

1st bead without 2nd.

©

$$\dot{R}_1 = -\mu_{11} k_1 (R_1 - R_1^0)$$

$$\dot{\Delta}_1 = -\mu_{11} k_1 \Delta_1 - \dot{R}_1^0$$

$$\dot{\Delta}_1 + \mu_{11} k_1 \Delta_1 = -\dot{R}_1^0$$

$$\Delta_1 = \left[ -\frac{i\omega}{i\omega + \mu_{11} k_1} \right] R_1^0(\omega)$$

$$\begin{aligned} R_1(\omega) &= \left[ 1 - \frac{i\omega}{i\omega + \mu_{11} k_1} \right] R_1^0(\omega) \\ &= \frac{\mu_{11} k_1}{i\omega + \mu_{11} k_1} R_1^0(\omega) \end{aligned}$$

As solution is  $A e^{i\delta}$

$$\delta = \tan^{-1} \left[ -\frac{\omega}{\mu_{11} k_1} \right]$$

①

$$\dot{R}_1 = -\mu_{11} k_1 (R_1 - R_1^0) - \mu_{12} k_2 (R_2 - R_2^0)$$

$$\dot{R}_2 = -\mu_{21} k_1 (R_1 - R_1^0) - \mu_{22} k_2 (R_2 - R_2^0).$$

$$\Delta_1 = R_1 - R_1^0$$

$$\dot{\Delta}_1 = \dot{R}_1 - \dot{R}_1^0 \longrightarrow$$

$$\Delta_2 = R_2 - R_2^0$$

$$\dot{\Delta}_2 = \dot{R}_2$$

$$\dot{\Delta}_1 = -\mu_{11} k_1 \Delta_1 - \mu_{12} k_2 \Delta_2 - \dot{R}_1^0$$

$$\dot{\Delta}_2 = -\mu_{21} k_1 \Delta_1 - \mu_{22} k_2 \Delta_2$$

## 2nd Bead

(2)

$$-\mu_{21} k_1 \Delta_1 = \dot{\Delta}_2 + \mu_{22} k_2 \Delta_2$$

$$-\mu_{21} k_1 \dot{\Delta}_1 = \ddot{\Delta}_2 + \mu_{22} k_2 \dot{\Delta}_2$$

$$\begin{aligned} \ddot{\Delta}_2 + \mu_{22} k_2 \dot{\Delta}_2 &= -\mu_{11} k_1 \left[ \dot{\Delta}_2 + \mu_{22} k_2 \Delta_2 \right] \\ &\quad + \mu_{21} \mu_{12} k_2 k_1 \Delta_2 + \mu_{21} k_1 \dot{R}_1^0 \end{aligned}$$

$$\begin{aligned} \Rightarrow \ddot{\Delta}_2 + (k_1 \mu_{11} + k_2 \mu_{22}) \dot{\Delta}_2 + k_1 k_2 (\mu_{11} \mu_{22} - \mu_{21} \mu_{12}) \Delta_2 \\ = + \mu_{21} k_1 \dot{R}_1^0 \end{aligned}$$

$$\Delta_2 = \frac{i\omega \mu_{21} k_1}{(\omega_0^2 - \omega^2) + i2\lambda\omega} R_1^0(\omega)$$

$$2\lambda = (k_1 \mu_{11} + k_2 \mu_{22})$$

$$\omega_0^2 = k_1 k_2 (\mu_{11} \mu_{22} - \mu_{21} \mu_{12})$$

$$\begin{aligned} \delta_2 &= \tan^{-1} \left[ \frac{ad - bc}{ac + bd} \right] = \tan^{-1} \left( \frac{a}{b} \right) \\ &= \tan^{-1} \left( \frac{\omega_0^2 - \omega^2}{2\lambda\omega} \right) \end{aligned}$$

Solving for the first bead.

$$\Delta_2 \rightarrow x$$

$$- \mu_{12} k_2 \Delta_2 = \dot{\Delta}_1 + \mu_{11} k_1 \Delta_1 + \dot{R}_1^o$$

$$- \mu_{12} k_2 \dot{\Delta}_2 = \ddot{\Delta}_1 + \mu_{11} k_1 \dot{\Delta}_1 + \ddot{R}_1^o$$

$$\Rightarrow \ddot{\Delta}_1 + \mu_{11} k_1 \dot{\Delta}_1 + \ddot{R}_1^o = + \mu_{12} \mu_{21} k_1 k_2 \Delta_1 - \mu_{22} k_2 x \left[ \begin{array}{l} + \dot{\Delta}_1 + \mu_{11} k_1 \Delta_1 + \dot{R}_1^o \end{array} \right]$$

$$\Rightarrow \ddot{\Delta}_1 + (\mu_{11} k_1 + \mu_{22} k_2) \dot{\Delta}_1 + (\mu_{11} \mu_{22} - \mu_{12} \mu_{21}) k_1 k_2 \Delta_1 = - \mu_{22} k_2 \ddot{R}_1^o - \ddot{R}_1^o$$

$$\Delta_1 = \left[ \frac{-i\omega \mu_{22} k_2 + \omega^2}{(\omega_0^2 - \omega^2) + i 2\lambda \omega} \right] R_1^o(\omega)$$

$$R_1(\omega) = \left[ 1 + \dots \right] R_1^o(\omega)$$

$$= \left[ \frac{\omega_0^2 + i(2\lambda - \mu_{22} k_2)\omega}{(\omega_0^2 - \omega^2) + i 2\lambda \omega} \right] R_1^o(\omega)$$

$$\tan \delta_1 = \frac{ad - bc}{ac + bd}$$

$$a = (\omega_0^2 - \omega^2)$$

$$b = 2\gamma\omega$$

$$c = \omega_0^2$$

$$d = (2\gamma - \mu_{22}k_2)\omega$$

Check in matlab code.