CSE331

Automata and Computability

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CSE331

OBE System .

CO1 → The fundamental concepts of computation, automation and compilation system and apply logical reasoning to solve a variety of problems .

CO2 → Various aspects of regular expression and identify different types of pattern using regular expression .

CO3 → DFA, NFA , e-NFA and sketch DFA, NFA , e-NFA diagrams of some problems .

CO4 → The limitations of Regular languages and apply pumping lemma upon different language classes .

CO5 → Design of CFG and construct parse trees of any string according to some CFG .

CO6 → Applicability of CFGs in developing programming languages and estimate the Chomsky Normal Form and CYK algorithms upon any CFG .

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Theory of Computation
OR Automata Theory

1| Introduction to TOC and DFA:-

Symbol - a, b, 0, 1, ...,

↓

"Σ" Alphabet - {a, b}

{a, b, c}, {0, 1}, {0, 1, ..., 9}.

↓

String - sequence of the symbols.

Ex: {a, b, aa, ab, bc ......}

A = {a, b}

how many strings of length 2 are possible of A?

a	a
a	b
b	a
b	b

how many strings of length n are possible of A?

_ _ _ _ ...... n

{a,b}, {a,b}, {a,b}, ... n

2 × 2 × 2 × ...... n

$\boxed{2^n}$

Language - collection of strings.

$\Sigma = \{a, b\}$

L_1 = set of all strings of length 2.

= {aa, ab, ba, bb}

L_1 is a finite language as it is a finite set.

L_2 = Set of all strings of length 3.

$= \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

L_2 is also finite.

L_3 = set of all strings where each string starts with 'a'

$= \{a, aa, ab, aaa, aab, aba, abb \ldots\ldots\}$

L_3 is infinite as it is an infinite set.

Powers of Σ : $\Sigma = \{a, b\}$

Σ^1 = set of all strings over Σ of length exactly '1'

$= \{a, b\}$

$\Sigma^2 = \Sigma . \Sigma = \{a, b\} . \{a, b\} = \{aa, ab, ba, bb\}$

set of all the strings of length '2'.

$\Sigma^3 = \Sigma . \Sigma . \Sigma = \{a, b\} . \{a, b\} . \{a, b\} =$

$|\Sigma^3| = 8$

$\vdots$

Σ^n = n length strings.

Σ^0 = set of all strings of length '0'

$= \{\epsilon\}$ → null string.

$|\epsilon| = 0$ → length of ϵ is 0.

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$\Sigma = \{a, b\}$

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \ldots\ldots$

$= \{\epsilon\} \cup \{a, b\} \cup \{aa, ab, ba, bb\} \ldots\ldots$

All possible strings of Σ of all lengths.

Σ^* $\boxed{L_1 \quad L_2 \quad L_3}$

$L_1 \subseteq \Sigma^*$

$L_2 \subseteq \Sigma^*$

$L_3 \subseteq \Sigma^*$

⋮

number of strings possible over Σ^* is infinite | number of languages possible over Σ^* is infinite.

Example: In 'c' programming language.

$\Sigma = \{a, b, \ldots z, A, B \ldots Z, 0, 1 \ldots 9, +, * \ldots\}$

```
void main ()
{ int a,b;
  :
}
```

In TOC, $\boxed{\text{program} = \text{String}}$

C - programming language = Set of all "valid" programs

$= \{P_1, P_2, \ldots\ldots\}$ P_m

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If, L is finite

$\Sigma = \{a, b\}$

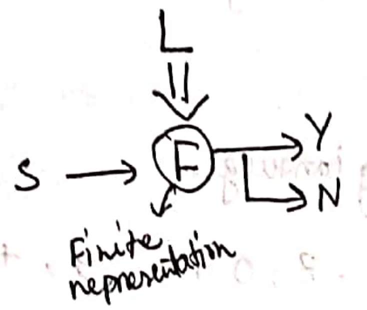
$L_1 = \{aa, ab, ba, bb\}$

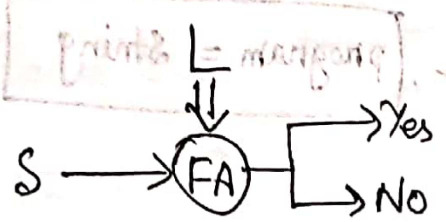
S = aaa, is it present in L_1?

If L is infinite,

$L_2 = \{a, aa, aaa, ab \text{ ----}\}$

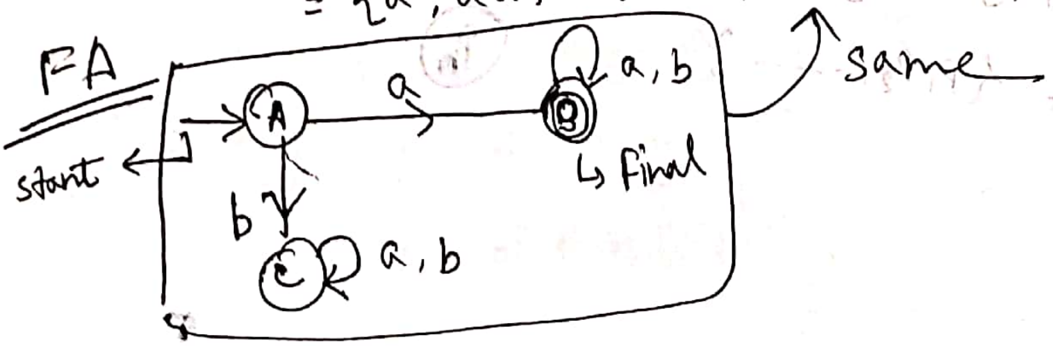
S = baba, is it present in L_2?





L_1 = set of all strings which starts with 'a'

$= \{a, aa, ab, aaa, \ldots\ldots\}$



S = aab, present or not ?

$A \xrightarrow{a} B \xrightarrow{a} B \xrightarrow{b} B$ → Final state

S is accepted in the FA.

$S_1 =$ $A \xrightarrow{b} C \xrightarrow{b} C \xrightarrow{a} \boxed{C}$ → not final state

S_1 is not accepted or not in the language.

Finite Automata

- FA with output
  - Moore
  - Mealy
- FA without output
  - DFA
  - NFA
  - ε-NFA

DFA :

$(Q, \Sigma, \delta, q_0, F)$

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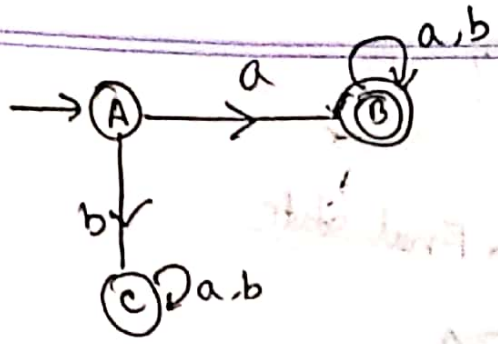


$Q = \{A, B, C\}$ → set of all states

$\Sigma = \{a, b\}$ → input alphabet

$q_0 = A$ → initial state

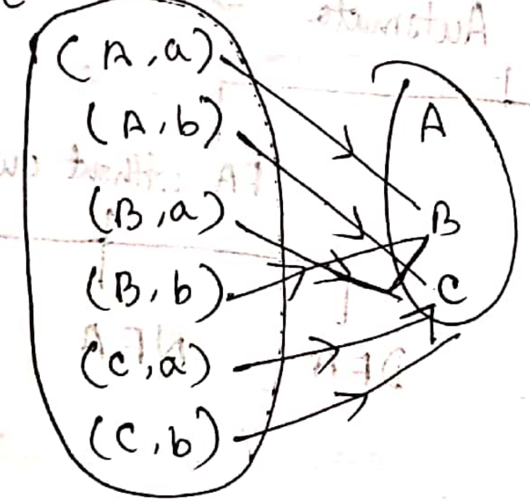
$F = \{B\}$ → set of final state

$\boxed{Q \supseteq F}$

→ Transition function

$\delta \quad Q \times \Sigma \rightarrow Q$

$\{A, B, C\} \times \{a, b\}$



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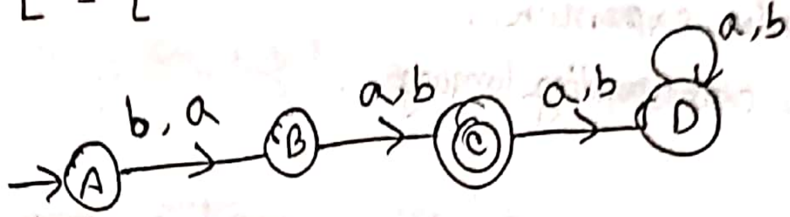
Ex: Construct a DFA, that accepts set of all Strings over $\{a,b\}$ of length 2.

$$\Sigma = \{a,b\}$$

$$L = \{aa, ab, ba, bb\}$$

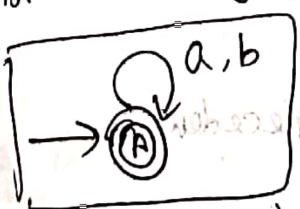


✓ $A \xrightarrow{a} B \xrightarrow{a} C$

✗ $A \xrightarrow{a} B \xrightarrow{b} C \xrightarrow{b} D \xrightarrow{a} \boxed{D} \rightarrow$ not final state

String Acceptance: A string is accepted by the finite automata if upon scanning the entire string, it is reached from initial to a final state.

Language Acceptance: A FA is said to accept a language if all the strings in the language are accepted and all the strings not in the language are ~~rege~~ rejected.



accepts all strings of L

$A \xrightarrow{a} A \qquad A \xrightarrow{a} A \xrightarrow{a} A$

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Regular Expression

A regular expression describes a language.

Notation:

r = regular expression.

L(r) = the corresponding language.

Example:

$r = a(b|c)d^*e$

L(r) = {abe
abde
abdde
abddde
....
ace
acde
acdde
.... }

Meta Symbols

( ) → Parantheses.

| → OR

* → Kleene Closure

ε → Epsilon

Precedence

1. * has the highest precedence.
2. Concatenation has the middle precedence.
3. | has the lowest precedence.
4. Use parantheses to override these rules.

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Examples :

(*) $ab^* = a(b^*)$

use parentheses $= (ab)^*$

(*) $a \mid bc = a \mid (bc)$

use parentheses $= (a \mid b)c$

Concatenation and | are associative :

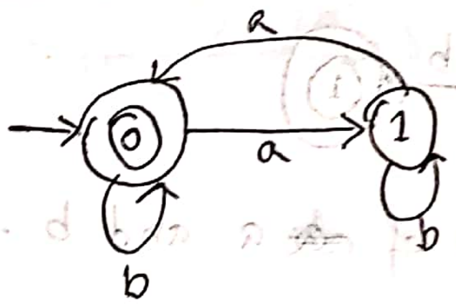
$(ab)c = a(bc) = abc$.

$(a|b)|c = a|(b|c) = a|b|c$.

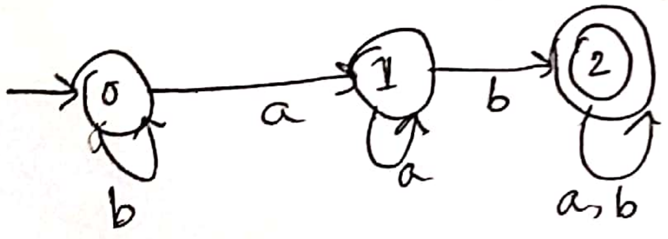
Designing Finite Automata From Regular Expression :

$\{a, b\}$

① Even number of a's : $(b \mid ab^*ab^*)^*$



② String with 'ab' as substring : $(a|b)^*ab(a|b)^*$



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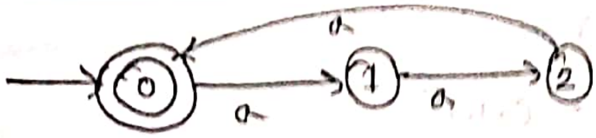
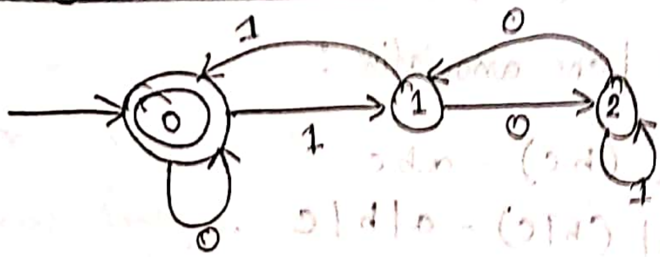
③ String with count of 'a' divisible by 3 : $(a^{3n} \mid n >= 0)$.

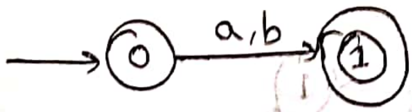


④ Binary numbers divisible by 3 : $(0 \mid 1(01^*0)^*1)^*$.

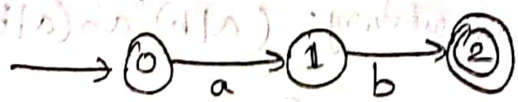


⑤ Alphabet 'a' : a.

⑥ 'a+b' : (Union process of two alphabets 'a' and 'b')



⑦ a.b : concatenation of ~~ab~~ a and b.



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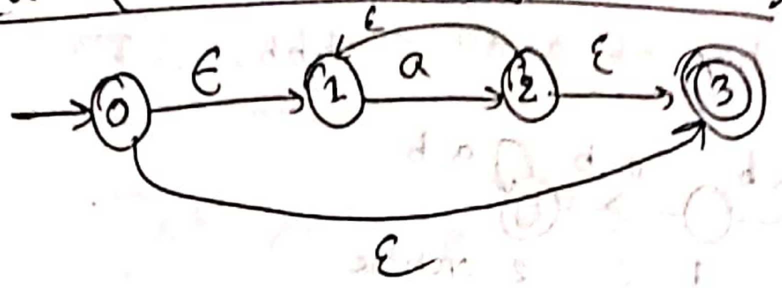
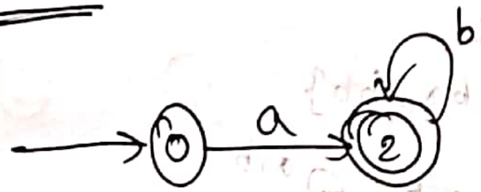
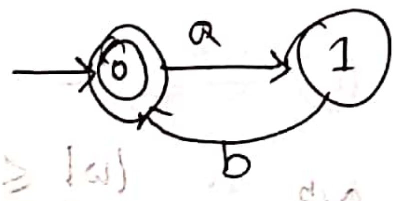
(8) a^* : (Kleene closure of alphabet 'a')



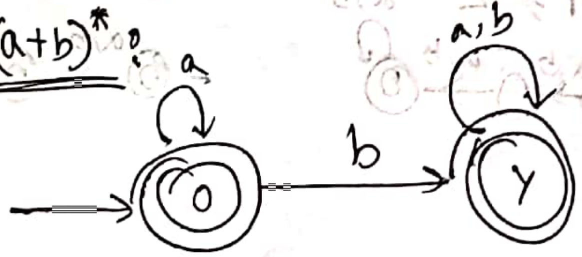
(9) ab^* :



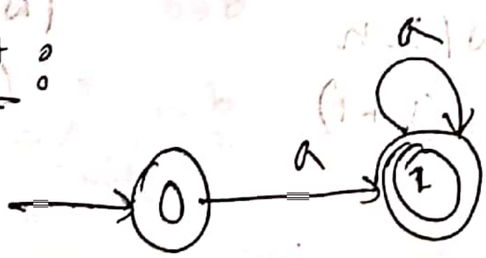
(10) $(ab)^*$



(11) $(a+b)^*$:



(12) a^+ :



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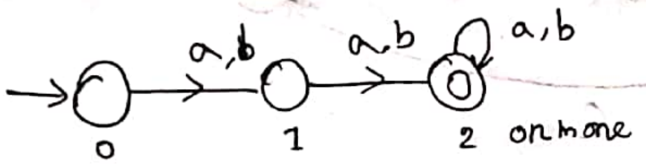
Construction of minimal DFA

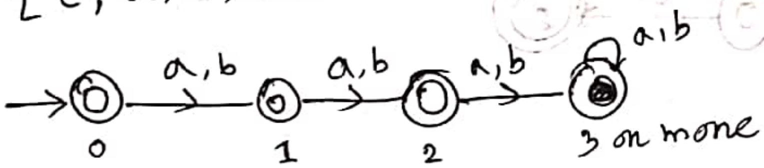
* DFA → $w \in \{a, b\}$: $|w| \geq 2$

$L = \{aa, ab, ba, bb, aaa \ldots bbb \ldots\}$



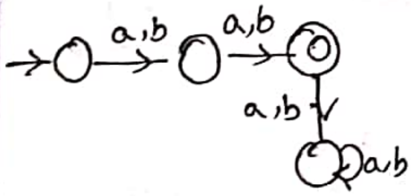
* DFA $w \in \{a, b\}$ $|w| \leq 2$

$L = \{\epsilon, a, b, aa, ab, ba, bb\}$

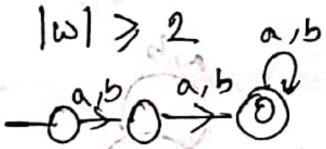


L = many DFA's.
only one minimal.

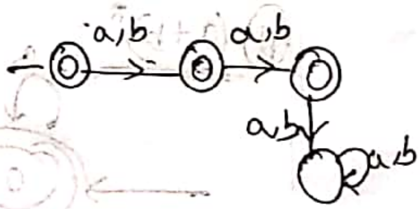
$|w| = 2$



$|w| \geq 2$



$|w| \leq 2$



$|w| = n$
states required to create minimal DFA = $(n+2)$

$|w| \geq n$
$(n+1)$

$|w| \leq n$
$(n+2)$

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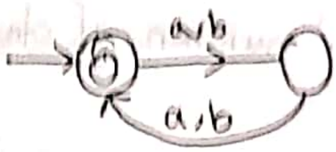
(8) minimal DFA, $w \in \{a,b\}$, $|w| \bmod 2 = 0$

$L = \{\epsilon, aa, ab, ba, bb$
$aaaa, \ldots, bbbb \ldots \}$

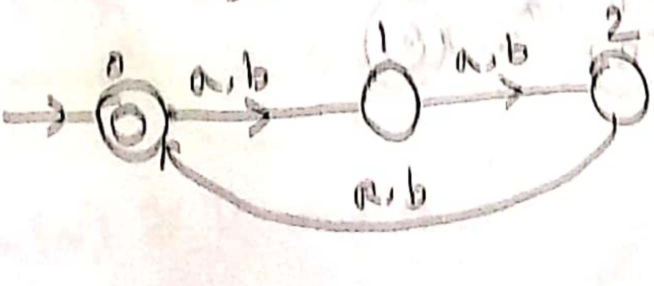
Minimal



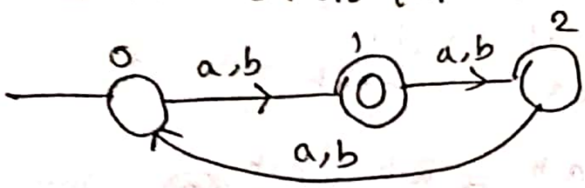
(9) minimal DFA, $w \in \{a,b\}$, $|w| \bmod 2 = 1$.

(10) $w \in \{a,b\}^*$ $|w| \bmod 3 = 0$.

$L = \{\epsilon, aaa, aab, \ldots \}$
aab
bbb



(*) $|w| \cong 1 \bmod 3$. or $|w| \bmod 3 = 1$



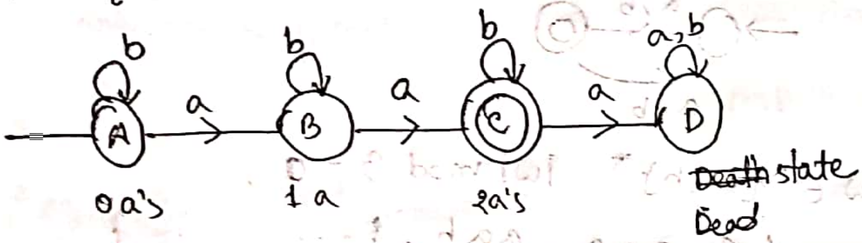
(*) $|w| \bmod n = 0$.

'n' number of states.

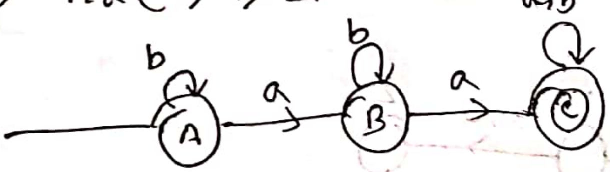
(*) Minimal DFA $w \in \{a,b\}^*$

$n_a(w) = 2$

$L = \{aa, baa, aba, aab, bbaa, \ldots\}$



(*) $n_a(w) \geq 2$



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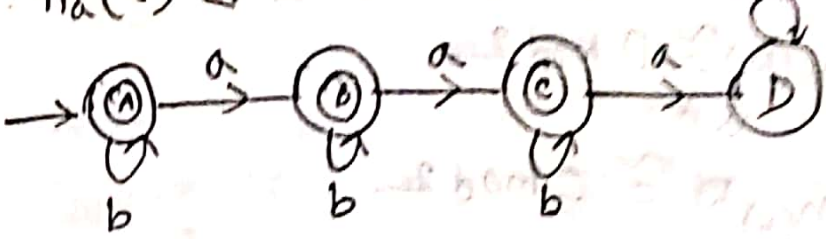
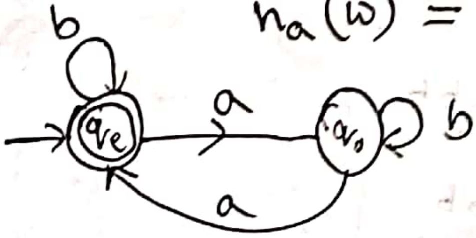
⊛ $n_a(w) \leq 2$.



⊛ Minimal DFA $w \in \{a,b\}^*$

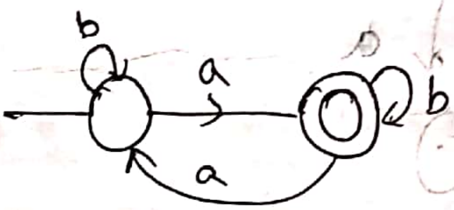
$n_a(w) \bmod 2 = 0$.

$n_a(w) \cong 0 \bmod 2$.

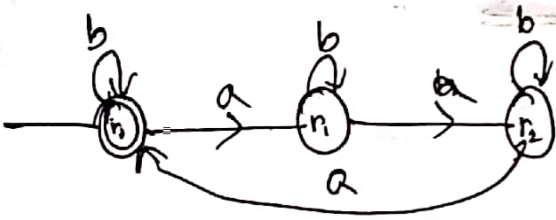


⊛ Minimal DFA $w \in \{a,b\}^*$

$n_a(w) \cong 1 \bmod 2$



⊛ $n_a(w) \bmod 3 = 0$.



⊛ $n_a(w) \cong k \bmod n$

n states

(8) Minimal DFA $w \in \{a, 2\}^*$

$n_a(w) \cong 0 \bmod 2$

$\&\&$

$n_{(b)} w \cong 0 \bmod 2$

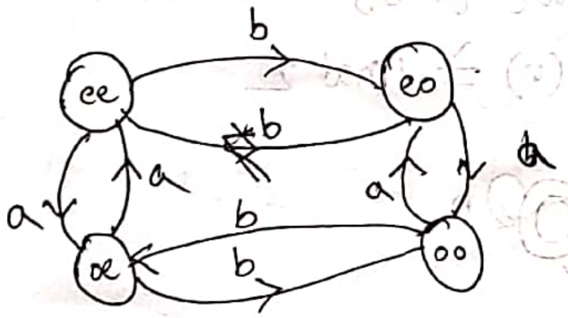
$L = \{ \epsilon, aa, bb, aabb, abab, babb, bbbb \ldots \}$

$n_a(w)$	$n_b(w)$	
e	e	$\rightarrow \epsilon, aa, bb$
e	o	$\rightarrow aab$
o	e	$\rightarrow aaabb$
o	o	$\rightarrow ab$

'a'
oe

'b'
eo

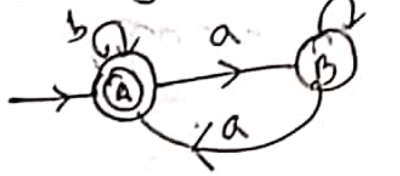
ab
o o



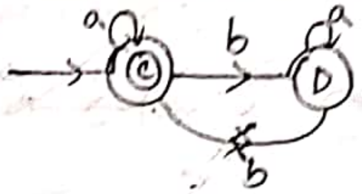
Method 1

Method - 2 (Cross Product)

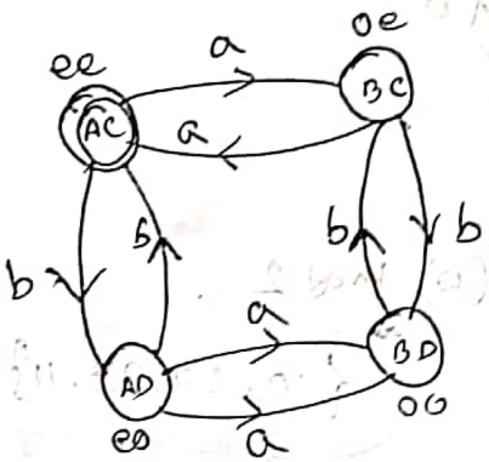
Counting a's



Counting b's



$\{A, B\} \times \{C, D\} = \{AC, AD, BC, BD\}$



$AC \xrightarrow{a} BC$

$AC \xrightarrow{b} AD$

$BC \xrightarrow{a} AC$

$BC \xrightarrow{b} BD$

$AD \xrightarrow{a} BD$

$AD \xrightarrow{b} AC$

$BD \xrightarrow{a} AD$

$BD \xrightarrow{b} BC$

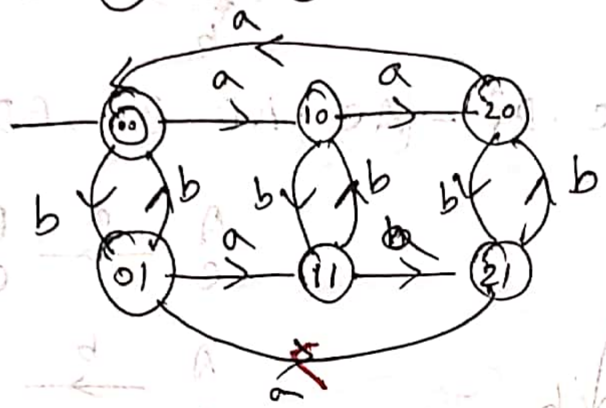
* <u>a's</u> <u>b's</u> = {ee}

e && e = {ee}

e or e = {ee, eo, oe}

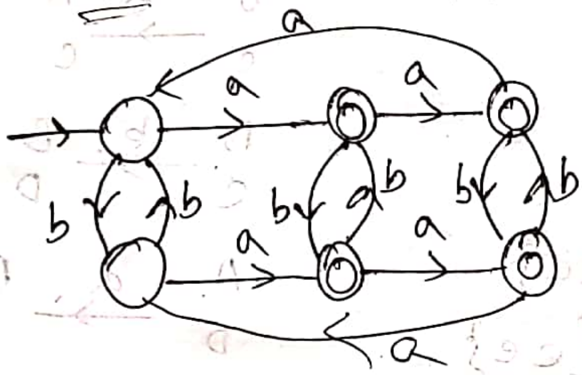
(*) minimal DFA $w \in \{a,b\}^*$ $n_a(w) \cong 0 \bmod 3$. $n_b(w) \cong 0 \bmod 2$

$\{00\}$



(*) $n_a(w) \bmod 3 \geq n_b(w) \bmod 2$.

$\{10, 20, 21, 11\}$



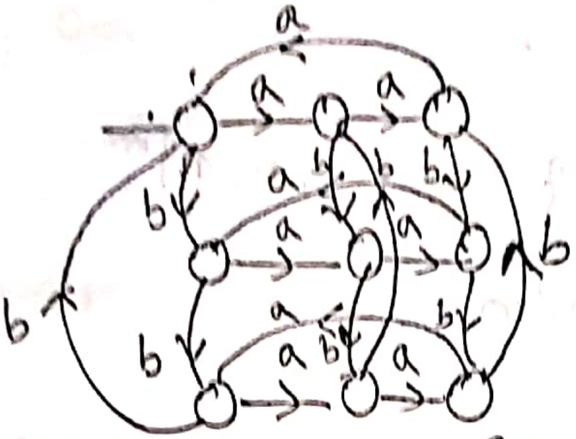
$n_{(a)} \bmod 3 = 0$
$\geq$
$n_{(b)} \bmod 2 = 0$

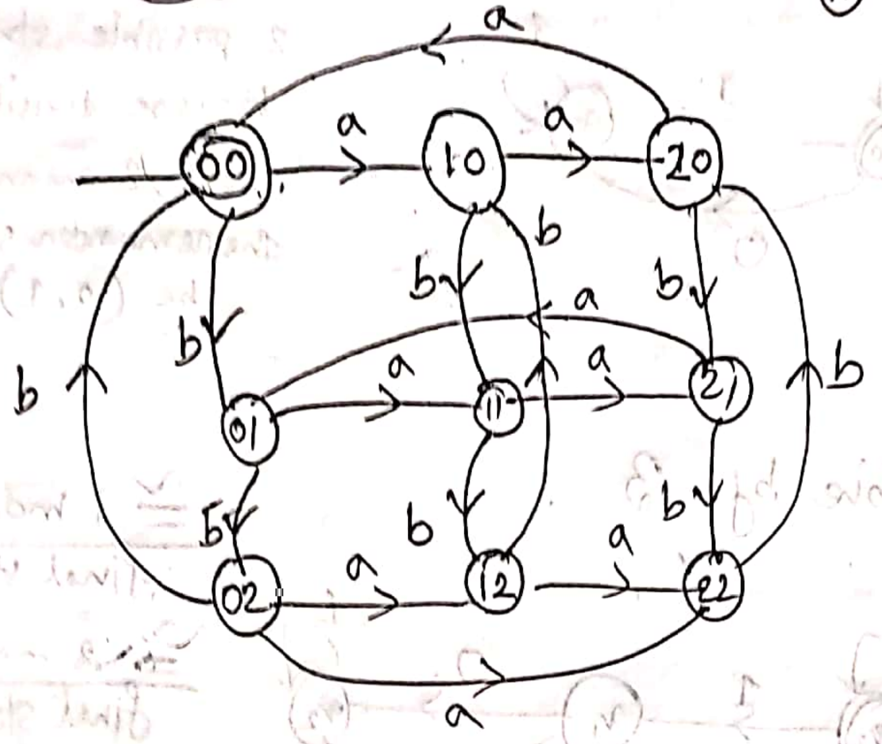
(*) $n_a(w) \bmod 3 = n_b(w) \bmod 2$

$\{00, 11\}$

(*) minimal DFA $w \in \{a, b\}^*$ $n_a(w) \cong 0 \mod 3$ &&

$n_b(w) \cong 0 \mod 3$. $\{00\}$





(*) $n_a(w) \mod 3 = 1$

&&

$n_b(w) \mod 3 = 2$

$\{12\}$

(*) $n_a(w) \mod 3 >$

$n_b(w) \mod 3$.

$\{10, 20, 21\}$

(*) $(n_a(w) >$

$n_b(w))$

mod 3

(*) $n_a(w) \cong 0 \mod n$

$n_b(w) \cong 0 \mod m$

states $(n \times m)$

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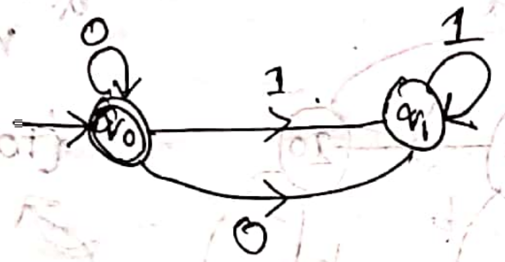
(*) Construct a minimal DFA, which accepts set of all strings over $\{0,1\}$, which when interpreted as binary number is divisible by 2.

$$\Sigma = \{0,1\} \quad w \in \{0,1\}^*.$$

$$(110)_2 = (6)_{10}$$

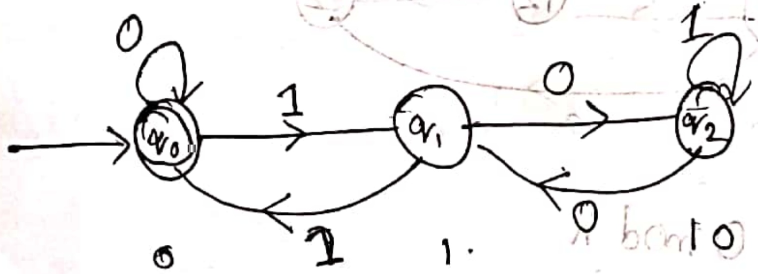


2 possible states because divisible by 2 means the remainder should be (0, 1).

(*) divisible by 3.

$\cong 1 \bmod 3$ — final state q_1

$\cong 2 \bmod 3$ — final state q_2



$(11)_2$
$3 \rightarrow$ rem 0

0	1	10
11	100	110
110	:
:	

$(100)_2$
$4 \rightarrow$ rem 1.

$(101)_2$
$5 \rightarrow$ rem 2

State transition table

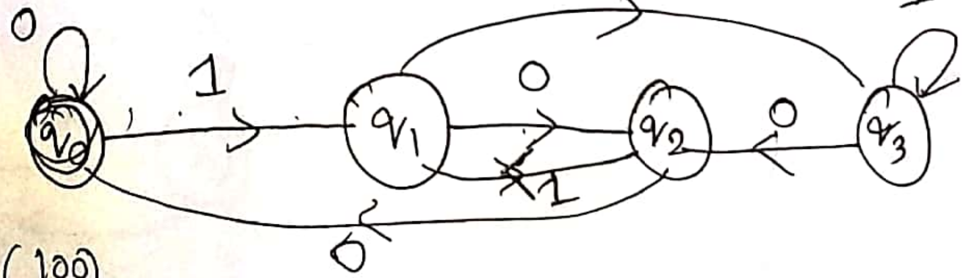
	0	1
$\rightarrow * q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

⊛ divisible by n → number of states (n)

⊛ divisible by 4

State transition table

	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_0	q_1
q_3	q_2	q_3



$\frac{(10)_2}{2}$ $\frac{(11)_2}{3}$ $\frac{(100)_2}{4}$ $\frac{(101)_2}{5}$ $\frac{(110)_2}{6}$ $\frac{(111)_2}{7}$

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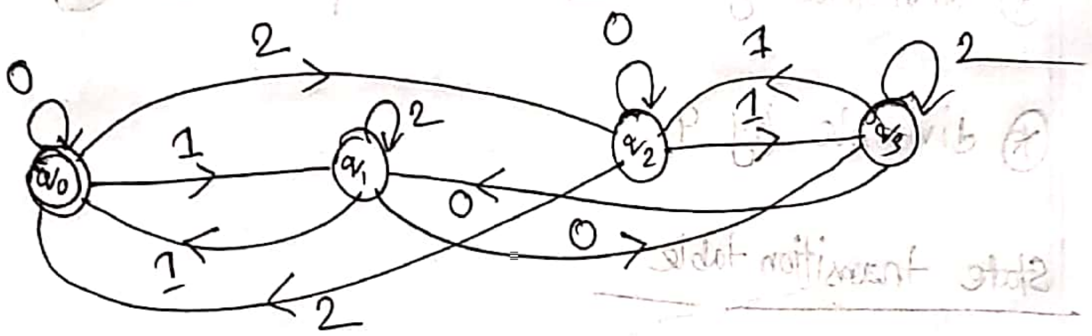
(*) $\Sigma = \{0, 1, 2\}$ divisible by 4

	0	1	2
$\rightarrow * q_0$	q_0	q_1	q_2
q_1	q_3	q_0	q_1
q_2	q_2	q_3	q_0
q_3	q_1	q_2	q_3



(*) base k number

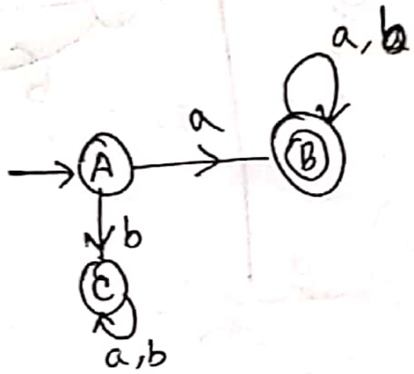
$\{0, 1, 2 \ldots, k-1\}$

% (m) → states.

divisible by.

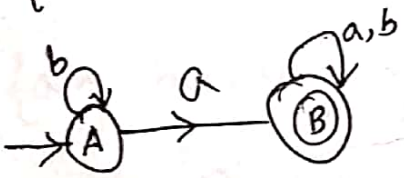
(*) Minimal DFA $\Sigma = \{a, b\}$,
set of all strings starting with 'a'.

$L = \{a, aa, ab, aaa \ldots \}$



(*) Minimal DFA $\Sigma = \{a, b\}$
set of all strings containing 'a'.

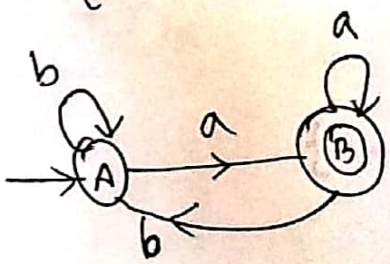
$L = \{a, aa, ab, ba, \ldots \}$



if contains
↓
no dead state.

(*) Minimal DFA $\Sigma = \{a, b\}$
set of all strings ending with 'a'.

$L = \{a, aa, ba, aaa, aba \ldots \}$



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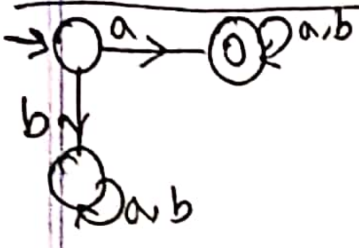
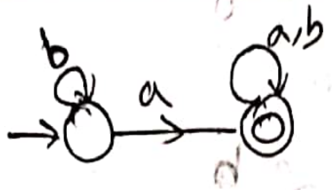
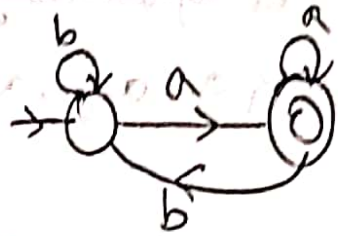
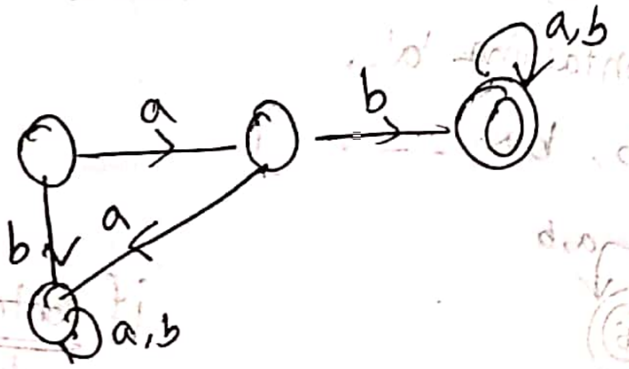
Starts with 'a'	Containing 'a'	Ends with 'a'
		

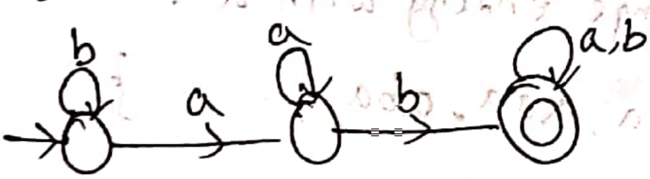
* startings with "ab"

$L = \{ab, aba, abb, \ldots\}$



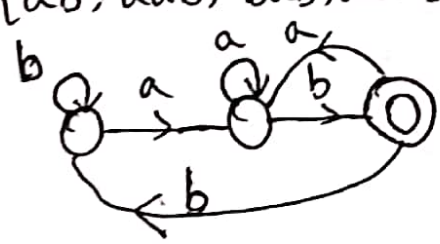
* Containing 'ab' as a substring.

$L = \{ab, aab, aba, bab, \ldots\}$



* Ending with 'ab'.

$L = \{ab, aab, bab, abab, \ldots\}$



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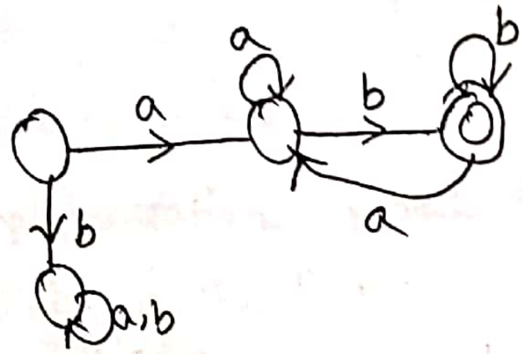
(f) starts with 'a' and ends with 'b'

$L_1 = \{a, ab, aa, aaa, \ldots\}$

$L_2 = \{b, ab, bb, aab, \ldots\}$

$L_1.L_2 = \{ab, abab, aabb, aaaab, \ldots\}$

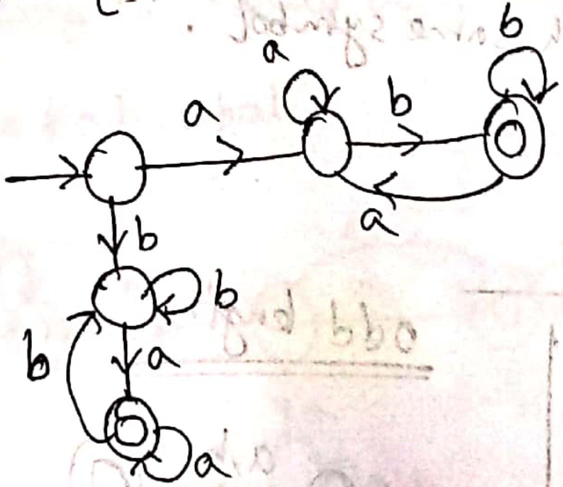


(*) $\Sigma = \{a, b\}$

starts and ends with different symbol. a —— b

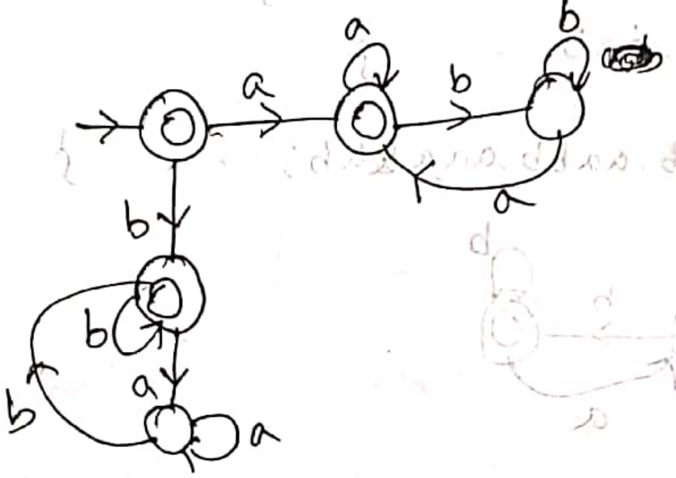
b —— a

$L = \{ab, ba, aab, abb, \ldots ba\}$



(*) starts and ~~symb~~ ends with same symbol

$L = \{\epsilon, a, b, aa, bb, \ldots\}$



(*) Complement $= \Sigma^* - L_1$

~~S and e diff~~

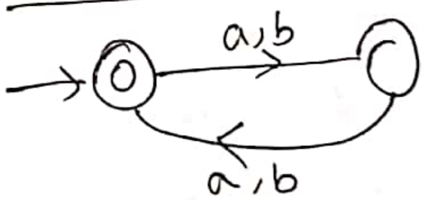
starts and ends with diff-symbol

{ and

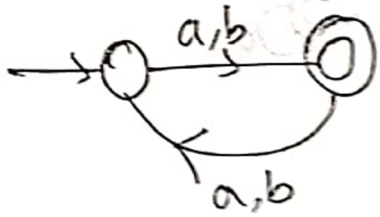
Starts and ends with same symbol.

Complementation of DFA

Even length.



odd length



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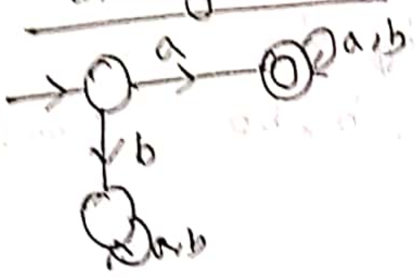
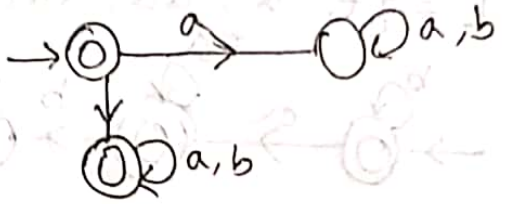
L_1	L_2
Starting with 'a'	~~Starting with 'b'~~ not starting with 'a'



$L = \{\epsilon, b, ba, \ldots\}$



$\boxed{L_1 = \overline{L_2}}$

i) Complementation: → only for DFA.

ii) $(Q, \Sigma, \delta, q_0, F)$

$\Downarrow$ comp

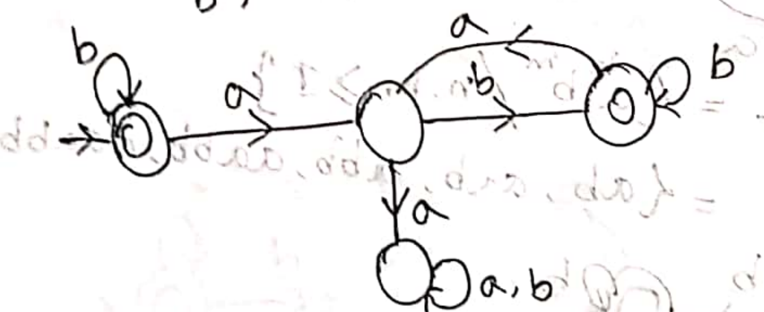
$(Q, \Sigma, \delta, q_0, Q-F)$

(*) DFA $w \in \{a,b\}^*$

Every 'a' should be followed by a 'b'

$L = \{\epsilon, ab, abab, \ldots$

$b, bb, \ldots\}$



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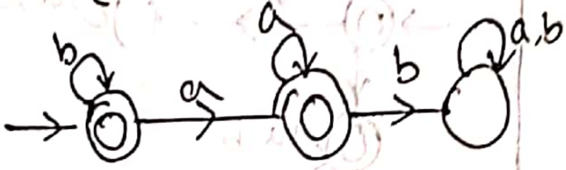
⊛ DFA $w \in \{a, b\}$

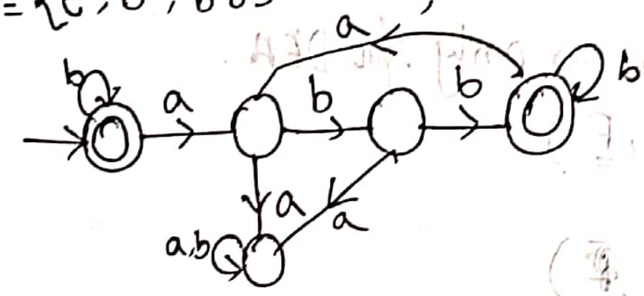
Every 'a' should never be followed by 'b'.

$L = \{\epsilon, a, aa, aaa, \ldots, b, bb, \ldots, ba, bba \ldots\}$



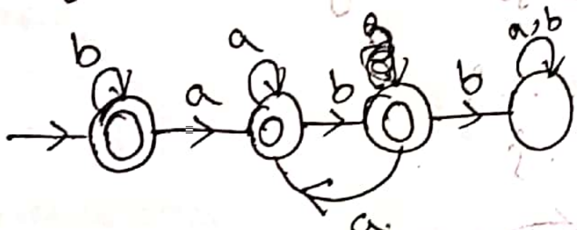
⊛ Every 'a' has to be followed by 'bb'

$L = \{\epsilon, b, bbb \ldots, abb, bbabb, \ldots\}$



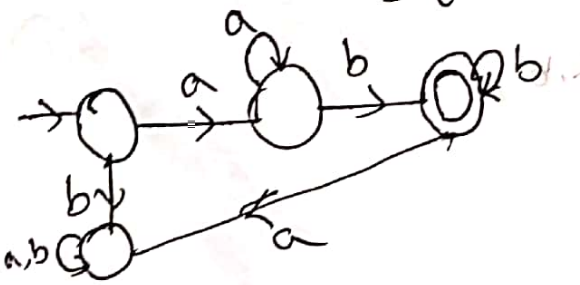
⊛ Every 'a' should never be followed by 'bb'.

$L_2 = \{\epsilon, a, aa, aaa, \ldots, b, bbb \ldots ab, aab \ldots\}$



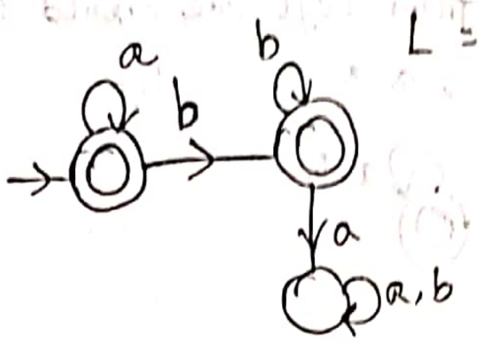
⊛ Minimal DFA, $L = \{a^n b^m / n, m \geq 1\}$.

$= \{ab, aab, abb, aabb, aaabb \ldots\}$

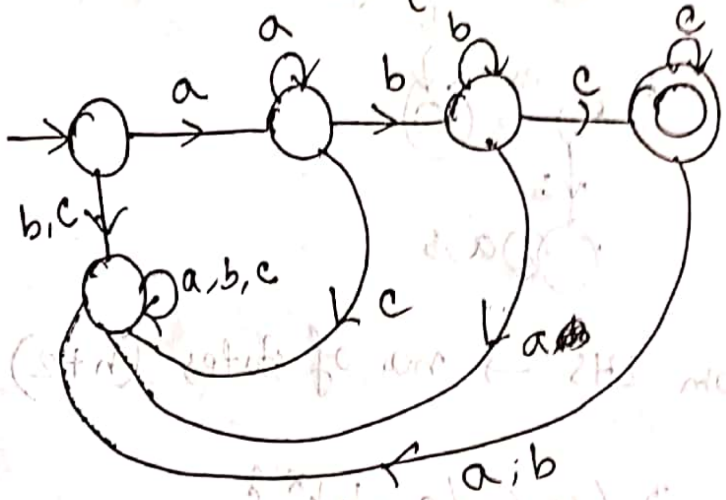


(*) Minimal DFA for, $L = \{a^n b^m / n, m \geq 0\}$

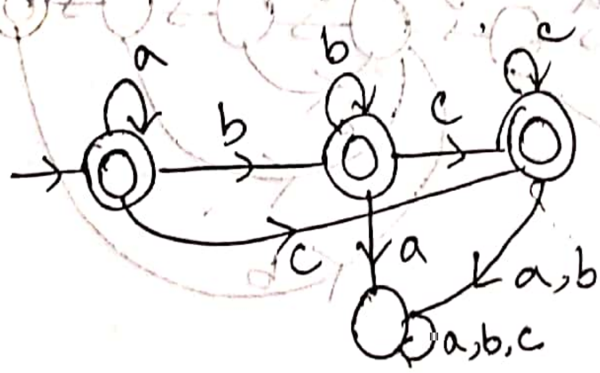
$L = \{\epsilon, a, aa, aaa \ldots, b, bb, bbb \ldots\}$
$ab, aab, aaabbbb \ldots$



(*) Minimal DFA for, $L = \{a^n b^m c^l / n, m, l \geq 1\}$
$= \{abc, aabc, abbc, abcc \ldots\}$



(*) Minimal DFA for. $L = \{a^n b^m c^l / n, m, l \geq 0\}$
$= \{e, a, aa, aaa, \ldots, b, bb, bb \ldots$
$c, cc \ldots abc, aabbcc,$
$aacc \ldots\}$



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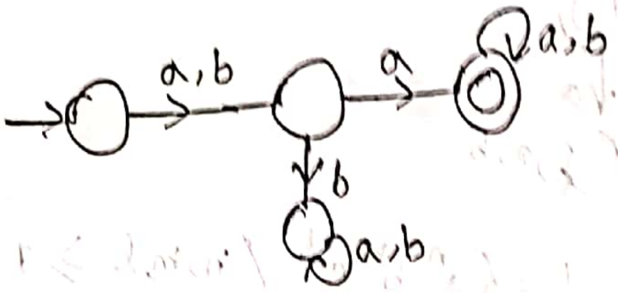
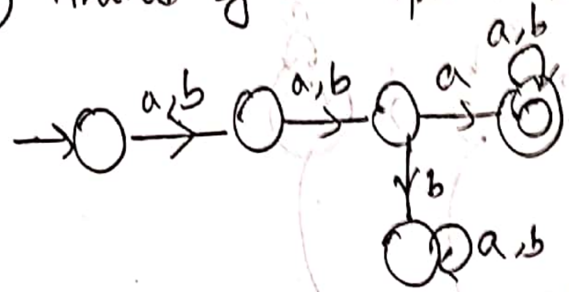
(*) $\Sigma = \{a, b\}$, DFA $w \in \{a, b\}^*$
second symbol in 'w' from Left side should be 'a'.
$L = \{aa, aaa, ba, aa \ldots, ba \ldots\}$

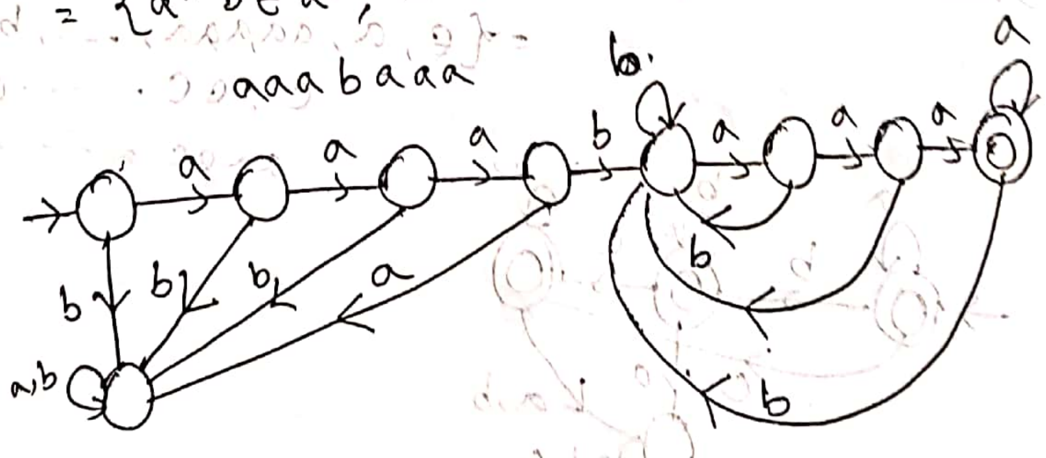


(*) Third symbol from Left side is 'a'.



n symbol from LHS → no. of states (n+2).

(*) $L = \{a^3 b w a^3 / w \in \{a, b\}^*\}$
$= \{a^3 b \epsilon a^3, a^3 b a a^3, a^3 b b a^3, a^3 b a a a^3 \ldots\}$
aaabaaa



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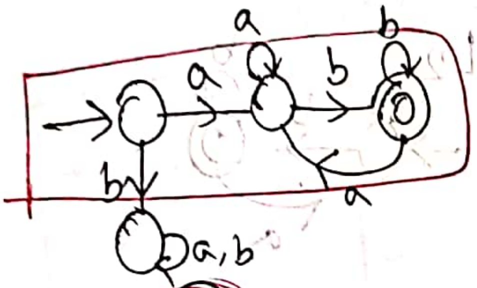
Operations of DFA

union → starts and ends with diff symbol.

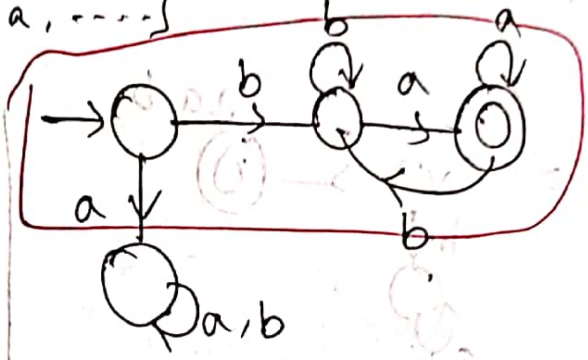
$\Sigma = \{a, b\}$

$L_1 = \{ab, aab, abb, \ldots\}$

$L_2 = \{ba, baa, bba, \ldots\}$

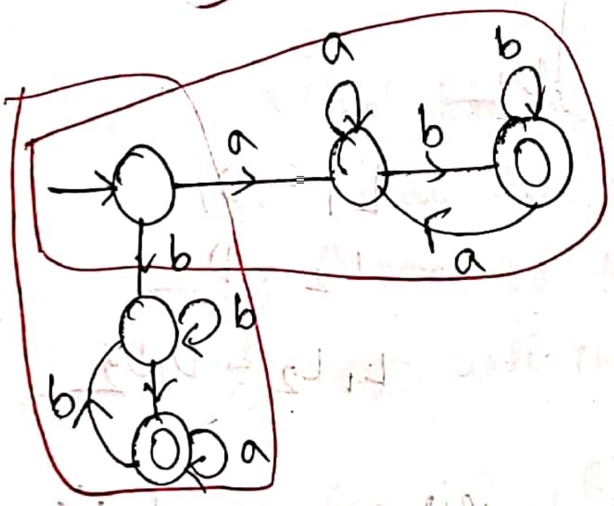


L_1



L_2

L_1

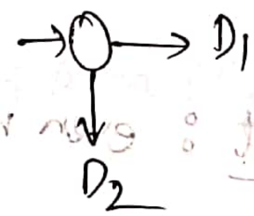


L_2

$L_1 = D_1$

$L_2 = D_2$

$L_1 \cup L_2$



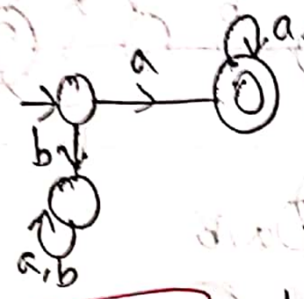
Concatenation → Starts with 'a' and ends with 'b'

$L_1 = \{$starting with 'a'$\} = \{a, aa, ab, aaa, \ldots\}$

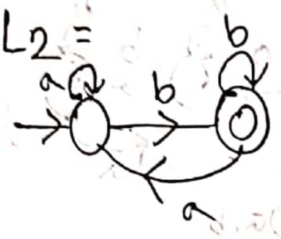
$L_2 = \{$ending with 'b'$\} = \{b, ab, bb, aab, \ldots\}$

$L_1 . L_2 = \{ab, aab, aaab, \ldots\}$

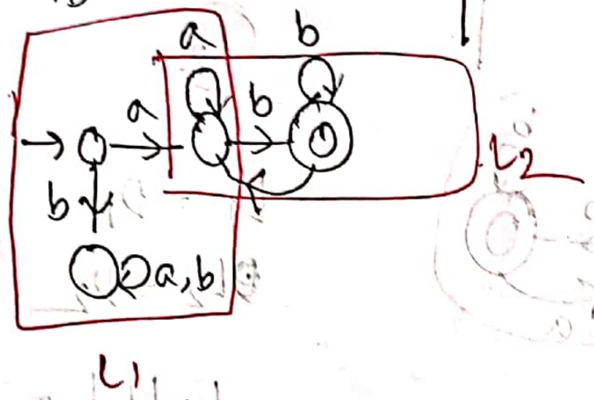
$L_1 =$



$L_2 =$



$L_1 L_2 =$



$L_1 - D_1$

$L_2 - D_2$

$L_1 L_2 - D_1 D_2$

Cross Product : even no. of 'a's ~~&&~~ even no. of b's.

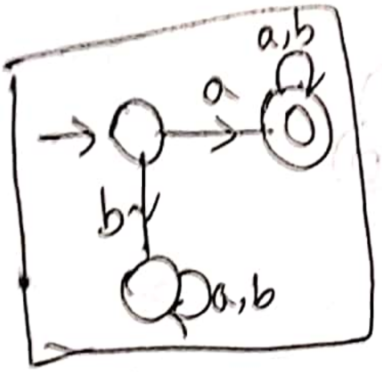
Complement : Does not contain 'a'.

L_1 = {starts with a}

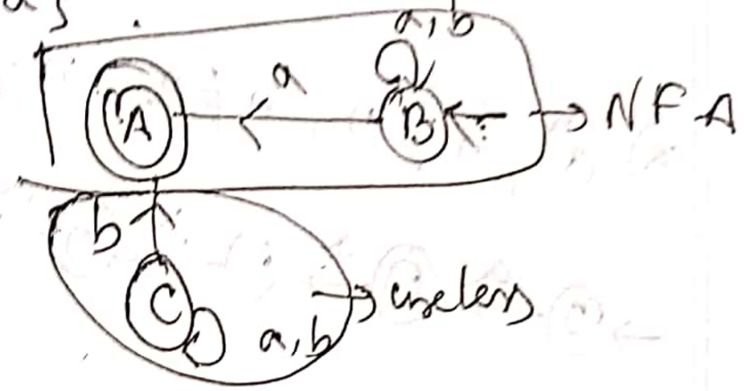
= {a, aa, ab, aaa, aab, aba, ...}

L_1^R = {a, aa, ba, aaa, baa, aba, ...}

= {ends with a}



L_1



① Final state as initial state.

② initial state as final state.

③ Reverse the transitions.

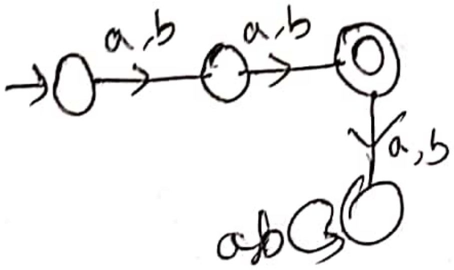
④ Loops will remain same.

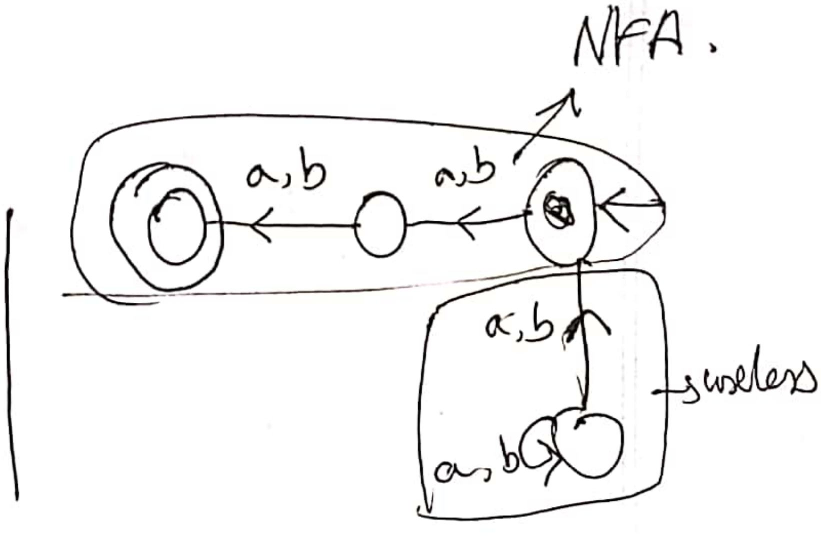
L_1 - $(DFA)^R$ - L_1^R - NFA or DFA.

Ⓐ L_1 = {aa, ab, ba, bb}

L_1^R = {aa, ba, ab, bb}

L_1





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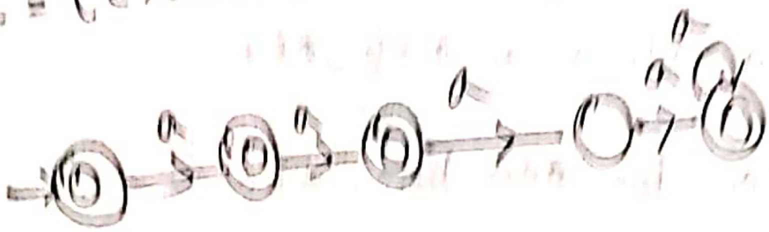
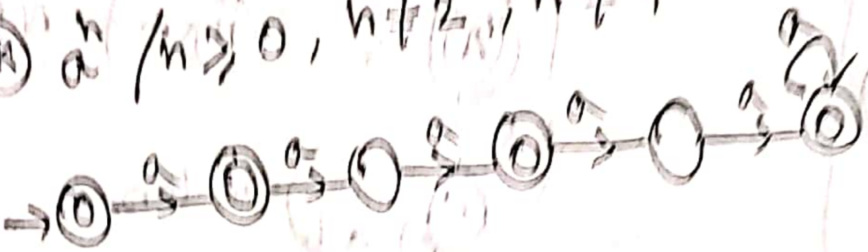
(7) $a^n / n \geq 0, n \neq 3.$ $\Sigma = \{a\}$

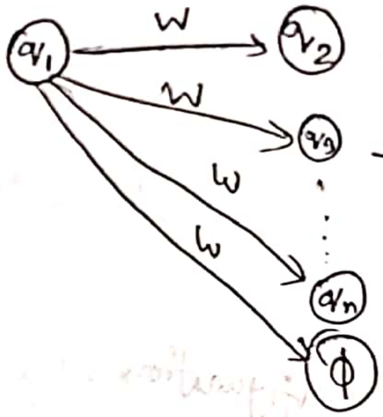
$L = \{\epsilon, a, aa, aaaa, aaaaa, \ldots\}$



(8) $a^n / n \geq 0, n \neq 2, n \neq 4$



$q_1 \xrightarrow{w} q_2 \rightarrow$ DFA



→ NFA

$(Q, \Sigma, \delta, q_0, F)$

$Q \rightarrow$ set of states (F.A)

$\Sigma \rightarrow$ input alphabet (F.A)

$q_0 \rightarrow$ start state

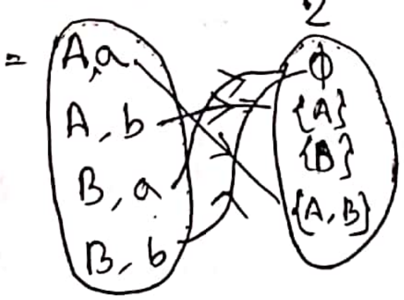
$F \rightarrow$ A set of final states.

$\delta \rightarrow$ Transition function.

NFA $\boxed{Q \times \Sigma \rightarrow 2^Q}$

$Q = \{A, B\}$ $\quad \Sigma = \{a, b\}$

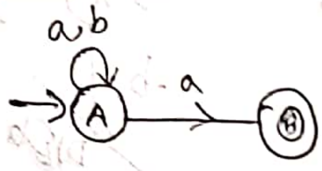
$Q \times \Sigma \quad 2^Q$



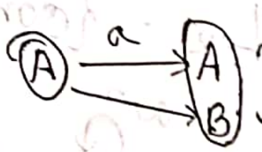
DFA = $Q \times \Sigma \Rightarrow Q$

L = {every string ends with 'a'}

$\Sigma = \{a, b\}$



L = {a, aa, ba .....}



} No determinism

ab =

$A \xrightarrow{a} A \xrightarrow{b} A$ ✓

$A \rightarrow B \rightarrow \phi$

Every DFA is NFA.

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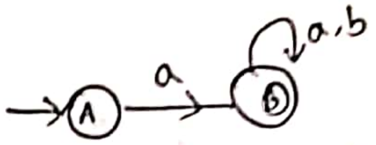
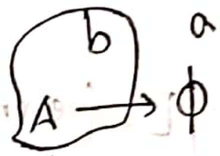
(1) L_1 = { starts with 'a' }

$\Sigma = \{a, b\}$

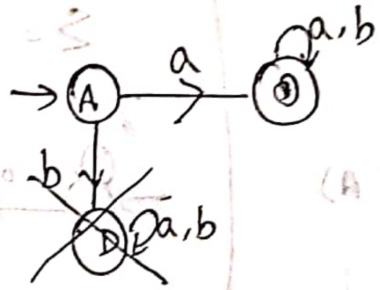


$$A \xrightarrow{a} B \xrightarrow{b} B$$

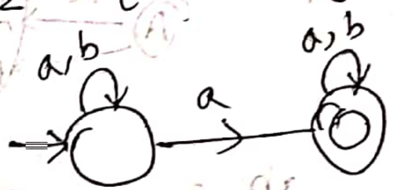


ϕ → Dead configuration.

DFA =



(*) L_2 = {containing a}



(*) L_3 = {Ends with a}

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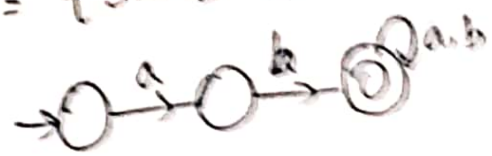
(2) L_4 = { starts with 'ab' }



(3) L_5 = { contains 'ab' }

(4) L_6 = { Ending with 'ab' }

Conversion of NFA to DFA

L_1 = { starts with 'a' }

$\Sigma = \{a, b\}$

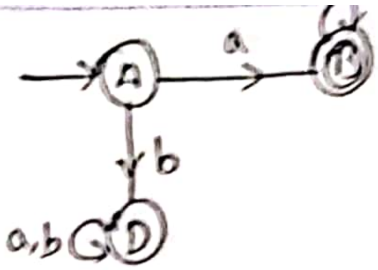
NFA :

State transition table

	a	b
→ A	B	ϕ
*B	B	B

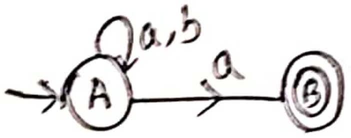
DFA transition table

	a	b	
A	B	D	→ Dead state
*B	B	B	
D	D	D	



⊛ L_1 = {ends with 'a'}

$\Sigma = \{a, b\}$



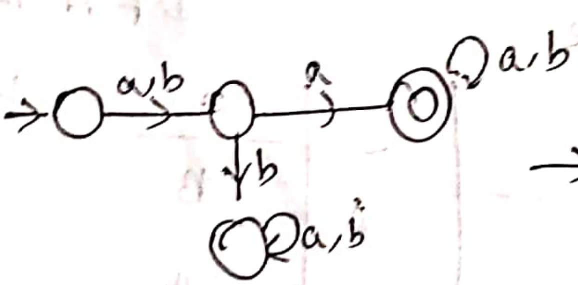
DFA

	a	b
→A	{A,B}	{A}
*B	{ }	{ }

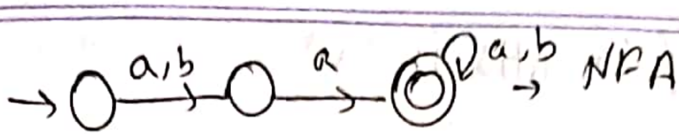
	a	b
→A	[AB] (new state)	[A]
AB	[AB]	[A]

A ∪ B

⊛ Second symbol from LHS 'a'

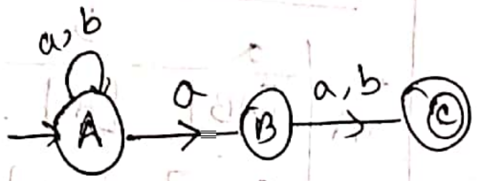


→ DFA



Second symbol from RHS 'a'

NFA = L = { aa , ab , aaa , aab . . . . . }



$\Sigma^* a (a+b)$

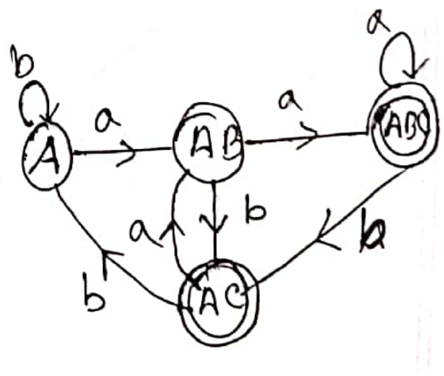
$\Sigma^* a a$ $\Sigma^* a b$

	a	b
→A	{A, B}	{A}
B	{C}	{C}
*C	{ }	{ }

A∪B

DFA → Table :

	a	b
→[A]	[AB]	[A]
[AB]	[ABC]	[AC]
*[AC]	[AB]	[A]
*[ABC]	[ABC]	[AC]



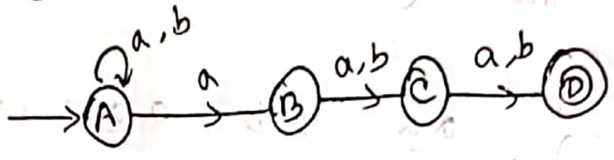
bbbaaaaaa

bbbaab@bba . . .

(i) 3rd symbol from RHS is 'a'

$\Sigma^* a$ aa
ab
ba
bb

NFA :-



	a	b
→A	{A,B}	{A}
B	{C}	{C}
C	{D}	{D}
*D	{ }	{ }

DFA

	a	b
→A	[AB]	[A]
AB	[ABC]	[AC]
AC	[ABD]	[AD]
ABC	[ABCD]	[ACD]
*AD	[AB]	[A]
*ABD	[ABC]	[AC]
*ABCD	[ABCD]	[ACD]
*ACD	[ABD]	[AD]

NFA → 'n' states in DFA → worst case 2^n states.

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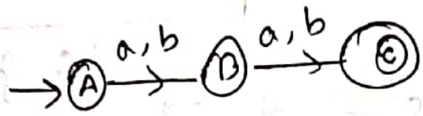
Construction of NFA

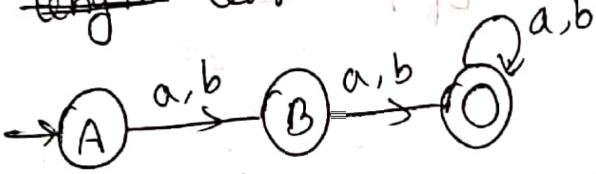
* strings of length "2"

$L = \{aa, ab, ba, bb\}$

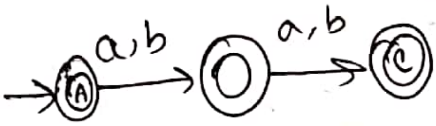
$\Sigma = \{a, b\}$



* at ~~length~~ least "2"



* at most "2"



Strings of length "n"

NFA → 'n' states.

NFA will have less no. of states.

L_1

$n(D_1) \geq n(N_1)$

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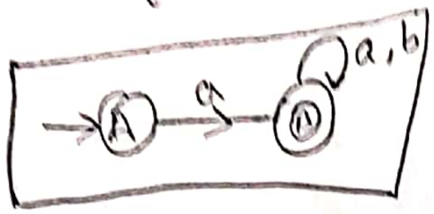
Complementation of NFA

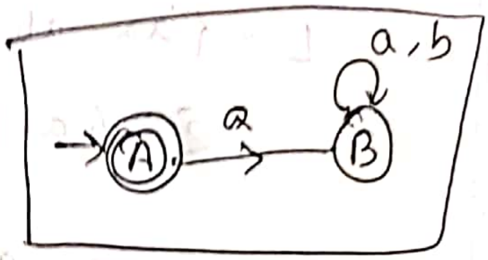
(8) NFA {a, b}

L = { starts with 'a' }

M =



$\overline{M}$ =



$L_1 = \{a, aa, ab, \dots\} \Longleftrightarrow L_2 = \{\epsilon\}$

$\overline{L_1} \neq L_2$

Σ^*

L_1	$\overline{L_1}$
Starts with a	Starts with 'b' and ϵ

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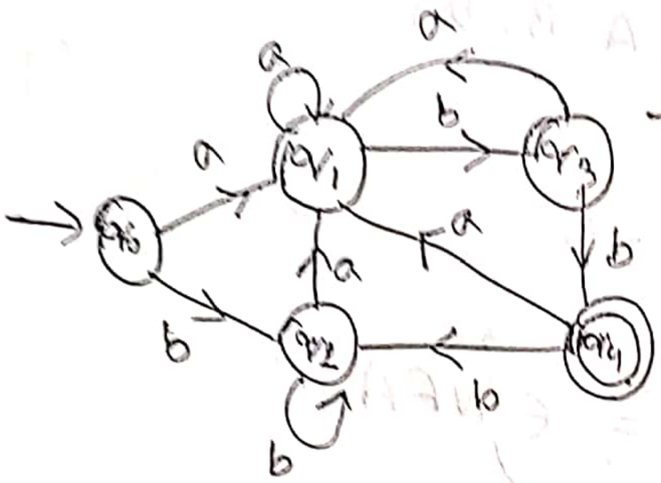
(P, q) equivalent :

$\delta(P, \omega) \in F \Rightarrow \delta(q, \omega) \in F$ $\qquad$ $\delta(P, \omega) \notin F \Rightarrow \delta(q, \omega) \notin F$.

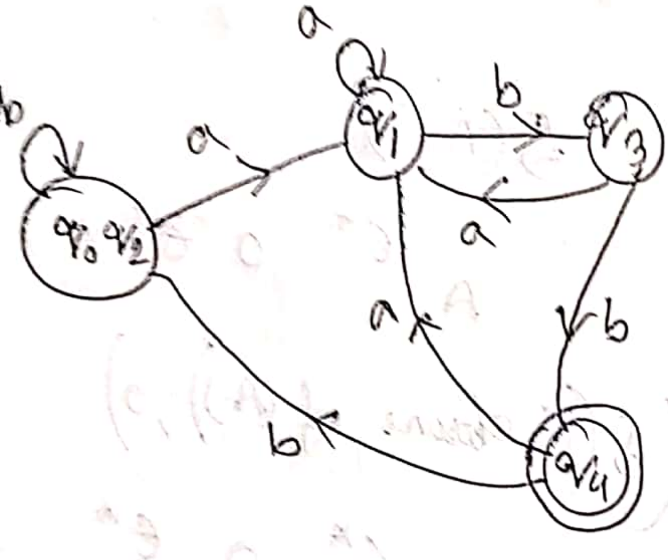
$|\omega| = 0$, 0 equivalent. $\qquad$ $|\omega| = n$, n equivalent.

$|\omega| = 1$, 1 equivalent.

$|\omega| = 2$, 2 equivalent.



	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	$*q_4$
$*q_4$	q_1	q_2



0 equivalent states $[q_0\ q_1\ q_2\ q_3]$ $[q_4]$

1 equivalent $[q_0, q_1, q_2]$ $[q_3]$ $[q_4]$

2 equi. $[q_0, q_2]$ $[q_1]$ $[q_3]$ $[q_4]$

3 equivalent $[q_0, q_2]$ $[q_1]$ $[q_3]$ $[q_4]$

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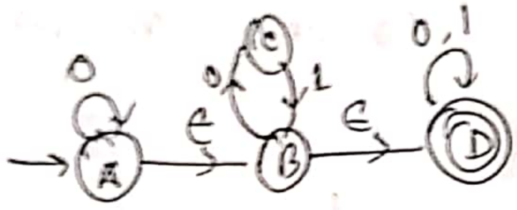
Epsilon NFA (ε-NFA)

$(Q, \Sigma, \delta, q_0, F)$

$\delta = Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^Q$



ε - closure (A) = {A, B, D}

$A \xrightarrow{\epsilon} A$

$A \xrightarrow{\epsilon} B \xrightarrow{\epsilon} D$

DFA ≅ NFA ≅ ε NFA

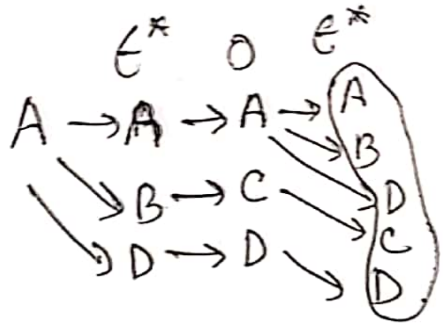
ε NFA → NFA.

~~δ(A, 0)~~

	0	1
A	{A, B, C, D}	
B		
C		
D		

A ϵ^* 0 ϵ^*

ε closure $(\delta((\epsilon \text{ closure of } (A)), 0))$



Context Free Grammar

Anisul Islam

A CFG is a set of rules / productions used to generate patterns of strings.

CFG → 4 tuples $(\underset{N}{V}, \underset{\Sigma}{T}, \underset{R}{P}, S)$

V ⇒ Variable / Non Terminals

T ⇒ set of terminals

$A \to \alpha$, $\alpha \in (V \cup T)^*$ ← P ⇒ productions / Rules.

S ⇒ Start variable.

⊛ CFG for $(0^n 1^n \mid n \geq 1)$

$0^1 1^1 = \{01\}$

$0^2 1^2 = \{0011\}$

$\underset{L.S}{\underline{S}} \Rightarrow \underset{R.S}{\underline{01 / 0S1}}$.

OR

$S \Rightarrow 01$

$S \Rightarrow 0S1$.

V = Left side এ যেগুলো use করা হয়। Capital letter

T = Right side এ যা কিছু থাকে সেগুলো Terminal. Variable ছাড়া. $\{0, 1\}$

P = $S \Rightarrow 01 / 0S1$

S = S

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① Identify the terminals, non terminals, start variable from the following grammar.

① $E \Rightarrow E + T \mid T$

$T \Rightarrow T * F \mid F$

$F \Rightarrow (E) \mid id$

② $S = (L) \mid a$

$L = L, S \mid S$

Non terminal on

$V = \{E, T, F\}$

$T = \{+, *, (,), id\}$

$S = E$

$V = \{S, L\}$

$T = \{\text{"a"}, \text{"("}, \text{")"}, \text{","}\}$

$S = S$.

$S \rightarrow 01 \mid 0S1$.

$S \rightarrow 0S1$

$\rightarrow 0011$

000111

$S \rightarrow 01 \mid 0S1$

$S \rightarrow 0S1$

$S \rightarrow 00S11$

$\rightarrow 000111$

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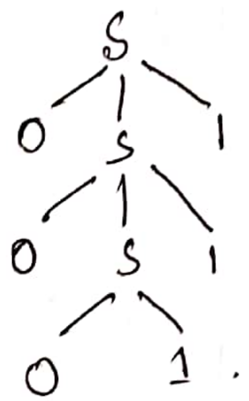
Parsing

$S \to 01 \mid 0S1$

0011

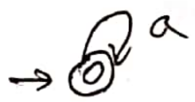
000111



Ex

$L = \{a^n \mid n \geq 0\}$

$= \{\epsilon, a, aa, \ldots\ldots\}$



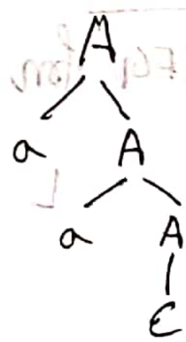
CFG $\boxed{A \Rightarrow aA \mid \epsilon}$ aa

$A \Rightarrow \epsilon$

$A \Rightarrow aA$

$\Rightarrow aaA$

$\Rightarrow aa\epsilon$



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Ex-2 $L = \{a^n \mid n \geq 1\}$ $\qquad a^+$

$= \{a, aa, aaa, \ldots\}$

$A \Rightarrow aA \mid a$.

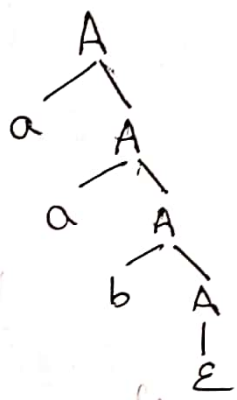
Ex-3

$L = \{$set of all strings over $a, b\}$

$(a \mid b)^*$.

$A \Rightarrow aA \mid bA \mid \varepsilon$

aab



Ex-4

CFG for set of strings which is of length at least 2.

$L = \{aa, ab, ba, bb, aaa, \ldots\}$

$$(a+b)\underbrace{(a+b)}_{A}\underbrace{(a+b)^*}_{B}$$

$A \to a|b$
$B \to aB|bB|\varepsilon$

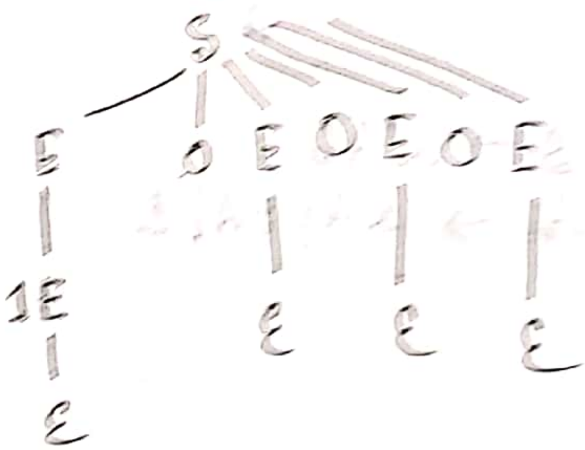
abb

Ex-5 CFG for set of all strings of at least 3 0s.

$$(0+1)^*0(0+1)^*0(0+1)^*0(0+1)^*$$

$S \to E0E0E0E$
$E \to 0E|1E|\varepsilon$

1000



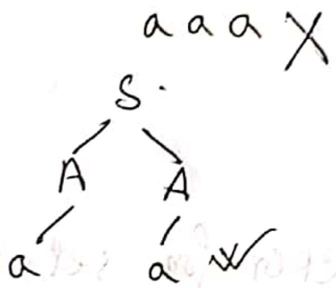
Ex-6 : set of all strings which is of length at most 2

$L = \{ \epsilon, a, b, aa, ab, ba, bb \}$

RE $\Rightarrow$ $\underbrace{(a+b+\epsilon)}_{A} \; \underbrace{(a+b+\epsilon)}_{A}$

$S \rightarrow AA$

$A \rightarrow a \mid b \mid \epsilon$



Ex-7

Starts with a and ends with b.

$a\underbrace{(a+b)^*}_{A}b$.

$S \rightarrow aAb$
$A \rightarrow aA \mid bA \mid \epsilon$

Scanned by CamScanner

Ex-8 Starts ~~as~~ and ends with different symbol.

$$a\,(a+b)^*\,b \quad | \quad b\,(a+b)^*\,a$$

$$= a\,\underbrace{(a+b)^*}_{A}\,b + b\,\underbrace{(a+b)^*}_{A}\,a$$

$S \Rightarrow aAb \mid bAa$

$A \Rightarrow aA \mid bA \mid \varepsilon$

Ex-9 Starts and ends with similar symbol.

$$a(a+b)^*a \mid b(a+b)^*b \mid \varepsilon \mid a \mid b$$

$S \Rightarrow aAa \mid bAb \mid \varepsilon \mid a \mid b$.

$A \Rightarrow aA \mid bA \mid \varepsilon$

Ex-10 : Even length string.

abab.

$$((a+b)(a+b))^*$$

$S \Rightarrow BS \mid \varepsilon$

$B \rightarrow AA$.

$A \rightarrow a \mid b$

Scanned by CamScanner

V → Set of all vertices

T → set of all terminals

P → set of all production

S → start symbol.

Ex

$$P = \left\{ \begin{array}{l} S \rightarrow aSb \\ S \rightarrow aB \\ B \rightarrow b \end{array} \right\}$$

→LHS

$V = \{S, B\}$

$T = \{a, b\}$

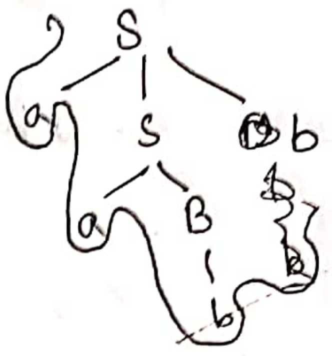
$S = S$

l.m **Derivation of aabb**

$$\left.\begin{array}{l} S \Rightarrow aSb \\ \Rightarrow \cancel{aaBb}\ aaBb \\ \Rightarrow aabb \end{array}\right\} \text{Sentential forms}$$

$\Rightarrow \boxed{aabb} \rightarrow$ string

Parse Tree



$\boxed{aabb}$

$L = \{aa, ab, ba, bb\}$

$S \rightarrow aa/ab/ba/bb$

$\underbrace{(a+b)}_{A}\ \underbrace{(a+b)}_{A}$

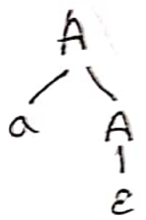
$S \rightarrow AA.$

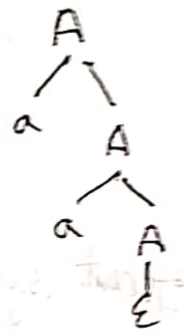
$A \rightarrow a/b$

⊛ $a^n / n \geq 0.$

$a, aa, aaaa, ------,$

$\boxed{A \rightarrow aA/\epsilon}$ $\boxed{A \rightarrow Aa/\epsilon}$

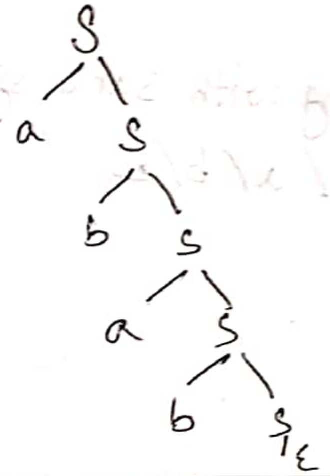




⊛ $(a+b)^*$

$S \rightarrow aS/bS/\epsilon$

abab



Scanned by CamScanner

* Set of all strings of length at least '2'

$(a+b)(a+b)(a+b)^*$

$S \rightarrow AAB$

$A \rightarrow a/b$

$B \rightarrow aB/bB/\epsilon$

* At most '2'

$(a+b+\epsilon)(a+b+\epsilon)$

$S \rightarrow AA$

$A \rightarrow a/b/\epsilon$

* starts with 'a and ends with 'b'

$a(a+b)^* b$.

$S \rightarrow aAb$

$A \rightarrow aA/bA/\epsilon$

* starting and ending with different symbols.

$a\underbrace{(a+b)^*}_{A} b \,|\, b\underbrace{(a+b)^*}_{A} a$

$S \rightarrow aAb \,|\, bAa$

$A \rightarrow aA \,|\, bA \,|\, \epsilon$

* starting and ending with same symbol.

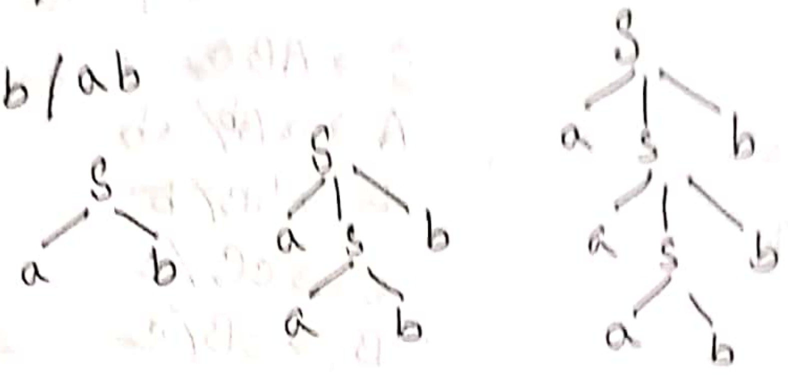
$S \rightarrow aAa / bAb / a / b / \epsilon$

$A \rightarrow aA / bA / \epsilon$

Scanned by CamScanner

(*) $a^n b^n / n \geq 1$.

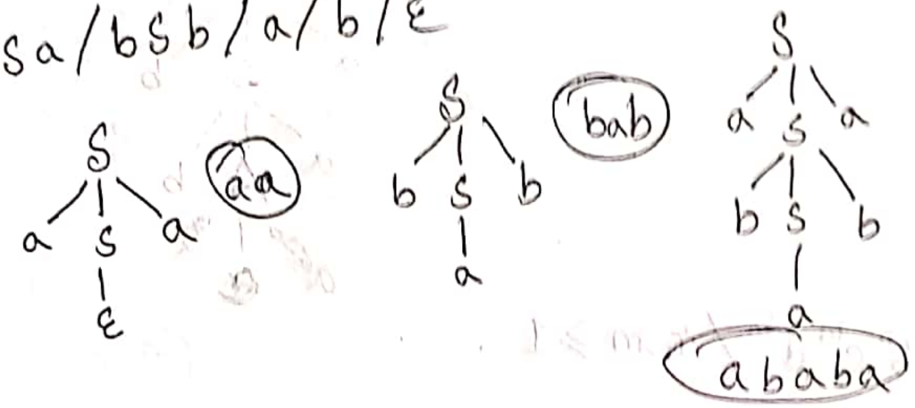
$S \to aSb / ab$

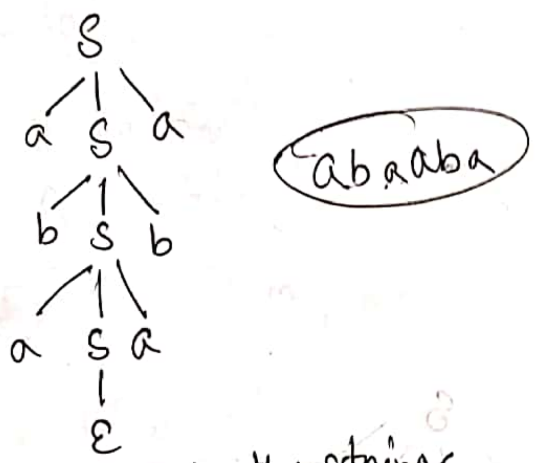


(*) ww^R, $\cup\ waw^R \cup wbw^R$ even length and odd length palindrome.

$w \in (a,b)^*$

$S \to aSa / bSb / a / b / \varepsilon$





(*) Even length strings $((a+b)(a+b))^*$

$B \to AA$ $S \to BS/\varepsilon$

$A \to a/b$

Scanned by CamScanner

* $a^n b^m / n, m \geq 1$.

$S \to AB$

$A \to aA / a$

$B \to bB / b$

* $a^n b^n c^m / n, m \geq 1$.

$S \to AB$

$A \to aAb / ab$

~~$B \to bB / b$.~~

~~$C \to cC / c$~~

$B \to cB / c$

* $a^n c^m b^n / n, m \geq 1$.

$S \to aSb / aAb$

$A \to cA / c$

aacbb

* $a^n b^n c^m d^m / n, m \geq 1$.

$S \to AB$

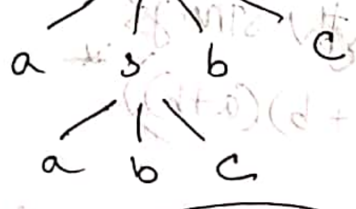
$A \to aAb / ab$

$B \to cBd / cd$

* $a^n b^n c^n > n \geq 1$.

$S \to aSbc / abc$

Not possible

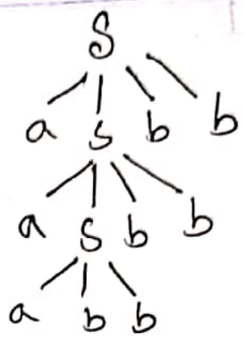


aabcbc

$a^n (bc)^n$

$\circledast$ $a^n b^{2n} / n \geq 1$.

$S \to aSbb / abb$.



$\underline{\underline{a^3 b^6}}$

$\circledast$ $a^n b^m c^m d^n / n, m \geq 1$.

$S \to aSd / aAd$

$A \to bAc / bc$

$\circledast$ $a^n b^m c^n d^m / n, m \geq 1$

$\otimes$ NOT Possible

$\circledast$ $a^{m+n} b^m c^n / m, n \geq 1$.

$a^n a^m b^m c^n$.

$S \to aSc / aAc$

$A \to aAb / ab$

$\circledast$ $a^n b^{n+m} c^m / n, m \geq 1$.

$\underline{a^n b^n}\ \underline{b^m c^m}$

$S \to AB$

$A \to aAb / ab$

$B \to bBc / bc$

Scanned by CamScanner

* $a^n b^m c^{n+m} / n,m \geq 1$.

$a^n b^m c^m c^n$.

$S \to aSc / aAc$

$A \to bAc / bc$

Classification of Grammar

chomsky

(i) Type 3

Type 2

Type 1

Type 0.

Type 3

$\boxed{\begin{array}{l} A \to \alpha B / \beta \\ A, B \in V \\ \alpha, \beta \in T^* \end{array}}$ RLG

$\boxed{\begin{array}{l} A \to B \alpha / \beta \\ A, B \in V \\ \alpha, \beta \in T^* \end{array}}$ LLG

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Type-2 (CFG)

Ambiguous — Unambiguous

CFG: D, N

CFG: LR, RR

* w ∈ L(G)

① step

$S \to \alpha$

② Steps

$S \Rightarrow aAb$

$\Rightarrow abb$

3 steps

$S \Rightarrow aABb$

$\Rightarrow aaBb$

$\Rightarrow aabb$

Scanned by CamScanner

PDA → Push down Automata

Finite Automata
+
stack.

$(Q, \Sigma, \delta, q_0, z_0, F, \Gamma)$

Q : Finite set of states

Σ : input symbol

δ : Transition function

q_0 : Initial state

z_0 : Bottom of the stack

F : set of final states

Γ : stack alphabet

PDA

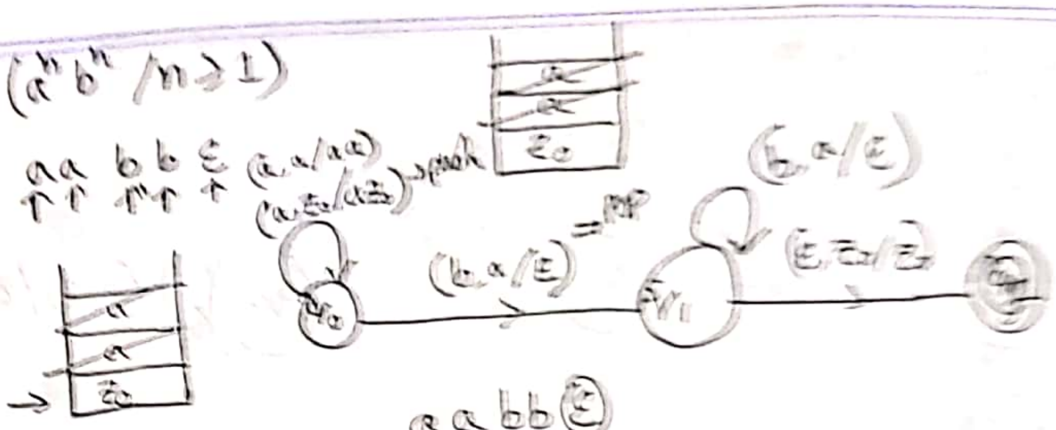
DPDA   NPDA

$\delta : Q \times \{\Sigma \cup \varepsilon\} \times \Gamma \rightarrow Q \times \Gamma^*$

$\delta : \{\Sigma \cup \varepsilon\} \times \Gamma \rightarrow 2^{(Q \times \Gamma^*)}$

Scanned by CamScanner

(*) $(a^n b^n / n \geq 1)$



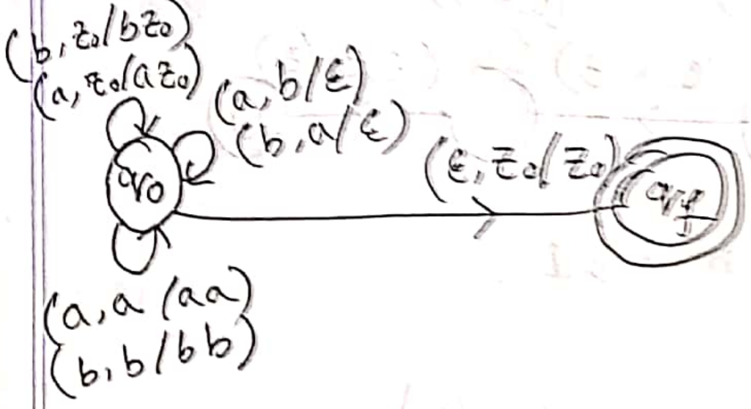
a a b b ε

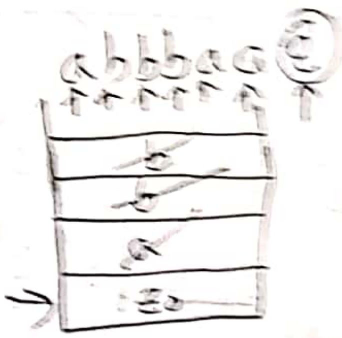
PDA
- Final state
- Empty stack

$$\delta(q_0, a, z_0) = (q_0, az_0)$$
$$\delta(q_0, a, a) = (q_0, aa)$$
$$\delta(q_0, b, a) = (q_1, \varepsilon)$$
$$\delta(q_1, b, a) = (q_1, \varepsilon)$$
$$\delta(q_1, \varepsilon, z_0) = (q_f, z_0)$$

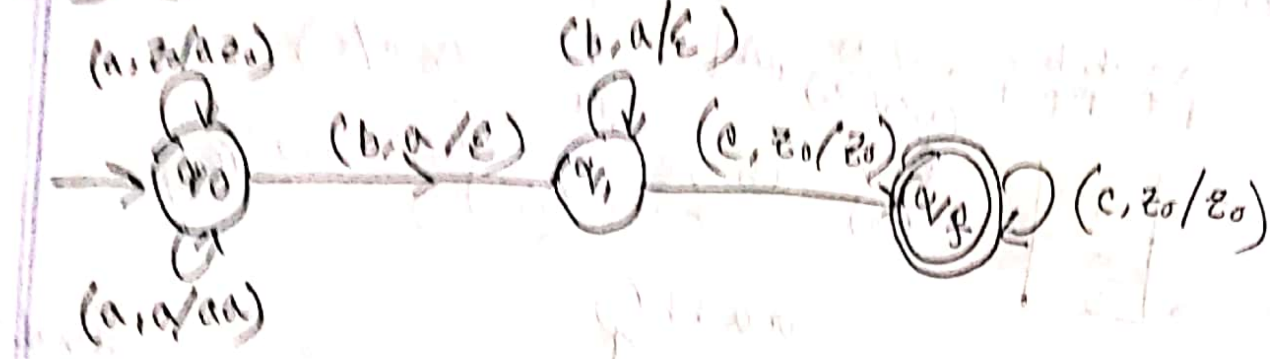
(*) $w / n_a(w) = n_b(w)$

ab bbaa abba
aabb baba
abab .

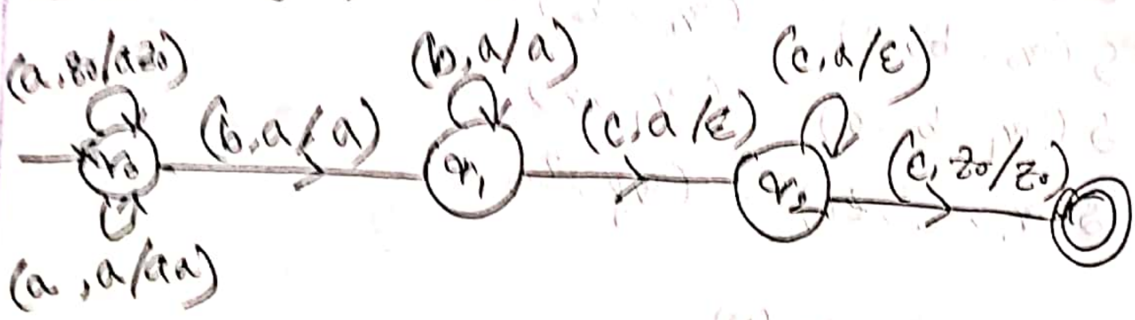




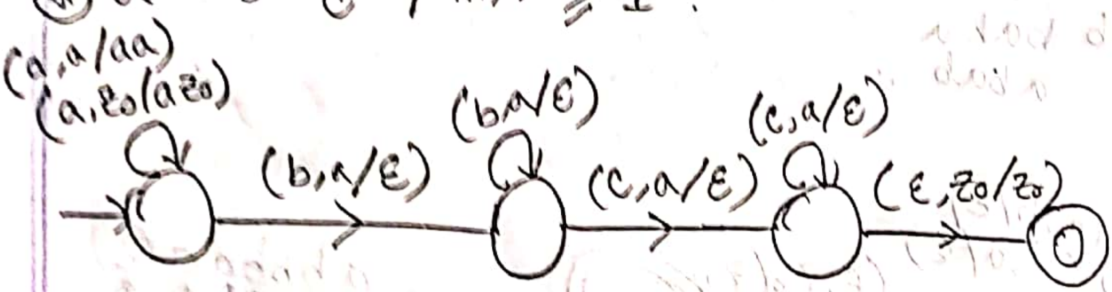
⊛ $a^n b^n c^m \,/\, n,m \geq 1$



⊛ $a^n b^m c^n \,/\, n,m \geq 1$

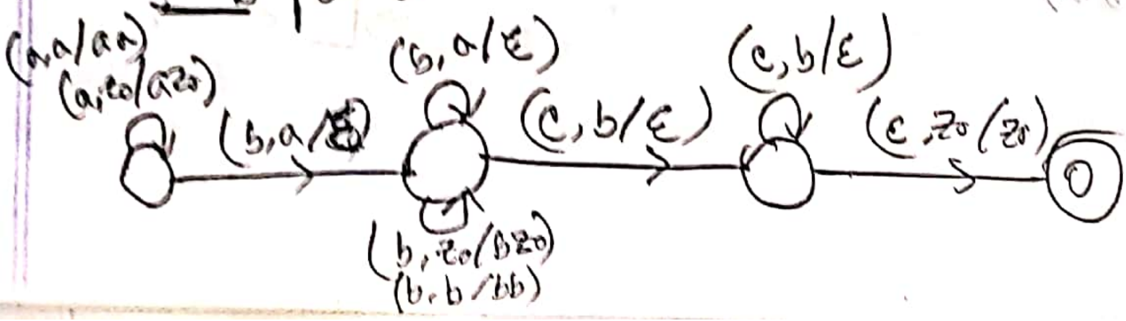


⊛ $a^{m+n} b^m c^n \,/\, m,n \geq 1$

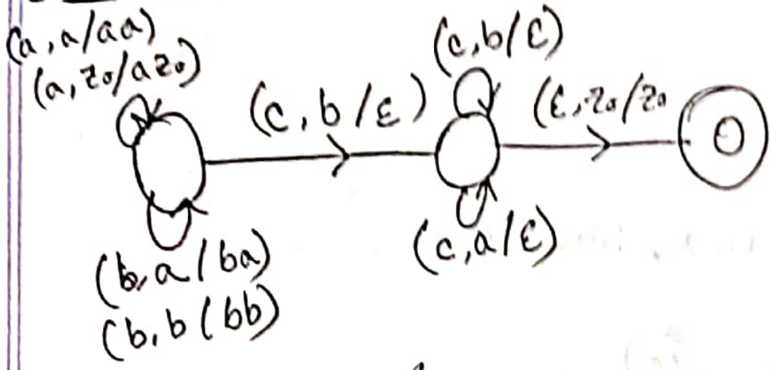


⊛ $a^n b^{m+n} c^m \,/\, n,m \geq 1$

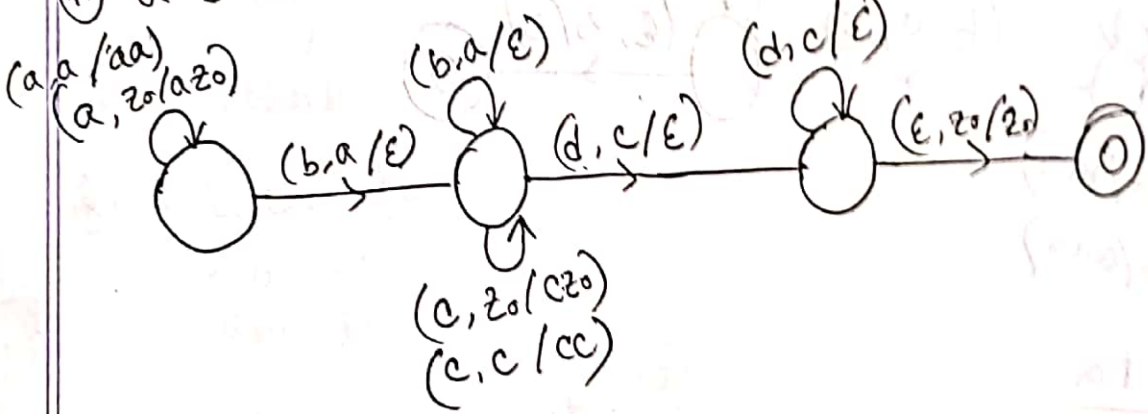
$a^n b^n \,|\, b^m c^m$



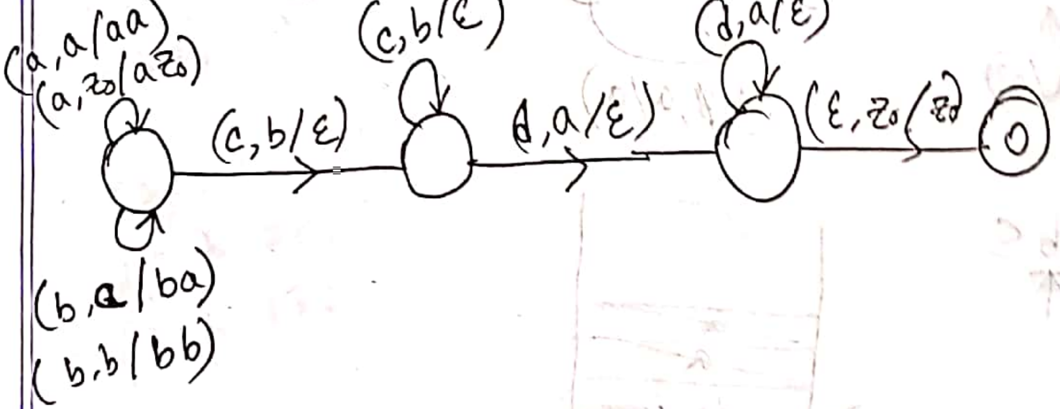
* $a^n b^m c^{n+m} / n,m \geq 1$.



* $a^n b^n c^m d^m / n,m \geq 1$.



* $a^n b^m c^m d^n / n,m \geq 1$.



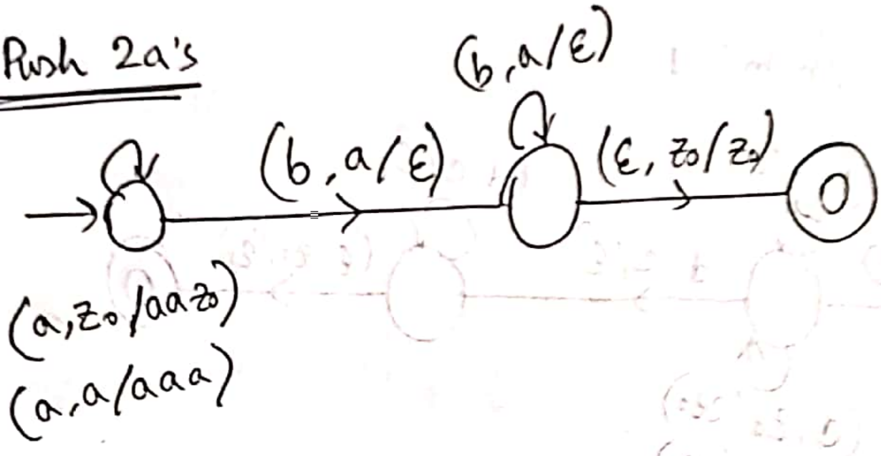
* $a^n b^m c^n d^m$ / $n,m \geq 1$.

Not possible

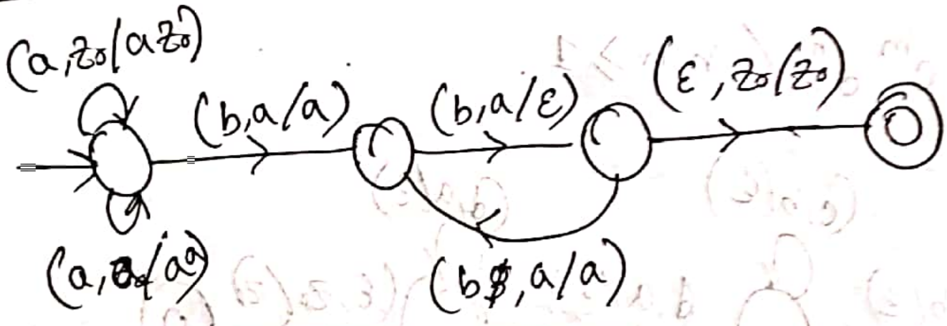
* $a^n b^{2n}$ / $n \geq 1$.

abb, aabbbb, aaa, bbbbbb, ------

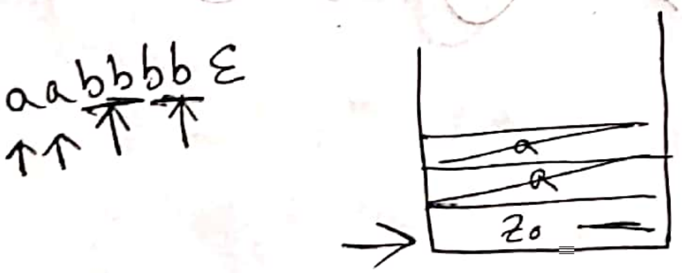
Push 2a's



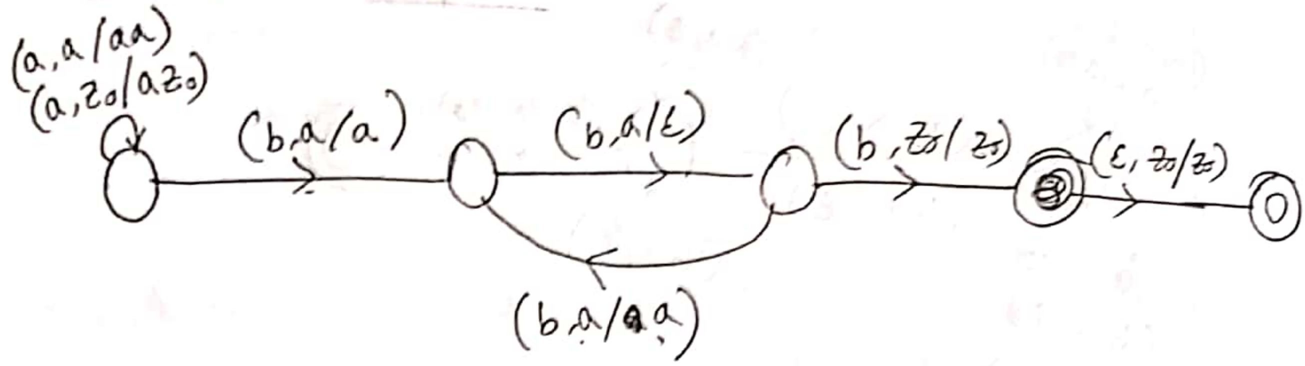
Push 1 a



aabbbbε



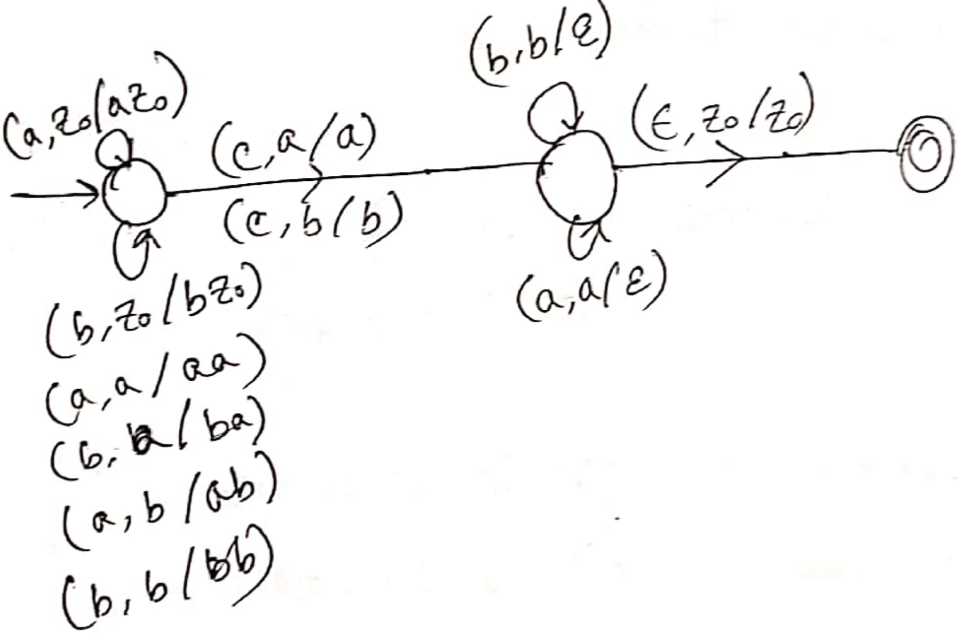
$a^n b^{2n+1}, n \geq 1$.



* $a^n b^n c^n / n \geq 1$.
Not possible.

* $wcw^R / w \in (a,b)^+$.

abcba
abbcbba



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a,b

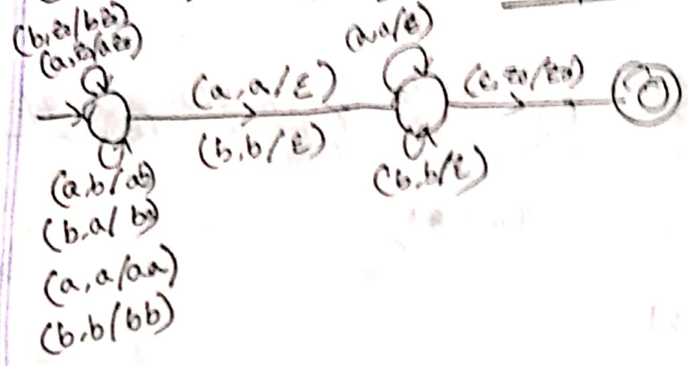
~~aaaabbbb~~

aaaa |b|

NPDA

(#) $ww^R / w \in (a,b)^+$

$\underline{ab|ba}$ $\underline{aa|aa}$



CNF

Also, Eliminate start symbol from RHS.

1. Eliminate ε - productions.
2. Eliminate unit productions.
3. Eliminate variables that derive no terminal string.
4. Eliminate variables not reached from the start symbol.

A CFG is said to be in CNF in every production is one of these two forms.

1. $A \rightarrow BC$ (right side is two variables)
2. $A \rightarrow a$ (right side is single terminal)

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$S \to A1B$

$A \to 0A \mid \epsilon$

$B \to 0B \mid 1B \mid \epsilon$.

$\Downarrow$

$S \to A1B \mid 1B$

$A \to 0A \mid 0$

$B \to 0B \mid 1B \mid \epsilon$.

$\Downarrow$

$S \to A1B \mid 1B \mid A1 \mid 1$.

$A \to 0A \mid 0$

$B \to 0B \mid 1B \mid 0 \mid 1$

$\Downarrow$

$S \to AXB \mid \cancel{X}B \mid A\cancel{X} \mid 1$.

$A \to 0A \mid 0$

$B \to 0B \mid \cancel{X}B \mid 0 \mid 1$.

$X \to 1$

$\cancel{Y \to A1}$

$\Downarrow$

$S \to AXB \mid XB \mid A\cancel{X} \mid 1$.

$A \to YA \mid 0$.

$B \to YB \mid XB \mid 0 \mid 1$

$S \to AXB \mid XB \mid AX \mid 1$.

$A \to YA \mid 0$

$B \to YB \mid XB \mid 0 \mid 1$.

$X \to 1$

$Y \to 0$

$\Downarrow$

$S \to PB \mid XB \mid AX \mid 1$.

$A \to YA \mid 0$

$B \to YB \mid XB \mid 0 \mid 1$.

$X \to 1$

$Y \to 0$

$P \to AX$.

$X \to 1$

$Y \to 0$

$S \to EcC \mid aAE \mid AU$

$A \to aA \mid \varepsilon$

$B \to bB \mid \varepsilon$

$C \to cC \mid \varepsilon$

$E \to aEc \mid F.$

$F \to bFc \mid \varepsilon$

$U \to aUc \mid V.$

$V \to bVc \mid bB.$

$w = aabcc.$

$\Downarrow$

~~S → EcC | aAE | AU~~

$S \to EcC \mid aAE \mid AU \mid aE \mid U$

$A \to aA \mid a$

$B \to bB \mid b.$

$C \to cC \mid \varepsilon$

$E \to aEc \mid F.$

$F \to bFc \mid \varepsilon.$

$U \to aUc \mid V.$

$V \to bVc \mid bB \mid b.$

Scanned by CamScanner

$S \to EcC \mid aAE \mid AU \mid aE \mid U \mid Ec$

$A \to aA \mid a$

$B \to bB \mid b.$

$C \to cC \mid c$

$E \to aEc \mid F.$

$F \to bFc \mid \varepsilon.$

$U \to aUc \mid V.$

$V \to bVc \mid bB \mid b.$

$\Downarrow$

$S \to EcC \mid aAE \mid AU \mid aE \mid U \mid Ec$

$A \to aA \mid a.$

$B \to bB \mid b.$

$C \to cC \mid c.$

$E \to aEc \mid F \mid \varepsilon.$

$F \to bFc \mid bF.$

$U \to aUc \mid V$

$V \to bVc \mid bB \mid b$

$\Downarrow$

$S \to EcC \mid aAE \mid AU \mid aE \mid U \mid Ec \mid cC \mid aA \mid \cancel{AU} \mid a \mid c$

$A \to aA \mid a.$

$B \to bB \mid b.$

$C \to cC \mid c.$

$E \to aEc \mid F \mid ac.$

$F \to bFc \mid bc.$

$U \to aUc \mid V.$

$V \to bVc \mid bB \mid b.$

Scanned by CamScanner

S → ERC | PAE | AU | ~~A~~ | PE | U | ER | RC | PA | a | c .

A → PA | a .

B → QB | b .

C → RC | c

E → PER | F | PR .

F → QFR | QR

U → PUR | V .

V → QVR | QB | b

P → a

Q → b

R → c

S → ERC | PAE | AU | PE | PUR | QVR | QB | b | ER | RC | PA | a | c .

A → PA | a .

B → QB | b .

C → RC | c

E → PER | QFR | PR | PR .

U → PUR | QVR | ~~QR~~ QB | b

V → QVR | QB | b .

P → a

Q → b

R → c

Scanned by CamScanner

$S \to MC \mid NE \mid AU \mid PE \mid OR \mid XR \mid QB \mid b \mid ER \mid RC \mid PA \mid a \mid c$

$A \to PA \mid a$.

$B \to QB \mid b$.

$C \to RC \mid c$

$E \to PM \mid YR \mid \cancel{QR} \mid QR$

$U \to OR \mid XR \mid QB \mid b$.

$V \to XR \mid QB \mid b$.

$P \to a$

$Q \to b$

$R \to c$

$M \to ER$

$N \to PA$

$O \to PU$

$X \to QV$

$Y \to QF$

$ER \to M$

$PA \to N$

$PU \to O$

$QV \to X$

~~$PE \to Y$~~

$QF \to Y$

Scanned by CamScanner

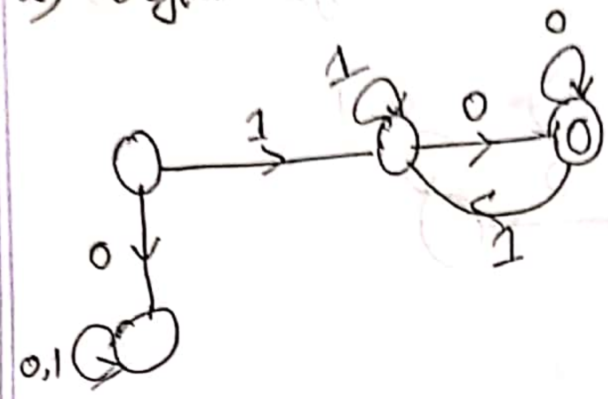
15				
14	25			
13	24	35		
{S, N, A} 12	23	34	45	
{P, A, S} 11	{P A S} 22	{S, B, U, V, Q} 33	{S, C, R} 44	{S, C, R} 55
a	a	b	c	c

PP, PA, PS, AP, AA, AS, SP, SA, SS

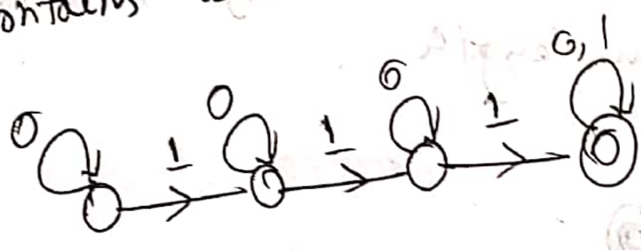
$\downarrow$

S, N, A

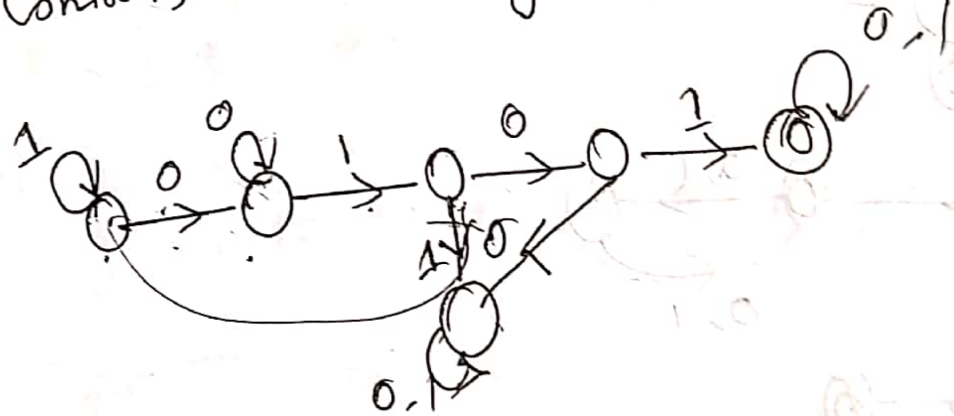
a) begins with 1 and ends with 0.



b) contains at least 3 1s.

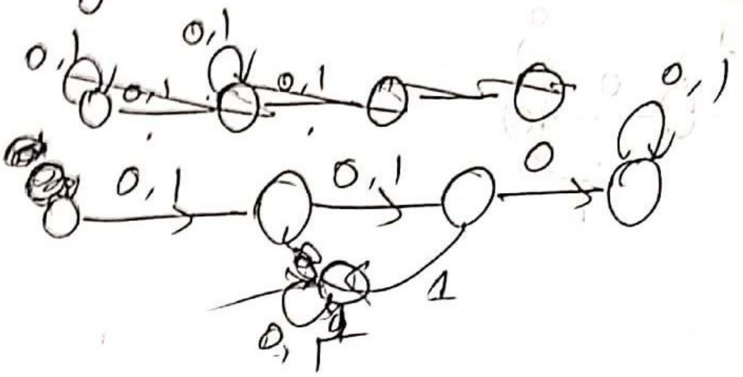


c) contains the substring 0101

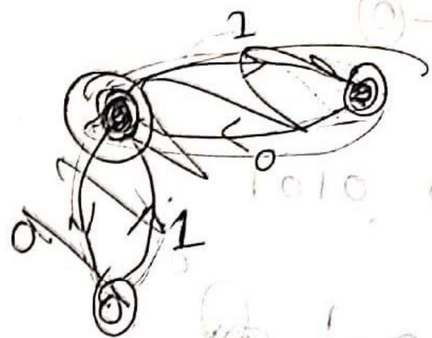


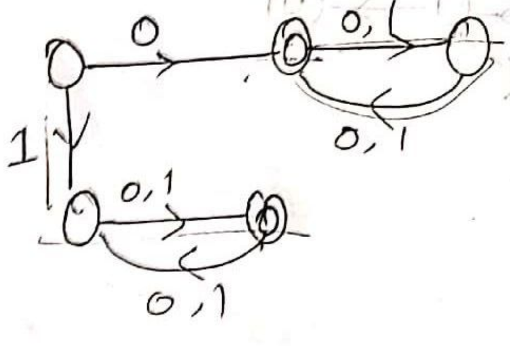
10001 0101

d) length at least 3 and 2nd symbol is 0.



e) starts with 0, and has odd length, or starts with 1 and even length.





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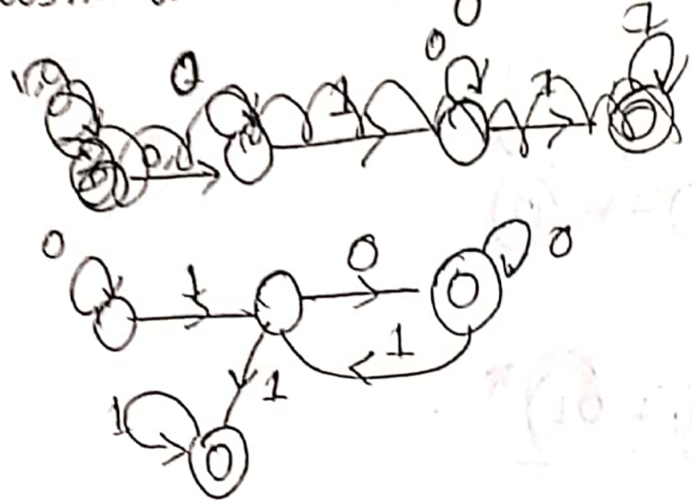
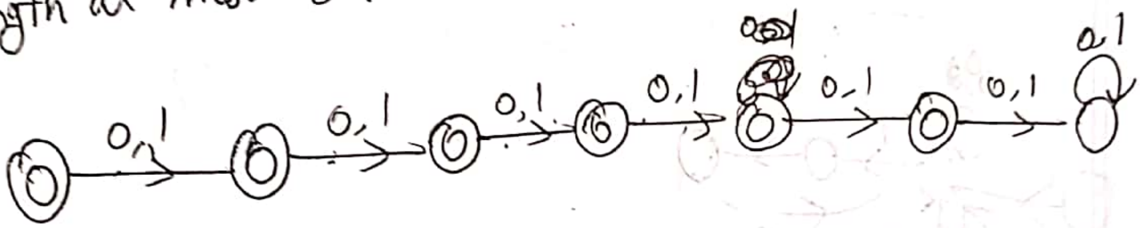
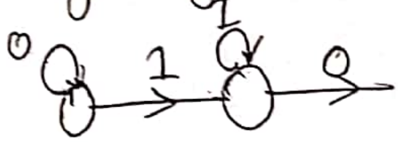
f) does not contain substring 110.



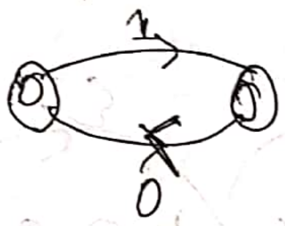
g) length at most 5.



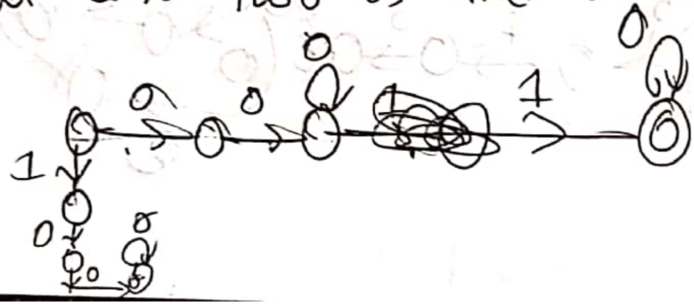
h) any string except 11 and 111.



i) every odd position is 1



j). at least two 0s and at most one 1.



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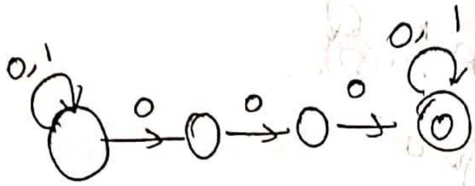
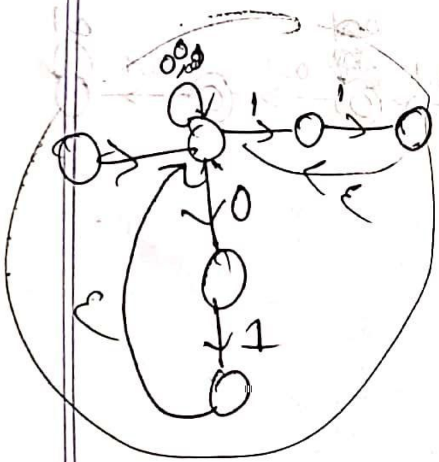
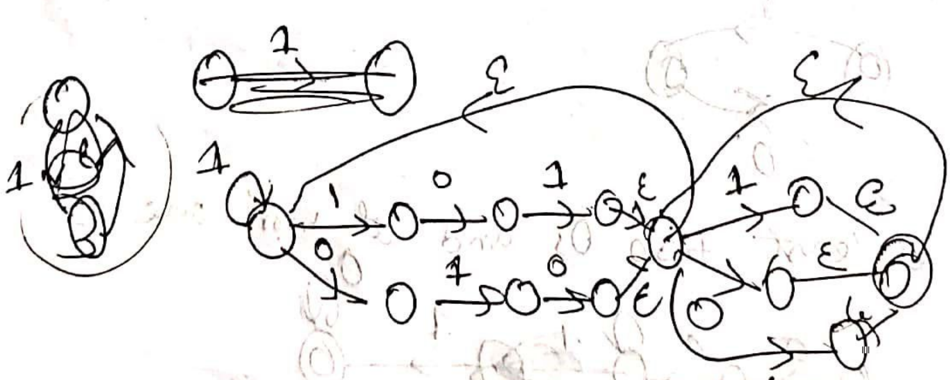
2) a) $(0+1)^* \, 000 \, (0+1)^*$



b) $(((00)^* (11)) + (01))^*$



c) $(1+\varepsilon)^* \, (101+010)^* \, (1+0+\varepsilon)^*$



$\{1\} \rightarrow A$.

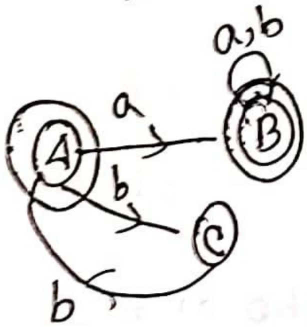
$\{A, a\} \rightarrow \{1, 2\} \rightarrow B$.

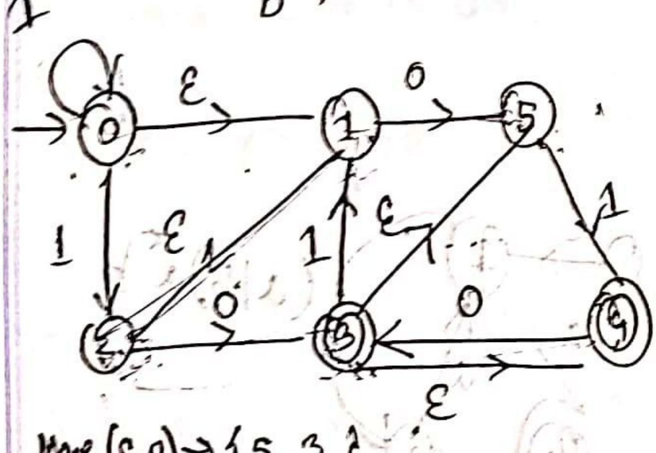
$\{A, b\} \rightarrow \{2\} \rightarrow C$

$\{B, a\} \rightarrow \{1, 2\} \rightarrow B$.

$\{B, b\} \rightarrow \{2, 1\} \rightarrow B$.

$\{C, b\} \rightarrow \{1\} \rightarrow A$. $\{C, a\} \rightarrow \{\emptyset\}$





ε of $0 \rightarrow \{0, 1, 2\} \rightarrow A$

Move$(A, 0) \rightarrow \{5, 3\}$

ε of $\{5, 3\} \rightarrow \{5, 3, 4\} \rightarrow B$.

Move$(A, 1) \rightarrow \{0, 2\}$

~~ε of $\{1, 2\} \rightarrow \{1, 2\} \rightarrow C$.~~

~~Move$(B, 0) \rightarrow \{3\}$~~

~~ε of $\{$~~

ε of $\{0, 2\} \rightarrow \{0, 1, 2\} \rightarrow A$.

Move$(B, 0) \rightarrow \{3\}$

ε of $\{3\} \rightarrow \{3, 4\} \rightarrow B$

Move$(B, 1) \rightarrow \{1, 4\}$

ε of $\{1, 4\} \rightarrow \{1, 2, 4\} \rightarrow C$

Move$(C, 0) \rightarrow \{5, 3\}$

ε of $\{5, 3\} \rightarrow \{5, 3, 4\} \rightarrow B$.

Move$(C, 1) \rightarrow \{\cancel{3}\}$

ε of $\{\cancel{3}\} \rightarrow$ ~~$\{3, 4\} \rightarrow B$~~

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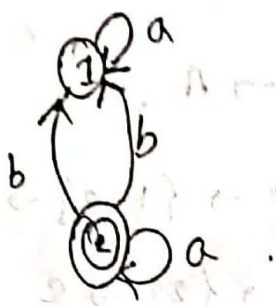

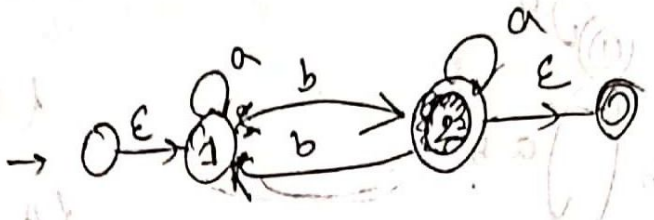

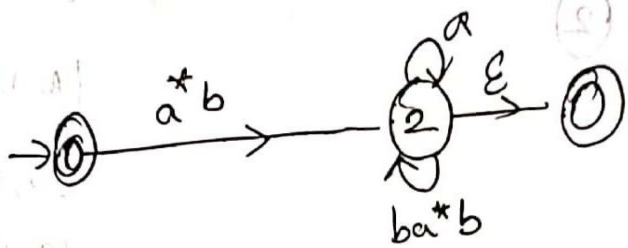

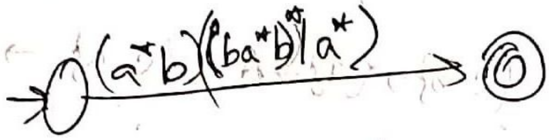

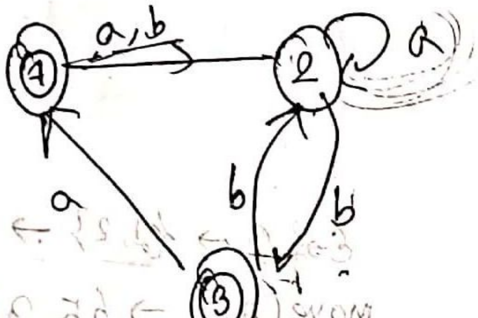

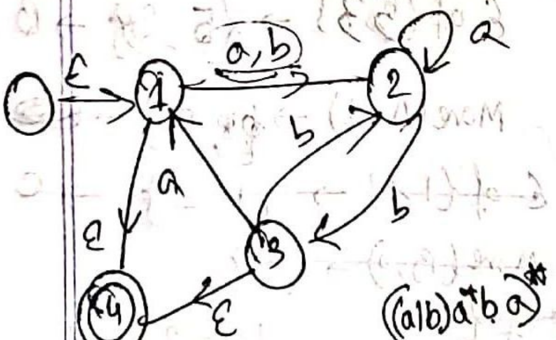

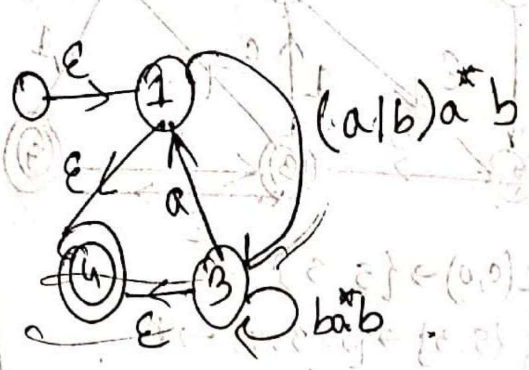

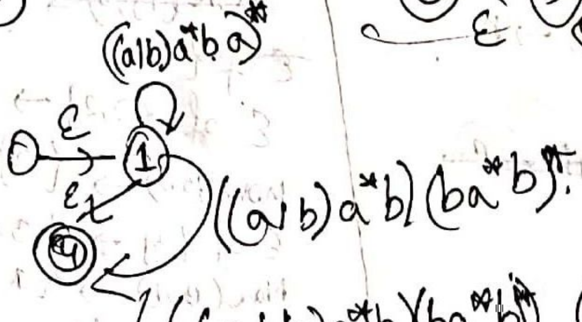

$(((a|b)a^*b)(ba^*b)^*)((a|b)a^*ba)^*)\,|\,\varepsilon$

5)

a) at least 3 1's.

{111, 0111, 0101010 . . . } $\cancel{(0|1)^* 1 0 1 (0|1)^*}$ $(0|1)^* 1 (0|1)^* 1 (0|1)^* 1 (0|1)^*$

$S \rightarrow A1A1A1A$.

$A \rightarrow 0A \mid 1A \mid \varepsilon$

b) starts and ends with same symbol.

$0(0|1)^*0 \mid 1(0|1)^*1$

$S \rightarrow 0A0 \mid 1A1$

$A \rightarrow 0A \mid 1A \mid \varepsilon$

c) length of w is odd.

$(0|1)(00|01|10|11)^*$.

$S \rightarrow 0A \mid 1A$

$A \rightarrow BA \mid CA \mid DA \mid EA \mid \varepsilon$

$B \rightarrow 00$

$C \rightarrow 01$

$D \rightarrow 10$

$E \rightarrow 11$

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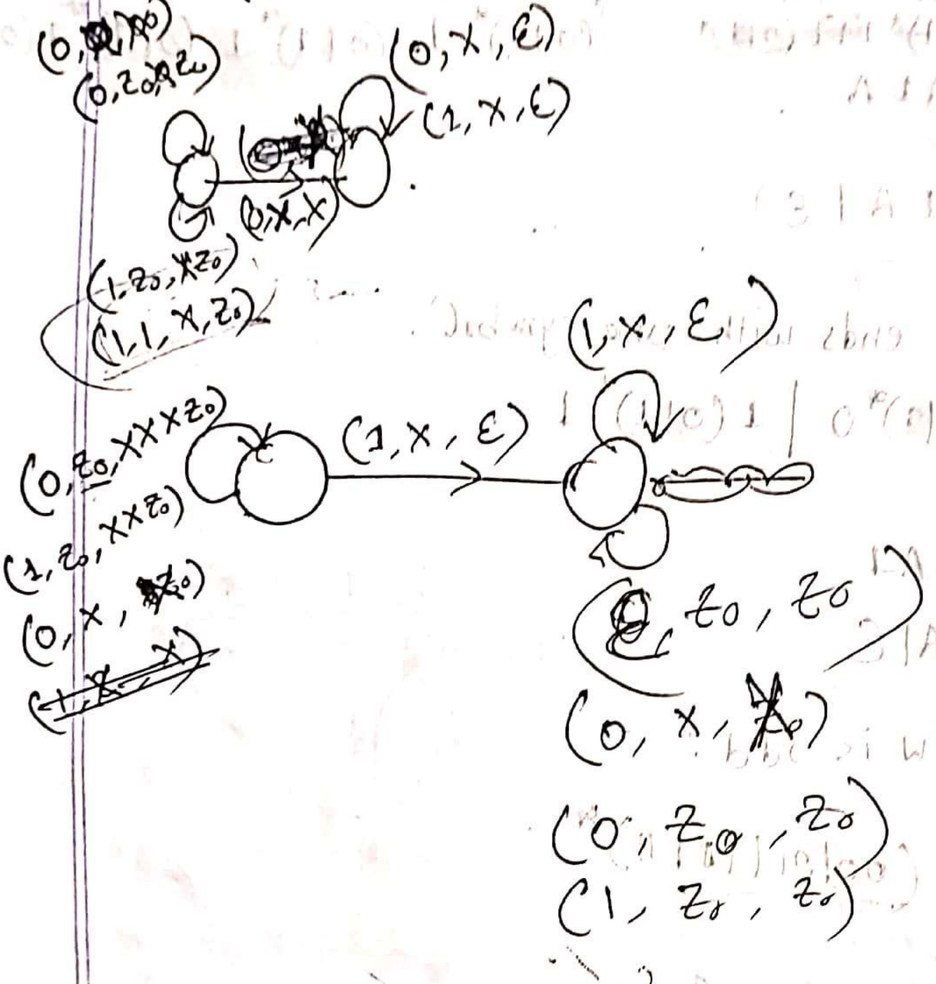
d) w is odd and middle is 0

$$((0+1)(0+1))^{n}\ 0\ ((0+1)(0+1))^{n}$$



$a^i b^j c^k \mid i, j, k >= 0$. and either $i = j$ or $j = k$.

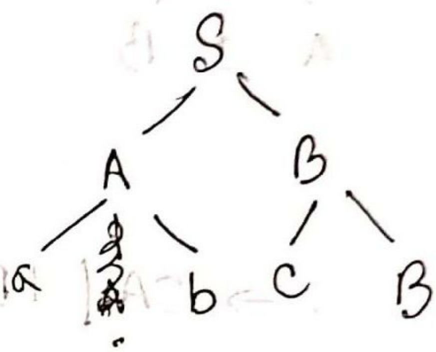
$i=1,\ j=2 \mid j=1,\ k=1$; abcc

$S \to \cancel{aSbc \mid abc}\ A \mid B$.

$A \to aAb \mid ab \mid \cancel{cA}\ Ac$

$B \to \cancel{aB}\ bBc \mid bc \mid \cancel{cB}\ Bc$

~~aAb~~



a) 0010101

LM

$S \to A1B$

$\to 0A1B$

$\to 00A1B$

$\to 001B$

$\to 0010B$

$\to 00101B$

$\to 001010B$

$\to 0010101B$

$\to 0010101$.

$S \to A1B$

$A \to 0A \mid \varepsilon$

$B \to 0B \mid 1B \mid \varepsilon$

RM

$S \to A1B$

$\to A10B$

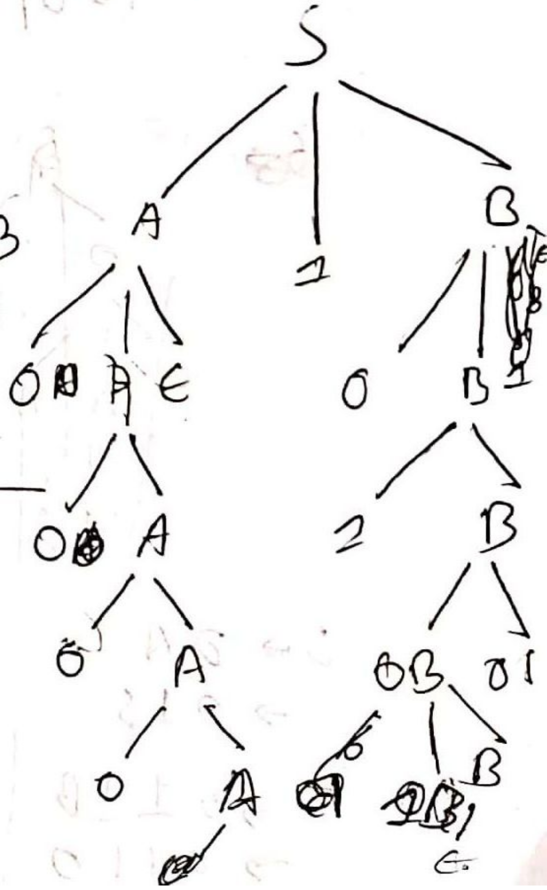
$\to A101B$

$\to A1010B$

$\to A10101$

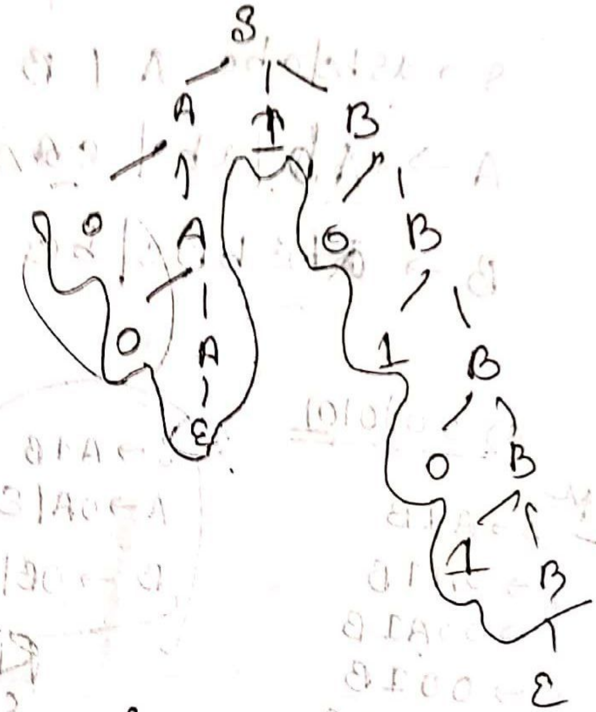
$\to 00A10101$

$\to 0010101$

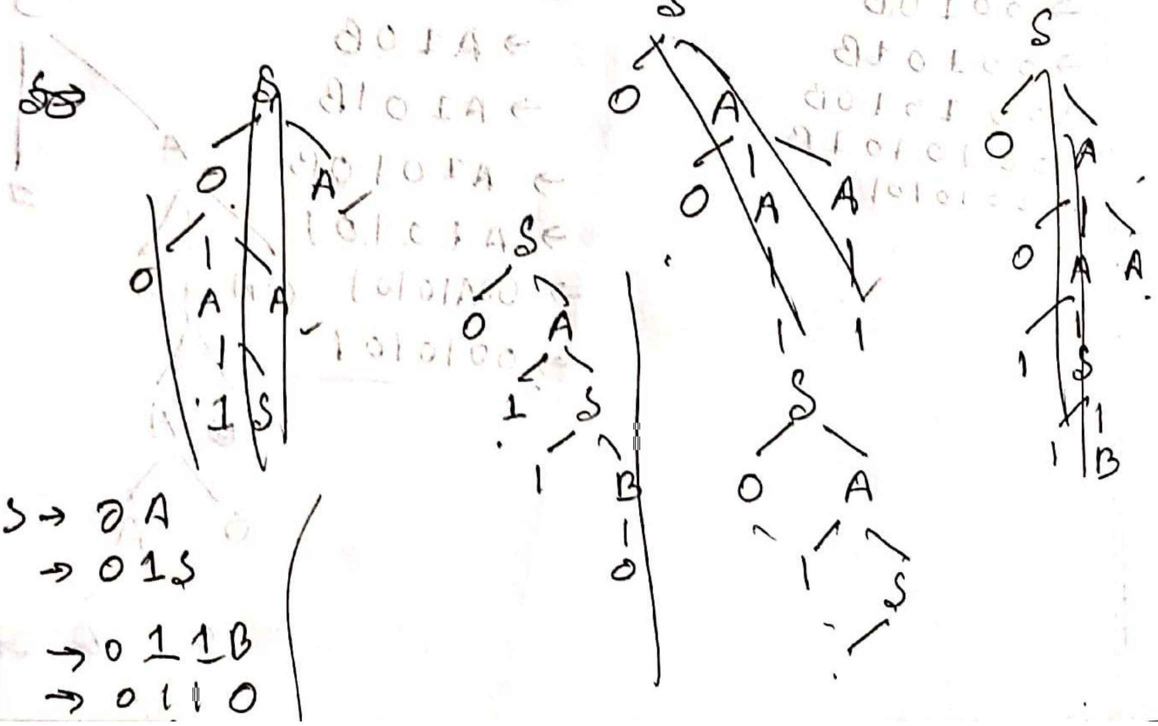


$S \to A1B$
$A \to 0A \mid \varepsilon$
$B \to 0B \mid 1B \mid \varepsilon$.

0 0 1 0 1 0 1



$S \to 0A \mid 1B$.
$A \to 0AA \mid 1S \mid 1$.
$B \to 1BB \mid 0S \mid 0$.



$S \to 0A$
$\to 01S$
$\to 011B$
$\to 0110$

$S \to N$
$VP \to$
$NP \to$
$N \to$

$S \to NP\ VP.$

$VP \to Vt\ NP.$

$NP \to Det\ N.$

$N \to Adj\ N.$

$Vt \to saw$

$Det \to the$

$Det \to a$

$N \to dragon$

$N \to boy$

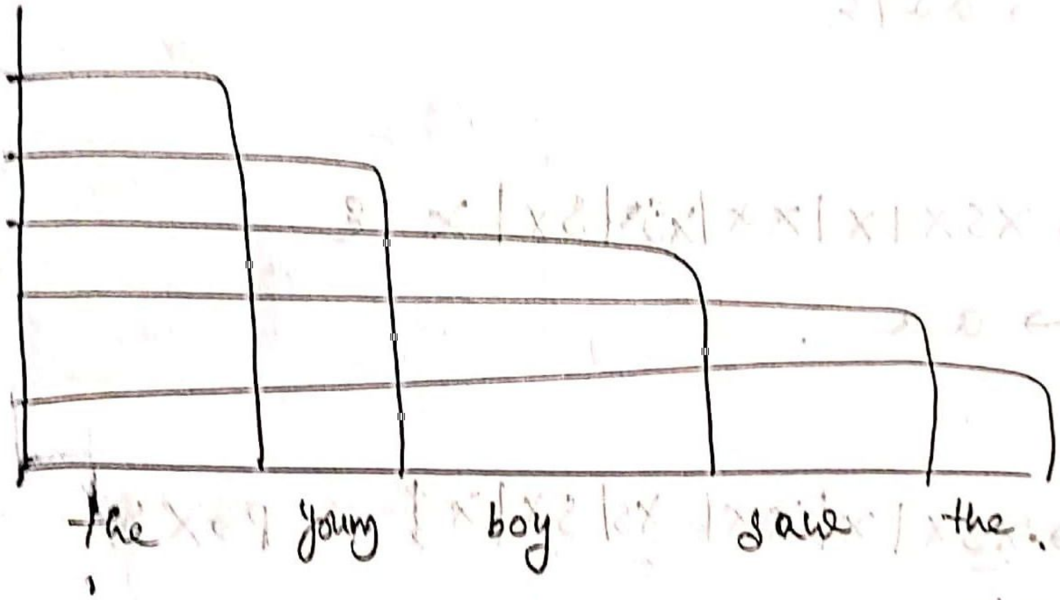
$Adj \to young.$

$S \to (a|the)(boy|dragon)\ saw\ (a|the)(boy|$

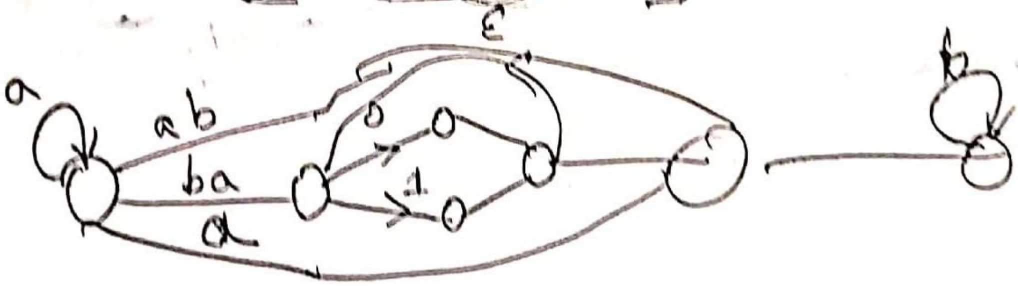
$VP \to saw(a|the).(boy/dragon)$

$NP \to (a|the).(boy|dragon)$

$N \to young.(boy|dragon)$



$a^* (ab + ba(0+1)^* + a) b^*.$



$S \to XSX \mid X \mid \varepsilon$

$X \to aa \mid \varepsilon$

$\downarrow$

$S_0 \to \cancel{XSX \mid X \mid \varepsilon}$ S

$S \to XSX \mid X \mid \varepsilon$

$X \to aa \mid \varepsilon$

$\Downarrow$

$S \to XSX \mid X \mid XX$

$X \to aa \mid \varepsilon$

$\Downarrow$

$S \to XSX \mid X \mid XX \mid XS \mid SX \mid X \mid \varepsilon$

$X \to aa$

$\Downarrow$

$S \to XSX \mid X \mid XX \mid XS \mid SX \mid X$ $\quad P \to XS$

$X \to aa$

$S_0 \to S$

$S \to PX \mid aa \mid XX \mid XS \mid SX \mid aa$

$X \to aa$

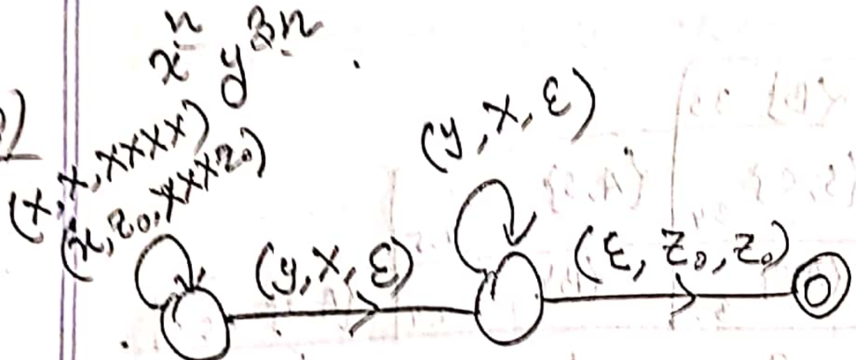
abbbcccc.

$(q_0, abbbcccc, z_0) \vdash (q_0, bbbcccc, az_0) \vdash (q_0, bbcccc, baz_0)$

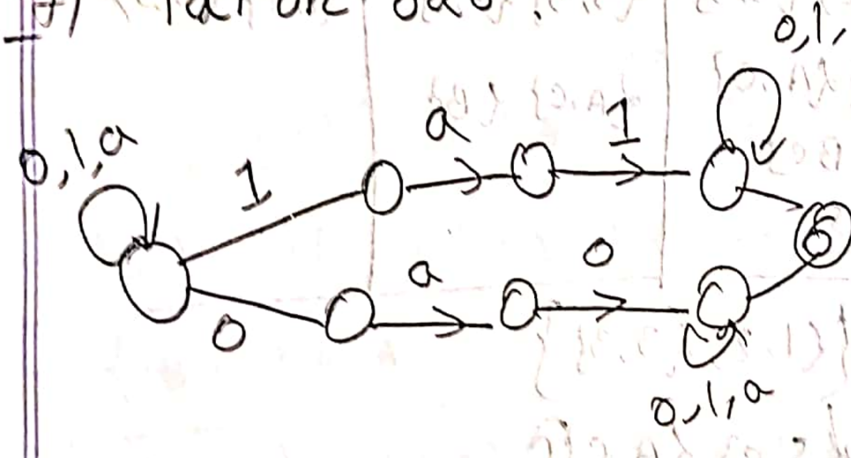
$\vdash (q_0, cccc, bbbaz_0) \vdash (q_2, ccc, bbaz_0) \vdash (q_2, cc, baz_0) \vdash$

$\vdash (q_2, c, az_0) \vdash (q_2, \varepsilon, z_0) \vdash (q_3, \varepsilon, \varepsilon)$

5) $x^n y^{3n}$.



7) 1a1 or 0a0



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$S \to AB / BC$

$A \to BA / a$ $\qquad w = \underline{ababa}$

$B \to CC \mid b$

$C \to AB / a$.

{S, A, C} 1,5				
{B} 1,4	{B} 2,5			
{B} 1,3	{S, C} 2,4	{B} 3,5		
{S, C} 1,2	{A, S} 2,3	{S, C} 3,4	{A, S} 4,5	
{A, C} 1,1	{B} 2,2	{A, C} 3,3	{B} 4,4	{A, C} 5,5
a	b	a	b	a

{1, 1} {2, 2}	{2, 2} {3, 3}	{3, 3} {4, 4}	{4, 4} {5, 5}
{A C} {B}	{B} {A, C}	{A, C} {B}	
{AB, CB}	{BA, BC}		
	A, S		

$\{\{1,1\}\{2,3\}\} \cup \{\{1,2\}\{3,3\}\}$

$\{\{A,C\} \ \{A,S\}\} \cup \{\{S,C\} \ \{A \ C\}\}$

$= \{AA, AS, CA, CS\} \cup \{SA, SC, CA, CC\}$

$= \{AA, AS, CA, CS, SA, SC, CA, \underline{CC}\}$

Scanned by CamScanner

$\{\{2,2\}\{3,4\}\} \cup \{\{2,3\}\{4,4\}\}$

$(\{B\}\{S,C\}) \cup \{\{A,S\}\{B\}\}$

$= \{BS, BC, AB, SB\}$

$= \{S, C\}$

$\{\{3,3\}\{4,5\}\} \cup \{\{3,4\}\{5,5\}\}$

$= \{\{A,C\}\{A,S\}\} \cup \{\{S,C\}\{A,C\}\}$

$= \{AA, AS, CA, CS, SA, SC, CA, CC\}$

$\{\{1,1\}\{2,4\}\} \cup \{\{1,2\}\{3,4\}\} \cup \{(1,3)(4,4)\}$

$= \{\{A,C\}\{S,C\}\} \cup \{\{S,C\}\{S,C\}\} \cup \{\{B\}\{B\}\}$

$= \{AS, AC, CS, CC, SS, SC, CS, CC, BB\}$

$= B.$

$\{(2,2)(3,5)\} \cup \{(2,3)(4,5)\} \cup \{(2,4)(5,5)\}$

$= \{\{B\}\{B\}\} \cup \{\{A,S\}\{A,S\}\} \cup \{\{S,C\}\{A,C\}\}$

$= \{BB, AA, AS, SA, SS, SA, SC, CA, CC\}$

$\{(1,1)(2,5)\} \cup \{(1,2)(3,5)\} \cup \{(1,3)(4,5)\} \cup \{(1,4)(5,5)\}$

$= (\{A,C\}\{B\}) \cup \{(S,C)(B)\} \cup \{\{B\}\{A,S\}\} \cup \{\{B\}\{A,C\}\}$

$= (AB, CB, SB, CB, BA, BS, BA, BC\}$

$= S, A, C$