

Name: _____

ID: _____

Department of Computer Science and Engineering**CSE 330: Numerical Methods****Quiz 2****SET B****Full Marks: 15**

Using Table 1, form a matrix representation for solving the coefficients for the polynomials having quadratic splines

Table 1

Sl. No.	V1	V2
1	10	29
2	12	33
3	15	37
4	17	44
5	19	47

Since there are 5 points, we will have a total of 4 splines. Also, we will have a total of $3 \times 4 = 12$ equations to solve for 12 unknowns.

$$\begin{aligned}
 f(v_1) = v_2 &= a_1 v_1^2 + b_1 v_1 + c_1; & 10 \leq v_1 \leq 12 \\
 &= a_2 v_1^2 + b_2 v_1 + c_2; & 12 \leq v_1 \leq 15 \\
 &= a_3 v_1^2 + b_3 v_1 + c_3; & 15 \leq v_1 \leq 17 \\
 &= a_4 v_1^2 + b_4 v_1 + c_4 & 17 \leq v_1 \leq 19
 \end{aligned}$$

Now, since each spline passes through 2 points,

$$a_1 v_1 + b_1 v_1 + c_1 \Rightarrow a_1(10) + b_1(10) + c_1 = 29 \quad \text{--- (1)}$$

$$a_1(10) + b_1(10) + c_1 = 33 \quad \text{--- (11)}$$

$$a_2 v_1 + b_2 v_1 + c_2 \Rightarrow a_2(12) + b_2(12) + c_2 = 33 \quad \text{--- (11)}$$

$$a_2(15) + b_2(15) + c_2 = 37 \quad \text{--- (14)}$$

$$a_3 v_1 + b_3 v_1 + c_3 \Rightarrow a_3(15) + b_3(15) + c_3 = 37 \quad \text{--- (5)}$$

$$a_3(17) + b_3(17) + c_3 = 44 \quad \text{--- (6)}$$

$$a_4 v_1 + b_4 v_1 + c_4 \Rightarrow a_4(17) + b_4(17) + c_4 = 44 \quad \text{--- (11)}$$

$$a_4(19) + b_4(19) + c_4 = 47 \quad \text{--- (111)}$$

That makes a total of $2 \times 4 = 8$ equations. We still need 4 more equations.

Since the first derivative of two consecutive splines are continuous at the interior points & 12, 15, 17 are the interior points,

$$2a_1(12) + b_1 - 2a_2(12) - b_2 = 0 \quad \text{--- (1x)} \quad [\text{since there are a total of 5 points, so we have 3 interior points}]$$

$$2a_2(15) + b_2 - 2a_3(15) - b_3 = 0 \quad \text{--- (x)} \quad [\text{so we have 2 boundary points}]$$

$$2a_3(17) + b_3 - 2a_4(17) - b_4 = 0 \quad \text{--- (x1)}$$

Assuming the first spline as linear,
 $a_1 = 0 \quad \text{--- (x11)} \quad [\because \text{linear spline is } b_1 v_1 + c_1]$

We now have a total of 12 equations. forming the coefficient matrix.

Coefficients
of: a_1 , b_1 , c_1 , a_2 , b_2 , c_2

$$100 \quad 10 \quad 1 \quad 0 \quad 0 \quad 0$$

$$144 \quad 12 \quad 1 \quad 0 \quad 0 \quad 0$$

$$6 \quad 0 \quad 0 \quad 144 \quad 12 \quad 1$$

$$0 \quad 0 \quad 0 \quad 225 \quad 15 \quad 1$$

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$$0 \quad 0 \quad 0 \quad 289 \quad 17 \quad 1$$

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$$0 \quad 0 \quad 0 \quad 361 \quad 19 \quad 1$$

$$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$24 \quad 1 \quad 0 \quad -24 \quad -1 \quad 0$$

$$0 \quad 0 \quad 0 \quad -30 \quad -1 \quad 0$$

$$0 \quad 0 \quad 0 \quad 30 \quad 1 \quad 0$$

$$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

Tips when writing the coefficient matrix:

1. Use a pencil to avoid errors

2. at the top, mention the coefficients (you can erase it afterwards if needed!)

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