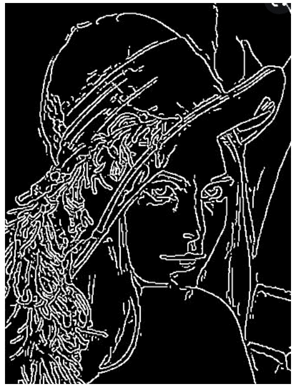
Edge:
Operators

Dr. Tushar Sandhan

Introduction

- Who am I?



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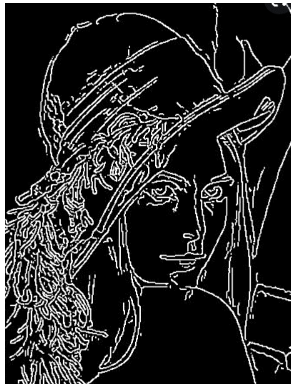

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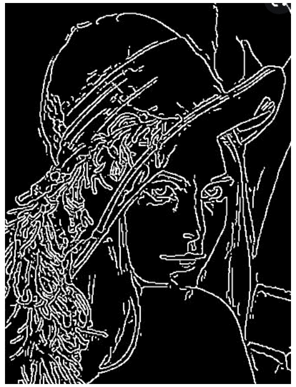

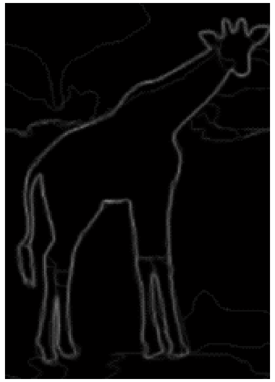

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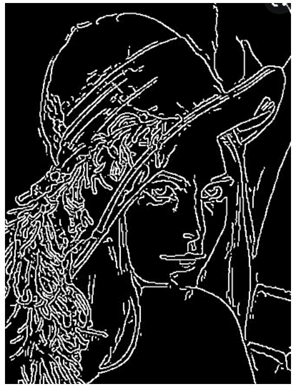

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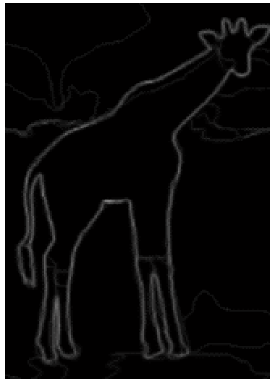

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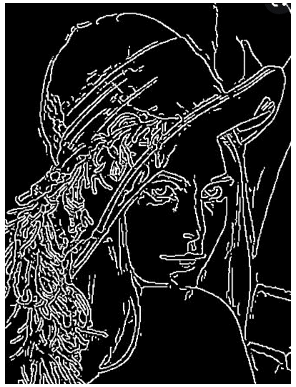

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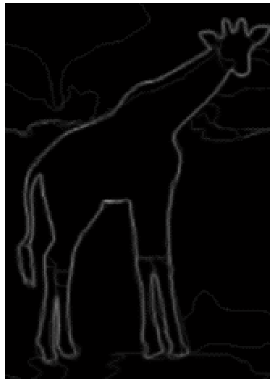

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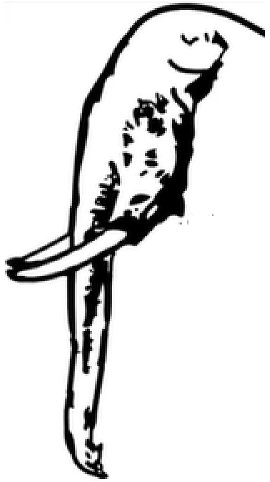

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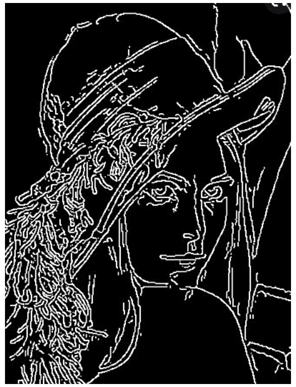

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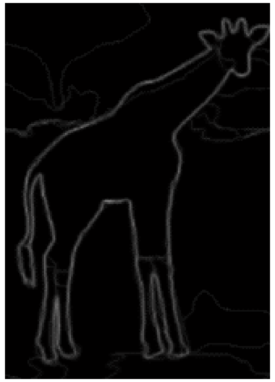

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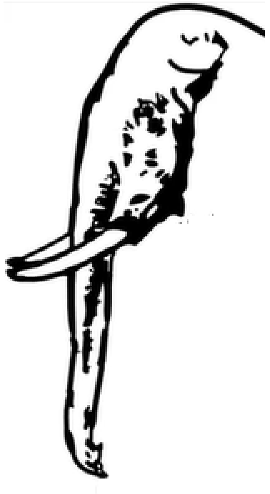

non-uniform

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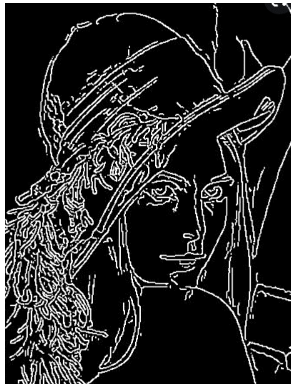

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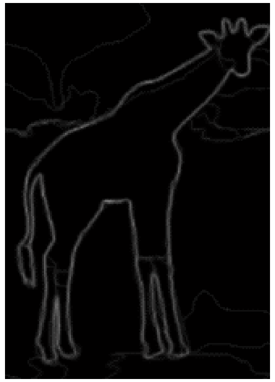

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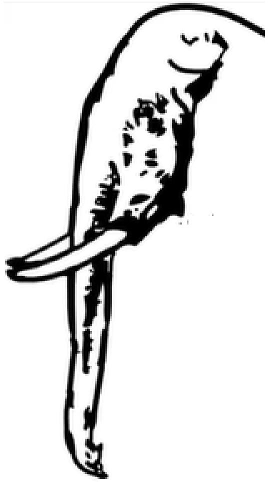

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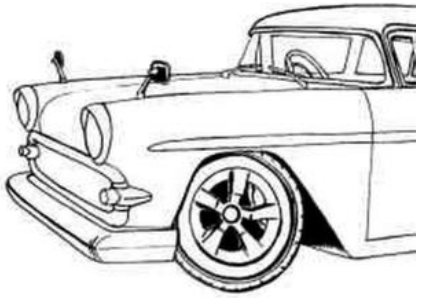

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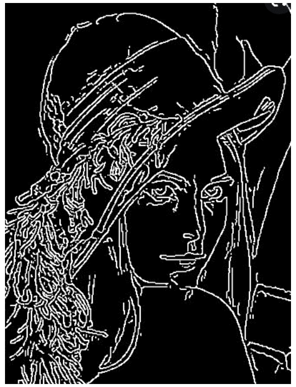

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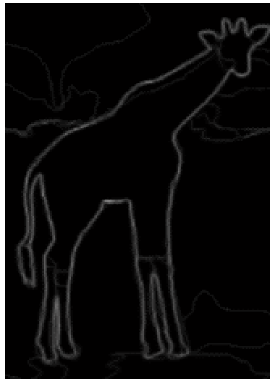

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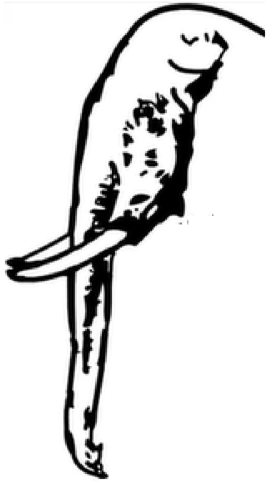

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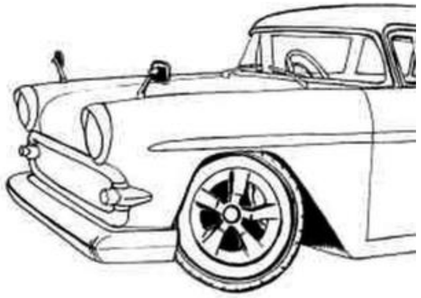

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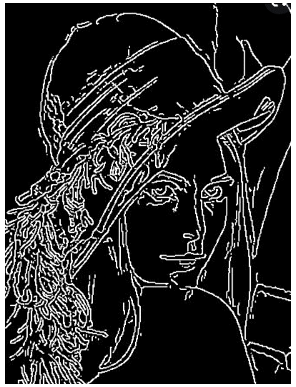

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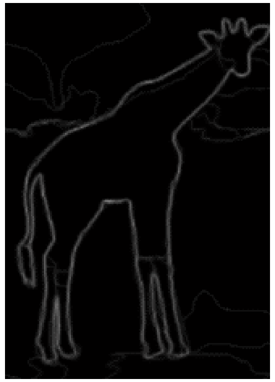

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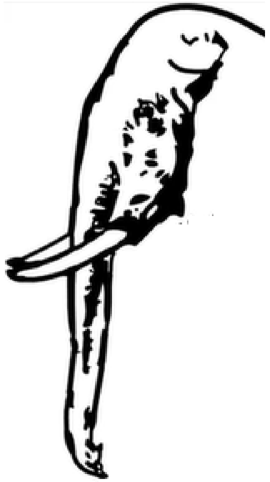

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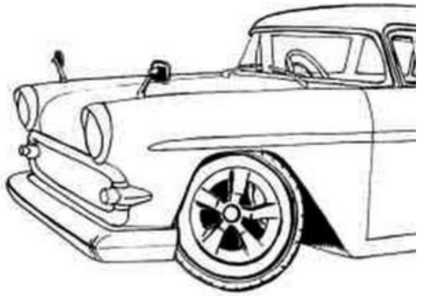

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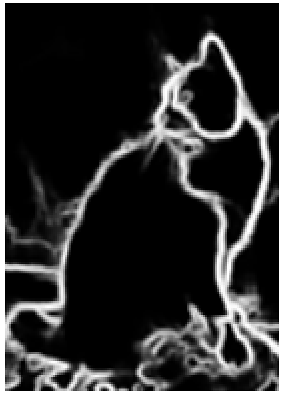

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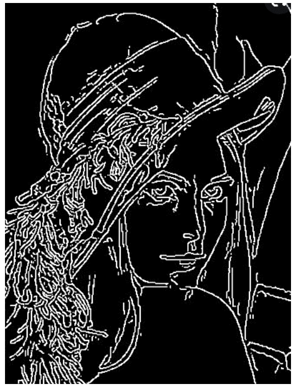

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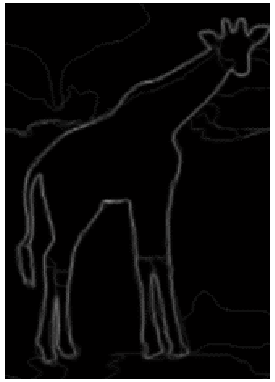

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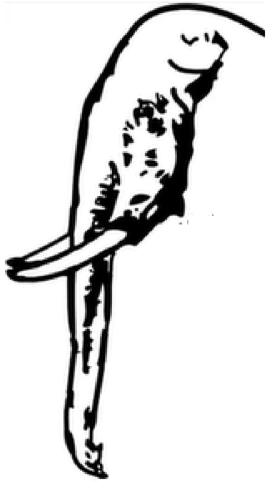

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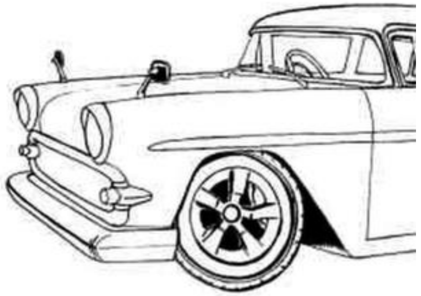

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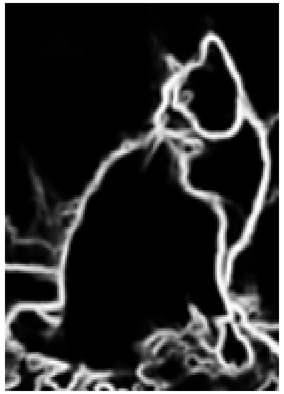

mewww~

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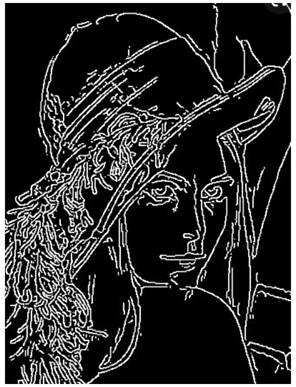

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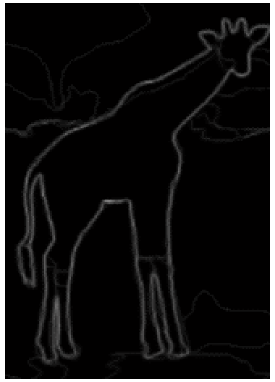

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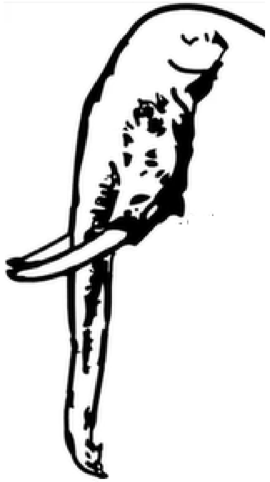

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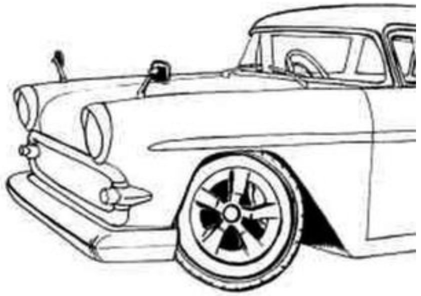

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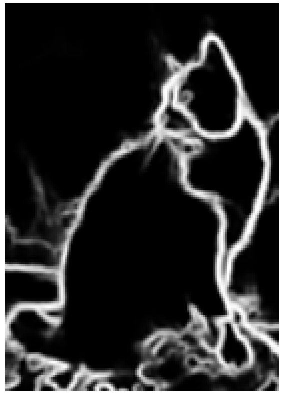

mewww~

- It implies: edges convey a lot of info.
- Lossy but extremely high compression

What makes edges

What makes edges

- Discontinuity in color

What makes edges

- Discontinuity in color
- Change in surface normal

What makes edges

- Discontinuity in color
- Change in surface normal
- Change in illumination

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- Reflectance change

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- Inoculation is an edge itself!

What makes edges

- Discontinuity in color
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Edge

- Taylor’s edge
  - expand $f(x + \Delta x)$

Edge

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  - expand $f(x+\Delta x)$

$$f(x+\Delta x) = f(x) + \Delta x \frac{\partial f(x)}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 f(x)}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 f(x)}{\partial x^3} + \cdots\cdots$$

$$= \sum_{n=0}^{\infty} \frac{(\Delta x)^n}{n!}\frac{\partial^n f(x)}{\partial x^n}$$

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$$f(x+\Delta x) = f(x) + \Delta x \frac{\partial f(x)}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 f(x)}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 f(x)}{\partial x^3} + \cdots\cdots$$
$$= \sum_{n=0}^{\infty} \frac{(\Delta x)^n}{n!}\frac{\partial^n f(x)}{\partial x^n}$$

$$f(x+1) = f(x) + \frac{\partial f(x)}{\partial x} + \frac{1}{2!}\frac{\partial^2 f(x)}{\partial x^2} + \frac{1}{3!}\frac{\partial^3 f(x)}{\partial x^3} + \cdots\cdots$$
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$$f(x+1) = f(x) + \frac{\partial f(x)}{\partial x} + \frac{1}{2!}\frac{\partial^2 f(x)}{\partial x^2} + \frac{1}{3!}\frac{\partial^3 f(x)}{\partial x^3} + \cdots\cdots$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!}\frac{\partial^n f(x)}{\partial x^n}$$

$$f(x-1) = f(x) - \frac{\partial f(x)}{\partial x} + \frac{1}{2!}\frac{\partial^2 f(x)}{\partial x^2} - \frac{1}{3!}\frac{\partial^3 f(x)}{\partial x^3} + \cdots\cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\frac{\partial^n f(x)}{\partial x^n}$$

Edge

- Forward

Edge

- Forward $$\frac{\partial f(x)}{\partial x} = f'(x) = f(x+1) - f(x)$$

Edge

- Forward $$\frac{\partial f(x)}{\partial x} = f'(x) = f(x+1) - f(x)$$

- Backward $$\frac{\partial f(x)}{\partial x} = f'(x) = f(x) - f(x-1)$$

Edge

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- Backward $$\frac{\partial f(x)}{\partial x} = f'(x) = f(x) - f(x-1)$$
- Central $$\frac{\partial f(x)}{\partial x} = f'(x) = \frac{f(x+1) - f(x-1)}{2}$$

Edge

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- Backward $$\frac{\partial f(x)}{\partial x} = f'(x) = f(x) - f(x-1)$$
- Central $$\frac{\partial f(x)}{\partial x} = f'(x) = \frac{f(x+1) - f(x-1)}{2}$$
- 2nd order central $$\frac{\partial^2 f(x)}{\partial x^2} = f''(x) = f(x+1) - 2f(x) + f(x-1)$$

Edge

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- Image $f(x, y)$

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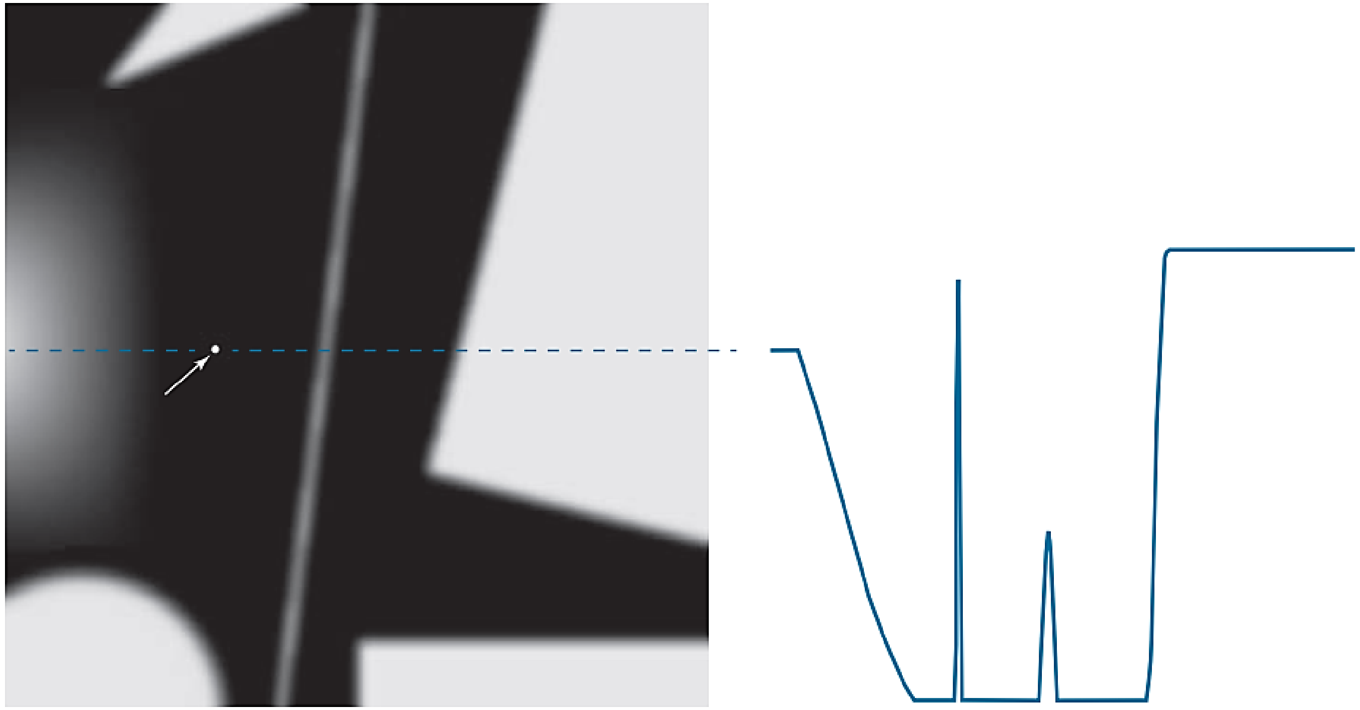
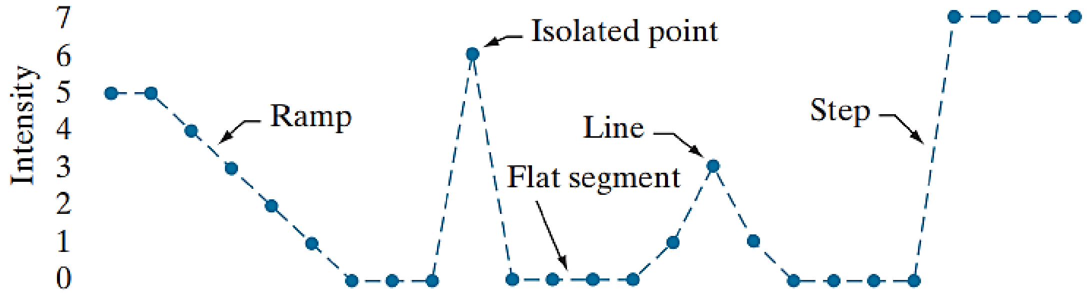
- Image $f(x,y)$

$$\frac{\partial^2 f(x,y)}{\partial x^2} = f(x+1,y) - 2f(x,y) + f(x-1,y)$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} = f(x,y+1) - 2f(x,y) + f(x,y-1)$$

Edge





Intensity values	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	•	•
First derivative		−1	−1	−1	−1	−1	0	0	6	−6	0	0	0	1	2	−2	−1	0	0	0	7	0	0	0			
Second derivative		−1	0	0	0	0	1	0	6	−12	6	0	0	1	1	−4	1	1	0	0	7	−7	0	0			

Obtained by f(x+1)-f(x)

Obtained by f(x+1)-2f(x)+f(x-1)

Isolated point

Point

- Isolated point
  - 2^{nd} derivatives
  - Laplacian

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Point

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$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

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$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

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0	1	0
1	−4	1
0	1	0

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0	1	0
1	−4	1
0	1	0

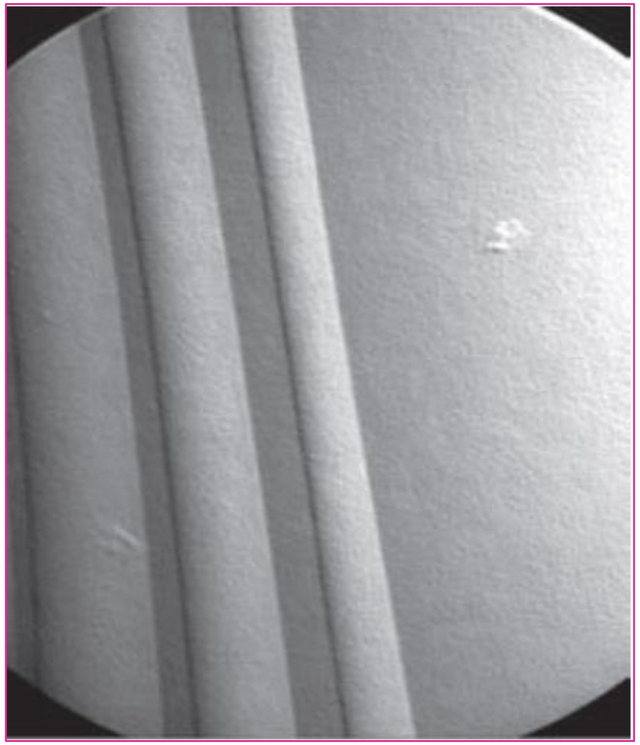
$\longrightarrow$

1	1	1
1	−8	1
1	1	1

Point

- Turbine blade under X-ray
  - Laplace operator

Input



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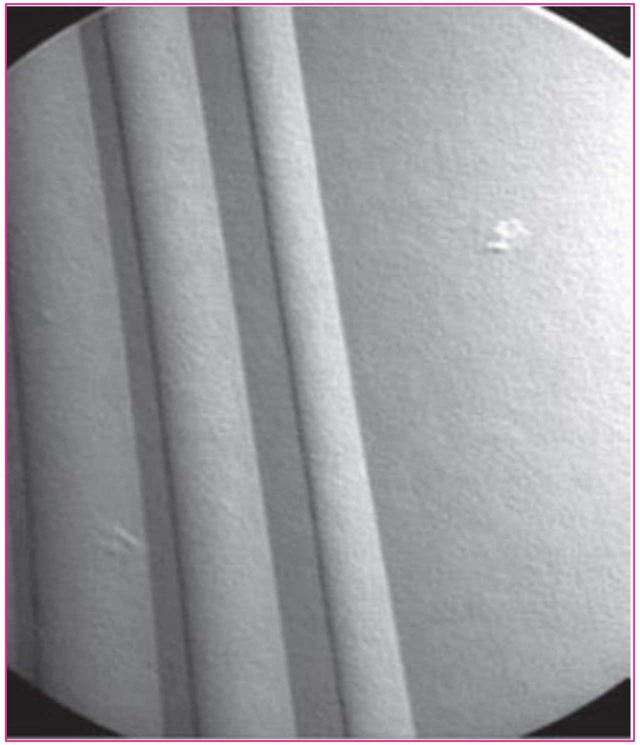
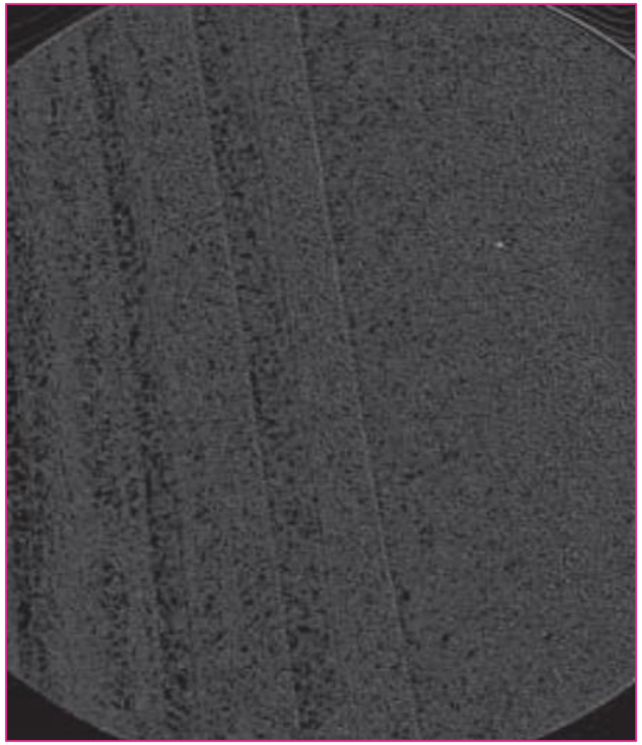


Laplacian



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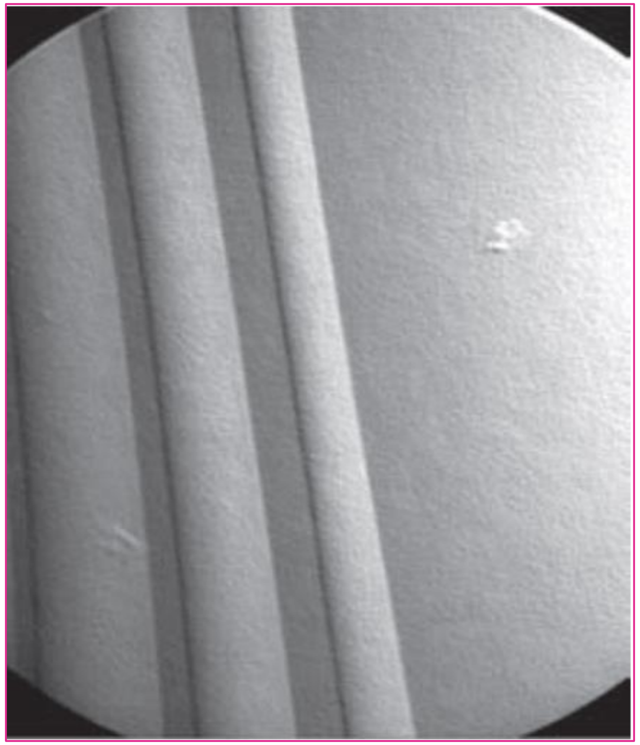

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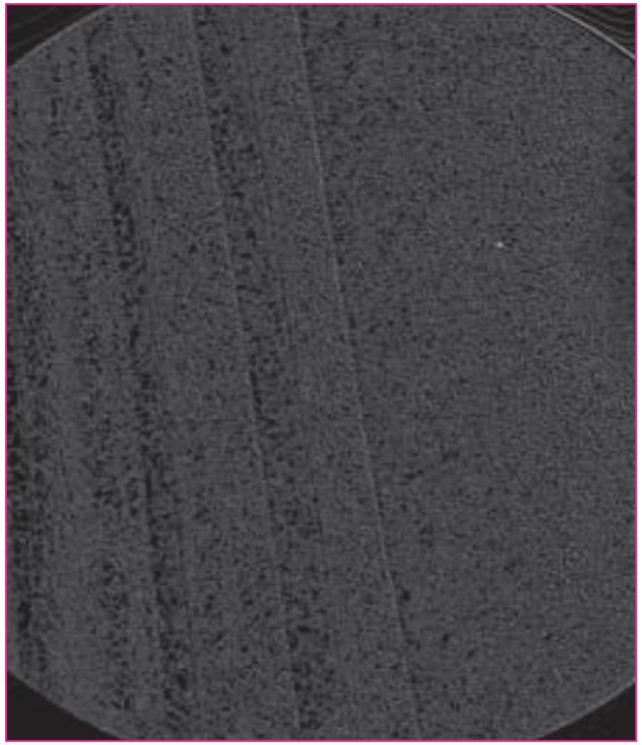
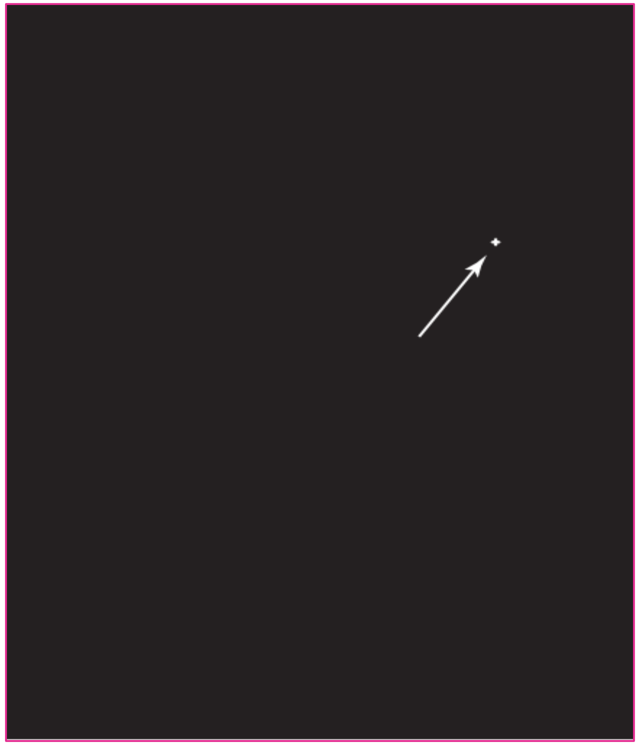


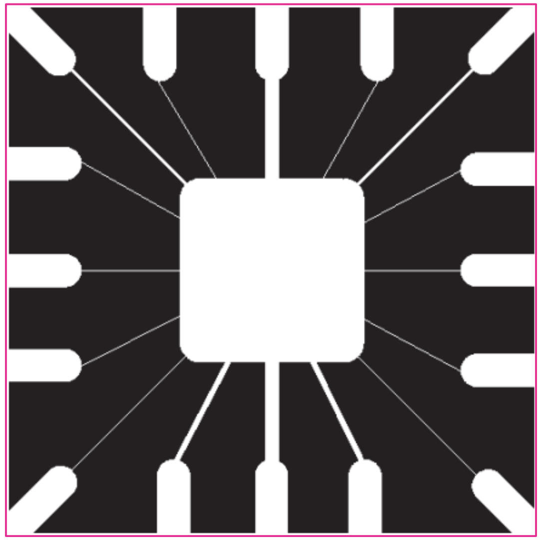
Thresholding



Line

- PCB wire bonds
  - Laplace operator

Input



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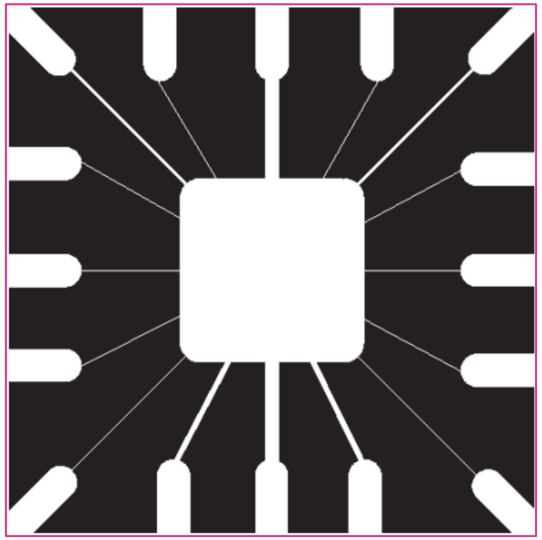
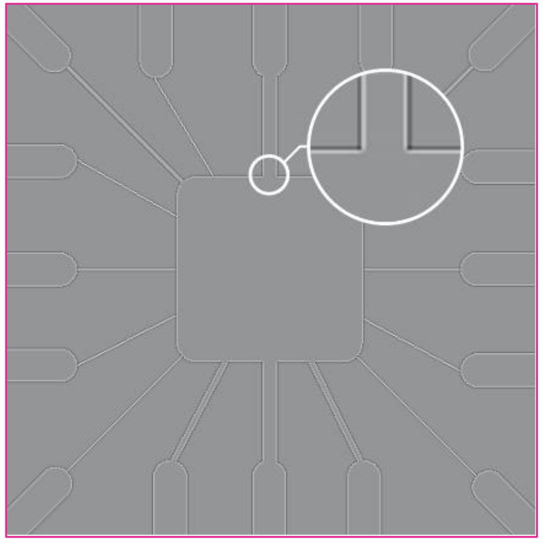
  - Laplace operator

Input



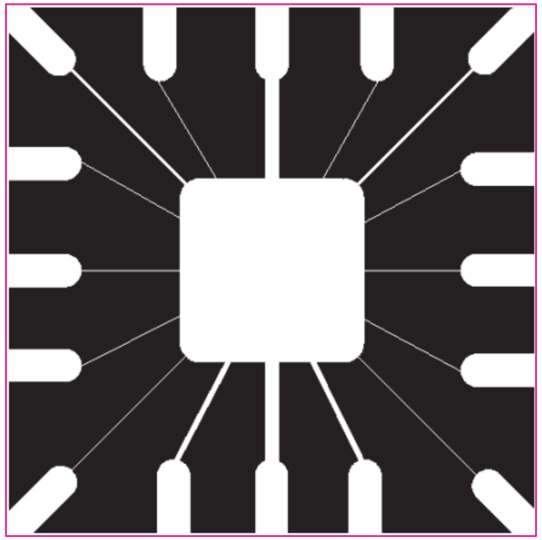
Laplacian



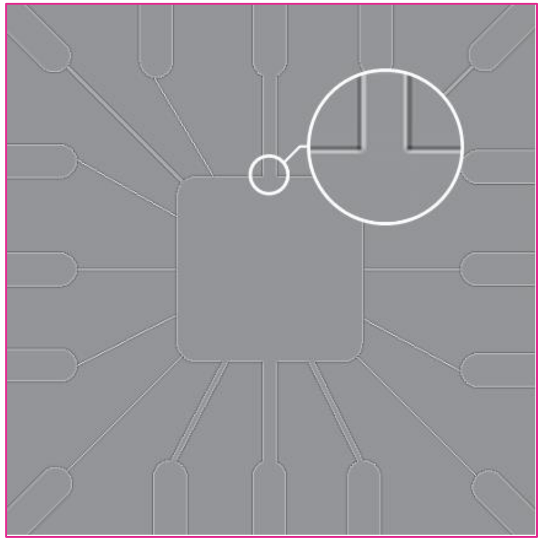
Line

- PCB wire bonds
  - Laplace operator

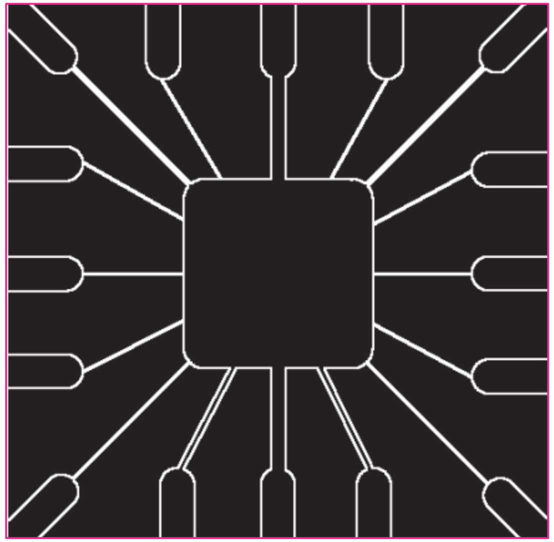
Input



Laplacian



|Laplacian|



Line

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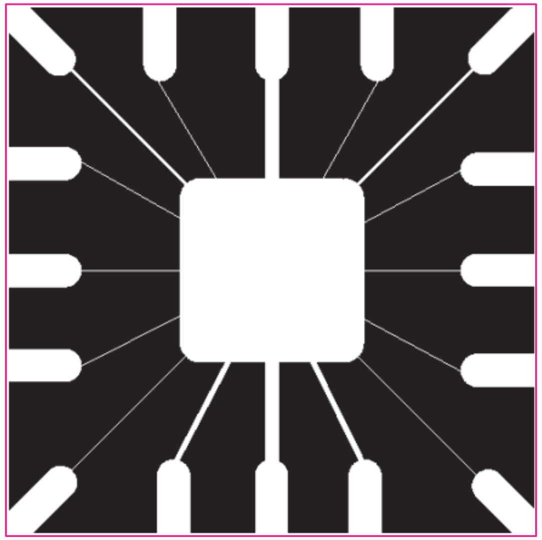
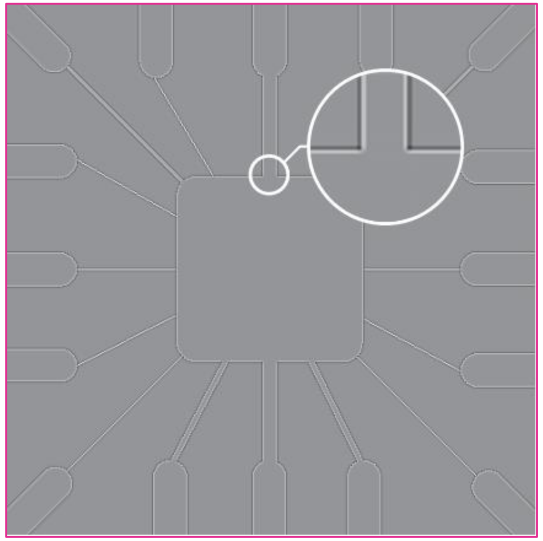
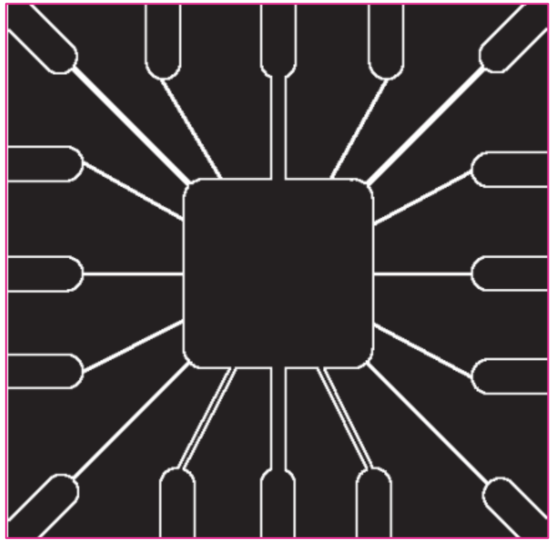


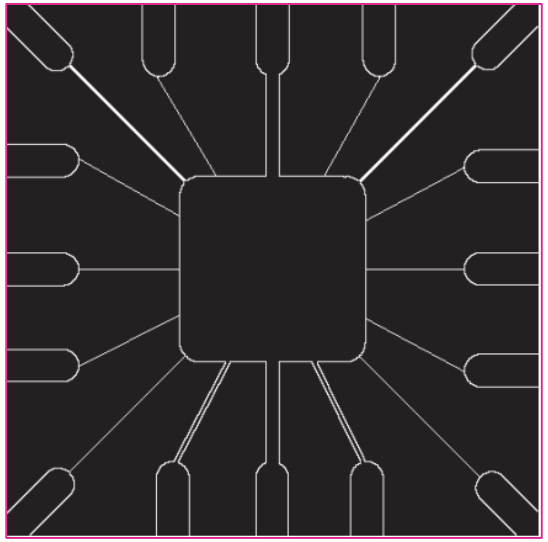
Laplacian



|Laplacian|



max(0, Laplacian)



Line

- With orientation

Horizontal

-1	-1	-1
2	2	2
-1	-1	-1

Line

- With orientation

Horizontal

-1	-1	-1
2	2	2
-1	-1	-1

+45 degrees

2	-1	-1
-1	2	-1
-1	-1	2

Line

- With orientation

Horizontal

−1	−1	−1
2	2	2
−1	−1	−1

+45 degrees

2	−1	−1
−1	2	−1
−1	−1	2

Vertical

−1	2	−1
−1	2	−1
−1	2	−1

Line

- With orientation

Horizontal

−1	−1	−1
2	2	2
−1	−1	−1

+45 degrees

2	−1	−1
−1	2	−1
−1	−1	2

Vertical

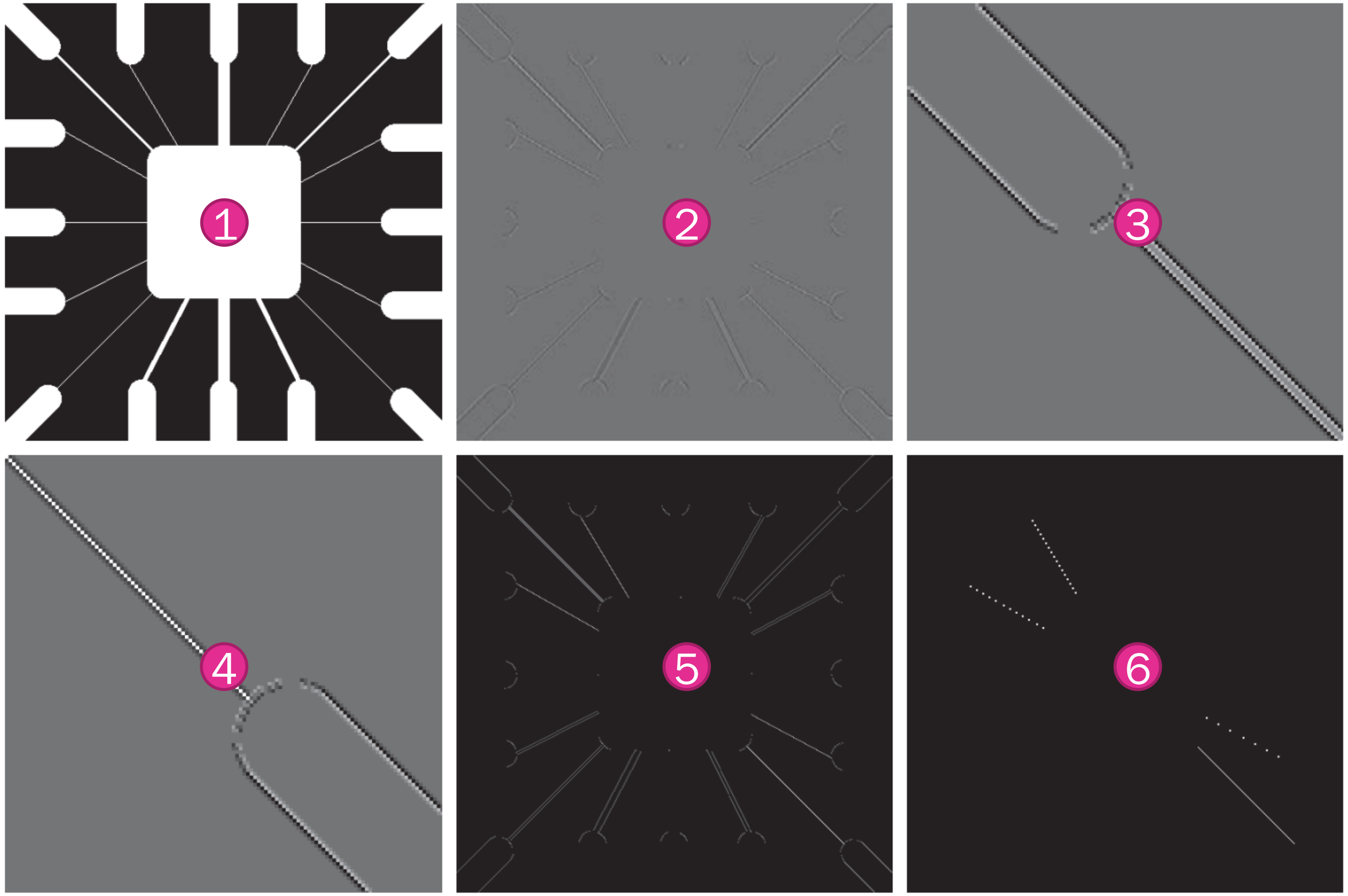
−1	2	−1
−1	2	−1
−1	2	−1

-45 degrees

−1	−1	2
−1	2	−1
2	−1	−1

Line

- With orientation
  - +45d line det kernel



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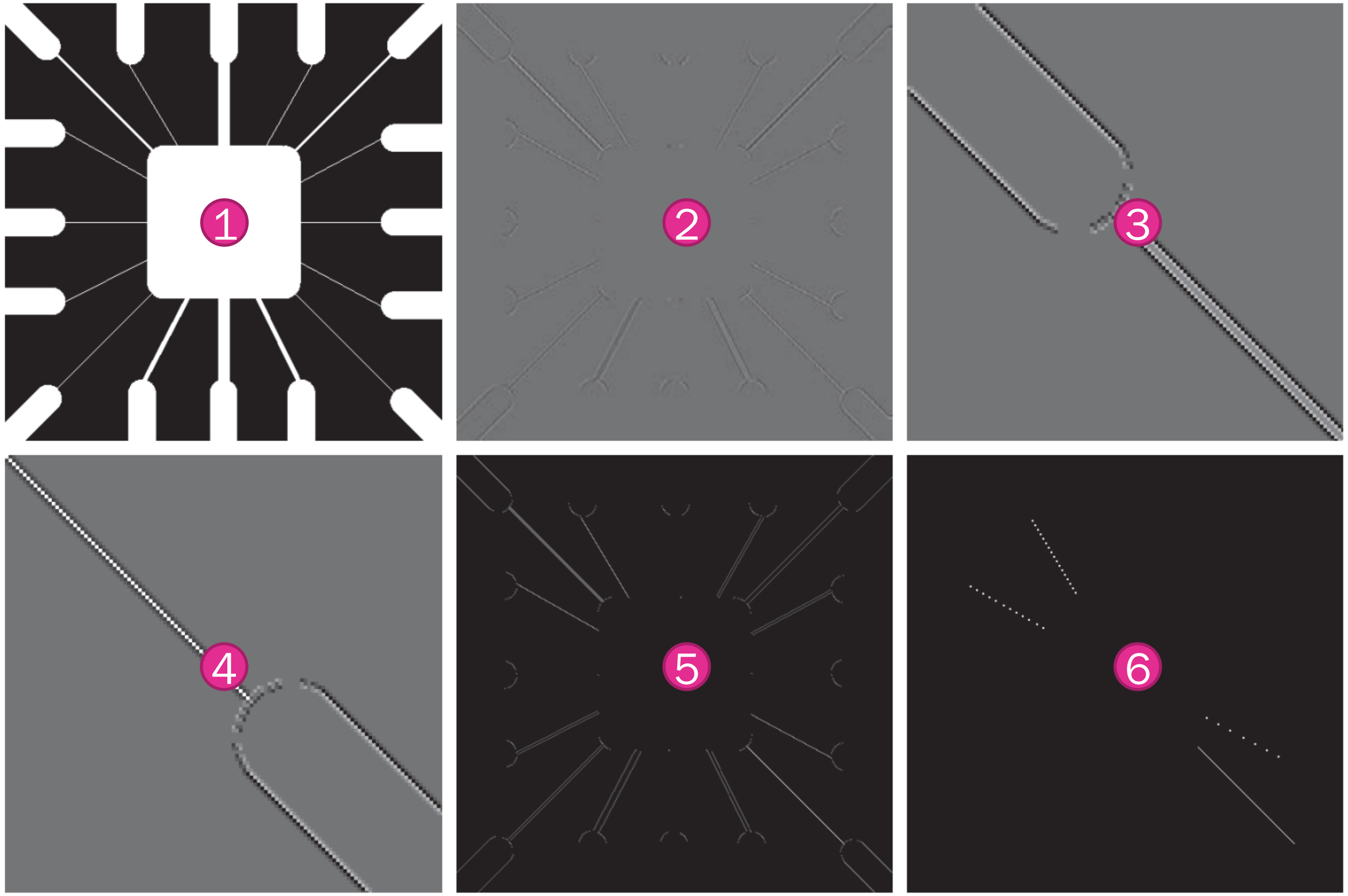
Line

- With orientation
  - +45d line det kernel



+45 degrees

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Line

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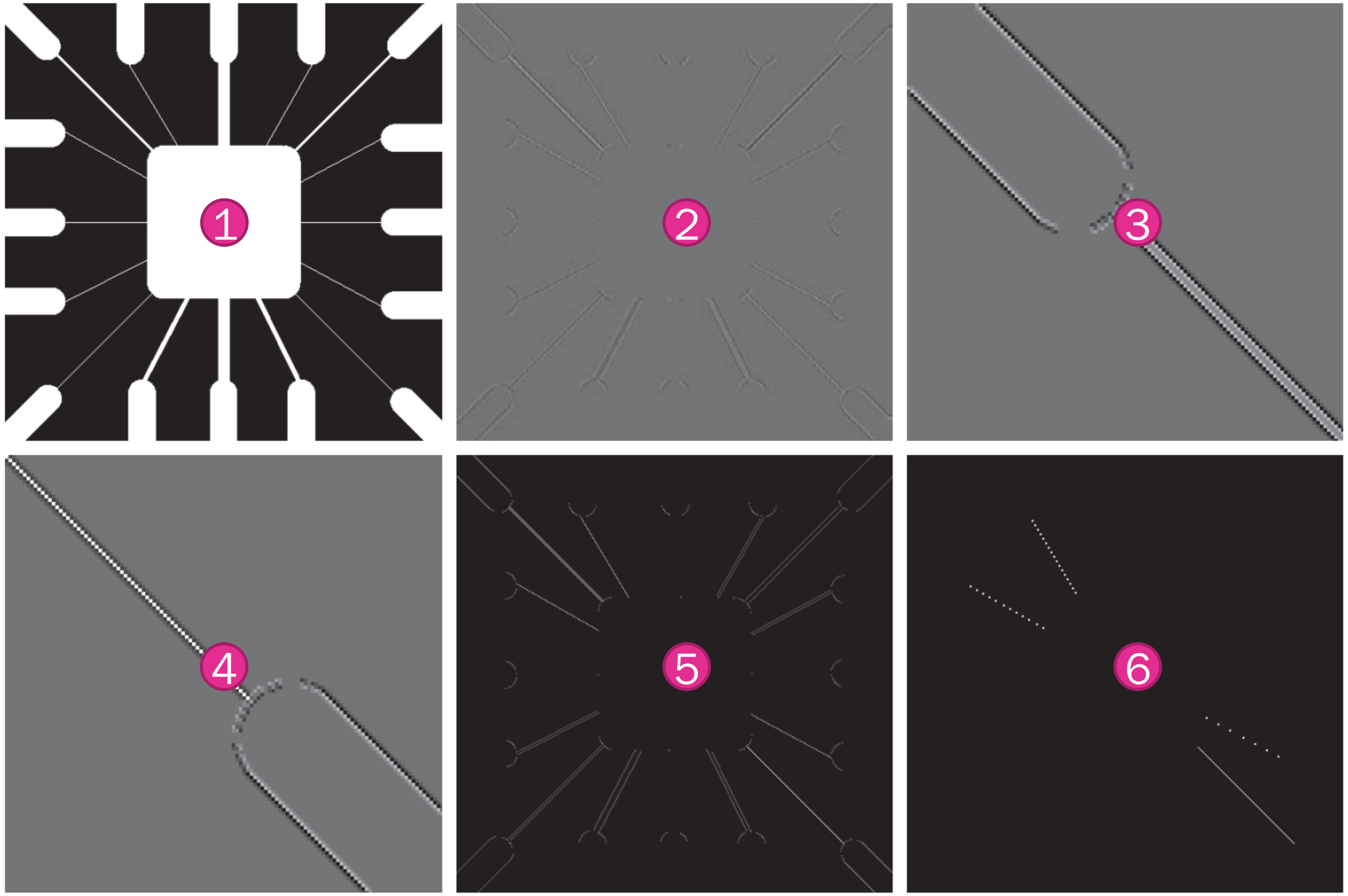


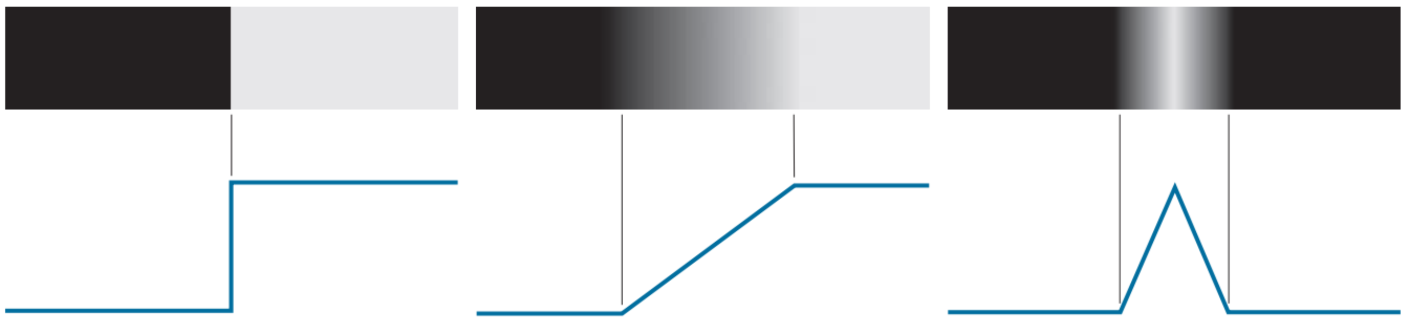
+45 degrees

2	−1	−1
−1	2	−1
−1	−1	2

1. Input
2. 45d det
3. Top zoom
4. Bottom zoom
5. Max(0, Lap)
6. Thresholding

Edge

- Types
  - step
  - ramp
  - roof



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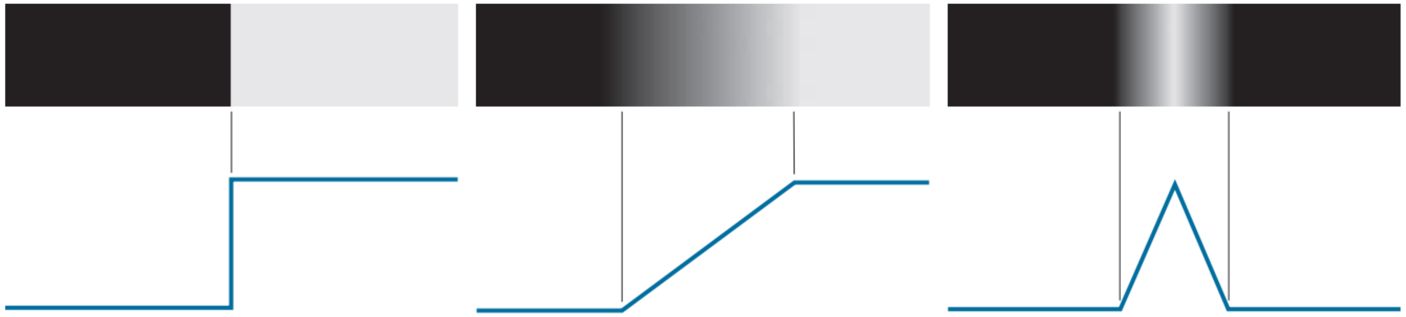

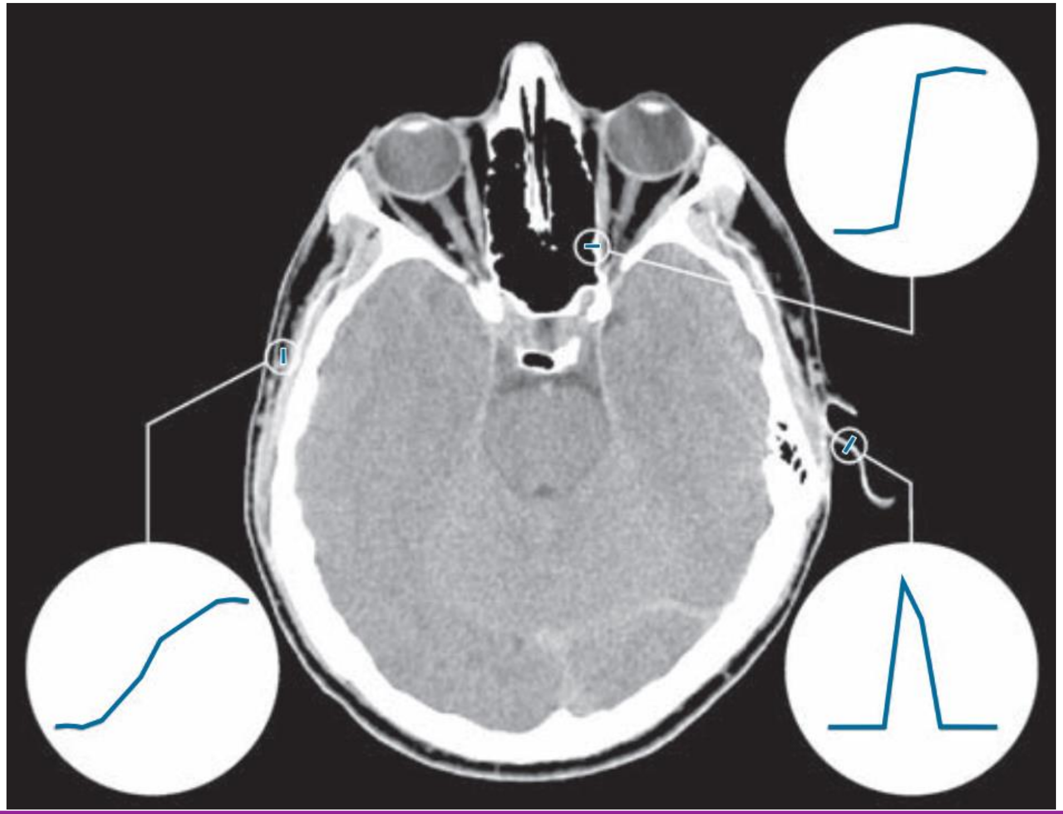

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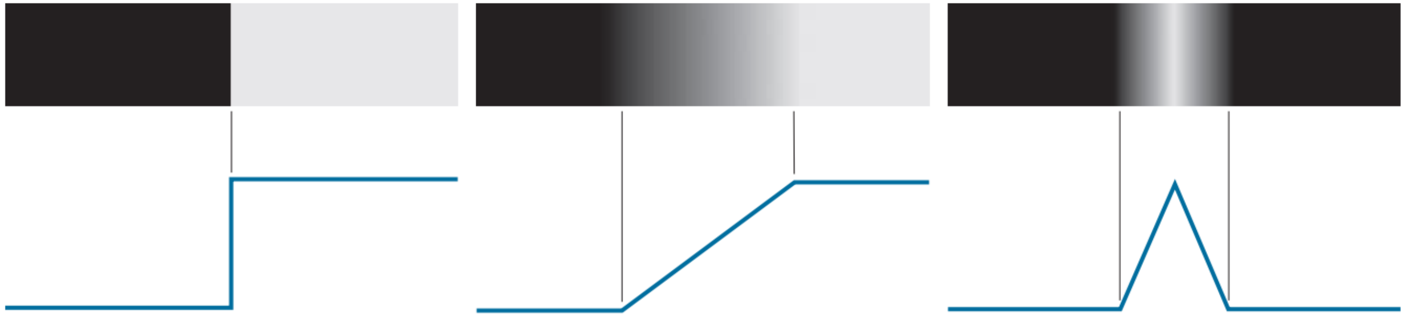

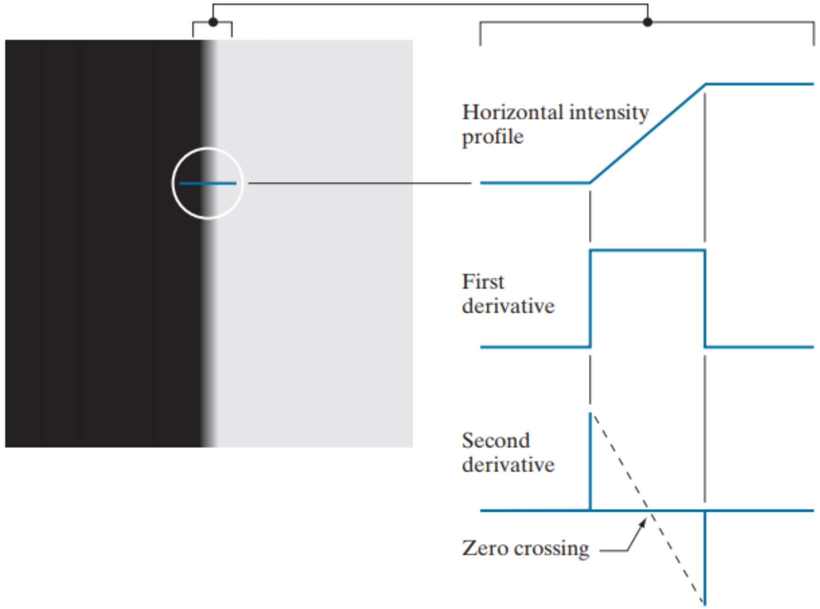

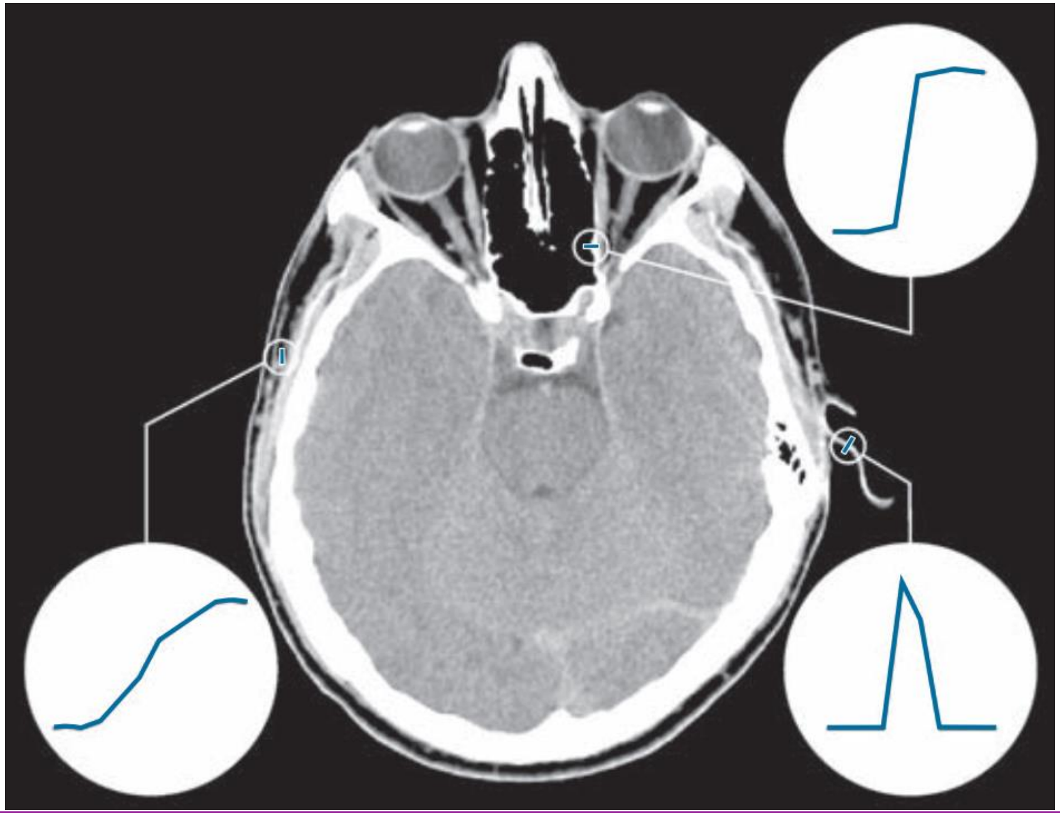

Edge

- Gradients

$$\nabla f(x,y) \equiv \operatorname{grad}[f(x,y)] \equiv \begin{bmatrix} g_x(x,y) \\ g_y(x,y) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f(x,y)}{\partial x} \\ \dfrac{\partial f(x,y)}{\partial y} \end{bmatrix}$$

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$$M(x,y) = \|\nabla f(x,y)\| = \sqrt{g_x^2(x,y) + g_y^2(x,y)}$$

$$\alpha(x,y) = \tan^{-1}\left[\frac{g_y(x,y)}{g_x(x,y)}\right]$$

Edge

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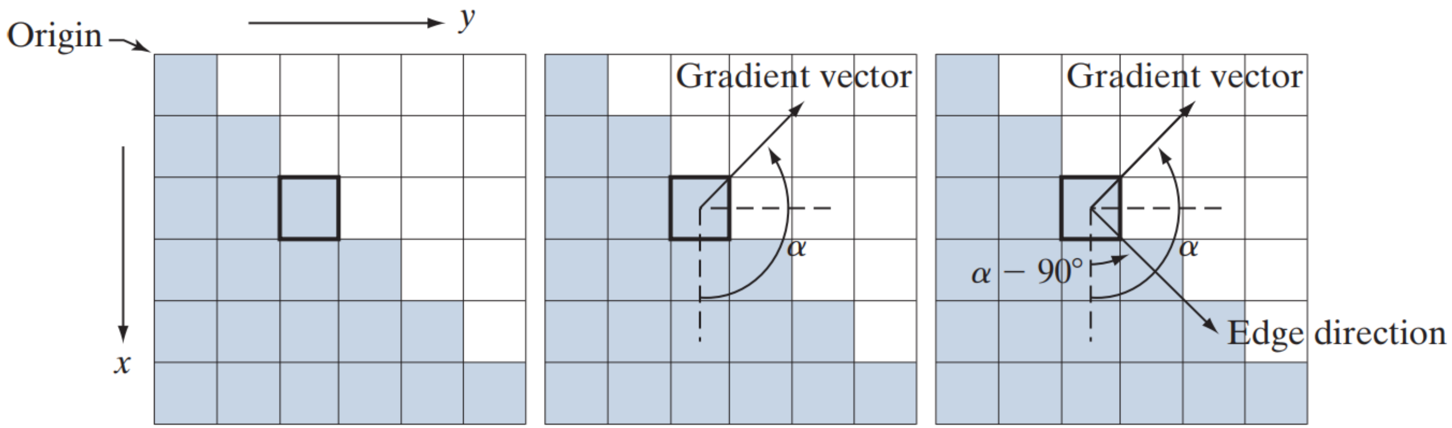

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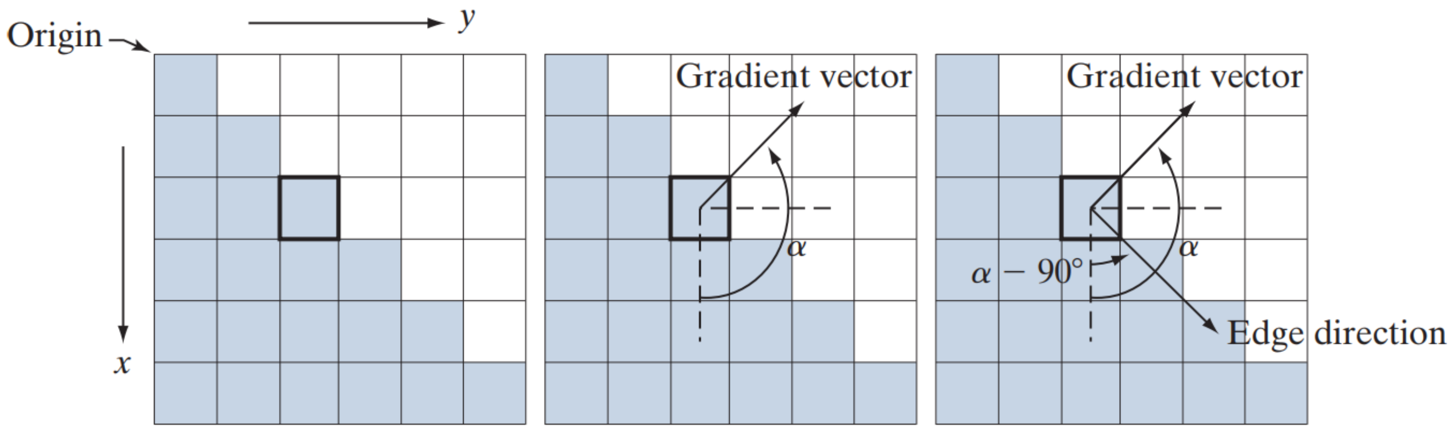
$$M(x,y) \approx |g_x| + |g_y|$$



Edge

- Sobel operator
  - derivatives via kernel
  - separable
  - diagonal direction points are not greatly discriminatory

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  - derivatives via kernel
  - separable
  - diagonal direction points are not greatly discriminatory

$$M_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} \quad M_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$M = \sqrt{(M_x^2 + M_y^2\ )}$$

$$\theta = \tan^{-1}(M_y, M_x)$$

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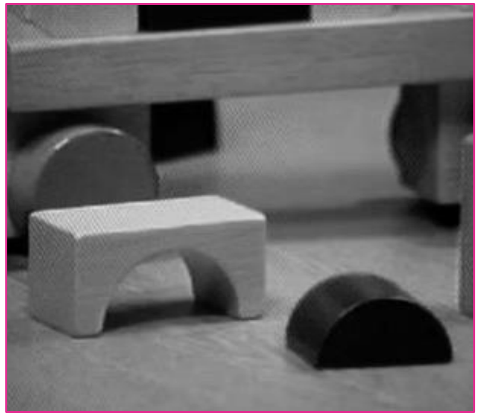
Input



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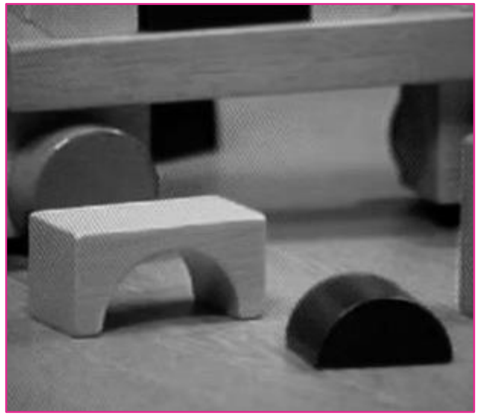
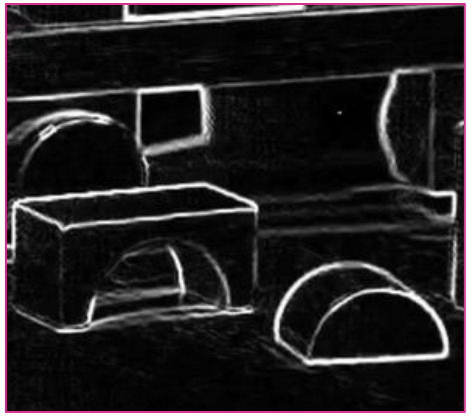


M



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Edge

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  - separable
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$$M = \sqrt{(M_x^2 + M_y^2)}$$

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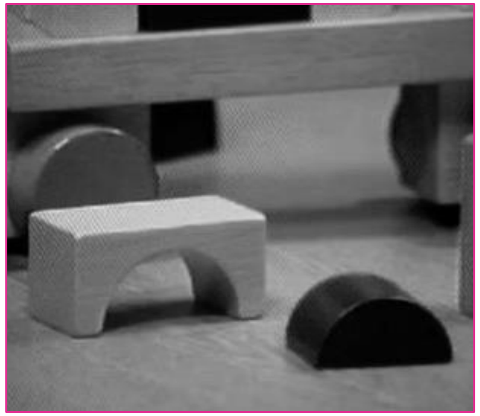
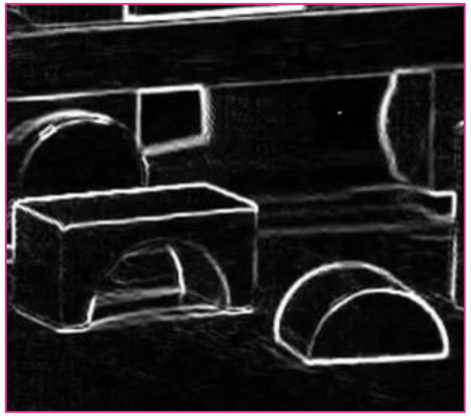
High robustness to noise

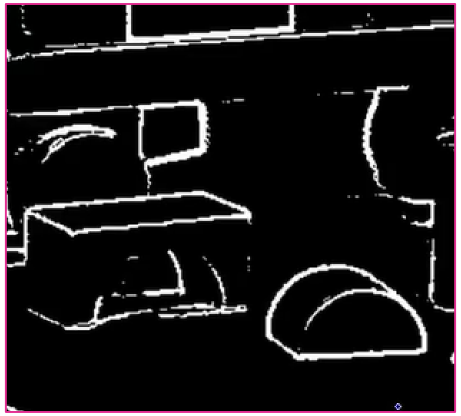
Input



M



Threshold on M



Edge

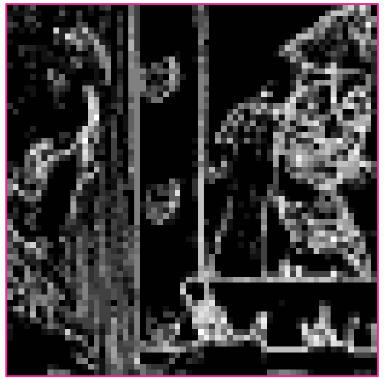
- Roberts operator
  - discriminatory diagonals
  - fast

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad M_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

input

no TH

with TH



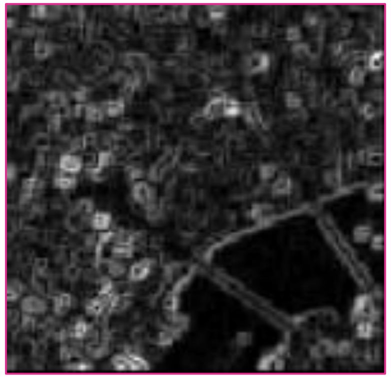
Edge

- Prewitt operator
  - high sensitivity than Sobel

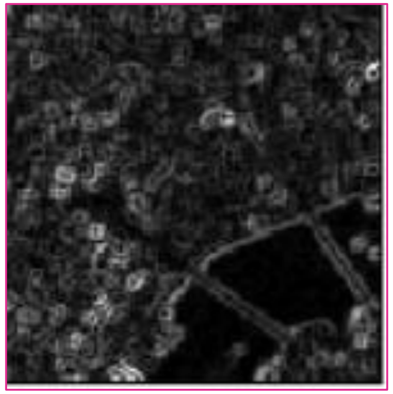
$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad M_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Input

Prewitt



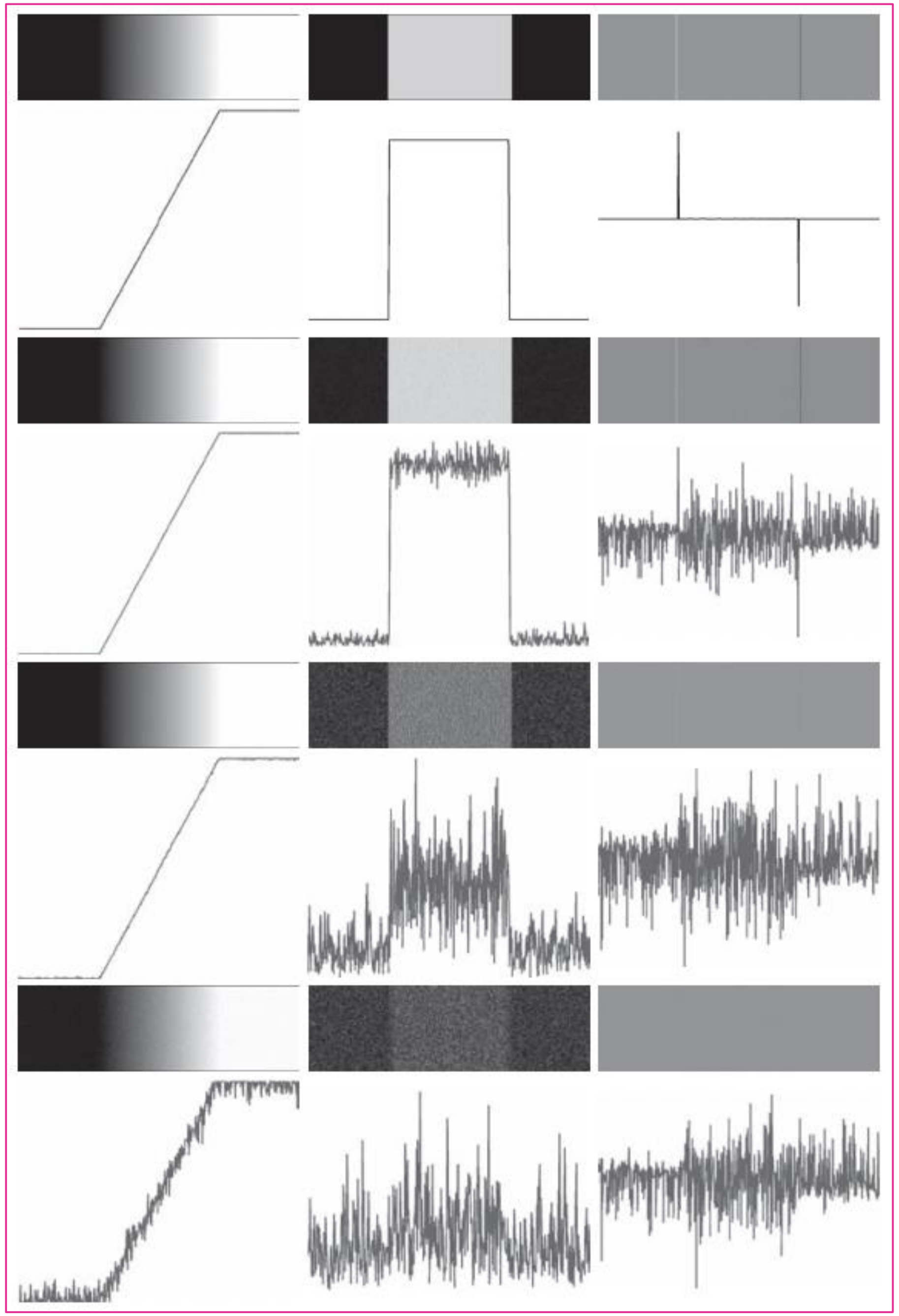
Sobel



Roberts

Edge

- Edge Sensitivity
    - Edge point is the peak in M in θ direction



Edge

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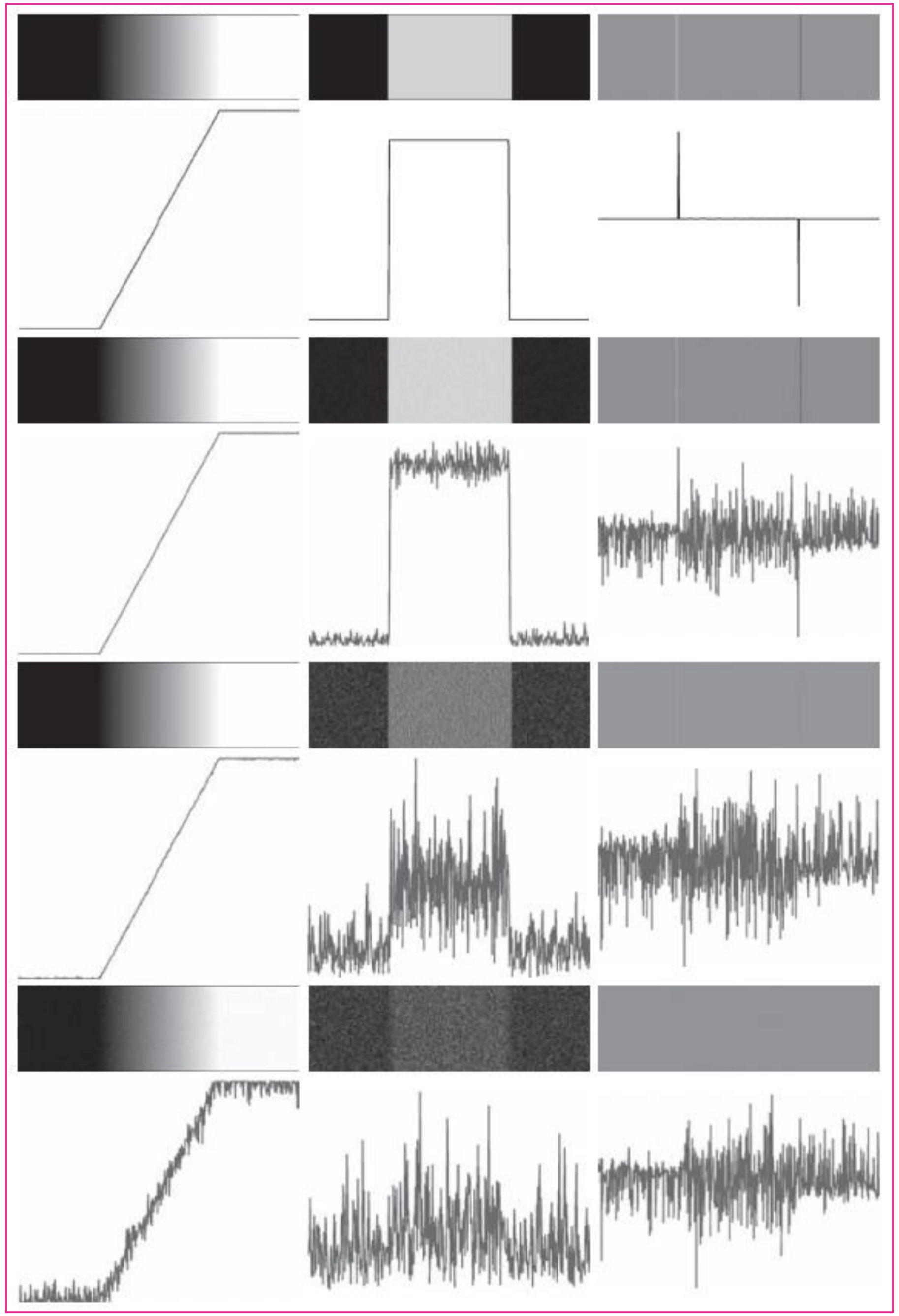
  - Edge point is the peak in M in θ direction
  - Edges are highly sensitive to the noise



Edge

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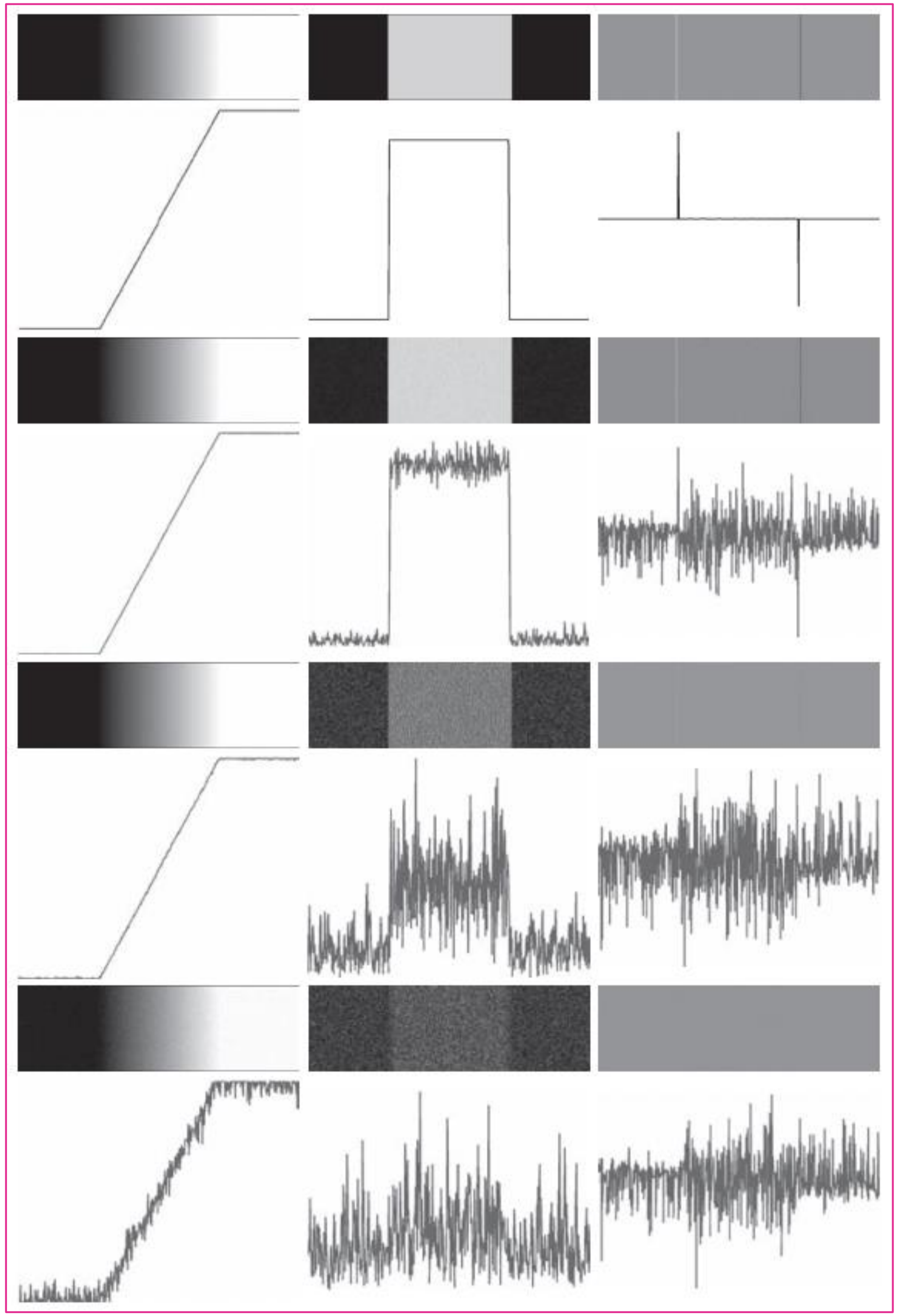
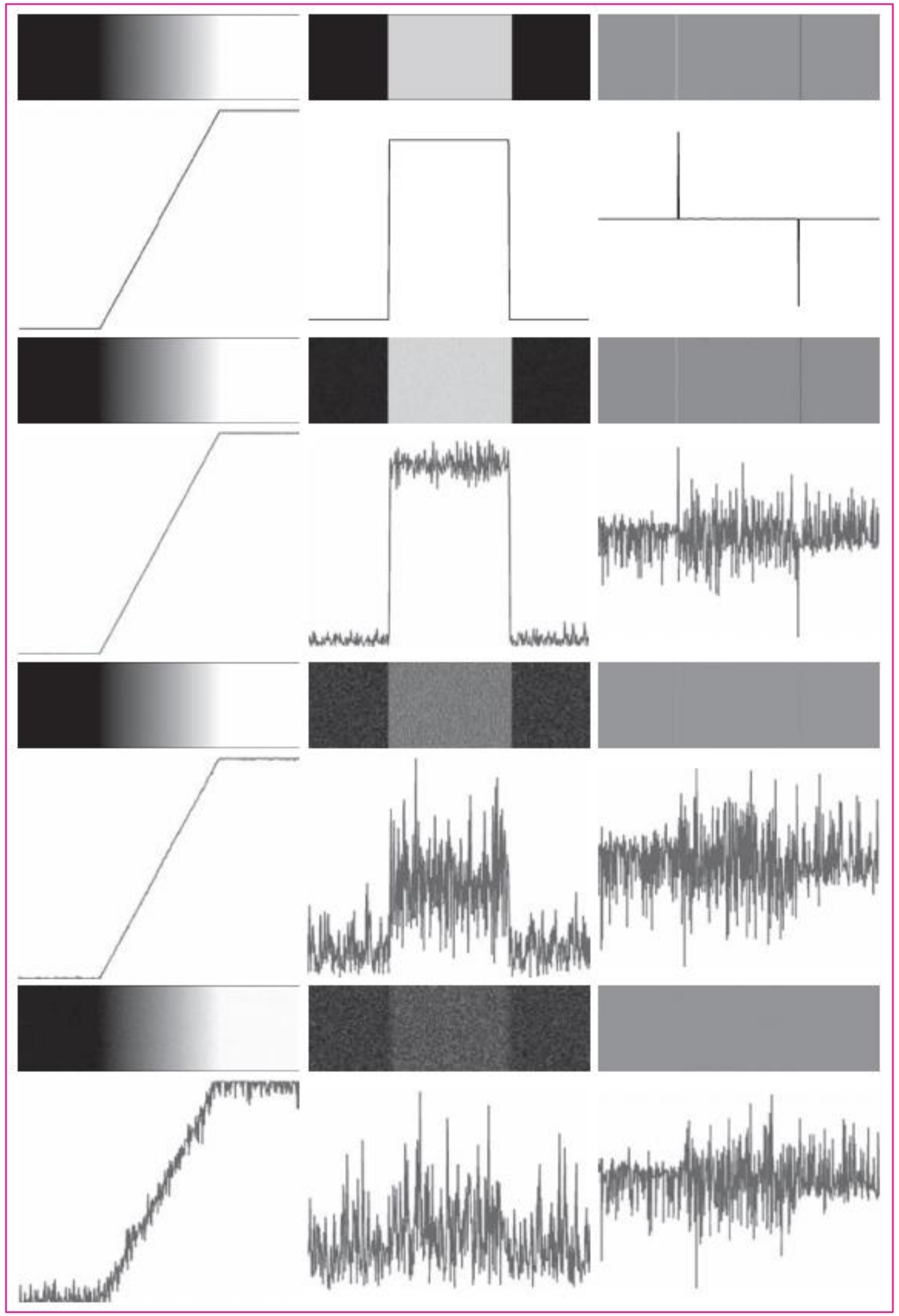
  - Edge point is the peak in M in θ direction
  - Edges are highly sensitive to the noise
  - Derivatives amplify noise



Edge

- Edge Sensitivity
  - Edge point is the peak in M in θ direction
  - Edges are highly sensitive to the noise
  - Derivatives amplify noise
  - How to reduce this sensitivity?



Edge

- Stability
  - refers to less sensitivity to noise

- Solution: apply smoothing filter G before finding edges

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$$\left\{ G_\sigma * I \right\}$$

Edge

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$$\Delta * \left\{ G_\sigma * I \right\}$$

… Δ is derivative (1^{st} or 2^{nd}) operator

Edge

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$$E = \Delta * \left\{ G_\sigma * I \right\}$$

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$$E = \left\{ \Delta * G_\sigma \right\} * I$$

Edge

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$$E = \Delta * \{G_\sigma * I\}$$

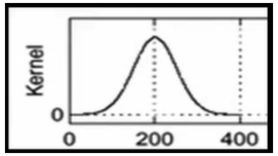
... Δ is derivative (1^{st} or 2^{nd}) operator

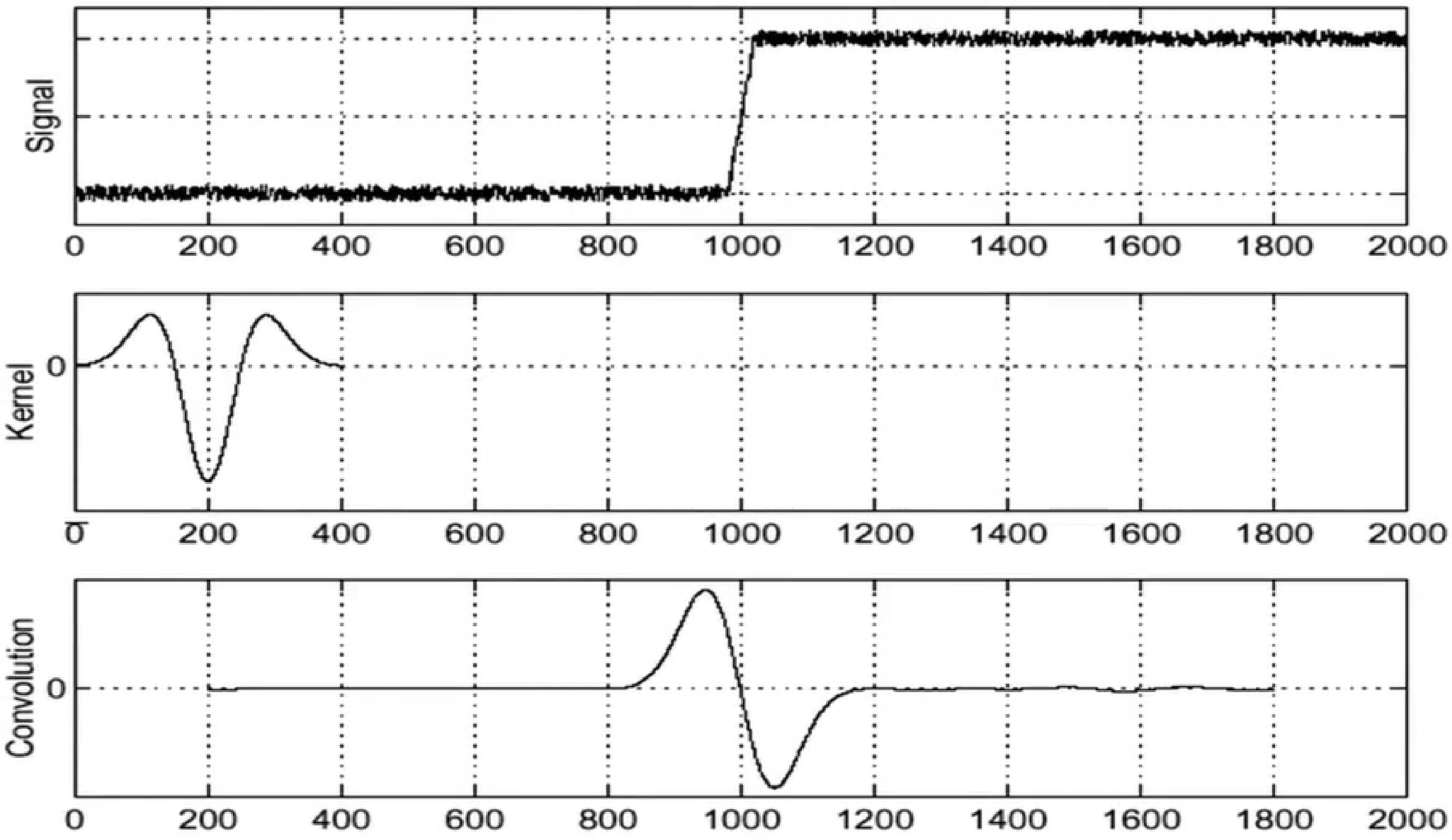
$$E = \{\Delta * G_\sigma\} * I$$

... Conv. is associative

Edge

- Edge at zero crossings

$$\frac{\delta^2}{\delta x^2}\left\{ \right.$$  $$\left. \right\} \longrightarrow$$



Edge

$$G(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- LoG
  - Laplacian of Gaussian

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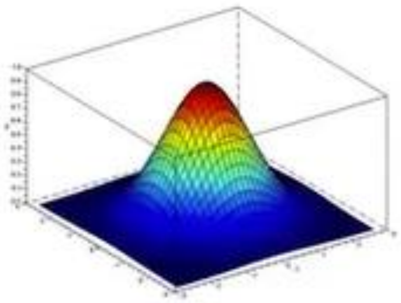

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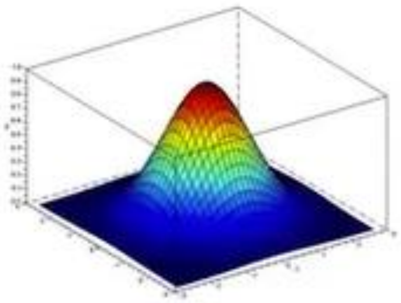
$$\nabla^2 G(x,y) = \frac{\partial^2 G(x,y)}{\partial x^2} + \frac{\partial^2 G(x,y)}{\partial y^2}$$



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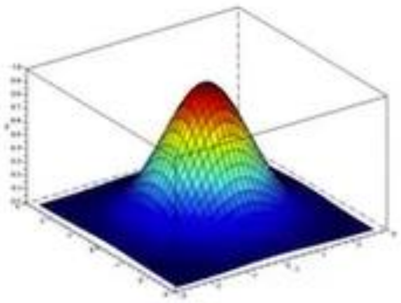

$$= \frac{\partial}{\partial x}\left(\frac{-x}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right) + \frac{\partial}{\partial y}\left(\frac{-y}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right)$$

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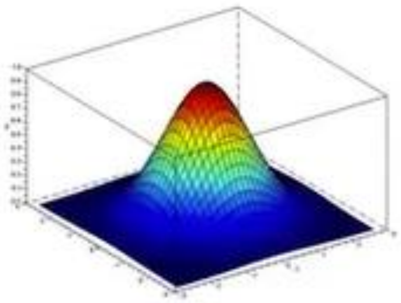

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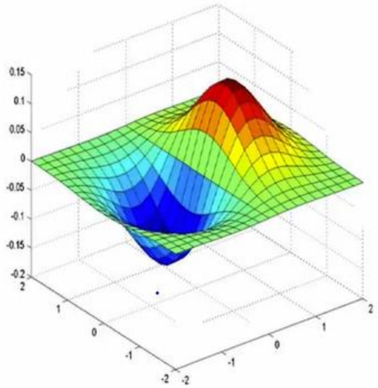

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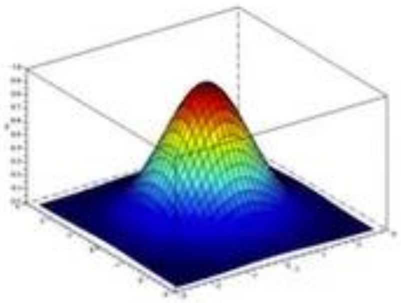

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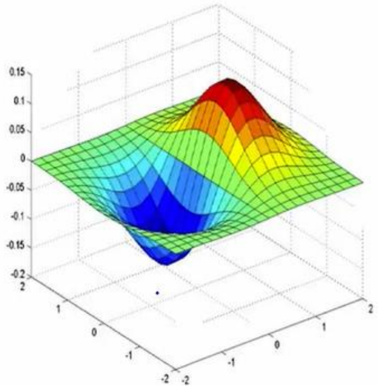
$$= \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}} + \left(\frac{y^2}{\sigma^4} - \frac{1}{\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$



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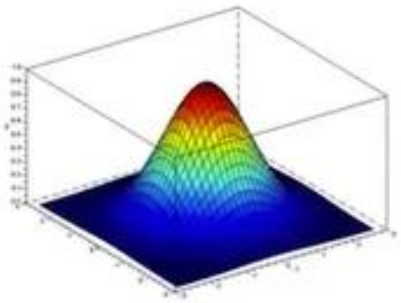
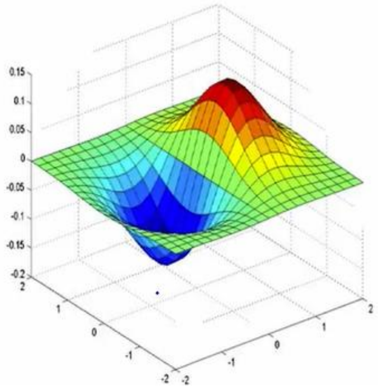


$$= \frac{\partial}{\partial x}\left(\frac{-x}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right) + \frac{\partial}{\partial y}\left(\frac{-y}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right)$$



$$= \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}} + \left(\frac{y^2}{\sigma^4} - \frac{1}{\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

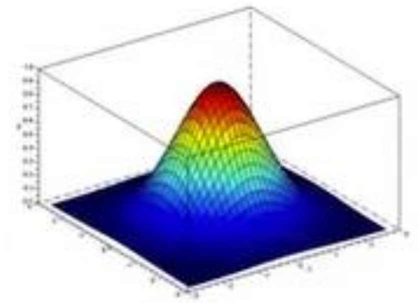
$$= \left(\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Edge

$$G(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- LoG
  - Laplacian of Gaussian

$$\nabla^2 G(x,y) = \frac{\partial^2 G(x,y)}{\partial x^2} + \frac{\partial^2 G(x,y)}{\partial y^2}$$



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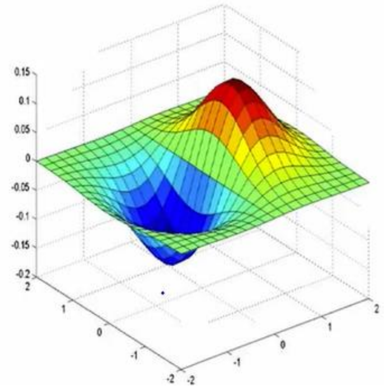

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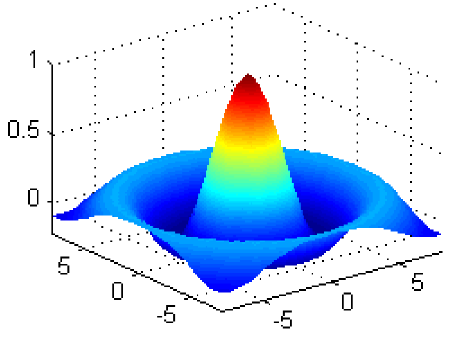
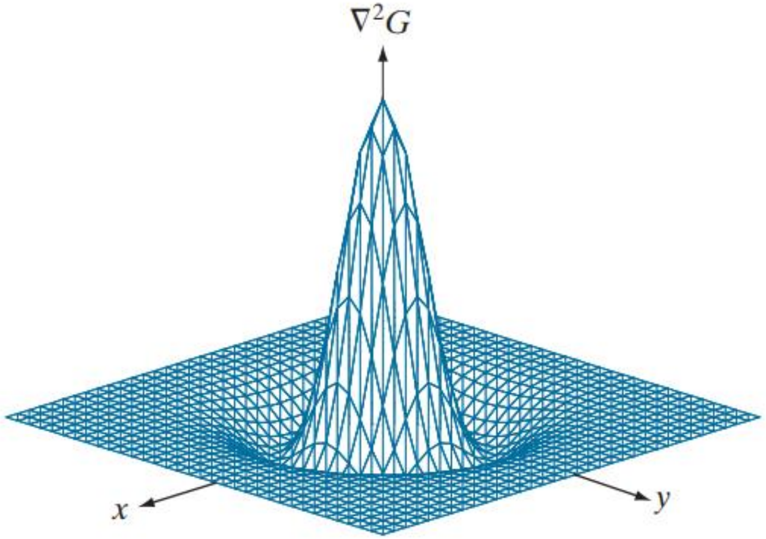
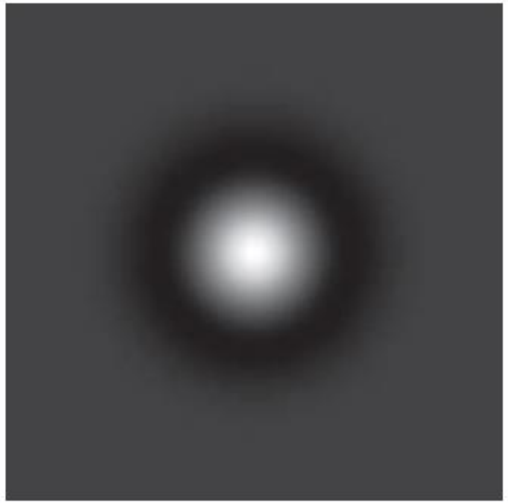
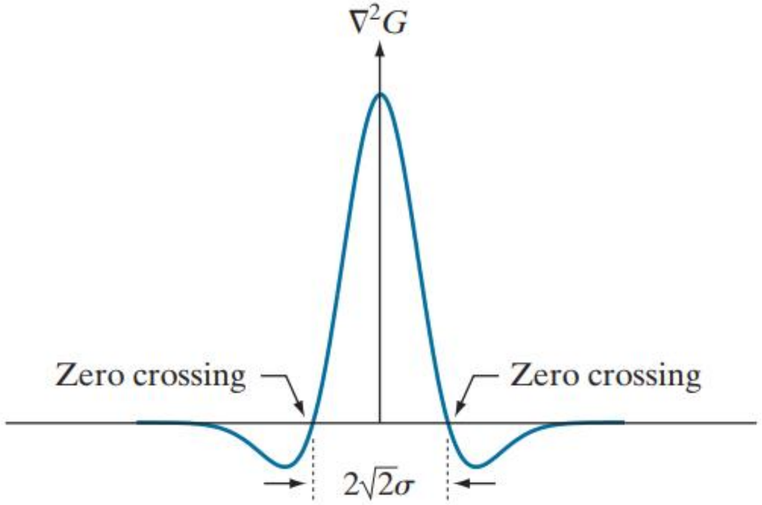


Edge

- LoG
  - Laplacian of Gaussian
  - for convenience negative of LoG are plotted







0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
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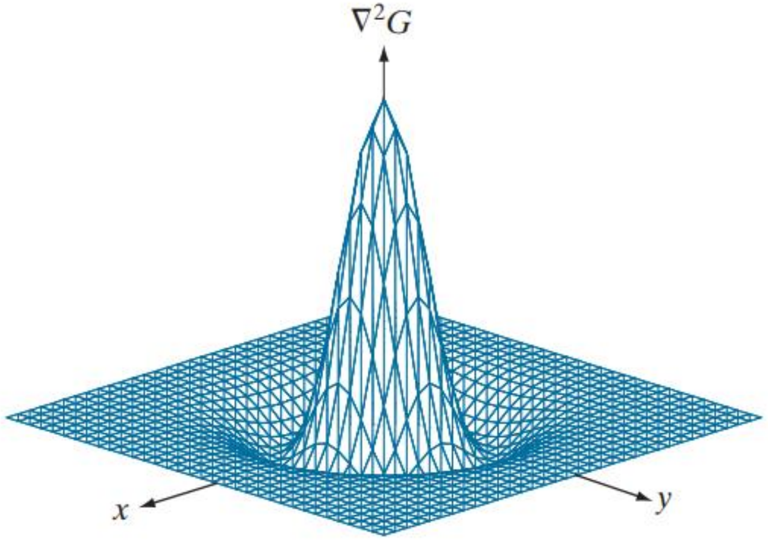

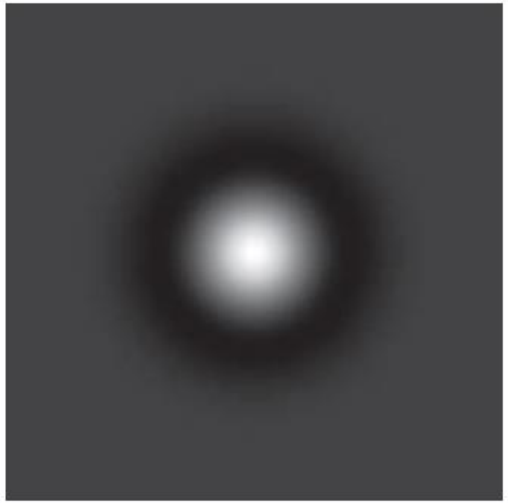

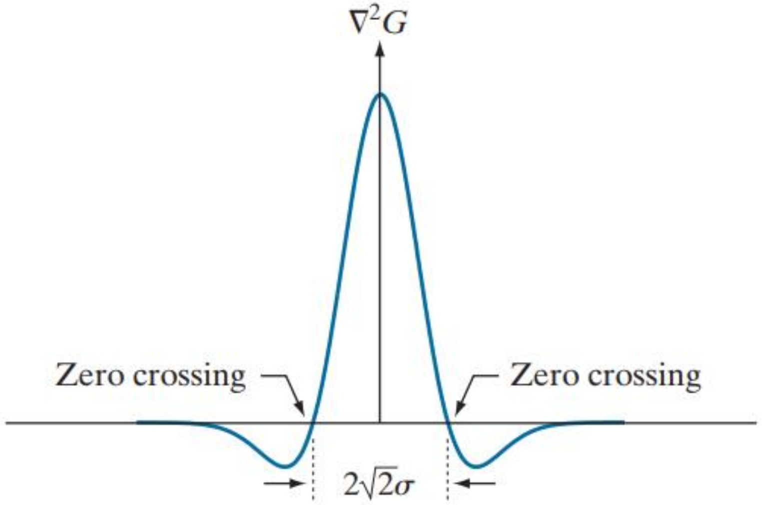

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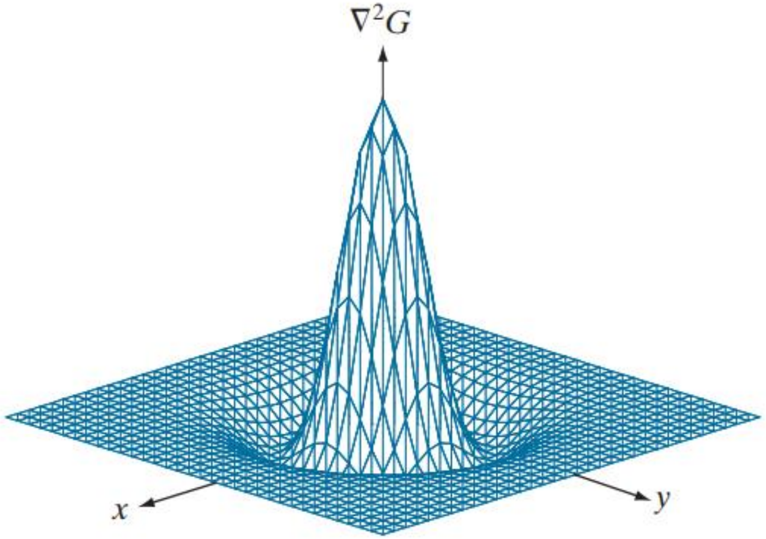

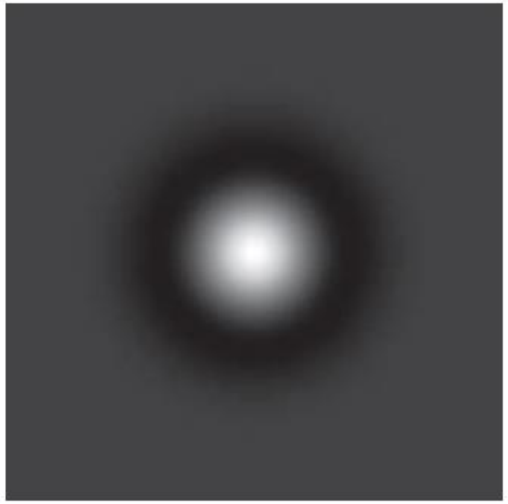

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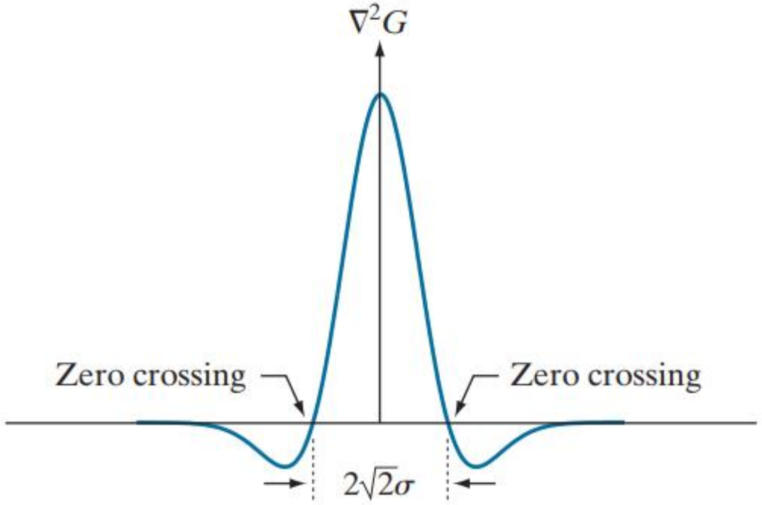

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0	−1	−2	−1	0
−1	−2	16	−2	−1
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Edge

- LoG

input

LoG

zero crossings

Edge

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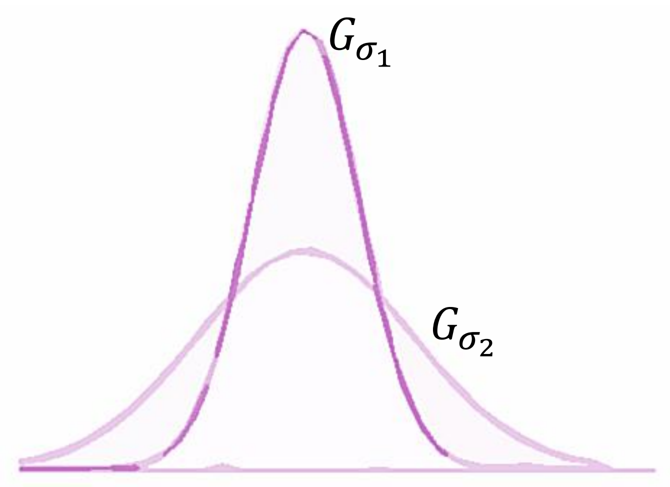

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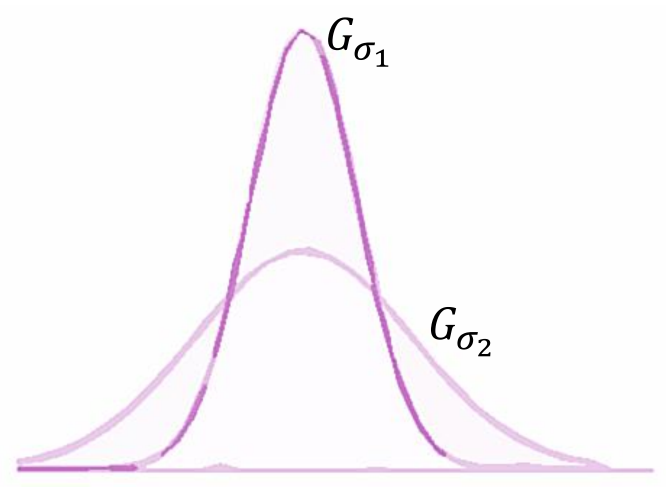
  - the one who obeys the LoG closely



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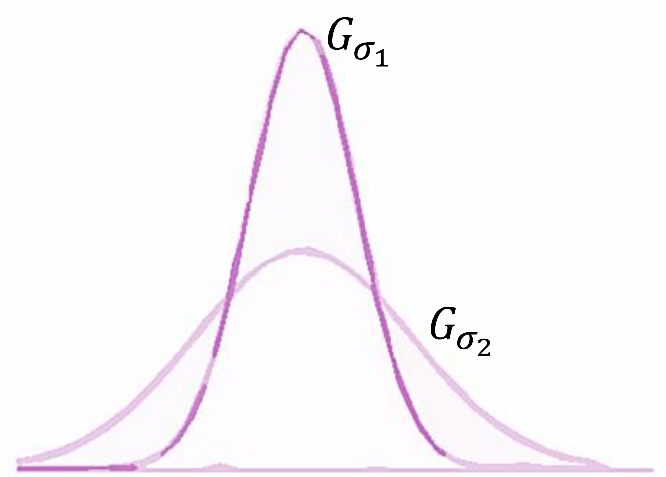
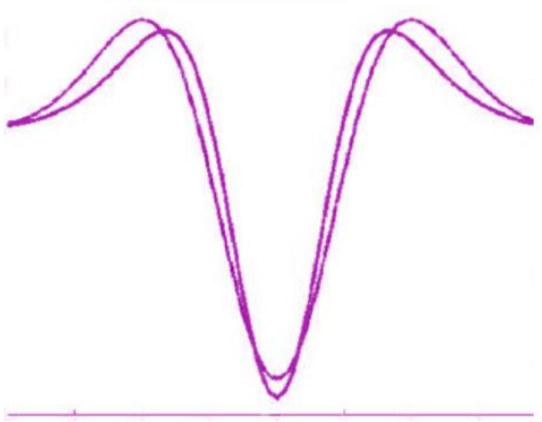
$$\sigma_1 = \frac{\sigma}{\sqrt{2}} \qquad \sigma_2 = \sqrt{2}\,\sigma$$





Conclusion

- Operators

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- There is no definition about what is a perfect edge

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  - Sobel
  - Roberts
  - Prewitt
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Who was I ?