The Binomial Model

- The binomial probability distribution is a discrete probability distribution function
- Useful in many situations where you have numerical variables that are counts or whole numbers
- Classic application of the binomial model is counting heads when flipping a coin

The Binomial Model

- The binomial model provides probabilities for random experiments in which you are counting the number of successes that occur. Four characteristics must be present:

  1) Fixed number of trials: n

  2) The only two outcomes are success and failure

  3) The probability of success, p, is the same at each trial

  4) The trials are independent

Binomial or Not?

- 40 randomly selected college students were asked if they selected their major in order to get a good job.
- 35 randomly selected Americans were asked what country their mothers were born.
- To estimate the probability that students will pass an exam, the professor records a study group's success on the exam.

Computing Binomial Probabilities

A Stats 10 test has 4 multiple choice questions with four choices with one correct answer each. If we just randomly guess on each of the 4 questions, what is the probability that you get exactly 3 questions correct?

- There are 4 different outcomes in which you could get 3 of 4 questions correct:

Correct, Correct, Correct, Wrong

Correct, Correct, Wrong, Correct

Correct, Wrong, Correct, Correct

Wrong, Correct, Correct, Correct

Computing Binomial Probabilities

A Stats 10 test has 4 multiple choice questions with four choices with one correct answer each. If we just randomly guess on each of the 4 questions, what is the probability that you get exactly 3 questions correct?

- The probability that you get one of these outcomes is
  - Correct, Correct, Correct, Wrong
  - $= 0.25 \times 0.25 \times 0.25 \times 0.75$
  - $= 0.25^3 \times 0.75$
  - $= 0.01172$
- The four outcomes all have the same probability so the probability that you get exactly 3 correct is
  - $4 \times 0.01172 = 0.04668$

Computing Binomial Probabilities

A Stats 10 test has 4 multiple choice questions with four choices with one correct answer each. If we just randomly guess on each of the 4 questions, what is the probability that you get exactly 1 question correct?

a) 0.04668

b) 0.42188

c) 0.10547

d) 0.25

Binomial Distribution Function

- The formula that finds the probabilities for the binomial distribution for probability of success p, fixed number of trials n, and k successes is as follows:

$$\binom{n}{k} p^k (1-p)^{n-k}$$

Binomial Coefficient

- The n over the k inside the parentheses can be read as "n choose k"
- Instead of writing all different combinations of outcomes and counting them all one-by-one this provides us the number of all those combinations.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Factorials

- ! - indicates a factorial
- n! = n x (n-1) x (n-2) x (n-3) x .... x 1

5! = 5 x 4 x 3 x 2 x 1 = 120

6! = 6 x 5 x 4 x 3 x 2 x 1 = 720

Binomial Coefficient Examples

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4\bullet 3\bullet 2\bullet 1}{(2\bullet 1)(2\bullet 1)} = 6$$

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5\bullet 4\bullet 3\bullet 2\bullet 1}{(3\bullet 2\bullet 1)(2\bullet 1)} = 10$$

Binomial Coefficient Hints

$\binom{n}{1} = n \quad \binom{9}{1} = 9 \quad \binom{n}{n-1} = n \quad \binom{17}{16} = 17$

$\binom{n}{0} = 1 \quad \binom{21}{0} = 1 \quad \binom{n}{n} = 1 \quad \binom{5}{5} = 1$

Computing Binomial Probabilities

- A Stats 10 test has 4 multiple choice questions with four choices with one correct answer each. If we just randomly guess on each of the 4 questions, what is the probability that you get exactly 2 questions correct?
- Using the binomial probability function:

$$\binom{4}{2} .25^2 (1 - .25)^2 = 6\ (0.0625)(0.5625)$$

$$= .2109$$

Computing Binomial Probabilities

- A Stats 10 test has 5 multiple choice questions with four choices with one correct answer each. If we just randomly guess on each of the 5 questions, what is the probability that you get exactly 2 questions correct?

  a) 0.6250

  b) 0.25

  c) 0.0625

  d) 0.2636

Computing Binomial Probabilities

- A Stats 10 test has 5 multiple choice questions with four choices with one correct answer each. If we just randomly guess on each of the 5 questions, what is the probability that you get 4 or more questions correct?

Computing Binomial Probabilities

- A Stats 10 test has 4 multiple choice questions with four choices with one correct answer each. If we just randomly guess on each of the 5 questions, what is the probability that you get at least 1 question correct?

  a) 0.2373

  b) 0.3955

  c) 0.7627

  d) 0.6045

Expected Value and Standard Deviation

- The mean and standard deviation of the binomial can be easily calculated
- Their interpretation is the same as with all distributions. Mean is the center and standard deviation tells us how far values typically are from the mean.
- Expected value or Mean = np
- Standard deviation = $\sqrt{np(1-p)}$

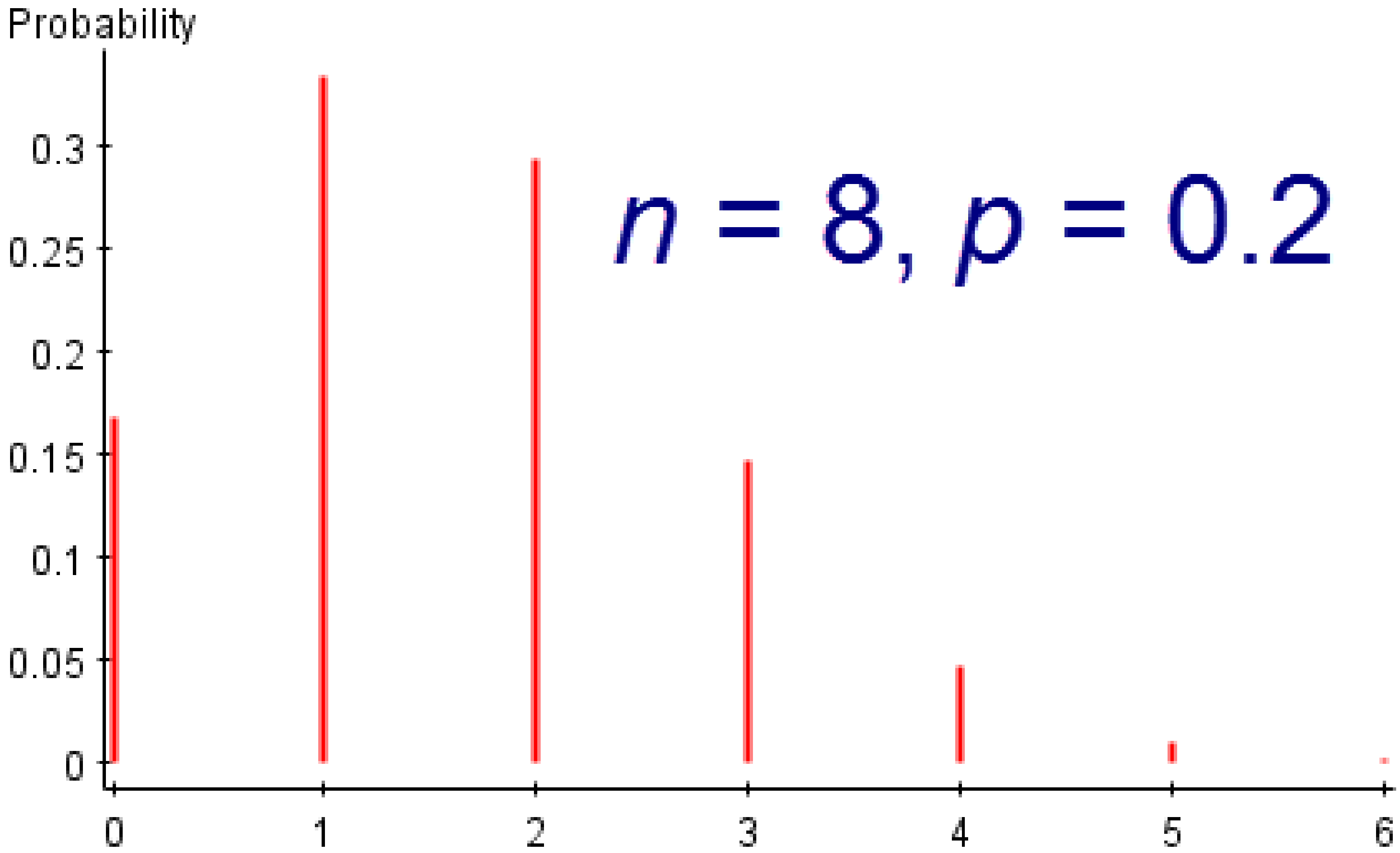
Expected Value Example

- A Stats 10 test has 4 multiple choice questions with one correct answer each. If we just randomly guess on each of the 4 questions, what is the expected number of questions we get correct?
- Expected value = np = 4 x 0.25 = 1
- Standard deviation = $\sqrt{np(1-p)}$ = $\sqrt{4 * 0.25(1-0.25)}$

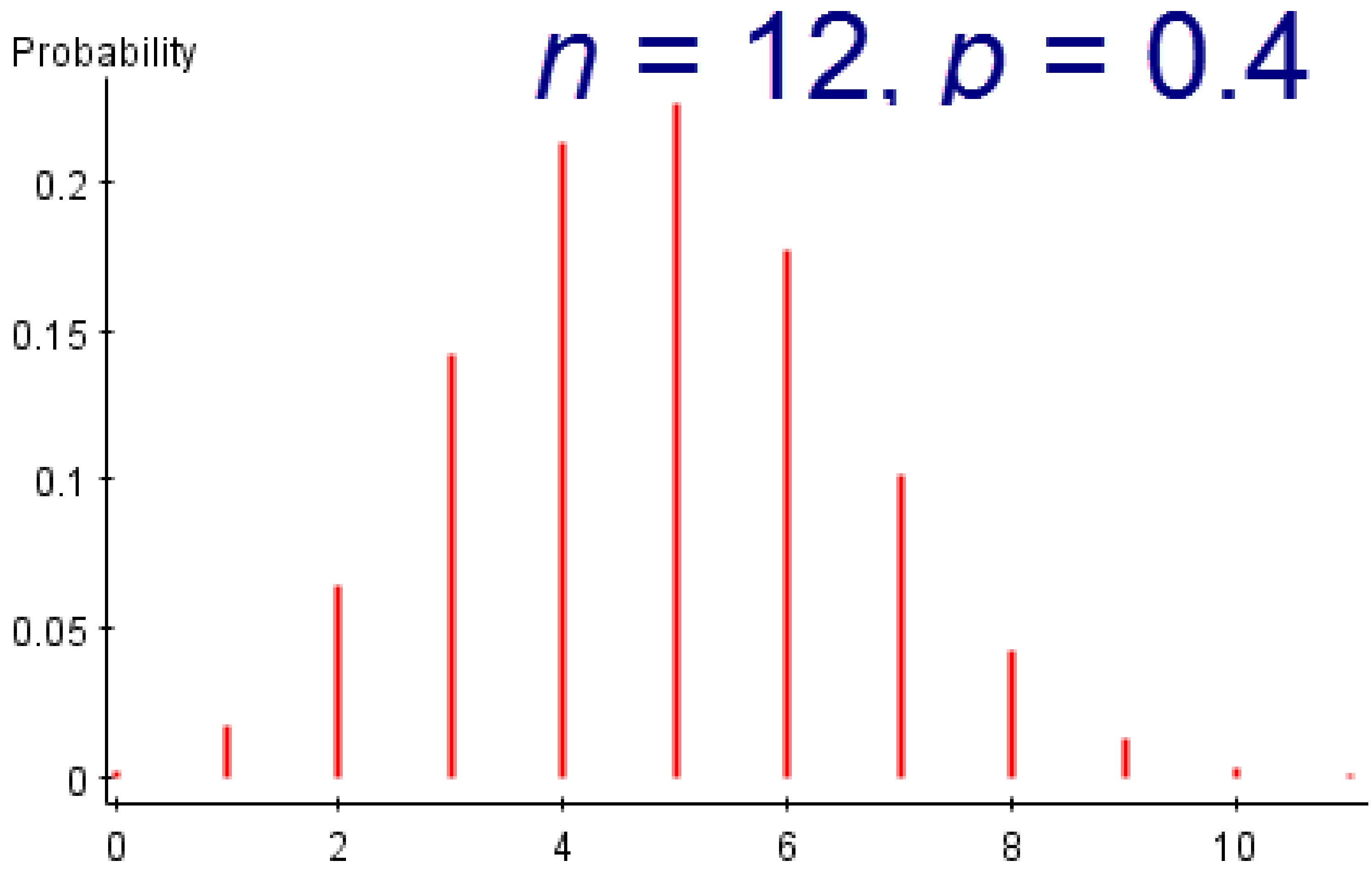
  = 0.866

- We are expected to only get 1 out of 4 questions correct if we just randomly guess.

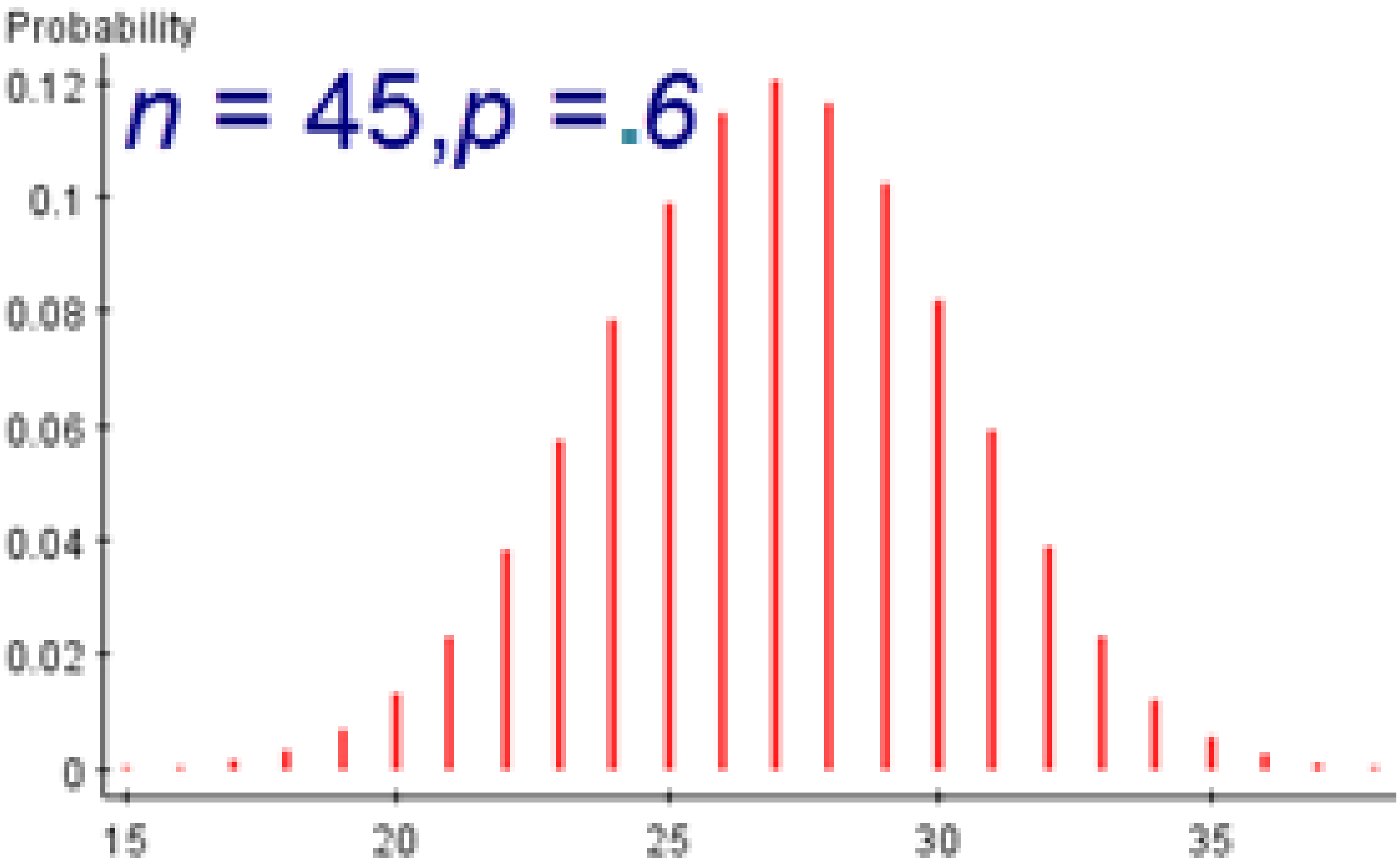
Visualizing the Binomial



Visualizing the Binomial



Visualizing the Binomial



Normal approximation to the Binomial

- The shape of the binomial distribution depends on both n and p.
- Binomial distributions are symmetric when p = 0.5, but they are also symmetric when n is large, even if p is close to 0 or 1.
- More specifically, when n x p is greater than or equal to 10 and n x (1-p) is greater than or equal to 10 then we can approximate the binomial distribution with a normal distribution with

  mean = np and standard deviation = $\sqrt{np(1-p)}$

Normal approximation to the Binomial

According to the website nationalbikeregistry.com, at UCLA only 3% of stolen bikes are returned to owners. If there are 335 bikes stolen at UCLA what is the probability that 12 or more stolen bikes will be returned?

- This is a binomial with $p = 0.03$ and $n = 335$
- $Np = 335(0.03) = 10.05$ and

  $n(1\text{-}p) = 335(0.97) = 324.95$

  These are both larger than or equal to 10 so we can approximate this binomial with a normal distribution

Normal approximation to the Binomial

According to the website nationalbikeregistry.com, at UCLA only 3% of stolen bikes are returned to owners. If there are 335 bikes stolen at UCLA what is the probability that 12 or more stolen bikes will be returned?

- This normal approximation has a mean of

$$np = 335(0.03) = 10.05$$

and a standard deviation of

$$\sqrt{np(1-p)} = \sqrt{335(0.03)(0.97)} = 3.12$$

Normal approximation to the Binomial

According to the website nationalbikeregistry.com, at UCLA only 3% of stolen bikes are returned to owners. If there are 335 bikes stolen at UCLA what is the probability that 12 or more stolen bikes will be returned?

- Now we can approach this as any other normal distribution and z-score problem with a N(10.05, 3.12)

Normal approximation to the Binomial

According to the website nationalbikeregistry.com, at UCLA only 3% of stolen bikes are returned to owners. If there are 335 bikes stolen at UCLA what is the probability that 5 or fewer stolen bikes will be returned?

a) 0.03

b) 0.15

c) 0.05

d) 0.015

Normal approximation to the Binomial

Approximately 97% of people own Snuggies. In our Stats 10 class of 171 students what is the probability that 150 students or fewer own a Snuggie?