Assignment #8

Due: 11:00pm on Saturday, July 20, 2013

You will receive no credit for items you complete after the assignment is due. Grading Policy

Center of Mass and External Forces

Learning Goal:

Understand that, for many purposes, a system can be treated as a point-like particle with its mass concentrated at the center of mass.

A complex system of objects, both point-like and extended ones, can often be treated as a *point particle*, located at the system's *center of mass*. Such an approach can greatly simplify problem solving.

Before you use the center of mass approach, you should first understand the following terms:

- System: *Any* collection of objects that are of interest to you in a particular situation. In many problems, you have a certain freedom in choosing your system. Making a wise choice for the system is often the first step in solving the problem efficiently.
- Center of mass: The point that represents the "average" position of the entire mass of a system. The postion of the center of mass $\vec{r}_{\rm cm}$ can be expressed in terms of the position vectors $\vec{r}_i$ of the particles as

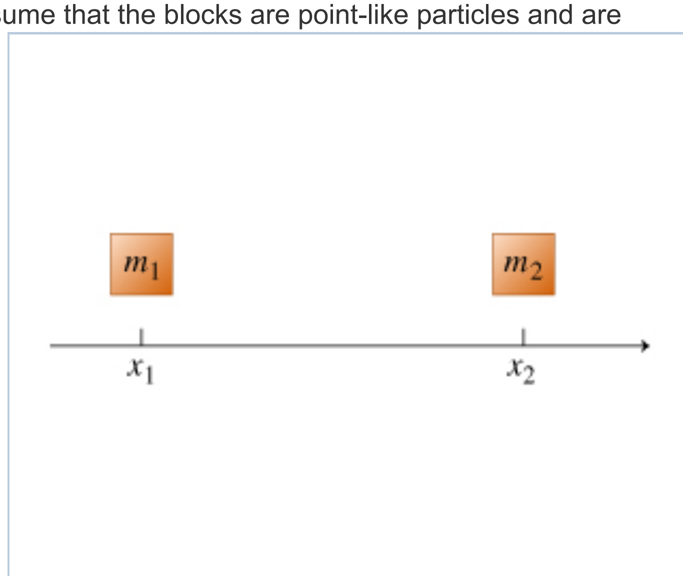
  $$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}.$$

  The *x* coordinate of the center of mass $x_{\rm cm}$ can be expressed in terms of the *x* coordinates $(r_x)_i$ of the particles as

  $$x_{cm} = \frac{\sum m_i (r_x)_i}{\sum m_i}.$$

  Similarly, the *y* coordinate of the center of mass can be expressed.
- Internal force: Any force that results from an interaction between the objects inside your system. As we will show, the internal forces do not affect the motion of the system's center of mass.
- External force: Any force acting on an object inside your system that results from an interaction with an object outside your system.

Consider a system of two blocks that have masses m_1 and m_2. Assume that the blocks are point-like particles and are located along the *x* axis at the coordinates x_1 and x_2 as shown .
In this problem, the blocks can only move along the *x* axis.



Part A

Find the x coordinate x_{cm} of the center of mass of the system.

Express your answer in terms of m_1, m_2, x_1, and x_2.

ANSWER:

x_{cm} = $\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

Correct

Part B

If $m_2 \gg m_1$, then the center of mass is located:

ANSWER:

- ○ to the left of m_1 at a distance much greater than $x_2 - x_1$
- ○ to the left of m_1 at a distance much less than $x_2 - x_1$
- ○ to the right of m_1 at a distance much less than $x_2 - x_1$
- ○ to the right of m_2 at a distance much greater than $x_2 - x_1$
- ○ to the right of m_2 at a distance much less than $x_2 - x_1$
- ◉ to the left of m_2 at a distance much less than $x_2 - x_1$

Correct

Part C

If $m_2 = m_1$, then the center of mass is located:

ANSWER:

- ○ at m_1
- ○ at m_2
- ◉ half-way between m_1 and m_2
- ○ the answer depends on x_1 and x_2

Correct

Part D

Recall that the blocks can only move along the x axis. The x components of their velocities at a certain moment are $v_{1\mathrm{x}}$ and $v_{2\mathrm{x}}$. Find the x component of the velocity of the center of mass $(v_{\mathrm{cm}})_x$ at that moment. Keep in mind that, in general: $v_x = dx/dt$.

Express your answer in terms of m_1, m_2, v_{1x}, and v_{2x}.

ANSWER:

$(v_{cm})_x$ = $\frac{m_1 v_{1x} + m_2 v_{2x}}{m_1 + m_2}$

Correct

Because v_{1x} and v_{2x} are the *x* components of the velocities of m_1 and m_2 their values can be positive or negative or equal to zero.

Part E

Suppose that v_{1x} and v_{2x} have equal magnitudes. Also, $\vec{v_1}$ is directed to the right and $\vec{v_2}$ is directed to the left. The velocity of the center of mass is then:

ANSWER:

- directed to the left
- directed to the right
- zero
- the answer depends on the ratio $\frac{m_1}{m_2}$

Correct

Part F

Assume that the *x* components of the blocks' momenta at a certain moment are p_{1x} and p_{2x}. Find the *x* component of the velocity of the center of mass $(v_{cm})_x$ at that moment.

Express your answer in terms of m_1, m_2, p_{1x}, and p_{2x}.

ANSWER:

$(v_{cm})_x$ = $\frac{p_{1x} + p_{2x}}{m_1 + m_2}$

Correct

Part G

Suppose that $\vec{v}_{cm} = 0$. Which of the following must be true?

ANSWER:

- (selected) $|p_{1x}| = |p_{2x}|$
- $|v_{1x}| = |v_{2x}|$
- $m_1 = m_2$
- none of the above

Correct

Part H

Assume that the blocks are accelerating, and the *x* components of their accelerations at a certain moment are $a_{1\mathrm{x}}$ and $a_{2\mathrm{x}}$. Find the *x* component of the acceleration of the center of mass $(a_{\mathrm{cm}})_x$ at that moment. Keep in mind that, in general, $a_x = dv_x/dt$.

Express your answer in terms of m_1, m_2, $a_{1\mathrm{x}}$, and $a_{2\mathrm{x}}$.

ANSWER:

$(a_{\mathrm{cm}})_x = \frac{a_{1\mathrm{x}}m_1 + a_{2\mathrm{x}}m_2}{m_1 + m_2}$

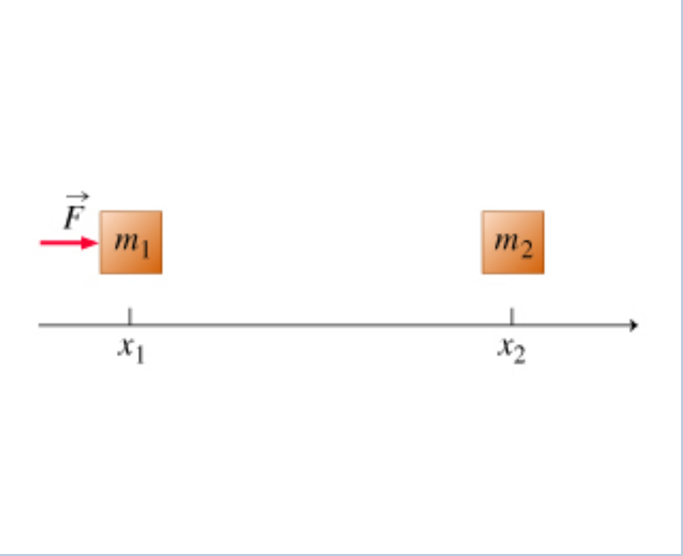
Correct

Because $a_{1\mathrm{x}}$ and $a_{2\mathrm{x}}$ are the *x* components of the velocities of m_1 and m_2 their values can be positive or negative or equal to zero.

We will now consider the effect of external and internal forces on the acceleration of the center of mass.

Part I

Consider the same system of two blocks. An *external* force $\vec{F}$ is now acting on block m_1. *No* forces are applied to block m_2 as shown . Find the *x* component of the acceleration of the center of mass $(a_{\mathrm{cm}})_x$ of the system.



Express your answer in terms of the *x* component F_x of the force, m_1 ,and m_2 .

Hint 1. Using Newton's laws

Find the acceleration of each block from Newton's second law and then apply the formula for $(a_{\mathrm{cm}})_x$ found earlier.

ANSWER:

$(a_{\mathrm{cm}})_x = \frac{F_x}{m_1+m_2}$

Correct

Part J

Consider the same system of two blocks. Now, there are two forces involved. An *external* force $\vec{F_1}$ is acting on block m_1 and another *external* force $\vec{F_2}$ is acting on block m_2. Find the *x* component of the acceleration of the center of mass $(a_{\mathrm{cm}})_x$ of the system.

Express your answer in terms of the *x* components $F_{1\mathrm{x}}$ and $F_{2\mathrm{x}}$ of the forces, m_1 and m_2.

ANSWER:

$(a_{\mathrm{cm}})_x = \frac{F_{1\mathrm{x}}+F_{2\mathrm{x}}}{m_1+m_2}$

Correct

Note that, in both cases, the acceleration of mass can be found as

$$(a_{\mathrm{cm}})_x = \frac{(F_{\mathrm{net}})_x}{M_{\mathrm{total}}}$$

where F_{net} is the net *external* force applied to the system, and M_{total} is the total mass of the system. Even though each force is only applied to *one* object, it affects the acceleration of the center of mass of the *entire system*.

This result is especially useful since it can be applied to a general case, involving *any* number of objects moving in *all* directions and being acted upon by *any* number of *external* forces.

Part K

Consider the previous situation. Under what condition would the acceleration of the center of mass be zero? Keep in mind that F_{1x} and F_{2x} represent *the components*, of the corresponding forces.

ANSWER:

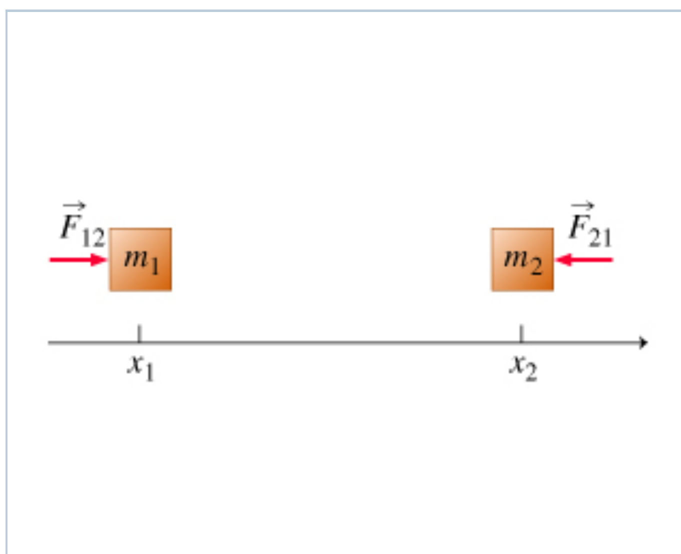
- $F_{1x} = -F_{2x}$
- $F_{1x} = F_{2x}$
- $m_1 = m_2$
- $m_1 \ll m_2$

Correct

Part L

Consider the same system of two blocks. Now, there are two *internal* forces involved. An *internal* force $\vec{F}_{12}$ is applied to block m_1 by block m_2 and another *internal* force $\vec{F}_{21}$ is applied to block m_2 by block m_1 . Find the *x* component of the acceleration of the center of mass $(a_{\rm cm})_x$ of the system.

Express your answer in terms of the *x* components F_{12x} and F_{21x} of the forces, m_1 and m_2.



ANSWER:

$(a_{\mathrm{cm}})_x$ = $\frac{F_{12\mathrm{x}}+F_{21\mathrm{x}}}{m_1+m_2}$

Correct

Newton's 3rd law tells you that $|F_{12x}| = -|F_{21x}|$. From your answers above, you can conclude that $(a_{\mathrm{cm}})_x = 0$ .The *internal forces* do *not* change the velocity of the center of mass of the system. In the absence of any *external* forces, $(a_{\mathrm{cm}})_x = 0$ and $(v_{\mathrm{cm}})_x$ is constant.

You just demonstrated this to be the case for the two-body situation moving along the *x* axis; however, it is true in more general cases as well.

A Game of Frictionless Catch

Chuck and Jackie stand on separate carts, both of which can slide without friction. The combined mass of Chuck and his cart, m_{cart}, is identical to the combined mass of Jackie and her cart. Initially, Chuck and Jackie and their carts are at rest.

Chuck then picks up a ball of mass m_{ball} and throws it to Jackie, who catches it. Assume that the ball travels in a straight line parallel to the ground (ignore the effect of gravity). After Chuck throws the ball, his speed relative to the ground is v_{c}. The speed of the thrown ball relative to the ground is v_{b}.

Jackie catches the ball when it reaches her, and she and her cart begin to move. Jackie's speed relative to the ground after she catches the ball is v_{j}.

When answering the questions in this problem, keep the following in mind:

1. The original mass m_{cart} of Chuck and his cart does not include the mass of the ball.
2. The speed of an object is the magnitude of its velocity. An object's speed will always be a nonnegative quantity.

Part A

Find the relative speed u between Chuck and the ball after Chuck has thrown the ball.

Express the speed in terms of v_{c} and v_{b}.

Hint 1. How to approach the problem

All this question is asking is: "How fast are Chuck and the ball moving away from each other?" If two objects are moving at the same speed (with respect to the ground) in the same direction, their relative speed is zero. If they are moving at the same speed, v, in opposite directions, their relative speed is $2v$. In this problem, you are given variables for the speed of Chuck and the ball with respect to the ground, and you know that Chuck and the ball are moving directly away from each other.

ANSWER:

u = $v_c + v_b$

Correct

Make sure you understand this result; the concept of "relative speed" is important. In general, if two objects are moving in opposite directions (either toward each other or away from each other), the relative speed between them is equal to the sum of their speeds with respect to the ground. If two objects are moving in the same direction, then the relative speed between them is the absolute value of the difference of the their two speeds with respect to the ground.

Part B

What is the speed v_{b} of the ball (relative to the ground) while it is in the air?

Express your answer in terms of m_{ball}, m_{cart}, and u.

Hint 1. How to approach the problem

Apply conservation of momentum. Equate the initial (before the ball is thrown) and final (after the ball is thrown) momenta of the system consisting of Chuck, his cart, and the ball. Use the result from Part A to eliminate v_{c} from this equation and solve for v_{b}.

Hint 2. Initial momentum of Chuck, his cart, and the ball

Before the ball is thrown, Chuck, his cart, and the ball are all at rest. Therefore, their total initial momentum is zero.

Hint 3. Find the final momentum of Chuck, his cart, and the thrown ball

What is the total momentum p_{final} of Chuck, his cart, and the ball after the ball is thrown?

Express your answer in terms of m_{ball}, m_{cart}, v_{c}, and v_{b}.
Remember that v_{c} and v_{b} are speeds, not velocities, and thus are positive scalars.

ANSWER:

p_{final} = $-m_{\mathrm{cart}}v_c + m_{\mathrm{ball}}v_b$

ANSWER:

v_{b} = $\frac{u m_{\mathrm{cart}}}{m_{\mathrm{ball}}+m_{\mathrm{cart}}}$

Correct

Part C

What is Chuck's speed v_{c} (relative to the ground) after he throws the ball?

Express your answer in terms of m_{ball}, m_{cart}, and u.

Hint 1. How to approach the problem

Use the answer to Part B to eliminate $v_{\rm b}$ from the equation derived in Part A. Then solve for $v_{\rm c}$.

ANSWER:

$v_{\rm c}$ = $\frac{um_{\rm ball}}{m_{\rm ball}+m_{\rm cart}}$

Correct

Part D

Find Jackie's speed $v_{\rm j}$ (relative to the ground) after she catches the ball, in terms of $v_{\rm b}$.

Express $v_{\rm j}$ in terms of $m_{\rm ball}$, $m_{\rm cart}$, and $v_{\rm b}$.

Hint 1. How to approach the problem

Apply conservation of momentum. Equate the initial (before Jackie catches the ball) and final (after the ball is caught) momenta of the system consisting of Jackie, her cart, and the ball, and solve for $v_{\rm j}$.

Hint 2. Initial momentum

Just before Jackie catches the ball, the momentum of the system consisting of Jackie, her cart, and the ball is equal to the momentum of the ball as it flies through the air: $p_{\rm initial} = m_{\rm ball}v_{\rm b}$.

Hint 3. Find the final momentum

What is the final momentum $p_{\rm final}$ of the system after Jackie catches the ball?

Express your answer in terms of $m_{\rm ball}$, $m_{\rm cart}$, and $v_{\rm j}$.

ANSWER:

$p_{\rm final}$ = $(m_{\rm cart} + m_{\rm ball})v_j$

ANSWER:

$v_{\rm j}$ = $\frac{m_{\rm ball}v_b}{m_{\rm cart}+m_{\rm ball}}$

Correct

Part E

Find Jackie's speed $v_{\rm j}$ (relative to the ground) after she catches the ball, in terms of u.

Express $v_{\rm j}$ in terms of $m_{\rm ball}$, $m_{\rm cart}$, and u.

Hint 1. How to approach the problem

In Part B, you found an expression for $v_{\rm b}$ in terms of u. You can substitute this expression for $v_{\rm b}$ into the equation you found in Part D, which will give you an expression for $v_{\rm j}$ in terms of the desired quantities.

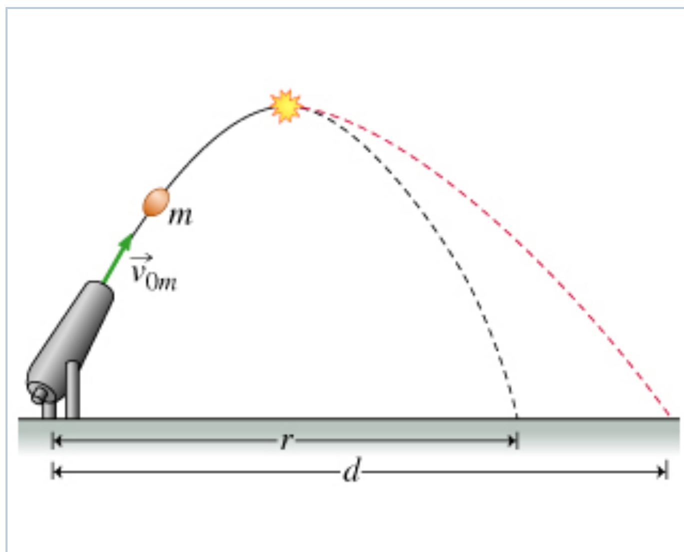
ANSWER:

$v_{\rm j}$ = $\frac{u m_{\rm ball} m_{\rm cart}}{\left(m_{\rm ball}+m_{\rm cart}\right)^2}$

Correct

Surprising Exploding Firework

A mortar fires a shell of mass m at speed v_0. The shell explodes at the top of its trajectory (shown by a star in the figure) as designed. However, rather than creating a shower of colored flares, it breaks into just two pieces, a smaller piece of mass $\frac{1}{5}m$ and a larger piece of mass $\frac{4}{5}m$. Both pieces land at exactly the same time. The smaller piece lands perilously close to the mortar (at a distance of zero from the mortar). The larger piece lands a distance d from the mortar. If there had been no explosion, the shell would have landed a distance r from the mortar. Assume that air resistance and the mass of the shell's explosive charge are negligible.



Part A

Find the distance d from the mortar at which the larger piece of the shell lands.

Express d in terms of r.

Hint 1. Find the position of the center of mass in terms of r

The two exploded pieces of the shell land at the same time. At the moment of landing, what is the distance $x_{\rm cm}$

from the mortar to the center of mass of the exploded pieces?

Express your answer in terms of r.

Hint 1. Key idea

The explosion only exerts *internal* forces on the particles. The only *external* force acting on the two-piece system is gravity, so the center of mass will continue along the original trajectory of the shell.

ANSWER:

x_{cm} = r

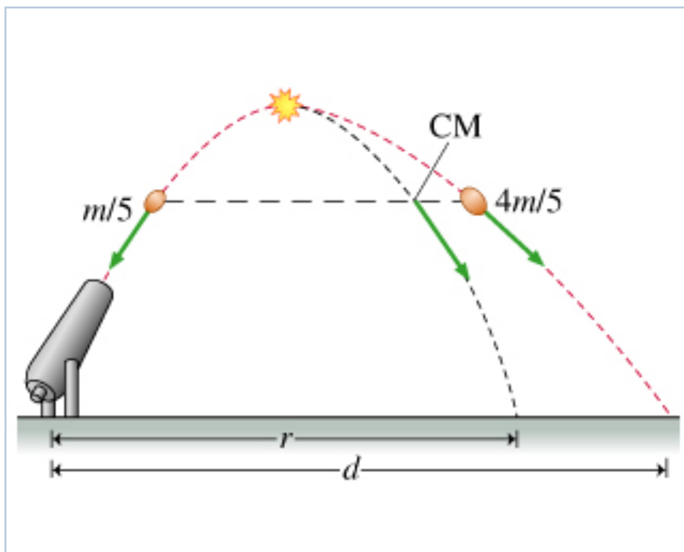
Hint 2. Find the position of the center of mass in terms of d

The larger piece of the shell lands a distance d from the mortar, and the smaller piece lands a distance zero from the mortar. What is x_{cm}, the final distance of the shell's center of mass from the mortar?

Express your answer in terms of d.

Hint 1. A helpful figure

Here is a figure to help you visualize the situation.



ANSWER:

x_{cm} = $\frac{4d}{5}$

ANSWER:

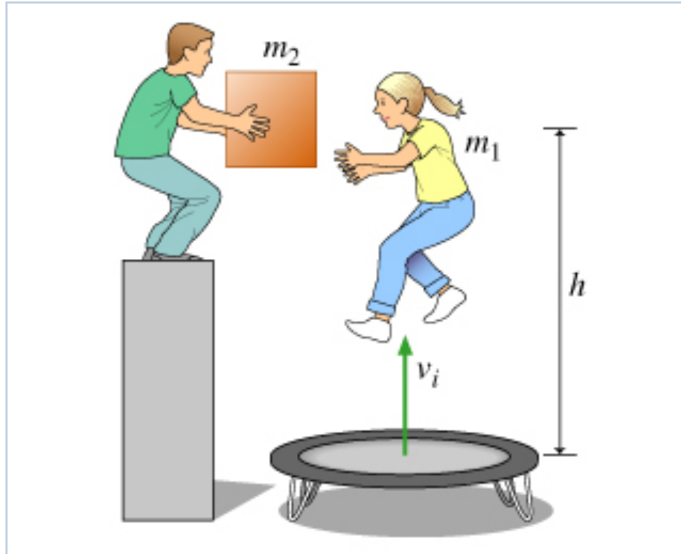
d = $\frac{5}{4}r$

Correct

A Girl on a Trampoline

A girl of mass $m_1 = 60$ kilograms springs from a trampoline with an initial upward velocity of $v_i = 8.0$ meters per second. At height $h = 2.0$ meters above the trampoline, the girl grabs a box of mass $m_2 = 15$ kilograms.

For this problem, use $g = 9.8$ meters per second per second for the magnitude of the acceleration due to gravity.



Part A

What is the speed v_{before} of the girl immediately before she grabs the box?

Express your answer numerically in meters per second.

Hint 1. How to approach the problem

Use conservation of energy. Find the initial kinetic energy K_{i} of the girl as she leaves the trampoline. Then find her gravitational potential energy U_{before} just before she grabs the box (define her initial potential energy to be zero). According to the principle of conservation of energy, $K_i = U_{\mathrm{before}} + K_{\mathrm{before}}$. Once you have K_{before}, use the definition of translational kinetic energy to find the girl's speed v_{before}.

Hint 2. Initial kinetic energy

What is the girl's initial kinetic energy K_{i} as she leaves the trampoline?

Express your answer numerically in joules.

ANSWER:

K_{i} = 1920 J

Hint 3. Potential energy at height h

What is the girl's gravitational potential energy U_{before} immediately before she grabs the box?

Express your answer numerically in joules.

ANSWER:

U_{before} = 1180 J

ANSWER:

v_{before} = 4.98 m/s

Correct

Part B

What is the speed v_{after} of the girl immediately after she grabs the box?

Express your answer numerically in meters per second.

Hint 1. How to approach the problem

Think of the process of grabbing the box as a collision. Though the girl and the box don't collide as such, any interaction between two objects that takes place extremely fast can be thought of as a collision. To find the velocity at a later time, which of the following principles could you use?

ANSWER:

- (selected) conservation of momentum alone
- conservation of energy alone
- both conservation of momentum and conservation of energy
- Newton's second law

Hint 2. Total initial momentum

What is the total momentum before the collision?

Answer in kilogram meters per second.

ANSWER:

p_{before} = 299 kg m/s

ANSWER:

v_{after} = 3.98 m/s

Correct

Part C

Is this "collision" elastic or inelastic?

Hint 1. Definition of an inelastic collision

If two objects move together with the same velocity after a collision, the collision is said to be inelastic.

ANSWER:

- ○ elastic
- ◉ inelastic

Correct

In inelastic collisions, some of the system's kinetic energy is lost. In this case the kinetic energy lost is converted to heat energy in the girl's muscles as she grabs the box, and sound energy.

Part D

What is the maximum height h_{max} that the girl (with box) reaches? Measure h_{max} with respect to the top of the trampoline.

Express your answer numerically in meters.

Hint 1. How to approach the problem

Use conservation of energy. From Part B you know the velocity of the girl/box system just after the girl grabs the box. Therefore, you can compute the kinetic energy K_{after} of the girl/box system just after the collision. You can also compute the gravitational potential energy U_{after} of the girl/box system at this point. The sum of these two quantities must equal the gravitational potential energy of the girl/box system at the height h_{max} (where their velocity, and therefore kinetic energy, will be zero).

Hint 2. Finding U_{after}

What is the girl/box system's gravitational potential energy U_{after} immediately after she grabs the box?

Express your answer numerically in joules.

ANSWER:

U_{after} = 1470 J

Hint 3. Finding K_{after}

What is the girl/box system's kinetic energy K_{after} immediately after she grabs the box?

Express your answer numerically in joules.

ANSWER:

K_{after} = 594 J

ANSWER:

h_{max} = 2.81 m

Correct

± A Rocket in Deep Space

A rocket is fired in deep space, where gravity is negligible. In the first second it ejects $\frac{1}{160}$ of its mass as exhaust gas and has an acceleration of 14.9m/s^2 .

Part A

What is the speed v_{gas} of the exhaust gas relative to the rocket?

Express your answer numerically in kilometers per second.

Hint 1. How to approach the problem

In deep space gravity is negligible and there is no air resistance; thus no external forces act on the rocket and the exhaust gas, and the total momentum of the system (rocket plus exhaust gas) is conserved. By applying conservation of momentum, one can derive a formula for the acceleration of the rocket given the speed of the exhaust gas relative to the rocket, the original mass of the rocket, and the rate of change of the rocket's mass with time.

Hint 2. The acceleration of the rocket

By applying conservation of momentum to the system that comprises the rocket and the exhaust gas, one can derive a formula for the acceleration of the rocket a in terms of the speed v_{gas} of the exhaust gas relative to the rocket and the mass of the rocket m. In symbols,

$$a = -\frac{v_{\text{gas}}}{m}\frac{dm}{dt},$$

where t represents time. The quantity $\frac{dm}{dt}$ is the time rate of change of the mass of the rocket, and it is a negative quantity (the mass of the rocket decreases continuously with time as it burns fuel). Since v_{gas} is a positive quantity (it is the *speed* of the exhaust gas), the acceleration of the rocket is also positive.

(Another way to derive the formula above, is to apply the relationship $\vec{F} = \frac{d\vec{p}}{dt}$ to the exhaust gas and note that the force on the rocket is equal and opposite to the force on the exhaust.)

Hint 3. Find the change in mass of the rocket

What is the rate of change of the mass of the rocket if m is its original mass before the launch?

ANSWER:

- ○ $-160m$
- ○ $\frac{m}{160}$
- ○ $m - \frac{m}{160}$
- ◉ $-\frac{m}{160}$

ANSWER:

v_{gas} = 2.38 $\mathrm{km/s}$

Correct

A message from your instructor...

Note: The problem numbers do not correspond to the end-of-chapter problems in the textbook (Knight). These problems are taken from a database provided by the publisher.

Problem 8.24

Two ice skaters, Daniel (mass 65.0 kg) and Rebecca (mass 45.0 kg), are practicing. Daniel stops to tie his shoelace and, while at rest, is struck by Rebecca, who is moving at 13.0 m/s before she collides with him. After the collision, Rebecca has a velocity of magnitude 8.00 m/s at an angle of 53.1° from her initial direction. Both skaters move on the frictionless, horizontal surface of the rink.

Part A

What is the magnitude of Daniel's velocity after the collision?

ANSWER:

7.198 m/s

Correct

Part B

What is the direction of Daniel's velocity after the collision?

ANSWER:

-37.97 ° from the Rebecca's original direction

Correct

Part C

What is the change in total kinetic energy of the two skaters as a result of the collision?

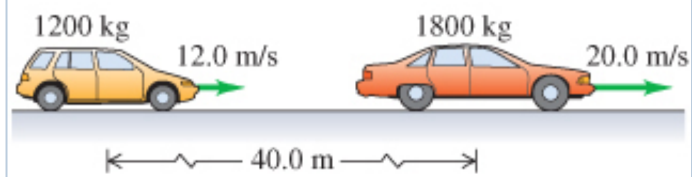
ANSWER:

-678 J

Correct

Problem 8.46

A 1200-kg station wagon is moving along a straight highway at 12.0 m/s. Another car, with mass 1800 kg and speed 20.0 m/s, has its center of mass 40.0 m ahead of the center of mass of the station wagon .



Part A

Find the position of the center of mass of the system consisting of the two automobiles.

ANSWER:

16.0 m behind the leading car

Correct

Part B

Find the magnitude of the total momentum of the system from the above data.

ANSWER:

5.04×10^4 $\mathrm{kg \cdot m/s}$

Correct

Part C

Find the speed of the center of mass of the system.

ANSWER:

16.8 m/s

Correct

Part D

Find the total momentum of the system, using the speed of the center of mass.

ANSWER:

5.04×10^4 $\mathrm{kg \cdot m/s}$

Correct

Problem 8.54

A rocket is fired in deep space, where gravity is negligible. In the first second it ejects $\frac{1}{160}$ of its mass as exhaust gas and has an acceleration of $14.9 \mathrm{m/s^2}$.

Part A

What is the speed of the exhaust gas relative to the rocket?

ANSWER:

2.38 km/s

Correct

Problem 8.56

A single-stage rocket is fired from rest from a deep-space platform, where gravity is negligible.

Part A

If the rocket burns its fuel in a time of 50.0 s and the relative speed of the exhaust gas is $v_{ex} = 2100\ \mathrm{m/s}$, what must the mass ratio m_0/m be for a final speed v of 8.00 km/s (about equal to the orbital speed of an earth satellite)?

ANSWER:

45.1

Correct

Problem 8.65

A railroad handcar is moving along straight, frictionless tracks with negligible air resistance. In the following cases, the car initially has a total mass (car and contents) of 170kg and is traveling east with a velocity of magnitude 4.80$\mathrm{m/s}$. Find the *final velocity* of the car in each case, assuming that the handcar does not leave the tracks.

Part A

An object with a mass of 22.0kg is thrown sideways out of the car with a speed of 2.30$\mathrm{m/s}$ relative to the car's initial velocity.

ANSWER:

4.80 m/s east

Correct

Part B

An object with a mass of 22.0kg is thrown backward out of the car with a velocity of 4.80$\mathrm{m/s}$ relative to the initial motion of the car.

ANSWER:

5.51 m/s east

Correct

Part C

An object with a mass of $22.0 \mathrm{kg}$ is thrown into the car with a velocity of $6.00 \mathrm{m/s}$ relative to the ground and opposite in direction to the initial velocity of the car.

ANSWER:

3.56 m/s east

Correct

Score Summary:

Your score on this assignment is 98.6%.
You received 98.59 out of a possible total of 100 points.