SET A ①

Part I Ans:-

No it is not possible because the matrix X has a dimension of 1×2 & is not a square matrix.

Part II ans:-

$$[A][x] = [c] \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & -5 \\ 1 & -3 & 2 \\ 5 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -13 \\ 10 \end{bmatrix}$$

Find $[A]^{-1}$

$$[A] = [L][U]$$

where

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{[A]} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}}_{[L]} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{[U]}$$

Step 1: find [L] & [U] using forward elimination of gaussian elimination.

1st step of forward elimination:

$$A = \begin{bmatrix} 3 & 2 & -5 \\ 1 & -3 & 2 \\ 5 & -1 & 4 \end{bmatrix}$$

pivot element $= a_{11} = 3$.

$$R_2' = R_2 - \frac{1}{3} \times R_1$$

$$a_{21}' = a_{21} - \frac{a_{21}}{a_{11}} \times a_{11} = 0; \quad l_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{3} = 0.33333$$

$$a_{22}' = a_{22} - \frac{a_{21}}{a_{11}} \times a_{12}$$

$$= -3 - \frac{1}{3} \times 2 = -3.6667$$

$$a_{23}' = a_{23} - \frac{a_{21}}{a_{11}} \times a_{13}$$

$$= 2 - \frac{1}{3} \times (-5) = 3.66667$$

$$A = \begin{bmatrix} 3 & 2 & -5 \\ 0 & -3.6667 & 3.66667 \\ 5 & -1 & 4 \end{bmatrix}$$

$$R_3' = R_3 - \frac{5}{3} \times R_1$$

$$a_{31}' = a_{31} - \frac{a_{31}}{a_{11}} \times a_{11} = 0 \quad ; \quad l_{31} = \frac{a_{31}}{a_{11}} = \frac{5}{3} = 1.6667$$

$$a_{32}' = a_{32} - \frac{a_{31}}{a_{11}} \times a_{12}$$

$$= (-1) - \frac{5}{3} \times (2)$$

$$= -4.3333$$

$$a_{33}' = a_{33} - \frac{a_{31}}{a_{11}} \times a_{13} = 4 - \frac{5}{3} \times (-5) = 12.3333$$

$$A = \begin{bmatrix} 3 & 2 & -5 \\ 0 & -3.6667 & 3.6667 \\ 0 & -4.3333 & 12.3333 \end{bmatrix}$$

2nd step of forward eliminations

pivot element $= -3.6667 = a_{22}$

$$R_3' = R_3 - \frac{-4.3333}{-3.6667} \times R_2$$

$$a_{32}' = a_{32} - \frac{a_{32}}{a_{22}} \times a_{22} = 0, \quad l_{32} = \frac{a_{32}}{a_{22}} = \frac{-4.3333}{-3.6667} = 1.1818$$

$$a_{33}' = a_{33} - \frac{a_{32}}{a_{22}} \times a_{23}$$

$$= 12.3333 - \frac{-4.3333}{-3.6667} \times 3.6667$$

$$= 8$$

$$A = \begin{bmatrix} 3 & 2 & -5 \\ 0 & -3.6667 & 3.6667 \\ 0 & 0 & 8 \end{bmatrix} = [U] = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ .33333 & 1 & 0 \\ 1.6667 & 1.1818 & 1 \end{bmatrix}$$

Step 2:

$$[A][A]^{-1} = [I]$$

$$\Rightarrow [A][B] = [I] \quad \text{Let } [A]^{-1} = B$$

& solve for [B] :-

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $[L][Z] = [I]$

& $[U][B] = [Z]$ for each column of $[B]$.

First column:

$$[A] \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

first, $[L][Z] = [I]$

$$\begin{bmatrix} 1 & 0 & 0 \\ .333333 & 1 & 0 \\ 1.6667 & 1.1818 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Using forward substitution,

$$z_1 = 1;$$

$$.33333 z_1 + z_2 = 0$$

$$\Rightarrow z_2 = -0.33333;$$

$$1.6667 z_1 + 1.1818 z_2 + z_3 = 0$$

$$\Rightarrow z_3 = -1.2728$$

$[U][B] = [Z]$

$$\begin{bmatrix} 3 & 2 & -5 \\ 0 & -3.6667 & 3.6667 \\ 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ -.33333 \\ -1.2728 \end{bmatrix}$$

Using back substitution:-

$8b_{31} = -1.2728$; $b_{31} = -0.1591$

$-3.6667 b_{21} + 3.6667 b_{31} = -.33333$

$\Rightarrow b_{21} =$; -0.068193

$3b_{11} + 2b_{21} - 5b_{31} = 1$

$\Rightarrow b_{11} =$ 0.11363

2nd column:

$$[A] \begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

first, $[L][Z] = [I]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.33333 & 1 & 0 \\ 1.6667 & 1.1818 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Forward substitution:

$z_1 = 0$

$z_2 = 1$

$z_3 = -1.1818$

Back Substitution:-

Next, $[U][B] = [Z]$

$$\begin{bmatrix} 3 & 2 & -5 \\ 0 & -3.6667 & 3.6667 \\ 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1.1818 \end{bmatrix}$$

Back Substitution:

$$b_{32} = \frac{-1.1818}{8} = -0.147725;$$

$$-3.6667\, b_{22} + 3.6667\, b_{32} = 1$$

$$\Rightarrow b_{22} = \cancel{-.420453}\ -0.42045$$

$$3b_{12} + 2b_{22} - 5b_{32} = 0$$

$$\therefore b_{12} = \cancel{2.8868}\ .034092$$

3rd column:

$$[A] \begin{bmatrix} b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

First, $[L][Z] = [I]$ [forward substitution]

$$\begin{bmatrix} 1 & 0 & 0 \\ .33333 & 1 & 0 \\ 1.6667 & 1.1818 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$z_1 = 0$; $z_2 = 0$; $z_3 = 1$

Then, $[U][B] = [Z]$ [Back Substitution]

$$\begin{bmatrix} 3 & 2 & -5 \\ 0 & -3.6667 & 3.6667 \\ 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$b_{33} = 1/8 = 0.125$$

$$-3.6667 b_{23} + 3.6667 b_{33} = 0$$

$$\Rightarrow b_{23} = 0.125$$

$$3b_{13} + 2b_{23} - 5b_{33} = 0$$

$$\therefore b_{13} = 0.125$$

So, the inverse matrix is

$$[B] = [A]^{-1} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} .11363 & .034092 & .125 \\ -.068193 & -.42045 & .125 \\ -.1591 & -.147725 & .125 \end{bmatrix}$$

Since $[A][X] = [C]$

$$\therefore [X] = [A]^{-1}[C]$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} .11363 & .034092 & .125 \\ -.068193 & -.42045 & .125 \\ -.1591 & -.147725 & .125 \end{bmatrix} \begin{bmatrix} 12 \\ -13 \\ 10 \end{bmatrix}$$

using calculator,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -40 \\ 71 \\ 113 \end{bmatrix}$$

$x = -40, y = 71, z = 113$

[check the inverse using your calculator & match your answer]

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2.1704 \\ 5.8977 \\ 1.2613 \end{bmatrix}$$

$x = 2.1704, y = 5.8977,$
$z = 1.2613$

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Using calculator to find (5) $[A]^{-1} * [C]$:-

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2.1704 \\ 5.8977 \\ 1.2613 \end{bmatrix}$$

$x = 2.1704$, $y = 5.8977$, $z = 1.2613$

[Check the inverse by typing in on your calculator & match answer]