Name: _____ ID: _____

Department of Computer Science and Engineering

CSE330: Numerical Methods

Summer 2018

Quiz-1 **SET A**

Full Marks: 15 **Time: 20minutes**

Solve ***cosx =2x*** using **Newton Raphson's** method upto ***3*** iterations and fill up the table below. Given initial guess, $x_i=0.5$.

Iteration Count	x_i	$\lvert\epsilon_a\rvert\%$	$f(x_i)$	$f'(x_i)$
1	0.45063	10.956%.	-1.0871×10^{-3}	-2.4355
2	0.45018	0.00996%.	8.794×10^{-6}	-2.4351
3	0.45018	0%.	8.794×10^{-6}	-2.4351

Given, $f(x) = \cos x - 2x = 0$

$\therefore f'(x) = -\sin x - 2$ $\left[\because \frac{d}{dx}(\cos x - 2x)\right.$

$\left. = \left[\frac{d}{dx}(\cos x) - 2\frac{d}{dx}(x)\right] = (-\sin x - 2)\right]$

Iteration 1: $x_0 = 0.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5 - \frac{\cos 0.5 - 2\times 0.5}{-\sin 0.5 - 2}$$

$$= 0.45063$$

$f(x_1) = \cos(0.45063) - 2 \times 0.45063 = -1.0871 \times 10^{-3}$

$f'(x_1) = -\sin(0.45063) - 2 = -2.43355$

$|\epsilon_a| = \left|\frac{x_1 - x_0}{x_1}\right| \times 100\% = \left|\frac{.45063 - 0.5}{.45063}\right| \times 100\% = 10.956\%$

Iteration 2:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= .45063 - \frac{-1.0871 \times 10^{-3}}{-2.43355}$$

$$= 0.45018$$

$f(x_2) = \cos(0.45018) - 2 \times 0.45018$

$= 8.794 \times 10^{-6}$

$f'(x_2) = -\sin(0.45018) - 2$

$= -2.4351$; $|\epsilon_a| = \left|\frac{x_2 - x_1}{x_2}\right| \times 100\% = \left|\frac{0.45018 - 0.45063}{.45018}\right| \times 100\% = 0.00996\%$

Iteration 3:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.45018 - \frac{8.794 \times 10^{-6}}{-2.4351}$$

$$= 0.45018$$

$|\epsilon_a| = \left|\frac{x_3 - x_2}{x_3}\right| \times 100\% = \left|\frac{0.45018 - 0.45018}{0.45018}\right| \times 100\% = 0\%$

$f(x_3) = f(0.45018) = \cos(0.45018) - 2 \times .45018 = 8.79397 \times 10^{-6}$

$f'(x_3) = f'(0.45018) = -\sin(0.45018) - 2 = -2.4351$