Name: ..... ID:.....

Department of Computer Science and Engineering

CSE 330: Numerical Methods

Quiz 2

Full Marks: 15 **SET A**

Using Table 1, form a matrix representation for solving the coefficients for the polynomials having quadratic splines

Table 1

Sl. No.	V1	V2
1	10	20
2	12	23
3	15	27
4	17	34
5	19	37

Since, there are 5 points, we will have a total of 4 splines. Also, we will have a total of $3 \times 4 = 12$ equations to solve for 12 unknowns.

$f(v_1) = v2 = a_1 v1^2 + b_1 v1 + c_1$, $10 \le v1 \le 12$

$= a_2 v1^2 + b_2 v1 + c_2$; $12 \le v1 \le 15$

$= a_3 v1^2 + b_3 v1 + c_3$; $15 \le v1 \le 17$

$= a_4 v1^2 + b_4 v1 + c_4$; $17 \le v1 \le 19$

Now, Since each spline pass through 2 points

$a_1 v1^2 + b_1 v1 + c_1 \Rightarrow a_1 (10)^2 + b_1 (10) + c_1 = 20$ —— (i)

$a_1 (12)^2 + b_1 (12) + c_1 = 23$ —— (ii)

$a_2 v1^2 + b_2 v1 + c_2 \Rightarrow a_2 (12)^2 + b_2 (12) + c_2 = 23$ —— (iii)

$a_2 (15)^2 + b_2 (15) + c_2 = 27$ —— (iv)

$a_3v_1^2 + b_3v_1 + c_3 \Rightarrow a_3(15)^2 + b_3(15) + c_3 = 27$ — (v)

$a_3(17)^2 + b_3(17) + c_3 = 34$ — (vi)

$a_4v_1^2 + b_4v_1 + c_4 \Rightarrow a_4(17)^2 + b_4(17) + c_4 = 34$ — (vii)

$a_4(19)^2 + b_4(19) + c_4 = 37$ — (viii)

That makes a total of $2 \times 4 = 8$ equations per spline. We still need 4 more equations.

Since the first derivative of two consecutive splines are continuous at the interior points & 12, 15, 17 are the interior points,

$2a_1(12) + b_1 - 2a_2(12) - b_2 = 0$ — (ix)

$2a_2(15) + b_2 - 2a_3(15) - b_3 = 0$ — (x)

$2a_3(17) + b_3 - 2a_4(17) - b_4 = 0$ — (xi)

[Since there are 5 points here, we have 3 interior points & 2 boundary points]

Assuming that the first spline is linear,

$a_1 = 0$ — (xii) [$\because$ linear spline is $b_1v_1 + c_1$]

We now have a total of 12 equations. forming the matrix of co-efficients :

coefficients of

a_1	b_1	c_1	a_2	b_2	c_2	a_3	b_3	c_3	a_4	b_4	c_4
100	10	1	0	0	0	0	0	0	0	0	0
144	12	1	0	0	0	0	0	0	0	0	0
0	0	0	144	12	1	0	0	0	0	0	0
0	0	0	225	15	1	0	0	0	0	0	0
0	0	0	0	0	0	225	15	1	0	0	0
0	0	0	0	0	0	289	17	1	0	0	0
0	0	0	0	0	0	0	0	0	289	17	1
0	0	0	0	0	0	0	0	0	361	19	1
24	1	0	-24	-1	0	0	0	0	0	0	0
0	0	0	30	1	0	-30	-1	0	0	0	0
0	0	0	0	0	0	34	1	0	-34	-1	0
1	0	0	0	0	0	0	0	0	0	0	0

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \\ a_4 \\ b_4 \\ c_4 \end{bmatrix} = \begin{bmatrix} 20 \\ 23 \\ 23 \\ 27 \\ 27 \\ 27 \\ 34 \\ 34 \\ 37 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Tips when writing the co-efficient matrix:

1. Use a pencil to avoid errors.
2. At the top, mention the co-efficients (You can erase it afterwards if needed!)