

Name: ID:

Department of Computer Science and Engineering

CSE 330: Numerical Methods

Quiz 1 FALL 2018

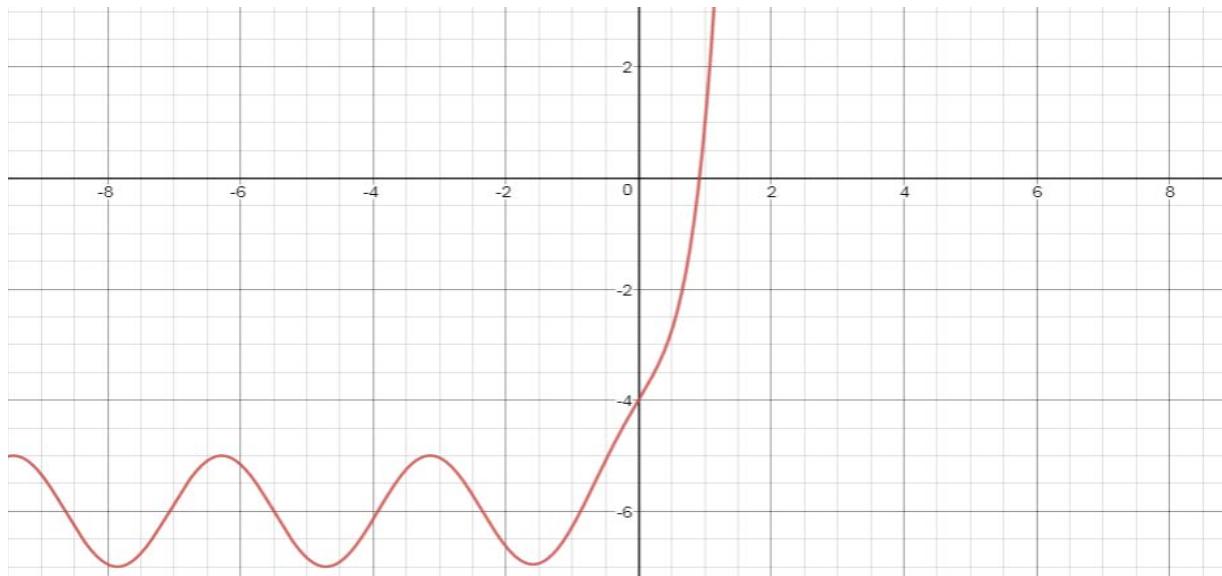
Full Marks: 10

SET A

Solve the following equation using an iterative approach upto **3** iterations. Also find the *absolute relative true error*, $|E_t| \%$ after each iteration. Suppose, you are given an initial guess of $x_0 = 2$.

$$e^{2x} + \cos(2x) - 6 = 0$$

Graphical representation of the given equation:



$$\text{Bisection Method: } x_m = \frac{x_l + x_u}{2}$$

$$\text{False Position Method: } x_r = \frac{x_u f(x_l) - x_l f(x_u)}{f(x_l) - f(x_u)}$$

$$\text{Newton Raphson's Method: } x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = e^{2x} + \cos(2x) - 6 = 0$$

$$x_0 = 21^\circ \approx 2$$

$$f'(x) = 2e^{2x} - 2\sin 2x$$

Iteration 1:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{e^{2 \times 2} + \cos(2 \times 2) - 6}{2e^{2 \times 2} - 2 \sin(2 \times 2)}$$

$$= 1.5669$$

Absolute True Error

$$| \epsilon_t | = \left| \frac{\text{True Value} - \text{Approximate value}}{\text{True Value}} \right| \times 100\%$$

Suppose, True value = 0.85 (from graph)

$$\therefore | \epsilon_t | = \left| \frac{0.85 - 1.5669}{0.85} \right| \times 100\% \\ = 84.34\%$$

Iteration 2:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5669 - \frac{e^{2 \times 1.5669} + \cos(2 \times 1.5669) - 6}{2e^{2 \times 1.5669} - 2 \sin(2 \times 1.5669)}$$

$$= 1.219$$

Absolute relative true error,

$$|e_t| = \left| \frac{.85 - 1.219}{.85} \right| \times 100\%.$$

$$= 43.41\%.$$

Iteration 3:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.219 - \frac{e^{2 \times 1.219} + \cos(2 \times 1.219) - 6}{2e^{2 \times 1.219} - 2 \sin(2 \times 1.219)}$$

$$= 1.002$$

$$|e_t| = \left| \frac{.85 - 1.002}{.85} \right| \times 100\%$$

$$= 17.88\%.$$