

# Boston School Choice Mechanism for User Association in Heterogeneous Networks

Fouad Ismael\*, Ahmed H. Abd El-Malek<sup>†</sup>, and Maha Elsabrouty<sup>†</sup>

<sup>†</sup>Department of Electronics and Communications Engineering, Egypt-Japan University of Science and Technology  
New Borg El-Arab City, Alexandria, Egypt

E-mail: {fouad-mis@hotmail.com, ahmed.abdelmalek@ejust.edu.eg, maha.elsabrouty@ejust.edu.eg}

**Abstract**—This paper presents the application of a particular class of matching game models, namely the Boston school choice approach, to the problem of user association in heterogeneous networks. Simulation results are provided to assess the performance of the Boston mechanism and the Gale-Shapley algorithm used in the college admission game concerning overall performance, average user utility and execution time.

## I. INTRODUCTION

The application of heterogeneous networks is one of the main enabling technologies in 5G [1]. The user association problem, in both the downlink and uplink direction, is considered to be one of the main challenges in heterogeneous communication systems. Several approaches can be used in the user association problem [2, 3].

Matching theory [4] is considered as one of the highly acclaimed distributed frameworks in resource allocation. It can be traced back to the noble winning work that is presented in the economic framework, more particularly in market design. The matching theory looks at the combinatorial matching of players of two different sets and provides a solid tractable mathematical framework in this context [5, 6]. Consequently, matching theory solves a lot of the shortcoming that may be found in the framework of game theory, where equilibrium, e.g., Nash equilibrium is calculated unilaterally on players of one set side [4].

There are numerous examples of using matching theory in the user association problem. The authors in [7] used it for solving an uplink association problem. The work in [8] used matching theory in a decoupled uplink-downlink association. The framework in [9] considered a joint user association and resource allocation problem using the matching theory. The work in [10] applied matching theory for a cache-aware user association. All these works adopted college admission matching in modeling the association problem. College admission enables both the college side (cell side) and the student side (users side) to change their association if the better matching candidate is available. The nature of the problem is evolving over several iterations that might affect the delay before the final association is reached. Such delay is a disadvantage in the delay-sensitive applications.

This work considers the application of the Boston mechanism (immediate-acceptance algorithm) to the problem of user association in heterogeneous networks which has not been addressed in the literature before. The results show that

the Boston mechanism could provide a QoS performance similar to that of Gale-Shapley mechanism in user association problems. Also, the Boston mechanism shows superiority under a high level of interference plus noise.

The rest of this paper is organized as follows: Section II gives a general explanation of the system model with detailed parameters. Section III is about using matching games to reach a precise matching between users and cells. Simulation setup and results are discussed in Section IV. Finally, Section V concludes the work contributions and future works.

## II. SYSTEM MODEL AND PRELIMINARIES

### A. System Model

In this section, we adopt a model of a two-tier heterogeneous wireless network which consists of standard MBSs  $m \in \{1, 2, \dots, M\}$ , as well as several SBSs  $l \in \{1, 2, \dots, L\}$ . This system is accessed by users  $n \in \{1, 2, \dots, N\}$ , where  $M$ ,  $L$ , and  $N$  represent the number of MBSs, SBSs and total users in the system, respectively. The system also uses time division multiple access (TDMA) in order to organize users associated to the BSs and equally distribute the available resources.

In the considered system model, each BS has a certain quota  $q_i$ , which is determined by its ability to support a certain number of users efficiently. According to [7], this quota can be either fixed or controlled by the network operator, i.e.  $q_m = N_{m_0}$ , where  $N_{m_0}$  is the number of users associated to the MBS  $m$  with no installed SBSs. As for small-cells; they have a constant quota  $q_l$ , which represents their ability only to serve a limited number of users. These SBSs are identical in their architecture with a same constant quota  $q_l$ .

### B. User Association Criteria

The focus of this paper is the uplink user association problem. This problem is different from the downlink association one as it is centered on users who have limits on their equipment like limited power and processing capabilities. It also has to consider channel conditions which are represented by the packet success rate (PSR), and the cells limitations which are represented in the service delays.

The system is governed in a distributed manner, where decisions are made by users, SBSs and MBSs. Users in this system try to maximize their quality of service (QoS). Therefore, the users receive the service delay  $d$  and the PSR given the signal-to-interference-plus-noise ratio (SINR) at the

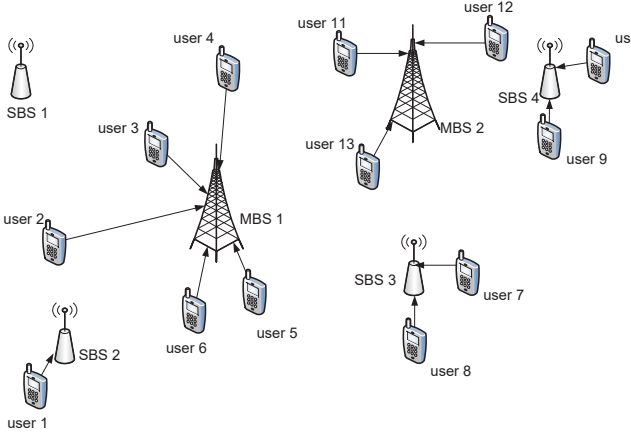


Fig. 1: A simplified system model with two MBSs and four SBSs; each with quota of 2, and accessed by a group of users.

cells. Hence, the users have all components to calculate the R-factor, which is a popular measure for QoS in voice-over IP networks described in [11, 12], and set their preference lists according to it. Mathematically, for any user  $n$ , and cells  $i$  and  $j$ , we have  $i \succeq_n j$  (which means cell  $i$  is preferred over cell  $j$  for user  $n$ ) if we have  $R_{i,n} > R_{j,n}$  (which means the R-factor between user  $n$  and cell  $i$  is greater than the one between user  $n$  and cell  $j$ ). On the other hand, the MBSs prefer to serve users according to their PSR values, i.e. they prefer users with high PSR values, over other users with low PSR values. The use of the PSR as a criterion for establishing MBSs preference instead of the R-factor can be justified as the R-factor contains the cell delay and is thus of interest to the user side only.

Concerning the SBSs, the PSR should take into account both the user quality, similar to the MBS, along with an incentive to offload users from the MBSs. In order to include the offloading “incentive” in the evaluation criterion for users, this criterion is changed to be a ratio between the PSR achieved by the users in SBSs and the best PSR achieved when connecting to MBSs so that the SBSs benefit can be formulated as [7]

$$\text{SBS Benefit} = \beta \frac{\rho_{l,n}}{\max_{m \in \{1,2,\dots,M\}} \rho_{m,n}}, \quad (1)$$

where  $\beta$  is a pricing factor that can be used to help including cell range expansion techniques.

Figure 1 represents a simplified system model. The illustrated network consists of two MBSs and four SBSs; each can handle up to two users. We have a group of users trying to access this network, with arrows indicating the uplink between users and cells. SBS 1 is idle as no users are near it. SBS 2 serves only one user (user 1) as it is the only one near SBS 2. User 2 is out of range for SBS 2 so it associates with MBS 1. User 3 is out of range for SBS 2 so it associates with MBS 1. User 4 is out of range for SBS 2 so it associates with MBS 1. User 5 is out of range for SBS 2 so it associates with MBS 1. User 6 is out of range for SBS 2 so it associates with MBS 1. User 7 is out of range for SBS 2 so it associates with MBS 1. User 8 is out of range for SBS 2 so it associates with MBS 1. User 9 is out of range for SBS 2 so it associates with MBS 1. User 10 is out of range for SBS 2 so it associates with MBS 1. User 11 is out of range for SBS 2 so it associates with MBS 1. User 12 is out of range for SBS 2 so it associates with MBS 1. User 13 is out of range for SBS 2 so it associates with MBS 1.

### C. R-factor: Definition and analysis

For any BS  $i \in \{1, \dots, M + L\}$ , the service delay  $d$  is

$$d_i = d_{i_w} + d_{i_b}, \quad (2)$$

where  $d_{i_w}$ , and  $d_{i_b}$  denote the wireless access delay, and the backhaul delay, respectively at any BS  $i$ . For a TDMA system, the wireless access delay at the  $i$ -th BS is given by

$$d_{i_w} = (q_i - 1)T. \quad (3)$$

As for the backhaul delay, we model it as a random value with a specific distribution, depending on the type of the BS. On the other hand, the PSR is computed by

$$\rho_{i,n} = [1 - P_{i,n}(\gamma_{i,n})]^B, \quad (4)$$

where  $P_{i,n}(\gamma_{i,n})$  is the bit error rate between base station  $i$  and user  $n$ , which is a function of the signal-to-interference-plus-noise ratio (SINR)  $\gamma_{i,n}$  between base station  $i$  and user  $n$ .  $B$  represents the number of bits per packet. In the considered model, the wireless channel gain is defined as

$$h_{i,n} = g_{i,n} x_{i,n}^\alpha, \quad (5)$$

where  $x_{i,n}$  is the distance between BS  $i$  and user  $n$ ,  $\alpha$  is the path loss exponent, and  $g_{i,n}$  is the factor representing fading and multipath effects. It is also assumed that the noise and interference at the BSs follow a zero-mean Gaussian distribution with a constant average power level over the all available BSs. Hence, the SINR is given by

$$\gamma_{i,n} = S_n h_{i,n} / \sigma_i^2, \quad (6)$$

where  $S_n$  is the power used by user  $n$  during uplink, and  $\sigma_i^2$  is the noise-plus-interference power at the  $i$ -th BS.

Based on [11], the service delay and PSR can be combined in the calculations of R-factor which is given by [7]

$$R = 94.2 - I_d - I_e, \quad (7)$$

where  $I_d$  is the delay impairment factor given by [11, (7-18)], and  $I_e$  is the equipment impairment factor, a function of PSR, which is generally approximated as

$$I_e = a + b \ln(1 + c \times (1 - \rho_{i,n})), \quad (8)$$

where  $a, b$ , and  $c$  are constants adjusted to suit a certain codec service. The R-factor would be very helpful in formulating matching games as discussed in the next section.

## III. MATCHING FRAMEWORKS: COLLEGE ADMISSION VS. BOSTON SCHOOL CHOICE SCENARIOS

### A. College Admission Mechanism

Considering the user association in the wireless networks, the Gale-Shapley algorithm depends on cells tentatively accepting applicants until either new or better candidate applies to the cells, or the algorithm terminates with convergence to a particular matching. The algorithm terminates when all students are categorized in only two groups: a group that is accepted in any of the colleges and a group that was rejected from all colleges in their lists.

In the considered model, the user associated problem can be formulated as in [7]. The problem was formulated by using the R-factor as a preference for users (who act as students in this game). For simplicity; the users calculate the minimum

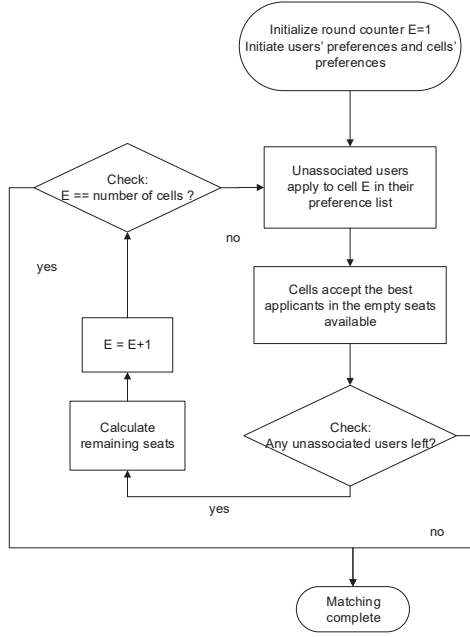


Fig. 2: The Boston school choice mechanism flowchart.

possible (i.e., guaranteed) R-factor if they join a certain cell and use that information to construct their preference lists. The R-factor guarantee comes from considering the maximum possible delay  $d_{i_{\max}}$  which is given by

$$d_{i_{\max}} = (q_i - 1)T + d_{i_b}. \quad (9)$$

The cells (who act as colleges) would estimate their backhaul delays and broadcast  $d_{i_{\max}}$  to users so they can construct their preferences according to (7).

One of the main drawbacks in this algorithm is that we have to wait until the algorithm terminates to know final users' association to their respective cells as there is no guarantee that any intermediate association of users will hold. Such drawback causes delays for all users and might affect the online operation. Also, the entrance of new users to the wireless scene will trigger the whole association to start over and might dislocate already associated user, thus rendering the matching inefficient. These shortcomings can be avoided through our proposal to apply the Boston School admission mechanism.

### B. Boston School Choice Mechanism

The Boston school choice problem is different from college admission. Schools are governed by "priority lists" instead of "preference lists" as in college admission. The priority lists are different in that they can discriminate among new applicants applying for the empty slots. A tie-breaking lottery if there were students with equal priorities applying to the same school at the same time. However, as soon as respective slots in the school are filled, schools are not allowed to defer already admitted students in favor of new applicants in subsequent rounds even if the new applicants would rank higher on the school list, i.e., criteria.

For this reason, Boston school admission is also known as the immediate-acceptance algorithm. Consequently, the algorithm is finished in a maximum number of rounds that does not exceed the total number of schools involved in the matching. Boston mechanism is described in the flowchart in Figure 2.

### C. Complexity Analysis and Comparison

Boston mechanism has a lower computational complexity compared to the college admission one. For simplicity and clarity, if we consider the stable marriage problem (which is a special case of college admission game where quota = 1 for all colleges) it has a complexity of  $O(N^2)$  according to [13]. However, the Boston mechanism under the same setup of the stable marriage problem has a complexity of  $O(N(M + L))$ , assuming that  $(M + L) < N$ , which means that the total number of cells is less than the number of users, the Boston mechanism has a lower complexity. Moreover, the complexity of the Boston mechanism in a general case is  $O(N(M + L) \log_2 N)$ .

## IV. SIMULATIONS AND DISCUSSION

Similar to [7], the considered system model consists of two adjacent square cells with an area  $1\text{km}^2$  each, and the MBS is in the center of each cell. Also, the system has 10 SBSs with quota  $q_l = 4$ , which are distributed randomly in the total area between the two cells. For simplicity purposes, the wireless channel is assumed to be only affected by the distances between users and base stations (i.e.,  $g_{i,n} = 1$ ). The uplink user power is set to 20 mW, time slot duration  $T = 20$  milliseconds, propagation loss = 3, the pricing factor  $\beta$  is set to 1, and the average interference plus noise power level is set to be -90 dBm. Binary phase shift keying (BPSK) modulation scheme is used in data transmission. The backhaul delays are random variables which follow a Pareto distribution with index = 1.16 and scale factors of 15 milliseconds and 40 milliseconds for MBSs and SBSs, respectively. The system works with the G.729 codec thereby, the values of a,b,c in (13) are 12, 15 and 60, respectively [7]. For time delay calculations in (8)-(11), the constant values are set as follows  $\psi = 177.3$  milliseconds,  $mT = 150$  milliseconds, and  $sT = 0.4$ . The number of bits per packet  $B$  is set to 160 bits.

Figure 3 presents a comparison between the Gale-Shapley algorithm and the Boston mechanism regarding average utility. In the case of -110 dBm noise power level, the figure shows that the average R-factor obtained is nearly the same for both the Gale-Shapley algorithm and Boston mechanism. Whereas, in the case of -90 dBm, the Boston mechanism has a better R-factor performance than that of the Gale-Shapley algorithm.

The discussion above is clearly shown in Figure 4, which represents the distribution of users according to their R-factor values. This figure shows that the Boston mechanism gets more users in the high R-factor region and the low R-factor region. However, this disadvantage does not affect the average R-factor values that much, so we can say that both algorithms can have similar average performance profiles.

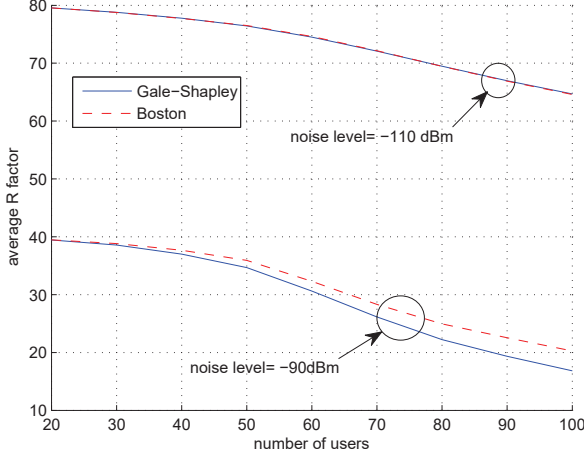


Fig. 3: Average utility while increasing number of users under noise levels of  $-90$  and  $-110$  dBm.

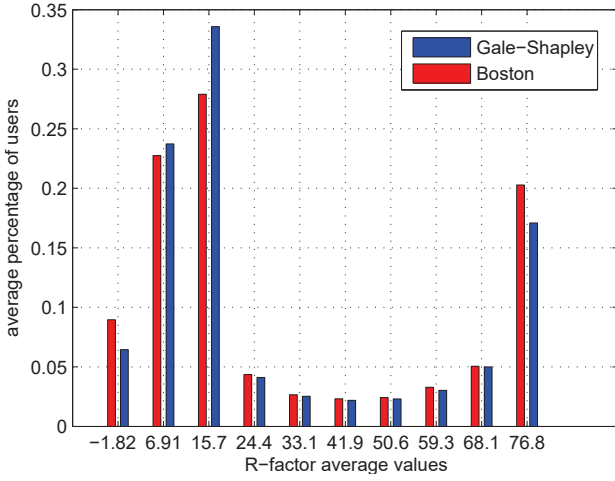


Fig. 4: Average percentage of users distributed over R-factor average values with 60 users and 10 SBSs.

Figure 5 considers the number of iterations to reach equilibrium. The results show that Boston mechanism generally converges with less number of iterations which provides a faster association and access to network service.

## V. CONCLUSION AND FUTURE WORK

The paper investigates the advantages of the application of the Boston school mechanism to the uplink user association problems in heterogeneous networks. The promising results regarding, the complexity, number of runs and performance in the low SNR regions prompt future extensions to the work. A deeper investigation of the preferences and priorities from the parameters offered in the 5G system is of high practical values.

## ACKNOWLEDGMENT

This work is done as part of the funded research project “Super-HETs, Empowering 5G Heterogeneous Networks for better Performance”, which is supported by the National Telecom Regulatory Authority in Egypt (NTRA)

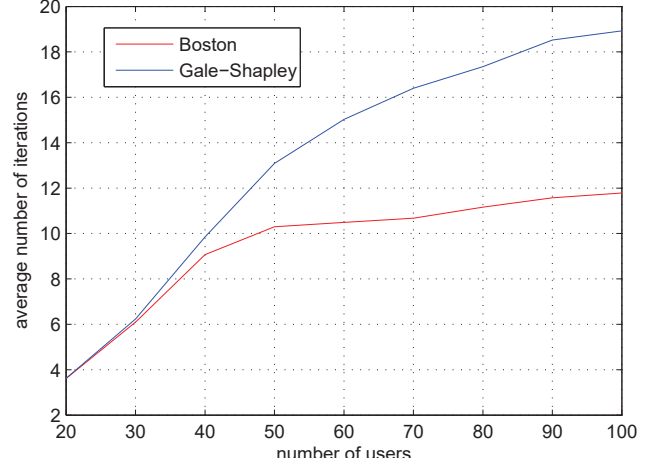


Fig. 5: Average number of iterations as a function of increasing number of users.

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