Algorithm 4.21: Left factoring a grammar.

INPUT: Grammar G.

OUTPUT: An equivalent left-factored grammar.

METHOD: For each nonterminal A, find the longest prefix α common to two or more of its alternatives. If $\alpha \neq \epsilon$ — i.e., there is a nontrivial common prefix — replace all of the A-productions $A \to \alpha\beta_1 \mid \alpha\beta_2 \mid \cdots \mid \alpha\beta_n \mid \gamma$, where γ represents all alternatives that do not begin with α, by

$$\begin{aligned} A &\to \alpha A' \mid \gamma \\ A' &\to \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n \end{aligned}$$

Here A' is a new nonterminal. Repeatedly apply this transformation until no two alternatives for a nonterminal have a common prefix. □

Example 4.22: The following grammar abstracts the "dangling-else" problem:

$$\begin{aligned} S &\to i\ E\ t\ S \mid i\ E\ t\ S\ e\ S \mid a \\ E &\to b \end{aligned} \tag{4.23}$$

Here, i, t, and e stand for **if**, **then**, and **else**; E and S stand for "conditional expression" and "statement." Left-factored, this grammar becomes:

$$\begin{aligned} S &\to i\ E\ t\ S\ S' \mid a \\ S' &\to e\ S \mid \epsilon \\ E &\to b \end{aligned} \tag{4.24}$$

Thus, we may expand S to $iEtSS'$ on input i, and wait until $iEtS$ has been seen to decide whether to expand S' to eS or to ϵ. Of course, these grammars are both ambiguous, and on input e, it will not be clear which alternative for S' should be chosen. Example 4.33 discusses a way out of this dilemma. □

Left factoring

Example 1: Left factorize the following grammar.

$$S \rightarrow \underbrace{a}_{\alpha}\underbrace{SSbS}_{\beta_1} \mid \underbrace{a}_{\alpha}\underbrace{SaSb}_{\beta_2} \mid \underbrace{a}_{\alpha}\underbrace{bb}_{\beta_3} \mid \underbrace{b}_{\gamma}$$

Solution:-

$$\left(\begin{array}{l} \text{if } A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \alpha\beta_3 \mid \cdots \mid \gamma \\ \text{then, } \quad A \rightarrow \alpha A' \mid \gamma \\ \qquad\quad\;\; A' \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \mid \cdots \end{array} \right)$$

$$S \rightarrow aS' \mid b$$

$$S' \rightarrow \underbrace{S}_{\alpha}\underbrace{SbS}_{\beta_1} \mid \underbrace{S}_{\alpha}\underbrace{aSb}_{\beta_2} \mid \underbrace{bb}_{\gamma}$$

Again left factoring,

$$S \rightarrow aS' \mid b$$

$$S' \rightarrow SS'' \mid bb$$

$$S'' \rightarrow SbS \mid aSb$$

Example 2: Left factorize the following grammar:

$$S \rightarrow bSSaaS \mid bSSaSb \mid bSb \mid a$$

Solution:

$$S \rightarrow \underbrace{bS}_{\alpha}\underbrace{SaaS}_{\beta_1} \mid \underbrace{bS}_{\alpha}\underbrace{SaSb}_{\beta_2} \mid \underbrace{bS}_{\alpha}\underbrace{b}_{\beta_3} \mid \underbrace{a}_{\gamma}$$

$$\left(\begin{array}{l} \text{If } A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \cdots \mid \alpha\beta_n \mid \gamma, \text{ then} \\ A \rightarrow \alpha A' \mid \gamma \\ A' \rightarrow \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n \end{array}\right)$$

$$S \rightarrow bSS' \mid a$$

$$S' \rightarrow \underbrace{Sa}_{\alpha}\underbrace{aS}_{\beta_1} \mid \underbrace{Sa}_{\alpha}\underbrace{Sb}_{\beta_2} \mid \underbrace{b}_{\gamma}$$

Again left factoring,

$$S' \rightarrow bSS' \mid a$$

$$S' \rightarrow SaS'' \mid b$$

$$S'' \rightarrow aS \mid Sb$$