On Application of the Correlation Vectors Subspace Method for 2-Dimensional Angle-Delay Estimation in Multipath OFDM Channels

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***Abstract*—Indoor localization systems often necessitate the estimation of signal parameters such as the angles or the delays of arrival. The paper examines two-dimensional (2D) joint angles and delays of arrival estimation using channel state information when the OFDM transmit symbol undergoes multipath fading and is received through multiple coherent signals using a uniform linear antenna array. Parameter estimation from coherent signals requires spatio-frequential smoothing receive preprocessing before the application of 2D subspace methods. The paper studies the estimation performance of such high-resolution algorithms combined with the recent method of correlation vectors subspace for improved parameter covariance matrix estimation. Different variants of the correlation vectors subspace method are considered. Simulation results show that the root mean square error for different number of snapshots and for low signal-to-noise ratio is reduced over the case where parameter estimation is performed without the correlation vectors subspace technique.**

***Index Terms*—2D parameter estimation, subspace estimation methods, delay of arrival, angle of arrival, channel state information, spatio-frequential smoothing, indoor localization, orthogonal frequency-division multiplexing (OFDM)**

I. Introduction

Location-aware systems with applications such as indoor navigation and smart home automation rely on the information about the location of people or electronic devices in order to control their features. To this end, indoor localization schemes to locate a device-bearing user are important, especially given that the radionavigation Global Positioning System (GPS) does not work well in an indoor environment due to the attenuation or blocking of line-of-sight (LOS) satellite signals by walls and ceilings. On the other hand, systems such as Wi-Fi 802.11 characterized by multi-carrier OFDM transmissions and multi-antenna transceiver architectures and utilized extensively in indoor areas for communications, could be exploited to offer the positioning service. The implementation of the indoor localization itself can take place using several reference wireless access points, or anchors to position a transmitting user by first estimating signal parameters, such as the angle of arrival (AoA) or the time delay of arrival (ToA) of the received signal, then performing triangulation or trilateration techniques to obtain an estimate of the user's location.

In this paper, we focus on the problem of jointly estimating the angle and the delay of arrival parameters, also known as two-dimensional (2D) space-time parameter identification [1]. We consider subspace-based algorithms to solve this estimation problem such as the 2D MUSIC (Multiple Signal Classification) and the 2D ESPRIT (Estimation of Signal Parameters by Rotational Invariance Techniques) in [2], [3]. Although these algorithms provide suboptimal estimation performance to maximum likelihood (ML) methods [4], they exhibit computational efficiency. The methods in [2], [3] were designed to jointly estimate the angles and the delays of arrival of multiple signals. However, for indoor scenarios characterized by rich multipath propagation, the received signal is a summation of scaled and delayed version of the transmitted signal, i.e., we are under the case of *coherent* sources. When such coherent multipath components exist, subspace methods fail to estimate parameters unless certain spatio-frequential smoothing preprocessing takes place [7].

The recent concept of correlation vectors subspaces was proposed in [9] to improve AoA estimation. For any given array geometry, in particular for uniform linear arrays (ULAs) available on most off-the shelf wireless routers, the correlation vectors subspace is uniquely determined. This fact, under certain assumptions, imposes a constraint on the structure of the parameter covariance matrix. The correlation vectors subspace is computed in [9] via a numerical approach and has been utilized in denoising the sample receive covariance matrix so as to provide better estimation. On the denoised covariance one may efficiently apply off-the-shelf AoA estimators, such as the MUSIC or ESPRIT algorithm. In [10], this idea is related to the difference coarray theory and analytical formulas for the correlation vectors subspace method are provided.

The present paper applies the theory of correlation vectors subspaces developed in [9], [10] on the 2D joint angles and delays of arrival estimation scenario of interest. Specifically, the presented method jointly calculates the angles and the delays of arrival of the multipath components leveraging a set of channel measurements, also known as channel state information (CSI), at the receiver. The idea of exploiting the

CSI may find application on several WiFi network interface controllers (NIC) from Atheros or the INTEL 5300 NIC which may expose the CSI to the user level [16], [17]. After presenting the system and channel model and related assumptions for multipath multi-carrier signal propagation, we shortly revisit the spatio-frequential smoothing preprocessing and the 2D high-resolution algorithms MUSIC and ESPRIT. Then we write the analytical formula for the correlation vectors subspaces for our 2D angle-delay estimation problem. We then present our performance evaluation framework which allows to combine the spatio-frequential smoothing preprocessign with the correlation vectors subspaces method and the 2D MUSIC and 2D ESPRIT. We approximate the model for the multipath case, i.e., the case of coherent sources, with the model for non-coherent sources and compare the simulated estimation performance of subspace-based estimators using the correlation vectors subspace method. Showing that the method improves results in particular for low signal-to-noise ratios (SNR) as compared to standard methods completes the contributions of the paper.

Notation: We denote matrices and vectors with boldface type, using capital letters for matrices and lower-case letters for vectors. An n-dimensional (column) vector $\mathbf{y}$ is denoted by $(y_1, y_2, \dots, y_n)$. $\mathbf{A}^{\mathrm{T}}$, $\mathbf{A}^{*}$, and $\mathbf{A}^{\mathrm{H}}$ denote the transpose, complex conjugate, and complex conjugate (Hermitian) transpose of the matrix $\mathbf{A}$.The Moore-Penrose inverse (pseudoinverse) of $\mathbf{A}$ is $\mathbf{A}^{\dagger}$ and $\mathrm{tr}(\mathbf{A})$ denotes the trace of $\mathbf{A}$. The vectorization operator is defined as $\mathrm{vec}([\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_n]) = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_n]^{\mathrm{T}}$, where $\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_n$ are column vectors of the same length. For two Hermitian matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \preceq \mathbf{B}$ is equivalent to $\mathbf{A} - \mathbf{B}$ being positive semidefinite. $\mathrm{col}(\mathbf{A})$ stands for the column space of $\mathbf{A}$. $\mathbf{I}_x$ defines the identity matrix of size x, $\mathbf{0}$ is a column vector of all zeros and $\|\mathbf{A}\|_{\mathrm{F}}$ denotes the Frobenius norm of $\mathbf{A}$. The Kronecker product of two matrices is denoted by the operator $\otimes$. We use the bracket notation as in [14] and [15]. We assume that the joint antenna (sensor) and OFDM subcarrier frequency indices are characterized by a 2D integer set, for instance, $\mathbb{S} = \{(1,1), (1,2), (2,1), (2,2)\}$ for the case of 2 antennas and 2 subcarriers. We also assume that for $(n_1, n_2) \in \mathbb{S}$ the triangular bracket $\langle S \rangle_{n_1,n_2}$ denotes the sample value on the support location (n_1, n_2). Hence, $\langle S \rangle_{1,2} = (1,2)$. Finally, $\mathbb{E}[X]$ denotes the expectation of a (complex) random variable X.

II. System Model and Assumptions

The case of a single source transmitting an OFDM symbol $g(t)$ containing N subcarriers under azimuth-only propagation is considered. The symbol arrives to a M-element ULA receiver through Q multipath components. Each component $q \in \{1, \dots, Q\}$, comes with AoA φ_q and after a delay time τ_q. The lth baseband received signal vector $\boldsymbol{x}^{(l)}(t)$, containing the signals received by all M antennas is

$$\boldsymbol{x}^{(l)}(t) = \sum_{q=1}^{Q} \boldsymbol{\alpha}(\varphi_q) \gamma_q^{(l)} g(t - \tau_q) + \boldsymbol{n}^{(l)}(t) \quad \in \mathbb{C}^{M\times 1}, \quad (1)$$

where $l \in \{1, \dots, L\}$ and L is the number of snapshots, defining the observation (measurement) interval $\{1, \dots, L\}$. In (1), $\gamma_q^{(l)}$ is the complex attenuation along the signal path q and the vector $\boldsymbol{n}^{(l)}(t) \in \mathbb{C}^{M\times 1}$ has as entries the additive noise for every antenna. The mth entry $\alpha_m(\varphi_q)$ of the array response vector $\boldsymbol{\alpha}(\varphi_q) \in \mathbb{C}^{M\times 1}$ is the response of the mth antenna to the qth path arriving from angle φ_q and depends on the antenna geometry. The OFDM symbol is expressed as $g(t) = \sum_{n=1}^{N} s_n e^{-\mathrm{j}\pi n \Delta_f t}$, $t \in [0, T]$, where $T = 1/\Delta_f$ is the symbol duration, Δ_f is the subcarrier spacing and s_n is the modulated information onto the nth subcarrier [5]. Substituting into (1) the received signal vector becomes

$$\boldsymbol{x}^{(l)}(t) = \sum_{q=1}^{Q} \boldsymbol{\alpha}(\varphi_q) \gamma_q^{(l)} \sum_{n=1}^{N} s_n e^{-\mathrm{j}\pi n \Delta_f (t-\tau_q)} + \boldsymbol{n}^{(l)}(t). \quad (2)$$

Following the approach in [2], [3], we sample the received signal at rate N times the symbol rate $(1/T)$, i.e., at time instants kT/N, obtaining

$$\boldsymbol{x}_k^{(l)} = \sum_{q=1}^{Q} \boldsymbol{\alpha}(\varphi_q) \gamma_q^{(l)} \sum_{n=1}^{N} b_n e^{-\mathrm{j}\pi n (k/N - \Delta_f \tau_q)} + \boldsymbol{n}_k^{(l)}. \quad (3)$$

We then collect N samples and apply N-point discrete Fourier transformation, resulting in

$$\mathbf{x}_n^{(l)} = \sum_{n=0}^{N-1} \boldsymbol{x}_k^{(l)} e^{-\mathrm{j}2\pi n \frac{k}{N}} = s_n \sum_{q=1}^{Q} \boldsymbol{\alpha}(\varphi_q) \gamma_q^{(l)} e^{-\mathrm{j}\pi n \Delta_f \tau_q} + \mathbf{n}_n^{(l)}. \quad (4)$$

Knowledge of s_n can be assumed from training symbols to recover a noisy channel measurement or CSI $\mathbf{h}_n^{(l)}$ from the receive data as

$$\mathbf{h}_n^{(l)} = \frac{s_n^*}{|s_n|^2} \mathbf{x}_n^{(l)} = \sum_{q=1}^{Q} \boldsymbol{\alpha}(\varphi_q) \gamma_q^{(l)} e^{-\mathrm{j}\pi n \Delta_f \tau_q} + \mathbf{w}_n^{(l)} \ \in \mathbb{C}^{M\times 1}. \quad (5)$$

The channel measurement noise at subcarrier n is expressed by $\mathbf{w}_n^{(l)}$, $n \in \{1, \dots, N\}$. It will be convenient to stack $\mathbf{w}_n^{(l)}, n \in \{1, \dots, N\}$ of all subcarriers in the tall vector

$$\mathbf{w}^{(l)} = \mathrm{vec}([\mathbf{w}_1^{(l)}, \dots, \mathbf{w}_N^{(l)}]) \quad \in \mathbb{C}^{MN\times 1} \quad (6)$$

as well as to collect the multipath coefficients in the vector $\boldsymbol{\gamma}^{(l)} = [\gamma_1^{(l)}, \dots, \gamma_Q^{(l)}]^{\mathrm{T}} \in \mathbb{C}^{Q\times 1}$. Then we can write the total channel measurement vector $\mathbf{h}^{(l)}$ in a compact form for all N frequencies, M antennas and Q paths as

$$\begin{aligned} \mathbf{h}^{(l)} &= \mathbf{U}(\varphi, \tau) \boldsymbol{\gamma}^{(l)} + \mathbf{w}^{(l)} \\ &= \sum_{q=1}^{Q} \mathbf{u}(\varphi_q, \tau_q) \gamma^{(l)} + \mathbf{w}^{(l)} \quad \in \mathbb{C}^{MN\times 1}, \end{aligned} \quad (7)$$

where $\mathbf{U}(\varphi, \tau) = [\mathbf{u}(\varphi_1, \tau_1), \dots, \mathbf{u}(\varphi_Q, \tau_Q)] \ \in \mathbb{C}^{MN\times Q}$ is the array-delay matrix (space-time manifold) with columns

$$\mathbf{u}(\varphi_q, \tau_q) = [\boldsymbol{\alpha}(\varphi_q) \otimes \mathbf{d}(\tau_q)] \ \in \mathbb{C}^{MN\times 1} \quad (8)$$

and is assumed full column rank [2]. For the case of a ULA with inter-antenna spacing d, the array response vector

$$\boldsymbol{\alpha}(\varphi_q) = [\alpha_1(\varphi_q), \dots, \alpha_M(\varphi_q)]^{\mathrm{T}} \in \mathbb{C}^{M\times 1} \quad (9)$$

has scalar entries [11]

$$\alpha_m(\varphi_q) = e^{-\mathrm{j}2\pi(m-1)\frac{d}{\lambda}\sin\varphi_q}, \tag{10}$$

where λ is the wavelength. This expresses the phase shift induced on the received signal along path q due to the additional distance $(m-1)d\,\sin\varphi_q$ the signal needs to travel to reach the mth antenna compared to the 1st [11]. Assuming the subcarriers are uniformly-spaced at each Δ_f in (7), the delay vector $\mathbf{d}(\tau_q)$ is expressed as

$$\mathbf{d}(\tau_q) = [d_1(\tau_q), \ldots, d_N(\tau_q)]^{\mathrm{T}} \in \mathbb{C}^{N\times 1} \tag{11}$$

and has scalar entries given by

$$d_n(\tau_q) = e^{-\mathrm{j}2\pi\Delta_f(n-1)\tau_q}. \tag{12}$$

It is noted that due to the spatial dimension, i.e., due to the existence of physically displaced antenna elements, the angle information φ_q manifests itself in the channel model (Eq. (7)-(10)). On the other hand, thanks to the frequency dimension, i.e., the existence of progressively increasing subcarrier frequencies, the delay information τ_q is present in the channel model (equations (7)-(12)), too.

The objective is to find estimates $\{(\hat{\varphi}_q, \hat{\tau}_q)\}$ of the channel parameters $\{(\varphi_q, \tau_q)\}$, $q \in \{1, .., Q\}$ using channel measurements $\mathbf{h}^{(l)}$. The noise vector $\mathbf{w}^{(l)}$ in (7) is additive Gaussian of zero mean and of covariance $\sigma^2\mathbf{I}_{MN}$ and is assumed to be white over space, subcarrier frequencies, and symbols. We also assume that the noise is independent from the multipath coefficients in $\boldsymbol{\gamma}^{(l)}$. Finally, the number of multipaths Q is assumed to be known and can be computed using techniques for estimating the number of coherent sources [12].

III. Subspace Methods for 2D Angle and Delay Estimation in OFDM Multipath Channels: A Recap

Essential to subspace techniques is the computation of the spatio-frequential channel covariance matrix

$$\mathbf{R} = \mathbb{E}\{\mathbf{h}^{(l)}\mathbf{h}^{(l)\mathrm{H}}\} = \mathbf{U}(\varphi,\tau)\mathbf{R}_\gamma\mathbf{U}(\varphi,\tau)^{\mathrm{H}} + \sigma^2\mathbf{I}_{MN} \tag{13}$$

where $\mathbf{R}_\gamma = \mathbb{E}[\boldsymbol{\gamma}^{(l)}\boldsymbol{\gamma}^{(l)\mathrm{H}}] \in \mathbb{C}^{Q\times Q}$ is the multipath covariance. The parameter covariance $\mathbf{U}(\varphi,\tau)\mathbf{R}_\gamma\mathbf{U}(\varphi,\tau)^{\mathrm{H}} = \mathbf{R} - \sigma^2\mathbf{I}_{MN}$ contains the parameters of interest. Moreover, the SNR at the receiver is calculated by

$$\mathrm{SNR} = \mathrm{tr}\left(\mathbf{U}(\varphi,\tau)\mathbf{R}_\gamma\mathbf{U}(\varphi,\tau)^{\mathrm{H}}\right)/\sigma^2. \tag{14}$$

Given a certain number of snapshots L, (13) is approximated by the sample channel covariance matrix

$$\widetilde{\mathbf{R}} = \frac{1}{L}\left(\sum_{l=1}^{L}\mathbf{h}^{(l)}\mathbf{h}^{(l)\mathrm{H}}\right) \in \mathbb{C}^{MN\times MN}. \tag{15}$$

Subspaces techniques like MUSIC and ESPRIT operate on the eigendecomposition of the channel covariance to form distinct parameter and noise subspaces. However their efficiency is dramatically reduced when $\mathbf{R}_\gamma$ is not full rank (i.e., $\mathbf{R}_\gamma < Q$) which is the case when multiple signals are coherent (e.g., in multipath propagation) or when the number of channel measurements is less then the number of multipaths ($L < Q$),

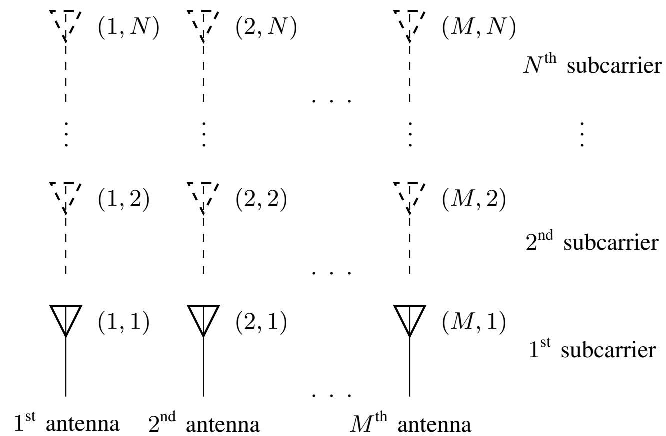

Fig. 1. A spatio-frequential array with M antennas and N subcarriers.

[2], [8]. Similarly, a practical assumption is that changes of the multipath OFDM channel parameters are slow enough so that in the observation interval $\{1, \ldots, L\}$ the state of channel remains quasistable (the number and values of angles and delays as well as of complex amplitudes of impinging paths are constant) [6]. In case of the model (7) with quasistable coefficients $\boldsymbol{\gamma}^{(l)} = \boldsymbol{\gamma}$ the matrix $\mathbf{R}_\gamma$ appears of rank one. Generally, the remedy for the rank-deficiency of $\mathbf{R}_\gamma$ is a preprocessing called smoothing [2], [6]–[8] which aims at "decorrelating" multipath components. A similar smoothing technique was utilized in [13] without being thoroughly analyzed.

A. Spatio-frequential smoothing

For the case of 2D angle-delay estimation a countermeasure to the problem of rank-deficiency of $\mathbf{R}_\gamma$ is the spatio-frequential smoothing which is the extension of spatial smoothing [8] to both space and (subcarrier) frequency dimensions. This is analytically proved in [7] and briefly described here.

The method considers the initial spatio-frequential array $\{(m,n)\}_{m=1,\ldots,M}^{n=1,\ldots N}$ of size MN formed jointly over space (antennas) and subcarrier frequencies as in Fig. 1. The array is divided into $K_{\mathrm{s}} \geq Q$ smaller overlapping spatio-frequential subarrays (Fig. 1 in [7]) over which channel covariance matrices can be computed. Each subarray has size $M_{\mathrm{s}}N_{\mathrm{s}}$ with M_{s} antennas and N_{s} subcarriers, where $M_{\mathrm{s}} < M$ and $N_{\mathrm{s}} < N$, so that $K_s = (M - M_{\mathrm{s}} + 1)(N - N_{\mathrm{s}} + 1)$. The channel equation (7) becomes

$$\mathbf{h}_{m,n}^{(l)} = \bar{\mathbf{U}}(\varphi,\tau)\mathbf{D}_\tau^{n-1}\mathbf{D}_\varphi^{m-1}\boldsymbol{\gamma}^{(l)} + \mathbf{w}_{m,n}^{(l)} \in \mathbb{C}^{M_{\mathrm{s}}N_{\mathrm{s}}\times 1}. \tag{16}$$

In equation (16), we have the matrices $\mathbf{D}_\varphi = \mathrm{diag}\{a_2(\varphi_1), \ldots, a_2(\varphi_Q)\}$, $\mathbf{D}_\tau = \mathrm{diag}\{d_2(\tau_1), \ldots, d_2(\tau_Q)\}$, and the matrix $\bar{\mathbf{U}}(\varphi,\tau) \in \mathbb{C}^{M_{\mathrm{s}}N_{\mathrm{s}}\times Q}$ is as in (8) with qth column $\bar{\mathbf{u}}(\varphi_q,\tau_q)$ but with vectors $\boldsymbol{\alpha}(\varphi_q)$ and $\mathbf{d}(\tau_q)$ of reduced sizes M_{s} and N_{s}, respectively [7]. Let $\mathbf{R}_{m,n} = \mathbb{E}\{\mathbf{h}_{m,n}^{(l)}\mathbf{h}_{m,n}^{(l)\mathrm{H}}\}$ denote the covariance of $\mathbf{h}_{m,n}^{(l)}$. Then, the smoothed channel covariance has similar structure to (13) and is given by the average [7]

$$\mathbf{R}_{\mathrm{s}} = \frac{1}{K_{\mathrm{s}}} \sum_{n=1}^{N-N_{\mathrm{s}}+1} \sum_{m=1}^{M-M_{\mathrm{s}}+1} \mathbf{R}_{m,n} \ \in \mathbb{C}^{M_{\mathrm{s}}N_{\mathrm{s}} \times M_{\mathrm{s}}N_{\mathrm{s}}} \quad (17)$$
$$= \bar{\mathbf{U}}(\varphi,\tau)\bar{\mathbf{R}}_{\gamma}\bar{\mathbf{U}}^{\mathrm{H}}(\varphi,\tau) + \sigma^2 \mathbf{I}_{M_{\mathrm{s}}N_{\mathrm{s}}}, \quad (18)$$

where

$$\bar{\mathbf{R}}_{\gamma} = \frac{1}{K_{\mathrm{s}}} \sum_{n=1}^{N-N_{\mathrm{s}}+1} \sum_{m=1}^{M-M_{\mathrm{s}}+1} \mathbf{D}_{\tau}^{n-1}\mathbf{D}_{\varphi}^{m-1}\mathbf{R}\mathbf{D}_{\tau}^{\mathrm{H}^{n-1}}\mathbf{D}_{\varphi}^{\mathrm{H}^{m-1}}. \quad (19)$$

The work in [7] proved that $\bar{\mathbf{R}}_{\gamma}$ attains full rank and thus subspace methods can be applied to the smoothed channel covariance matrix of reduced dimensions.

In practice, the sample covariance matrix of $\mathbf{h}_{m,n}^{(l)}$ is

$$\widetilde{\mathbf{R}}_{m,n} = \frac{1}{L}\left(\sum_{l=1}^{L} \mathbf{h}_{m,n}^{(l)}\mathbf{h}_{m,n}^{(l)\mathrm{H}}\right) \ \in \mathbb{C}^{M_sN_s \times M_sN_s} \quad (20)$$

and, hence, the sample smoothed covariance $\widetilde{\mathbf{R}}_{\mathrm{s}}$ is computed similarly to (17) using $\widetilde{\mathbf{R}}_{m,n}$.

B. 2D MUSIC and ESPRIT Estimation Methods

Subspace methods start with the eigendecomposition of the (smoothed) channel covariance $\widetilde{\mathbf{R}}_{\mathrm{s}}$ whose eigenvalues are $\lambda_1 > \lambda_2 > \cdots > \lambda_{M_{\mathrm{s}}N_{\mathrm{s}}}$ and the corresponding eigenvectors are $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{M_{\mathrm{s}}N_{\mathrm{s}}}$. The 2D MUSIC [3] for angle and delay of arrival estimation requires to form the noise subspace matrix $\mathbf{V}_{\mathrm{n}} = [\mathbf{v}_{Q+1}, \ldots \mathbf{v}_{M_{\mathrm{s}}N_{\mathrm{s}}}] \in \mathbb{C}^{M_sN_s \times (M_sN_s - Q)}$. Then, a 2D search in order to estimate the angle and delay parameters follows as

$$\{(\hat{\varphi}_q, \hat{\tau}_q)\}_{q=1}^{Q} = \arg\max_{\varphi,\tau}(||\mathbf{V}_{\mathrm{n}}^{\mathrm{H}}[\boldsymbol{\alpha}(\varphi) \otimes \mathbf{d}(\tau)]||_{\mathrm{F}}^2)^{-1}. \quad (21)$$

It is noted that the search is performed across a 2D grid for the (φ, τ) parameters, where $\varphi \in [-\pi/2, \pi/2]$, $\tau \in [0, \mathcal{T}]$ and $\mathcal{T}$ depends on the maximum path delay.

The 2D ESPRIT algorithm was introduced as a computationally attractive estimation algorithm that exploits the rotational invariance of the parameter subspace $\mathbf{V} = [\mathbf{v}_1 \ldots, \mathbf{v}_Q] \in \mathbb{C}^{M_sN_s \times Q}$ available with a spatio-frequential array comprised of a ULA and equidistant subcarrier frequencies [2]. The parameters are estimated directly from

$$\{\hat{\varphi}_q\}_{q=1}^{Q} = \{\cos^{-1}[\arg(\lambda_q(\mathbf{\Phi}))c/(-2\pi f d)]\}_{q=1}^{Q}$$
$$\{\hat{\tau}_q\}_{q=1}^{Q} = \{\arg(\lambda_q(\mathbf{T}))/(-2\pi\Delta_f)\}_{q=1}^{Q}, \quad (22)$$

where $\lambda_q(\mathbf{\Phi})$ and $\lambda_q(\mathbf{T})$ denotes the qth eigenvalue of the matrices $\mathbf{\Phi}$ and $\mathbf{T}$. These are calculated by solving the system of equations $\mathbf{J}_{\varphi}\mathbf{V}\mathbf{\Phi} = \mathbf{J}'_{\varphi}\mathbf{V}$ and $\mathbf{J}_{\tau}\mathbf{V}\mathbf{T} = \mathbf{J}'_{\tau}\mathbf{V}$, respectively, where $\mathbf{J}_{\varphi} = \mathbf{I}_{N_{\mathrm{s}}} \otimes [\mathbf{I}_{M_{\mathrm{s}}-1} \ \mathbf{0}]$, $\mathbf{J}'_{\varphi} = \mathbf{I}_{N_{\mathrm{s}}} \otimes [\mathbf{0} \ \mathbf{I}_{M_{\mathrm{s}}-1}]$, $\mathbf{J}_{\tau} = [\mathbf{I}_{N_{\mathrm{s}}-1} \ \mathbf{0}] \otimes \mathbf{I}_{M_{\mathrm{s}}}$ and $\mathbf{J}'_{\tau} = [\mathbf{0} \ \mathbf{I}_{N_{\mathrm{s}}-1}] \otimes \mathbf{I}_{M_{\mathrm{s}}}$.

IV. Correlation Vectors Method for 2D Angle and Delay Estimation in OFDM Multipath Channels

The recent work in [9] found that for a given array geometry the parameter covariance can be described by a subspace defined through all possible correlation vectors of the received signals. This so-called *correlation vectors subspace* is computed in [9] through a numerical approximation and is utilized to denoise the sample covariance matrix. Then the angles of arrival are estimated from the denoised covariance matrix using estimators such as the MUSIC or ESPRIT algorithm. [10] further studies this new technique providing analytical, closed-form expressions for the basis of the correlations vectors subspace. In this section, we focus on the 2D angles and delays of arrival estimation problem in multipath OFDM channels, and based on suggestions given in [10] write the analytical expressions of the correlation vectors subspace for the system model at hand. It is noted that the correlation vectors method assumes full rank parameter covariance, this is why we apply the technique on the smoothed covariance matrix of Section III.A.

First, we define the normalized angle of arrival of source q as $\phi_q = (d/\lambda)\sin\varphi_q, \in [-0.5, 0.5]$. Similarly, it is possible to find Δ_f so that the delay $\tau_q = \Delta_f \tau_q$ is normalized in the interval $[-0.5, 0.5]$. Then we can rewrite our system model with the normalized parameters, since (10) and (12) simply become $\alpha_m(\phi_q) = e^{-\mathrm{j}2\pi(m-1)\phi_q}$ and $d_n(\tau_q) = e^{-\mathrm{j}2\pi(n-1)\tau_q}$, respectively. Moreover, under the assumption that $\bar{\mathbf{R}}_{\gamma} = \mathrm{diag}(|\bar{\gamma}_1|^2, \ldots, |\bar{\gamma}_Q|^2)$, we can rewrite the model in (18) as

$$\mathbf{R}_{\mathrm{s}} = \sum_{q=1}^{Q} |\bar{\gamma}_q|^2 \bar{\mathbf{u}}(\phi_q, \tau_q)\bar{\mathbf{u}}^{\mathrm{H}}(\phi_q, \tau_q) + \sigma^2 \mathbf{I}_{M_{\mathrm{s}}N_{\mathrm{s}}}, \quad (23)$$

where $|\bar{\gamma}_q|$ expresses the amplitude of the qth complex attenuation. It is noted that for high σ^2 (low SNR), the error in approximating (18) as in (23) due to not fulfilling the assumption $\bar{\mathbf{R}}_{\gamma} = \mathrm{diag}(|\bar{\gamma}_1|^2, \ldots, |\bar{\gamma}_Q|^2)$, is decreased. By rearranging the elements in (23) we have

$$\mathrm{vec}(\mathbf{R}_{\mathrm{s}} - \sigma^2 \mathbf{I}_{M_{\mathrm{s}}N_{\mathrm{s}}}) = \sum_{q=1}^{Q} |\bar{\gamma}_q|^2 \bar{\mathbf{u}}(\phi_q, \tau_q)\bar{\mathbf{u}}^{\mathrm{H}}(\phi_q, \tau_q). \quad (24)$$

The correlation vectors are defined as

$$\mathbf{c}(\phi_q, \tau_q) := \mathrm{vec}(\bar{\mathbf{u}}(\phi_q, \tau_q)\bar{\mathbf{u}}^{\mathrm{H}}(\phi_q, \tau_q)) \in \mathbb{C}^{(M_{\mathrm{s}}N_{\mathrm{s}})^2 \times 1}. \quad (25)$$

We recall the following definition from [9], [10] adopted to our 2D angle-delay estimation context as:

Definition 1: The subspace $\mathcal{CVS}$ defined as

$$\mathcal{CVS} := \mathrm{span}\{\mathbf{c}(\phi_q, \tau_q) | -0.5 \le \phi_q \le 0.5, -0.5 \le \tau_q \le 0.5\}$$

is called the *correlation vectors subspace*. The matrix of (column) basis vectors of $\mathcal{CVS}$ is denoted by $\mathbf{Q}_{\mathcal{CVS}}$, i.e., $\mathcal{CVS} = \mathrm{col}(\mathbf{Q}_{\mathcal{CVS}})$. Equation (24) implies that

$$\mathrm{vec}(\mathbf{R}_{\mathrm{s}} - \sigma^2 \mathbf{I}_{M_{\mathrm{s}}N_{\mathrm{s}}}) \subseteq \mathcal{CVS}, \quad (26)$$

i.e., that the quantity $\mathrm{vec}(\mathbf{R}_\mathrm{s} - \sigma^2 \mathbf{I}_{M_\mathrm{s} N_\mathrm{s}})$, which is equal to the parameter covariance, is constrained by $\mathcal{CVS}$. Moreover, in [9] it was found that

$$\mathcal{CVS} = \mathrm{col}(\mathbf{S}), \tag{27}$$

i.e., $\mathbf{Q}_{\mathcal{CVS}}$ is calculated from the eigenvectors of $\mathbf{S}$ associated with the non-zero eigenvalues, where

$$\mathbf{S} = \int_{-\pi/2}^{\pi/2} \int_{0}^{\mathcal{T}} \mathbf{c}(\phi_q, \tau_q) \mathbf{c}^\mathrm{H}(\phi_q, \tau_q) d\varphi d\tau \in \mathbb{C}^{(M_\mathrm{s} N_\mathrm{s})^2 \times (M_\mathrm{s} N_\mathrm{s})^2} \tag{28}$$

Then, it was proposed to denoise the sample covariance matrix by the following convex optimization procedure

$$\begin{aligned} \widetilde{\mathbf{R}}_{\mathcal{CVS}} = \arg\min_{\mathbf{X}} \quad & \left\| \widetilde{\mathbf{R}}_\mathrm{s} - \sigma^2 \mathbf{I}_{M_\mathrm{s} N_\mathrm{s}} - \mathbf{X} \right\|_\mathrm{F}^2 \\ \text{subject to } & \left(\mathbf{I}_{(M_\mathrm{s} N_\mathrm{s})^2} - \mathbf{Q}_{\mathcal{CVS}} \mathbf{Q}_{\mathcal{CVS}}^\dagger \right) \mathrm{vec}(\mathbf{X}) = 0, \\ & \mathbf{X} \succeq 0. \end{aligned} \tag{29}$$

Subsequently, conventional estimation algorithms like 2D MUSIC and 2D ESPRIT can be applied to the denoised covariance $\widetilde{\mathbf{R}}_{\mathcal{CVS}}$ to estimate the parameters if interest[1].

Instead of solving the convex optimization problem (29), it was suggested in [9] to solve the two-step approximation

$$\begin{aligned} \widetilde{\mathbf{R}}_{\mathcal{CVS}}^1 = \arg\min_{\mathbf{X}} \quad & \left\| \widetilde{\mathbf{R}}_\mathrm{s} - \sigma^2 \mathbf{I}_{M_\mathrm{s} N_\mathrm{s}} - \mathbf{X} \right\|_\mathrm{F}^2 \\ \text{subject to } & \left(\mathbf{I}_{(M_\mathrm{s} N_\mathrm{s})^2} - \mathbf{Q}_{\mathcal{CVS}} \mathbf{Q}_{\mathcal{CVS}}^\dagger \right) \mathrm{vec}(\mathbf{X}) = 0, \end{aligned} \tag{30}$$

$$\begin{aligned} \widetilde{\mathbf{R}}_{\mathcal{CVS}} = \arg\min_{\mathbf{X}} \quad & \left\| \widetilde{\mathbf{R}}_{\mathcal{CVS}}^1 - \mathbf{X} \right\|_\mathrm{F}^2 \\ \text{subject to } & \mathbf{X} \succeq 0. \end{aligned} \tag{31}$$

Note, that the solution to the problem (30) can be solved by an orthogonal projection as follows (we refer to [9] for details)

$$\mathrm{vec}(\widetilde{\mathbf{R}}_{\mathcal{CVS}}^1) = \mathbf{Q}_{\mathcal{CVS}} \mathbf{Q}_{\mathcal{CVS}}^\dagger \, \mathrm{vec}(\widetilde{\mathbf{R}}_\mathrm{s} - \sigma^2 \mathbf{I}_{M_\mathrm{s} N_\mathrm{s}}) \tag{32}$$

and then $\widetilde{\mathbf{R}}_{\mathcal{CVS}}$ is obtained from the eigendecomposition of $\widetilde{\mathbf{R}}_{\mathcal{CVS}}^1$.

To calculate the correlation subspace, the matrix $\mathbf{S}$ in (28) is approximated by numerical integration via choosing a certain grid for the parameters (φ, τ). This introduces in turn perturbations on zero eigenvalues of $\mathbf{S}$, thus the subspace spanned by eigenvectors of $\mathbf{S}$ corresponding to non-zero eigenvalues and hence the matrix $\mathbf{Q}_{\mathcal{CVS}}$ cannot be estimated precisely. To overcome this, [10] derived analytical formulas for the correlation vectors subspace $\mathcal{CVS}$ using the theory of difference coarrays for a variety of antenna configurations. For the reader's convenience we recall the fundamental definitions.

Assume the set of locations $\{1, 2\}$ for $M_\mathrm{s} = 2$ antennas and the set $\{1, 2\}$ for the indices of $N_\mathrm{s} = 2$ subcarriers. Let $\mathbb{S}$ be the 2D set of M_s antenna and N_s subcarrier indices for a spatio-frequential array, where $|\mathbb{S}| = (M_\mathrm{s} N_\mathrm{s})^2$. E.g., for the 2 antennas and the 2 subcarriers we denote

$$\mathbb{S} = \{(1, 1), (2, 1), (1, 2), (2, 2)\}. \tag{33}$$

This example configuration is similar to Fig. 1 but with $M = 2$ and $N = 2$.

Definition 2: The difference coarray $\mathbb{D}$ contains the differences between the elements in $\mathbb{S}$, i.e., $\mathbb{D} := \{n_1 - n_2 : \forall n_1, n_2 \in \mathbb{S}\}$.
For the example in (33) we get

$$\begin{aligned} \mathbb{D} = \{ & (-1, -1), (0, -1), (1, -1), (-1, 0), \\ & (0, 0), (1, 0), (-1, 1), (0, 1), (1, 1)\} \end{aligned} \tag{34}$$

Definition 3: The binary matrix $\mathbf{J}$ has size $|\mathbb{S}|^2 \times |\mathbb{D}|$. Its columns satisfy $\langle \mathbf{J} \rangle_{:}, m = \mathrm{vec}(\mathbf{I}(m))$ for $m \in \mathbb{D}$, where the matrix $\mathbf{I}(m)$ is given by

$$\langle \mathbf{I} \rangle_{n_1, n_2} := \begin{cases} 1, & \text{if } n_1 - n_2 = m, \\ 0, & \text{otherwise.} \end{cases} \tag{35}$$

For the example above, the matrix $\mathbf{J}$ can be calculated as follows (note, that the 9 columns correspond to the 9 elements of $\mathbb{D}$ and the 16 rows to 4^2 combination of sensors according to $|\mathbb{S}| = 4$):

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{36}$$

The importance of $\mathbf{J}$ stems from then fact that it allows for an analytical calculation of the subspace $\mathcal{CVS}$, specifically,

$$\mathbf{S} = \mathbf{J} \cdot \mathbf{J}^\mathrm{H} \tag{37}$$

and we recall the following Theorem from [10]:

Theorem 1: $\mathcal{CVS} = \mathrm{col}(\mathbf{J})$.
Moreover [10] suggested a modified version to the optimization problem in (29), where knowledge of the noise variance is not required

$$\begin{aligned} \widetilde{\mathbf{R}}_{\mathcal{CVS}} = \arg\min_{\mathbf{X}} \quad & \left\| \widetilde{\mathbf{R}}_\mathrm{s} - \mathbf{X} \right\|_\mathrm{F}^2 \\ \text{subject to } & \left(\mathbf{I}_{(M_\mathrm{s} N_\mathrm{s})^2} - \mathbf{Q}_{\mathcal{CVS}} \mathbf{Q}_{\mathcal{CVS}}^\dagger \right) \mathrm{vec}(\mathbf{X}) = 0. \end{aligned} \tag{38}$$

The problem in (38) has analytical solution

$$\mathrm{vec}(\widetilde{\mathbf{R}}_{\mathcal{CVS}}) = \mathbf{Q}_{\mathcal{CVS}} \mathbf{Q}_{\mathcal{CVS}}^\dagger \mathrm{vec}(\widetilde{\mathbf{R}}_\mathrm{s}). \tag{39}$$

[1] Note, that in [9] the spectral norm in the optimization procedure was proposed. However, we have found the Frobenius norm to work better in our simulations. We remark, that the arguments in [9] and [10] do not depend on the actual norm chosen.

Correlation vectors subspace methods	
Method	Description
CVS1-numerical	$\mathbf{S}$ is computed (numerically) from (28) and $\widetilde{\mathbf{R}}_{\mathcal{CVS}}$ is calculated using the approximation optimization in (30)-(32)
CVS2-analytical	$\mathbf{S}$ is computed analytically from $\mathbf{J}$ and $\widetilde{\mathbf{R}}_{\mathcal{CVS}}$ is calculated using the optimization problem in (29)
CVS3-analytical	$\mathbf{S}$ is computed analyically from $\mathbf{J}$ and $\widetilde{\mathbf{R}}_{\mathcal{CVS}}$ is calculated using the optimization problem in (38)-(39)

TABLE I
THE CORRELATION VECTORS SUBSPACE METHODS CONSIDERED IN THE COMPUTER SIMULATIONS FOR 2D ANGLE-DELAY ESTIMATION

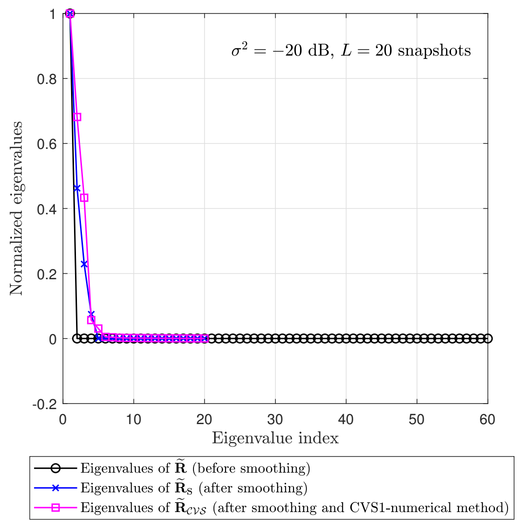

Fig. 2. Eigenvalues of the sample covariance matrix and of the estimated parameter covariance matrix.

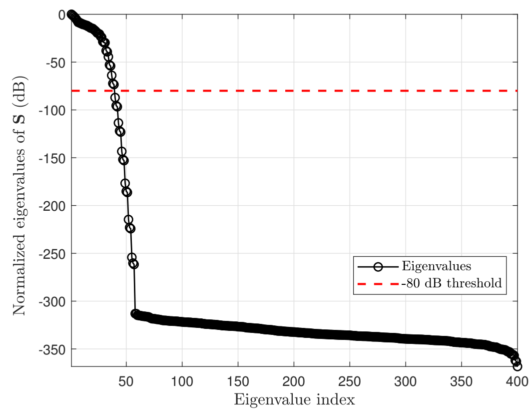

Fig. 3. The normalized eigenvalues of $\mathbf{S}$ calculated via numericall integration using (28). There are 39 significant eigenvalues, i.e., higher than the -80 dB threshold.

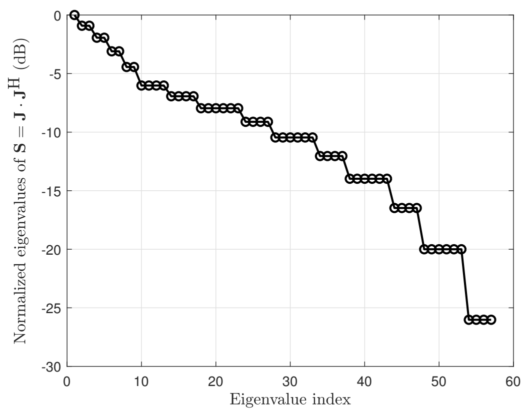

Fig. 4. The $|\mathbb{D}| = 57$ (non-zero) normalized eigenvalues of $\mathbf{S}$ calculated analytically from (37) in dB.

V. PERFORMANCE EVALUATION

In this section we present the performance evaluation framework that assesses the capability of various subspace methods to detect the angles and delays of arrival parameters in the multipath OFDM scenario. To estimate the parameters of interest, we follow the steps below:

1) We collect the channel measurement data $\mathbf{h}^{(l)}$, $l = \{1, \ldots, L\}$, assuming certain N and M, specific ground truth angles and delays of arrival $\{(\varphi_q, \tau_q)\}$, $q \in \{1, .., Q\}$, certain noise conditions and that the number of multipaths Q is known.
2) We apply the spatio-frequential smoothing preprocessing described in Section III.A with M_s and N_s to satisfy $(M - M_s + 1)(N - N_s + 1) \geq Q$. In this way, we obtain the smoothed covariance matrix $\widetilde{\mathbf{R}}_s$. Subspace-based parameter estimation follows.
3) In case the correlation vectors subspace methods is used (Section IV), then first we calculate the denoised covariance $\widetilde{\mathbf{R}}_{\mathcal{CVS}}$.
4) We estimate the parameters $\{(\hat{\varphi}_q, \hat{\tau}_q)\}$, $q \in \{1, .., Q\}$ using the MUSIC or the ESPRIT estimation methods using $\widetilde{\mathbf{R}}_s$ (if step 3 was omitted) or $\widetilde{\mathbf{R}}_{\mathcal{CVS}}$ (if the correlation vectors subspace method was used).
5) We assess the estimation performance of the proposed methods using the RMSE metric for various conditions.

Regarding step 3 above, we consider the variants of the correlations subspace methods described in Table I. To solve the optimization problems of the methods CVS1-numerical and CVS3-analytical, the equations (32) and (39), respectively, can be efficiently utilized. However, in order to solve the optimization of the method CVS2-analytical, we will utilize the `cvx` package [18].

A. Eigenvalues

Our simulation results have been done with $M = 3$ antennas and $N = 20$ subcarriers. We have fixed $Q = 4$ paths, where their corresponding angles and delays of arrival are $(\varphi_1, \tau_1) = (15°, 40$ nsec$)$, $(\varphi_2, \tau_2) = (60°, 100$ nsec$)$, $(\varphi_3, \tau_3) = (-20°, 150$ nsec$)$, $(\varphi_4, \tau_4) = (50°, 200$ nsec$)$. The complex attenuation vector on every simulation run $\boldsymbol{\gamma}$ is set to a constant arbitrary value. Spatio-frequential smoothing

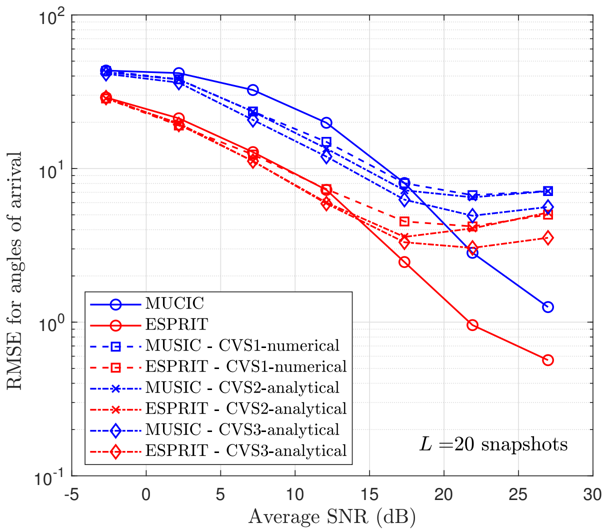

Fig. 5. RMSE of the angles of arrival versus average SNR. There are $Q = 4$ multipath components (correlated sources) and subspace-based methods are performed on $L = 20$ channel measurements.

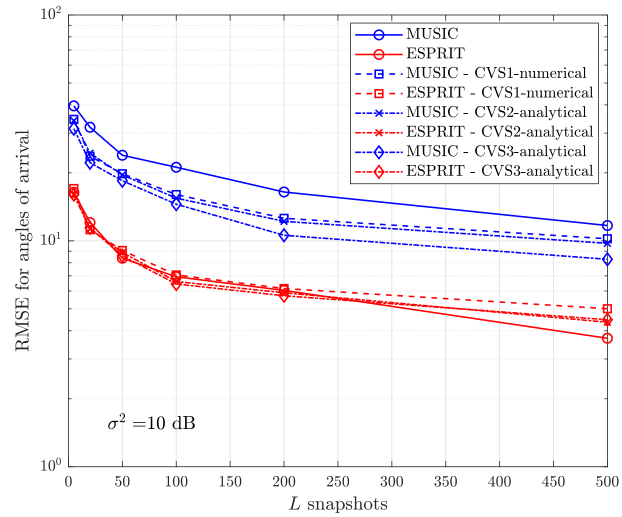

Fig. 7. MSE of the angles of arrival with respect to number of snapshots. There are $Q = 4$ multipath components (correlated sources) and the noise variance is equal to 10 dB.

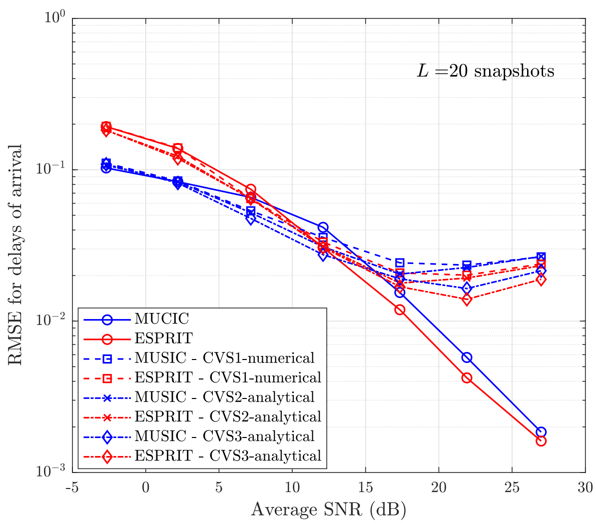

Fig. 6. RMSE of the delays of arrival with respect to average SNR. There are $Q = 4$ multipath components (correlated sources) and subspace-based methods are performed on $L = 20$ channel measurements.

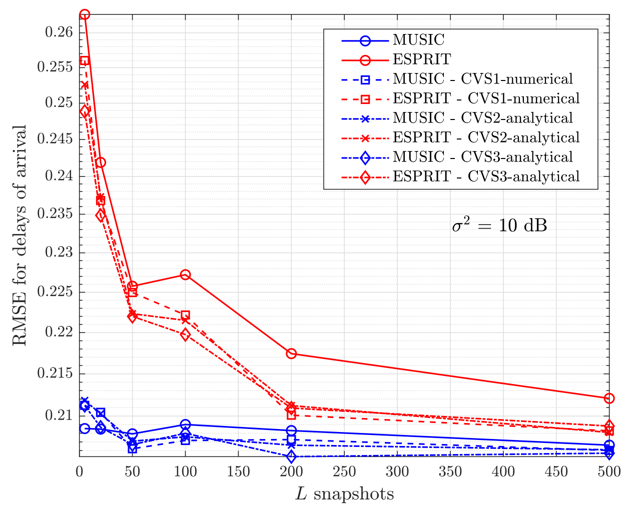

Fig. 8. RMSE of the delays of arrival with respect to number of snapshots. There are $Q = 4$ multipath components (correlated sources) and the noise variance is equal to 10 dB.

is applied by setting $M_s = 2$ and $N_s = 10$, i.e., there are $K_s = (M - M_s + 1)(N - N_s + 1) = 22$ spatio-frequential subarrays to smooth over, enough for distinguishing 4 paths.

For $\sigma^2 = -20$ dB and $L = 20$ snapshots, Fig. 2 shows the eigenvalues of the sample covariance matrix $\widetilde{\mathbf{R}}$, the smoothed covariance matrix $\widetilde{\mathbf{R}}_s$ and of the estimated parameter covariance $\widetilde{\mathbf{R}}_{\mathcal{CVS}}$ using the method CVS1-numerical of Table I, normalized to the interval $[0, 1]$. It is observed that due to the rank-one condition of $\mathbf{R}_\gamma$, only one out of the $MN = 60$ eigenvalues of $\widetilde{\mathbf{R}}$ is significantly greater than zero. In other words, the parameter space is projected in the noise space due to the rank deficiency of the parameter covariance $\mathbf{R}_\gamma$. With spatio-frequential smoothing however, $\widetilde{\mathbf{R}}_s$ becomes full rank and out of the $M_s N_s = 20$ eigenvalues now at least 4 eigenvalues appear to be clearly greater than zero, and thus the parameter subspace can now be resolved more accurately. This situation is preserved even after applying the correlations vectors subspaces method, and especially CVS1-numerical from Table I.

Under the same conditions, Fig. 3 and Fig. 4 show the eigenvalues of $\mathbf{S}$ computed either numerically from (28) or analytically from (37), respectively. The determination of the number of non-zero eigenvalues of $\mathbf{S}$ is needed in order to find the matrix $\mathbf{Q}_{\mathcal{CVS}}$ describing the correlation vectors subspace. In order to compute the number of non-zero eigenvalues of $\mathbf{S}$ from (28), this is done by counting the eigenvalues higher than a threshold (e.g., -80 dB in [9]) to find them equal to

39 for the considered spatio-frequential array. On the other hand, the number of non-zero eigenvalues of $\mathbf{S}$ from (37) can be computed directly as the size of the difference coarray $|\mathbb{D}| = 57$ without the need of a subjective threshold. It is noted that the matrix $\mathbf{S}$ and, thus, $\mathbf{Q}_{\mathcal{CVS}}$ need to be computed only once for a certain spatio-frequential array topology.

B. Estimation Error Performance

In the following, 200 simulation runs where executed. The estimation performance in terms of root mean square error (RMSE) is as follows

$$\mathrm{RMSE} = \sqrt{\frac{1}{Q}\sum_{q=1}^{Q}(\hat{p}_q - p_q)^2}, \qquad (40)$$

where $p = \varphi$ for the angles and $p = \tau$ for the delays of arrival, respectively, and $\hat{p}$ denotes the estimated parameter.

Fig. 5 and Fig. 6 show the RMSE for the angles and the delays of arrival, respectively, depending on the SNR for a fixed set of 20 snapshots. The result is compared for the three subspace algorithms described in Table I as well as for the case where no correlation vectors subspace method was used. As one can see, for low SNR, all correlation vectors subspace methods consistently outperform the case where 2D MUSIC or 2D ESPRIT alone where used. It is also remarkable, that the benefit seems to be higher for 2D MUSIC than 2D ESPRIT. From Fig. 5, achieving an RMSE of 11° requires around 15 dB average SNR when using 2D MUSIC alone, whereas with the CVS3-analytical method this error performance is achieved with an average SNR of 12 dB. It is noted that the 2D MUSIC and the 2D ESPRIT methods alone perform better than when combined with the correlation vectors subspace methods for high SNR, since then the error of approximating (18) with (23) increases, as discussed in Section IV. When comparing the three correlation vectors subspace techniques, the methods computing $\mathbf{S}$ analytically from $\mathbf{J}$ perform better than the numerical one, with the method CVS3-analytical providing the best error performance.

Fig. 7 and Fig. 8 depict the RMSE for the angles and the delays of arrival depending on the number of snapshots with a fixed noise variance for the three subspace algorithms shown in Table I. As one can see in both figures, the correlation vectors subspace algorithms outperform the 2D MUSIC alone for almost all the range of L. Only when utilizing 2D ESPRIT and for a high number of snapshots ($L > 260$), not using the correlations vectors techniques, i.e., the 2D ESPRIT method alone, exhibits better RMSE performance in estimating the angles of arrival (Fig. 7).

C. Algorithmic Complexity and Performance Implication

Although we have not systematically studied performance, it is clear that CVS1-numerical is worst as it requires a numerical calculation. However, the matrix can be precomputed and stored if the device is not too memory-constraint. The main difference then stems from the different optimization schemes used, where CVS3-analytical (39) is suitable for real-time use cases as it does not require a convex optimization.

VI. Conclusion and Outlook

The present paper applies the theory of correlation vectors subspace developed in [9], [10] on the 2D joint angles and delays of arrival estimation scenario under indoor multipath and multi-carrier transmissions. Besides explicitly writing an analytical formula for this use case and combing the algorithm with the existing ones for the case of coherent multipath component signals, our contribution is a thorough comparison of the estimation performance of subspace-based estimators using the correlation vectors subspace method. As the results of the simulations are encouraging, denoting error performance improvement with the correlation vectors subspace method, we intend to mathematically justify applying the method for the coherent signals case as a future work.

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