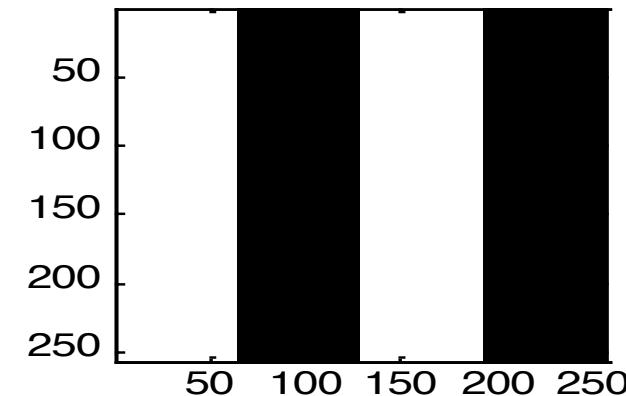


Negative of Images

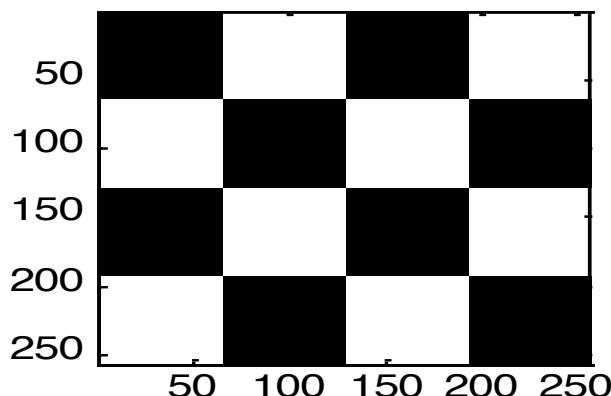
binary image1



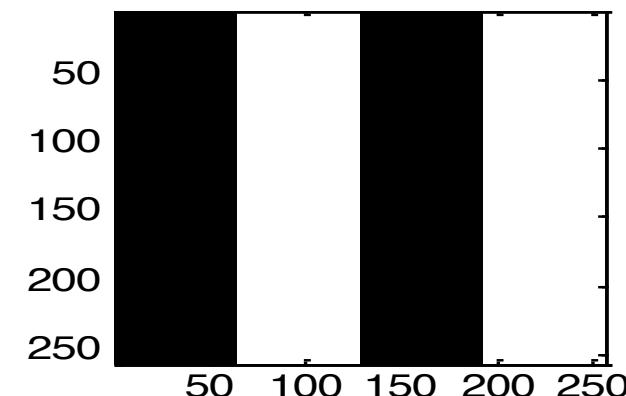
binary image2



Negative of binary image1



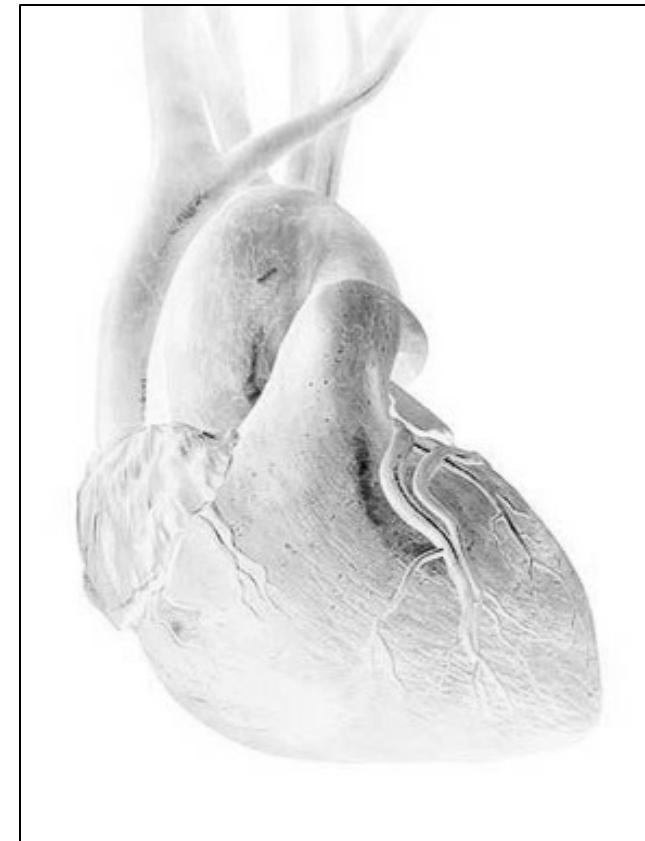
Negative of binary image2



Negative of an Image



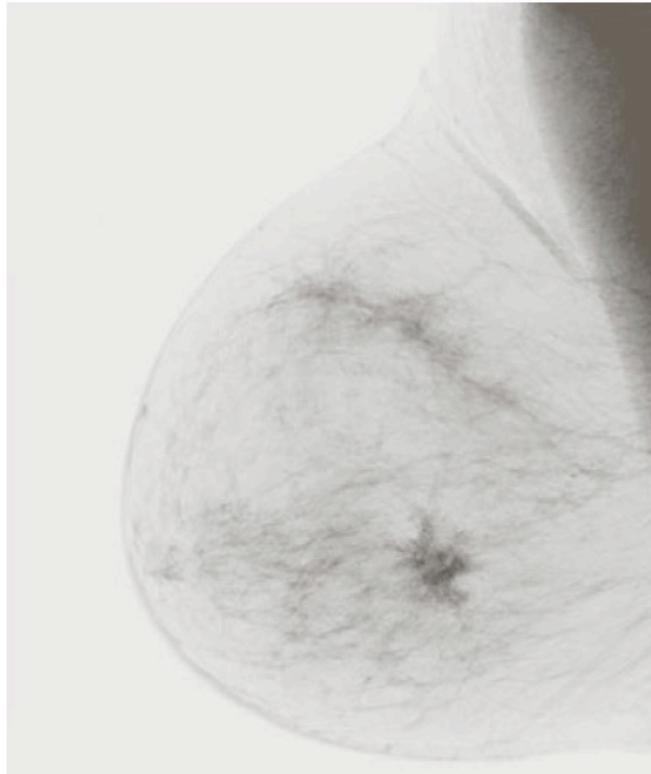
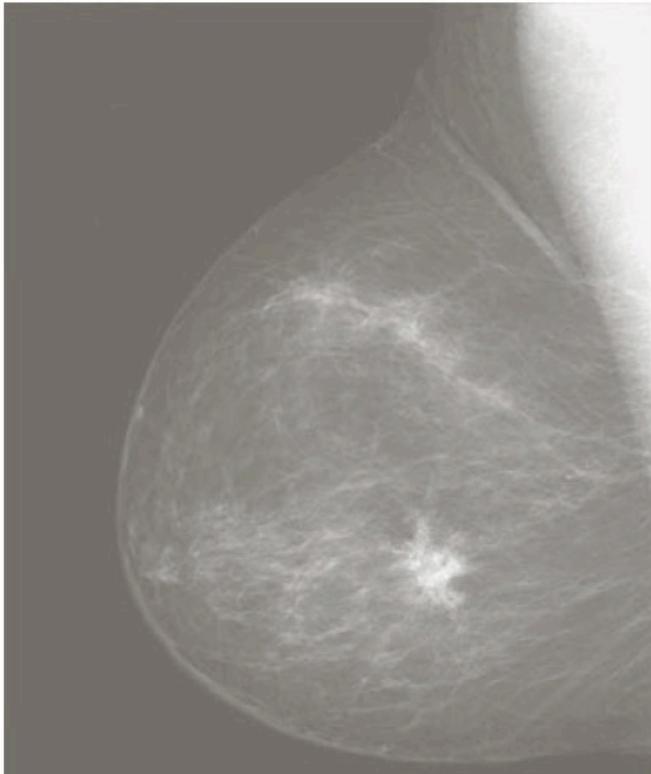
Original
Image



Negative of
an Image



Negative



a b

FIGURE 3.4

(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).
(Courtesy of G.E. Medical Systems.)

Cont..

3. Log Transformation:

- The general form of the log transformation:
 - $S = c \cdot \log(1+r)$ where c is a constant, and $r \geq 0$
- Log transformation maps a **narrow range** of low gray-level values in the input image into a **wider range** of the output levels.
- Used to **expand** the values of **dark pixels** in an image while **compressing** the **higher-level** values.
- It **compresses** the dynamic range of images with large variations in pixel values.
- Example of image with dynamic range: **Fourier spectrum image**.
- It can have intensity range from 0 to 10^6 or higher.
- We can't see the significant degree of detail as it will be lost in the display.



Algorithm

INPUT : Gray scale Image input ($M \times N$)

OUTPUT : Log transformed gray scale image output ($M \times N$)

STEPS :

1. Read the Input gray scale Image ($M \times N$)

2. Calculate Log of the input image by applying the Log transform

for ($i=0; i < M; i++$)

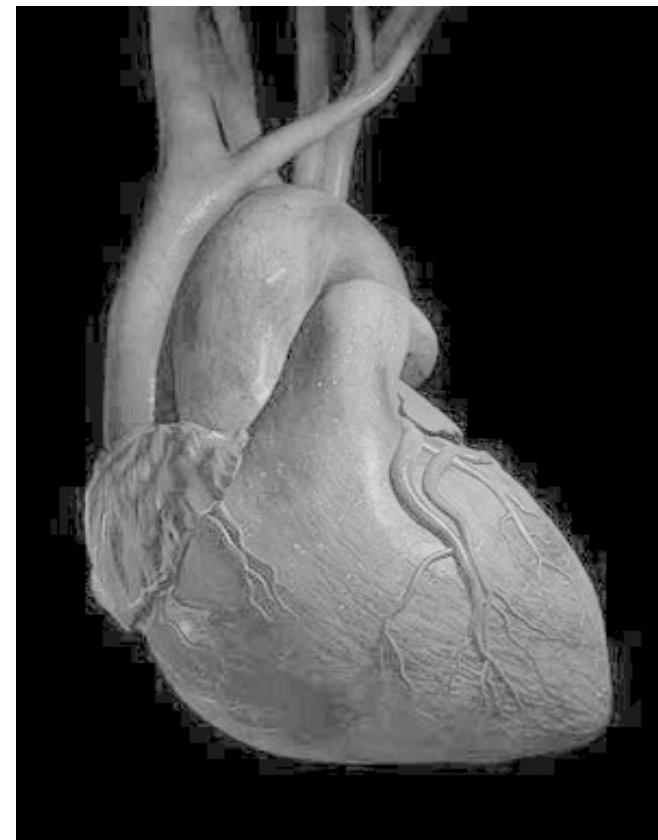
 for ($j=0; j < N; j++$)

$output[i][j] = c * \log(1 + input[i][j]);$

Where **c** is the constant



Log Transformations



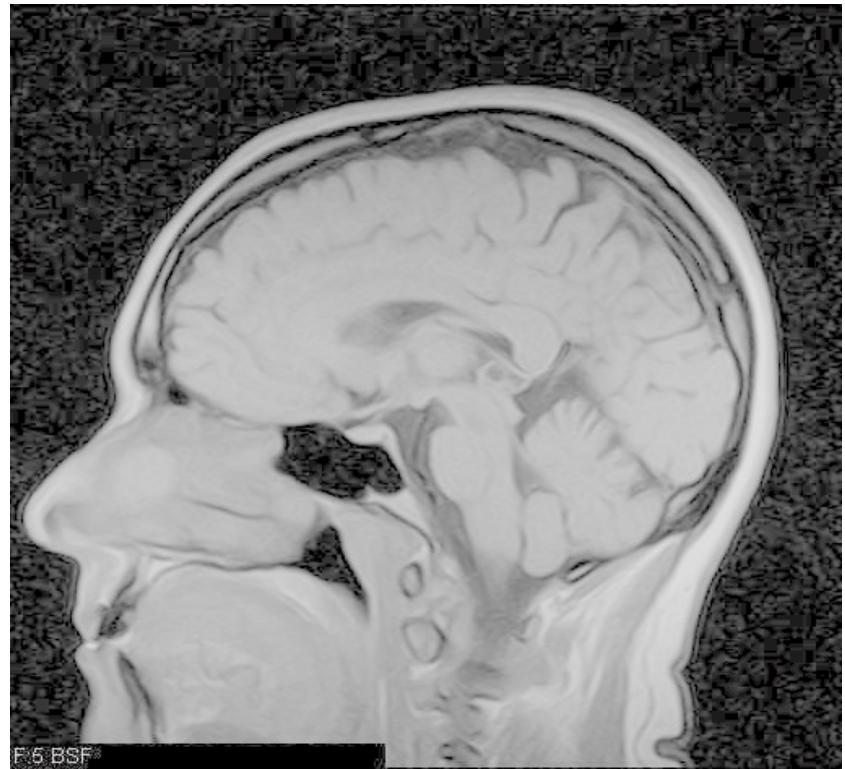
- Log Transformations maps a narrow range of low intensities values of input levels into a wider range of output levels – expands the values of dark

Cont..

- A logarithmic transform is appropriate when we want to enhance the low pixel values at the expense of loss of information in the high pixel values
- **Example,** the man in fig 1 is photographed in front of a bright background
- The dynamic range of the film material is too small, so that the gray levels on the subject's face are clustered in a small pixel value range
- A logarithmic transform spreads them over a wider range, while the higher values are compressed



Cont..



- On the other hand, applying a logarithmic transform to bright images is less appropriate, because most of its details are contained in the high pixel values
- Applying the logarithmic operator yields the image that shows a lot of information is lost during the transform

Cont..

4. *Inverse logarithm Transformation:*

- Do opposite to the log transformations.
- Used to *expand the values of high pixels in an image while compressing the darker-level values.*

5. *Power-law (Gamma) Transformation:*

- The general form of the power-law transformation: $S = c.r^\delta$, where **c** and **$\delta \geq 0$**
- Different transformations curves are obtained by varying **δ (gamma)**.
- By convention, the exponent in the power-law equation is referred to as **gamma**
- Many image capturing, printing and display devices use **gamma** correction, which enhances the given image by power-law response phenomena.



Algorithm

INPUT : Gray scale Image input ($M \times N$)

OUTPUT : Log transformed gray scale image output ($M \times N$)

STEPS :

1. Read the Input gray scale Image ($M \times N$)

2. Calculate Log of the input image by applying the Log transform

for ($i=0; i < M; i++$)

 for ($j=0; j < N; j++$)

$output[i][j] = c * (input[i][j])^\delta;$

Where **c** is the constant and **$\delta \geq 0$**



Gamma Correction

- The process used to correct this power-law response phenomena is called **gamma correction**.
- As in the case of the log transformation, power-law curves with fractional values of δ map a **narrow range** of dark I/P values into a **wider range** of O/P values, with the opposite being true for higher values of I/P levels.
- Unlike the log function, however, we notice here a family of possible transformation curves obtained simply by varying δ .
- We see in Fig. 3.6 that curves generated with values of $\delta > 1$ have exactly the opposite effect as those generated with values of $\delta < 1$.
- Finally, we note that $S = c \cdot r^\delta$ reduces to the identity transformation when $c = \delta = 1$.



Gamma Correction

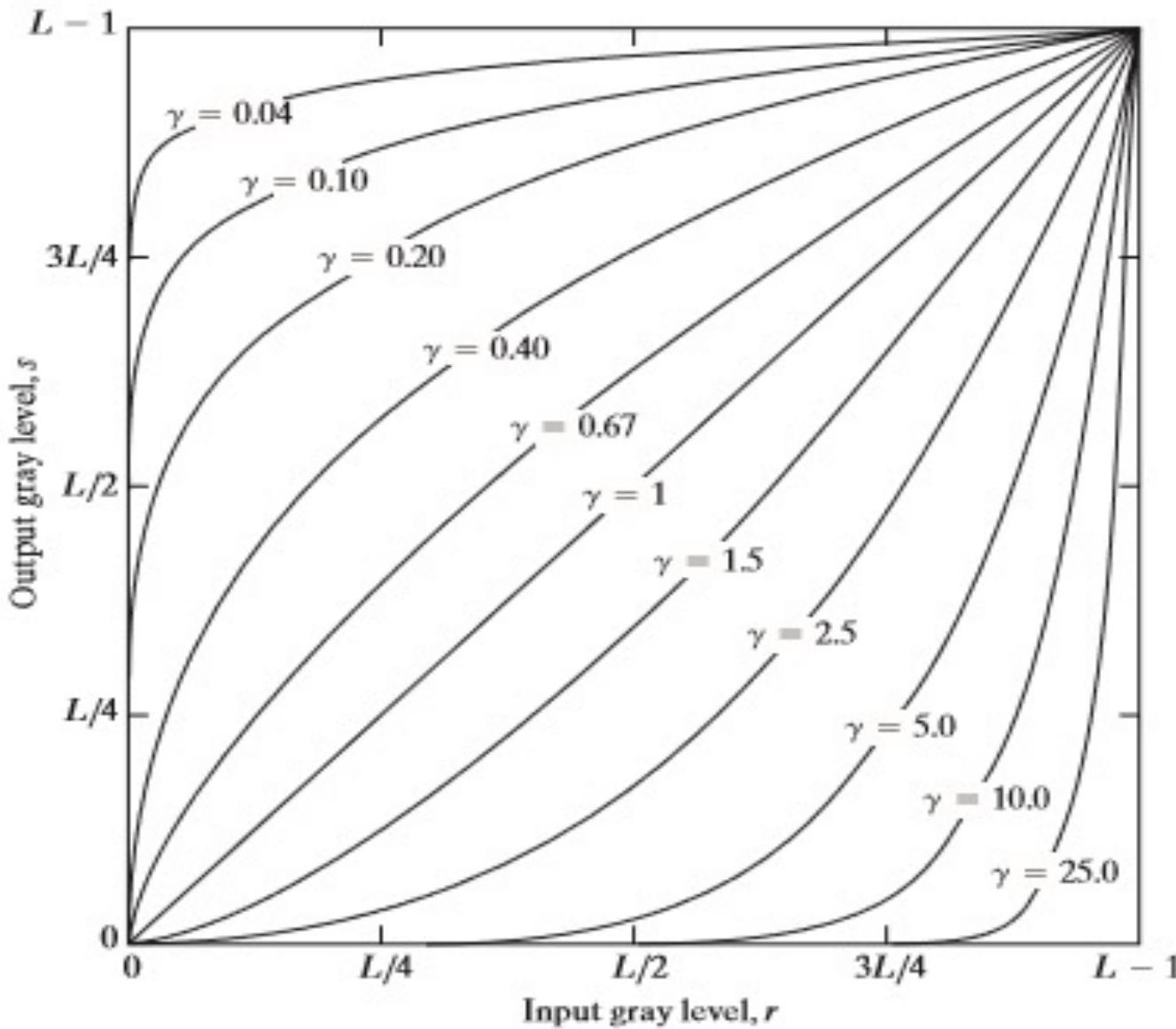


FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Plots of s versus r for various values of δ or γ are shown in Fig. 3.6

Gamma Correction

- **Example** CRT devices have an intensity-to-voltage response i.e. a power function, with δ varying from 1.8 to 2.5.
- The curve for $\delta = 2.5$ in Fig. 3.6, we see that such display systems would tend to produce **images that are darker than intended**. This effect is illustrated in Fig. 3.7.
- Figure 3.7(a) shows a simple gray-scale linear wedge I/P into a CRT monitor. The O/P of the monitor appears darker than the I/P, as shown in Fig. 3.7(b). Gamma correction in this case is straightforward.
- Now pre-process the I/P image before putting it into the monitor by performing the transformation $s = r^{1/2.5} = r^{0.4}$. The result is in Fig. 3.7(c).
- When I/P into the same monitor, this gamma-corrected I/P produces an O/P that is close in appearance to the original image, as shown in Fig. 3.7(d). A similar analysis would apply to other imaging devices such as scanners and printers.



Example: Gamma Transformations

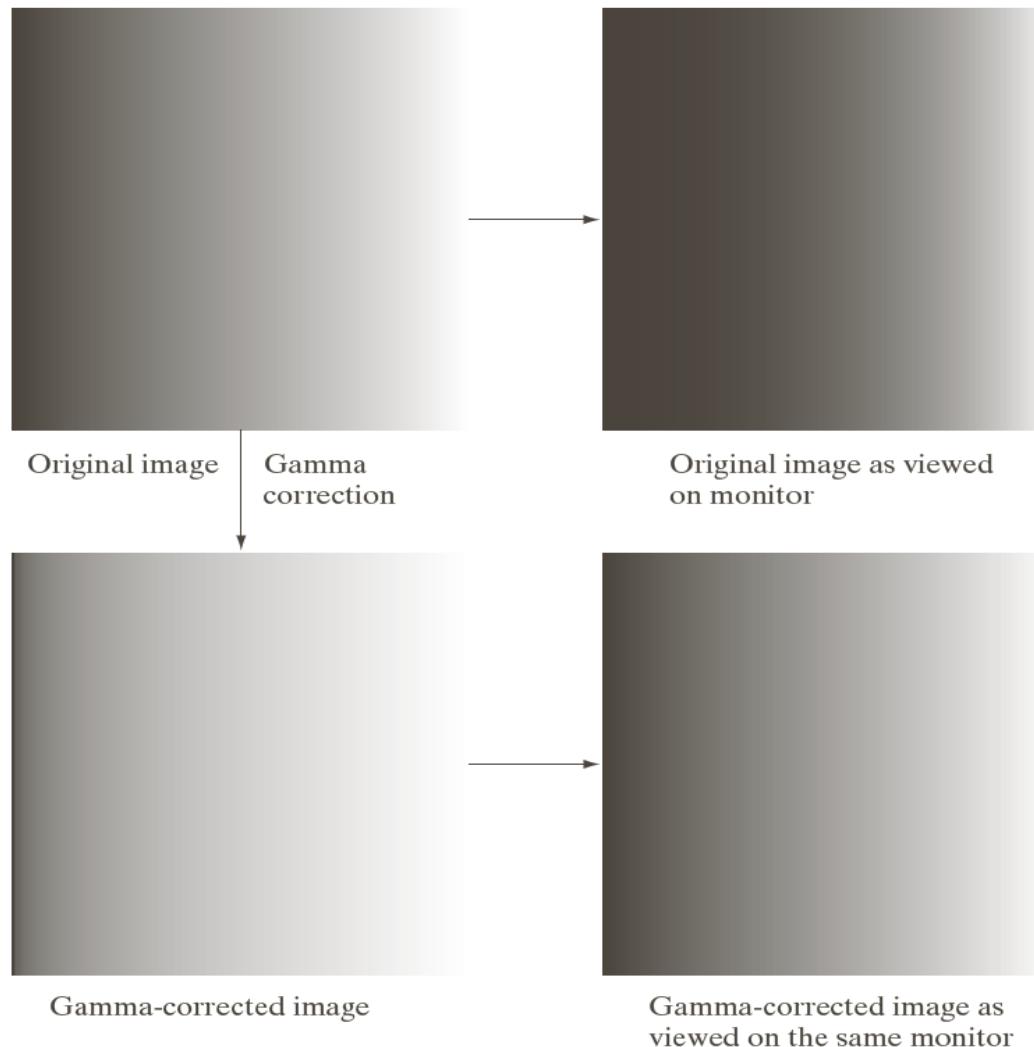
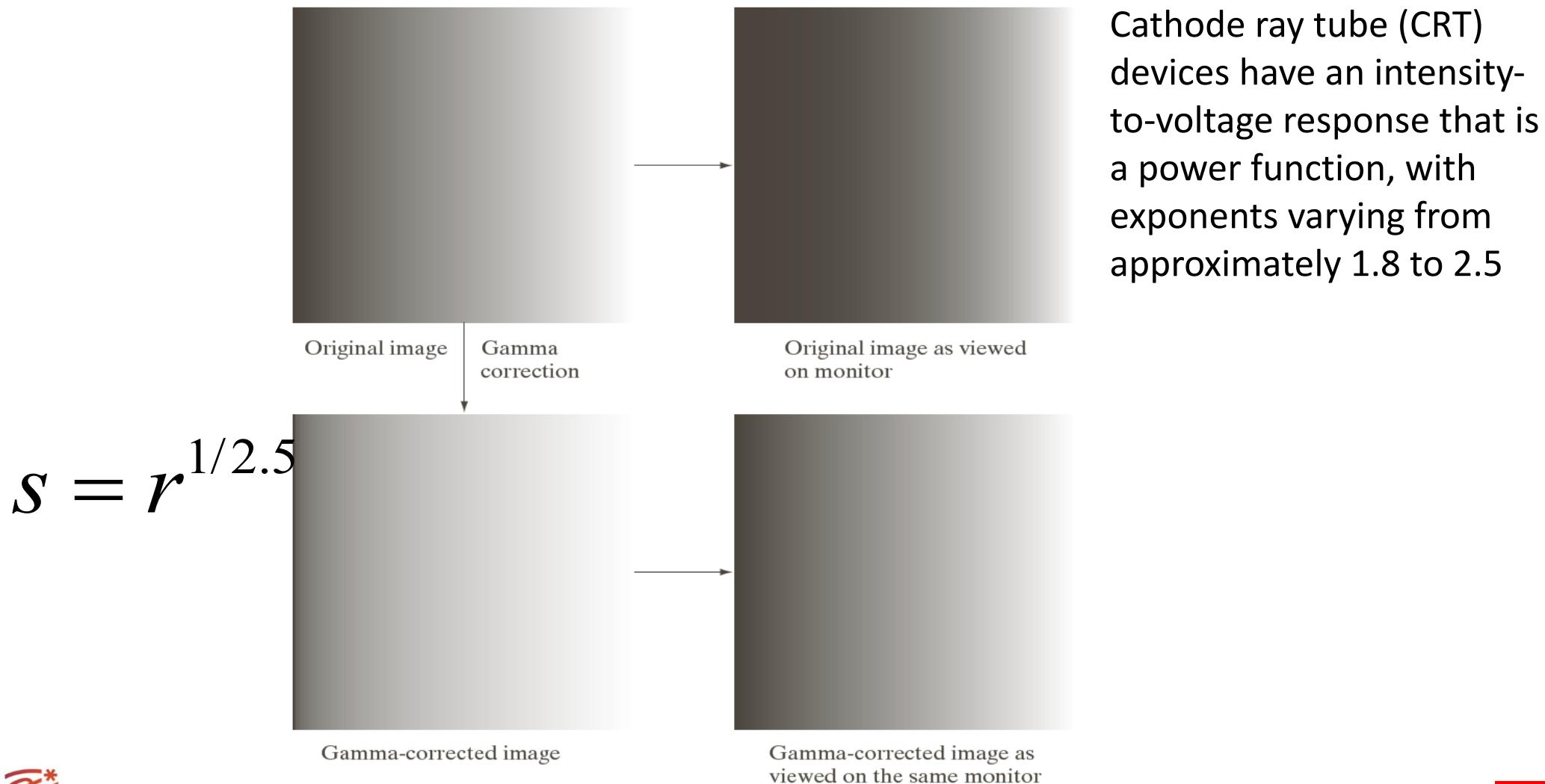


FIGURE 3.7
(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).



Example: Gamma Transformations



Cont..

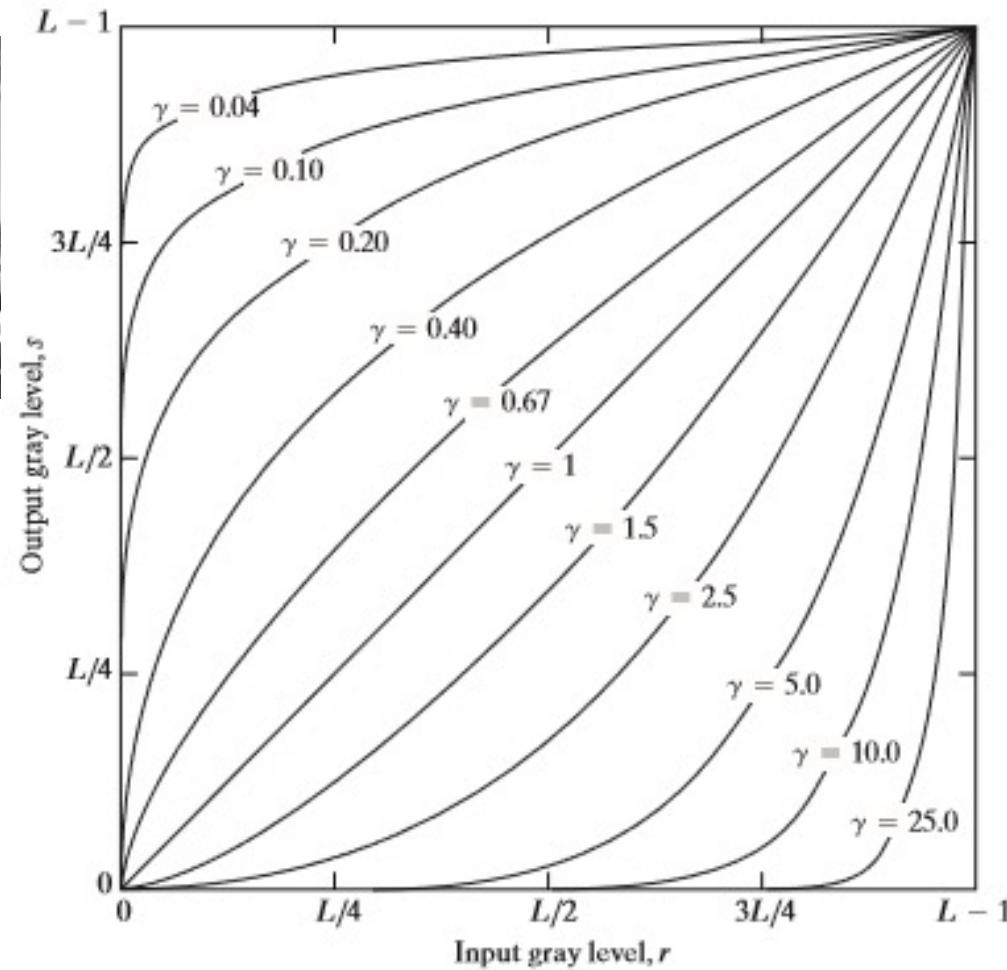
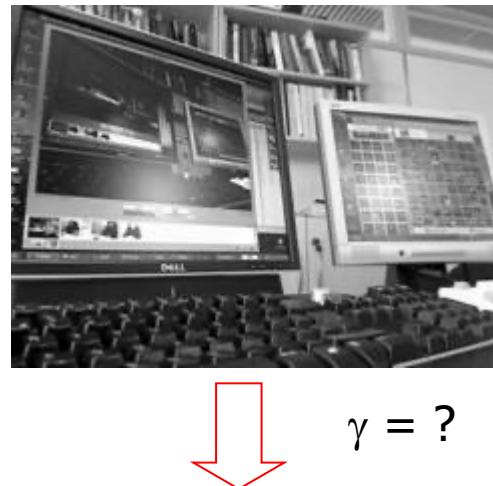


FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Gamma Correction

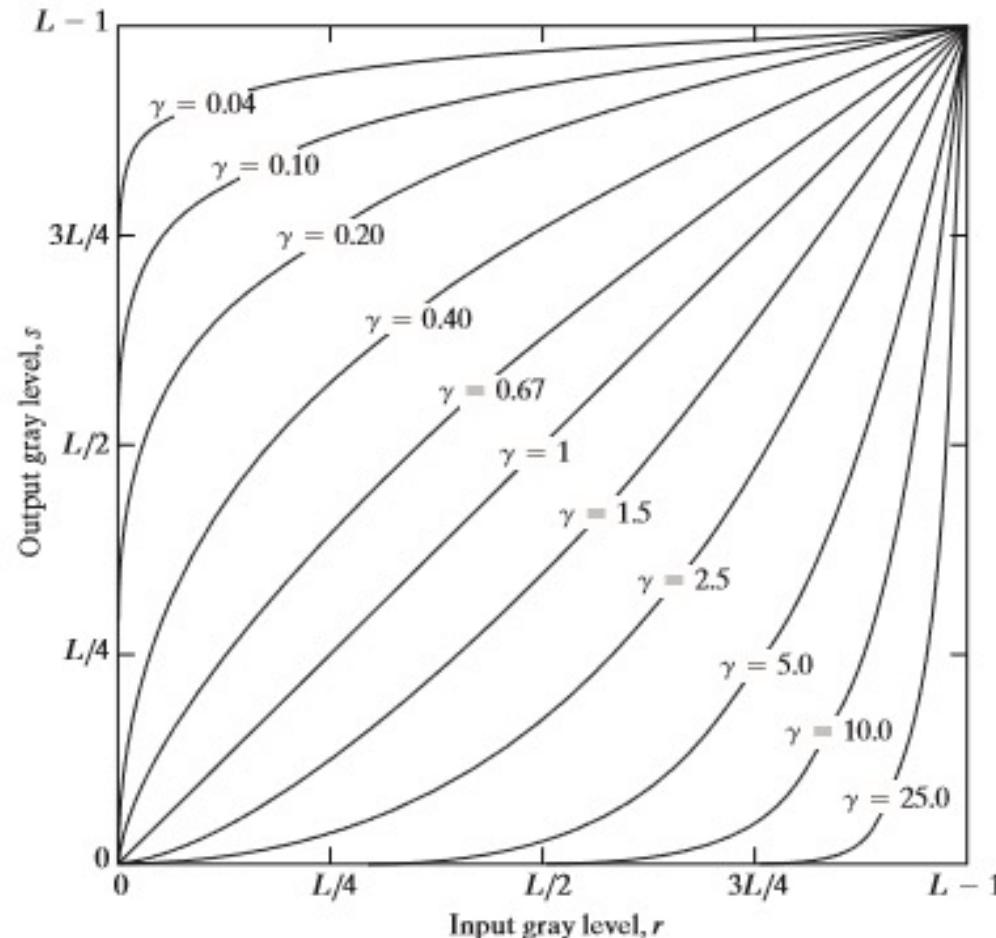
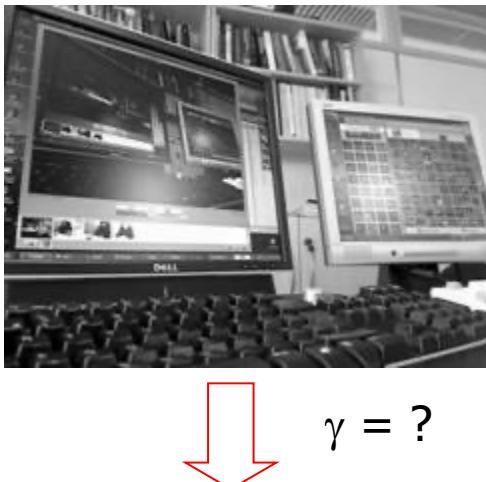
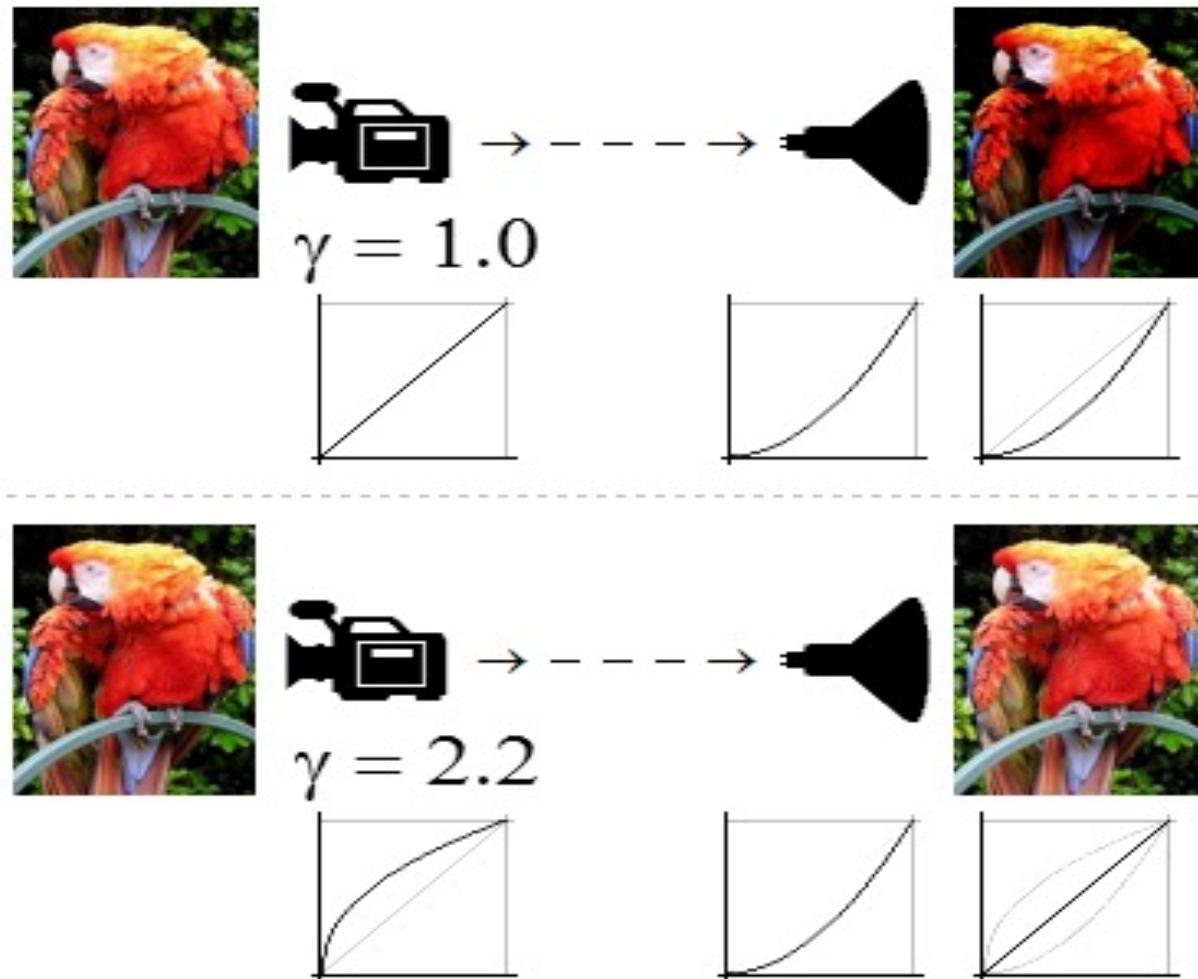


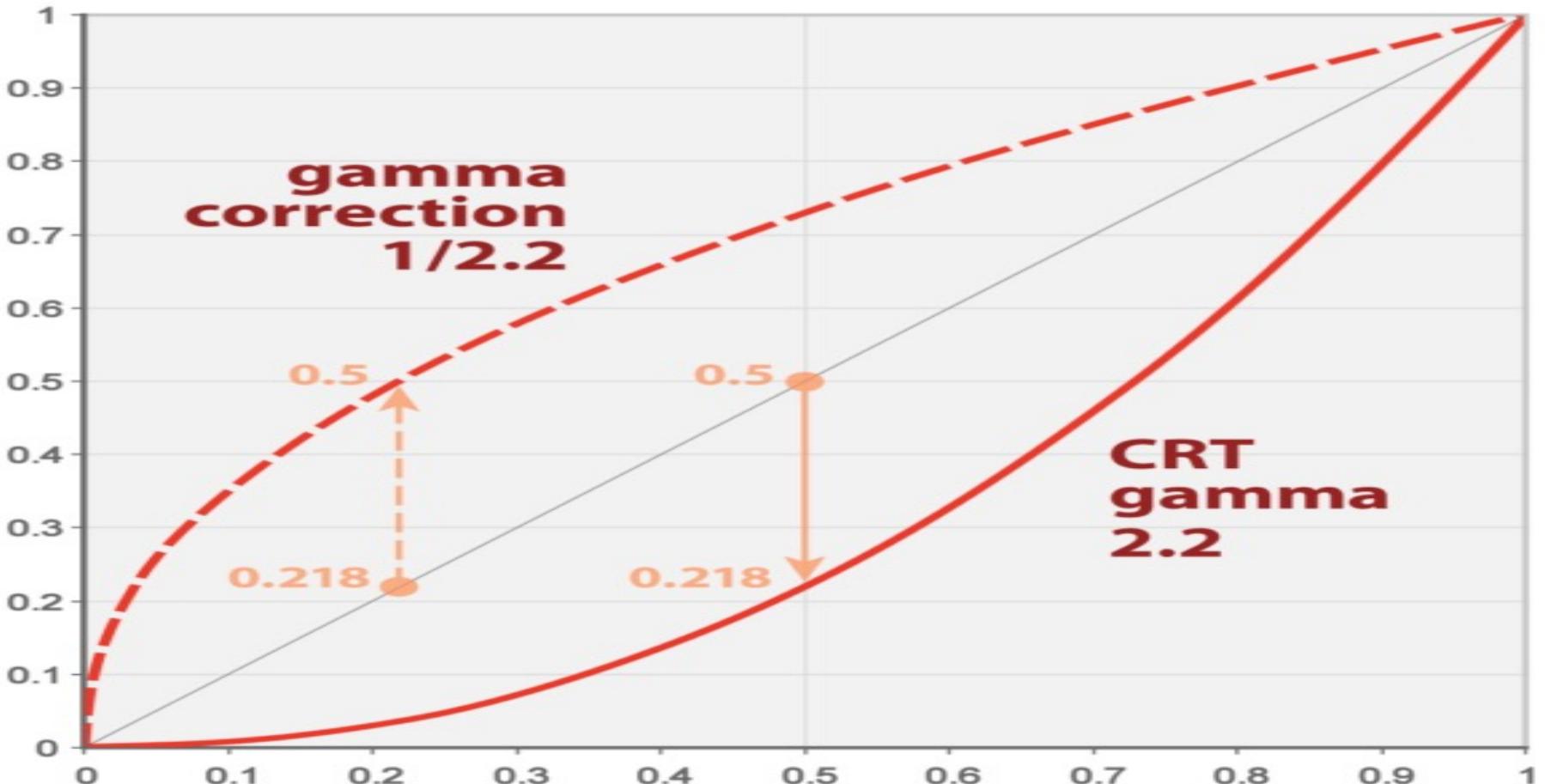
FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Gamma Compression (gamma encoding)

- Systems with linear and gamma-corrected cameras
- The dashes in the middle represent the storage and transmission of image signals or data files
- The 3 curves represent I/P –O/P functions of the camera, the display, and the overall system, respectively.

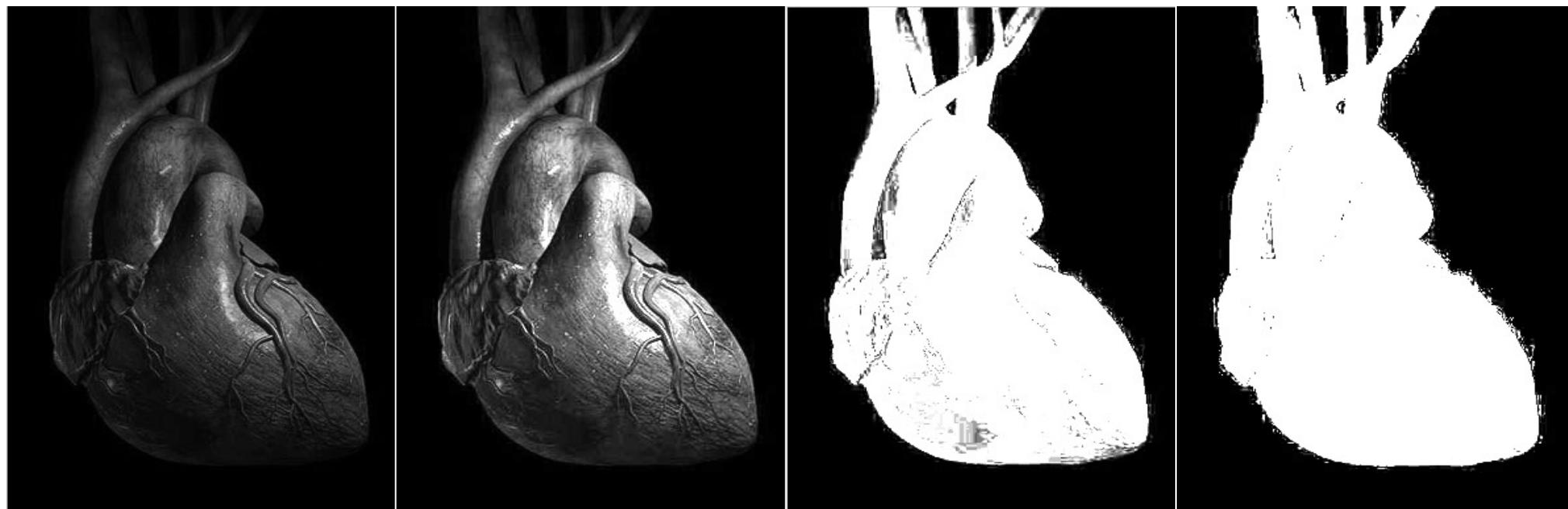


Example of CRT gamma correction



Gamma Correction

- Each panel shows the display gamma that the pixel values have been adjusted for



Input Image

Gamma(γ) = 1, c=2

Gamma(γ) = 2, c=2

Gamma(γ) = 3, c=2

What is gamma correction ?

- Some of the images on the server, especially the older images, are not gamma corrected
- In other words, they are not corrected for the nonlinear relationship between pixel value and displayed intensity that is typical for a color television monitor
- This nonlinear relationship is roughly a power function, i.e.
 $displayed_intensity = pixel_value^{\gamma}$
- Most monitors have a gamma between 1.7 and 2.7. Gamma correction consists of applying the inverse of this relationship to the image before display, i.e. by computing $new_pixel_value = old_pixel_value^{(1.0/\gamma)}$.



Why is gamma correction is important?

- Displaying an image accurately on a computer screen is of concern. Images that are not corrected properly can look either bleached out, or too dark.
- Trying to reproduce colors accurately also requires some knowledge of gamma correction because varying the value of gamma correction changes not only the brightness, but also the ratios of red to green to blue.
- Gamma correction has become increasingly important in the past few years, as use of DI for commercial purposes has increased.
- It is not unusual that images created for millions of people, the majority have different monitors or monitor settings. Some computer systems even have partial gamma correction built in.



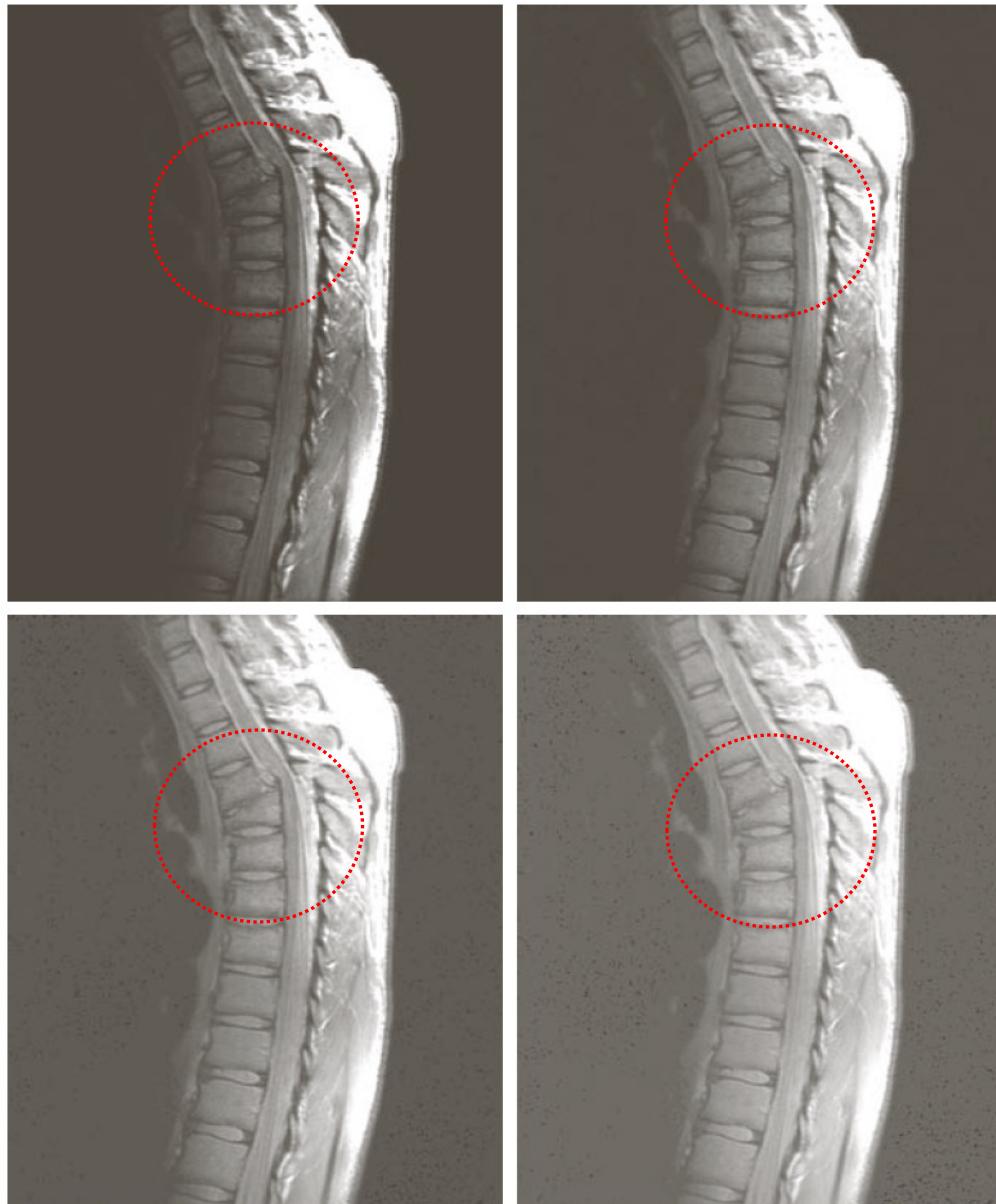
Cont..

- Current image standards do not contain the value of gamma with which an image was created, thus complicating the issue. Given these constraints, a reasonable approach when storing images in a Web site is to preprocess the images with a gamma that represents an “average” of the types of monitors and CS that one expects in the open market at any given point in time.
- It is useful for GP contrast manipulation. Figure 3.8(a) shows a MRI of an upper thoracic human spine with a fracture dislocation and spinal cord impingement. The fracture is visible near the vertical center of the spine, approximately one-fourth of the way down from the top of the picture.
- Since the given image is dark, details are desirable. This can be done by power-law transformation with a fractional exponent.



Example: Gamma Transformations

as γ decreased from 0.6 to 0.4, more detail became visible.



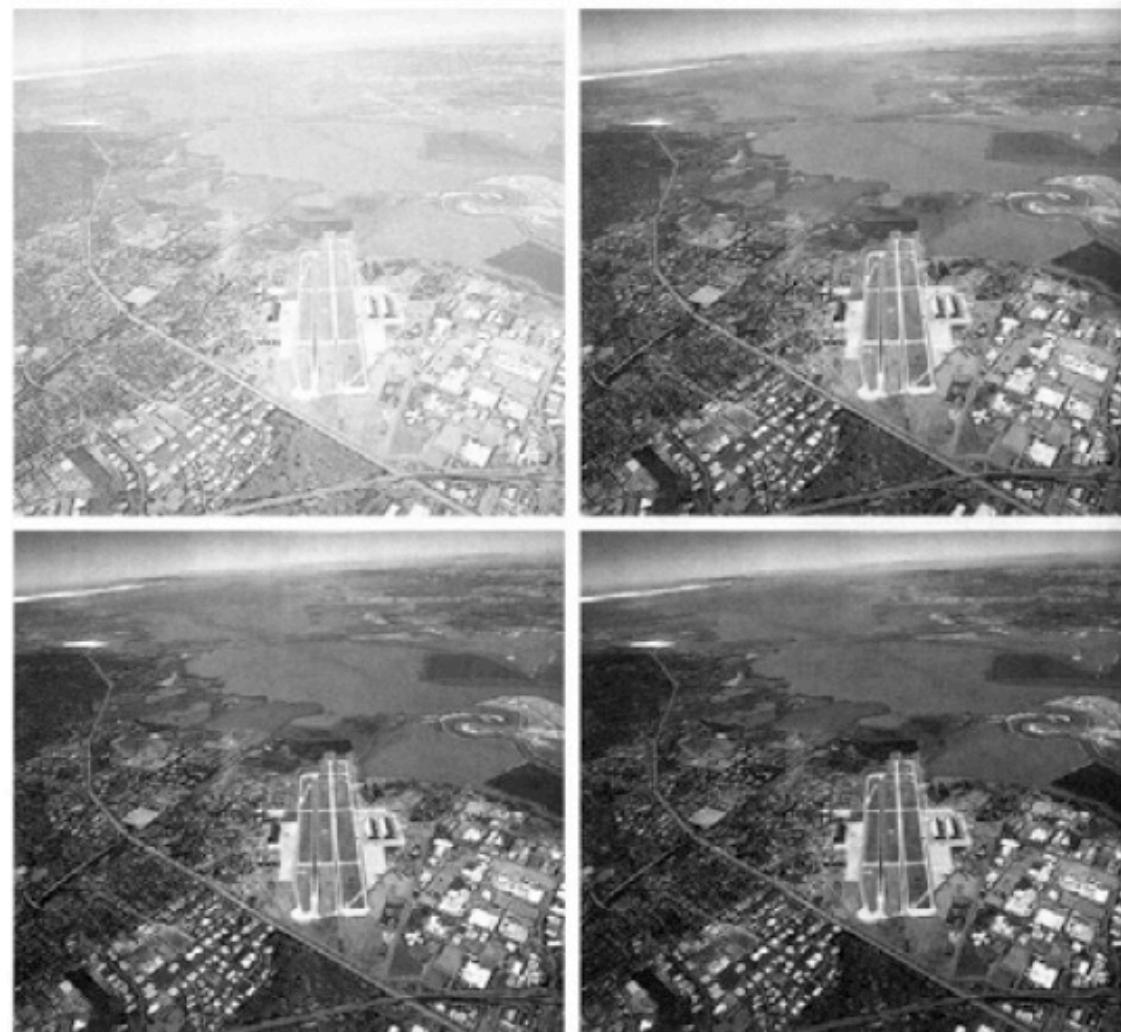
a	b
c	d

FIGURE 3.8
(a) Magnetic resonance image (MRI) of a fractured human spine.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3 , respectively.
(Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)



Power-Law Transformations

The original “washed-out” image out and gamma transformed with $c = 1$ and $\gamma = 3.0, 4.0$, and 5.0 respectively. The last image may appear having areas that are too dark



Cont..

- The difference between the **log function** and the **power-law functions** is that using the power-law function a family of possible transformation curves can be obtained just by varying the λ .
- Power-law transformations are much more **versatile** for spreading/compressing of gray levels in an image than the log transformation.
- Log function has the important characteristic that it compresses the dynamic range of images with large variations in pixel values.



Algorithm

INPUT : Gray scale Image input($M \times N$)

OUTPUT: Power-Law transformed Gray scale image output($M \times N$)

STEPS :

1. Read the Input Gray scale Image ($M \times N$)

2. Calculate Power-Law of the input image by applying the equation $s = cr^\gamma$

Where s is input and r is output image

for ($i=0; i < M; i++$)

 for ($j=0; j < N; j++$)

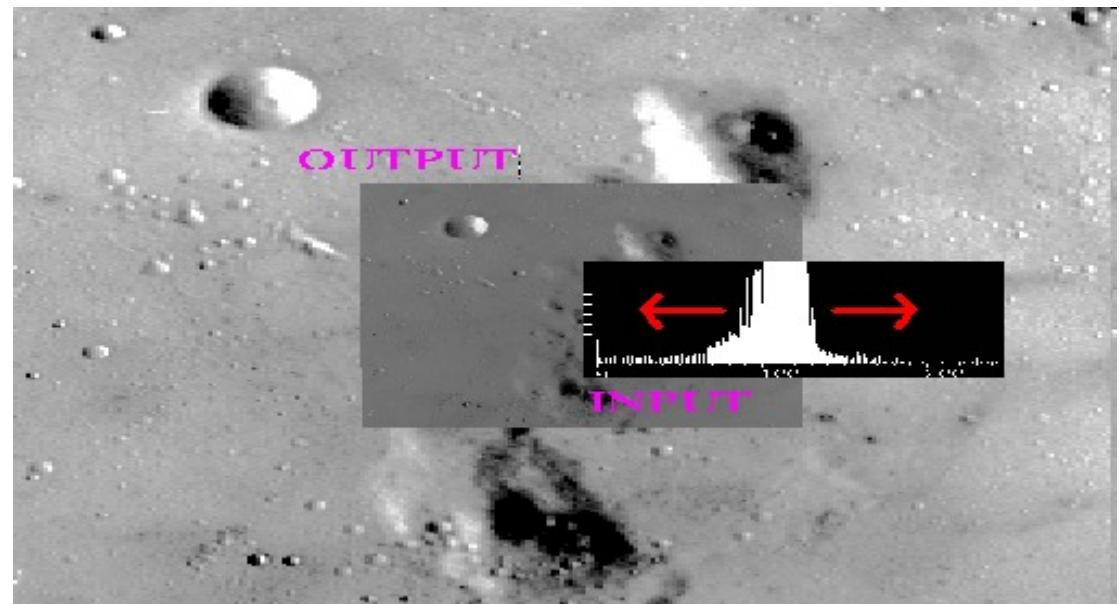
 Calculate $output[i][j] = c * pow(input[i][j], gamma);$

Where **c** is the constant gamma is the power of the input pixel



Piecewise Linear Transformations

- Another approach is to use a piecewise linear function of arbitrary complexity
 - **Contrast stretching**



- **Intensity-level slicing**



Cont..

6. Piecewise-linear Transformation Functions

- A complementary approach to the methods discussed is to use piecewise linear functions.
 - **Advantage:** the form of piecewise function is *arbitrarily complex* (more options to design), some important transformations can be formulated only as piecewise functions.
 - **Disadvantage:** their specification requires more user input.
- A. **Contrast stretching:** One of the most (simplest) piecewise functions is the *contrast stretching, which is* used to enhance the low contrast images.



Cont..

A. Contrast stretching:

- Low-contrast images can result from poor illumination, lack of dynamic range in the imaging sensor, or even wrong setting of a lens aperture during image acquisition.
- The idea of **contrast stretching** is to increase the dynamic range of the gray levels in the image being processed.
- Figure 3.10(a) shows a transformation used for contrast stretching. The locations of points (r_1, s_1) and (r_2, s_2) control the shape of the transformation function.
 - a) $s_1 = r_1$ and $s_2 = r_2$ (*Identity transformation*) (**no change** in the image).
 - b) $r_1 = r_2$, $s_1 = 0$, and $s_2 = L-1$ (*thresholding function*) [*Image converted* to black and white i.e. it creates a binary image, as illustrated in Fig. 3.2(b).]
 - c) Intermediate values of (r_1, s_1) and (r_2, s_2) produces various degree of spread in the intensity level of the O/P image. Thus affecting its contrast.



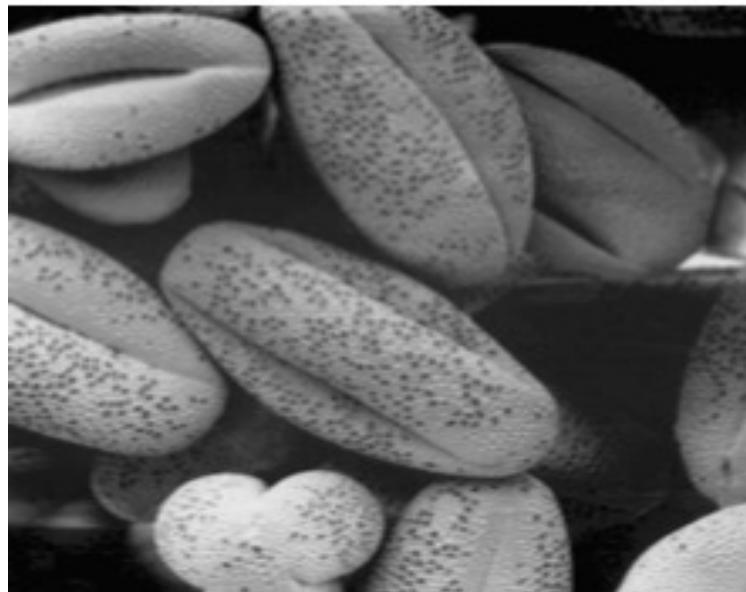
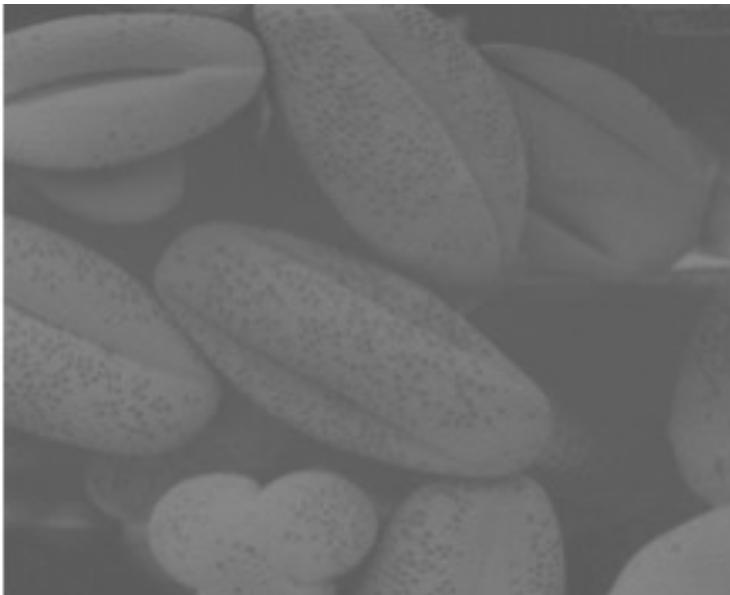
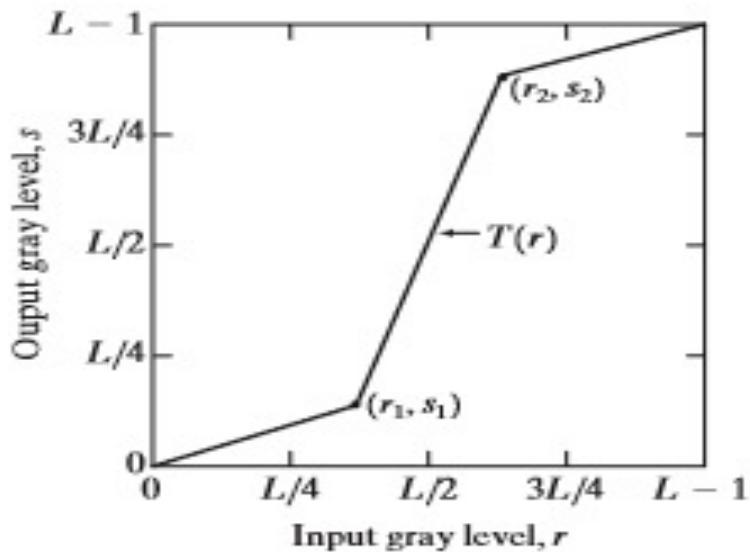
Cont..

A. Contrast stretching:

- In general, $r_1 \leq r_2$ and $s_1 \leq s_2$, so the function is **single valued** and **monotonically increasing**.
 - This condition preserves the order of gray levels, thus preventing the creation of intensity artifacts in the processed image.
- **Example:** To enhance an 8-bit image with **low contrast** using **contrast stretching**, and obtained by setting:
 - $(r_1, s_1) = (r_{\min}, 0)$ and $(r_2, s_2) = (r_{\max}, L-1)$ where r_{\min} and r_{\max} denote the *minimum and the maximum intensity level in the image*. $[5, 40] \Rightarrow [0, 255]$
- So the transformation function stretches the levels linearly from their original range to the full range **[0, L-1]**.



Contrast Stretching



a
b
c
d

FIGURE 3.10
Contrast stretching.
(a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Cont..

- **Example:**
- Figure 3.10(b) shows an 8-bit image with **low contrast**.
- Fig. 3.10(c) shows the result of contrast stretching, obtained by setting
 - $(r_1, s_1) = (r_{\min}, 0)$ and $(r_2, s_2) = (r_{\max}, L-1)$
 - where r_{\min} and r_{\max} denote the *minimum and the maximum intensity level in the image*.
 - So the transformation function stretches the levels linearly from their original range to the full range $[0, L-1]$.
- Finally, Fig. 3.10(d) shows the result of using the **thresholding function**, with $r_1 = r_2 = m$, the mean gray level in the image.
- The original image on which these results are based is a scanning electron microscope image of pollen, magnified approximately 700 times.

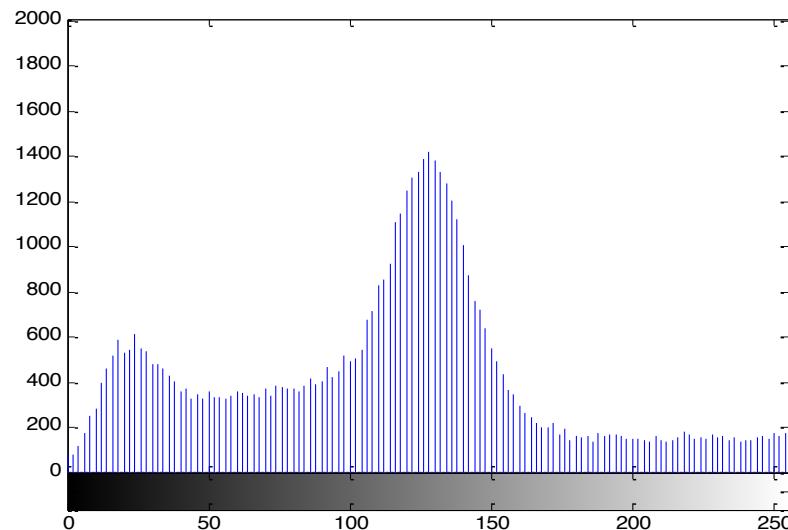
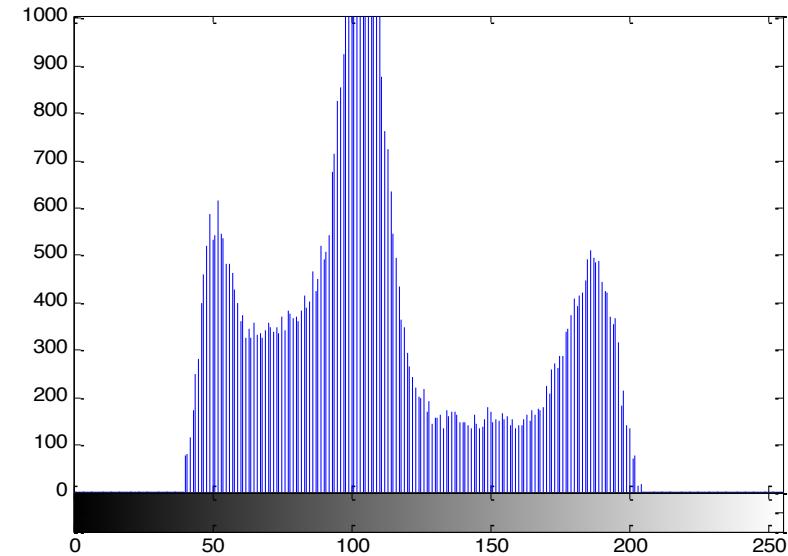
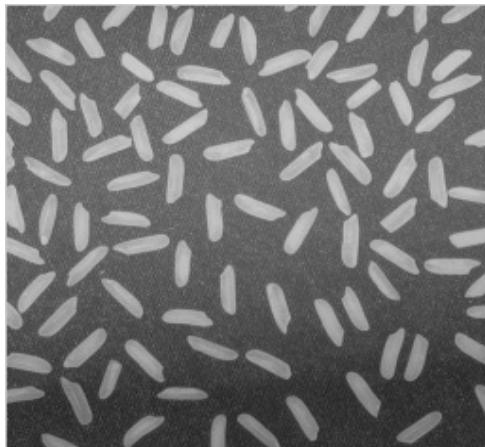
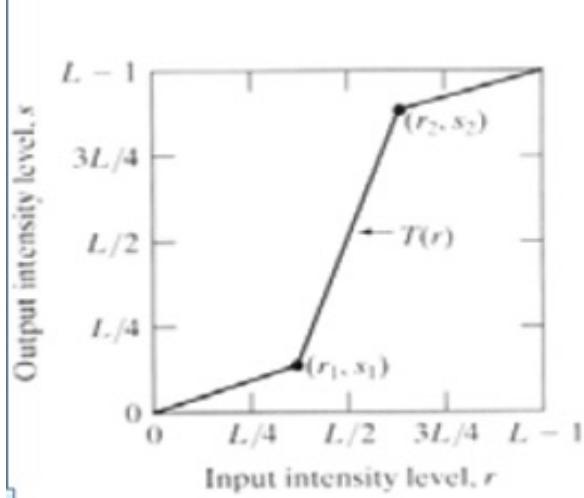


Contrast Stretching

Expands the range of intensity levels in an image to utilize the entire dynamic range of recording media

$$(r_1, s_1) = (r_{\min}, 0)$$

$$(r_2, s_2) = (r_{\max}, L-1)$$



Contrast Stretching

$$s = (r - c) \left[\frac{b - a}{d - c} \right] + a$$

Lower and **upper** limits for **a(s_1)** and **b(s_2)** are **0** and **255**

c (r_{min}) and **d**(r_{max}) are lower and upper values of input image

r is the input , **s** is the output



Algorithm

INPUT : Gray scale Image input(M x N)

OUTPUT: Contrast Stretched Gray scale image output(M x N)

STEPS :

1. Read the Input Gray scale Image (M x N)
2. Calculate Contrast Stretching of the input image by applying the equation

$$s = (r - c) \left[\frac{b - a}{d - c} \right] + a$$

Where Lower and upper limits for a and b are 0 and 255; c and d are lower and upper values of input image, r is the input, s is the output

```
I=imread('1.png'); [M,N]=sizeof(I);  
for (i=0; i<M; i++)  
    for (j=0; j<N; j++)  
        output[i][j] =[input[i][j] – c][(b-a)/(d-c)]+a ;
```



Cont..

- B. Gray-level slicing (Intensity-level slicing):** This technique is used to *highlight a specific range of gray levels in a given image*, it is called **intensity-level slicing**, and can be implemented in several ways.
- **Applications:** enhancing features such as masses of water in satellite imagery and enhancing flaws in X-ray images. There are several ways of doing level slicing, but most of them are variations of two basic themes.
 - I. 1st approach is to display a **high value** for all gray levels in the range of interest(ROI) and a **low value** for all other gray levels. This transformation, shown in Fig. 3.11(a), produces a **binary image**.
 - II. 2nd approach, **transformation** shown in Fig. 3.11(b), **brightens** the desired range of gray levels but preserves the background and gray-level tonalities in the image.
 - Variations of the two transformations shown in Fig. 3.11 are easy to formulate.

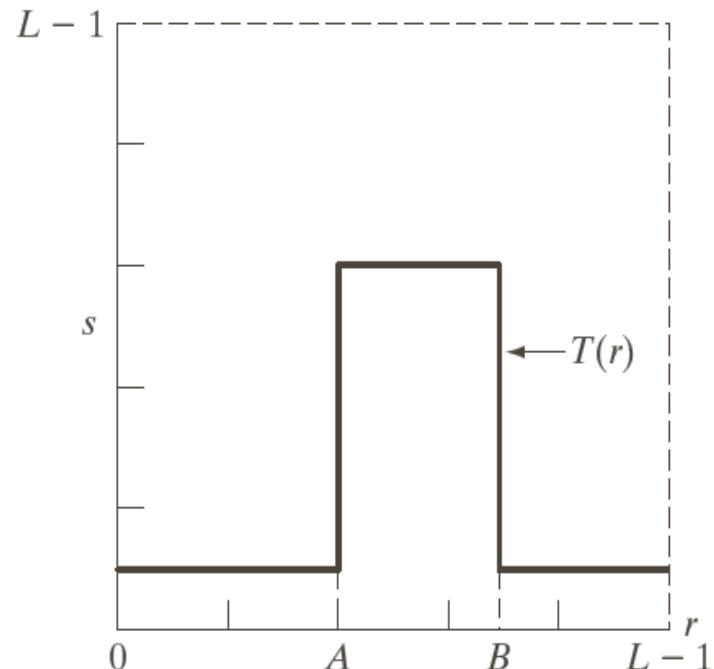


Grey-level Slicing

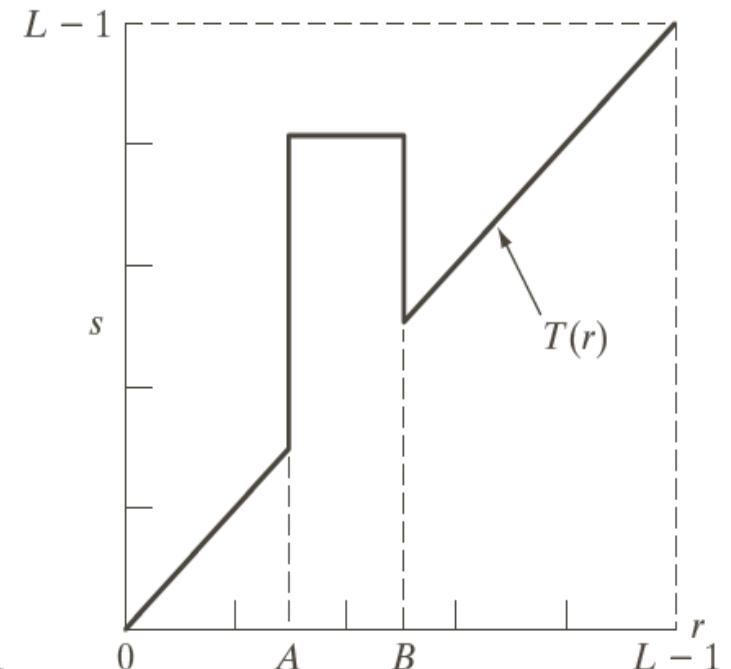
- Highlighting a specific level of intensities (range of gray levels) .
- Used to enhance features in images

a b

FIGURE 3.11 (a) This transformation highlights intensity range $[A, B]$ and reduces all other intensities to a lower level. (b) This transformation highlights range $[A, B]$ and preserves all other intensity levels.



Highlights the intensity range $[A,B]$ and reduces all other intensities to a lower level



Highlights the intensity range $[A,B]$ and preserves all other intensity levels



Intensity-level slicing

$$s = \begin{cases} 50 & \text{for } 0 \leq r < A \\ 200 & \text{for } A \leq r < B \\ 50 & \text{for } B \leq r \leq 255 \end{cases}$$

- Where A & B are the suitable to be identified from the specific input image

$$s = \begin{cases} r & \text{for } 0 \leq r < A \\ 200 + r & \text{for } A \leq r < B \\ r & \text{for } B \leq r \leq 255 \end{cases}$$



Algorithm

INPUT : Gray scale Image input($M \times N$)

OUTPUT : Intensity-level sliced Gray scale image output($M \times N$)

STEPS :

1. Read the Input Gray scale Image ($M \times N$)
2. Calculate Intensity-level slicing of the input image by applying

the equation

$$s = \begin{cases} 50 & \text{for } 0 \leq r < A \\ 200 & \text{for } A \leq r < B \\ 50 & \text{for } B \leq r \leq 255 \end{cases}$$

for ($i=0; i < M; i++$)

 for ($j=0; j < N; j++$)

 { Apply formula

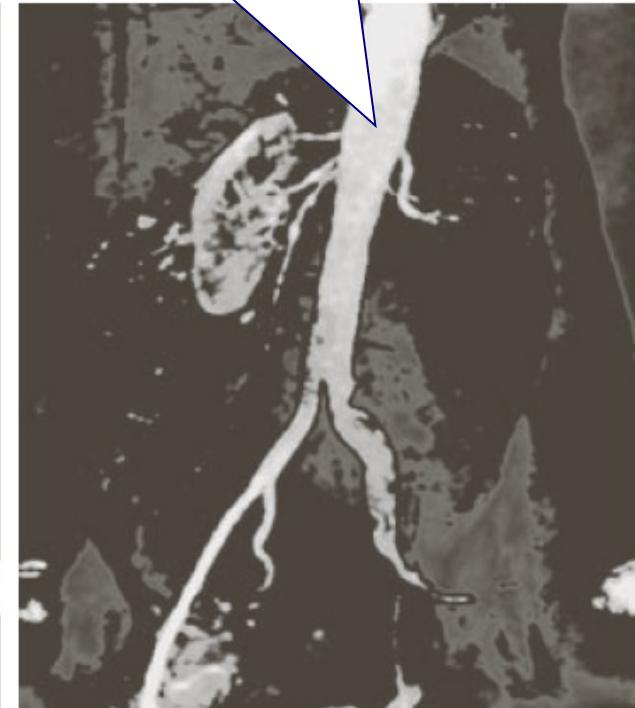
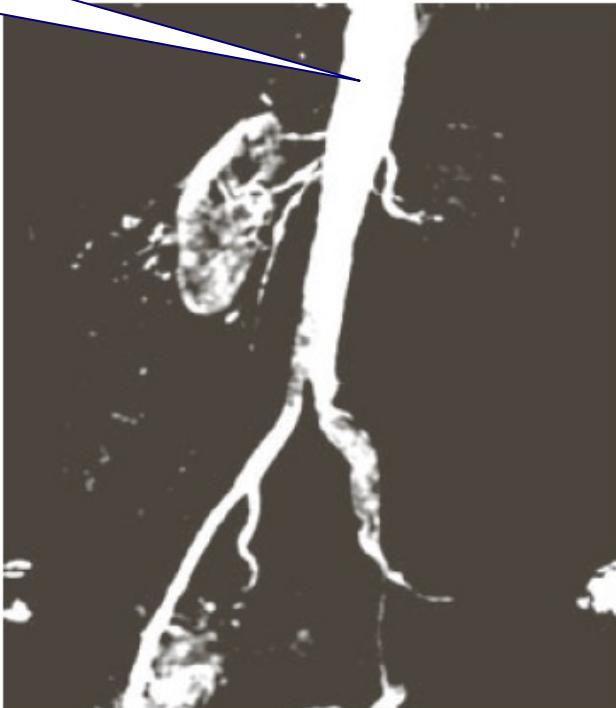
 }



Highlight the major blood vessels and study the shape of the flow of the contrast medium (to detect blockages, etc.)

Cont..

Measuring the actual flow of the contrast medium as a function of time in a series of images



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Cont..

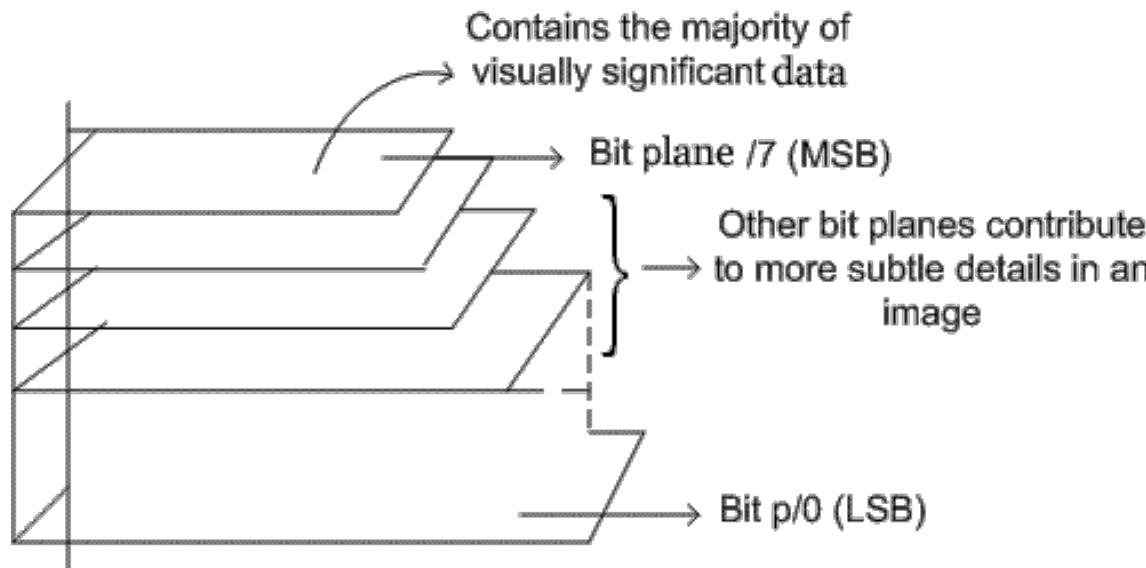
C. Bit-plane slicing: Pixels are digital number composed of bits.

- Instead of highlighting intensity-level range, we could **highlight** the contribution made by each bit, this method is useful and used in image compression.
- Let each pixel in an image is represented by 8 bits Imagine the image is composed of 8, 1-bit planes ranging from bit plane1-0 (LSB)to bit plane 7 (MSB).
- In terms of 8-bits bytes, plane 0 contains all lowest order bits in the bytes comprising the pixels in the image and plane 7 contains all high order bits.
- Figure 3.13 illustrates these ideas, and Fig. 3.14 shows the various bit planes for the image shown in Fig. 3.13.
- **Note:** The higher-order bits (especially the top four) contain the majority of the visually significant data. The other bit planes contribute to more subtle details in the image.



Cont..

C. Bit-plane slicing:



- In terms of bit-plane extraction for a 8-bit image, it is seen that binary image for bit plane 7 is obtained by proceeding the I/P image with a thresholding gray-level transformation function that maps all levels between 0 and 127 to one level (e.g 0)and maps all levels from 129 to 253 to another (eg. 255).



Bit-plane Slicing

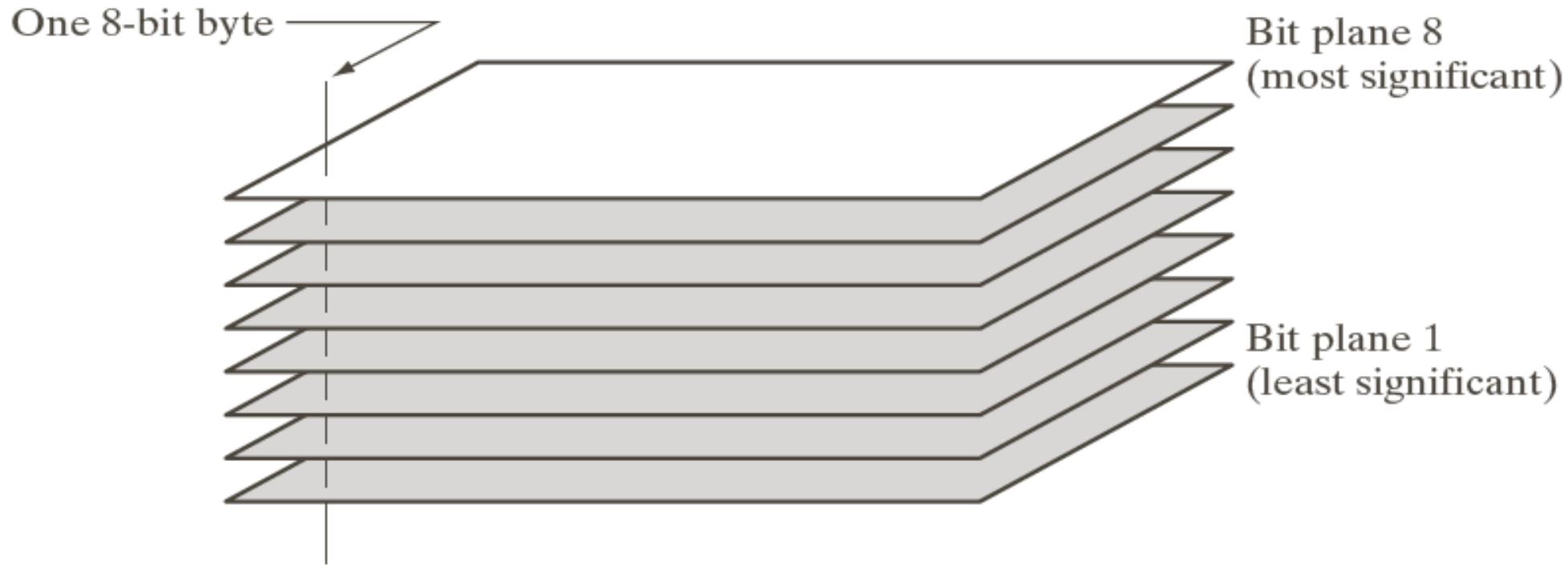


FIGURE 3.13
Bit-plane
representation of
an 8-bit image.



Bit-plane Slicing

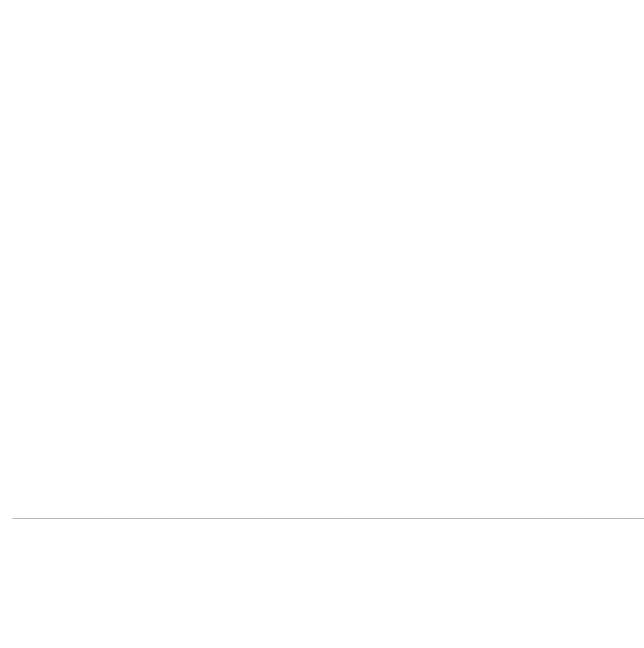


Figure 3.14(a) shows the original 8-bit gray-scale image of size 500×1192 pixels. This image is composed of 256 different gray levels, ranging from black (0) to white (255).

The image is divided into 8 horizontal bands, each representing a bit plane. Bit plane 1 corresponds to the least significant bit (LSB), while bit plane 8 corresponds to the most significant bit (MSB).

Figure 3.14(b) through (i) show the 8 bit planes corresponding to the original image. Each bit plane is a binary image, where the value of each pixel is either 0 or 1.

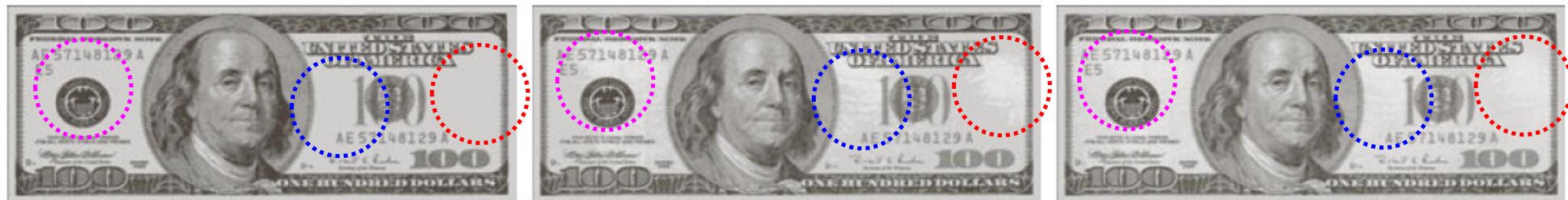
For example, if a pixel has a value of 255 in the original image, it will have a value of 1 in all 8 bit planes. If a pixel has a value of 0, it will have a value of 0 in all 8 bit planes.

Figure 3.14(j) shows the original image reconstructed from its 8 bit planes. The image is identical to the original image.

a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500×1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Bit-plane Slicing



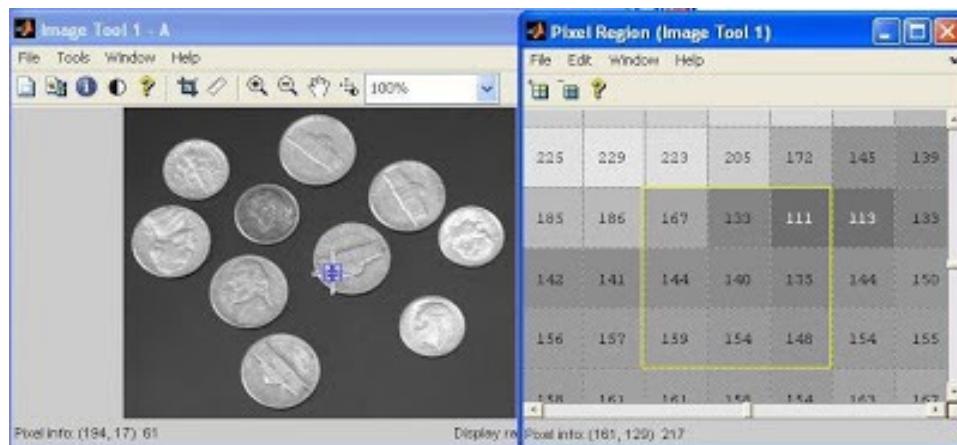
a b c

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).



Cont..

- **Example(Bit- plane slicing):**
- Digitally, an image is represented in terms of pixels. These pixels can be expressed further in terms of bits.
- Consider the image ‘coins.png’ and the pixel representation of the image.



10100111	10000101	01101111
10010000	10001100	10000111
10011111	10011010	10010100

- Consider the pixels that are bounded within the yellow line. The binary formats for those values are (8-bit representation)

Cont..

- The binary format for the pixel value 167 is 10100111
- Similarly, for 144 it is 10010000
- This 8-bit image is composed of eight 1-bit planes.
- Plane 1 contains the lowest order bit of all the pixels in the image.

10100111	10000101	01101111
10010000	10001100	10000111
10011111	10011010	10010100

Fig 1.

010100111	00000101	01101111
00010000	00001100	00000111
00011111	00011010	00010100

Fig 2.

- And plane 8 contains the highest order bit of all the pixels in the image.



Cont..

- Let's see how we can do this using MATLAB
- A=[167 133 111;
 144 140 135;
 159 154 148]
- B = bitget(A,1); %LSB of all pixels 'bitget' is a MATLAB function used to fetch a bit from the specified position from all the pixels.
- B=[1 1 1
 0 0 1
 1 0 0]
- B = bitget(A,8); %Highest order bit of all pixels
- B=[1 1 0
 1 1 1
 1 1 1]



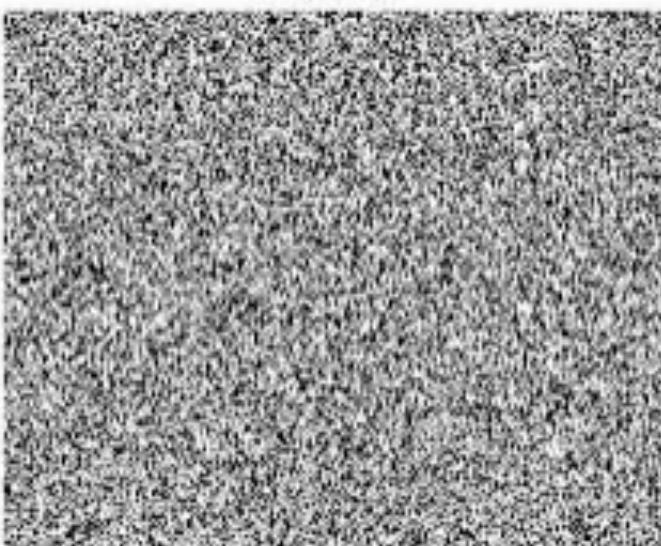
Cont..

- **MATLAB CODE:**
- %Bit Planes from 1 to 8. Output Format: Binary
- A = imread('coins.png');
- B = bitget(A,1);
- figure, subplot(2,2,1); imshow(logical(B));title('Bit plane 1');
- B= bitget(A,2);
- subplot(2,2,2); imshow(logical(B));title('Bit plane 2');
- B = bitget(A,3);
- subplot(2,2,3); imshow(logical(B));title('Bit plane 3');
- B = bitget(A,4);
- subplot(2,2,4); imshow(logical(B));title('Bit plane 4');

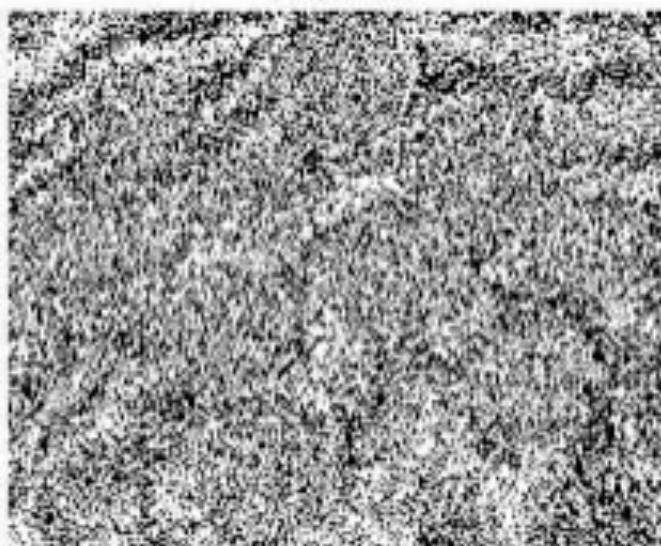


Cont..

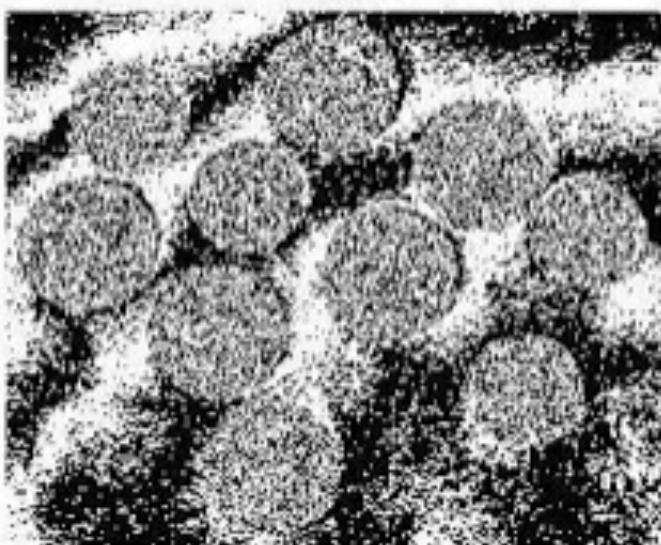
Bit plane 1



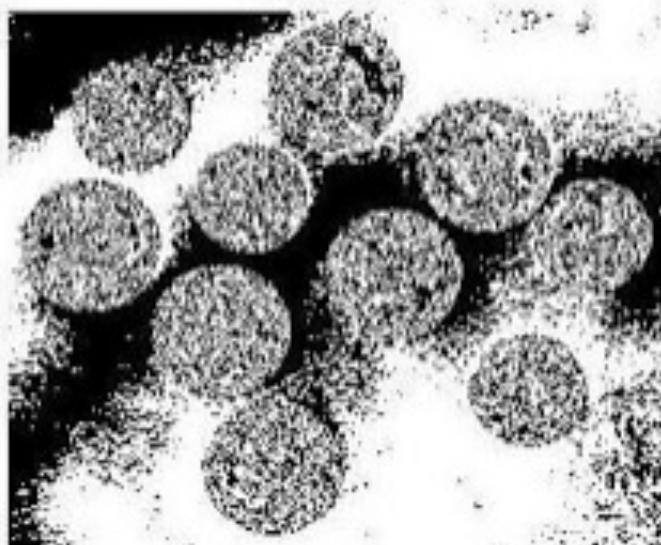
Bit plane 2



Bit plane 3



Bit plane 4



Cont..

- `B = bitget(A,5);`
- `figure, subplot(2,2,1); imshow(logical(B));title('Bit plane 5');`
- `B = bitget(A,6);`
- `subplot(2,2,2); imshow(logical(B));title('Bit plane 6');`
- `B = bitget(A,7);`
- `subplot(2,2,3); imshow(logical(B));title('Bit plane 7');`
- `B = bitget(A,8);`
- `subplot(2,2,4); imshow(logical(B));title('Bit plane 8');`



Cont..

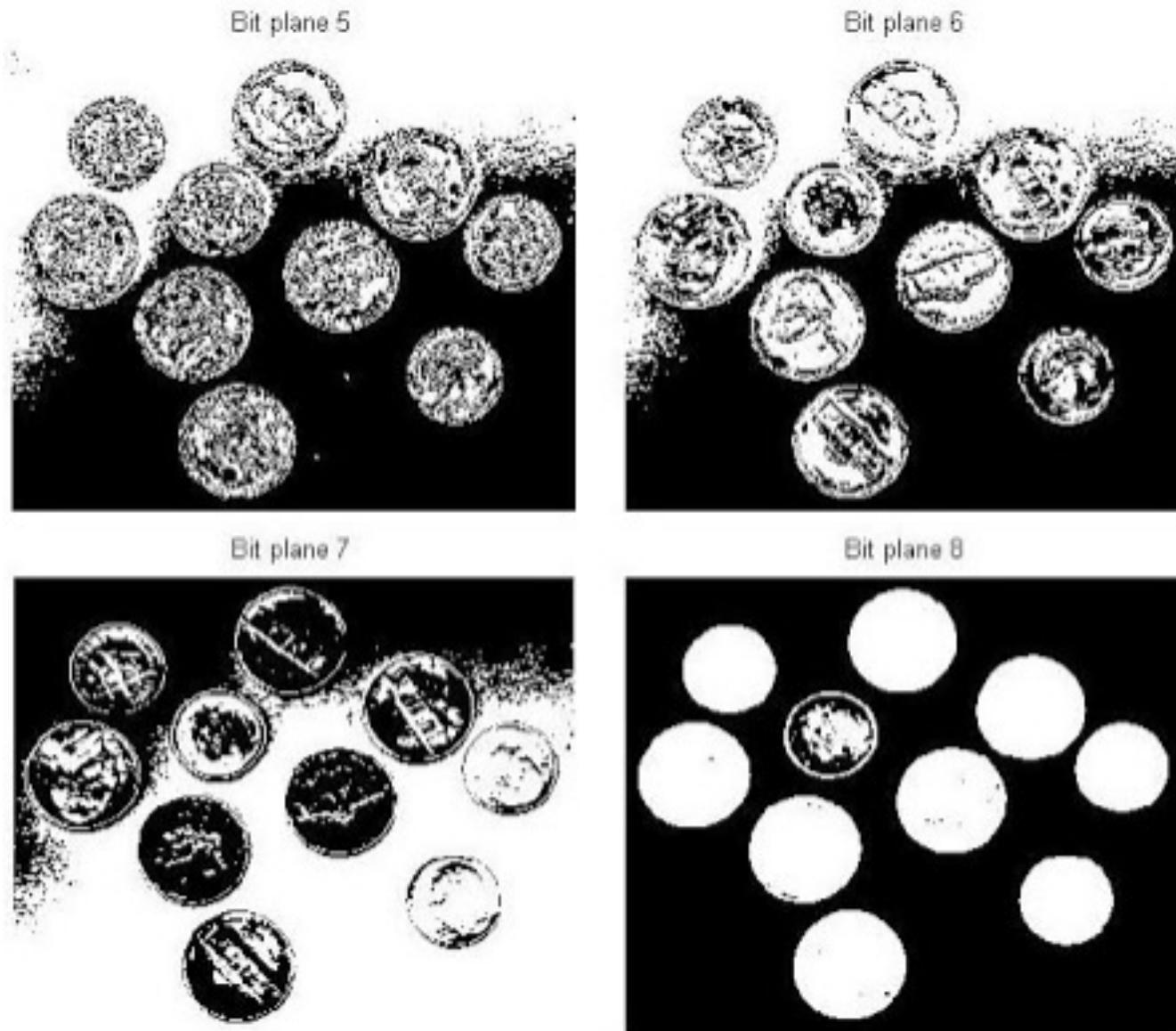


Image reconstruction using n bit planes

1. The nth plane in the pixels are multiplied by the constant $2^{(n-1)}$
2. For instance, consider the matrix
3. $A = [167 \ 133 \ 111 \ 144 \ 140 \ 135 \ 159 \ 154 \ 148]$ and the respective bit format

10100111	10000101	01101111
10010000	10001100	10000111
10011111	10011010	10010100



Cont..

4. Combine the 8 bit plane and 7 bit plane.

For 10100111, multiply the 8 bit plane with 128 and 7 bit plane with 64.

$$(1 \times 128) + (0 \times 64) + (1 \times 0) + (0 \times 0) + (0 \times 0) + (1 \times 0) + (1 \times 0) + (1 \times 0) = 128$$

5. Repeat this process for all the values in the matrix and the final result will be

[128 128 64

128 128 128

128 128 128]



Cont..

- **MATLAB CODE:**

```
%Image reconstruction by combining 8 bit plane and 7 bit plane
A=imread('coins.png');
B=zeros(size(A));
B=bitset(B,7,bitget(A,7));
B=bitset(B,8,bitget(A,8));
B=uint8(B);
figure, imshow(B);
```
- **Explanation:**
 - ‘bitset’ is used to set a bit at a specified position. Use ‘bitget’ to get the bit at the positions 7 and 8 from all the pixels in matrix A and use ‘bitset’ to set these bit values at the positions 7 and 8 in the matrix B.



Cont..

- **MATLAB CODE:**

- %Image reconstruction by combining 8,7,6 and 5 bit planes
- A= imread('coins.png');
- B= zeros(size(A));
- B= bitset(B,8,bitget(A,8));
- B= bitset(B,7,bitget(A,7));
- B= bitset(B,6,bitget(A,6));
- B= bitset(B,5,bitget(A,5));
- B= uint8(B);
- figure, imshow(B);



Image reconstructed using 5,6,7 and 8 bit planes

Histogram Processing

- Histogram Equalization
- Histogram Matching
- Local Histogram Processing
- Using Histogram Statistics for Image Enhancement



The Histogram of a Grayscale Image

Gray level histogram of an image is the distribution of the gray levels in an image

Examination of the histogram is one of the most useful tools for image enhancement, as it makes easy to see the modifications that may improve an image

The histogram can be modified by a mapping function, which will **stretch**, **shrink** (compress), or **slide** the histogram

Histogram stretching and histogram shrinking are forms of gray scale modification, sometimes referred to as ***histogram scaling***



Histogram Processing

- The histogram of a digital image with gray levels in the range $[0, L-1]$ is a discrete function

$$p(r_k) = n_k/n,$$

where,

- r_k is the k^{th} gray level,
- n_k is the number of pixels in the image with that gray level,
- n is the total number of pixels in the image, $k = 0, 1, \dots, L-1$



Example

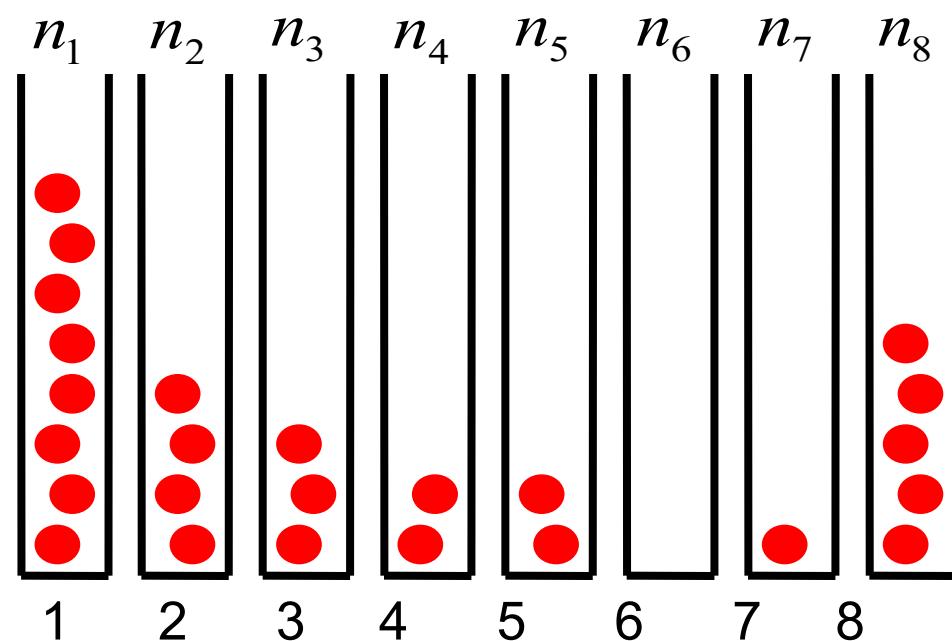
Consider a 5x5 image with integer intensities in the range between one and eight:

1	8	4	3	4
1	1	1	7	8
8	8	3	3	1
2	2	1	5	2
1	1	8	5	2



Cont..

$$h(r_k) = n_k$$



Cont..

$$h(r_1) = 8$$

$$h(r_2) = 4$$

$$h(r_3) = 3$$

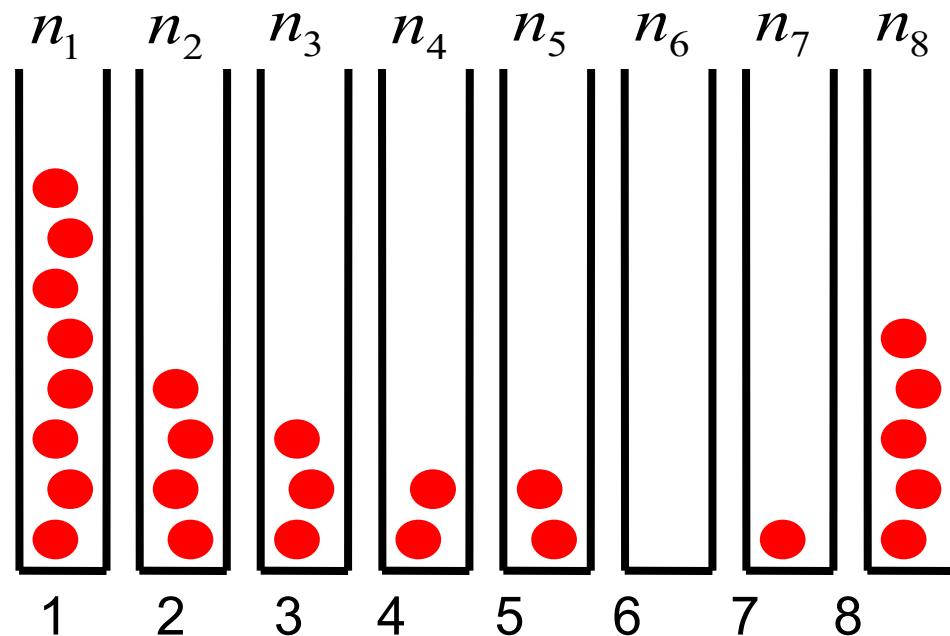
$$h(r_4) = 3$$

$$h(r_5) = 2$$

$$h(r_6) = 0$$

$$h(r_7) = 1$$

$$h(r_8) = 5$$



Normalised Histogram function

The **normalised histogram function** is the histogram function divided by the total number of the pixels of the image:

$$p(r_k) = \frac{h(r_k)}{n} = \frac{n_k}{n}$$

It gives a measure of how likely is for a pixel to have a certain intensity. That is, it gives the **probability** of occurrence the intensity.

The sum of the normalised histogram function over the range of all intensities is 1.



Normalised Histogram function

$$h(r_1) = 8$$

$$h(r_2) = 4$$

$$h(r_3) = 3$$

$$h(r_4) = 2 \quad \longrightarrow$$

$$h(r_5) = 2$$

$$h(r_6) = 0$$

$$h(r_7) = 1$$

$$h(r_8) = 5$$

$$p(r_1) = 8 / 25 = 0.32$$

$$p(r_2) = 4 / 25 = 0.16$$

$$p(r_3) = 3 / 25 = 0.12$$

$$p(r_4) = 2 / 25 = 0.08$$

$$p(r_5) = 0 / 25 = 0.00$$

$$p(r_6) = 1 / 25 = 0.04$$

$$p(r_7) = 5 / 25 = 0.20$$



Histogram Processing

Histogram $h(r_k) = n_k$

r_k is the k^{th} intensity value

n_k is the number of pixels in the image with intensity r_k

Normalized histogram $p(r_k) = \frac{n_k}{MN}$

n_k : the number of pixels in the image of
size $M \times N$ with intensity r_k



Histogram Processing

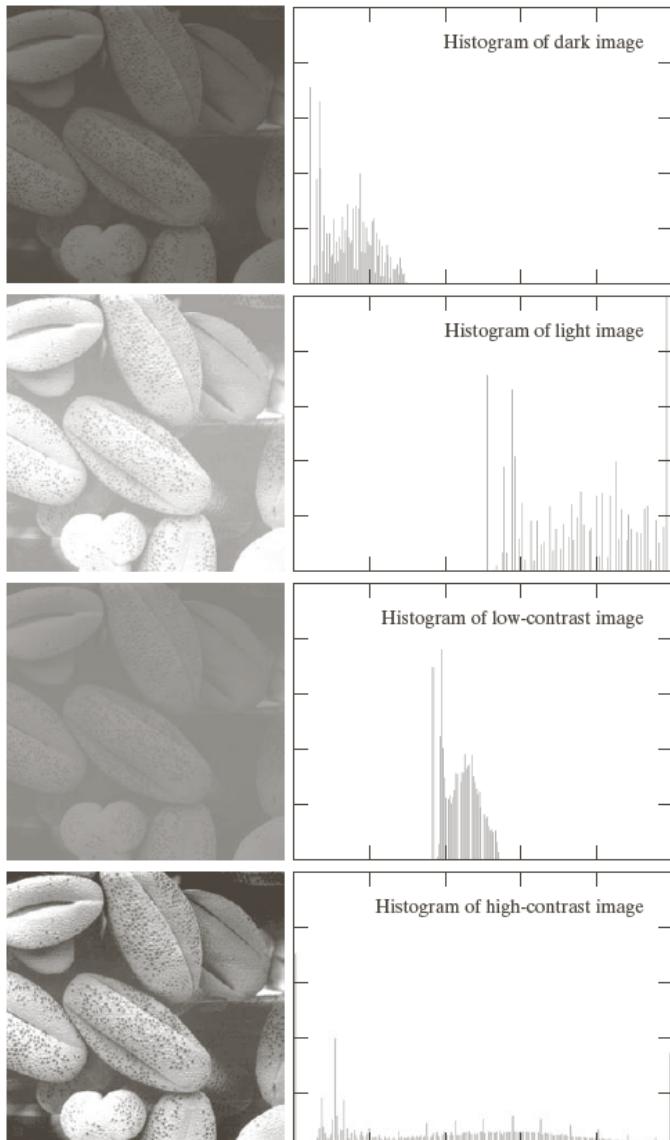


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Histogram Equalization

- The histogram of an image represents the relative frequency of occurrence of the various gray levels in the image.
- It provides a total description of the appearance of an image.
- The type and degree of enhancement obtained depends on the nature of the specified histogram.
- Let the variable r represent the grey level of the pixels in the image to be enhanced.
- Assume that the pixel values are normalized to lie in the range $0 \leq r \leq 1$ with $r = 0$ represents black and $T(r)$ represents white in the gray scale



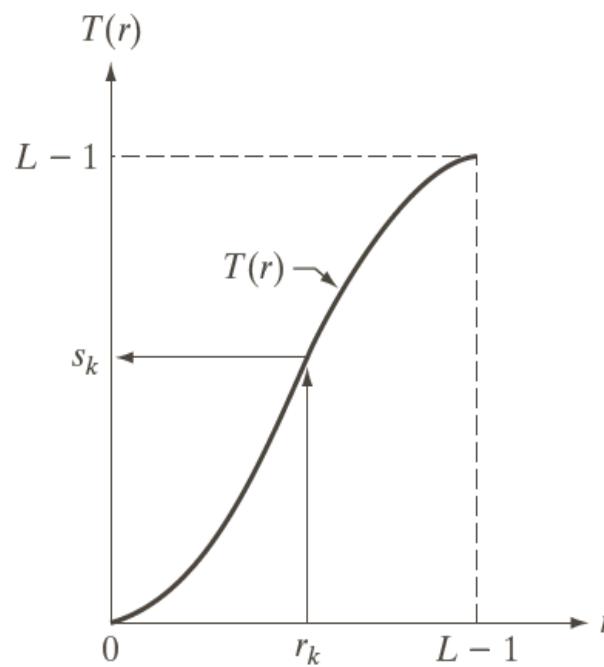
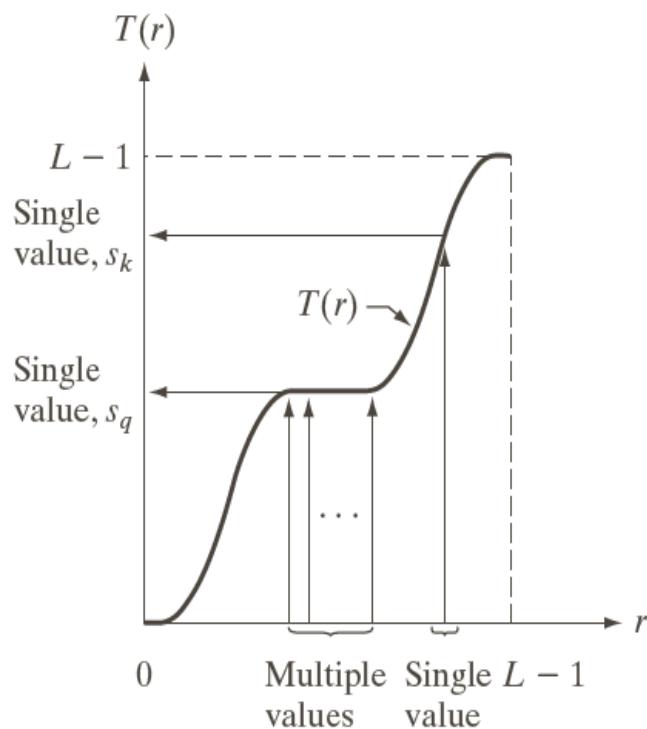
Cont..

- For any r in $[0, 1]$, we consider transformations of the form $s = T(r)$ which produce a level S for every pixel value r in the original image.
- It is assumed that the transformation function satisfies the conditions:
 - The following transformation
 $s = T(r)$ satisfies
 1. $T(r)$ is single-valued and monotonically increasing in the interval $0 \leq r \leq 1$, and
 2. $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$
 - Condition (1) preserve the order from black to white in the grayscale while condition (2) guarantees a mapping that is consistent with the allowed range of the pixel values



Cont..

- Gray-level transformation function



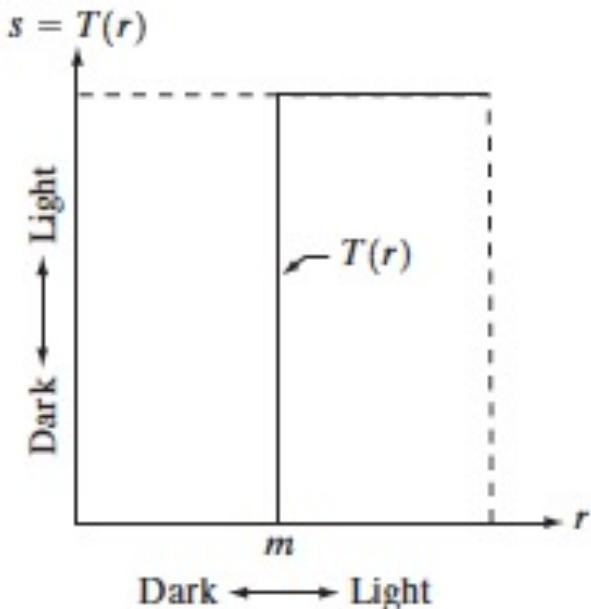
a b

FIGURE 3.17
(a) Monotonically increasing function, showing how multiple values can map to a single value.
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.



Cont..

- Example of such a transformation is:



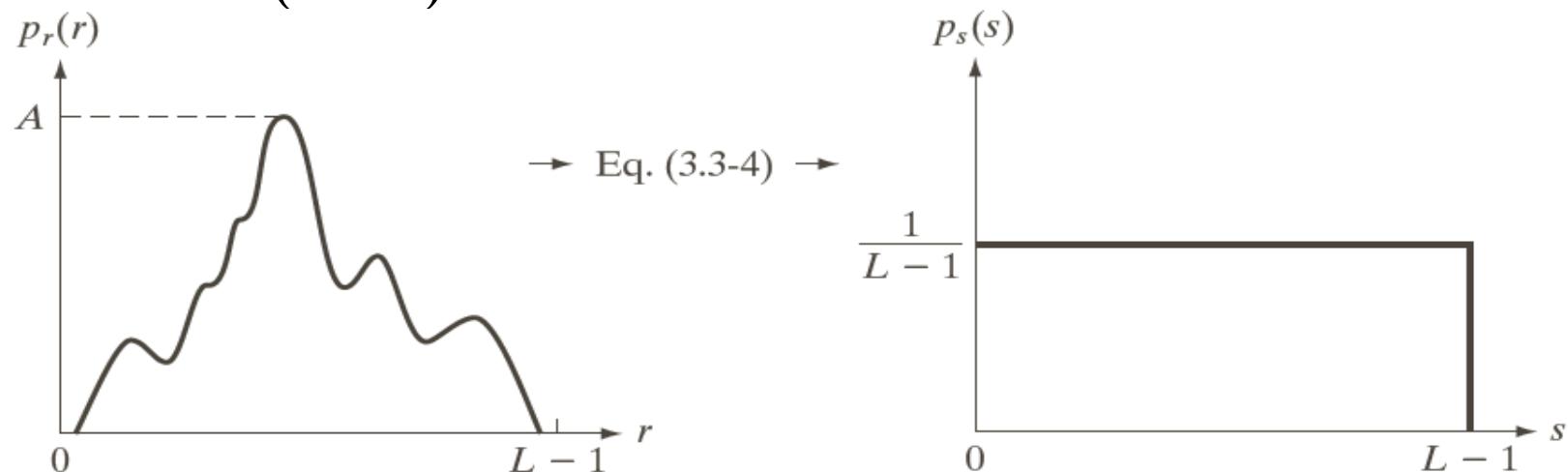
- The inverse transform $r = T^{-1}(s)$ for $0 \leq r \leq 1$, where it is assumed $T^{-1}(s)$ satisfies conditions (1) or (2) wrt variables. The gray levels in an image are random quantities in the interval $[0,1]$.
- Assuming that they are continuous variables the original and transformed gray levels can be characterized by their probability density functions(PDF) $P_r(r)$ and $P_s(s)$.
- A great deal can be said about the general characteristics of an image from the density functions of its gray levels.



Cont..

The intensity levels in an image may be viewed as random variables in the interval $[0, L-1]$.

Let $p_r(r)$ and $p_s(s)$ denote the probability density function (PDF) of random variables r and s .



a | b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r . The resulting intensities, s , have a uniform PDF, independently of the form of the PDF of the r 's.

Cont..

$$s = T(r) \quad 0 \leq r \leq L - 1$$

- a. $T(r)$ is a strictly monotonically increasing function in the interval $0 \leq r \leq L - 1$;
- b. $0 \leq T(r) \leq L - 1$ for $0 \leq r \leq L - 1$.

a b

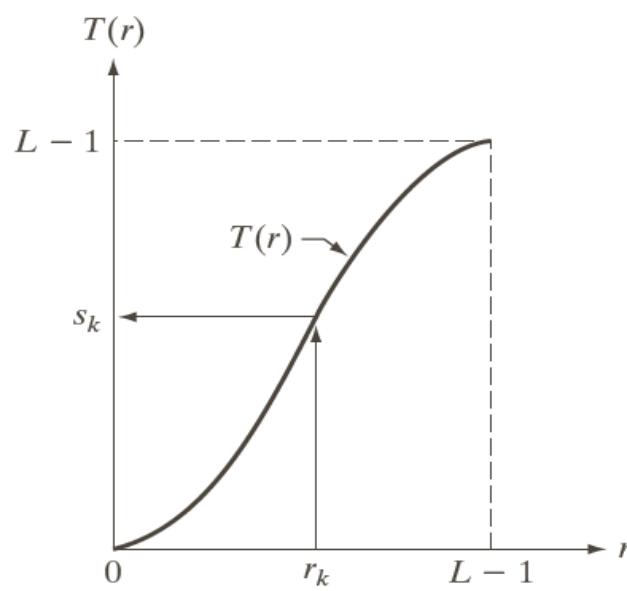
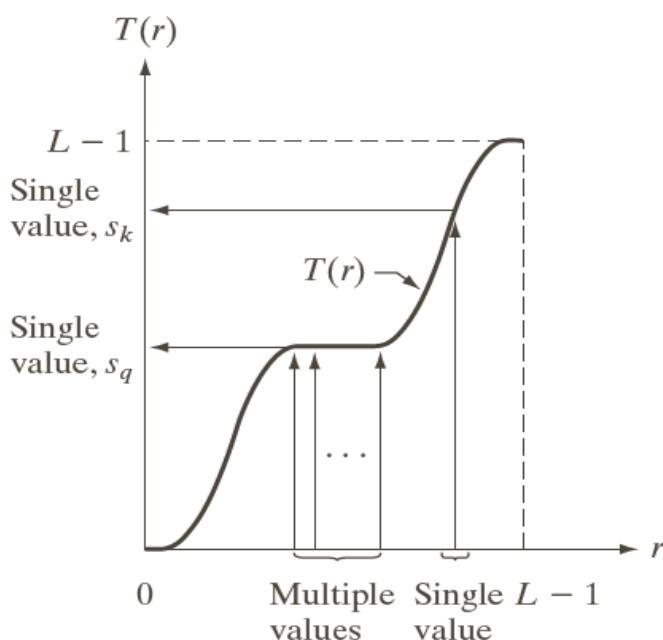


FIGURE 3.17

(a) Monotonically increasing function, showing how multiple values can map to a single value.
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Equalization

$$s = T(r) \quad 0 \leq r \leq L - 1$$

- a. $T(r)$ is a strictly monotonically increasing function in the interval $0 \leq r \leq L - 1$;
- b. $0 \leq T(r) \leq L - 1$ for $0 \leq r \leq L - 1$.

$T(r)$ is continuous and differentiable.

$$p_s(s)ds = p_r(r)dr$$



Histogram Equalization (Understanding)

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$

$$\begin{aligned} \frac{ds}{dr} &= \frac{dT(r)}{dr} = (L-1) \frac{d}{dr} \left[\int_0^r p_r(w) dw \right] \\ &= (L-1) p_r(r) \end{aligned}$$

$$p_s(s) = \frac{p_r(r) dr}{ds} = p_r(r) \cancel{\left(\frac{ds}{dr} \right)} = p_r(r) \cancel{\left((L-1) p_r(r) \right)} = \frac{1}{L-1}$$



Example

Suppose that the (continuous) intensity values in an image have the PDF

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2}, & \text{for } 0 \leq r \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Find the transformation function for equalizing the image histogram.



Example

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$



Histogram Equalization

Continuous case:

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

Discrete values:

$$\begin{aligned} s_k &= T(r_k) = (L - 1) \sum_{j=0}^k p_r(r_j) \\ &= (L - 1) \sum_{j=0}^k \frac{n_j}{MN} = \frac{L - 1}{MN} \sum_{j=0}^k n_j \quad k=0,1,\dots, L-1 \end{aligned}$$



Histogram Equalization

The formula for histogram equalisation in the discrete case is given

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j) = \frac{(L-1)}{MN} \sum_{j=0}^k n_j$$

where

- r_k : input intensity
- s_k : processed intensity
- n_j : the frequency of intensity j
- MN : the number of image pixels.



Example 1: Histogram Equalization

$$s_k = T(r_k) = (L - 1) \sum_{j=0}^k p_r(r_j)$$

$$p(r_1) = 8/25 = 0.32$$

$$p(r_2) = 4/25 = 0.16$$

$$p(r_3) = 3/25 = 0.12$$

$$p(r_4) = 3/25 = 0.08$$

$$p(r_5) = 2/25 = 0.08$$

$$p(r_6) = 0/25 = 0.00$$

$$p(r_7) = 1/25 = 0.04$$

$$p(r_8) = 5/25 = 0.20$$

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) \\ = 7 \times 0.32 = 2.24 = 2$$

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) \\ = 7 \times (0.32 + 0.16) = 7 \times 0.48 = 3.36 = 3$$

$$s_2 = T(r_2) = 7 \sum_{j=0}^2 p_r(r_j) \\ = 7 \times (0.32 + 0.16 + 0.12) = 7 \times 0.6 = 4.2 = 4$$



Example 1: Histogram Equalization (Cont..)

r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	New n_k
1	8	0.32	0.32	2.24	2
2	4	0.16	0.48	3.36	3
3	3	0.12	0.6	4.2	4
4	3	0.08	0.68	4.76	5
5	2	0.08	0.76	5.32	5
6	0	0.00	0.76	5.32	5
7	1	0.04	0.8	5.6	6
8	5	0.20	1	7.00	7

Observe that there are 6 distinct intensity levels. r_1 is mapped to $s_1=2$ and No of pixels are 8.

Equating Gray Levels to No. of Pixels: $1 \rightarrow 0$; $8 \rightarrow 0$;

$s_2 = 3 \rightarrow 4$; $s_3 = 4 \rightarrow 3$; $r_4 \& r_5 \& r_6 = s_4 = 5 \rightarrow (3+2)5$ $s_5 = 6 \rightarrow 1$
 $s_6 = 7 \rightarrow 5$; Total : 25



Example 2: Histogram Equalization (Cont..)

r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	New n_k
0	2	0.11	0.11	0.77	1
1	4	0.11	0.22	1.54	2
2	6	0.33	0.55	3.85	4
3	8	0.11	0.66	4.62	5
4	10	0.11	0.77	5.39	5
5	12	0.11	0.88	6.16	6
6	14	0.11	0.99	6.93	7
7	16	0.01	1	7.00	7

Observe that there are 6 distinct intensity levels. r_0 is mapped to $s_0=1$ and No of pixels are 2.

Equating Gray Levels to No. of Pixels: $0 \rightarrow 0$; $3 \rightarrow 0$;

$s_1=2 \rightarrow 4$; $s_2 = 4 \rightarrow 6$; $r_3 \& r_4=s_3 = 5 \rightarrow 18$; $s_4 = 6 \rightarrow 12$
 $r_6 \& r_7=s_5 = 7 \rightarrow (14+16)=30$ Total : 70



Example 3

Perform histogram equalization on the following 3x3, 8-level image :-

1	3	5
4	4	3
5	2	2

Gray Level (r_k)	0	1	2	3	4	5	6	7
No of Pixels (n_k)	0	1	2	2	2	2	0	0

r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	New n_k
0	0	0	0	0	0
1	1	1/9	1/9	7/9=0.777	1
2	2	2/9	3/9 = 1/3	7/3=2.333	2
3	2	2/9	5/9	35/9=3.888	4
4	2	2/9	7/9	49/9=5.444	5
5	2	2/9	1	7	7
6	0	0	1	7	7
7	0	0	1	7	7

Observe that there are 5 distinct intensity levels. r_1 is mapped to $s_1=1$ and No of pixels are 1. Last column New n_k shows the final distribution of gray levels pixels, where pixels 2 & 0 corresponds to levels 5 & 6 are map to level 7 in O/P image.

Equating Gray Levels to No. of Pixels: 0 -> 0; 3 -> 0; 6->0

$s_2=3-> 2;$ $s_3 = 4 -> 2;$ $s_4 = 5 -> 2;$ $r_5 \& r_6 \& r_7 = s_5 = 6 -> (2+0+0)=2$

Total : 9



Example 3

Image after equalization is as follows :-

1	3	5
4	4	3
5	2	2

old

1	4	7
5	5	4
7	2	2

new

Example 4

Consider a 8x8 image with integer intensities in the range between one and eight:

0	1	7	4	3	4	4	5
0	1	1	1	7	5	4	6
0	5	5	1	5	1	4	6
0	2	2	1	5	2	4	6
0	1	1	5	5	2	4	6
0	5	5	4	3	5	4	1
0	2	2	5	5	2	4	2
0	5	5	4	5	2	4	2

Gray Level (r_k)	0	1	2	3	4	5	6	7
No of Pixels (n_k)	8	10	10	2	12	16	4	2



r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	New n_k
0	8	$8/64=0.125$	0.125	$0.125 \times 7 = 0.875$	1
1	10	$10/64=0.156$	0.281	1.967	2
2	10	$10/64=0.156$	0.437	3.059	3
3	2	$2/64=0.03$	0.467	3.269	3
4	12	$12/64=0.187$	0.654	4.578	5
5	16	$16/64=0.25$	0.904	6.328	6
6	4	$4/64=0.0625$	0.97	6.79	7
7	2	$2/64=0.03$	1	7.00	7

Observe that there are 6 distinct intensity levels. r_0 is mapped to $s_0=1$ and No of pixels are 8. Last column New n_k shows the final distribution of gray levels pixels, where pixels 10 & 2 corresponds to levels 2&3 are map to level 3 in O/P image.

Equating Gray Levels to No. of Pixels: $0 \rightarrow 0$; $4 \rightarrow 0$;

$s_1=2 \rightarrow 10$; $r_2 \& r_3=s_2=3 \rightarrow 12$; $s_3=5 \rightarrow 12$; $s_4=6 \rightarrow 16$

$r_6 \& r_7=s_5=7 \rightarrow (4+2)=6$ Total : 64



Example 5: Histogram Equalization

Suppose that a 3-bit image ($L=2^3=8$) of size 64×64 pixels ($MN = 4096$) has the intensity distribution shown in following table.

Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k .

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



Example 5: Histogram Equalization (Cont..)

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j)$$

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) = 7 \times 0.19 = 1.33 \rightarrow 1$$

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) = 7 \times (0.19 + 0.25) = 3.08 \rightarrow 3$$

$$s_2 = 4.55 \rightarrow 5$$

$$s_3 = 5.67 \rightarrow 6$$

$$s_4 = 6.23 \rightarrow 6$$

$$s_5 = 6.65 \rightarrow 7$$

$$s_6 = 6.86 \rightarrow 7$$

$$s_7 = 7.00 \rightarrow 7$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



Cont..

r_k	n_k	$P_r(r_k) = n_k/MN$	$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j)$
0	790	0.19	1.33 = 1
1	1023	0.25	3.08 = 3
2	850	0.21	4.55 = 5
3	656	0.16	5.67 = 6
4	329	0.08	6.23 = 6
5	245	0.06	6.65 = 7
6	122	0.03	6.86 = 7
7	81	0.02	7.00 = 7



Cont..

Gray Levels (r_k)	No. of Pixels n_k	(PDF) $Pr(rk) = nk/n$	(CDF) $S_k = \sum P_k(rk)$	$(L-1) Sk = 7 \times Sk$	Rounding off
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	830	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1	7	7
$n = 4096$		1			

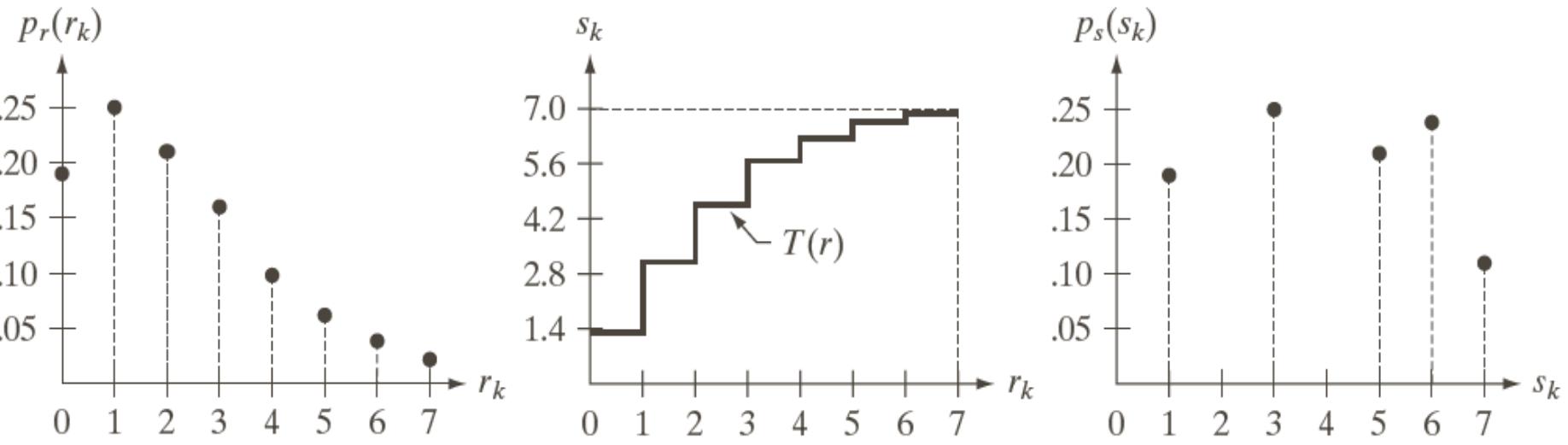
These are the values for equalized Histogram. Observe that there are 5 distinct intensity levels. r_0 is mapped to $s_0=1$ and No of pixels are 790. Equating Gray Levels to No. of Pixels: $0 \rightarrow 0$; $2 \rightarrow 0$; $4 \rightarrow 0$;

$$s_0 = 1 \rightarrow 790; \quad s_1 = 3 \rightarrow 1023; \quad s_2 = 5 \rightarrow 830 \quad r_3 \& r_4 = 6 \rightarrow (656+329) = 985$$

$$r_5 \& r_6 \& r_7 = 7 \rightarrow (245+122+81)=448; \quad \text{Total : } 4096 \quad \text{Hence Verified!!} \boxed{103}$$



Example: Histogram Equalization



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

These are the values for equalized Histogram. Observe that there are 5 distinct intensity levels. r_0 is mapped to $s_0=1$ and No of pixels are 790. Equating Gray Levels to No. of Pixels:

$$0 \rightarrow 0; \quad 4 \rightarrow 0; \quad 2 \rightarrow 0;$$

$$S_0 = 1 \rightarrow 790; \quad s_1 = 3 \rightarrow 1023; \quad s_2 = 5 \rightarrow 850 \quad r_3 \& r_4 = 6 \rightarrow (656+329) = 985$$

$$r_5 \& r_6 \& r_7 = 7 \rightarrow (245+122+81)=448; \quad \text{Total : } 4096$$

Now dividing these numbers by $MN=4096$ gives the equalized Hist. in fig 3.19(c).



Cont..

- It is clearly seen that
 - Histogram equalization distributes the gray level to reach the maximum gray level (white) because the cumulative distribution function equals 1 when $0 \leq r \leq L-1$
 - If the cumulative numbers of gray levels are slightly different, they will be mapped to little different or same gray levels as we may have to approximate the processed gray level of the output image to integer number
 - Thus, the discrete transformation function can't guarantee the one to one mapping relationship
 - Notice that due to discretization, the resulting histogram will rarely be perfectly flat. However, it will be extended.



Histogram Equalization

- Example

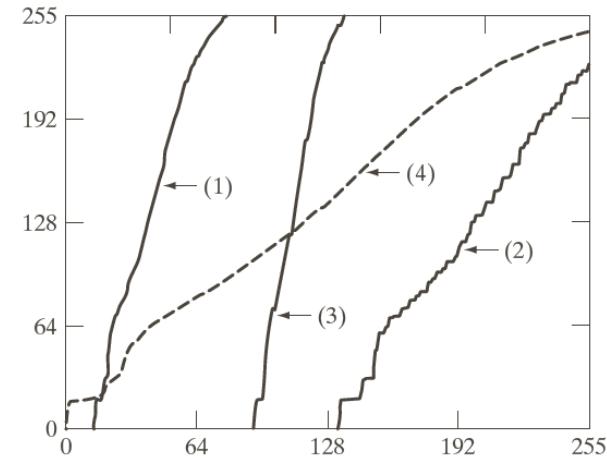
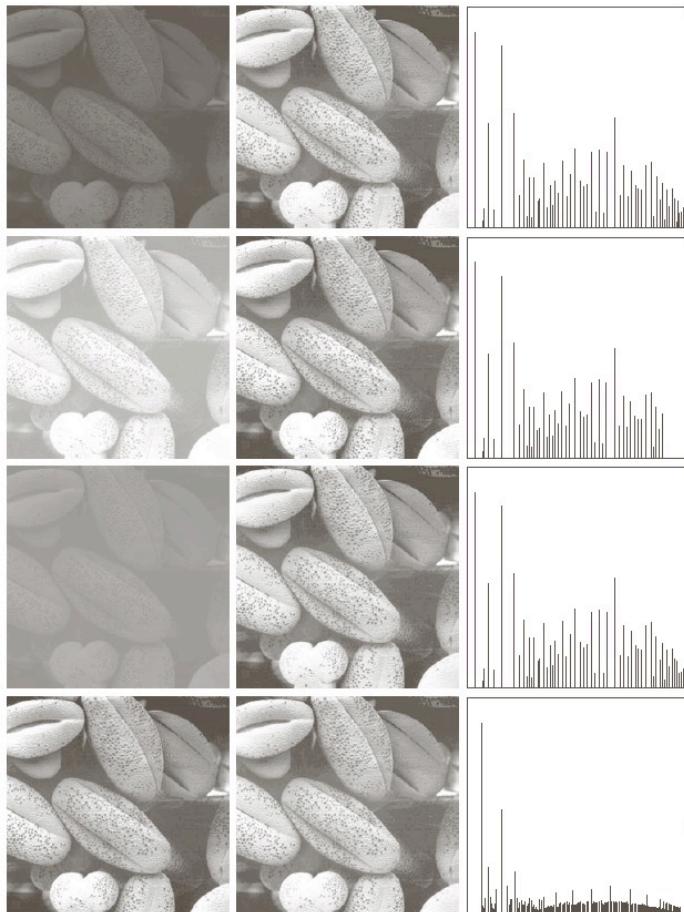


FIGURE 3.21
Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).



FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Difference

- The goal of HE is to produce an output image that has a **flattened histogram**.
- HM is to take an I/P image and generate an O/P image that is based upon the **shape** say $p_z(z)$ of a specific (or **reference**) **histogram**.
- In HE (also known as histogram flattening), the goal is to **improve contrast** in images that might be either **blurry** or have a **background** and **foreground** that are either/ both **bright** or both **dark**. **Low contrast** images typically have histograms that are concentrated within a tight range of values.
- HE can improve the contrast in these images by spreading out the histogram so that the intensity values are **distributed uniformly** over a larger intensity range. Ideally, the histogram of the output image will be **perfectly flat**.



Histogram Specification/Matching

- Equalize the levels of the original image.
- Histogram matching is the transformation of an image.
- The process of Histogram Matching takes in an input image and produces an output image that is based upon a specified histogram.
- The well-known histogram equalization method is a special case in which the specified histogram is uniformly distributed.



Histogram Specification or Matching

- Sometimes, this may not be desirable.
- Instead, we may want a transformation that yields an output image with a pre-specified histogram.
- Applying the transformation H to the original image yields an image with histogram.
- Again, the actual histogram of the output image does not exactly but only approximately matches with the specified histogram.
- This is because we are dealing with discrete histograms.



Histogram Matching

Histogram matching (histogram specification)

- generate a processed image that has a specified histogram

Let $p_r(r)$ and $p_z(z)$ denote the continuous probability density functions of the variables r and z . $p_z(z)$ is the specified probability density function.

Let s be the random variable with the probability

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

Define a random variable z with the probability

$$G(z) = (L - 1) \int_0^z p_z(t) dt = s$$



Histogram Matching

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$

$$G(z) = (L-1) \int_0^z p_z(t) dt = s$$

$$z = G^{-1}(s) = G^{-1} [T(r)]$$



Histogram Matching: Procedure

- Obtain $p_r(r)$ from the input image and then obtain the values of s

$$s = (L - 1) \int_0^r p_r(w) dw$$

- Use the specified PDF and obtain the transformation function $G(z)$

$$G(z) = (L - 1) \int_0^z p_z(t) dt = s$$

- Mapping from s to z

$$z = G^{-1}(s)$$



Histogram Matching: Example

Assuming continuous intensity values, suppose that an image has the intensity PDF

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2}, & \text{for } 0 \leq r \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Find the transformation function that will produce an image whose intensity PDF is

$$p_z(z) = \begin{cases} \frac{3z^2}{(L-1)^3}, & \text{for } 0 \leq z \leq (L-1) \\ 0, & \text{otherwise} \end{cases}$$



Histogram Matching: Example

Find the histogram equalization transformation for the input image

$$s = T(r) = (L-1) \int_0^r p_r(w) dw = (L-1) \int_0^r \frac{2w}{(L-1)^2} dw = \frac{r^2}{L-1}$$

Find the histogram equalization transformation for the specified histogram

$$G(z) = (L-1) \int_0^z p_z(t) dt = (L-1) \int_0^z \frac{3t^2}{(L-1)^3} dt = \frac{z^3}{(L-1)^2} = s$$

The transformation function

$$z = \left[(L-1)^2 s \right]^{1/3} = \left[(L-1)^2 \frac{r^2}{L-1} \right]^{1/3} = \left[(L-1)r^2 \right]^{1/3}$$



Histogram Matching: Discrete Cases

- Obtain $p_r(r_j)$ from the input image and then obtain the values of s_k , round the value to the integer range [0, L-1].

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j) = \frac{(L-1)}{MN} \sum_{j=0}^k n_j$$

- Use the specified PDF and obtain the transformation function $G(z_q)$, round the value to the integer range [0, L-1].

$$G(z_q) = (L-1) \sum_{i=0}^q p_z(z_i) = s_k$$

- Mapping from s_k to z_q $z_q = G^{-1}(s_k)$

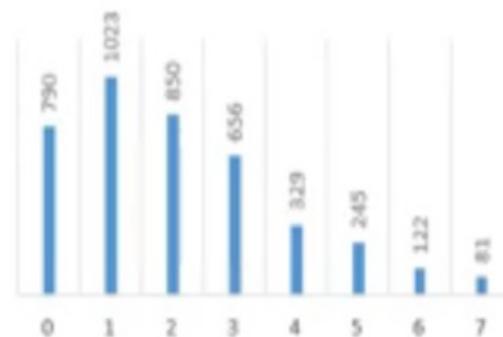


Example 1: Histogram Matching

Given histogram (a) & (b), modify histogram (a) as given by histogram (b)

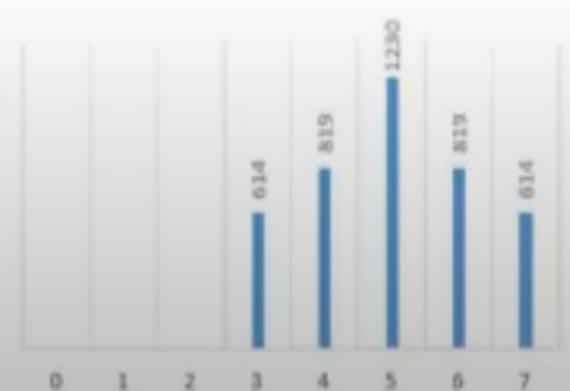
(a)

Gray level.	0	1	2	3	4	5	6	7
No. of pixels	790	1023	850	656	329	245	122	81



(b)

Gray level.	0	1	2	3	4	5	6	7
No. of pixels	0	0	0	614	819	1230	819	614



Cont..

Equalize histogram (a)

Gray level	nk	PDF	CDF	Sk x 7	Round off	New nk.
0	790	0.19	0.19	1.33	1	790
1	1023	0.25	0.44	3.08	3	1023
2	850	0.21	0.65	4.55	5	850
3	656	0.16	0.81	5.67	6	656+329
4	329	0.08	0.89	6.23	6	
5	245	0.06	0.95	6.65	7	
6	122	0.03	0.98	6.86	7	245 +122+81
7	81	0.02	1	7	7	
N=4096						



Cont..

Now Equalize histogram (b)

Gray level	nk	PDF	CDF	Sk x 7	Round off
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	614	0.149	0.149	1.05	1
4	819	0.20	0.35	2.50	3
5	1230	0.30	0.65	4.55	5
6	819	0.20	0.85	5.97	6
7	614	0.15	1	7	7
N=4096					



Cont..

Mapping

First and last columns
of histogram (b)

Gray level	Round off
0	0
1	0
2	0
3	1
4	3
5	5
6	6
7	7

Last two columns
of histogram (a)

Round off	New nk.
1	790
3	1023
5	850
6	985
7	448

Gray Level	0	1	2	3	4	5	6	7
No of Pixels								



Cont..

Mapping

First and last columns
of histogram (b)

Gray level	Round off
0	0
1	0
2	0
3	1
4	3
5	5
6	6
7	7

Last two columns
of histogram (a)

Round off	New nk.
1	790
3	1023
5	850
6	985
7	448

Gray Level	0	1	2	3	4	5	6	7
No of Pixels				790				



Cont..

Mapping

First and last columns
of histogram (b)

Gray level	Round off
0	0
1	0
2	0
3	1
4	3
5	5
6	6
7	7

Last two columns
of histogram (a)

Round off	New nk.
1	790
3	1023
5	850
6	985
7	448

Gray Level	0	1	2	3	4	5	6	7
No of Pixels				790	1023			



Cont..

Mapping

First and last columns
of histogram (b)

Gray level	Round off
0	0
1	0
2	0
3	1
4	3
5	5
6	6
7	7

Last two columns
of histogram (a)

Round off	New nk.
1	790
3	1023
5	850
6	
6	985
7	
7	448
7	

Gray Level	0	1	2	3	4	5	6	7
No of Pixels				790	1023	850		



Cont..

Mapping

First and last columns
of histogram (b)

Gray level	Round off
0	0
1	0
2	0
3	1
4	3
5	5
6	6
7	7

Last two columns
of histogram (a)

Round off	New nk.
1	790
3	1023
5	850
6	985
7	448

Gray Level	0	1	2	3	4	5	6	7
No of Pixels				790	1023	850	985	



Cont..

Mapping

First and last columns
of histogram (b)

Gray level	Round off
0	0
1	0
2	0
3	1
4	3
5	5
6	6
7	7

Last two columns
of histogram (a)

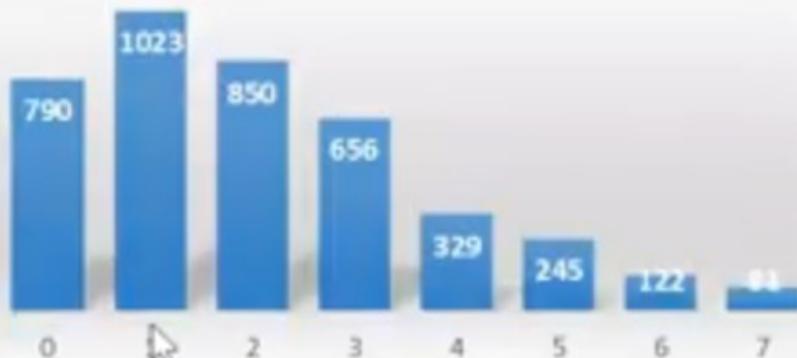
Round off	New nk.
1	790
3	1023
5	850
6	985
7	448

Gray Level	0	1	2	3	4	5	6	7
No of Pixels				790	1023	850	985	448



Cont..

Histogram of input image

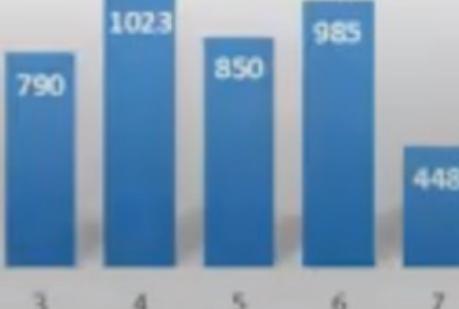


Specified Histogram



Plot histogram for modified image.

Histogram of output image



Example : Histogram Matching

Suppose that a 3-bit image ($L=8$) of size 64×64 pixels ($MN = 4096$) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.

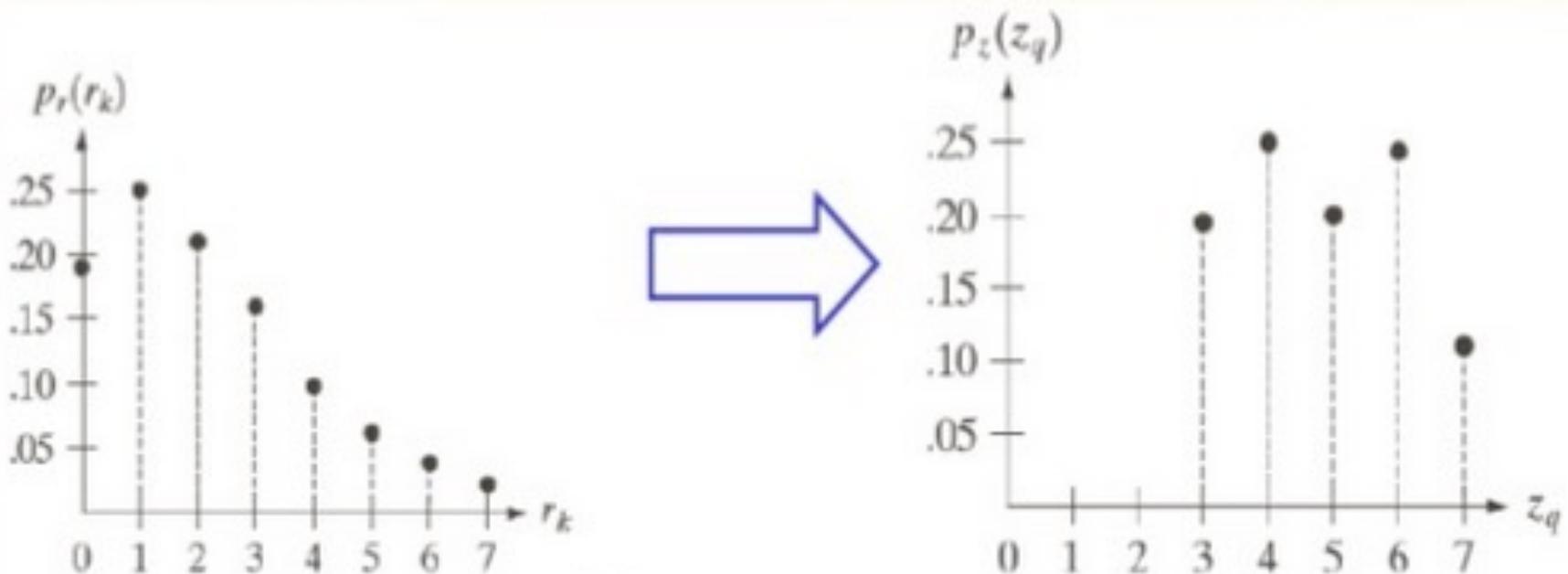
r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15



Example: Histogram Matching

Suppose that a 3-bit image ($L=8$) of size 64×64 pixels ($MN = 4096$) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.



Example: Histogram Matching

Original (given)			Reference(given)	
r_k	n_k	$p_r(r_k) = n_k/MN$	z_k	$p(z_k)=n_k/MN$
0	790	0.19	0	0
1	1023	0.25	1	0
2	850	0.21	2	0
3	656	0.16	3	0.15
4	329	0.08	4	0.2
5	245	0.06	5	0.3
6	122	0.03	6	0.2
7	81	0.02	7	0.15



Example: Histogram Matching(Equalization)

r_k	n_k	PDF $p_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	Round off	New n_k
0	790	0.19	$0+0.19=0.19$	1.33	1	790
1	1023	0.25	$0.19+0.25=0.44$	3.08	3	1023
2	850	0.21	$0.44+0.21=0.65$	4.55	5	850
3	656	0.16	$0.65+0.16=0.81$	5.67	6	985
4	329	0.08	0.89	6.23	6	
5	245	0.06	0.95	6.65	7	448
6	122	0.03	0.98	6.86	7	
7	81	0.02	1.00	7.00	7	

Original Image (given)



Example: Histogram Matching

z_k	n_k	PDF $p_r(z_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	Round off
0	790	0	0	0	0
1	1023	0	0	0	0
2	850	0	0	0	0
3	656	0.15	0.15	1.05	1
4	329	0.2	0.35	2.45	2
5	245	0.3	0.65	4.55	5
6	122	0.2	0.85	5.95	6
7	81	0.15	1.00	7	7

Reference Image (given)



Cont..

Mapping

1st and last column of HE of Reference Image Table and last 2 columns of source Table

r_k	Round off(S)	Round off (H)	New n_k	Map
0	0	1	790	
1	0	3	1023	
2	0	5	850	
3	1	6	985	
4	2	6		
5	5	7	448	
6	6	7		
7	7	7		

Gray Level	0	1	2	3	4	5	6	7
No of Pixels	0	0	0	790				



Cont..

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$

$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

$$G(z_0) = 7 \sum_{j=0}^0 p_z(z_j) = 0.00 \rightarrow 0 \quad G(z_1) = 0.00 \rightarrow 0 \quad G(z_2) = 0.00 \rightarrow 0$$

$$G(z_3) = 1.05 \rightarrow 1 \quad G(z_4) = 2.45 \rightarrow 2$$

$$G(z_5) = 4.55 \rightarrow 5 \quad G(z_6) = 5.95 \rightarrow 6$$

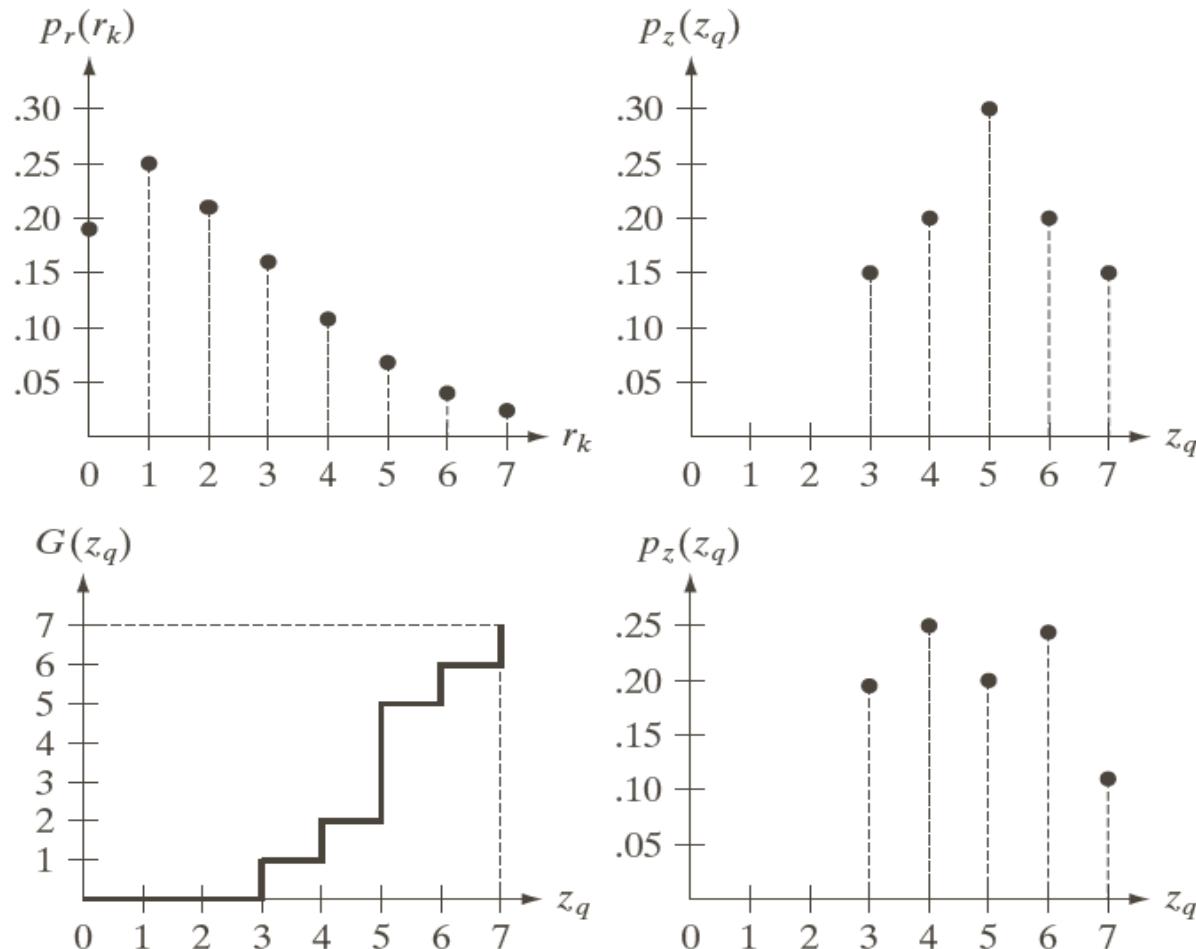
$$G(z_7) = 7.00 \rightarrow 7$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11



Cont..



a	b
c	d

FIGURE 3.22

- (a) Histogram of a 3-bit image.
- (b) Specified histogram.
- (c) Transformation function obtained from the specified histogram.
- (d) Result of performing histogram specification. Compare (b) and (d).

Example: Histogram Matching

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7, \\ s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

$$G(z_0) = 7 \sum_{j=0}^0 p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0 \quad G(z_2) = 0.00 \rightarrow 0$$

$$G(z_3) = 1.05 \rightarrow 1 \quad s_0 \quad G(z_4) = 2.45 \rightarrow 2 \quad s_1$$

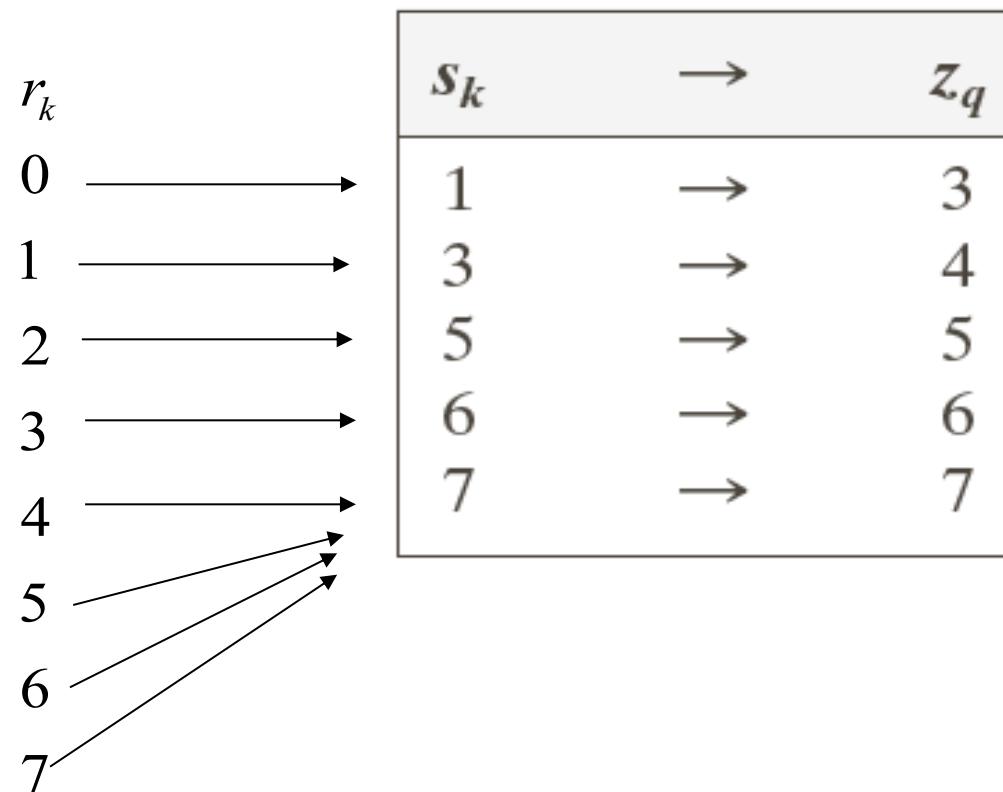
$$G(z_5) = 4.55 \rightarrow 5 \quad s_2 \quad G(z_6) = 5.95 \rightarrow 6 \quad s_3$$

$$G(z_7) = 7.00 \rightarrow 7 \quad s_4 \quad s_5 \quad s_6 \quad s_7$$



Cont..

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$



Cont..

$r_k \rightarrow z_q$

$0 \rightarrow 3$

$1 \rightarrow 4$

$2 \rightarrow 5$

$3 \rightarrow 6$

$4 \rightarrow 7$

$5 \rightarrow 7$

$6 \rightarrow 7$

$7 \rightarrow 7$



Cont..

Source



Reference



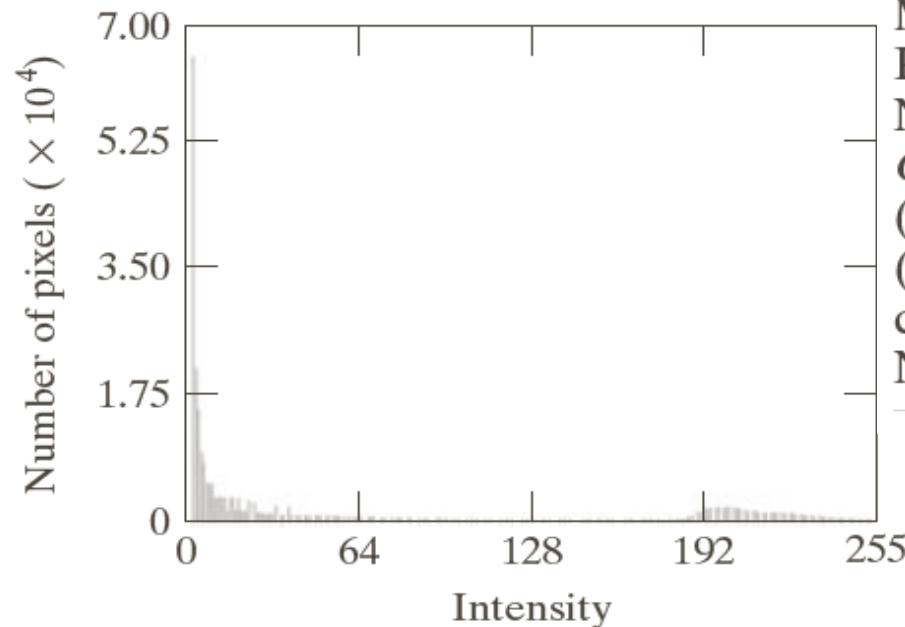
Matched



Example: Histogram Matching



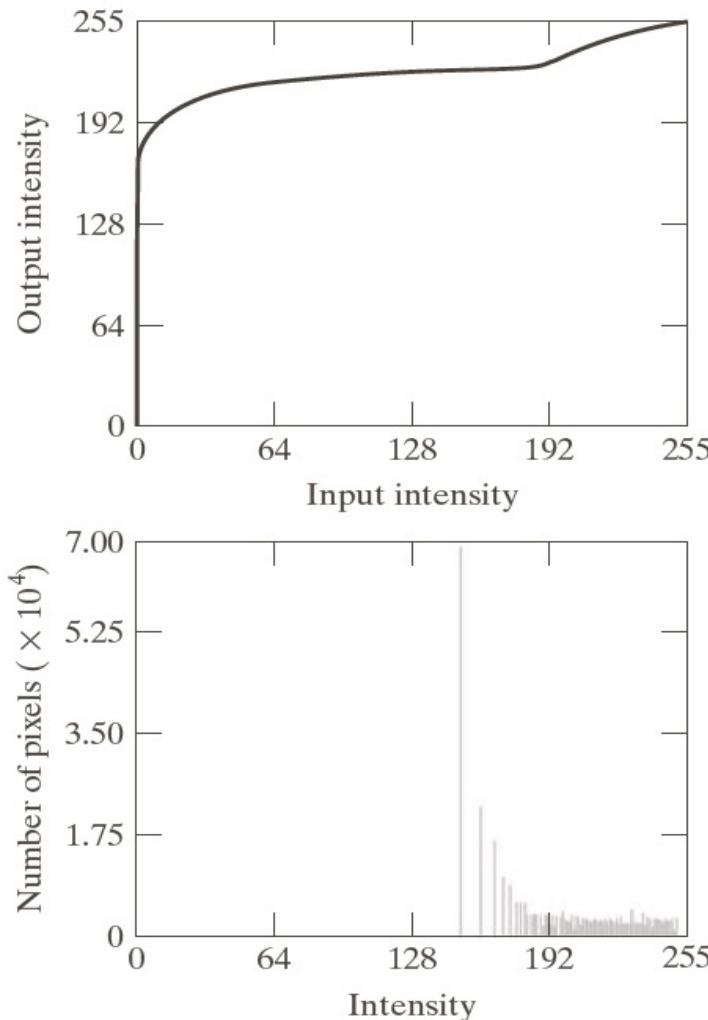
Drawbacks/Artefacts



a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*.
(b) Histogram.
(Original image courtesy of NASA.)

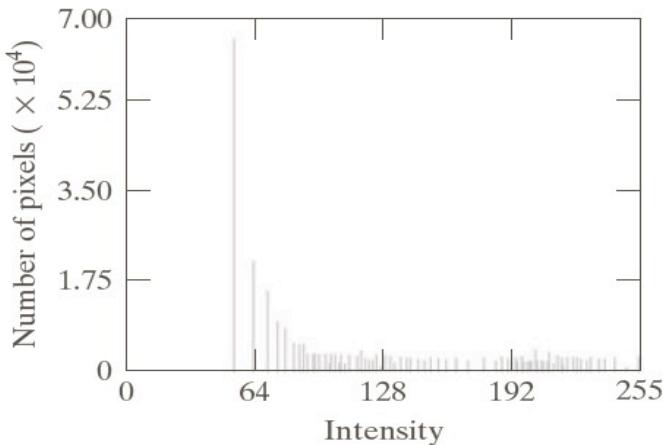
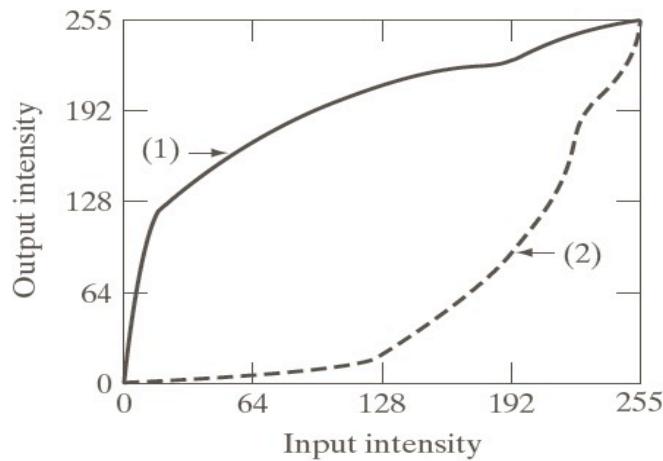
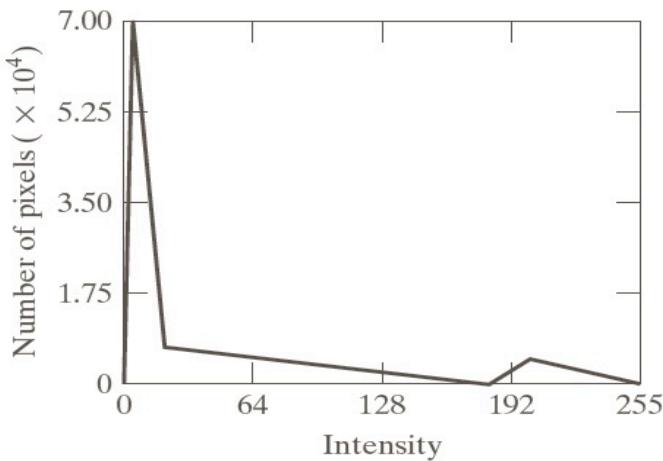
Example: Histogram Matching



Drawbacks/Artefacts

a
b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).



a
b
c
d

FIGURE 3.25
 (a) Specified histogram.
 (b) Transformations.
 (c) Enhanced image using mappings from curve (2).
 (d) Histogram of (c).

Cont..

- 3.25(a) shows a *manually specified function* of original histogram, has a smoother transition of levels in the dark region of the gray scale.
- The transformation function $G(z)$ obtained from this histogram using is labeled transformation (1) in Fig. 3.25(b). Similarly, $G^{-1}(s)$ is labeled transformation (2) in Fig. 3.25(b).
- The enhanced image in Fig. 3.25(c) was obtained by applying transformation (2) to the pixels of the histogram-equalized image in Fig. 3.24(b). The improvement of the histogram-specified image over the result obtained by histogram equalization is evident by comparing these two images.
- The histogram of Fig. 3.25(c) is shown in Fig. 3.25(d). The most distinguishing feature of this histogram is how its low end has shifted right toward the lighter region of the gray scale, as desired.



Example 2

Perform histogram specification on the 8x8, 8-level image described in the table:

Table 1: Pixel Distribution of the image

Gray Level (r_k)	0	1	2	3	4	5	6	7
No of Pixels (n_k)	8	10	10	2	12	16	4	2

The target histogram is as shown in table:

Table 2: Pixel Distribution of the image

Gray Level (r_k)	0	1	2	3	4	5	6	7
No of Pixels (n_k)	0	0	0	0	20	20	16	8



Example 2: Histogram Matching

Given histogram (a) & (b), modify histogram (a) as given by histogram (b)

Original Image

Gray level	0	1	2	3	4	5	6	7
No. of Pixels	8	10	10	2	12	16	4	2

Desired Image

Gray level	0	1	2	3	4	5	6	7
No. of Pixels	0	0	0	0	20	20	16	8



Cont..

Histogram Equalization of Input Image (a)

Gray level(r_k)	No. of Pixels(n_k)	PDF(n_k/N) $P_r(r_k)$	CDF	$(L-1)*CDF$	H_k
0	8	0.13	0.13	0.91	1
1	10	0.16	0.29	2.03	2
2	10	0.16	0.45	3.15	3
3	2	0.03	0.48	3.36	3
4	12	0.18	0.66	4.62	5
5	16	0.25	0.91	6.37	6
6	4	0.06	0.97	6.79	7
7	2	0.03	1.0	7	7
		64	1		



Cont..

Histogram Equalization of Input Image (a)

Gray level r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	Round off (H_k)	New n_k
0	8	0.13	0.13	0.91	1	8
1	10	0.16	0.29	2.03	2	10
2	10	0.16	0.45	3.15	3	12
3	2	0.03	0.48	3.36	3	
4	12	0.18	0.66	4.62	5	12
5	16	0.25	0.91	6.37	6	16
6	4	0.06	0.97	6.79	7	6
7	2	0.03	1	7	7	

64



Cont..

Histogram Equalization of Reference Image (b)

Gray level(r_k)	No. of Pixels(n_k)	PDF(n_k/N) $P_r(r_k)$	CDF	$(L-1)*CDF$	s_k
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	20	0.31	0.31	2.17	2
5	20	0.31	0.62	4.34	4
6	16	0.25	0.87	6.09	6
7	8	0.13	1.0	7	7
64		1			



Cont..

Mapping

1st and last column of HE of Reference Image Table and last 2 columns of source Table

r_k	Round off(S)
0	0
1	0
2	0
3	0
4	2
5	4
6	6
7	7

Round off (H)	New n_k
1	8
2	10
3	12
3	
5	12
6	16
7	6
7	

Map
4
4
5
5
6
6
7
7

Gray Level	0	1	2	3	4	5	6	7
No of Pixels	0	0	0	0	18	12	28	6



Example 3: Histogram Matching

- Apply the histogram specification to the image given here. Assume the target given in table. Show the resultant final mapping.

1	3	4	5
5	6	6	6
7	7	7	7
5	5	5	5

Gray Level (r_k)	0	1	2	3	4	5	6	7
No of Pixels (n_k)	0	0	0	0	20	20	16	8

Cont..

Histogram Equalization of Input Image

r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	Round off (H)	New n_k
0	0	0	0	0	0	1
1	1	1/16	1/16	0.4375	0	
2	0	0	1/16	0.4375	0	
3	1	1/16	2/16	0.875	1	2
4	1	1/16	3/16	1.3125	1	
5	6	6/16	9/16	3.94	4	6
6	3	3/16	12/16	5.25	5	3
7	4	4/16	16/16=1	7	7	4

16



Cont..

Histogram Equalization (HE) of Reference Image Table

r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	Round off (S)
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	20	0.3125=0.31	0.31	2.17	2
5	20	0.3125=0.31	0.62	4.34	4
6	16	0.25	0.87	6.09	6
7	8	0.125=0.13	1	7.00	7

64



Cont..

Mapping

1st and last column of HE of Reference Image Table and last 2 columns of source Table

r_k	Round off(S)
0	0
1	0
2	0
3	0
4	2
5	4
6	6
7	7

Round off (H)	New n_k	Map
0	1	1
0		1
0		1
1	2	4
1		4
4	6	5
5	3	6
7	4	7

Gray Level	0	1	2	3	4	5	6	7
No of Pixels	0	1	0	0	2	6	3	4



Example 4: Histogram Matching

- Apply the histogram specification to the image given here. Assume the target given in table. Show the resultant final mapping.

1	3	4	5
5	6	6	6
7	7	7	7
5	5	5	5

Gray Level	0	1	2	3	4	5	6	7
Target Mapping(S)	0	0	1	2	2	3	6	7



Cont..

Histogram Equalization of Input Image

r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	Round off (H)	New n_k
0	0	0	0	0	0	1
1	1	1/16	1/16	0.44	0	
2	0	0	1/16	0.44	0	
3	1	1/16	2/16	0.88	1	2
4	1	1/16	3/16	1.31	1	
5	6	6/16	9/16	3.94	4	6
6	3	3/16	12/16	5.25	5	3
7	4	4/16	16/16=1	7	7	4

16



Cont..

Mapping

1st and last column of HE of Reference Image Table and last 2 columns of source Table

r_k	Round off(S)
0	0
1	0
2	1
3	2
4	2
5	3
6	6
7	7

Round off (H)	New n_k
0	1
0	
0	
1	2
1	
4	6
5	3
7	4

Map
1
1
1
2
2
5
6
7

Gray Level	0	1	2	3	4	5	6	7
No of Pixels	0	1	2	0	0	6	3	4



Cont..

- By applying these mapping the final image will be:-

1	3	4	5
5	6	6	6
7	7	7	7
5	5	5	5

1	2	2	5
5	6	6	6
7	7	7	7
5	5	5	5

Gray Level	0	1	2	3	4	5	6	7
No of Pixels	0	1	2	0	0	6	3	4



Example 5: Histogram Matching

- Apply the histogram specification to the image given here. Assume the target given in table. Show the resultant final mapping.

1	2	3	4
5	6	7	0
7	7	7	7
5	5	5	5

Gray Level (r_k)	0	1	2	3	4	5	6	7
No of Pixels (n_k)	0	0	0	0	20	20	16	8



Cont..

Histogram Equalization of Input Image

r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	Round off (H)	New n_k
0	1	1/16	1/16	0.44	0	1
1	1	1/16	2/16	0.88	1	2
2	1	1/16	3/16	1.31	1	
3	1	1/16	4/16	1.75	2	2
4	1	1/16	5/16	2.19	2	
5	5	5/16	10/16	4.38	4	5
6	1	1/16	11/16	4.81	5	1
7	5	5/16	16/16=1	7	7	5

16



Cont..

Histogram Equalization (HE) of Reference Image Table

r_k	n_k	PDF $P_r(r_k) = n_k/MN$	CDF	$S_k = CDF(L-1)$	Round off (S)
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	20	0.3125=0.31	0.31	2.17	2
5	20	0.3125=0.31	0.62	4.34	4
6	16	0.25	0.87	6.09	6
7	8	0.125=0.13	1	7.00	7

64



Cont..

Mapping

1st and last column of HE of Reference Image Table and last 2 columns of source Table

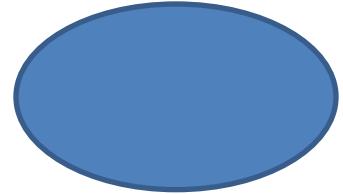
r_k	Round off(S)
0	0
1	0
2	0
3	0
4	2
5	4
6	6
7	7

Round off (H)	New n_k	Map
0	1	1
1	2	4
1		4
2	2	4
2		4
4	5	5
5	1	6
7	5	7

Gray Level	0	1	2	3	4	5	6	7
No of Pixels	0	1	0	0	4	5	1	5



Local Histogram Processing



- Define a neighborhood and move its center from pixel to pixel
- At each location, the histogram of the points in the neighborhood is computed. Either histogram equalization or histogram specification transformation function is obtained
- Map the intensity of the pixel centered in the neighborhood
- Move to the next location and repeat the procedure

N_4 of $(x,y) \rightarrow (x-1,y), (x+1,y), (x,y-1), (x, y+1)$



Local Histogram Processing

- Local Histogram Processing
- The two histogram processing methods discussed:
- Global
 - pixels are modified by a transformation function based on the gray-level content of an entire image.
- Global approach is suitable for overall enhancement
 - there are cases in which it is necessary to enhance details over small areas in an image



Histogram Processing

- Global vs Local Processing
- The histogram processing methods discussed above are **global**, in the sense that pixels are modified by a transformation function based on the gray-level content of an entire image.
- However, there are cases in which it is necessary to enhance details over **small areas** in an image.

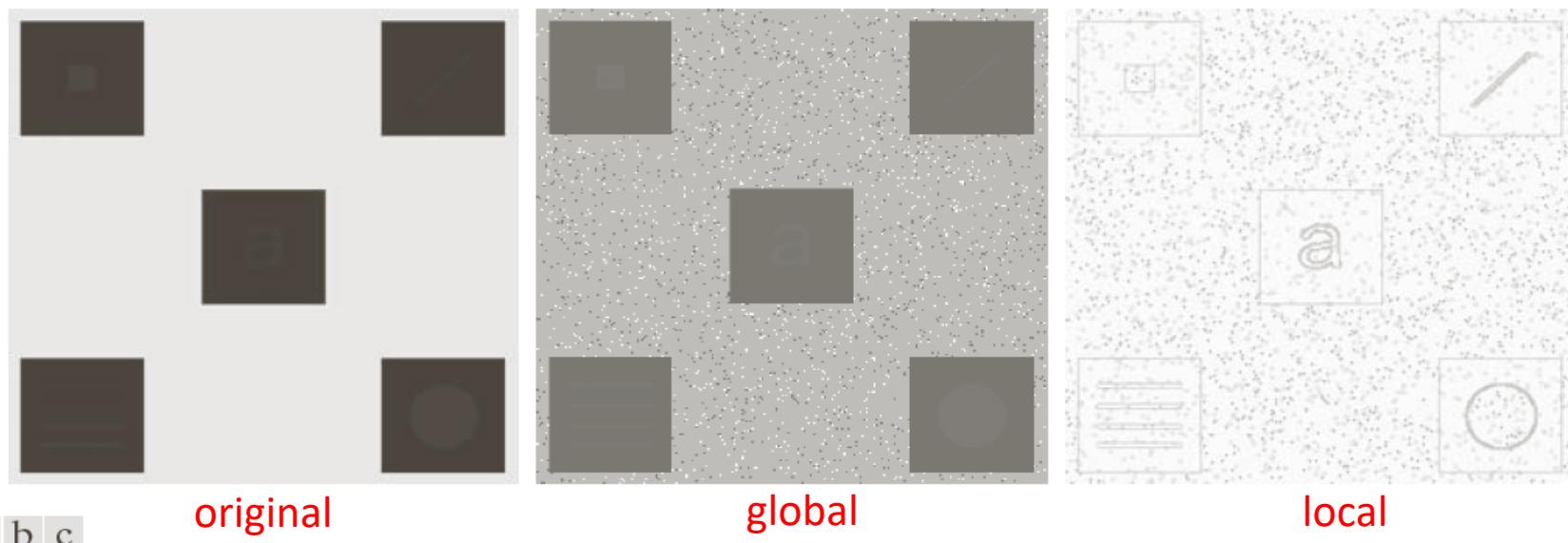


FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3 .

Use of Histogram Statistics for Image Enhancement

- Moments can be determined directly from a histogram much faster than they can from the pixels directly.
- Let r denote a discrete random variable representing discrete gray-levels in the range $[0, L-1]$, and $p(r_i)$ denote the normalized histogram component corresponding to the i^{th} value of r , then the n^{th} moment of r about its mean is defined as

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

where m is the mean (average intensity) value of r

$$m = \sum_{i=0}^{L-1} r_i p(r_i)$$

- For example, the second moment (also the variance of r) is

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$



Cont..

Average Intensity

$$m = \sum_{i=0}^{L-1} r_i p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

$$u_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

Variance

$$\sigma^2 = u_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - m]^2$$



Cont..

- **Global mean** and **variance** are computed on an entire image and useful for gross adjustment in overall intensity and contrast.
- More powerful use is in **local enhancement**, where **local mean** and **variance** are used as the basis for making changes.
- S_{xy} denote a neighborhood of a specified useful (x, y) . The mean value of the pixel is given by

Local average intensity

$$m_{s_{xy}} = \sum_{i=0}^{L-1} r_i p_{s_{xy}}(r_i)$$

Local variance

$$\sigma_{s_{xy}}^2 = \sum_{i=0}^{L-1} (r_i - m_{s_{xy}})^2 p_{s_{xy}}(r_i)$$

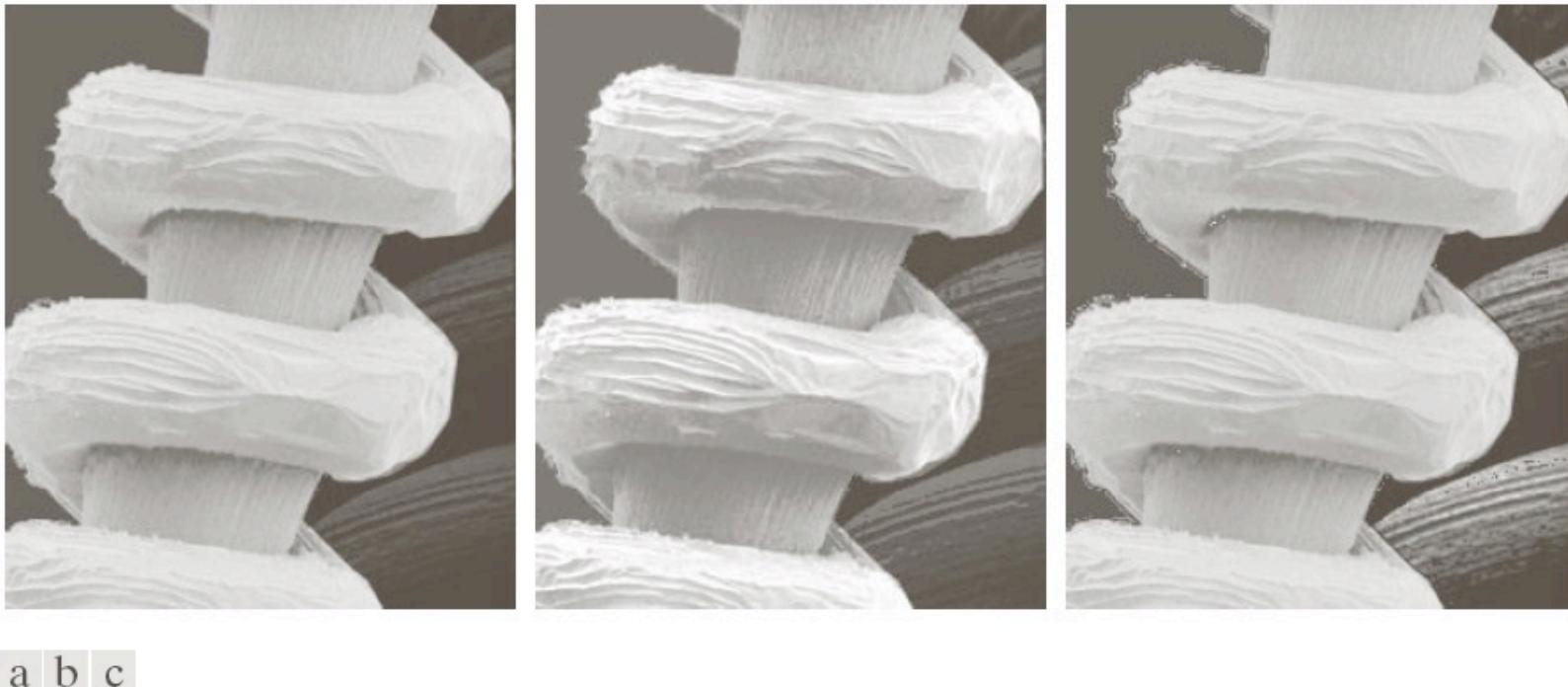
s_{xy} denotes a neighborhood

- Enhance the dark area while leaving the light area unchanged as possible.



Histogram Processing

- Global vs Local Processing



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Cont..

- Two uses of the **mean and variance** for enhancement purposes:
 - The **global** mean and variance (global means for the entire image) are useful for adjusting overall contrast and intensity.
 - The mean and standard deviation for a **local** region are useful for correcting for large-scale changes in intensity and contrast.



Enhancement using Arithmetic/Logic Operations

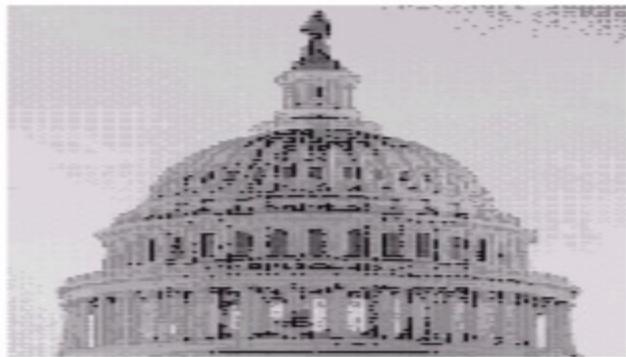


Enhancement using Arithmetic/Logic Operations

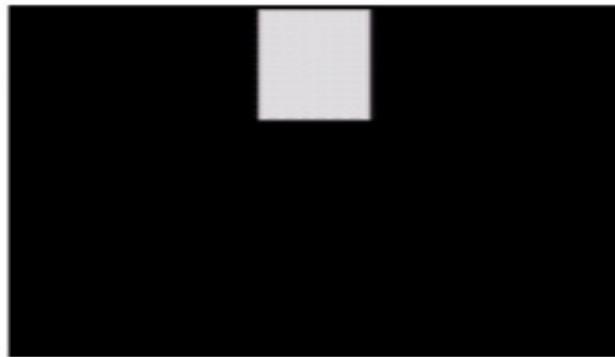
- Arithmetic/ Logic operations perform on pixel by pixel basis between two or more images
- Except NOT operation which perform only on a single image
- **Logic Operations**
- Logic operation performs on gray-level images, the pixel values are processed as binary numbers
- light represents a binary 1, and dark represents a binary 0
- NOT operation = negative transformation



Example of AND Operation



original image

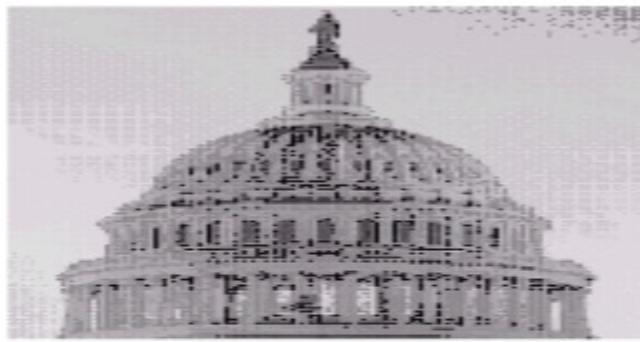


AND image
mask

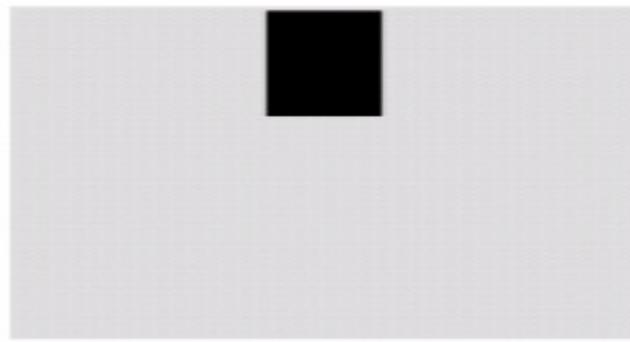


result of AND
operation

Example of OR Operation



original image



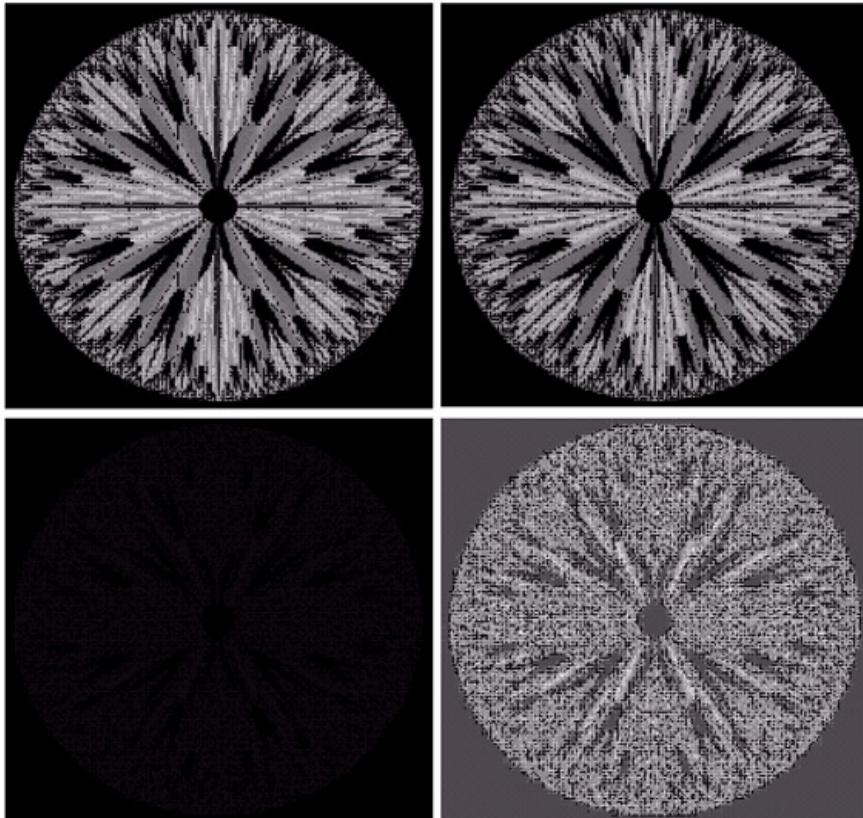
OR image
mask



result of OR
operation

Example of Image Subtraction

$$g(x,y) = f(x,y) - h(x,y)$$



a	b
c	d

- a) original fractal image
- b) result of setting the 4 lower-order bit planes to zero
 - refer to the bit-plane slicing
 - the higher planes contribute significant detail
 - the lower planes contribute more to fine detail
 - image b). is nearly identical visually to image a), with a very slight drop in overall contrast due to less variability of the gray-level values in the image.
- c) difference between a) and b) (nearly black)
- d) histogram equalization of c). (perform contrast stretching transformation)

Spatial Filtering



Spatial Filtering

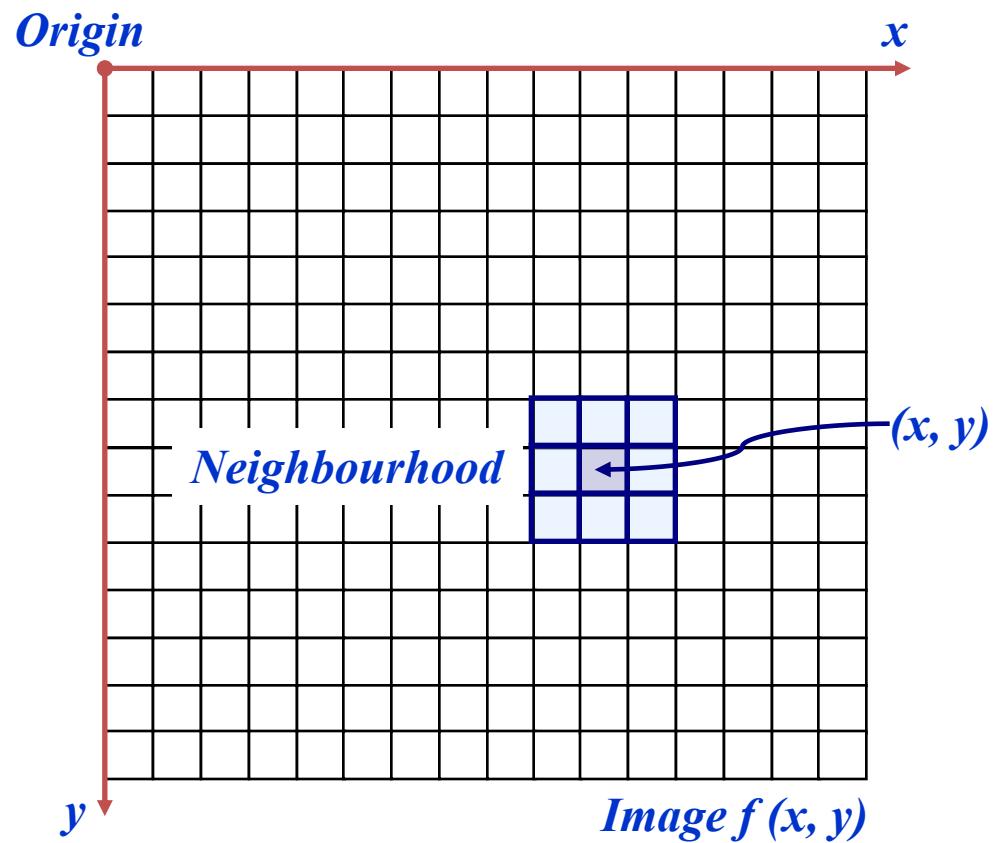
In this lecture we will look at spatial filtering techniques:

- Neighbourhood operations
- What is spatial filtering?
- Smoothing operations
- What happens at the edges?
- Correlation and convolution
- Sharpening filters
- Combining filtering techniques



Neighbourhood Operations

- Neighbourhood operations simply operate on a larger neighbourhood of pixels than point operations
- Neighbourhoods are mostly a rectangle around a central pixel
- Any size rectangle and any shape filter are possible



Simple Neighbourhood Operations

- Some simple neighbourhood operations include:
 - **Min:** Set the pixel value to the minimum in the neighbourhood
 - **Max:** Set the pixel value to the maximum in the neighbourhood
 - **Average:** Average of the pixel values
 - **Median:** The median value of a set of numbers is the midpoint value in that set (e.g. from the set [1, 7, 15, 18, 24] 15 is the median). Sometimes the median works better than the average



Simple Neighbourhood Operations Example

Original Image

123	127	128	119	115	130
140	145	148	153	167	172
133	154	183	192	194	191
194	199	207	210	198	195
164	170	175	162	173	151

x

• • •

y

•
•
•

Enhanced Image

x

• • •

y

•
•
•



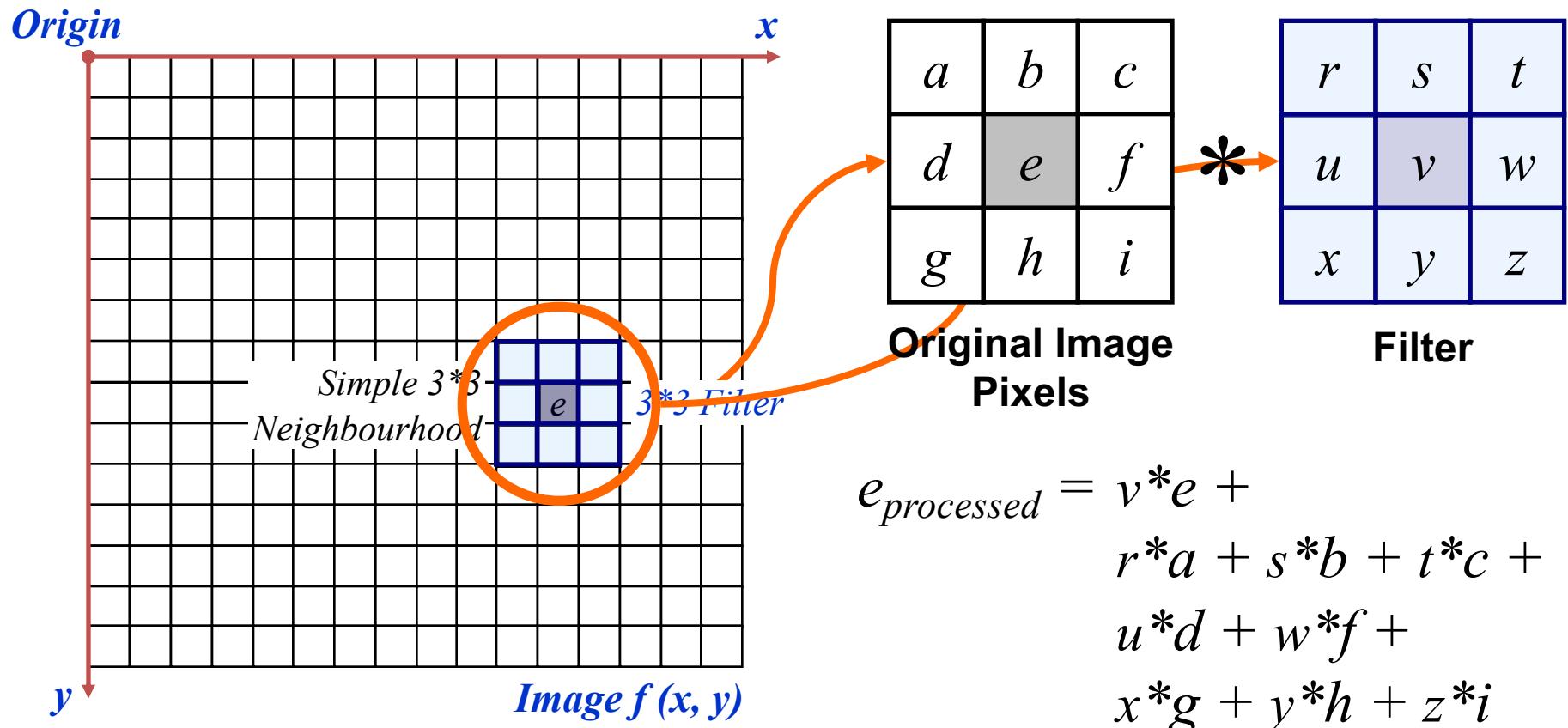
Spatial Filtering

- Fundamentals of spatial filtering:-
 - Filtering is useful for amplifying(passing) or rejecting certain frequency components.
- A spatial filter consists of (a) **a neighborhood**, and (b) **a predefined operation**
- Linear spatial filtering of an image of size $M \times N$ with a filter of size $m \times n$ is given by the expression

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

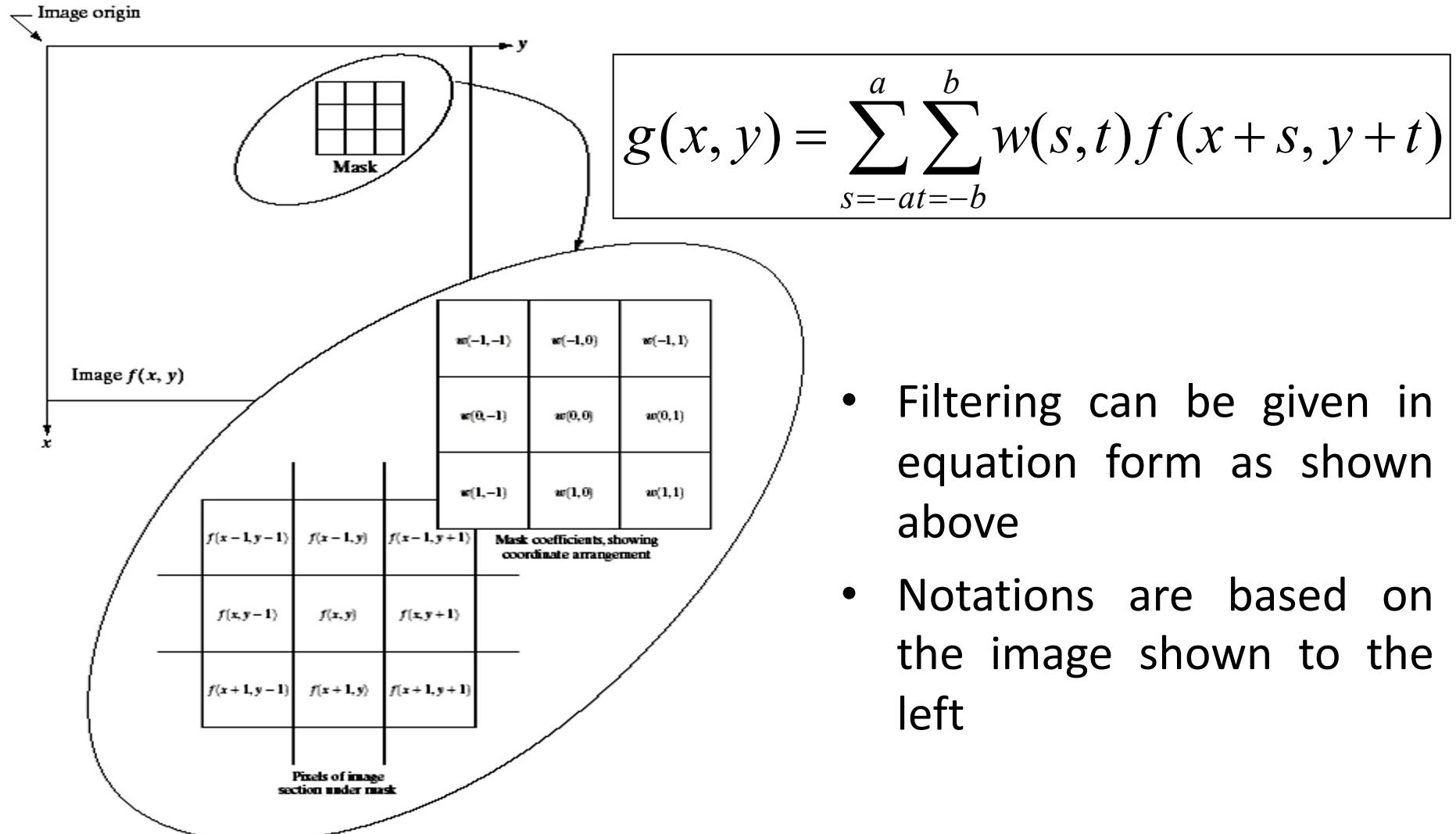


The Spatial Filtering Process



The above is repeated for every pixel in the original image to generate the filtered image

Spatial Filtering: Equation Form



- Filtering can be given in equation form as shown above
- Notations are based on the image shown to the left

Smoothing Spatial Filters

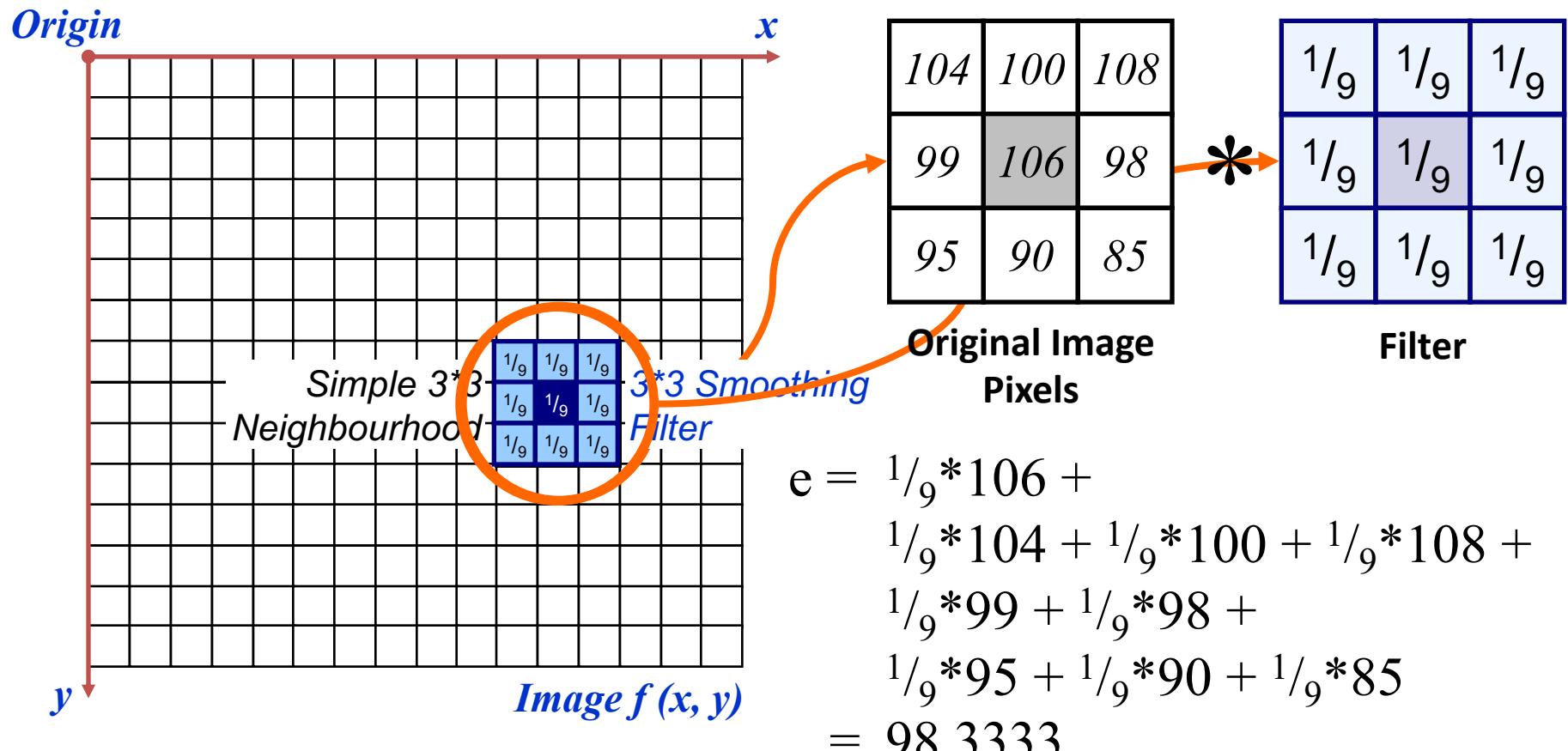
- One of the simplest spatial filtering operations we can perform is a smoothing operation
 - Simply average all of the pixels in a neighbourhood around a central value
 - Especially useful in removing noise from images
 - Also useful for highlighting gross detail

$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$

Simple averaging filter



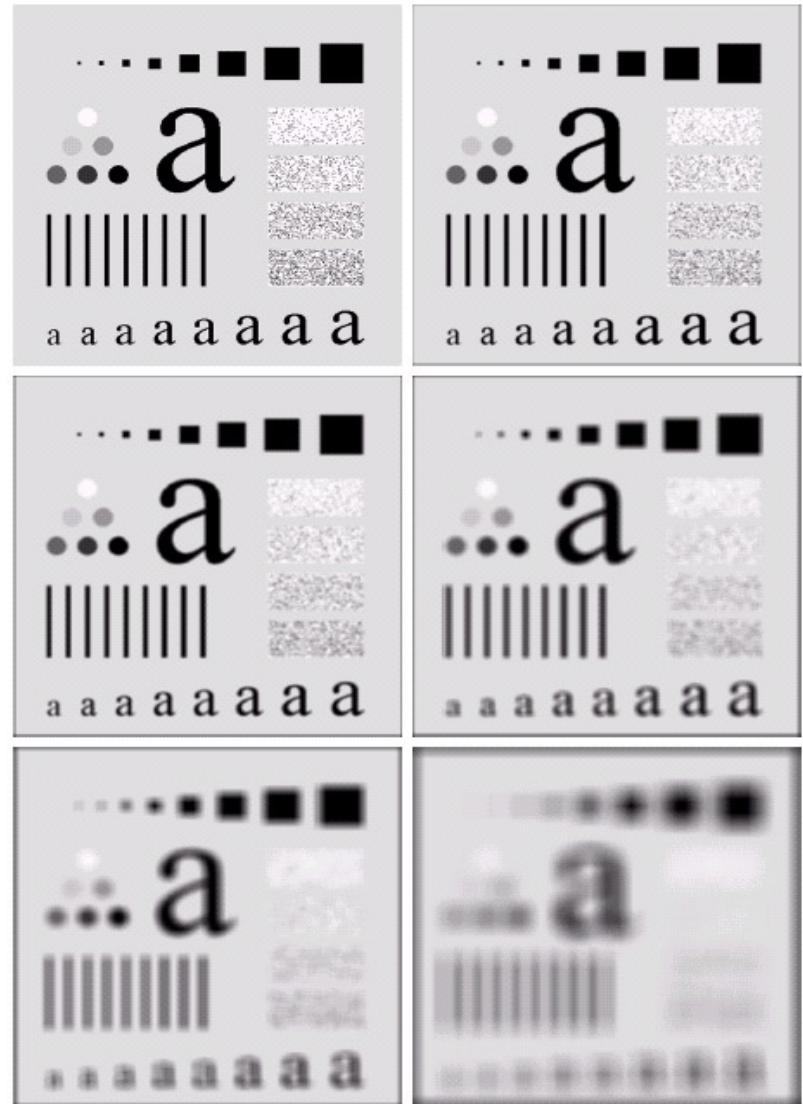
Smoothing Spatial Filtering



The above is repeated for every pixel in the original image to generate the smoothed image.

Image Smoothing Example

- The image at the top left is an original image of size 500*500 pixels
- The subsequent images show the image after filtering with an averaging filter of increasing sizes
 - 3, 5, 9, 15 and 35
- Notice how detail begins to disappear



Weighted Smoothing Filters

- More effective smoothing filters can be generated by allowing different pixels in the neighbourhood different weights in the averaging function
 - Pixels closer to the central pixel are more important
 - Often referred to as a *weighted averaging*

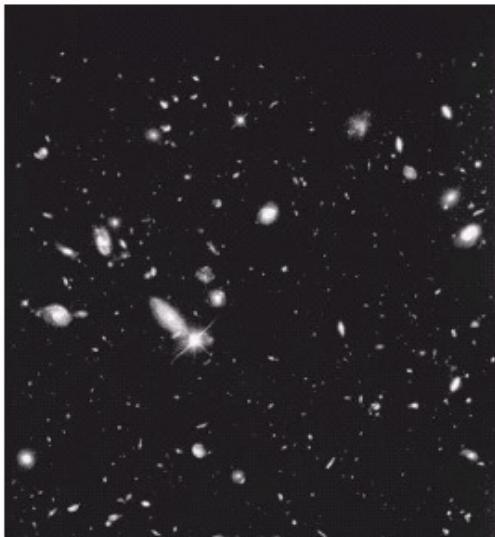
$1/_{16}$	$2/_{16}$	$1/_{16}$
$2/_{16}$	$4/_{16}$	$2/_{16}$
$1/_{16}$	$2/_{16}$	$1/_{16}$

Weighted averaging filter

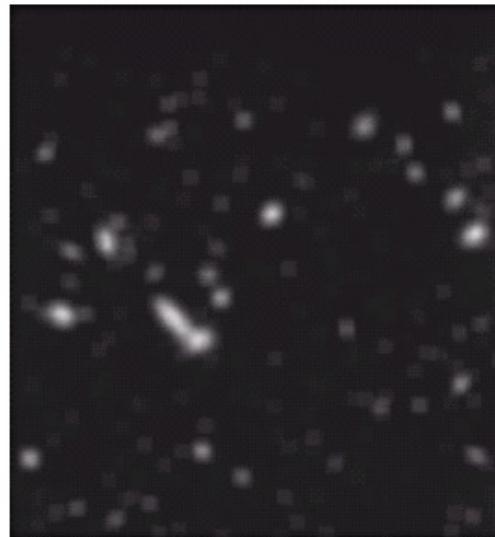


Another Smoothing Example

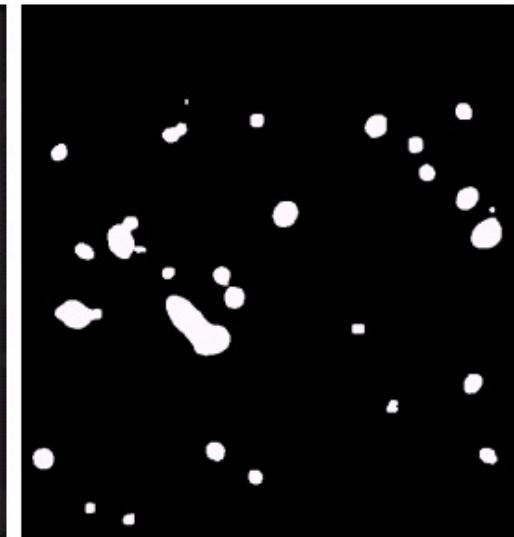
- By smoothing the original image we get rid of lots of the finer detail which leaves only the gross features for thresholding



Original Image



Smoothed Image



Thresholded Image

Order-statistic (Nonlinear) Filters

- **Nonlinear**
- Based on ordering (ranking) the pixels contained in the filter mask
- Replacing the value of the center pixel with the value determined by the ranking result
- E.g., median filter, max filter, min filter



Median Filtering

- Median filtering is particularly effective in the presence of impulse noise (salt and pepper noise).
- Unlike average filtering, median filtering does not blur too much image details.



Median Filtering

- The median filter is non linear:
$$\text{median}\{x + y\} \neq \text{median}\{x\} + \text{median}\{y\}$$
- It works well for impulse noise (e.g. salt and pepper).
- It requires sorting of the image values.
- It preserves the edges better than an average filter in the case of impulse noise.
- It is robust to impulse noise at 50%.

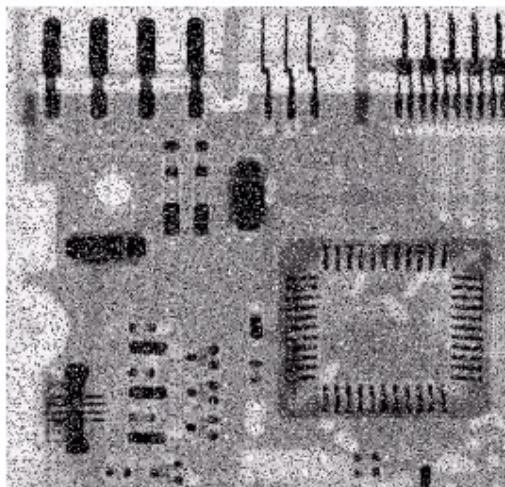


Median Filtering

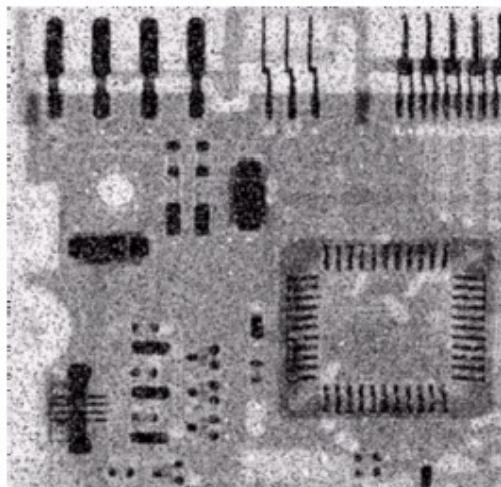
- Advantages:
 - Removes impulsive noise
 - Preserves edges
- Disadvantages:
 - poor performance when # of noise pixels in the window is greater than 1/2 # in the window
 - poor performance with Gaussian noise



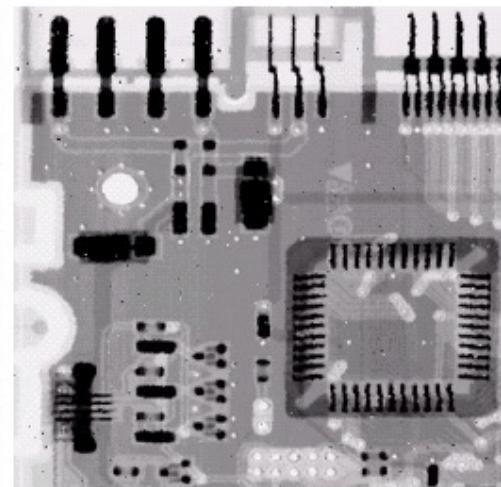
Averaging Filter Vs. Median Filter Example



**Original Image
With Noise**



**Image After
Averaging Filter**

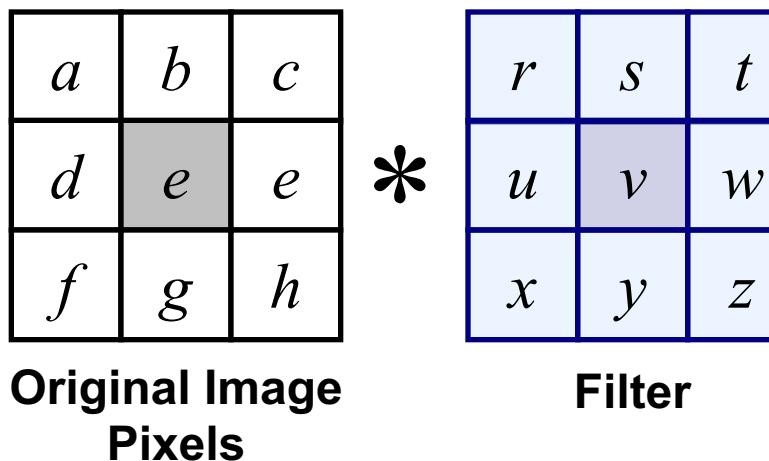


**Image After
Median Filter**

- Filtering is often used to remove noise from images
- Sometimes a median filter works better than an averaging filter

Correlation & Convolution

- The filtering we have been talking about so far is referred to as *correlation* with the filter itself referred to as the *correlation kernel*
- Convolution* is a similar operation, with just one subtle difference

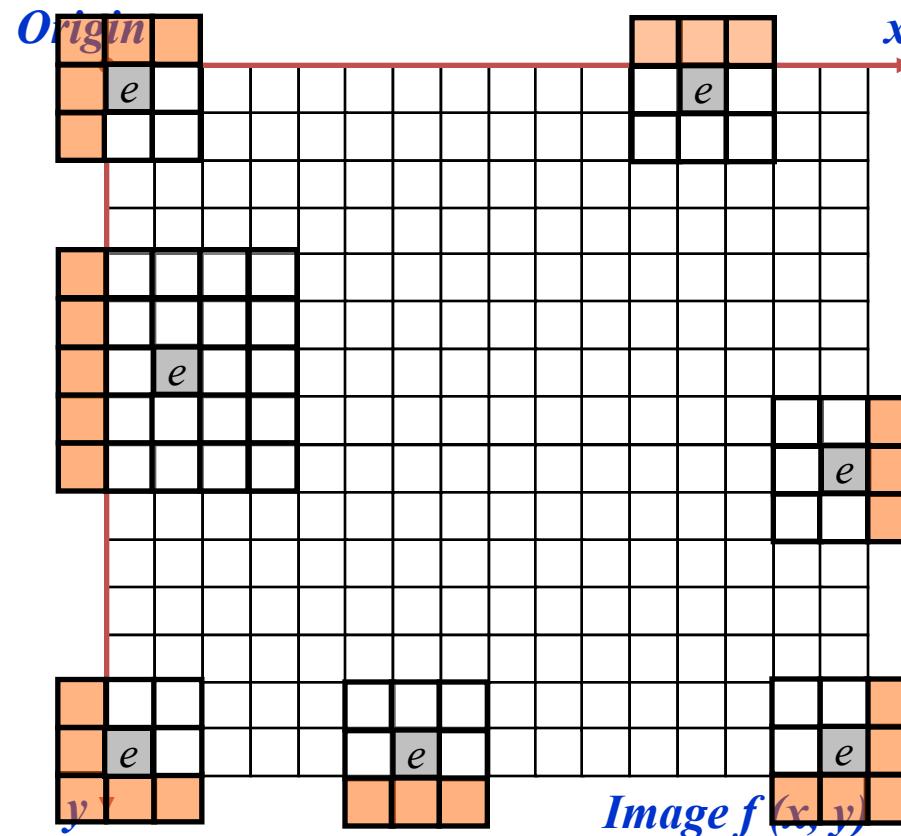


$$e_{processed} = v^*e + z^*a + y^*b + x^*c + w^*d + u^*e + t^*f + s^*g + r^*h$$

- For symmetric filters it makes no difference.

Strange Things Happen At The Edges!

At the edges of an image we are missing pixels to form a neighbourhood



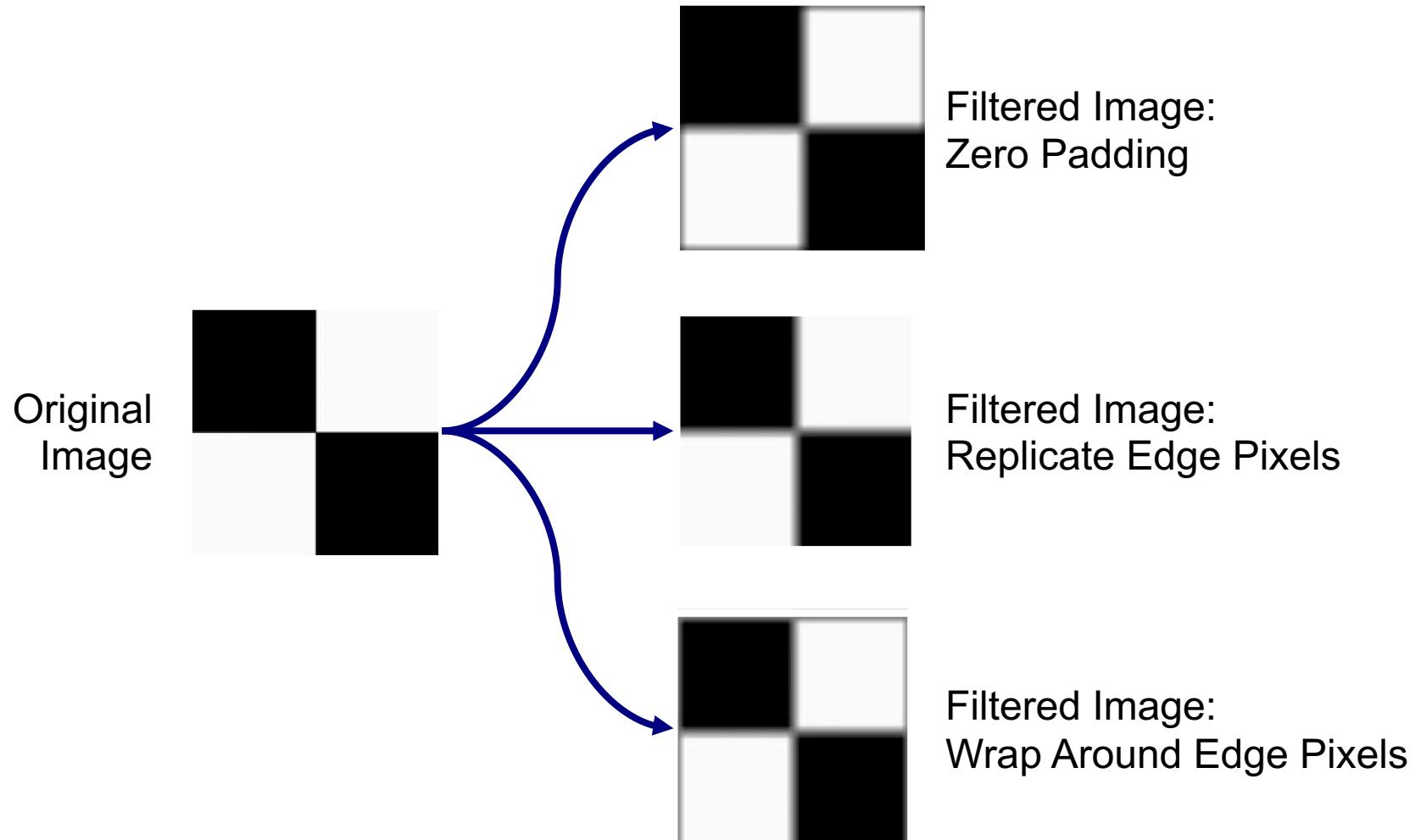
Strange Things Happen At The Edges! (cont...)

There are a few approaches to dealing with missing edge pixels:

- Omit missing pixels
 - Only works with some filters
 - Can add extra code and slow down processing
- Pad the image
 - Typically with either all white or all black pixels
- Replicate border pixels
- Truncate the image
- Allow pixels *wrap around* the image
 - Can cause some strange image artefacts



Strange Things Happen At The Edges! (cont...)



Fundamentals of Spatial Filtering

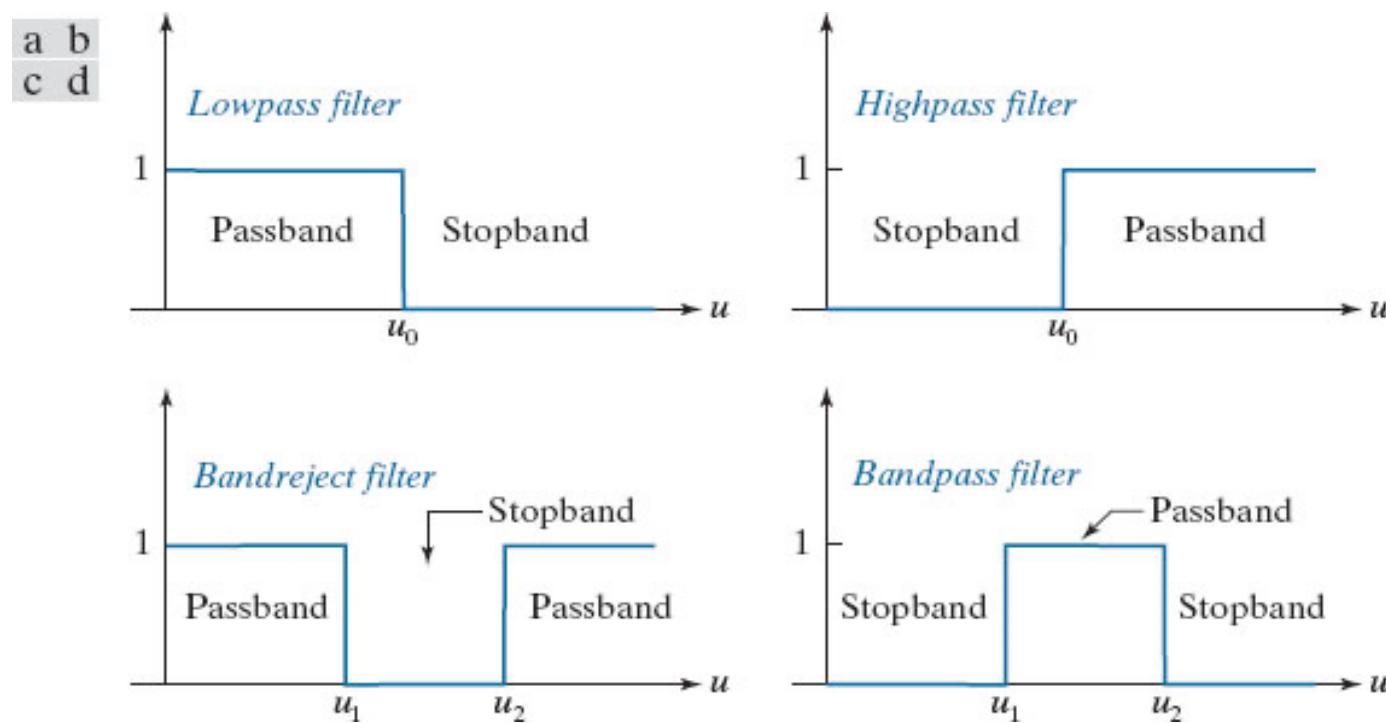
- Filtering refers to accepting(passing) or rejecting certain frequency components. This effectively smoothens or sharpens the image.
- E.g. Low pass filter, high pass filter, etc.
- Such operations can be directly carried out on image in spatial domain also by using spatial filters (kernels, spatial masks, templates, & windows).
- Spatial filters are more versatile as they are used in linear as well as non-linear filtering (Difficult in frequency domain).



Cont..

Transfer functions of ideal 1-D filters in the frequency domain (u denotes frequency). (a) Lowpass filter. (b) Highpass filter. (c) Bandreject filter. (d) Bandpass filter. (As before, we show only positive frequencies for simplicity.)

Figure
3.58



Cont..

Summary of the four principal spatial filter types expressed in terms of lowpass filters. The centers of the unit impulse and the filter kernels coincide.

Filter type	Spatial kernel in terms of lowpass kernel, lp
Lowpass	$lp(x, y)$
Highpass	$hp(x, y) = \delta(x, y) - lp(x, y)$
Bandreject	$br(x, y) = lp_1(x, y) + hp_2(x, y)$ $= lp_1(x, y) + [\delta(x, y) - lp_2(x, y)]$
Bandpass	$bp(x, y) = \delta(x, y) - br(x, y)$ $= \delta(x, y) - [lp_1(x, y) + [\delta(x, y) - lp_2(x, y)]]$

Table 3.7



Fundamentals of Spatial Filtering

- Types of Spatial Filtering
 1. Point to point (pixel to pixel) operation (discussed so far)
 2. Mask based (Neighborhood) operations
 - i. Operation with 3x3 filter (E.g. Mean, max, min, etc)
 - ii. Correlation or Convolution
- Linear vs Non-Linear Filter If the operation performed on the image pixels is linear, then the filter is called a linear spatial filter, otherwise nonlinear.



Operation with 3x3 Filter

- At any point (x, y) in the image, the response, $g(x, y)$, of the filter is the sum of products of the filter coefficients and the image pixels encompassed by the filter:

$$g(x, y) = f(x-1, y-1).w1 + f(x-1, y).w2 + f(x-1, y+1).w3 + f(x, y-1).w4 + f(x, y).w5 + f(x, y+1).w6 + f(x+1, y-1).w7 + f(x+1, y).w8 + f(x+1, y+1).w9$$

- For the mask of size $m \times n$, we assume

$$m = 2a + 1;$$

$$n = 2b + 1;$$

where a & b are positive integers.

X-1,y+1	X,y+1	
x-1,y	x,y	
x-1, y-1	X,y-1	

- 3x3 is the smallest filter.



Operation with 3x3 Filter

- Generalized equation:

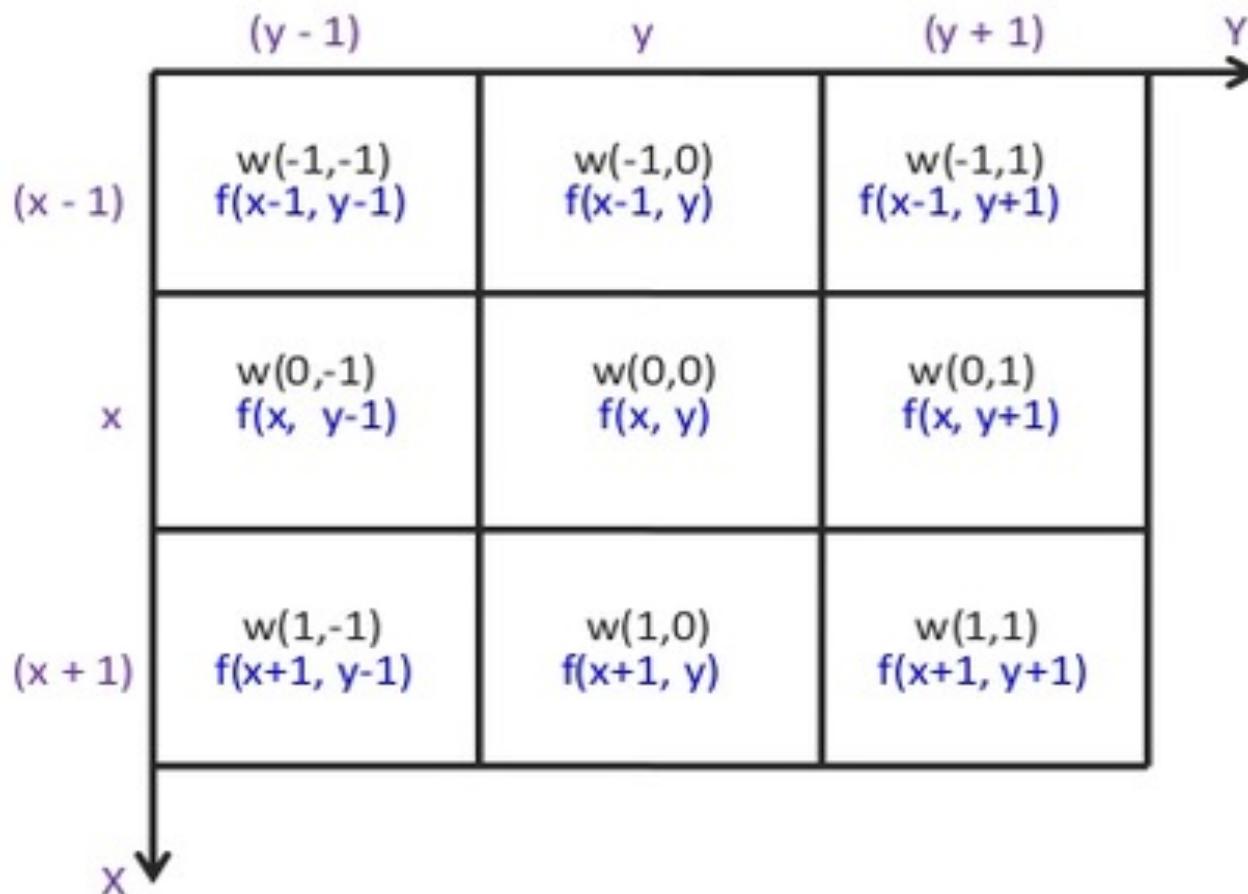
$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

- where, x & y are varied so that each pixel in w visits every pixel in f.



Operation with 3x3 Filter

- 3 x 3 Neighborhood / Mask / Window / Template:



Correlation & Convolution

- Correlation & Convolution are two closely related concepts used in linear spatial filtering.
- **Correlation:** It is a process of moving a filter mask over an image & computing the sum of products at each location.
- **Convolution:** Here, the mechanics are same, except that the filter is first rotated by 180° .
- Correlation & Convolution are function of displacement. Correlation & Convolution are exactly same if the filter mask is symmetric.
- 1D correlation and convolution of a filter with a discrete unit impulse is shown below.

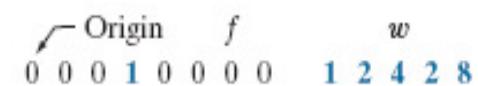
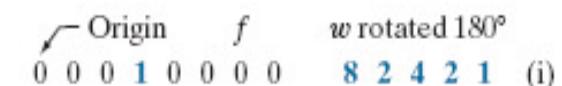
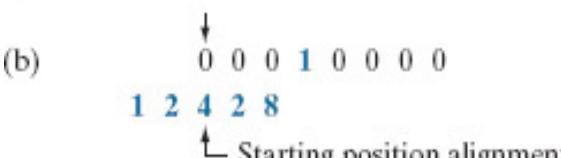
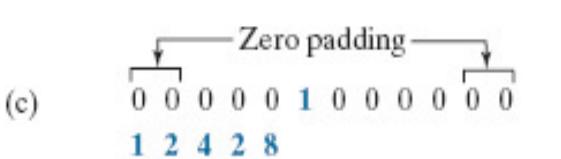
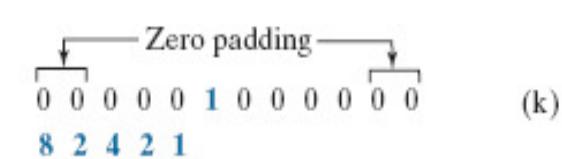
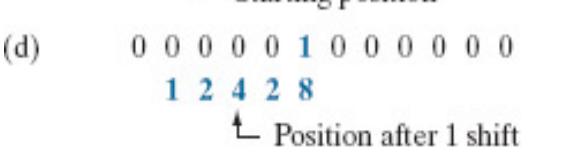
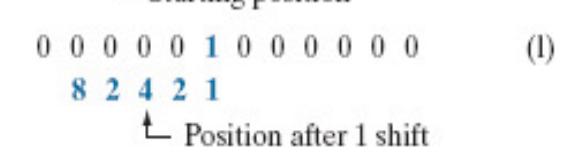
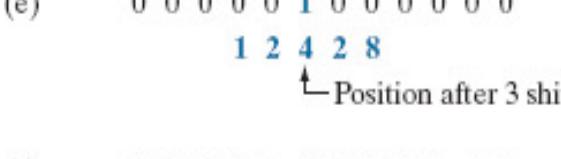
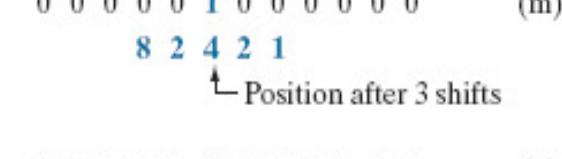
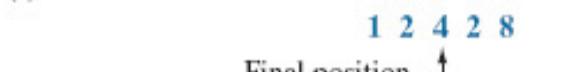


Cont..

Illustration of 1-D correlation and convolution of a kernel, w, with a function f consisting of a discrete unit impulse.

Note that correlation and convolution are functions of the variable x, which acts to **displace** one function with respect to the other.

For the extended correlation and convolution results, the starting configuration places the rightmost element of the kernel to be coincident with the origin of f. Additional padding must be used.

	<i>Correlation</i>	<i>Convolution</i>
(a)		
(b)		
(c)		
(d)		
(e)		
(f)		
	Correlation result	Convolution result
(g)	0 8 2 4 2 1 0 0	0 1 2 4 2 8 0 0
(h)	0 0 0 8 2 4 2 1 0 0 0 0	0 0 0 1 2 4 2 8 0 0 0 0

	Extended (full) correlation result	Extended (full) convolution result
(i)	0 0 0 8 2 4 2 1 0 0 0 0	0 0 0 1 2 4 2 8 0 0 0 0



Correlation & Convolution

- Correlation is a function of displacement of the filter.
- Correlating a filter w with a function that contains all '0' & single '1' yields a 180° rotated copy of w .
- Correlating a function with discrete unit impulse yields a rotated (time inverted) version of the function.
- Convolving a function with a unit impulse yields the same function.
- Thus, to perform convolution all we have to do is rotate one function by 180° & perform same operation as in correlation.



Origin	$f(x, y)$
0 0 0 0 0	
0 0 0 0 0	$w(x, y)$
0 0 1 0 0	1 2 3
0 0 0 0 0	4 5 6
0 0 0 0 0	7 8 9

(a)

FIGURE 3.30
 Correlation
 (middle row) and
 convolution (last
 row) of a 2-D
 filter with a 2-D
 discrete, unit
 impulse. The 0s
 are shown in gray
 to simplify visual
 analysis.



Correlation & Convolution

- Summarizing in equation form, we have that
- The **Correlation** of a filter $w(x, y)$ of size $m \times n$ with an image $f(x, y)$ is given by:

$$w(x, y) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x+s, y+t)$$

- The **Convolution** of $w(x, y)$ and $f(x, y)$ is given by:

$$w(x, y) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x-s, y-t)$$



Vector representation of Linear Filtering

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

FIGURE 3.31

Another representation of a general 3×3 filter mask.

- $R = w_1z_1 + w_2z_2 + \dots + w_9z_9$
 $= \sum_{k=1}^9 w_k z_k$
 $= w^T z$
- Where, w & z are 9-dimensional vectors formed from coefficients of the mask & image intensities encompassed by the mask, resp.



Generating Spatial Filter Masks

1. Average Mean Filter

- The average value at any location (x, y) in the image is the sum of the nine intensity values in the 3×3 neighborhood centered on (x, y) divided by 9.
- If $z_i, i = 1, 2, \dots, 9$ denote these intensities, then the average is:
- $R = \frac{1}{9} \sum_{i=1}^9 z_i$

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$
$$\frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.



Generating Spatial Filter Masks

- General implementation for filtering an $M \times N$ image with a weighted average filter of size $m \times n$ is given by:

$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}$$

2. Exponential Filter

- Some applications have a continuous function of 2 variables.
- E.g. Gaussian function Spatial filter mask has the basic form:

$$h(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- where, σ is standard deviation



Smoothing Spatial Filters

Smoothing filters are used for blurring and for noise reduction

Blurring is used in removal of small details and bridging of small gaps in lines or curves

Smoothing spatial filters include linear filters and nonlinear filters.



Spatial smoothing and image approximation

- Spatial smoothing may be viewed as a process for estimating the value of a pixel from its neighbours.
- What is the value that “best” approximates the intensity of a given pixel given the intensities of its neighbours?
- We have to define “best” by establishing a criterion.



Generating Spatial Filter Masks

- In the resultant image the Low frequency region has remained unchanged.
- Sharp transition between 10 & 50 has changed from 10 to 23.33 to 36.66 and finally to 50.
- Thus, Sharp edges has become blurred.
- Best result when used over image corrupted by Gaussian noise.
- Other types of low pass averaging mask are:

1	0	1	0
----	1	2	1
6	0	1	0

1	1	1	1
----	1	2	1
10	1	1	1

Order-Statistic Filters

- These are non-linear spatial filters whose response is based on ordering (increasing / decreasing) the pixels contained in the image area encompassed by the filter.
- Then replacing the value of the center with the middle value determined by ranking result.
- E.g. Median filter, Max filter, Min Filter

i. **Median Filter:**

- Popular with certain random noise and impulse noise (Salt & Pepper noise).
- They provide excellent noise reduction
- Comparatively less blurring than linear smoothing filter of same size.



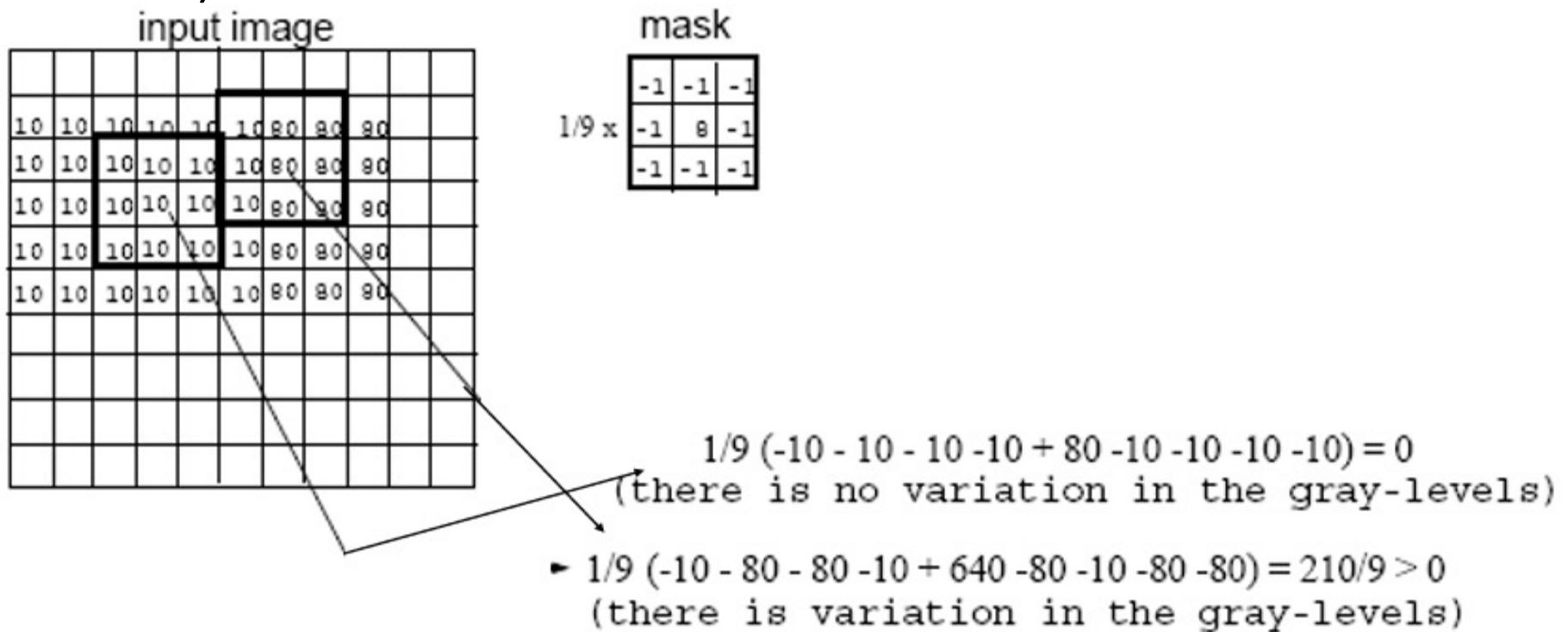
Spatial smoothing and image approximation

- **Background:** To highlight fine detail in an image or to enhance blurred detail
- **Applications:** electronic printing, medical imaging, industrial inspection, autonomous target detection (smart weapons)etc.,
- **Foundation (Blurring vs Sharpening):**
- Blurring/smoothing is performed by spatial averaging (equivalent to integration)
- Sharpening is performed by noting only the gray level changes in the image that is the differentiation



Cont..

- Operation of Image Differentiation
 - Enhance edges and discontinuities (magnitude of output gray level $>>0$)
 - De-emphasize areas with slowly varying gray-level values (output gray level: 0)



Cont..

- Common Sharpening Filters

- Unsharp masking

- High Boost filter

- Derivative



Cont..

- Unsharp Masking
 - Obtain a sharp image by subtracting a lowpass filtered (i.e., smoothed) image from the original image:
$$\text{Highpass} = \text{Original} - \text{Lowpass}$$

$$\text{TEXT} - \text{TEXT} = \text{TEXT}$$

(with some contrast
enhancement)



Cont..

- High Boost
 - Image sharpening emphasizes edges but details are lost.
 - Idea: amplify input image, then subtract a lowpass image.

$$\text{Highboost} = A \text{ Original} - \text{Lowpass}$$

$$= (A-1) \text{ Original} + \text{Original} - \text{Lowpass}$$

$$= (A-1) \text{ Original} + \text{Highpass}$$

$$(A-1) \text{ TEXT} + \text{TEXT} = \text{TEXT}$$



Cont..

- High Boost
 - If $A=1$, the result is unsharp masking.
 - If $A>1$, part of the original image is added back to the high pass filtered image.

$$\text{Highboost} = (A-1) \text{ Original} + \text{Highpass}$$

- One way to implement high boost filtering is using these masks:

A >= 1 w = 9A - 1			A = 2 w = 17		
-1	-1	-1	-1	-1	-1
-1	w	-1	-1	17	-1
-1	-1	-1	-1	-1	-1



Sharpening Spatial Filter

- The principal objective of sharpening is to highlight transitions in intensity.
- Applications: Electronics Printing, Medical Imaging, Industrial Inspection, Autonomous guidance in military systems, etc.
- Derivatives of a digital function are defined in terms of differences:
- **First order derivative** of 1D function $f(x)$ is difference:

$$\frac{\delta f}{\delta x} = f(x + 1) - f(x)$$

- **Second order derivative** of $f(x)$ as the difference:

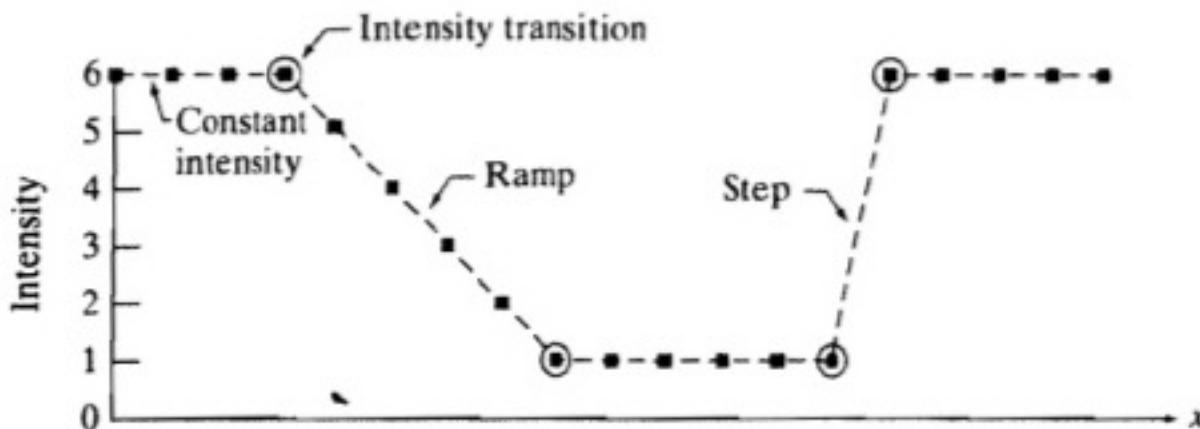
$$\frac{\delta^2 f}{\delta x^2} = f(x + 1) + f(x - 1) - 2f(x)$$



Sharpening Spatial Filter

First Derivative:	Second derivative:
must be zero in areas of constant intensity.	must be zero in constant areas
must be nonzero at the onset of an intensity step / ramp.	Must be nonzero at the onset and end of an intensity step / ramp
Must be nonzero along ramps.	Must be zero along ramps of constant slope.





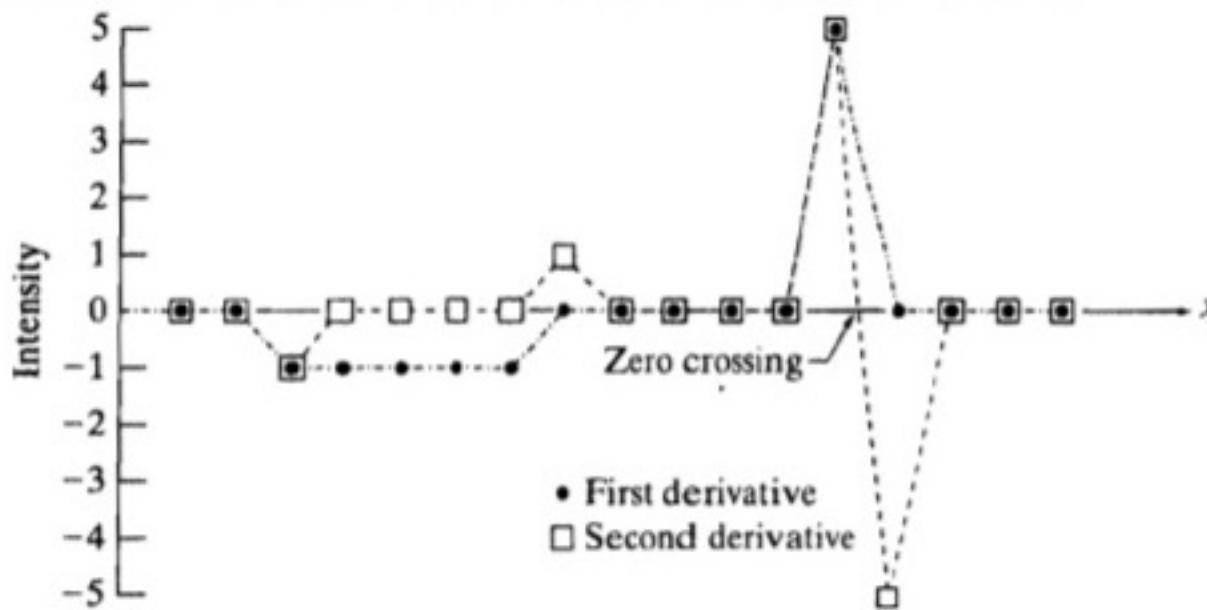
Scan
line

6	6	6	6	5	4	3	2	1	1	1	1	1	6	6	6	6
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

 $\rightarrow x$

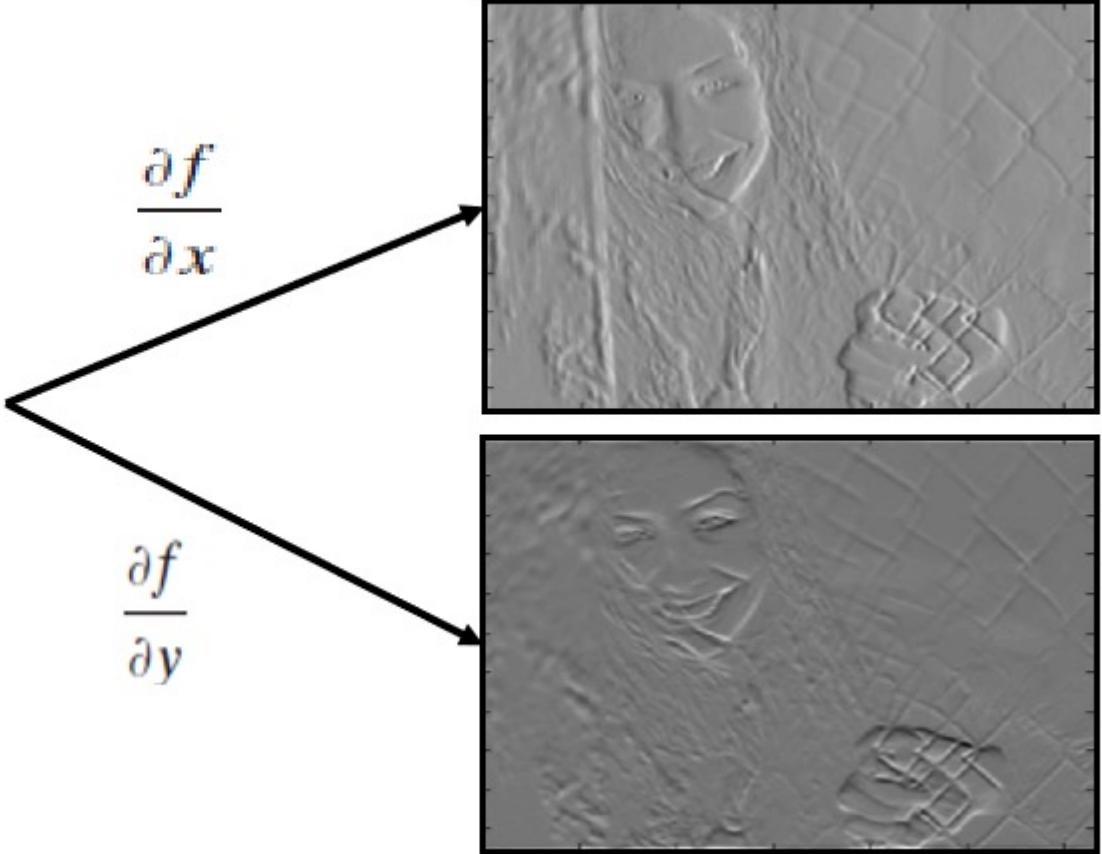
1st derivative 0 0 -1 -1 -1 -1 -1 0 0 0 0 0 0 5 0 0 0 0

2nd derivative 0 0 -1 0 0 0 0 1 0 0 0 0 0 5 -5 0 0 0 0



Example: visualize partial derivatives

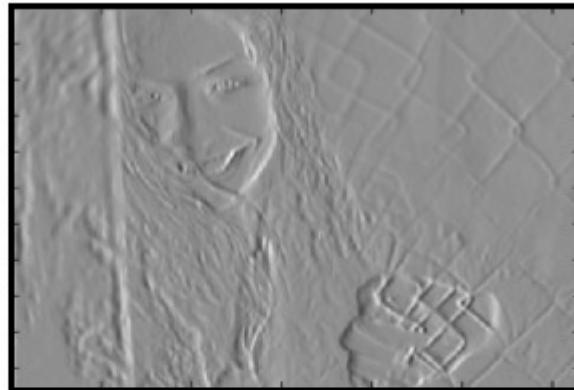
- The gradient can be visualized by mapping the values to [0, 255]



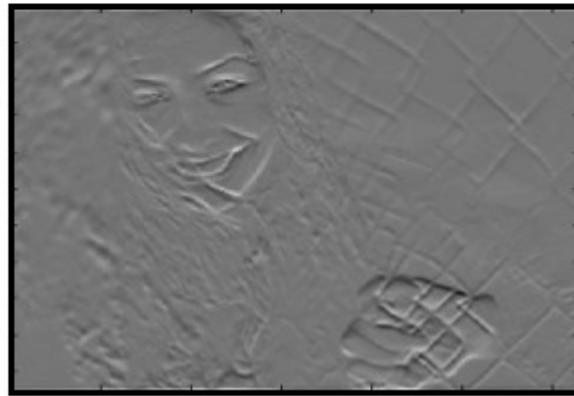
Example: visualize partial derivatives



$$\frac{\partial f}{\partial x}$$



$$\frac{\partial f}{\partial y}$$



Gradient Magnitude

$$\sqrt{\frac{\partial f^2}{\partial x} + \frac{\partial f^2}{\partial y}}$$



(**isotropic**, i.e.,
edges in all directions)

Implement Gradient Using Masks

- We can implement $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ using masks:

$$\begin{array}{|c|c|} \hline -1 & 1 \\ \hline \end{array}$$

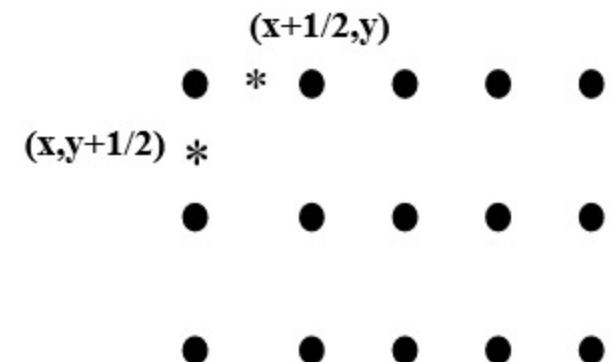
$$\begin{array}{|c|} \hline 1 \\ \hline -1 \\ \hline \end{array}$$

$$\frac{\partial f}{\partial x}$$

good approximation
at $(x+1/2, y)$

$$\frac{\partial f}{\partial y}$$

good approximation
at $(x, y+1/2)$



$$\frac{\partial f}{\partial x} : f(x + 1, y) - f(x, y).$$

$$\frac{\partial f}{\partial y} : f(x, y) - f(x, y + 1)$$



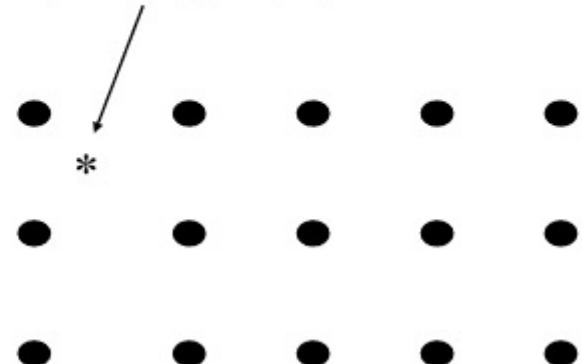
Implement Gradient Using Masks

- A different approximation of the gradient:

$$\frac{\partial f}{\partial x}(x, y) = f(x, y) - f(x + 1, y + 1)$$

$$\frac{\partial f}{\partial y}(x, y) = f(x + 1, y) - f(x, y + 1),$$

good approximation
 $(x+1/2, y+1/2)$



1st Derivative Filtering

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Roberts Cross Gradient

$$G_x = (z_9 - z_5) \quad G_y = (z_8 - z_6)$$

-1	0	0	-1
0	1	1	0

Sobel Gradient

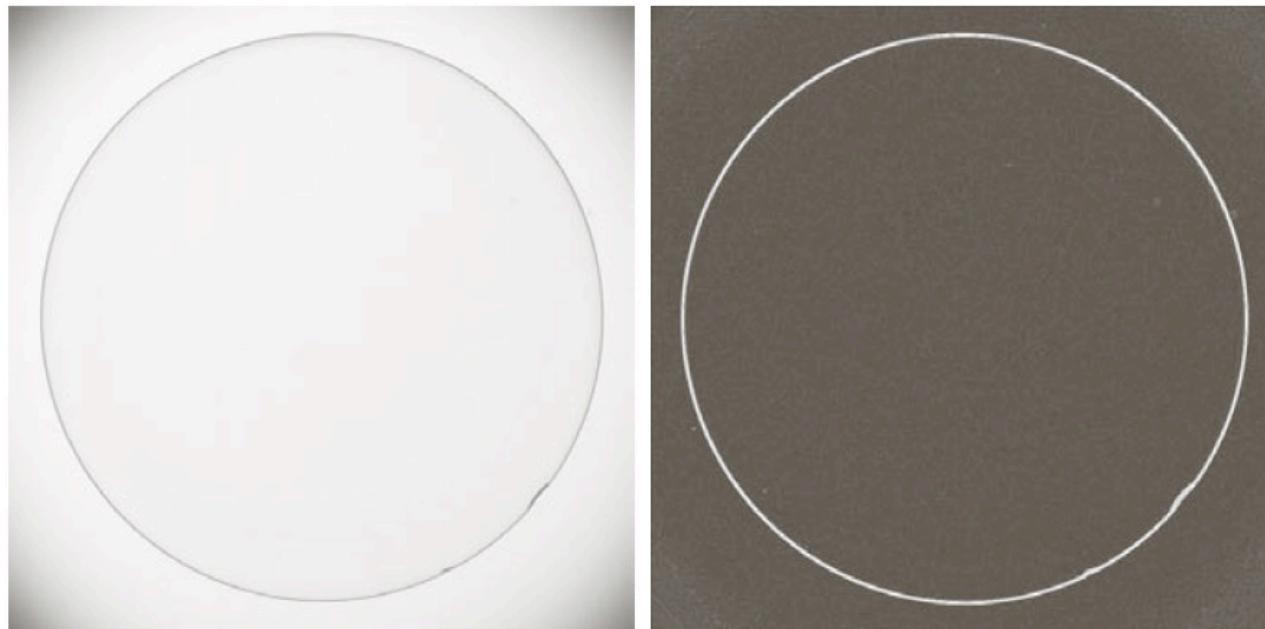
$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1



Example



a b

FIGURE 3.42
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Pete Sites, Perceptics Corporation.)

Second Order Derivative



The Laplacian

- To express the equation in discrete form,
- In x-direction,

$$\frac{\delta}{\delta x} \left[\frac{\delta f}{\delta x} \right] = \frac{\delta^2 f}{\delta x^2} = f(x + 1, y) + f(x - 1, y) - 2f(x, y)$$

- In y-direction,

$$\frac{\delta^2 f}{\delta y^2} = f(x, y + 1) + f(x, y - 1) - 2f(x, y)$$

- Thus, discrete Laplacian of 2 variables is,

$$\begin{aligned}\Delta^2 f(x, y) &= \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2} \\ &= f(x + 1, y) + f(x - 1, y) + f(x, y + 1) + f(x, y - 1) - 4f(x, y)\end{aligned}\quad (3-53)$$



The Laplacian

- **Isotropic Filter:** They are rotation invariant.
- **Laplacian** is simplest Isotropic derivative operator. It is defined as:

$$\Delta^2 f = \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}$$

- Laplacian is a **linear operator**.



Cont..

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad 90^\circ \textit{isotropic}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad 45^\circ \textit{isotropic}$$



The Laplacian

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

a b c d

FIGURE 3.45 (a) Laplacian kernel used to implement Eq. (3-53). (b) Kernel used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other Laplacian kernels.



The Laplacian

- If background features need to be recovered by still preserving the sharpening effect of the Laplacian, then
- The basic way to use the Laplacian for image sharpening is:
- $g(x, y) = f(x, y) + c[\Delta^2 f(x, y)]$

Where,

- $f(x, y)$ – input image
- $g(x, y)$ – sharpened image
- c – (-1) or (+1) for filters with negative & positive center resp.



Unsharp Masking & Highboost Filtering

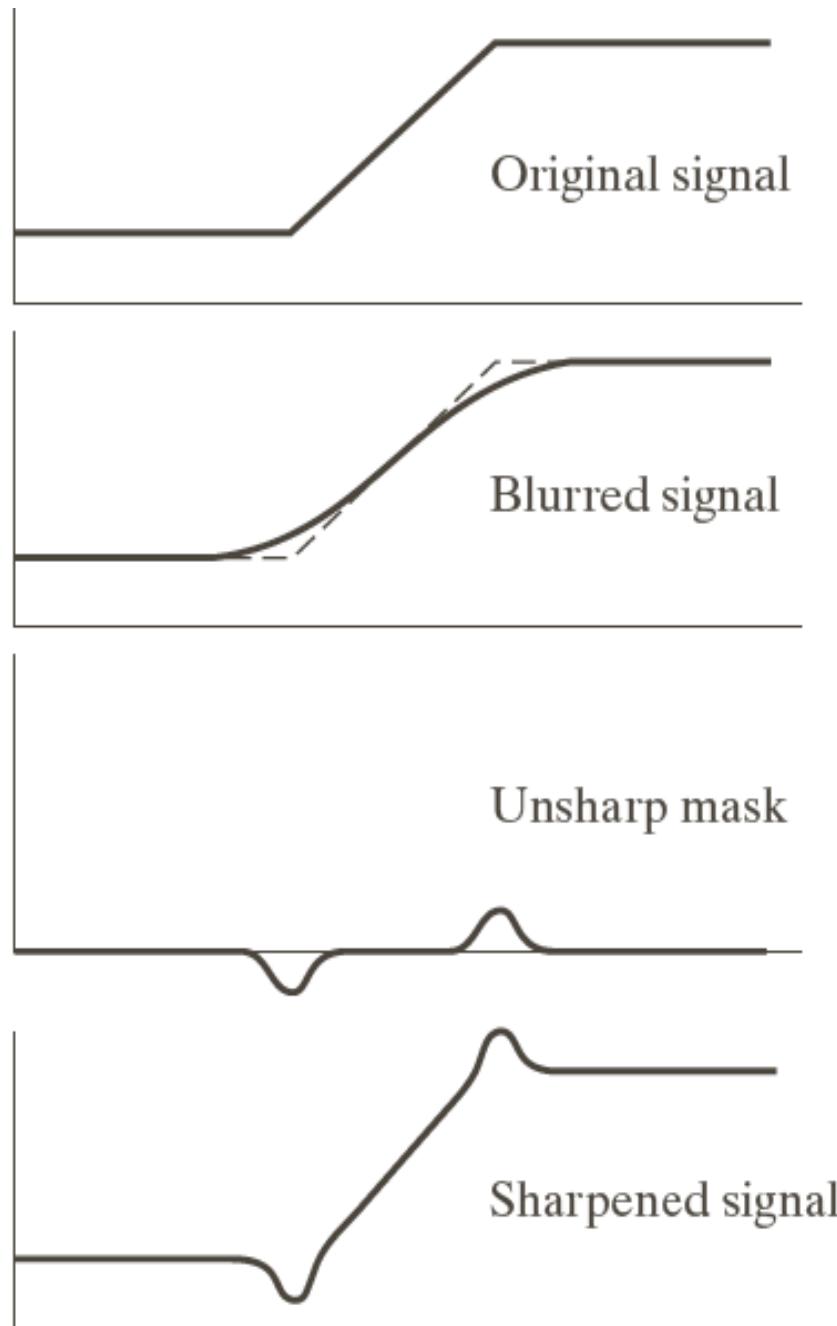
- The process of subtracting an unsharp (smoothed) version of an image from the original image is called unsharp masking.
- It consists of three steps:
 - i. Blur the original image. $\bar{f}(x, y)$
 - ii. Subtract the blurred image from the original (results in mask).
$$g_{mask}(x, y) = f(x, y) - \bar{f}(x, y)$$
 - iii. Add the mask to the original. where if, $k = 1$, unsharp masking, $k > 1$, highboost filtering, $k < 1$, de-emphasizes the contribution of unsharp mask.

$$g(x, y) = f(x, y) + k * g_{mask}(x, y)$$



Cont..

Mechanics of Unsharp masking



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking.
(a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).



Cont..

- Unsharp masking



a
b
c
d
e

FIGURE 3.40
(a) Original image.
(b) Result of blurring with a Gaussian filter.
(c) Unsharp mask. (d) Result of using unsharp masking.
(e) Result of using highboost filtering.

Using First-Order Derivative (Gradient)

- First derivative are implemented using the magnitude of the gradient.
- For image $f(x, y)$, the gradient of f at (x, y) is given by:

$$\nabla f = \text{grad}(f) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\delta f}{\delta x} \\ \frac{\delta f}{\delta y} \end{bmatrix}$$

- This vector points in the direction of the greatest rate of change of f at (x, y) .
- Its magnitude is given by: $M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$
- Sometimes $M(x, y) \approx |g_x| + |g_y|$



Cont..

- More Derivative Filters

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-1	0	0	-1
0	1	1	0

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

a
b c
d e

FIGURE 3.41
A 3×3 region of an image (the z s are intensity values).
(b)–(c) Roberts cross gradient operators.
(d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.



Miscellaneous content Low Pass Median Filtering.

- Averaging Filter removes the noise by blurring till it is no longer seen.
- It blurs the edges too.
- Bigger the averaging mass more the blurring.
- Sometimes the image contains ‘salt & pepper noise’.
- If averaging filter is used then it will remove the noise at the cost of ruined edges.
- Thus a nonlinear filter Median filter is required.
- They are also called as order statistics filter since their response is based on ordering or ranking of pixels contained within the mask.



Cont..

- Here we are going to discuss another method of dealing with images.
- This other method is known as convolution. Usually the black box(system) used for image processing is an LTI system or linear time invariant system.
- By linear we mean that such a system where output is always linear, neither log nor exponent or any other. And by time invariant we means that a system which remains same during time.
- So now we are going to use this third method.
 - It can be mathematically represented as two ways



$$g(x,y) = h(x,y) * f(x,y)$$



Cont..

- It can be explained as the “mask convolved with an image”.
- Or
$$g(x,y) = f(x,y) * h(x,y)$$
- It can be explained as “image convolved with mask”.
- There are two ways to represent this because the convolution operator(*) is commutative. The $h(x,y)$ is the mask or filter.
- **What is mask?**
- Mask is also a signal. It can be represented by a two dimensional matrix. The mask is usually of the order of 1x1, 3x3, 5x5, 7x7 . A mask should always be in odd number, other wise you cannot find the mid of the mask. Why do we need to find the mid of the mask. The answer lies below, in topic of, how to perform convolution?



Cont..

- **How to perform convolution?**
 - In order to perform convolution on an image, following steps should be taken.
 - Flip the mask (horizontally and vertically) only once
 - Slide the mask onto the image.
 - Multiply the corresponding elements and then add them
 - Repeat this procedure until all values of the image has been calculated.
- **Example of convolution**
 - Let's perform some convolution. Step 1 is to flip the mask.
- **Mask**
 - Let's take our mask to be this.

1	2	3
4	5	6
7	8	9



Cont..

- **Mask**

- Flipping the mask horizontally

3	2	1
6	5	4
9	8	7

1	2	3
4	5	6
7	8	9

Original Mask

- Flipping the mask vertically

9	8	7
6	5	4
3	2	1

- **Image**

- Let's consider an image to be like this

2	4	6
8	10	12
14	16	18



Convolution

- Convolving mask over image. It is done in this way. Place the center of the mask at each element of an image. Multiply the corresponding elements and then add them and paste the result onto the element of the image on which you place the center of mask.

9	8	7	
6	2	5	4
3	8	2	10
14			16



Cont..

9	8	7		
6	2	5	4	4
3	8	2	10	1
14			16	18

- The box in red color is the mask, and the values in the orange are the values of the mask. The black color box and values belong to the image. Now for the first pixel of the image, the value will be calculated as
- First pixel = $(5*2) + (4*4) + (2*8) + (1*10)$
 $= 10 + 16 + 16 + 10 = 52$
- Place 52 in the original image at the first index and repeat this procedure for each pixel of the image.
- **Why Convolution**
 - Convolution can achieve something, that the previous two methods of manipulating images can't achieve. Those include the blurring, sharpening, edge detection, noise reduction etc.



Cont..

- **Example 1:** What is the effect of applying Laplacian filter on an image.
- **Mask**
 - Let's take Laplacian mask to be this.
- **Image**
 - Let's consider an image to be like this

1	1	1
1	- 8	1
1	1	1

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

Cont..

- **Mask**

- Let's take Laplacian mask to be this.

1	1	1
1	- 8	1
1	1	1

- **Image**

- Let's consider an image to be like this

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

50
50
50
100
100
100

Padded with Boundary elements

100
100
100

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0					

Case 1: 1st Block :-

$$\begin{aligned}
 & 8 \times 50 - 8 \times 50 \\
 & = 400 - 400 \\
 & = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0					

Case 2: 2nd Block :-

$$\begin{aligned}
 & 8 \times 50 - 8 \times 50 \\
 & = 400 - 400 \\
 & = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0	0			

Case 3: 3rd Block :-

$$8 \times 50 - 8 \times 50 = 0$$



Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0	0	0		

Case 4: 4th Block :-

$$8 \times 50 - 8 \times 50 = 0$$



Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0	0	0	0	

Case 5: 5th Block :-

$$8 \times 50 - 8 \times 50 = 0$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0	0	0	0	0

Case 6: 6th Block :-

$$8 \times 50 - 8 \times 50 = 0$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0	0	0	0	0
0					

Case 7: 7th Block :-

$$8 \times 50 - 8 \times 50 = 0$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
150					

Similarly we filled for block 8th to 12th

Case 13: 13th Block :-

$$\begin{aligned}
 & 5 \times 50 - 8 \times 50 + 3 \times 100 \\
 & = 250 - 400 + 300 = 550 - 400 \\
 & = 150
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
150	150				

Case 14: 14th Block :-

$$\begin{aligned}
 & 5 \times 50 - 8 \times 50 + 3 \times 100 \\
 & = 250 - 400 + 300 = 550 - 400 \\
 & = 150
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
150	150	150	150	150	150
-150					

Similarly we filled for block 13th to 18th

Case 19: 19th Block :-

$$\begin{aligned}
 & 3 \times 50 - 8 \times 100 + 5 \times 100 \\
 & = 150 - 800 + 500 = 650 - 800 \\
 & = -150
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

1	1	1
1	-8	1
1	1	1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
150	150	150	150	150	150
-150	-150				

Case 20: 20th Block :-

$$\begin{aligned}
 & 3 \times 50 - 8 \times 100 + 5 \times 100 = 650 - 400 \\
 & = 150 - 800 + 500 = 650 - 800 \\
 & = -150
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	- 8	1
1	1	1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
150	150	150	150	150	150
-150	-150	-150	-150	-150	-150
0					

Similarly we filled for block 20th to 24th

Case 25: 25th Block :-

$$8 \times 100 - 8 \times 100$$

$$= 800 - 800$$

$$= 0$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	-8	1
1	1	1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
150	150	150	150	150	150
-150	-150	-150	-150	-150	-150
0	0	0	0	0	0
0					

Similarly we filled for block 25th to 30th

Case 31: 31st Block :-

$$\begin{aligned}
 8 \times 100 - 8 \times 100 \\
 = 800 - 800 \\
 = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	-8	1
1	1	1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
150	150	150	150	150	150
-150	-150	-150	-150	-150	-150
0	0	0	0	0	0
0	0	0	0	0	0

Similarly we filled for block 32th to 36th

Case 36: 36th Block :-

$$8 \times 100 - 8 \times 100$$

$$= 800 - 800$$

$$= 0$$



Cont..

- **Example 2:** What is the effect of applying Laplacian filter on an image.
- **Mask**
 - Let's take Laplacian mask to be this.
- **Image**
 - Let's consider an image to be like this

- 1	- 1	- 1
- 1	8	- 1
-1	-1	- 1

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

Cont..

- **Mask**

- Let's take Laplacian mask to be this.

- 1	- 1	- 1
- 1	8	- 1
- 1	- 1	- 1

- **Image**

- Let's consider an image to be like this

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

50
50
50
100
100
100

Padded with Boundary elements

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

- 1	- 1	- 1
- 1	8	- 1
- 1	- 1	- 1

Mask

0					

Case 1: 1st Block :-

$$\begin{aligned}
 & - 8 \times 50 + 8 \times 50 \\
 & = - 400 + 400 \\
 & = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

- 1	- 1	- 1
- 1	8	- 1
- 1	- 1	- 1

Mask

0	0					

Case 2: 2nd Block :-

$$\begin{aligned}
 & - 8 \times 50 + 8 \times 50 \\
 & = - 400 + 400 \\
 & = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

- 1	- 1	- 1
- 1	8	- 1
- 1	- 1	- 1

Mask

0	0	0	0	0	0
0					

Case 7: 7th Block :-

$$- 8 \times 50 - 8 \times 50 = 0$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

Similarly we filled for block 8th to 12th

Case 13: 13th Block :-

$$\begin{aligned}
 & - 5 \times 50 + 8 \times 50 - 3 \times 100 \\
 & = - 250 + 400 - 300 \\
 & = - 550 + 400 \\
 & = - 150
 \end{aligned}$$

- 1	- 1	- 1
- 1	8	- 1
- 1	- 1	- 1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
- 150					



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

- 1	- 1	- 1
- 1	8	- 1
- 1	- 1	- 1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
- 150	- 150				

Case 14: 14th Block :-

$$\begin{aligned}
 & - 5 \times 50 + 8 \times 50 - 3 \times 100 \\
 & = - 250 + 400 - 300 \\
 & = - 550 + 400 \\
 & = - 150
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

- 1	- 1	- 1
- 1	8	- 1
- 1	- 1	- 1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
- 150	- 150	- 150	- 150	- 150	- 150
150					

Similarly we filled for block 13th to 18th

Case 19: 19th Block :-

$$\begin{aligned}
 & - 3 \times 50 + 8 \times 100 - 5 \times 100 \\
 & = - 150 + 800 - 500 = - 650 + 800 \\
 & = 150
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

- 1	- 1	- 1
- 1	8	- 1
- 1	- 1	- 1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
- 150	- 150	- 150	- 150	- 150	- 150
150	150				

Case 20: 20th Block :-

$$\begin{aligned}
 & - 3 \times 50 + 8 \times 100 - 5 \times 100 \\
 & = - 150 + 800 - 500 = - 650 + 800 \\
 & = 150
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

- 1	- 1	- 1
- 1	- 8	- 1
- 1	- 1	- 1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
- 150	- 150	- 150	- 150	- 150	- 150
150	150	150	150	150	150
0					

Similarly we filled for block 20th to 24th

Case 25: 25th Block :-

$$\begin{aligned}
 & - 8 \times 100 + 8 \times 100 \\
 & = - 800 + 800 \\
 & = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

-1	-1	-1
-1	-8	-1
-1	-1	-1

Mask

50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

0	0	0	0	0	0
0	0	0	0	0	0
150	150	150	150	150	150
-150	-150	-150	-150	-150	-150
0	0	0	0	0	0
0					

Similarly we filled for block 25th to 30th

Case 31: 31st Block :-

$$\begin{aligned}
 & - 8 \times 100 + 8 \times 100 \\
 & = - 800 + 800 \\
 & = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

1	1	1
1	-8	1
1	1	1

Mask

0	0	0	0	0	0
0	0	0	0	0	0
150	150	150	150	150	150
-150	-150	-150	-150	-150	-150
0	0	0	0	0	0
0	0	0	0	0	0

Similarly we filled for block 32th to 36th

Case 36: 36th Block :-

$$\begin{aligned}
 & - 8 \times 100 + 8 \times 100 \\
 & = - 800 + 800 \\
 & = 0
 \end{aligned}$$



Cont..

- **Example 3:** What is the effect of applying Laplacian filter on an image.
- **Mask**
 - Let's take Laplacian mask to be this.
- **Image**
 - Let's consider an image to be like this

0	1	0
1	- 4	1
0	1	0

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

Cont..

- **Mask**

- Let's take Laplacian mask to be this.

0	1	0
1	- 4	1
0	1	0

- **Image**

- Let's consider an image to be like this

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

50
50
50
50
100
100
100

Padded with Boundary elements

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

0	1	0
1	-4	1
0	1	0

Mask

0					

Case 1: 1st Block :-

$$\begin{aligned}
 4 \times 50 - 4 \times 50 \\
 = 200 - 200 \\
 = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

0	1	0
1	- 4	1
0	1	0

Mask

0	0					

Case 2: 2nd Block :-

$$\begin{aligned}
 & 4 \times 50 - 4 \times 50 \\
 & = 200 - 200 \\
 & = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

0	1	0
1	- 4	1
0	1	0

Mask

0	0	0	0	0	0
0					

Case 7: 7th Block :-

$$4 \times 50 - 4 \times 50 = 0$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

Similarly we filled for block 8th to 12th

Case 13: 13th Block :-

$$\begin{aligned}
 & 3 \times 50 - 4 \times 50 + 1 \times 100 \\
 & = 150 - 200 + 100 \\
 & = 250 - 200 \\
 & = 50
 \end{aligned}$$

0	1	0
1	- 4	1
0	1	0

Mask

0	0	0	0	0	0
0	0	0	0	0	0
50					



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

0	1	0
1	- 4	1
0	1	0

Mask

0	0	0	0	0	0
0	0	0	0	0	0
50	50				

Case 14: 14th Block :-

$$\begin{aligned}
 & 3 \times 50 - 4 \times 50 + 1 \times 100 \\
 & = 150 - 200 + 100 \\
 & = 250 - 200 \\
 & = 50
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

0	1	0
1	- 4	1
0	1	0

Mask

0	0	0	0	0	0
0	0	0	0	0	0
50	50	50	50	50	50
- 50					

Similarly we filled for block 13th to 18th

Case 19: 19th Block :-

$$\begin{aligned}
 & 1 \times 50 - 4 \times 100 + 3 \times 100 \\
 & = 50 - 400 + 300 = 350 - 400 \\
 & = - 50
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

0	1	0
1	- 4	1
0	1	0

Mask

0	0	0	0	0	0
0	0	0	0	0	0
- 150	- 150	- 150	- 150	- 150	- 150
150	150				

Case 20: 20th Block :-

$$\begin{aligned}
 & 1 \times 50 - 4 \times 100 + 3 \times 100 \\
 & = 50 - 400 + 300 = 350 - 400 \\
 & = - 50
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

0	1	0
1	- 4	1
0	1	0

Mask

0	0	0	0	0	0
0	0	0	0	0	0
- 150	- 150	- 150	- 150	- 150	- 150
150	150	150	150	150	150
0					

Similarly we filled for block 20th to 24th

Case 25: 25th Block :-

$$4 \times 100 - 4 \times 100$$

$$= 400 - 400$$

$$= 0$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

0	1	0
1	-4	1
0	1	0

Mask

50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

0	0	0	0	0	0
0	0	0	0	0	0
150	150	150	150	150	150
-150	-150	-150	-150	-150	-150
0	0	0	0	0	0
0					

Similarly we filled for block 25th to 30th

Case 31: 31st Block :-

$$\begin{aligned}
 4 \times 100 - 4 \times 100 \\
 = 400 - 400 \\
 = 0
 \end{aligned}$$



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

0	1	0
1	-4	1
0	1	0

Mask

0	0	0	0	0	0
0	0	0	0	0	0
150	150	150	150	150	150
-150	-150	-150	-150	-150	-150
0	0	0	0	0	0
0	0	0	0	0	0

Similarly we filled for block 32th to 36th

Case 36: 36th Block :-

$$4 \times 100 - 4 \times 100$$

$$= 400 - 400$$

$$= 0$$



Cont..

- **Example 4:** What is the effect of applying Laplacian filter on an image.
- **Mask**
 - Let's take Laplacian mask to be this.
- **Image**
 - Let's consider an image to be like this

0	-1	0
-1	4	-1
0	-1	0

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

- i. Padded with Boundary elements
- ii. Padded with zero



Cont..

- **Mask**

- Let's take Laplacian mask to be this.

0	-1	0
-1	4	-1
0	-1	0

- **Image**

- Let's consider an image to be like this

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

50
50
50
100
100
100

Padded with Boundary elements

Cont..

- **Mask**

- Let's take Laplacian mask to be this.

0	-1	0
-1	4	-1
0	-1	0

- **Image**

- Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	50	50	50	50	50	50	0
0	50	50	50	50	50	50	0
0	50	50	50	50	50	50	0
0	100	100	100	100	100	100	0
0	100	100	100	100	100	100	0
0	100	100	100	100	100	100	0
0	0	0	0	0	0	0	0

Padded with zero

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

100					

Case 1: 1st Block :-

$$\begin{aligned}
 4 \times 50 - 2 \times 50 \\
 = 200 - 100 \\
 = 100
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

100	50				

Case 2: 2nd Block :-

$$\begin{aligned}
 4 \times 50 - 3 \times 50 \\
 = 200 - 150 \\
 = 50
 \end{aligned}$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

0
0
0
0
0

100	50	50			

Case 3: 3rd Block :-

$$\begin{aligned}
 4 \times 50 - 3 \times 50 \\
 = 200 - 150 \\
 = 50
 \end{aligned}$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

0
0
0
0
0

100	50	50	50		

Case 4: 4th Block :-

$$\begin{aligned}
 4 \times 50 - 3 \times 50 \\
 = 200 - 150 \\
 = 50
 \end{aligned}$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0
0
0
0
0
0

0	-1	0
-1	4	-1
0	-1	0

Mask

100	50	50	50	50	

Case 5: 5th Block :-

$$\begin{aligned}
 4 \times 50 - 3 \times 50 \\
 = 200 - 150 \\
 = 50
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

100	50	50	50	50	100

Case 6: 6th Block :-

$$\begin{aligned}
 4 \times 50 - 2 \times 50 \\
 = 200 - 100 \\
 = 100
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

0
0
0
0
0

100	50	50	50	50	100
50					

Case 7: 7th Block :-

$$\begin{aligned}
 4 \times 50 - 3 \times 50 \\
 = 200 - 150 \\
 = 50
 \end{aligned}$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	-1	0	0	0	0
-1	4	-1	0	0	0
0	-1	0	0	0	0
0	0	0	0	0	0

Mask

100	50	50	50	50	100
50	0				

Case 8: 8th Block :-

$$\begin{aligned}
 4 \times 50 - 4 \times 50 \\
 = 200 - 200 \\
 = 0
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0	0
0	50	50	50	50	50	50	0
0	50	50	50	50	50	50	0
0	50	50	50	50	50	50	0
0	100	100	100	100	100	100	0
0	100	100	100	100	100	100	0
0	100	100	100	100	100	100	0
0	0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50

Similarly 9th to 11th block values are same

Case 12: 12th Block :-

$$\begin{aligned}
 4 \times 50 - 3 \times 50 \\
 = 200 - 150 \\
 = 50
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

0
0
0
0
0

100	50	50	50	50	100
50	0	0	0	0	50
0					

Case 13: 13th Block :-

$$\begin{aligned}
 & 4 \times 50 - 2 \times 50 - 1 \times 100 \\
 & = 200 - 200 \\
 & = 0
 \end{aligned}$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0
0	0	0
0	0	0
0	0	0
0	0	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50
0	-50				

Case 14: 14th Block :-

$$4 \times 50 - 3 \times 50 - 1 \times 100$$

$$= 200 - 250$$

$$= 0$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0
0	0	0
0	0	0
0	0	0
0	0	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50			

Case 15: 15th Block :-

$$4 \times 50 - 3 \times 50 - 1 \times 100$$

$$= 200 - 250$$

$$= 0$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

0	100	50	50	50	50	100
0	50	0	0	0	0	50
0	0	-50	-50	-50	-50	
0						
0						

Case 16: 16th Block :-

$$\begin{aligned}
 & 4 \times 50 - 3 \times 50 - 1 \times 100 \\
 & = 200 - 250 \\
 & = 0
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0

Case 18: 18th Block :-

$$4 \times 50 - 2 \times 50 - 1 \times 100$$

$$= 200 - 200$$

$$= 0$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

0
0
0
0

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0
150					

Case 19: 19th Block :-

$$\begin{aligned}
 & 4 \times 100 - 1 \times 50 - 2 \times 100 \\
 & = 400 - 250 \\
 & = 150
 \end{aligned}$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	-1	0	0	0	0
-1	4	-1	0	0	0
0	-1	0	0	0	0
0	0	0	0	0	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0
150	50				

Case 20: 20th Block :-

$$\begin{aligned}
 4 \times 100 - 1 \times 50 - 3 \times 100 \\
 = 400 - 350 \\
 = 50
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0
150	50	50	50	50	150

Case 24: 24th Block :-

$$\begin{aligned}
 & 4 \times 100 - 1 \times 50 - 2 \times 100 \\
 & = 400 - 250 \\
 & = 150
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0
0	0	0
0	0	0
0	0	0
0	0	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0
150	50	50	50	50	150
100					

Case 25: 25th Block :-

$$\begin{aligned}
 4 \times 100 - 3 \times 100 \\
 = 400 - 300 \\
 = 100
 \end{aligned}$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

0
0
0
0
0

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0
150	50	50	50	50	150
100	0				

Case 26: 26th Block :-

$$\begin{aligned}
 & 4 \times 100 - 4 \times 100 \\
 & = 400 - 400 \\
 & = 0
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0
150	50	50	50	50	150
100	0	0	0	0	100

Similarly block 27 to 29 will have zero

Case 30: 30th Block :-

$$4 \times 100 - 3 \times 100$$

$$= 400 - 300$$

$$= 100$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0
0	0	0
0	0	0
0	0	0
0	0	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0
150	50	50	50	50	150
100	0	0	0	0	100
200					

Case 31: 31st Block :-

$$\begin{aligned}
 4 \times 100 - 2 \times 100 \\
 = 400 - 200 \\
 = 200
 \end{aligned}$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

0
0
0
0
0
0

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0
150	50	50	50	50	150
100	0	0	0	0	100
200	100				

Case 32: 32th Block :-

$$\begin{aligned}
 4 \times 100 - 3 \times 100 \\
 = 400 - 300 \\
 = 100
 \end{aligned}$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Image

0	-1	0
-1	4	-1
0	-1	0

Mask

100	50	50	50	50	100
50	0	0	0	0	50
0	-50	-50	-50	-50	0
150	50	50	50	50	150
100	0	0	0	0	100
200	100	100	100	100	200

Case 36: 36th Block :-

$$\begin{aligned}
 4 \times 100 - 2 \times 100 \\
 = 400 - 200 \\
 = 200
 \end{aligned}$$



Cont..

- **Example 5:** What is the effect of applying 3x3 Median filter on an image.
- **Image**
 - Let's consider an image to be like this

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

Cont..

- **Image**
 - Let's consider an image to be like this

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100



Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

100 100 100 100 100 100 100

Image

Case 1: 1st Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

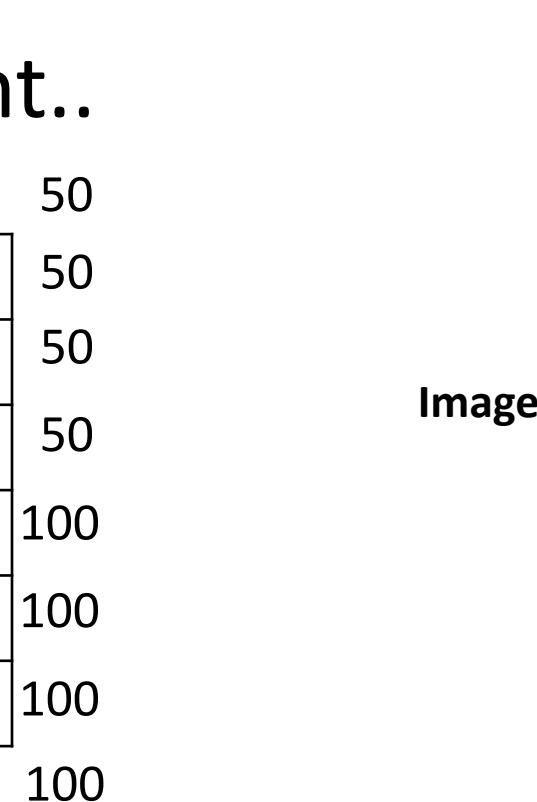
100 100 100 100 100 100 100

Image

50						

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100



Case 2: 2nd Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

50						

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

50	50					

Case 2: 2nd Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

50	50	50			

Case 3: 3rd Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

50
50
50
50
100
100
100
100
100

Image

50	50	50	50		

Case 4: 4th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

50
50
50
50
100
100
100
100

Image

50	50	50	50	50	50

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

50	50	50	50	50	50	50

Image

Case 7: 7th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

50	50	50	50	50	50

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

50
50
50
50
100
100
100
100

Image

50	50	50	50	50	50
50					

Case 7: 7th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

50
50
50
50
100
100
100
100

Image

50	50	50	50	50	50
50					

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

50	50	50	50	50	50
50	50				

Case 8: 8th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

50	50	50	50	50	50
50	50				

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

50
50
50
50
100
100
100
100

Image

50	50	50	50	50	50
50	50				

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

50	50	50	50	50	50
50	50	50			

Case 9: 9th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

50	50	50	50	50	50
50	50	50			

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

100 100 100 100 100 100 100

Image

50	50	50	50	50	50
50	50	50			

Case 10-12: 10th - 12th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

50	50	50	50	50	50
50	50	50	50	50	50

Case 10-12: 10th - 12th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

50	50	50	50	50	50
50	50	50	50	50	50

Case 13: 13th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50						

Image

Case 13: 13th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

50	50	50	50	50	50
50	50	50	50	50	50
50					

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

100 100 100 100 100 100 100

Image

50	50	50	50	50	50
50	50	50	50	50	50
50	50				

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50

Case 15-18: 15th - 18th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

100 100 100 100 100 100 100 100

Image

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50

Case 19: 19th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100						

Image

Case 19: 19th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100					

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

100 100 100 100 100 100 100 100

Image

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100					

Case 25: 25th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

Image

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100					

Case 31: 31st Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	50	50	50
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

Image

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

Case 36: 36th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

0	0	0	0	0	0	0	0
0	50	50	50	50	50	50	50
0	50	50	50	50	50	50	50
0	50	50	50	50	50	50	50
0	100	100	100	100	100	100	100
0	100	100	100	100	100	100	100
0	100	100	100	100	100	100	100
0	0	0	0	0	0	0	0

Padded with
Zero elements

Image

Case 1: 1st Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

0	0	0	0	0	0	0	0
0	0	50	50	50	50	50	50
50	50	50	50	50	50	50	50
0	50	50	50	50	50	50	50
0	100	100	100	100	100	100	100
0	100	100	100	100	100	100	100
0	100	100	100	100	100	100	100
0	0	0	0	0	0	0	0

Padded with
Zero elements

Image

0					

Case 1: 1st Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0					

Case 2: 2nd Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50				

Case 2: 2nd Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50			

Case 3: 3rd Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50		

Case 4: 4th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

0
0
0
0
0
0
0

Padded with
Zero elements

Image

0	50	50	50	50	

Case 5: 5th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0
0
0
0

0	50	50	50	50	

Case 6: 6th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0
0	50	50	50	0	0	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0
0
0
0

0	50	50	50	50	0

Case 6: 6th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0	0
0	50	50	50	50	50	50	50
0	50	50	50	50	50	50	50
0	50	50	50	50	50	50	50
0	100	100	100	100	100	100	100
0	100	100	100	100	100	100	100
0	100	100	100	100	100	100	100
0	0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50	50	0

Case 7: 7th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0	0
0	0	0	50	50	50	0	0
50	50	50	50	50	50	50	0
50	50	50	50	50	50	50	0
0	100	100	100	100	100	100	0
0	100	100	100	100	100	100	0
0	100	100	100	100	100	100	0
0	0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50	50	0
50					

Case 7: 7th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50	50	0
50					

Case 8: 8th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50	50	0
50	50				

Case 8: 8th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50	50	0
50	50	50	50	50	

Case 12: 12th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50	50	0
50	50	50	50	50	50

Case 12: 12th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50	50	0
50	50	50	50	50	50
50					

Case 13: 13th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50	50	0
50	50	50	50	50	50
50	50	50	50	50	50
50					

Case 19: 19th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Padded with
Zero elements

Image

0	50	50	50	50	0
50	50	50	50	50	50
50	50	50	50	50	50
50	100				

Case 20: 20th Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Cont..

- **Example 6:** What is the effect of applying Sobel filter on an image.
- **Mask**
 - Let's take Laplacian mask to be this.
- **Image**
 - Let's consider an image to be like this

- 1	- 2	- 1
0	0	0
1	2	1

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

Cont..

- **Example 7:** What is the effect of applying Sobel filter on an image.
- **Mask**
 - Let's take Laplacian mask to be this.
- **Image**
 - Let's consider an image to be like this

- 1	0	1
- 2	0	2
- 1	0	1

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

Cont..

- **Example 8:** What is the effect of applying Robert Cross Gradient filter on an image.

- **Mask**

- Let's take Laplacian mask to be this.

- 1	0
0	1

- **Image**

- Let's consider an image to be like this

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

Cont..

- **Mask**

- Let's take Laplacian mask to be this.

-1	0
0	1

- **Image**

- Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	50	50	50	50	50	50	0
0	50	50	50	50	50	50	0
0	50	50	50	50	50	50	0
0	100	100	100	100	100	100	0
0	100	100	100	100	100	100	0
0	100	100	100	100	100	100	0
0	0	0	0	0	0	0	0

Padded with zero

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
0	0	1
0		
0		
0		
0		
0		
0		

Mask

50					

Case 1: 1st Block :-

$$1 \times 50 = 50$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

-1	0
0	1

Mask

50	50					

Case 2: 2nd Block :-

$$1 \times 50 = 50$$

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0				
0	0	1				
0	0	0				
0	0	0				
0	0	0				
0	50	50	50			

Mask

Case 2: 2nd Block :-

$$1 \times 50 = 50$$

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Image

-1	0
0	1

Mask

50	50	50	50	50	0

Case 6: 6th Block :-

0

Cont..

0	0	0	0	0	0	0	0
0	50	50	50	50	50	50	50
0	50	50	50	50	50	50	50
0	50	50	50	50	50	50	50
0	100	100	100	100	100	100	100
0	100	100	100	100	100	100	100
0	100	100	100	100	100	100	100
0	0	0	0	0	0	0	0

Image

-1	0
0	1

Mask

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
50	50	50	50	50	50	0
50						

Case 7: 7th Block :-

$$1 \times 50 = 50$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
0	0	1
0	0	0
0	0	0
0	0	0
0	50	50
0	50	0
0	0	0
0	0	0
0	0	0

Mask

Case 8: 8th Block :-

$$-1 \times 50 + 1 \times 50 = 0$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Image

-1	0
0	1

Mask

50	50	50	50	50	0
50	0	0	0	0	-50

Case 12: 12th Block :-

$$-1 \times 50 = -50$$

Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
0	0	1
0	0	0
0	0	0
0	0	0
0	50	50
50	0	0
50		

Mask

Case 13: 13th Block :-

$$1 \times 50 = 50$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
0	0	1
0		
0		
0		
0		
0		
0		

Mask

50	50	50	50	50	0
50	0	0	0	0	-50
50	0	0	0	0	-50

Case 14: 14th Block :-

$$1 \times 50 - 1 \times 50 = 0$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
0	0	1

Mask

0	50	50	50	50	50	0
0	50	0	0	0	0	-50
0	50	0	0	0	0	-50
0	100					
0						
0						

Case 19: 19th Block :-
1x100



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
0	0	1

Mask

0	50	50	50	50	50	0
0	50	0	0	0	0	-50
0	50	0	0	0	0	-50
0	100	50				
0						
0						

Case 20: 20th Block :-

$$-1 \times 50 + 1 \times 100 = 50$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Image

-1	0
0	1

Mask

50	50	50	50	50	0
50	0	0	0	0	-50
50	0	0	0	0	-50
100	50	50	50	50	-50

Case 24: 24th Block :-
 $-1 \times 50 = -50$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
0	0	1

Mask

0	50	50	50	50	50	0
0	50	0	0	0	0	-50
0	50	0	0	0	0	-50
100	50	50	50	50	50	-50
100						

Case 25: 25th Block :-

$$1 \times 100 = -50$$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
0	0	1
0		
0		
0		
0		
0		

Mask

50	50	50	50	50	0
50	0	0	0	0	-50
50	0	0	0	0	-50
100	50	50	50	50	-50
100	0	0	0	0	-100

Case 26: 26th Block :-

$$1 \times 100 - 1 \times 100 = 0$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	50	50	50	50	50	50
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	100	100	100	100	100	100
0	0	0	0	0	0	0

Image

0	-1	0
0	0	1

Mask

0	50	50	50	50	50	0
0	50	0	0	0	0	-50
0	50	0	0	0	0	-50
0	100	50	50	50	50	-50
0	100	0	0	0	0	-100
0	0					

Case 31: 31th Block :-
 $= 0$



Cont..

0	0	0	0	0	0
0	50	50	50	50	50
0	50	50	50	50	50
0	50	50	50	50	50
0	100	100	100	100	100
0	100	100	100	100	100
0	100	100	100	100	100
0	0	0	0	0	0

Image

0	-1	0
0	0	1
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
50	50	50
50	0	0
50	0	0
100	50	50
100	0	0
0	-100	-100

Mask

Case 32: 32th Block :-

$$-1 \times 100 = -100$$



Cont..

0	0	0	0	0	0	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	50	50	50	50	50	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	100	100	100	100	100	0
0	0	0	0	0	0	0

Image

-1	0
0	1

Mask

50	50	50	50	50	0
50	0	0	0	0	-50
50	0	0	0	0	-50
100	50	50	50	50	-50
100	0	0	0	0	-100
0	-100	-100	-100	-100	-100

Case 36: 36th Block :-

$$-1 \times 100 = -100$$

Cont..

- **Example 9:** What is the effect of applying Robert Cross Gradient filter on an image.

- **Mask**

- Let's take Laplacian mask to be this.

0	-1
1	0

- **Image**

- Let's consider an image to be like this

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

Cont..

- **Example 10:** What is the effect of applying 3x3 Mean Filter filter on an image.
- **Image**
 - Let's consider an image to be like this

50	25	30	40
15	50	25	20
35	15	50	20
10	30	20	40



Cont..

- **Example 11:** What is the effect of applying 3x3 Median filter on an image.
- **Image**
 - Let's consider an image to be like this

5	5	6	6	6	7
5	5	2	2	2	2
5	1	2	1	2	1
1	1	1	1	2	2

Cont..

- **Example 11:** What is the effect of applying 3x3 Median filter on an image.

- **Image**

- Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0					



Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0
0	5	5	6	6	6	7
0	5	5	2	2	2	2
0	5	1	2	1	2	1
0	1	1	1	1	2	2
0	0	0	0	0	0	0

0	5				

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0
0	5	5	6	6	6	7
0	5	5	2	2	2	2
0	5	1	2	1	2	1
0	1	1	1	1	2	2
0	0	0	0	0	0	0

0	5	2			

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2		

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	0

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	0
5					

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	0
5	5				

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	0
5	5	2			

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	0
5	5	2	2		

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	0
5	5	2	2	2	

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	0
5	5	2	2	2	2

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1					

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2				

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1			

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1	2		

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1	2	2	

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1	2	2	2

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1	2	2	2
0					

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1	2	2	2
0	1				

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1	2	2	2
0	1	1			

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1	2	2	2
0	1	1	1		

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1	2	2	2
0	1	1	1	1	

Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0	0	0
0	5	5	6	6	6	7	0
0	5	5	2	2	2	2	0
0	5	1	2	1	2	1	0
0	1	1	1	1	2	2	0
0	0	0	0	0	0	0	0

0	5	2	2	2	2
5	5	2	2	2	2
1	2	1	2	2	2
0	1	1	1	1	0

Cont..

- **Example 11:** What is the effect of applying 3x3 Median filter on an image.
- **Image**
 - Let's consider an image to be like this

50	25	30	40
15	50	25	20
35	15	50	20
10	30	20	40



Cont..

- **Image**
 - Let's consider an image to be like this

0	0	0	0	0	0
0	50	25	30	40	0
0	15	50	25	20	0
0	35	15	50	20	0
0	10	30	20	40	0
0	0	0	0	0	0

Padded with
Boundary
elements

Cont..

0	0	0	0	0	0
0	50	25	30	40	0
0	15	50	25	20	0
0	35	15	50	20	0
0	10	30	20	40	0
0	0	0	0	0	0

Image

Case 1: 1st Block :-

- first sort all the pixel values from the surrounding neighborhood into numerical order
- Replacing the pixel being considered with the middle pixel value.

Generating Spatial Filter Masks

1. Average Mean Filter

- The average value at any location (x, y) in the image is the sum of the nine intensity values in the 3×3 neighborhood centered on (x, y) divided by 9.
- If $z_i, i = 1, 2, \dots, 9$ denote these intensities, then the average is:
- $R = \frac{1}{9} \sum_{i=1}^9 z_i$

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$
$$\frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.



Cont..

- General implementation for filtering an $M \times N$ image with a weighted average filter of size $m \times n$ is given by:

$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}$$



Cont..

- **Example 12** 8x8 Pseudo image with a single edge (High Frequency) of 10 & 50. Remove using a 3x3 size averaging mask.

10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50

- 8x8 Image



Cont..

10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50

$$\begin{array}{r} 1 \\ \hline 9 \end{array} \quad \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$



Cont..

0	0	0						
0	10	10	10	10	10	10	10	10
0	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50

$$\begin{array}{r} 1 \\ \hline 9 \end{array} \quad \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$



Cont..

0	0	0						
0	4.44	10	10	10	10	10	10	10
0	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50

$$\begin{array}{r} 1 \\ \hline 9 \end{array} \quad \begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 10 & 10 \\ \hline 0 & 10 & 10 \\ \hline \end{array}$$



Cont..

0	0	0	0	10	10	10	10	10
0	4.44	6.66	10	10	10	10	10	10
0	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50

1	0	0	0
-----	10	10	10
9	10	10	10



Cont..

0	0	0	0	0	10	10	10	10
0	4.44	6.66	6.66	10	10	10	10	10
0	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50

1	0	0	0
-----	10	10	10
9	10	10	10



Cont..

0	0	0	0	0	0			
0	4.44	6.66	6.66	6.66	10	10	10	10
0	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50

1	0	0	0
-----	10	10	10
9	10	10	10



Cont..

0	0	0	0	0	0	0	0	0	0
0	4.44	6.66	6.66	6.66	6.66	6.66	6.66	4.44	0
0	6.66	10	10	10	10	10	10	10	0
0	10	10	10	10	10	10	10	10	0
10	10	10	10	10	10	10	10	10	0
50	50	50	50	50	50	50	50	50	0
50	50	50	50	50	50	50	50	50	0
50	50	50	50	50	50	50	50	50	0
50	50	50	50	50	50	50	50	50	0

1	10	10	10
-----	10	10	10
9	10	10	10



Cont..

0	0	0	0	0	0	0	0	0	0
0	4.44	6.66	6.66	6.66	6.66	6.66	6.66	4.44	0
0	6.66	10	10	10	10	10	10	10	0
0	10	10	10	10	10	10	10	10	0
10	10	10	10	10	10	10	10	10	0
50	50	50	50	50	50	50	50	50	0
50	50	50	50	50	50	50	50	50	0
50	50	50	50	50	50	50	50	50	0
50	50	50	50	50	50	50	50	50	0

1	10	10	10
-----	10	10	10
9	10	10	10



Cont..

0	0	0	0	0	0	0	0	0	0	0
0	4.44	6.66	6.66	6.66	6.66	6.66	6.66	4.44	0	0
0	6.66	10	10	10	10	10	10	6.66	0	0
0	6.66	10	10	10	10	10	10	6.66	0	0
0	15.55	10	10	10	10	10	10	10	0	0
0	50	50	50	50	50	50	50	50	0	0
	50	50	50	50	50	50	50	50		
	50	50	50	50	50	50	50	50		
	50	50	50	50	50	50	50	50		

1	0	10	10
-----	0	10	10
9	0	50	50



Cont..

0	0	0	0	0	0	0	0	0	0	0
0	4.44	6.66	6.66	6.66	6.66	6.66	6.66	6.66	4.44	0
0	6.66	10	10	10	10	10	10	10	6.66	0
0	6.66	10	10	10	10	10	10	10	6.66	0
0	15.55	23.33	10	10	10	10	10	10	10	0
0	50	50	50	50	50	50	50	50	50	0
	50	50	50	50	50	50	50	50	50	
	50	50	50	50	50	50	50	50	50	
	50	50	50	50	50	50	50	50	50	

$$\begin{array}{r} 1 \\ \hline 9 \end{array} \quad \begin{array}{|c|c|c|} \hline & 10 & 10 & 10 \\ \hline & 10 & 10 & 10 \\ \hline & 50 & 50 & 50 \\ \hline \end{array}$$



Cont..

0	0	0	0	0	0	0	0	0	0	0
0	4.44	6.66	6.66	6.66	6.66	6.66	6.66	6.66	4.44	0
0	6.66	10	10	10	10	10	10	10	6.66	0
0	6.66	10	10	10	10	10	10	10	6.66	0
0	15.55	23.33	23.33	23.33	23.33	23.33	23.33	23.33	15.55	0
0	24.44	36.66	36.66	36.66	36.66	36.66	36.66	36.66	24.44	0
0	33.33	50	50	50	50	50	50	50	50	0
	50	50	50	50	50	50	50	50	50	
	50	50	50	50	50	50	50	50	50	

1	50	50	50
-----	50	50	50
9	50	50	50



Cont..

0	0	0	0	0	0	0	0	0	0
0	4.44	6.66	6.66	6.66	6.66	6.66	6.66	4.44	0
0	6.66	10	10	10	10	10	10	6.66	0
0	6.66	10	10	10	10	10	10	6.66	0
0	15.55	23.33	23.33	23.33	23.33	23.33	23.33	15.55	0
0	24.44	36.66	36.66	36.66	36.66	36.66	36.66	24.44	0
0	33.33	50	50	50	50	50	50	33.33	0
0	33.33	50	50	50	50	50	50	33.33	0
0	22.22	33.33	33.33	33.33	33.33	33.33	33.33	22.22	0
0	0	0	0	0	0	0	0	0	0
	1	50	50	0					
	-----	50	50	0					
	9	0	0	0					



Cont..

10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	23.33	23.33	23.33	23.33	23.33	23.33	10
50	36.66	36.66	36.66	36.66	36.66	36.66	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50

$$\begin{array}{r} 1 \quad \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \\ \hline \hline \end{array}$$



Cont..

- In the resultant image the Low frequency region has remained unchanged.
- Sharp transition between 10 & 50 has changed from 10 to 23.33 to 36.66 and finally to 50.
- Thus, Sharp edges has become blurred.
- Best result when used over image corrupted by Gaussian noise.
- Other types of low pass averaging mask are:

1	0	1	0
----	1	2	1
6	0	1	0

1	1	1	1
----	1	2	1
10	1	1	1

Thank You

