Analysis of Widely Linear Equalization over Frequency Selective Channels with Multiple Interferences

Hayfa Fhima[1,2], Hmaied Shaïek[1], Rafik Zayani[2], Daniel Roviras[1], Bruno Sens Chang[3] and Ridha Bouallegue[2]
[1] CEDRIC/LAETITIA Laboratory, CNAM, Paris, France
[2] Innov'COM Laboratory, Sup'Com, Carthage University, Tunisia
[3] CPGEI/Electronics Department, Federal University of Technology - Paraná Curitiba, Brazil

***Abstract*—In this paper, the error performance of linear (MMSE-LE) and widely linear (MMSE-WLE) equalizers is analyzed in terms of mean square error (MSE) and bit error rate (BER). This analysis is done in a system using improper modulations of type Pulse Amplitude Modulations (PAM) having inter-symbol-interferences (ISI) and/or inter-user-interferences (IUI). For this scenario, we derive theoretical expressions of both MSE and BER for the two equalizers. Analytical expressions developed in this paper are verified by computer simulations. It is worth mentioning that these expressions are valid for any number of interferers and any channel delay spread.**

Index Terms: Widely linear, mean square error, bit error rate, improper modulation, ISI and IUI.

I. Introduction

It is well known that the equalization process [1] is the way to fight inter-symbol interferences (ISI) [2] and inter-user interferences (IUI) caused by, respectively, frequency-selective multipath fading channels and external interferences [3]. In systems where the modulation is improper (i.e the pseudo-covariance of the signal in non-zero) [3], [4], the overall information carried by the signal, is distributed between the covariance matrix and the pseudo-covariance one [5], [6]. This is why both should be considered in the equalization process, in order to improve the error performance. This scheme is called widely linear processing (WL) [1], and it outperforms the linear one (LE) when using improper modulations as demonstrated in [2], [3], [7], [8], [9] and [10].

The performance gain of the MMSE-WLE when compared to MMSE-LE has encouraged researchers to evaluate analytically the behavior of the two equalizers when using one-dimensional modulation formats. Indeed, authors in [11] have developed an expression for the mean square error (MSE) of the linear equalizer in a system considering only ISI with the diversity combining technique. Based on this result, the MSE for the WLE is introduced in [2] in a system where the modulation is improper over a frequency selective channel.

Authors in [11] and [2] have dealt in a scenario with only ISI , whereas authors in [3] have dealt in a scenario with only IUI, in both single input single output (SISO) and single input multiple output (SIMO) systems. In this case, the signal to interference plus noise ratio (SINR) is derived in order to characterize the behavior of the two equalizers.

Authors in [8] have presented a new method to characterize the behavior of widely linear equalizer as a function of only IUI. In presence of IUI and ISI and based on an adaptive filter, authors in [9], have analyzed theoretically the convergence behavior of the adaptive equalizers. In [10], authors, have given an approximation of symbol error rate (SER) with WL receivers in SIMO systems with the presence of co-channel interferers. Moreover, the analysis of the adaptive filters' convergence and complexity is done in [12] through the development of the analytical steady-state MSE.

Authors in [9], [10] and [12], have dealt with many types of interferences using WL receivers; however, in these papers no theoretical analysis has been carried out for the MSE or BER of improper modulations in the presence of IUI and ISI, which brings us to the main originality of this work. Thus, in this work we develop analytical expressions of both MSE and BER for linear and widely linear equalizers. This analysis is done in a system using PAM modulation with the corruption of multiple interferences over a frequency-selective channel. This scenario has been chosen in order to cope with a Filter Bank Multi Carrier/Orthogonal Quadrature Amplitude Modulation (FBMC/OQAM) system [14]. With this kind of multi-carrier system, QAM symbols are transmitted using two PAM modulations staggered by half a symbol period. Hence, the useful transmitted signal can be seen as a PAM signal. When the transmission channel is frequency selective, adjacent carriers will interfere with the useful signal and are modeled, in our scenario, by IUI PAM interferences. Afterward, we compare the results of both analytical and simulated BER for MMSE-LE and MMSE-WLE for different channel lengths and number of interferers. It was seen that the theoretical results match the simulated ones.

This paper is organized as follows: the system model is introduced in Section II. Section III presents a theoretical performance analysis of the two equalizers in terms of MSE and BER. Analytical and simulation results are given in Section IV, followed by the conclusion in Section V.

In this article, we use bold capital letters to represent matrix, bold lowercase letters to denote vectors and scalars are represented by lowercase letters. $\mathbb{E}[\cdot]$ is the expectation operator, $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote respectively the conjugate, transpose and the hermitian operations. The time and frequency domain elements are represented respectively by lowercase and capital letters.

II. System model

We consider the system model shown by Figure 1 with K interferers.

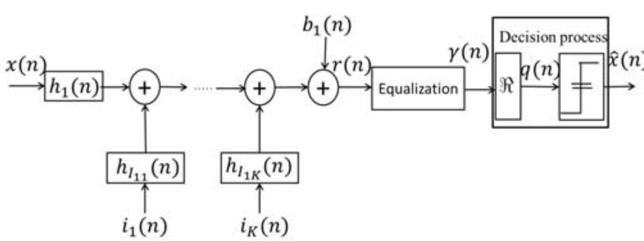

Fig. 1: System model with K interferers.

In this Figure, $x(n)$ is the useful signal at time n, with variance σ_x^2 and symbol-duration T. $i_k(n)$ is the k^{th} interfering signal ($k = 1, ..., K$) with variance σ_I^2. Without loss of generality, the studied modulation is the M-ary pulse amplitude modulation (M-PAM) where the signal amplitude $\in \{\pm A, \pm 3A, .., \pm(M-1)A\}$. It is assumed that all transmitted signals are independent, and $h_1(l)$ and $h_{I_{1k}}(l)$, $l \in \{0, 1, .., L-1\}$ are the discrete time channel impulse responses of, respectively, the useful signal and the k^{th} interferer. $b_1(n)$ is a proper Complex White Gaussian noise with variance equal to σ_b^2. $r(n)$ is the equalizer input signal, $\gamma(n)$ and $q(n)$ are the equalizer output before and after taking its real part, respectively. Finally, $\hat{x}(n)$ is the estimated signal after decision.

The received signal $r(n)$ can be written as follows:

$$r(n) = \sum_{l=0}^{L-1} h_1(l)x(n-l) + \sum_{k=1}^{K} \left[\sum_{l=0}^{L-1} h_{I_{1k}}(l) i_k(n-l) \right] + b_1(n). \tag{1}$$

III. Performance Analysis

A. Time domain equalizers

As mentioned before, the criterion adopted for the equalization process is the minimization of the mean square error (MSE) between the transmitted and the equalized signals given by the following expression:

$$MSE_\gamma = \mathbb{E}\left[|\gamma(n) - x(n-k_0)|^2 \right], \tag{2}$$

where k_0 is the decision delay that should be well chosen [2], [13].

Consider the vector $\boldsymbol{r}(n)$, of length N given by $\boldsymbol{r}(n) = [r(n)\ r(n-1) ...\ r(n-N+1)]^T$. With classical linear processing [3], [14] of the received signal, the equalizer of length N (satisfying $\gamma(n) = \boldsymbol{f_L}^H \boldsymbol{r}(n)$) is given by

$$\boldsymbol{f_L} = \boldsymbol{R}_{\boldsymbol{rr}}^{-1} \boldsymbol{r}_{\boldsymbol{r}x}, \tag{3}$$

where $\boldsymbol{R_{rr}} = \mathbb{E}[\boldsymbol{r}(n)\boldsymbol{r}^H(n)]$ is the autocorrelation function matrix and $\boldsymbol{r}_{\boldsymbol{r}x} = \mathbb{E}[\boldsymbol{r}(n)x^H(n-k_0)]$ represents the inter-correlation vector of length N.

When applying widely linear processing [1], [3], [7] to the received signal $\boldsymbol{r}(n)$ and to its complex conjugate version $\boldsymbol{r}^*(n)$, the equalizing filters (satisfying $\gamma(n) = \boldsymbol{f_{1W}}^H \boldsymbol{r}(n) + \boldsymbol{f_{2W}}^H \boldsymbol{r}^*(n)$) are given as following:

$$\boldsymbol{f_{1W}} = \left[\boldsymbol{R_{rr}} - \boldsymbol{C_{rr}} \boldsymbol{R_{rr}}^{-1*} \boldsymbol{C_{rr}}^*\right]^{-1} \left[\boldsymbol{r_{rx}} - \boldsymbol{C_{rr}} \boldsymbol{R_{rr}}^{-1*} \boldsymbol{r_{rx}}^*\right] \quad (4)$$

$$\boldsymbol{f_{2W}} = \boldsymbol{f^*_{1W}}, \quad (5)$$

where $\boldsymbol{C_{rr}} = \mathbb{E}[\boldsymbol{r}(n)\boldsymbol{r}^T(n)]$ is the pseudo-autocorrelation matrix of rank N.

If we assume that the matrix $\boldsymbol{R_{rr}}$ and $\boldsymbol{C_{rr}}$, and the vector $\boldsymbol{r_{rx}}$ are already computed. The complexity to compute Eq.(3) (for the LE) and Eq.(4) (for the WLE) is asymptotically of order N^2 and N^3.

B. MSE expressions

In order to evaluate the system performance, we consider MSE_q according to the real part of the equalized signal:

$$\begin{aligned} MSE_q &= \mathbb{E}\left[|\Re(\gamma(n)) - x(n-k_0)|^2\right] \\ &= \mathbb{E}\left[|q(n) - x(n-k_0)|^2\right]. \end{aligned} \quad (6)$$

It is worth noticing that the difference between (6) and (2) (which allows the derivation of the equalizers' responses) is only for the linear case, since the output of the WLE is real.

When the LE ($\boldsymbol{f_L}$) and WLE ($\boldsymbol{f_{1W}}$ and $\boldsymbol{f_{2W}}$) are computed, $q(n)$ can be written as $q(n) = \gamma'(n) + \gamma'^*(n) = \boldsymbol{f_1}\boldsymbol{r}(n) + \boldsymbol{f_2}\boldsymbol{r}^*(n)$, where $\boldsymbol{f_1} = \boldsymbol{f_{1W}}^H$ for WLE, $\boldsymbol{f_1} = \frac{1}{2}\boldsymbol{f_L}^H$ for LE and $\boldsymbol{f_2} = \boldsymbol{f_1}^*$ for both equalizers' case. Therefore, $q(n)$ can be generated by using both the transmitted signal and its complex conjugate. This configuration can be modeled as shown in Figure 2, where signals carried by the lower branch are the complex conjugate of those in the upper one. Note that this system model remains valid for both equalizers cases, by taking $f_1^*(n)$ and $f_2^*(n)$ equal to $f_{1W}(n)$ and $f_{2W}(n)$ given by (4) and (5) for the WLE, and $f_1^*(n) = \frac{1}{2}f_L(n)$ and $f_2^*(n) = \frac{1}{2}f_L^*(n)$ for the LE, where $f_L(n)$ is given by (3).

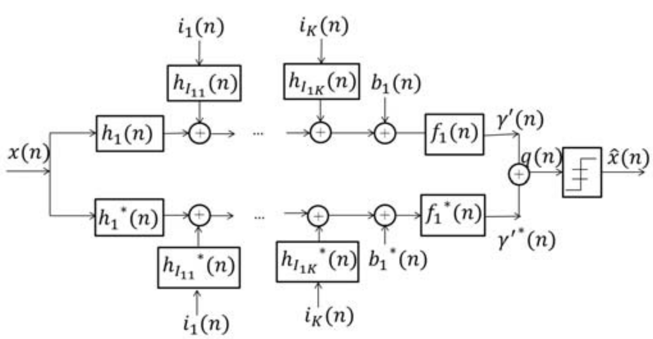

Fig. 2: Equivalent system model scheme with K interferers.

For the determination of the MSE in time domain, we consider the real part of the equalized signal that can be written as follows:

$$q(n) = \alpha\ x(n-k_0) + n_E(n) + b_E(n), \quad (7)$$

where α is a real multiplicative coefficient applied to the useful signal. $n_E(n)$ and $b_E(n)$ are, respectively, the residual $ISI + IUI$ and noise, of variances σ^2_{NE} and σ^2_{BE}.
Thus, (6) can be rewritten as:

$$MSE_q = |\alpha - 1|^2 \sigma_x^2 + \sigma^2_{NE} + \sigma^2_{BE}. \quad (8)$$

C. Frequency domain equalizers and MSE expressions

In order to compute the value of MSE_q expressed in (8), we will first compute the power spectral density (PSD) of the error and then integrate it over the whole spectrum. To compute this PSD we must first derive the equalizers in the frequency domain.

As in the time domain, we distinguish two errors given by the following expressions:

$$E_\Gamma(\omega) = \Gamma(\omega) - X(\omega)\, e^{-j\omega k_0 T} \quad (9)$$

$$\begin{aligned} E_Q(\omega) &= Q(\omega) - X(\omega)\, e^{-j\omega k_0 T} \\ &= \left[F_1(\omega)H_1(\omega) + F_1^*(-\omega)H_1^*(-\omega) - e^{-j\omega k_0 T}\right] X(\omega) \\ &+ \sum_{k=1}^{K} [F_1(\omega)H_{I_{1k}}(\omega) + F_1^*(-\omega)H^*_{I_{1k}}(-\omega)] I_k(\omega) \\ &+ F_1(\omega)B_1(\omega) + F_1^*(-\omega)B_1^*(-\omega), \end{aligned} \quad (10)$$

where $E_\Gamma(\omega)$, $E_Q(\omega)$, $\Gamma(\omega)$, $X(\omega)$, $Q(\omega)$, $F_1(\omega)$, $H_1(\omega)$, $H_{I_{1k}}(\omega)$, $B_1(\omega)$ and $I_k(\omega)$, $k \in \{1,..,K\}$ are the Fourier transforms of respectively $e_\gamma(n)$, $e_q(n)$, $\gamma(n)$, $x(n)$, $q(n)$, $f_1(n)$, $h_1(n)$, $h_{I_{1k}}(n)$, $b_1(n)$ and $i_k(n)$.
For the WLE case, $E_\Gamma(\omega) = E_Q(\omega)$ (since the equalizer output signal is real ($\gamma(n) = q(n)$)), whereas for the LE case, it is equal to:

$$\begin{aligned} E_\Gamma(\omega) &= \left[F_L(\omega)H_1(\omega) - e^{-j\omega k_0 T}\right] X(\omega) \\ &+ \sum_{k=1}^{K} \left[F_L(\omega)H_{I_{1k}}(\omega)\right] I_k(\omega) + F_L(\omega)B_1(\omega). \end{aligned} \quad (11)$$

As in the time domain, to derive the expressions of the equalizers, we proceed by minimizing the power spectral density Se_Γ of the error $e_\gamma(n) = \gamma(n) - x(n-k_0)$ where

$$Se_\Gamma(\omega) = \mathbb{E}[|E_\Gamma(\omega)|^2] = \mathbb{E}\left[|\Gamma(\omega) - X(\omega)e^{-j\omega k_0 T}|^2\right]. \quad (12)$$

For the linear case, $E_\Gamma(\omega)$ is given by equation (11) whereas, for the widely linear case, it is expressed by equation (10) with $F_1(\omega) = F_{1W}(\omega)$.

Therefore, the expressions of MMSE-LE, MMSE-WLE are given by equations (13), ((14) and (15)), respectively.

$$F_L(\omega) = \frac{\sigma_x^2 H_1^*(\omega)\, e^{-j\omega k_0 T}}{\sigma_x^2 |H_1(\omega)|^2 + \sigma_I^2 \sum_{k=1}^{K} |H_{I_{1k}}(\omega)|^2 + \sigma_b^2}, \quad (13)$$

$$F_{1W}(\omega) = \frac{\sigma_x^2 H_1^*(\omega) e^{-j\omega k_0 T}\left[\sigma_b^2 + \sigma_I^2 \sum_{k=1}^{K} |H_{I_{1k}}(-\omega)|^2\right]}{O(\omega)} - \frac{\sigma_x^2 \sigma_I^2 H_1(-\omega) e^{-j\omega k_0 T} \sum_{k=1}^{K} H_{I_{1k}}^*(\omega) H_{I_{1k}}^*(-\omega)}{O(\omega)} \quad (14)$$

where $O(\omega)$ is expressed in equation (16), and

$$F_{2W}(\omega) = F_{1W}^*(-\omega). \quad (15)$$

Note that F_L, F_{1W} and F_{2W} (given by (13), (14) and (15)), are the Fourier transforms of respectively f_L^H, f_{1W}^H and f_{2W}^H (given by (3), (4) and (5)). Once the equalizers are computed, the power spectral density Se_Q of the error $e_q(n) = q(n) - x(n - k_0)$ is expressed as $Se_Q(\omega) = \mathbb{E}[|E_Q(\omega)|^2]$, where $E_Q(\omega)$ is given by (10). Therefore, Se_Q is expressed as:

$$\begin{aligned} Se_Q(\omega) &= |F_1(\omega)H_1(\omega) + F_1^*(-\omega)H_1^*(-\omega) - e^{-j\omega k_0 T}|^2 \sigma_x^2 \\ &+ \sum_{k=1}^{K} |F_1(\omega)H_{I_{1k}}(\omega) + F_1^*(-\omega)H_{I_{1k}}^*(-\omega)|^2 \sigma_I^2 \\ &+ \left[|F_1(\omega)|^2 + |F_1^*(-\omega)|^2\right] \sigma_b^2, \end{aligned} \quad (17)$$

where $F_1(\omega) = F_{1W}(\omega)$ for the MMSE-WLE and for the MMSE-LE $F_1(\omega) = \frac{1}{2}F_L(\omega)$.

Referring to [11], the mean square error of the signal described by Figure 2 is:

Hence, the equality between (8) and (18) gives result to the following formula:

$$\alpha = \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} e^{j\omega k_0 T} \left[F_1(\omega)H_1(\omega) + F_1^*(-\omega)H_1^*(-\omega)\right] dw. \quad (19)$$

Therefore, the BER expression of the studied system will be derived.

D. BER

The BER corresponding to the transmission of a M-PAM signal is given by the following expression:

$$BER = \left[\frac{2(M-1)}{M \log_2(M)}\right] Pr(n_E(n) + b_E(n) > \alpha A) \quad (20)$$

Since $n_E(n)$ and $b_E(n)$ are Gaussian, zero mean with variances, respectively, equal to σ_{NE}^2 and σ_{BE}^2, the BER expression can be written as:

$$\begin{aligned} BER &= \left[\frac{2(M-1)}{M \log_2(M)}\right] Q\left(\frac{\alpha A}{\sqrt{\sigma_{NE}^2 + \sigma_{BE}^2}}\right) \\ &= \left[\frac{2(M-1)}{M \log_2(M)}\right] Q\left(\frac{\alpha A}{\sqrt{MSE_q - \sigma_x^2(\alpha - 1)^2}}\right) \end{aligned} \quad (21)$$

where A is equal to:

$$A = \sqrt{\frac{3}{M^2 - 1}\sigma_x^2}. \quad (22)$$

Note that the proposed method of computing the different elements given by (10), (17), (18), (19), (20) and (21) is valid for both MMSE-LE (when $F_1(\omega) = \frac{1}{2}F_L(\omega)$) and MMSE-WLE (when $F_1(\omega) = F_{1W}(\omega)$).

IV. Numerical results

In this section, we present a comparison between the analytical expressions of BERs and MSEs given by (21) and (8), respectively and the simulated ones for both equalizers. This comparison is done as a function of the number of interferers K and the channel length L.

For the simulations, the L taps of the channel impulse responses are chosen randomly having a Rayleigh modulus and a phase uniformly distributed in $[0, 2\pi]$ for each one. Likewise, these taps are kept constant during the transmission of the 10^6 M-PAM symbols. The equalizer length is equal to $N = 30$, with the delay $k_0 = 20$. The simulation results are the average of 100 different channels realizations. The simulation parameters for Figure 3 are $K = 1$, $L = 3$ and $M = 4$, for Figure 4, $K = 5$, $L = 10$ and $M = 2$ and for Figure 5, $K = 1$, $L = 3$ and $M = 16$.

Through Figures 3 to 5, we show that the results provided by the theoretical expressions (21) and (8) match the simulated results for a smaller or larger number of interfering signals, for shorter or longer channels, for $2-$PAM, $4-$PAM and $16-$PAM and this remains valid for any PAM order, i.e. for all cases $(\forall K, \forall L, \forall M)$. These results also show the degradation of the performance when using a WLE when the number of interferences increases.

In the considered scenario, with the existence of both ISI and IUI, the error performance of the MMSE-LE is limited; even with a large number of interferers the system using a MMSE-WLE outperforms a system using a MMSE-LE.

V. Conclusion

In this article, we have proposed analytical expressions for the MSE and BER for a system having an improper modulation of type $M-$PAM ($M = 2$, $M = 4$ and $M = 16$) over frequency selective channels, and in presence of K rectilinear interferences of type $M-$PAM. These expressions are calculated in order to evaluate the performances of both linear and widely linear equalizers. Thus, the MSE/BER results obtained by the proposed equations match the simulated results ($\forall L$, $\forall K$ and for any PAM order). Likewise, we have shown that, when the number of interferences K is low, the performance gap between the linear and widely linear equalizers is high. However, when K increases, this gap is gradually reduced and we have shown a degradation in the performance of the widely linear equalizer.

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$$O(\omega) = \sigma_x^2 |H_1(-\omega)|^2 \left[\sigma_I^2 \sum_{k=1}^{K} |H_{I_{1k}}(\omega)|^2 + \sigma_b^2 \right] + \sigma_x^2 |H_1(\omega)|^2 \left[\sigma_I^2 \sum_{k=1}^{K} |H_{I_{1k}}(-\omega)|^2 + \sigma_b^2 \right]$$
$$+ \sigma_b^2 \left[\sigma_b^2 + \sigma_I^2 \sum_{k=1}^{K} \left(|H_{I_{1k}}(\omega)|^2 + |H_{I_{1k}}(-\omega)|^2 \right) \right] - 2\sigma_I^2 \sigma_x^2 \Re \left(H_1(\omega) H_1(-\omega) \sum_{k=1}^{K} H_{I_{1k}}^*(\omega) H_{I_{1k}}^*(-\omega) \right)$$
$$+ \sigma_I^4 \left[\left(\sum_{k=1}^{K} |H_{I_{1k}}(\omega)|^2 \right) \left(\sum_{k=1}^{K} |H_{I_{1k}}(-\omega)|^2 \right) - \left| \sum_{k=1}^{K} H_{I_{1k}}(\omega) H_{I_{1k}}(-\omega) \right|^2 \right] \quad (16)$$

$$MSE_q = \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} Se_Q(\omega)\, dw.$$
$$= \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} |F_1(\omega) H_1(\omega) + F_1^*(-\omega) H_1^*(-\omega) - e^{-j\omega k_0 T}|^2 \sigma_x^2$$
$$+ \sum_{k=1}^{K} |F_1(\omega) H_{I_{1k}}(\omega) + F_1^*(-\omega) H_{I_{1k}}^*(-\omega)|^2 \sigma_I^2 + \left[|F_1(\omega)|^2 + |F_1^*(-\omega)|^2 \right] \sigma_b^2 \, dw.$$

$$= \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \left| F_1(\omega) e^{j\omega k_0 T} H_1(\omega) + F_1^*(\omega) e^{j\omega k_0 T} H_1^*(-\omega) - 1 \right|^2 \sigma_x^2$$
$$+ \sum_{k=1}^{K} |F_1(\omega) H_{I_{1k}}(\omega) + F_1^*(-\omega) H_{I_{1k}}^*(-\omega)|^2 \sigma_I^2 + \left[|F_1(\omega)|^2 + |F_1^*(-\omega)|^2 \right] \sigma_b^2 \, dw. \quad (18)$$

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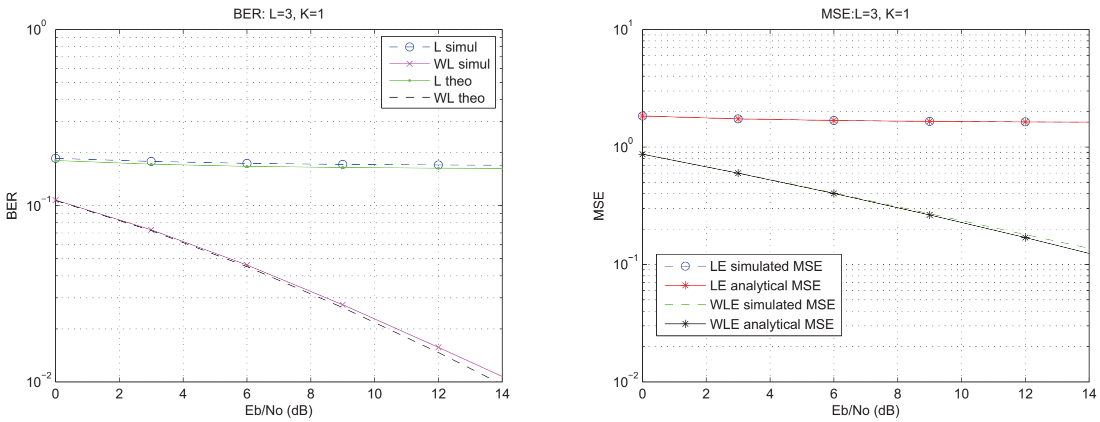
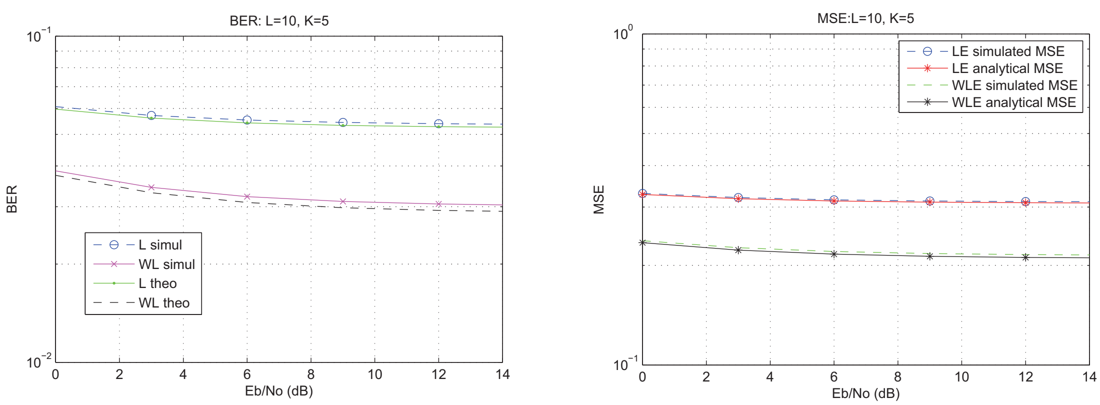


(a) BER

(b) MSE

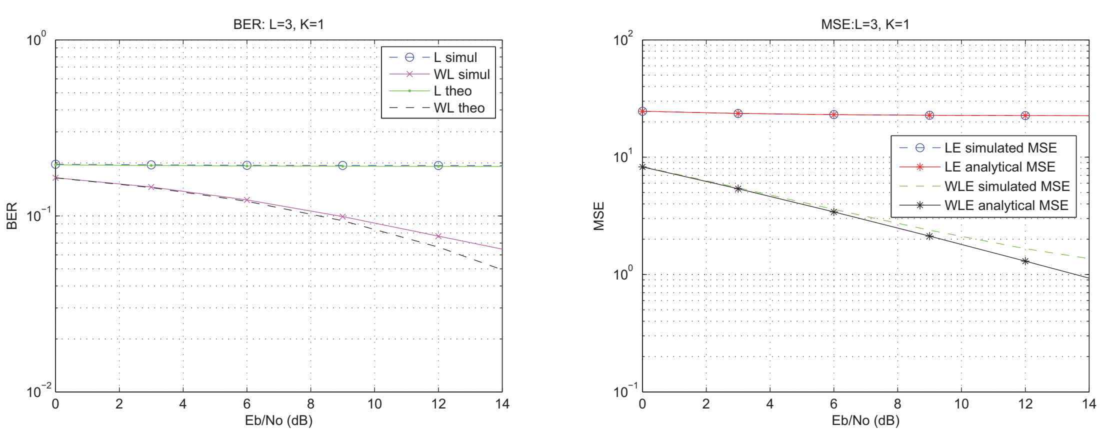
Fig. 3: Error performance for $K = 1$, $L = 3$ and $M = 4$.



(a) BER

(b) MSE

Fig. 4: Error performance for $K = 5$, $L = 10$ and $M = 2$.



(a) BER

(b) MSE

Fig. 5: Error performance for $K = 1$, $L = 3$ and $M = 16$.