

Massive MIMO Heterogeneous Networks: Downlink Sum Rate Maximization under Power Control

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Abstract— This paper studies the rate maximization problem of a heterogeneous network composed of a massive multi-input-multi-output macro base station overlaid with small cells. The rate maximization is formulated as an optimization problem that maximizes the total sum rate of all users while maintaining an acceptable quality of service per user and total system power threshold. Numerical results show that the presence of small cells improves the total sum rate compared to relying on the macro base station alone under the same total system power constraint.

Keywords—massive MIMO, Heterogeneous Networks, rate maximization.

I. INTRODUCTION

To meet the dramatic increasing rate demands in the fifth-generation (5G) cellular networks, two major network architecture technologies have been under research in the recent few years, namely, massive multi-input-multi-output (M-MIMO) and small cells (SCs) [1].

In M-MIMO, the base station is equipped with hundreds of antennas that serves tens of users in the same time-frequency resource. The huge number of antennas increases the beam focus leading to almost no intra-cell interference and high spectral efficiency. M-MIMO has increased the spectral efficiency 20 times more than the fourth-generation networks [2]. On the other hand, SCs are micro/femto cells with limited number of antennas to increase the coverage and improve the capacity in highly populated environments [3]. They are of low radio frequency range and low power output.

The presence of both macro base station (MBS) equipped with M-MIMO and SCs in heterogeneous Networks (HetNets) has been studied a lot in recent works [4-10]. According to [4], overlaying the MBS with SCs is the more practical network architecture that can adapt to several types of users' densities and mobilities. Because the M-MIMO antennas can sustain the quality of service (QoS) for high mobility users in large coverage areas while intelligent SCs can enhance coverage and augment overall network capacity in hotspot regions with large number of low mobility users. The authors in [5] studied the rate and coverage analysis of the M-MIMO coexisting with SCs operating in the millimeter wave frequency band. The work in [6] proposed using M-MIMO for the backhaul links in HetNets and studied the sum rate analysis in full duplex mode where both the transmitting and receiving operations occur on the same frequency band.

The coexistence of M-MIMO and SCs makes the downlink beamforming problem more challenging due to the interference introduced between the served users by different tiers in the same

coverage area. The authors in [4] studied the downlink beamforming problem for the M-MIMO MBS where the optimization problem is concerned with signal-to-interference-plus-noise ratio balancing for the case of imperfect channel state information (CSI) at the transmitter. Moreover, the beamforming in [7] was concerned with improving the users' quality of experience expressed as the mean opinion score (MOS) of the users under constraints on the MBS power and the required QoS of the users. On the other hand, the work in [8-9] tackled the problem of energy efficient beamforming of M-MIMO and SCs. The authors in [8] proposed the objective of the beamforming problem as a minimization of the total transmit power under constraints on the QoS of the users. While the work in [9] studied the maximization of a fractional objective of the total sum rate to the total power of the MBS and SCs.

This paper studies the possible improvements in spectral efficiency when a M-MIMO MBS is overlaid with SCs to serve users simultaneously. The main target of the paper is exploring which scenario yields a higher total sum rate; the M-MIMO MBS alone or overlaying it with several SCs under the same power threshold. The objective is to maximize the total sum rate of all users with a total power constraint, a QoS constraint for each user and a power per-antenna constraint for each transmitter (MBS and SCs). Numerical results show that the cooperative operation involving transmission from both the MBS and the SCs provides better rates under the same total power constraint.

Notations: Lower-case and upper-case boldface letters denote vectors and matrices. The $(\cdot)^T$, $(\cdot)^H$, ∇ and $\|\cdot\|$ operators present the transpose, conjugate transpose, gradient and Frobenius norm, respectively. Finally, $\mathcal{CN}(\mathbf{a}, \sigma^2 \mathbf{I}_K)$ denotes the circularly symmetric complex Gaussian distribution with mean \mathbf{a} and covariance matrix $\sigma^2 \mathbf{I}_K$, where σ^2 is the variance and \mathbf{I}_K is identity matrix of size K .

II. SYSTEM MODEL

A single-cell downlink scenario is considered with one MBS. The MBS is equipped with N_{BS} antennas and delivers information to K single-antenna users. Moreover, there are S SCs that overlay the MBS. Each SC is equipped with N_{SC} antennas. N_{BS} is assumed to be much larger than K , which is the case of M-MIMO. The TDD operation will be adopted in this paper, as it is based on the concept of channel reciprocity. In the downlink transmission, by using the estimated CSI from the uplink, the transmitted signals will then be precoded to avoid intracell-interference between the served users. The precoding stage can be facilitated by the presence of a centralized-radio access network (C-RAN) that enables swift exchange of CSI between the baseband processing units (BBUs)

of the MBS and the SCs co-located in centralized RAN model [10]. The received signal at user k can be described [12] as;

$$y_k = \mathbf{h}_{0,k} \mathbf{x}_0 + \sum_{i=1}^S \mathbf{h}_{i,k} \mathbf{x}_i + n_k \quad (1)$$

where $\mathbf{x}_0 \in \mathbb{C}^{N_{BS} \times 1}$, $\mathbf{x}_i \in \mathbb{C}^{N_{SC} \times 1}$ are the transmit signals and $\mathbf{h}_{0,k} \in \mathbb{C}^{1 \times N_{BS}}$, $\mathbf{h}_{i,k} \in \mathbb{C}^{1 \times N_{SC}}$ are the random channel vectors from the MBS and the i^{th} SC to user k , respectively. The term $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ is the circular symmetric complex gaussian noise with zero mean and σ_k^2 receiver noise variance.

The channels defined in (1) are assumed to be Rayleigh block-fading with $\mathbf{h}_{0,k} \sim \mathcal{CN}(0, \mathbf{R}_{k,0})$ and $\mathbf{h}_{i,k} \sim \mathcal{CN}(0, \mathbf{R}_{k,i})$. The covariance matrices $\mathbf{R}_{k,0} \in \mathbb{C}^{N_{BS} \times N_{BS}}$ and $\mathbf{R}_{k,i} \in \mathbb{C}^{N_{SC} \times N_{SC}}$ describe the spatial channel correlation [11]. In this type of channel, the random realization changes every coherence period. The resulting transmitted signals after precoding, \mathbf{x}_0 and \mathbf{x}_i at the MBS and the i^{th} SC, are:

$$\mathbf{x}_i = \sum_{k=1}^K \mathbf{w}_{k,i} s_{k,i} \quad i = 0, \dots, S \quad (2)$$

where $\mathbf{w}_{k,0} \in \mathbb{C}^{N_{BS} \times 1}$ and $\mathbf{w}_{k,i} \in \mathbb{C}^{N_{SC} \times 1}$ are the beamforming vectors of user k at the MBS ($i = 0$) and the i^{th} SC, respectively. The term $s_{k,i} \sim \mathcal{N}(0,1)$ denotes the information symbol from transmitter i to user k and generated independently from different gaussian code books and assumed to be of unit power.

As we can see from (1), the user k receives from the MBS and the SCs different signals simultaneously using coordinated multi-point joint downlink transmission [8]. Although, the system model may be similar to the one in [8] which was concerned with the total power minimization, the addressed problem in this paper is concerned with sum rate maximization of all users under a constraint on the total power threshold which is a completely different problem.

A. Problem Formulation

The problem under study is finding the beamforming weights to maximize the total sum rate of all served users while satisfying several constraints that can ensure a good QoS and guarantee energy efficiency at the same time.

The QoS constraints ensure that the signal-to-interference and noise ratio (SINR) per user must be greater than or equal to a certain specified value. Hence, a minimum information rate [bits/s/Hz] per user is predefined by the problem to avoid outage and ensure fairness. In other words, avoid maximization of the total sum rate at the expense of low rate at some users. Thus, these constraints can be mathematically formulated as follows;

$$\text{SINR}_k \geq \gamma_k \quad \forall k \quad (3)$$

where,

$$\text{SINR}_k = \frac{|\mathbf{h}_{0,k} \mathbf{w}_{k,0}|^2 + \sum_{i=1}^S |\mathbf{h}_{i,k} \mathbf{w}_{k,i}|^2}{\sum_{j=1, j \neq k}^K (|\mathbf{h}_{0,k} \mathbf{w}_{j,0}|^2 + \sum_{i=1}^S |\mathbf{h}_{i,k} \mathbf{w}_{j,i}|^2) + \sigma_k^2} \quad (4)$$

The energy efficiency is guaranteed by specifying a maximum power limit that can be consumed by all transmitters (the MBS and the S SCs) during transmission. The power emitted P can be modeled as;

$$P = \sum_{k=1}^K (\rho_0 \|\mathbf{w}_{k,0}\|^2 + \sum_{i=1}^S \rho_i \|\mathbf{w}_{k,i}\|^2) \leq P_{max} \quad (5)$$

where, ρ_0 and $\rho_i \geq 1$ are constants that account for the inefficiencies in power amplifiers at the MBS and the i^{th} SC, respectively.

Each transmitter of the MBS and the SCs is subject to a maximum power limit per antenna constraint which can be modeled as follows;

$$\sum_{k=1}^K \mathbf{w}_{k,i}^H \mathbf{Q}_{i,l} \mathbf{w}_{k,i} \leq p_{i,l}, \forall l = \begin{cases} 1, \dots, N_{BS} & i = 0 \\ 1, \dots, N_{SC} & i = 1, \dots, S \end{cases} \quad (6)$$

where, $\mathbf{Q}_{0,l} \in \mathbb{C}^{N_{BS} \times N_{BS}}$ and $\mathbf{Q}_{i,l} \in \mathbb{C}^{N_{SC} \times N_{SC}}$ are weighting matrices that select one antenna at each constraint with value 1 at the l^{th} entry and zeros elsewhere. The term $p_{i,l}$ is the maximum power limit at antenna l and transmitter i . These constraints ensure that each antenna in transmitter i does not consume more power than the maximum power limit.

The sum rate optimization problem is now ready. The objective is to maximize the total sum rate under QoS constraints, power per-antenna constraints and total power consumed constraint. The problem can be presented as follows;

$$\text{maximize}_{\mathbf{w}_{k,i}} \sum_{k=1}^K \log_2(1 + \text{SINR}_k) \quad (7a)$$

Subject to (s.t.)

$$\text{SINR}_k \geq \gamma_k \quad \forall k \quad (7b)$$

$$\sum_{k=1}^K (\rho_0 \|\mathbf{w}_{k,0}\|^2 + \sum_{i=1}^S \rho_i \|\mathbf{w}_{k,i}\|^2) \leq P_{max} \quad (7c)$$

$$\sum_{k=1}^K \mathbf{w}_{k,i}^H \mathbf{Q}_{i,l} \mathbf{w}_{k,i} \leq p_{i,l} \quad \forall i, l \quad (7d)$$

III. PROPOSED ITERATIVE BEAMFORMING DESIGN

Unfortunately, problem (7) is not convex. In this section, we will follow suit of the framework presented in [12], in formulating the problem under study into a Second Order Cone (SOC) by replacing the non-convex constraints by its convex upper bound and hence solve the resulting problem iteratively until convergence. However, it should be noted that our problem is different from [12] due to the presence of SCs that can transmit simultaneously with the M-MIMO MBS which changes the definition of the SINR_k as shown in (4) leading to a different objective function. Also, the QoS constraints in (3) are added to the problem to guarantee fairness. It is worth noting that without these QoS constraints, there is no guarantee that all the users have acceptable information rate as the objective function cares about the sum rate of all users not each user individually. This will affect the mathematical formulation as will be shown here after.

A. SOC formulation

Problem (7) can be modified to;

$$\text{maximize}_{\mathbf{w}_{k,i}} \prod_{k=1}^K (1 + \text{SINR}_k) \quad (8a)$$

s.t.

$$\text{SINR}_k \geq \gamma_k \quad \forall k \quad (8b)$$

$$\sum_{k=1}^K (\rho_0 \|\mathbf{w}_{k,0}\|^2 + \sum_{i=1}^S \rho_i \|\mathbf{w}_{k,i}\|^2) \leq P_{max} \quad (8c)$$

$$\sum_{k=1}^K \mathbf{w}_{k,i}^H \mathbf{Q}_{i,l} \mathbf{w}_{k,i} \leq p_{i,l} \quad \forall i, l \quad (8d)$$

Problem (8) can be simplified by introducing t_k ;

$$\text{maximize}_{\mathbf{w}_{k,i}, t_k} \prod_{k=1}^K t_k \quad (9a)$$

s.t.

$$\text{SINR}_k \geq t_k - 1 \quad \forall k \quad (9b)$$

$$\text{SINR}_k \geq \gamma_k \quad \forall k \quad (9c)$$

$$\sum_{k=1}^K \left(\rho_0 \|\mathbf{w}_{k,0}\|^2 + \sum_{i=1}^S \rho_i \|\mathbf{w}_{k,i}\|^2 \right) \leq P_{\max} \quad (9d)$$

$$\sum_{k=1}^K \mathbf{w}_{k,i}^H \mathbf{Q}_{i,l} \mathbf{w}_{k,i} \leq p_{i,l} \quad \forall i, l \quad (9e)$$

Now, substituting with (4) in (9b) and introducing a slack variable β_k , the constraints (9b) and (9c) are modified to;

$$|\mathbf{h}_{0,k} \mathbf{w}_{k,0}|^2 + \sum_{i=1}^S |\mathbf{h}_{i,k} \mathbf{w}_{k,i}|^2 \geq (t_k - 1) \beta_k^2 \quad \forall k \quad (10a)$$

$$\sqrt{\sum_{j \neq k}^K \left(|\mathbf{h}_{0,k} \mathbf{w}_{j,0}|^2 + \sum_{i=1}^S |\mathbf{h}_{i,k} \mathbf{w}_{j,i}|^2 \right) + \sigma_k^2} \leq \beta_k \quad \forall k \quad (10b)$$

$$(t_k - 1) \geq \gamma_k \quad \forall k \quad (10c)$$

Note that the slack variable β_k introduced, keeps (10) equivalent to (9b) and (9c). This is because at an optimal solution, (10) will be active i.e. satisfied at equality.

Then, each constraint in (10a) can be decomposed as follows;

$$\mathbf{h}_{i,k} \mathbf{w}_{k,i} \geq \sqrt{a_{k,i}} \beta_k \quad \forall k, i \quad (11a)$$

$$\left(\sum_{i=0}^S a_{k,i} \right) \geq (t_k - 1) \quad \forall k \quad (11b)$$

$$\text{Im}(\mathbf{h}_{i,k} \mathbf{w}_{k,i}) = 0 \quad \forall k, i \quad (11c)$$

where $a_{k,i}$ is the SINR from transmitter i to user k .

Let us define $f(a_{k,i}, \beta_k) = \sqrt{a_{k,i}} \beta_k \quad \forall a_{k,i}, \beta_k \geq 0$. Since $f(a_{k,i}, \beta_k)$ is non-convex, the constraints in (11a) are not convex constraints. Thus, according to [13], if the non-convex function is replaced by its convex upper bound then the resulting convex problem can be solved iteratively and at each new iteration the variables can be updated till convergence. For a fixed $\varphi_k > 0$, define a new convex function;

$$G(a_{k,i}, \beta_k, \varphi_k) \triangleq \frac{\varphi_{k,i}}{2} \beta_k^2 + \frac{a_{k,i}}{2\varphi_{k,i}} \quad \forall a_{k,i}, \beta_k \geq 0 \quad (12)$$

For a given value of $\varphi_{k,i}$, $G(a_{k,i}, \beta_k, \varphi_{k,i})$ is convex (a function is convex if and only if its hessian matrix is positive semi-definite). As a result, $G(a_{k,i}, \beta_k, \varphi_{k,i})$ can be a convex upper bound for $f(a_{k,i}, \beta_k)$.

$$\text{At } \varphi_{k,i} = \frac{\sqrt{a_{k,i}}}{\beta_k}, \quad G(a_{k,i}, \beta_k, \varphi_k) = f(a_{k,i}, \beta_k) \quad (13a)$$

$$\nabla G(a_{k,i}, \beta_k, \varphi_k) = \nabla f(a_{k,i}, \beta_k) \quad (13b)$$

Accordingly, $G(a_{k,i}, \beta_k, \varphi_k)$ replaces $f(a_{k,i}, \beta_k)$ in (11a);

$$\mathbf{h}_{i,k} \mathbf{w}_{k,i} \geq \frac{\varphi_{k,i}}{2} \beta_k^2 + \frac{a_{k,i}}{2\varphi_{k,i}} \quad \forall k, i \quad (14)$$

The constraints in (14) can be summed up per user, and convexity is still satisfied since the sum of two convex functions is still convex.

$$\sum_{i=0}^S \mathbf{h}_{i,k} \mathbf{w}_{k,i} \geq \sum_{i=0}^S \frac{\varphi_{k,i}}{2} \beta_k^2 + \sum_{i=0}^S \frac{a_{k,i}}{2\varphi_{k,i}} \quad \forall k \quad (15)$$

The SOC representation is based on converting any hyperbolic constraint into a SOC constraint as follows [14];

$$uv \geq \omega^2 \Leftrightarrow \left\| \begin{bmatrix} 2\omega \\ u - v \end{bmatrix} \right\| \leq u + v \quad \forall u, v > 0, \omega \in R \quad (16)$$

The SOC formulation defined in (17) can be summarized in the following steps;

- 1- The non-linear objective function in (9a) is formulated into a linear objective and $K - 1$ SOC constraints as shown in

(17a) and (17b) by collecting every two variables of t_k in (9a) and applying their corresponding hyperbolic constraint identity in (16). The collection of every two variables into a new variable needs some extra variables $\tau_z^{(m)}$, where z is the count of the extra variable and (m) is the level of grouping.

- 2- The constraints in (15) are formulated into SOC constraints (17c) using the identity in (16).
- 3- The rest of the problem complies with the SOC representation.

Finally, the SOC representation of problem (8) can be defined as follows;

$$\text{maximize}_{\mathbf{w}_{k,i}, t_k, \beta_k, a_{k,i}, \tau_z^{(k)}} \quad \tau^{(0)} \quad (17a)$$

s.t.

$$\left\| [2\tau^{(0)} \quad (\tau_1^{(1)} - \tau_2^{(1)})]^T \right\| \leq (\tau_1^{(1)} + \tau_2^{(1)}) \quad (17b)$$

$$\vdots$$

$$\left\| [2\tau_z^{(K-1)} \quad (t_{2z-1} - t_{2j})]^T \right\| \leq (t_{2z-1} + t_{2z}), z = 1, \dots, 2^{K-1}$$

$$\left\| \left[\frac{1}{2} \left(\sum_{i=0}^S \mathbf{h}_{i,k} \mathbf{w}_{k,i} - \sum_{i=0}^S \frac{a_{k,i}}{2\varphi_{k,i}^{(n)}} - 1 \right) \quad \sqrt{\sum_{i=0}^S \frac{\varphi_{k,i}^{(n)}}{2}} \beta_k \right]^T \right\| \quad (17c)$$

$$\leq \frac{1}{2} \left(\sum_{i=0}^S \mathbf{h}_{i,k} \mathbf{w}_{k,i} - \sum_{i=0}^S \frac{a_{k,i}}{2\varphi_{k,i}^{(n)}} + 1 \right) \quad \forall k$$

$$\left(\sum_{i=0}^S a_{k,i} \right) \geq t_k - 1 \quad \forall k \quad (17d)$$

$$\left\| [\mathbf{h}_{0,k} \mathbf{w}_{1,0} \dots \mathbf{h}_{i,k} \mathbf{w}_{1,i} \dots \mathbf{h}_{0,k} \mathbf{w}_{j,0} \dots \mathbf{h}_{i,k} \mathbf{w}_{j,i} \quad \sigma_k]^T \right\| \leq \beta_k \quad \forall k, j = 1 \dots k-1 \dots k+1 \dots K \text{ and } i = 0 \dots S \quad (17e)$$

$$t_k - 1 \geq \gamma_k \quad \forall k \quad (17f)$$

$$\sum_{k=1}^K \left(\rho_0 \|\mathbf{w}_{k,0}\|^2 + \sum_{i=1}^S \rho_i \|\mathbf{w}_{k,i}\|^2 \right) \leq P_{\max} \quad (17g)$$

$$\sum_{k=1}^K \mathbf{w}_{k,i}^H \mathbf{Q}_{i,l} \mathbf{w}_{k,i} \leq p_{i,l} \quad \forall i, l \quad (17i)$$

B. Iterative algorithm

At each iteration, $\varphi_{k,i}$ is updated using its definition in (13a) using the optimal values of β_k and $a_{k,i}$ of the previous iteration. The algorithm is summarized in Algorithm 1. The symbols $\beta_k^{(n)}$, $a_{k,i}^{(n)}$ and $\varphi_{k,i}^{(n)}$ denote the variables β_k , $a_{k,i}$ and $\varphi_{k,i}$ at the n^{th} iteration. A KKT point is achieved based on the justification presented in [12].

Algorithm 1: Algorithm for Problem (17)

Given: Channel vectors $\mathbf{h}_{k,i} \quad \forall i, k$

Initialize $n = 0$, $\varphi_{k,i}^{(n)} = \text{random} \quad \forall k, i$

Repeat

- Use $\varphi_{k,i}^{(n)}$ to solve the SOC problem in (17), then the optimal solution of $(\beta_k^{(n)}, a_{k,i}^{(n)})$ is denoted by $(\beta_k^*, a_{k,i}^*)$
- Update $\varphi_{k,i}^{(n+1)} = \sqrt{a_{k,i}^*} / \beta_k^*$ and $n = n + 1$.

Stop if the increase in the objective function $\leq \epsilon$

IV. NUMERICAL RESULTS

In this section, the scenario under test is composed of 1 M-MIMO MBS equipped with N_{BS} antennas that is located at the

center of a cell whose radius is 0.5 Km. The MBS is overlaid with 4 SCs with N_{SC} antennas and located at 0.35 Km from the MBS. There are 8 served users, 4 of them are uniformly distributed in the coverage area while each one of the other 4 is uniformly distributed within 40 meters around one of the 4 SCs.

The channel model follows case 1 for HetNets in the 3GPP LTE standard [15]. The beamformer weights were obtained by solving the sum rate maximization problem in (7). Firstly, the problem is reformulated into SOC problem in (17) that is solved iteratively by Algorithm 1 using the modeling language in [16].

Figure 1 shows the average sum rate of all users in two different scenarios. Scenario 1: the power is completely dedicated to the MBS. Scenario 2: the power is distributed between the MBS and 4 SCs with N_{SC} antennas to serve users simultaneously. Each user has a QoS constraint as in (7b) such that the rate per user must exceed 2 bits/s/Hz. The total power consumption threshold was set to 19 dBm. The maximum power per antenna for the MBS and SCs were set to 18.2 dBm and -11 dBm and the parameters ρ_0 and ρ_i were set to 2.58 and 19.2, respectively [8]. The parameter ϵ in Algorithm 1 was set to 10^{-2} in simulations.

The allocation of the MBS and one or more of the 4 SCs to each user is based on the received power at the user from each transmitter (MBS and SCs). If the received power from a certain transmitter exceeds a predefined threshold (-40 dBm) then this transmitter is allocated for this user.

It is clear from Figure 1 that the presence of SCs greatly improves the total sum rate. It is also interesting to conclude that adding 4 SCs can increase the total sum rate even better than adding greater number of antennas to the M-MIMO MBS. Moreover, it is obvious that as the number of antennas of the SCs increases from 1 to 3, the total sum rate is highly improved since the SC is more likely to be allocated to more users.

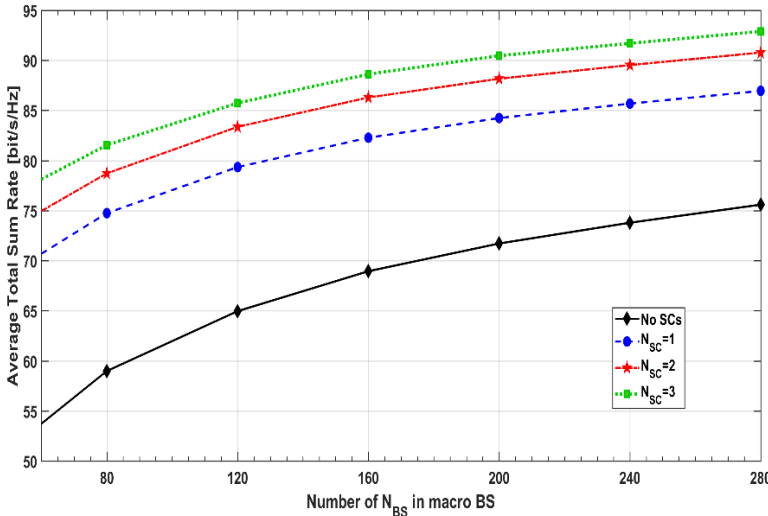


Fig. 1. Average total sum rate of 8 users served by 1 MBS and 4 SCs under QoS constraint per user of 2 bits/s/Hz and 19 dBm total transmit power.

V. CONCLUSION AND FUTURE WORK

The spectral efficiency of a M-MIMO heterogeneous network is improved when overlaid with small cells to serve users simultaneously under the same threshold of total power. The beamforming weights were designed by a joint optimization

problem. The objective is maximizing the total sum rate under quality of service, power per-antenna and total consumed power constraints. As a future work, extending the optimization model to account for the effect of channel uncertainty and develop robust optimization technique to obtain the beamforming weights is important to overcome the problem of quantization errors and outdated channel estimation. Moreover, expanding the scenario to a multi-cell system to add the effect of inter-cell interference should also be considered towards a more realistic network system model.

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