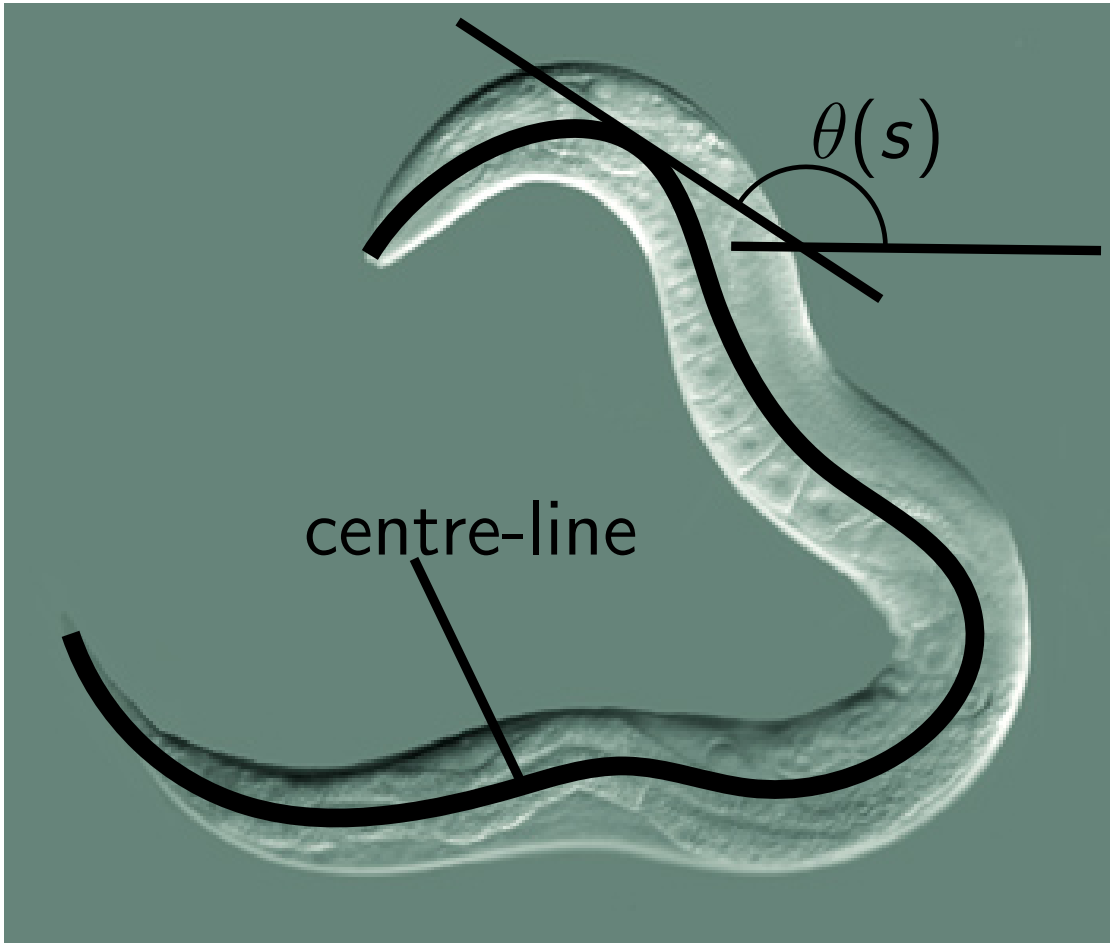
Rejected headsweep

Accepted headsweep

Eigen-worms



s = fractional distance along centre-line.

Parameterise posture as $\theta(s) - \int \theta(s')\mathrm{d}s'$

PCA $\Longrightarrow$ 4 modes.

[Stephens et al.]

Eigen-larvae?

Posture not captured by centre-line.

Length is dynamical.

Principal Component Analysis (PCA)

Our data-set consists of vectors (each member from one frame of the videos):

$$\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots$$

Make change of variables (describe each frame by α^n):

$$\mathbf{x}_i = \langle \mathbf{x} \rangle + \sum_n \alpha_i^n \mathbf{u}_n.$$

Choose directions $\mathbf{u}_n$ to be of decreasing importance so we can ignore all but first few α^n.

Principal Component Analysis (cont'd)

- Choose direction $\mathbf{u}_1$ to account for the maximum variance possible.
- Choose direction $\mathbf{u}_2$ to account for as much of the remaining variance as possible.
- And so on.

$\Longrightarrow$ $\mathbf{u}_n$ are eigenvectors of the covariance matrix:

$$\mathbf{C} = \left\langle (\mathbf{x} - \langle \mathbf{x} \rangle) \cdot (\mathbf{x} - \langle \mathbf{x} \rangle)^{\mathrm{T}} \right\rangle, \qquad \mathbf{C} \cdot \mathbf{u}_n = \sigma_n^2 \mathbf{u}_n.$$

where σ_n^2 are decreasing – variance accounted for by n^{th} direction.

PCA with constraints

Suppose each datum satisfies a linear constraint:

$$\mathbf{b}\cdot\mathbf{x}_i = \beta.$$

We have

$$\mathbf{b}\cdot\langle\mathbf{x}\rangle = \beta,$$

$$\mathbf{b}\cdot\mathbf{C} = 0 \qquad \Longrightarrow \mathbf{b}\cdot\mathbf{u}_n = 0 \quad \text{if} \quad \sigma_n^2 \neq 0.$$

So, if we choose $\alpha^n = 0$ in null directions:

$$\mathbf{b}\cdot\left[\langle\mathbf{x}\rangle + \sum_n \alpha^n \mathbf{u}_n\right] = \beta.$$

i.e. the constraint is satisfied for any choice of $\{\alpha^n\}$ in non-null directions.

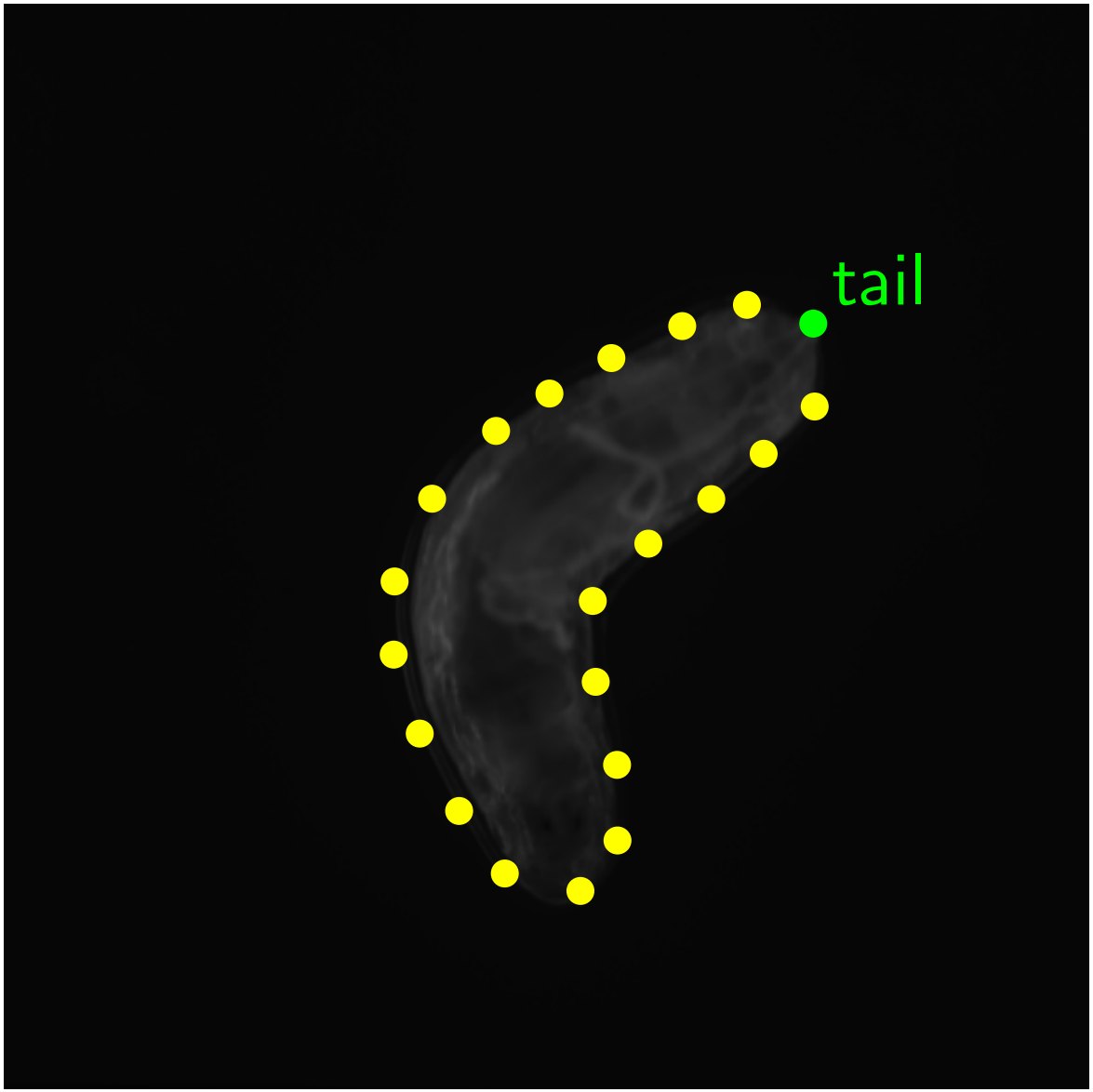
Analysing larva image

1. Extract boundary

Analysing larva image

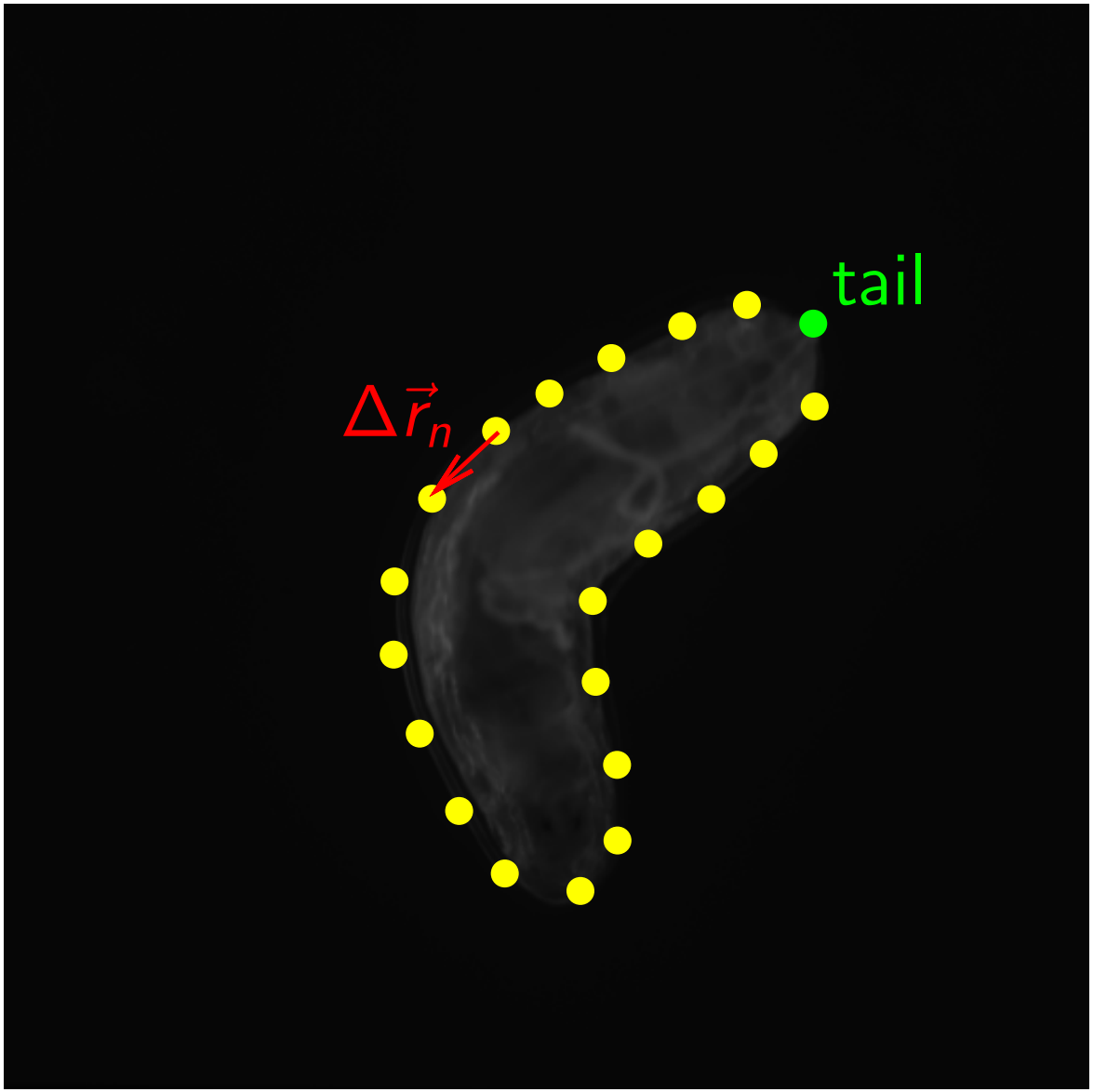
1. Extract boundary
2. Find tail

Analysing larva image



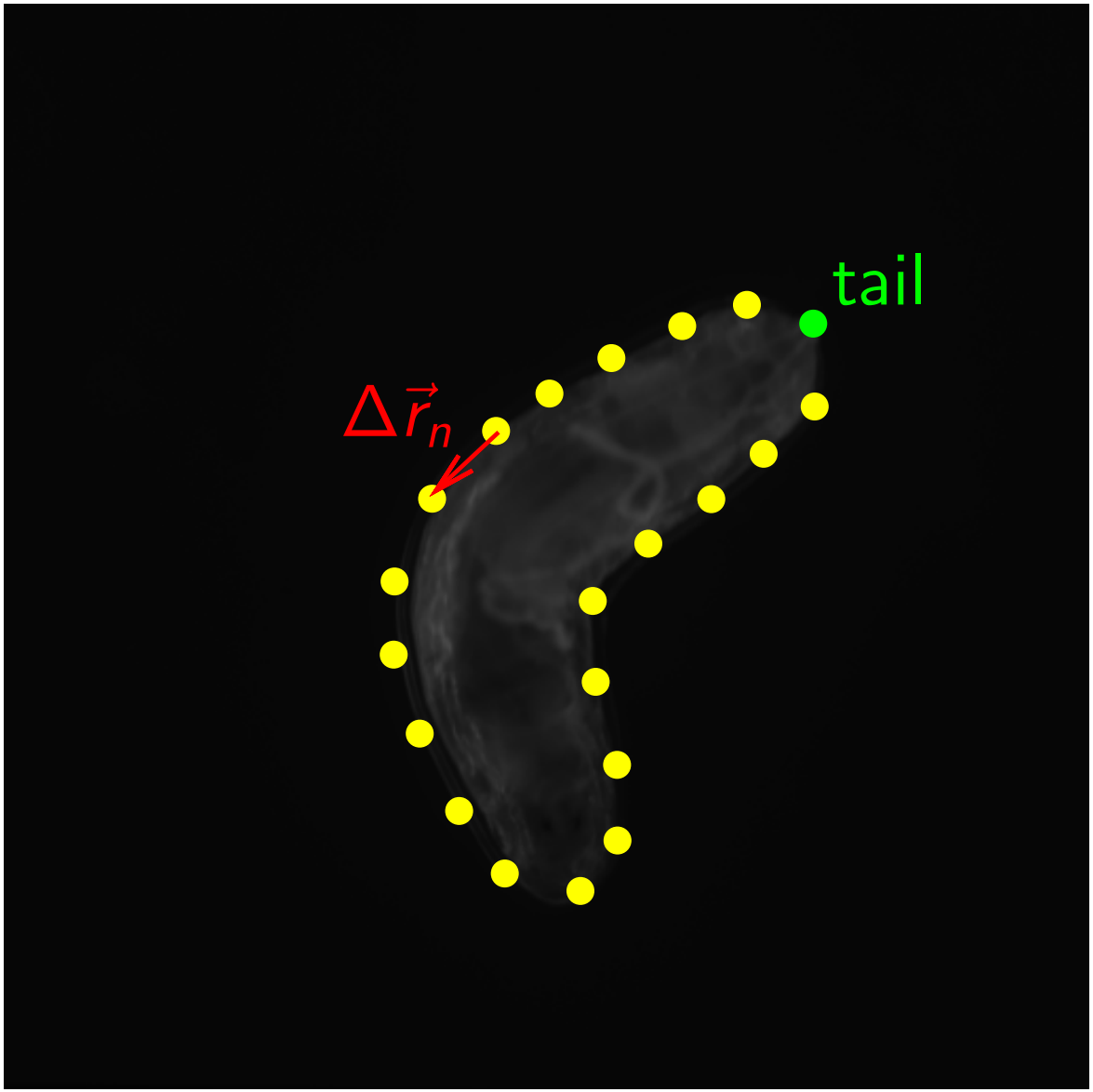
1. Extract boundary
2. Find tail
3. Mark N points along boundary

Analysing larva image



1. Extract boundary
2. Find tail
3. Mark N points along boundary
4. Record difference vectors

Analysing larva image



1. Extract boundary
2. Find tail
3. Mark N points along boundary
4. Record difference vectors
5. Rotate so that tangent horizontal at tail

Parameterising posture

We record:

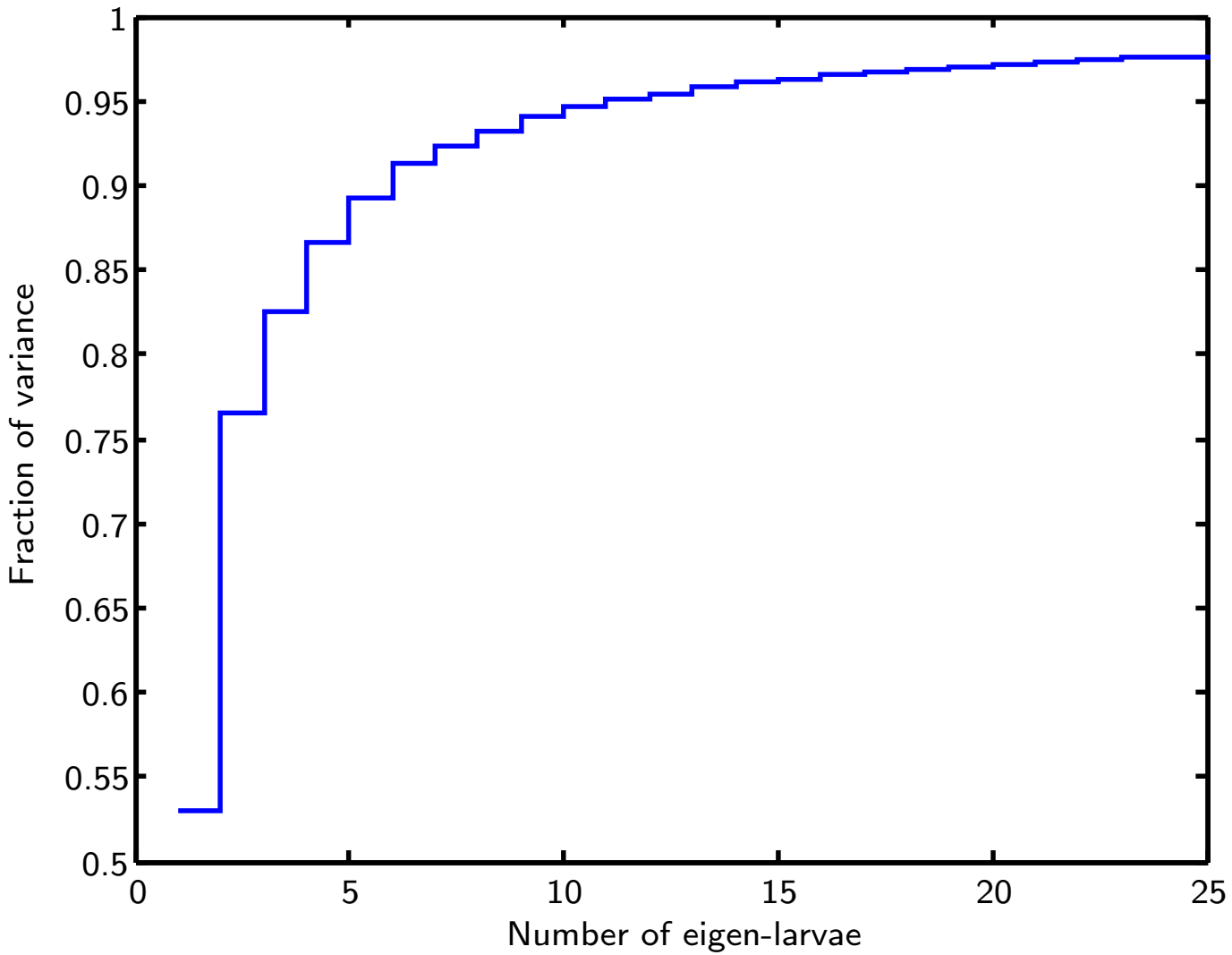
$$\mathbf{x} = (\Delta x_1, \ldots, \Delta x_N, \Delta y_1, \ldots, \Delta y_N)^{\mathrm{T}}$$

Constraints:

- $\sum_n \Delta x_n = 0$
- $\sum_n \Delta y_n = 0$ (closed curve)
- $\Delta y_1 + \Delta y_N = 0$. (overall rotations)

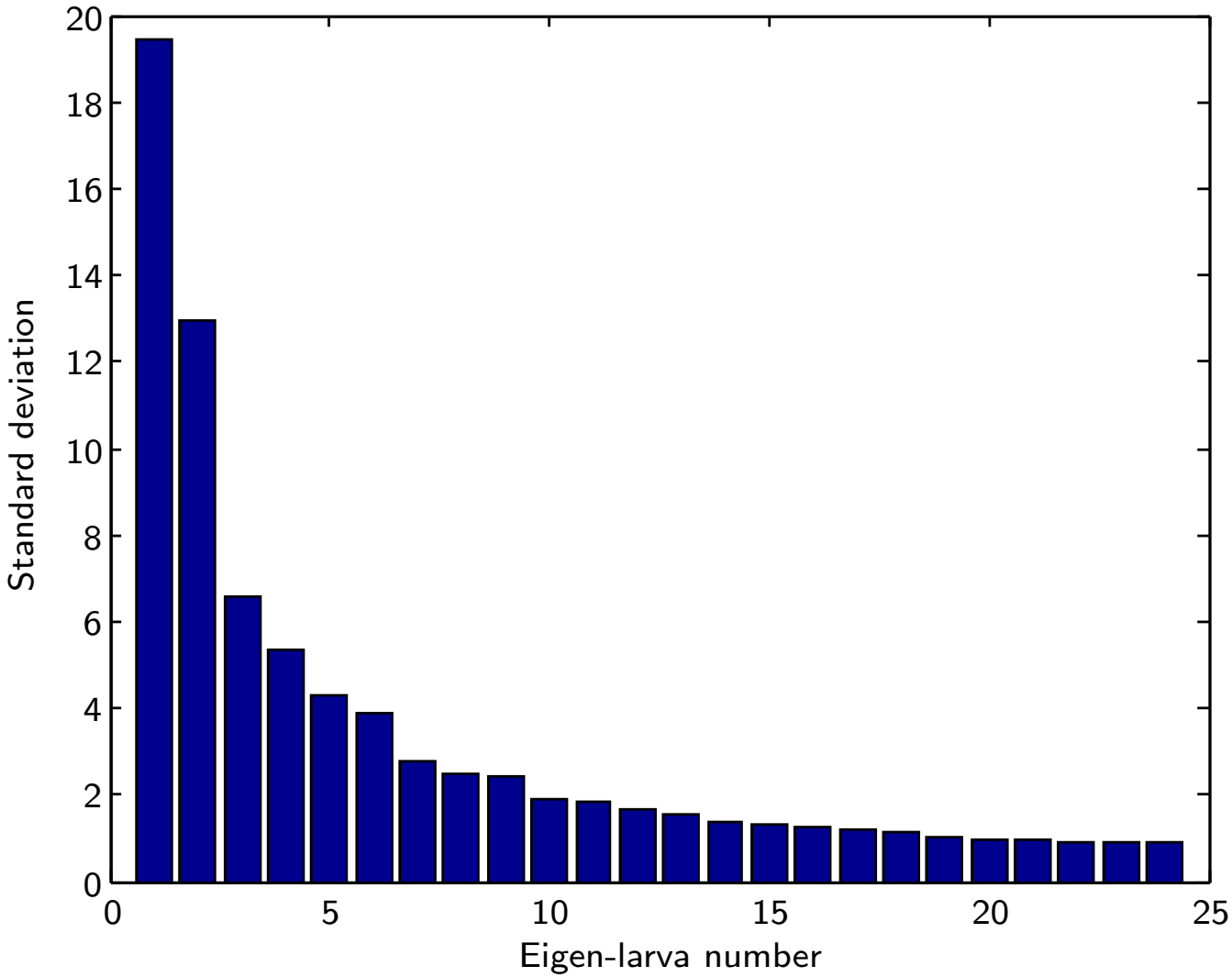
All linear $\implies$ respected by PCA.

Dimensionality reduction

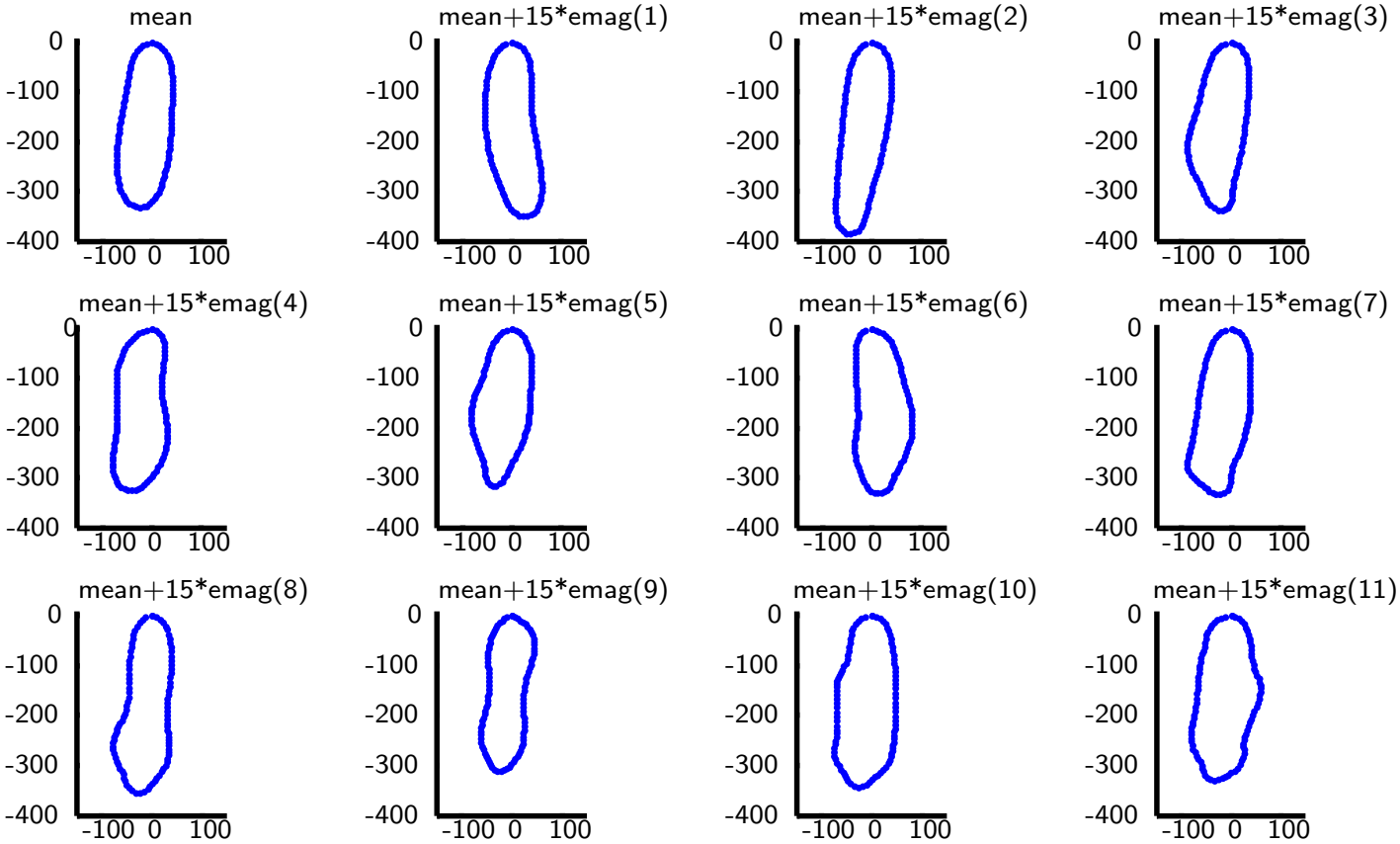


cf. for C.elegans, 95% of variance from 4 eigen-worms.

Dimensionality reduction



Eigen-larvae



Trajectories for accepted headsweeps