Question 1:- a)

Categories	Fever	cough	Breathing Difficulties	Covid -19
1	No	No	No	No
2	Yes	Yes	Yes	Yes
3	Yes	Yes	No	No
4	Yes	No	Yes	Yes
5	Yes	Yes	Yes	Yes
6	No	Yes	No	No
7	Yes	No	Yes	Yes
8	Yes	No	Yes	Yes
9	No	Yes	~~No~~ Yes	Yes
10	Yes	Yes	No	Yes
11	No	Yes	No	No
12	No	Yes	Yes	Yes
13	No	Yes	Yes	No
14	Yes	Yes	No	No

$$\text{Entropy}(\text{Covid}) = -\frac{8}{14}\log_2\left(\frac{8}{14}\right) - \frac{6}{14}\left(\log_2 \frac{6}{14}\right)$$

$$= 0.985$$

$$\text{Entropy}(\text{Fever} = \text{No})$$

$$= -\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\left(\log_2\left(\frac{2}{6}\right)\right)$$

$$= 0.918$$

$$\text{Entropy}(\text{Fever} = \text{Yes})$$

$$= -\frac{6}{8}\log_2\left(\frac{6}{8}\right) - \frac{2}{8}\log_2\left(\frac{2}{8}\right)$$

$$= 0.811$$

$$\therefore \text{Gain}(\text{Fever}) = 0.985 - \left[0.918 \times \frac{6}{14}\right] - \left[0.811 \times \frac{8}{14}\right]$$

$$= 0.128$$

For, cough,

$$\text{Entropy}(\text{cough}=\text{No}) = -\frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{3}{4}\log_2\left(\frac{3}{4}\right)$$

$$= 0.811$$

$$\text{Entropy}(\text{cough}=\text{Yes}) = -\frac{5}{10}\log_2\left(\frac{5}{10}\right) - \frac{5}{10}\log_2\left(\frac{5}{10}\right)$$

$$= 1$$

$$\text{Gain}(\text{cough}) = 0.785 - \left[0.811 \times \frac{4}{14}\right] - \left[1 \times \frac{10}{14}\right]$$

$$= 0.039$$

For, breathing difficulty

~~Entropy(BD = Yes) = $-\frac{8}{14}\log$~~

$$\text{Entropy}(BD = Yes) = -\frac{7}{8}\log_2\left(\frac{7}{8}\right) - \frac{1}{8}\log_2\left(\frac{1}{8}\right)$$

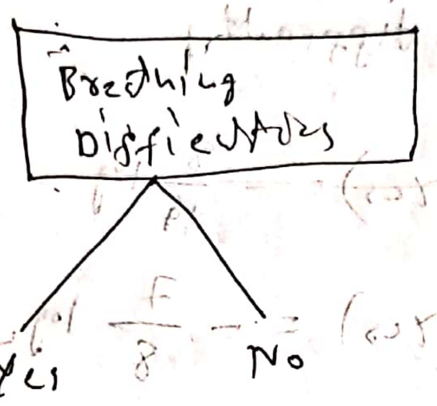
$$= 0.543$$

$$\text{Entropy}(BD = No) = -\frac{5}{6}\log_2\left(\frac{5}{6}\right) - \frac{1}{6}\log_2\left(\frac{1}{6}\right)$$

$$= 0.650$$

$$\text{Gain}(BD) = 0.985 - \left[\frac{8}{14}\ 0.543\right] - \left[\frac{6}{14} \times 0.650\right]$$

$$= 0.396$$



For, Gain(Breathing Diff = Yes | Fever)

$$Entropy(Covid) = -\frac{7}{8}\log_2\left(\frac{7}{8}\right) - \frac{1}{8}\log_2\left(\frac{1}{8}\right)$$

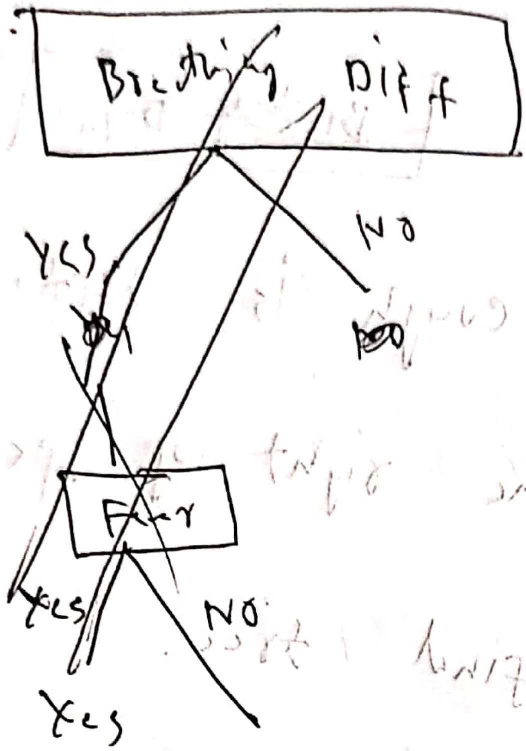
$$= 0.543$$

$$Entropy(Fever = yes) = 0$$

$$Entropy(Fever = no)$$

$$= -\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \log_2\left(\frac{1}{3}\right)$$

$$= 0.918$$



$- \frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3}\}$

∴ Gain(Fever) = 0.543 ~~0.344~~

= 0.199

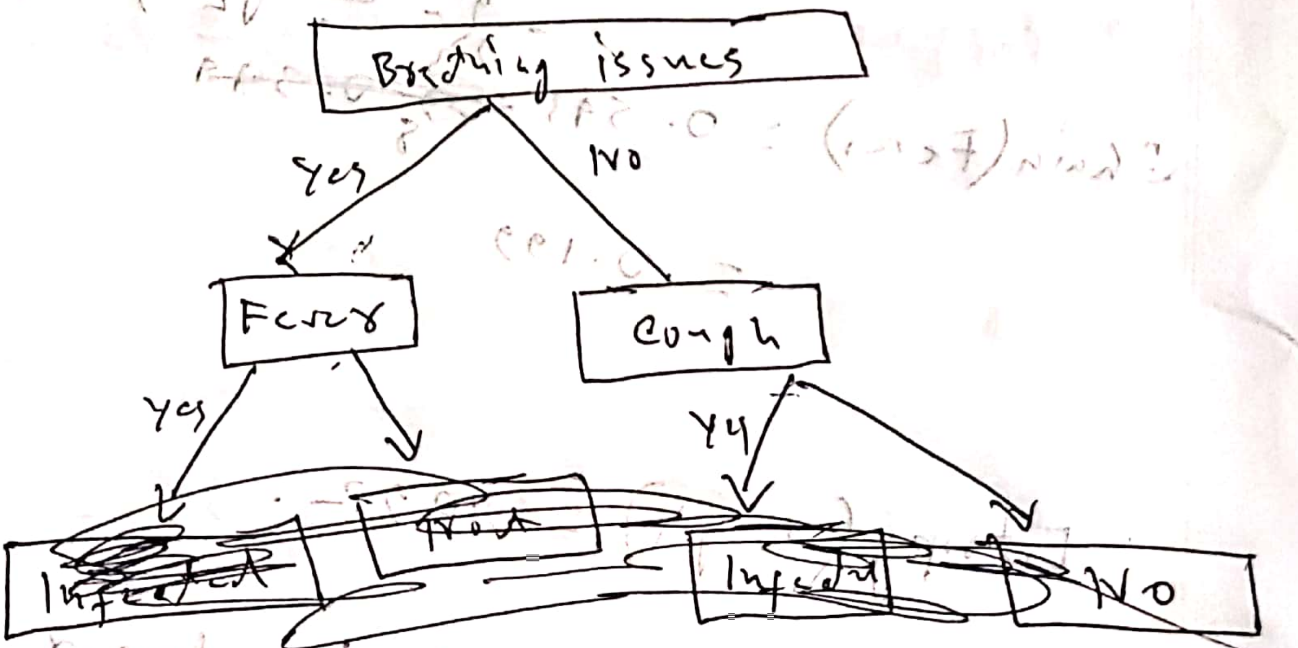
∴ Entropy(cough) = 0.092.

∴ Hence, Gain(Fever) is larger

~~Breathing Diff~~

But, only cough is left. So, cough will go to the right of fever.

So, the final tree.



b) Learning phase:

Fever	Yes	No
Yes	$\frac{6}{8}$	$\frac{2}{6}$
No	$\frac{2}{8}$	$\frac{4}{6}$

cough	Yes	No
Yes	$\frac{5}{8}$	$\frac{5}{6}$
No	$\frac{3}{8}$	$\frac{1}{6}$

Breathing	Yes	No
Yes	$\frac{7}{8}$	$\frac{1}{6}$
No	$\frac{1}{8}$	$\frac{5}{6}$

$\cancel{p(Yes)}$

$$p(covid = Yes) = \frac{8}{14}$$

$$p(covid = No) = \frac{6}{14}$$

$p(Fever = No, cough = No, Breathing = Yes \mid covid = Yes)$

$$= \frac{2}{8} \times \frac{3}{8} \times \frac{7}{8} = 0.082 \times \frac{8}{14} = 0.046$$

For, covid = No,

$$= \frac{4}{6} \times \frac{1}{6} \times \frac{1}{6} = 0.018 \times \frac{6}{14} = 7.93 \times 10^{-3}$$

As, we can see propability of covid = Yes higher.

∴ The decision is covid positive.

Question 2:

Part 1:

i) $P(A) = \frac{2}{6} = \frac{1}{3}$

ii) $P(B) = \frac{3}{6} = \frac{1}{2}$

Part 2:

i) $A \cap B = \{2\}$

$\therefore P(A \cap B) = \frac{1}{6}$

ii) Here, $P(A) * P(B) = \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{6}$

$\therefore P(A \cap B) = \frac{1}{6}$

$\therefore P(A \cap B) = P(A) * P(B)$; So, they are independent

Part 3:

i) ~~P(A∩B|C)~~ =

$P(C) = \frac{2}{6} = \frac{1}{3}$; $P(A \cap B \cap C) = 0$

$\therefore P(A \cap B \mid C) = \frac{P(A \cap B \cap C)}{P(C)}$

$= \frac{0}{\frac{1}{3}} = 0$

(Ans)

ii) $P(A \mid C) \times P(B \mid C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

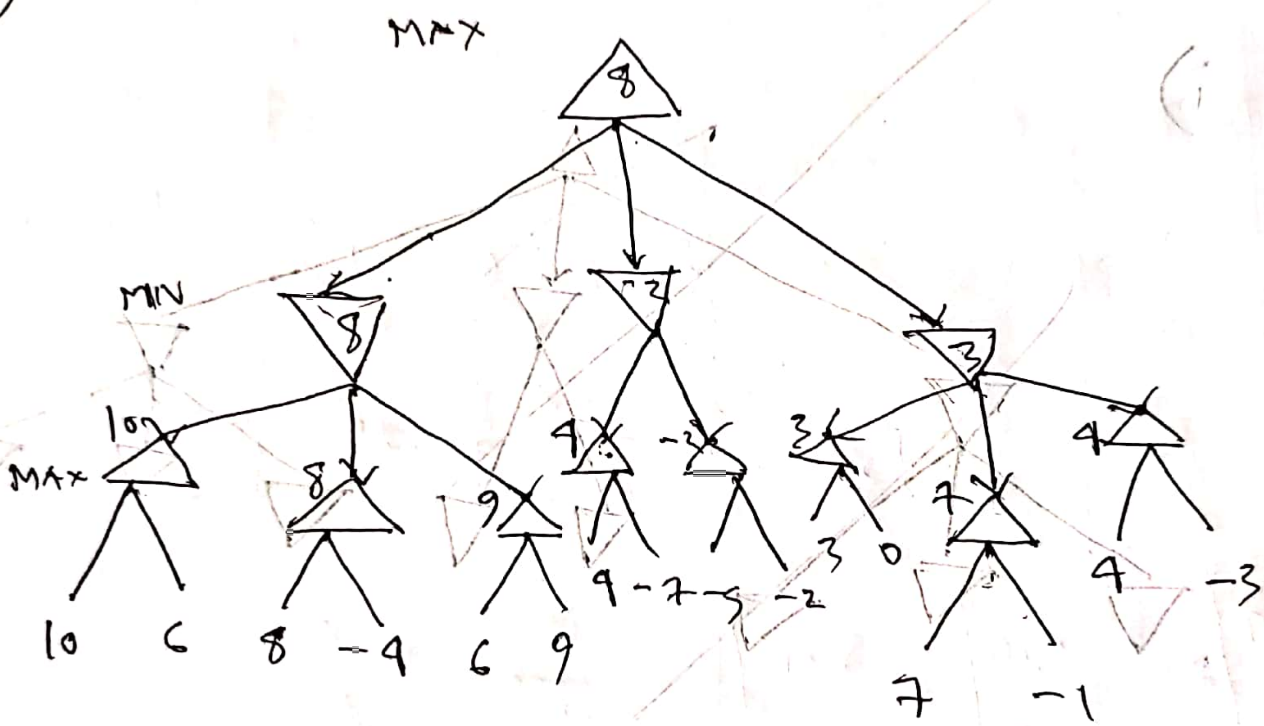
$\therefore P(A \cap B \mid C) = 0$

$\therefore$ ~~The~~ $P(A \mid C) \times P(B \mid C) \neq P(A \cap B \mid C)$.

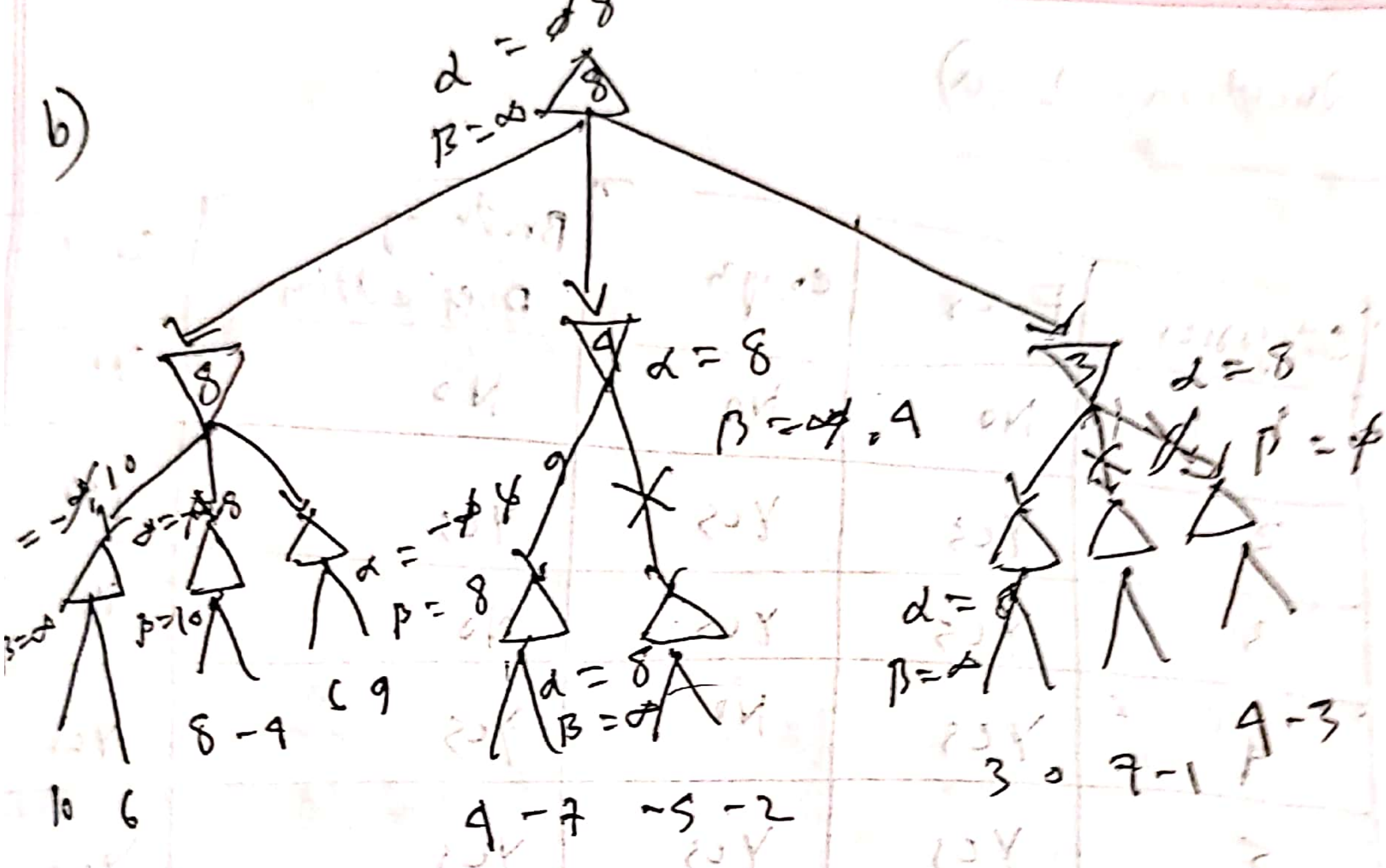
$\therefore$ They are not conditionally independent.

Question 3:

a)

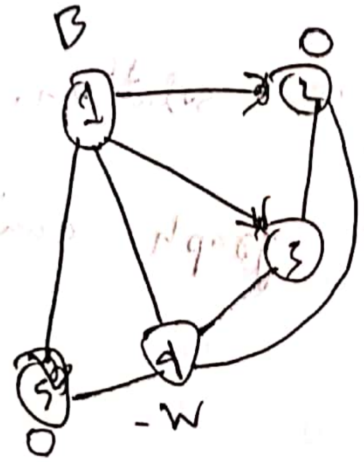


b)



4. variables = {1, 2, 3, 4, 5}

Domain = {Black, White, Orange}



The highest degree is of 1 and 4. Their degree is 4. So, I can start from them

				Degree
1 →	B	~~W~~	~~O~~	4
2 →	~~B~~	~~W~~	O	~~3~~ ~~2~~ 1
3 →	~~B~~	~~W~~	~~O~~	~~3~~ ~~2~~ 1
4 →	~~B~~	W	O	~~4~~ ~~3~~ 2
5 →	~~B~~	~~W~~	O	~~2~~ ~~1~~ 0

After picking 1 and 4, if we pick 2 then it doesn't give a solution.

Again, if we backtrack and pick 3 it also doesn't give a solution. So, we can see that this graph can't satisfy the constraint.