

## Exercise Sheet 1

### Exercise 1

Let A: "Aldo is Italian" and B: "Bob is English". Formalize the following sentences:

1. Aldo is not Italian.
2. Aldo is Italian while Bob is English.
3. Aldo is Italian or if Aldo is not Italian then Bob is English.
4. Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English.

### Solution

1.  $\neg A$
2.  $A \wedge B$
3.  $A \vee (\neg A \Rightarrow B)$
4.  $(A \wedge B) \vee \neg(A \vee B)$

### Exercise 2

Angelo, Bruno and Carlo are three students that took the Logic exam. Consider a propositional language where:

1. p: Angelo passed the exam.
2. q: Bruno passed the exam.
3. r: Carlo passed the exam.

Write these propositions using p, q, and r and logical connectives (including negations).

1. Carlo is the only one passing the exam.
2. Only one among Angelo, Bruno and Carlo, passed the exam.
3. At least one among Angelo, Bruno and Carlo passed.
4. At least two among Angelo, Bruno and Carlo passed the exam.
5. Exactly two, among Aldo, Bruno and Carlo passed the exam.
6. At most two among Angelo, Bruno and Carlo passed the exam.

### Solution

1.  $r \wedge \neg p \wedge \neg q$
2.  $(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$   
or  
 $(p \vee q \vee r) \wedge \neg(p \wedge q) \wedge \neg(p \wedge r) \wedge \neg(q \wedge r)$
3.  $p \vee q \vee r$
4.  $(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)$
5.  $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$
6.  $\neg(p \wedge q \wedge r)$   
or  
 $(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$

**Exercise 3**

Construct a truth table for each of these compound propositions.

1.  $(p \vee \neg q) \Rightarrow q$
2.  $(p \Rightarrow q) \vee (p \Rightarrow \neg q)$
3.  $(\neg p \vee q) \wedge (q \Rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$

**Solution**

1.

$p$	$q$	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \Rightarrow q$
$\perp$	$\perp$	<b>T</b>	<b>T</b>	$\perp$
$\perp$	<b>T</b>	$\perp$	$\perp$	<b>T</b>
<b>T</b>	$\perp$	<b>T</b>	<b>T</b>	$\perp$
<b>T</b>	<b>T</b>	$\perp$	<b>T</b>	<b>T</b>

2.

$p$	$q$	$\neg q$	$p \Rightarrow q$	$(p \Rightarrow \neg q)$	$(p \Rightarrow q) \vee (p \Rightarrow \neg q)$
$\perp$	$\perp$	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>
$\perp$	<b>T</b>	$\perp$	<b>T</b>	<b>T</b>	<b>T</b>
<b>T</b>	$\perp$	<b>T</b>	$\perp$	<b>T</b>	<b>T</b>
<b>T</b>	<b>T</b>	$\perp$	<b>T</b>	$\perp$	<b>T</b>

3.  $F = (\neg p \vee q) \wedge (q \Rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$

$p$	$q$	$r$	$\neg p$	$\neg r$	$\neg p \vee q$	$\neg r \wedge \neg p$	$q \Rightarrow \neg r \wedge \neg p$	$p \vee r$	F
$\perp$	$\perp$	$\perp$	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	$\perp$	$\perp$
$\perp$	$\perp$	<b>T</b>	<b>T</b>	$\perp$	<b>T</b>	$\perp$	<b>T</b>	<b>T</b>	<b>T</b>
$\perp$	<b>T</b>	$\perp$	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	$\perp$	$\perp$
$\perp$	<b>T</b>	<b>T</b>	<b>T</b>	$\perp$	<b>T</b>	$\perp$	$\perp$	<b>T</b>	$\perp$
<b>T</b>	$\perp$	$\perp$	$\perp$	<b>T</b>	$\perp$	$\perp$	<b>T</b>	<b>T</b>	$\perp$
<b>T</b>	$\perp$	<b>T</b>	$\perp$	$\perp$	$\perp$	$\perp$	<b>T</b>	<b>T</b>	$\perp$
<b>T</b>	<b>T</b>	$\perp$	$\perp$	<b>T</b>	<b>T</b>	$\perp$	$\perp$	<b>T</b>	$\perp$
<b>T</b>	<b>T</b>	<b>T</b>	$\perp$	$\perp$	<b>T</b>	$\perp$	$\perp$	<b>T</b>	$\perp$

**Exercise 4**

Show that each of these conditional statements is a tautology by/without using truth tables.

1.  $[\neg p \wedge (p \vee q)] \Rightarrow q$
2.  $[p \wedge (q \Rightarrow q)] \Rightarrow q$
3.  $[(p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow r)] \Rightarrow r$

**Solution**

1.

$p$	$q$	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \Rightarrow q$
$\perp$	$\perp$	<b>T</b>	$\perp$	$\perp$	<b>T</b>
$\perp$	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>
<b>T</b>	$\perp$	$\perp$	<b>T</b>	$\perp$	<b>T</b>
<b>T</b>	<b>T</b>	$\perp$	<b>T</b>	$\perp$	<b>T</b>

$$[\neg p \wedge (p \vee q)] \Rightarrow q \equiv \neg[\neg p \wedge (p \vee q)] \vee q$$

$$\begin{aligned}
&\equiv [p \vee \neg(p \vee q)] \vee q \\
&\equiv [p \vee (\neg p \wedge \neg q)] \vee q \\
&\equiv [(p \vee \neg p) \wedge (p \vee \neg q)] \vee q \\
&\equiv [T \wedge (p \vee \neg q)] \vee q \\
&\equiv (p \vee \neg q) \vee q \\
&\equiv p \vee \neg q \vee q \\
&\equiv p \vee T \\
&\equiv T
\end{aligned}$$

### Exercise 5

Show that the following conditional statements are logically equivalent:

1.  $(p \Rightarrow q) \wedge (p \Rightarrow r)$  and  $p \Rightarrow (q \wedge r)$
2.  $(p \Rightarrow r) \wedge (q \Rightarrow r)$  and  $(p \vee q) \Rightarrow r$
3.  $(p \Rightarrow q) \vee (p \Rightarrow r)$  and  $p \Rightarrow (q \vee r)$
4.  $\neg p \Rightarrow (q \Rightarrow r)$  and  $q \Rightarrow (p \vee r)$

### Solution

1. Build the truth tables for the both conditional statements and compare the truth values for each possible situation.

### Exercise 6

Construct DNF and CNF for the examples in Exercise 3.

### Solution

1. CNF:  $(p \vee q) \wedge (\neg p \vee q)$   
DNF:  $(\neg p \wedge q) \vee (p \wedge q)$
2. CNF: -  
DNF:  $(\neg p \wedge \neg q) \vee (\neg p \wedge q) \vee (p \wedge \neg q) \vee (p \wedge q)$
3. CNF:  $(p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$   
DNF:  $\neg p \wedge \neg q \wedge r$