

$$\dot{R}_1 = -\mu_{11} k_1 (R_1 - R_1^0)$$

Only 1st bead

(0)

$$\dot{\Delta}_1 = -\mu_{11} k_1 \Delta_1 - \dot{R}_1^0$$

$$\dot{\Delta}_1 + \mu_{11} k_1 \Delta_1 = -\dot{R}_1^0$$

$$\Delta_1 = \left[\frac{-i\omega}{i\omega + \mu_{11} k_1} \right] R_1^0(\omega)$$

$$\rightarrow R_1 = \left[1 - \frac{i\omega}{i\omega + \mu_{11} k_1} \right] R_1^0(\omega)$$

$$\delta_1 = \tan^{-1}$$

$$\frac{c + id}{a + ib} = \frac{(ac + bd) + i(ad - bc)}{a^2 + b^2}$$

$$\tan^{-1} \left(\frac{ad - bc}{ac + bd} \right)$$

$$\delta_1 = \tan^{-1} \left(\frac{a}{b} \right) = \tan^{-1} \left(\frac{\mu_{11} k_1}{\omega} \right)$$

(1)

$$\dot{R}_1 = -\mu_{11} k_1 (R_1 - R_1^0) - \mu_{12} k_2 (R_2 - R_2^0)$$

$$\dot{R}_2 = -\mu_{21} k_1 (R_1 - R_1^0) - \mu_{22} k_2 (R_2 - R_2^0).$$

$$\Delta_1 = R_1 - R_1^0$$

$$\dot{\Delta}_1 = \dot{R}_1 - \dot{R}_1^0 \longrightarrow$$

$$\Delta_2 = R_2 - R_2^0$$

$$\dot{\Delta}_2 = \dot{R}_2$$

$$\dot{\Delta}_1 = -\mu_{11} k_1 \Delta_1 - \mu_{12} k_2 \Delta_2 - \dot{R}_1^0$$

$$\dot{\Delta}_2 = -\mu_{21} k_1 \Delta_1 - \mu_{22} k_2 \Delta_2$$

2nd Bead

(2)

$$-\mu_{21} k_1 \Delta_1 = \dot{\Delta}_2 + \mu_{22} k_2 \Delta_2$$

$$-\mu_{21} k_1 \dot{\Delta}_1 = \ddot{\Delta}_2 + \mu_{22} k_2 \dot{\Delta}_2$$

$$\begin{aligned} \ddot{\Delta}_2 + \mu_{22} k_2 \dot{\Delta}_2 &= -\mu_{11} k_1 \left[\dot{\Delta}_2 + \mu_{22} k_2 \Delta_2 \right] \\ &\quad + \mu_{21} \mu_{12} k_2 k_1 \Delta_2 + \mu_{21} k_1 \dot{R}_1^0 \end{aligned}$$

$$\begin{aligned} \Rightarrow \ddot{\Delta}_2 + (k_1 \mu_{11} + k_2 \mu_{22}) \dot{\Delta}_2 + k_1 k_2 (\mu_{11} \mu_{22} - \mu_{21} \mu_{12}) \Delta_2 \\ = + \mu_{21} k_1 \dot{R}_1^0 \end{aligned}$$

$$\Delta_2 = \frac{i\omega \mu_{21} k_1}{(\omega_0^2 - \omega^2) + i2\gamma\omega} R_1^0(\omega)$$

$$2\gamma = (k_1 \mu_{11} + k_2 \mu_{22})$$

$$\omega_0^2 = k_1 k_2 (\mu_{11} \mu_{22} - \mu_{21} \mu_{12})$$

$$\begin{aligned} \delta_2 &= \tan^{-1} \left[\frac{ad - bc}{ac + bd} \right] = \tan^{-1} \left(\frac{a}{b} \right) \\ &= \tan^{-1} \left(\frac{\omega_0^2 - \omega^2}{2\gamma\omega} \right) \end{aligned}$$

1st bead

(3)

$$-\mu_{12} k_2 \dot{\Delta}_2 = \ddot{\Delta}_1 + \mu_{11} k_1 \dot{\Delta}_1 + \dot{R}_1^0$$

$$-\mu_{12} k_2 \dot{\Delta}_2 = \ddot{\Delta}_1 + \mu_{11} k_1 \dot{\Delta}_1 + \ddot{R}_1^0$$

$$\ddot{\Delta}_1 + \mu_{11} k_1 \dot{\Delta}_1 + \ddot{R}_1^0 = \mu_{12} \mu_{21} k_1 k_2 \Delta_1$$

$$- \mu_{22} k_2 \left[\dot{\Delta}_1 + \mu_{11} k_1 \Delta_1 + \dot{R}_1^0 \right]$$

$$\Rightarrow \ddot{\Delta}_1 + (\mu_{11} k_1 + \mu_{22} k_2) \dot{\Delta}_1 + k_1 k_2 (\mu_{11} \mu_{22} - \mu_{12} \mu_{21}) \Delta_1 = -\mu_{22} k_2 \dot{R}_1^0 - \ddot{R}_1^0$$

$$\Delta_1 = \left[\frac{-i\omega \mu_{22} k_2 + \omega^2}{(\omega_0^2 - \omega^2) + i2\gamma\omega} \right] R_1^0(\omega)$$

$$\delta_1 = \tan^{-1} \left[\frac{ad - bc}{ac + bd} \right] = \tan^{-1} \left[- \frac{\omega \mu_{22} k_2 (\omega_0^2 - \omega^2) + 2\gamma\omega \cdot \omega^2}{(\omega_0^2 - \omega^2)\omega^2 - 2\gamma\omega^2 \mu_{22} k_2} \right]$$

$$R_1 - R_1^0(\omega) = \begin{bmatrix} \dots \end{bmatrix} R_1^0(\omega)$$

$$R_1(\omega) = R_1^0(\omega) + \begin{bmatrix} \end{bmatrix} R_1^0(\omega).$$

$$= \left\{ 1 + \begin{bmatrix} \end{bmatrix} \right\} R_1^0(\omega).$$

$$= \frac{(a+c) + i(b+d)}{a+ib}$$

$$\delta_1 = \tan^{-1} \left[\frac{a(b+d) - b(a+c)}{a(a+c) + b(b+d)} \right].$$