#### ck rings from fluid mech

Subhaneil Lahiri

based on arXiv:0905.3404 [kep-th] with Shiraz Minusila and arXiv:0903.4734 [kep-th] with Jyotivnoy Bhattachaya May 1, 2009

#### Black rings from fluid mechanics

#### Subhaneil Lahiri

based on arXiv:0705.3404 [hep-th] with Shiraz Minwalla and arXiv:0903.4734 [hep-th] with Jyotirmoy Bhattacharya

May 1, 2009

Black rings from fluid mechanics

Motivation
Higher dimensional gravity
Higher dimensional gravity

- 1. Why higher D gravity?
- 2. 1/D expansion

2009-05-01

3. perturbation theory. SU(2), SU(infinity).

Motivation Higher dimensional gravity

General relativity makes sense in any # of dimensions. D is a parameter. We vary parameter to understand the theory better: c.f. coupling constants, gauge groups, ... Sometimes we need extra dimensions: string theory, large extra dimensions scenarios, ... is grown the same in D > 47

ligher dimensional gravity

#### Higher dimensional gravity

General relativity makes sense in any # of dimensions.

D is a parameter.

We vary parameters to understand the theory better: c.f. coupling constants, gauge groups, ...

Sometimes we need extra dimensions: string theory, large extra dimensions scenarios, . . .

Is gravity the same in D > 4?



1/2 Black holes have thermodynamics. Plot phase diagBlack holes have thermodynamics. Plot phase diag

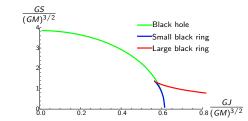
#### Black holes in four vs. five dimensions

In four dimensions there are horizon topology and black hole uniqueness theorems.

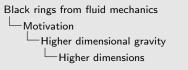
Motivation

Higher dimensional gravity

In five dimensions, we are allowed an  $S^1 \times S^2$  horizon as well – the black ring. [Emparan, Reall]



For a range of energies and angular momenta, it is possible to have two black ring and one black hole solutions - violating uniqueness.



2009-05-01

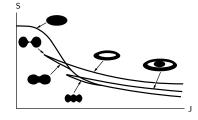
1/3 One possible phase diag.  $\exists$  another

Higher dimensions Motivation Higher dimensional gravity



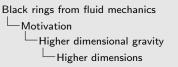
#### Higher dimensions

For  $D \ge 6$ : no exact solutions (except Myers-Perry). Approximate solutions for  $R_{S^1} \gg R_{S^3}$ .



[Emparan et al.]





2009-05-01

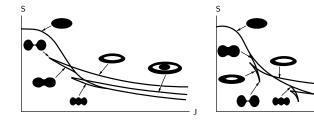
2/3 Common features: robust

Higher dimensions Motivation Higher dimensional gravity



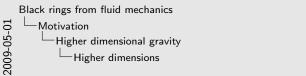
#### Higher dimensions

For  $D \ge 6$ : no exact solutions (except Myers-Perry). Approximate solutions for  $R_{S^1} \gg R_{S^3}$ .



[Emparan et al.]





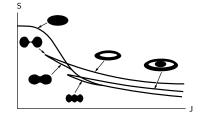
3/3 Positive Yamabe. Not every allowed exists:  $S^3/\Gamma$ 

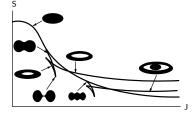
#### Motivation Higher dimensional gravity

# For $D \ge 6$ no exact collisions (except Myern-Perry). Approximate solutions for $R_D \supset R_D$ . Other topologies? Globar Spolutions: Solution (except Myern-Perry). Approximate solutions for $R_D \supset R_D$ . [Collect Spolutions]

#### Higher dimensions

For  $D \ge 6$ : no exact solutions (except Myers-Perry). Approximate solutions for  $R_{S^1} \gg R_{S^3}$ .





Other topologies?

[Emparan et al.] [Galloway, Schön]



Black rings from fluid mechanics

Motivation

The AdS/CFT correspondence

The AdS/CFT correspondence

2009-05-01

### The AdS/CFT correspondence

1. We'll look at deformed version: confining

Gravitational theory  $\Leftrightarrow$  Non-gravitational theory

The AdS/CFT correspondence

Low curvature Strong coupling

Deconfinement Black holes



Black rings from fluid mechanics

Motivation

The AdS/CFT correspondence
Black holes and fluid mechanics

ong wavelengths, deconfined plasma described by fluid mechanics.

y input: equation of state, transport coefficients. Also works at strong ling.

Black holes and fluid mechanics

#### Black holes and fluid mechanics

1/2

- 1. Compute eos, trans, difficult in CFT.
- 2. Once we know them, strong coupling doesn't matter

At long wavelengths, deconfined plasma described by fluid mechanics.

Only input: equation of state, transport coefficients. Also works at strong coupling.

The AdS/CFT correspondence

ng wavelengths, deconfined plasma described by fluid mechanics.
input: equation of state, transport coefficients. Also works at strong
leg.

Black holes and fluid mechanics

Black holes and fluid mechanics

2/2

1. Compute eos, trans, difficult in CFT.

Black holes and fluid mechanics

- 2. Once we know them, strong coupling doesn't matter
- 3. Take data from static. Predict features of non-static

At long wavelengths, deconfined plasma described by fluid mechanics.

Only input: equation of state, transport coefficients. Also works at strong coupling.

The AdS/CFT correspondence

Equation of state: black hole thermodynamics, static case. Transport coefficients: small fluctuations, e.g.  $\eta/s = 1/4\pi$ . [Son,Starinets]

⇒ universal features of black holes at long wavelengths.

2009-05-01

Outline

Outline

Maintain

Plannabil steep

Relicionist field mechanics

Of Time democrate configurations

Office democrate configurations

Office democrate generalisations

Office democrating generalisations

Office democrating generalisations

Outline

- Motivation
- 2 Plasmaball setup
- 3 Relativistic fluid mechanics
- 4 Three dimensional configurations
- 5 Higher dimensional generalisations
- 6 Summary

Outline

Plasmaballs in confining theories

Plasmaballs in confining theories

-Plasmaballs in confining theories

- 1. Why 1st order
- 2. with gravity dual
- 3.  $l_{\rm mfp} \sim R_{\theta}$ . Reduce D = d + 2. n 1 angular momenta.

Plasmaballs are a bubbles of deconfined phase, surrounded by confined phase, held together by surface tension.

Focus on theories that come from compactifying conformal theories on a Scherk-Schwarz circle.

Leads to confining theory.



#### Confined phase

At low temperatures, gravity dual: AdS soliton:

Plasmaball setup

$$\mathrm{d}s^2 = rac{R_{\mathsf{AdS}}^2}{z^2} \left( -\mathrm{d}t^2 + F_{R_{ heta}}(z) \, \mathrm{d} heta^2 + \mathrm{d}ec{x}^2 + rac{1}{F_{R_{ heta}}(z)} \, \mathrm{d}z^2 
ight),$$

Scherk-Schwarz AdS

where 
$$F_{a}(u)=1-\left(rac{\pi z}{a}
ight)^{4}$$
 and  $R_{\mathsf{AdS}}^{2}=\sqrt{\lambda}lpha'$ .

[Witten]

Small z: Poincaré AdS<sub>5</sub> with one compact direction.

At  $z = R_{\theta}/\pi$ , the  $\theta$  circle contracts: space stops.

$$z=0$$
  $z=\frac{R}{\pi}$ 



 $ds^2 = \frac{R_{\Delta fS}^2}{r^2} \left( -F_{\beta}(x) dt^2 + d\theta^2 + d\vec{x}^2 + \frac{1}{F_{\gamma}f(x)} dx^2 \right).$ 

Horizon at  $x = \frac{\beta}{\pi}$ . Temperature:  $T = 1/\beta$ .

2009-05-01

1. Double Wick rotation

#### Deconfined phase

At high temperatures: the black brane:

$$\mathrm{d}s^2 = rac{R_\mathsf{AdS}^2}{z^2} \left( -F_eta(z) \, \mathrm{d}t^2 + \mathrm{d}\theta^2 + \mathrm{d}ec{x}^2 + rac{1}{F_eta(z)} \, \mathrm{d}z^2 
ight).$$

Plasmaball setup

Horizon at 
$$z = \frac{\beta}{\pi}$$
. Temperature:  $\mathcal{T} = 1/\beta$ .

2009-05-01

- 1. Conformal.
- 2. Different reference.
- 3. Reduction.

Deconfined phase Plasmaball setup Scherk-Schwarz AdS



#### Deconfined phase

At high temperatures: the black brane:

$$\mathrm{d}s^2 = \frac{R_{\mathsf{AdS}}^2}{z^2} \left( -F_\beta(z) \, \mathrm{d}t^2 + \mathrm{d}\theta^2 + \mathrm{d}\vec{x}^2 + \frac{1}{F_\beta(z)} \, \mathrm{d}z^2 \right).$$

Horizon at  $z = \frac{\beta}{\pi}$ . Temperature:  $\mathcal{T} = 1/\beta$ .

Dominant phase above transition temperature,  $\mathcal{T}_{\mathrm{c}}=\frac{1}{R_{\theta}}.$ 

The equation of state of the dual plasma can be found from this gravity solution.

$$\mathcal{P} = \frac{\alpha}{\mathcal{T}_c} \left( \mathcal{T}^4 - \mathcal{T}_c^4 \right).$$



Black rings from fluid mechanics

Plasmaball setup

Properties of plasmaballs

Plasmaball solutions

2009-05-01

- 1. Vertical Hawking AdS
- 2. Horizontal Hawking flat
- 3. between flat and AdS. C < 0.
- 4. Only  $\sigma(\mathcal{T}_c)$ . Ignore temp dep

Plasmaball setup Properties of plasmaballs

Plasmaball solutions

#### Plasmaball solutions

On the bulk side, deep interior looks like black brane. Far from the plasmaball, it looks like the AdS soliton. There is a domain wall that interpolates between the two.

Boundary

AdS soliton Black brane AdS soliton

In the limit of infinitely large radius, a numerical domain wall solution has been found. The surface tension and thickness can be computed from this solution.

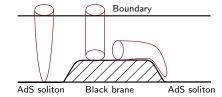
[Aharony, Minwalla, Wiseman]



#### Topology

The Scherk-Schwarz circle does not contract in the black brane region but does contract in the AdS soliton region.

Plasmaball setup Properties of plasmaballs



Horizon topology: fibre circle over the plasmaball, contracting at surfaces.



Fluid mechanics

The equation of notion we  $\nabla_{\mu} T^{\mu} = 0$ . The dynamical input is in specifying  $T^{\mu}$ .

For long wavelengths, we need only go up to one derivation terms:  $T^{\mu} = T^{\mu}_{\mu h h h} T^{\mu}_{\mu h h h} T^{\mu}_{\mu h h}$ Cellificates depend on T. Determined from state black bases.

Cellificates depend on  $T^{\mu}_{\mu}$  Determined from state black bases. This is preprinted based down at suffered, which is the state of t

Fluid mechanics

Benefit of fluid pic. Know deriv exp in gravity. Surf, puzzle solved centuries ago

- 1. equilibrium -¿ dissipative =0
- 2. rest frame temp

The equations of motion are  $\nabla_{\mu}T^{\mu\nu}=0$ . The dynamical input is in specifying  $T^{\mu\nu}$ .

Relativistic fluid mechanics

For long wavelengths, we need only go up to one derivative terms:  $T^{\mu\nu} = T^{\mu\nu}_{\rm perfect} + T^{\mu\nu}_{\rm dissipative}$ .

Coefficients depend on  $\mathcal{T}$ . Determined from static black brane.

This approximation breaks down at surfaces – but at scales  $\gg$  surface thickness we can replace these regions with a  $\delta$ -function localised surface tension.

Black rings from fluid mechanics 2009-05-01 Three dimensional configurations Solutions

Rigid rotation:  $(u^{\epsilon}, u^{\epsilon}, u^{\phi}) = \gamma(1, 0, \Omega)$ , where  $\gamma = \frac{1}{\sqrt{1-u^{\epsilon}}}$ 

#### Three dimensional configurations

1/4

1.  $\theta, \eta = 0$  - rotation + boost.

Three dimensional configurations

2. press grad - temp grad.

Rigid rotation:  $(u^t, u^r, u^\phi) = \gamma(1, 0, \Omega)$ , where  $\gamma = \frac{1}{\sqrt{1-v^2}}$ . Centripetal force provided by pressure gradient.

Black rings from fluid mechanics

Three dimensional configurations

Solutions

Three dimensional configurations

2/4

- 1.  $\theta, \eta = 0$  rotation + boost.
- 2. press grad temp grad.
- 3. extra term in q

Three dimensional configurations

Rigid rotation:  $(u^i, u^i, u^a) = \gamma(1, 0, \Omega)$ , where  $\gamma = \frac{1}{\sqrt{1-v^2}}$ . Centripetal force provided by pressure gradient. We find  $T_{\rm dissipative}^{\rm dissipative} \propto \vec{\nabla}(T/\gamma)$ .

#### Three dimensional configurations

Rigid rotation:  $(u^t, u^r, u^\phi) = \gamma(1, 0, \Omega)$ , where  $\gamma = \frac{1}{\sqrt{1-v^2}}$ . Centripetal force provided by pressure gradient.

Three dimensional configurations Solutions

We find 
$$T_{
m dissipative}^{\mu 
u} \propto \vec{
abla}(\mathcal{T}/\gamma)$$
.



- 1.  $\theta, \eta = 0$  rotation + boost.
- 2. press grad temp grad.
- 3. extra term in q

Three dimensional configurations

Rigid rotation:  $(u^i,u^i,u^i)=\gamma(1,0,\Omega)$ , where  $\gamma=\frac{1}{\sqrt{1-u^i}}$ . Contripietal force provided by pressure gradient.

We find  $T_{\rm descipations}^{\rm acc} \propto \bar{\nabla}(T/\gamma)$ .

Interior: e.o.m.  $\nabla_{\mu}T_{\rm prefers}^{\rm acc} \propto \bar{\nabla}(T/\gamma)=0$ .

#### Three dimensional configurations

Rigid rotation:  $(u^t, u^r, u^\phi) = \gamma(1, 0, \Omega)$ , where  $\gamma = \frac{1}{\sqrt{1-v^2}}$ . Centripetal force provided by pressure gradient.

Three dimensional configurations Solutions

We find 
$$T_{
m dissipative}^{\mu
u} \propto ec{
abla}(\mathcal{T}/\gamma).$$

Interior: e.o.m. 
$$abla_{\mu} T^{\mu\nu}_{\mathrm{perfect}} \propto \vec{\nabla} (\mathcal{T}/\gamma) = 0.$$



#### 4/4

- 1.  $\theta, \eta = 0$  rotation + boost.
- 2. press grad temp grad.
- 3. extra term in q
- 4. inner / outer.
- 5. 2 param.

Rigid rotation:  $(u^t, u^t, u^o) = \gamma(1, 0, \Omega)$ , where  $\gamma = \frac{1}{\sqrt{1-u^2}}$ . Centripotal force provided by pressure gradient.

faces:  $P = \pm 4$ . Relates  $(T/\gamma)$  to  $\Omega$  and position of surface.

# Three dimensional configurations

Rigid rotation:  $(u^t, u^r, u^{\phi}) = \gamma(1, 0, \Omega)$ , where  $\gamma = \frac{1}{\sqrt{1-v^2}}$ . Centripetal force provided by pressure gradient.

We find 
$$T_{
m dissipative}^{\mu
u} \propto ec{
abla}(\mathcal{T}/\gamma).$$

Interior: e.o.m. 
$$\nabla_{\mu} T^{\mu\nu}_{\mathrm{perfect}} \propto \vec{\nabla} (\mathcal{T}/\gamma) = 0.$$

Surfaces:  $\mathcal{P} = \pm \frac{\sigma}{r}$ . Relates  $(\mathcal{T}/\gamma)$  to  $\Omega$  and position of surface.

2009-05-01

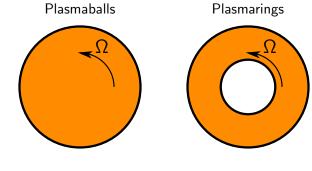
1/2



#### Solutions

714110115

We find two types of solution:

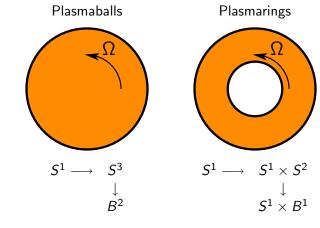


Three dimensional configurations Solutions

 $B^2$   $S^1 \times B^1$ 

#### Solutions

We find two types of solution:



2009-05-01

2009-05-01

dynamics

We compute the thermodynamic properties of the whole solution with

 $E = \int d^2x \left(T^H\right),$   $L = \int d^2x \left(r^2T^{4\phi}\right),$  $S = \int d^2x \left(\gamma s\right).$  Three dimensional configurations

Thermodynamics

#### Thermodynamics

We compute the thermodynamic properties of the whole solution with

$$E = \int \mathrm{d}^2 x \left( T^{tt} \right),$$
  $L = \int \mathrm{d}^2 x \left( r^2 T^{t\phi} \right),$   $S = \int \mathrm{d}^2 x \left( \gamma s \right).$ 

#### 2/2

2009-05-01

- 1. include KE. Different to  ${\mathcal E}$
- 2. Different to  $\mathcal{T}$
- 3. Identify with Hawking temp. const on horizon.
- 4. redshift glueballs

Thermodynamics

odynamic properties of the whole solution with  $E = \int d^2x \left(T^B\right),$   $L = \int d^2x \left(r^2T^{bb}\right),$   $S = \int d^2x \left(r^3\right).$  overall temperature and angular velocity via  $dE = T dS + \Omega M.$ 

 $T = \frac{T}{\gamma} \,, \qquad \Omega \mbox{ as before} \,. \label{eq:T}$ 

Three dimensional configurations

Thermodynamics

#### Thermodynamics

We compute the thermodynamic properties of the whole solution with

$$E = \int d^2x \left(T^{tt}\right),$$

$$L = \int d^2x \left(r^2 T^{t\phi}\right),$$

$$S = \int d^2x \left(\gamma s\right).$$

Then we compute an overall temperature and angular velocity via

$$dE = TdS + \Omega dL$$

we find

$$T=rac{\mathcal{T}}{\gamma}\,,\qquad \Omega$$
 as before .

Black rings from fluid mechanics

Three dimensional configurations

Thermodynamics

Phase diagram

1/2

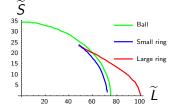
2009-05-01

- 1. approx break down at ext
- 2. c.f. flat

Phase diagram Three dimensional configurations Thermodynamics



#### Phase diagram



Black rings from fluid mechanics

Three dimensional configurations

Thermodynamics

Phase diagram

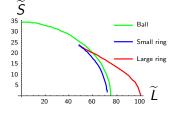
2/2

2009-05-01

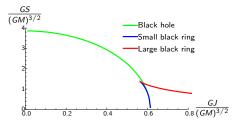
1. approx break down at ext

Phase diagram

- 2. c.f. flat
- 3. upper bound on L
- 4. Precise and rigorous for SSAdS. Qualitative for flat.
- 5. Left, easy. Right hard.



Three dimensional configurations



Thermodynamics

1. not full list. just ones we can imagine as fluids

2009-05-01

1/3

Higher dimensional generalisations Six dimensional gravity

## Topologies in six dimensions

$$S^4$$

$$S^3 \times S^1$$

 $S^2 \times S^2$ 

$$S^2 \times T^2$$



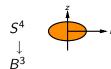
# 2/3

2009-05-01

- 1. not full list. just ones we can imagine as fluids
- 2. cylindrical coords. Suppress  $\phi$

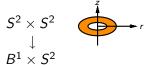
Topologies in six dimensions

#### Topologies in six dimensions









$$S^2 \times T^2$$
  $\downarrow$   $B^1 \times T^2$ 

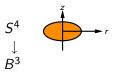
# 3/3

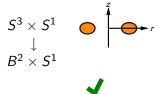
2009-05-01

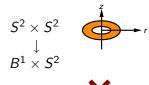
- 1. not full list. just ones we can imagine as fluids
- 2. cylindrical coords. Suppress  $\phi$

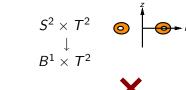
Topologies in six dimensions

#### Topologies in six dimensions











#### Solving equations of motion

Again: rigid rotation  $(u^t, u^r, u^{\phi}, u^z) = \gamma(1, 0, \Omega, 0)$ .

Again: 
$$\frac{T}{\gamma} = T = \text{constant}$$
.

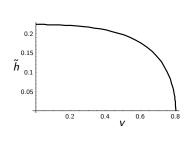
Now: surface satisfies  $\mathcal{P} = \sigma K^{\mu}_{\mu}$ .

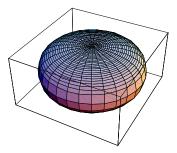
# Ordinary balls



Increase  $\Omega$ ,  $\mathcal{P}(0) \rightarrow 0$ 

2009-05-01



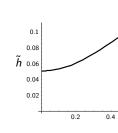


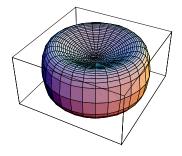
Black rings from fluid mechanics

Higher dimensional generalisations

Solutions in six dimensions
Pinched balls

#### Pinched balls





- 1. Emparan, Myers, wavy BH.
- 2. Increase  $\Omega$ ,  $h(0) \rightarrow 0$

2009-05-01

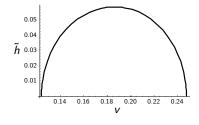
0.6

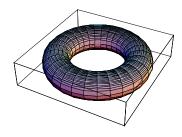
0.8

Black rings from fluid mechanics Higher dimensional generalisations Solutions in six dimensions -Rings

2009-05-01







Higher dimensional generalisations Solutions in six dimensions

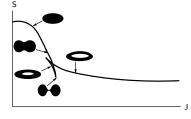
Phase diagram

# 1/3

2009-05-01

 $1. \ \ compare \ with \ proposals$ 

### Phase diagram



[Bhardwaj,Bhattacharya]

Black rings from fluid mechanics

Higher dimensional generalisations

Solutions in six dimensions

Phase diagram

#### 2/3

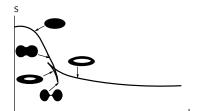
2009-05-01

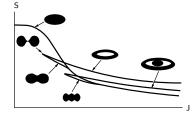
- 1. compare with proposals
- 2. not good

Phase diagram



#### Phase diagram





[Bhardwaj,Bhattacharya]

Higher dimensional generalisations Solutions in six dimensions

Black rings from fluid mechanics

Higher dimensional generalisations

Solutions in six dimensions

Phase diagram

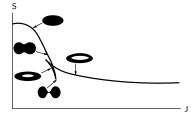


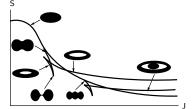
# Phase diagram

3/3

2009-05-01

- 1. compare with proposals
- 2. not good
- 3. good. Myers-Perry continues past wavy BH





[Bhardwaj,Bhattacharya]

Higher dimensional generalisations Solutions in six dimensions

# Topologies in seven dimensions

1/2

2009-05-01

1. again not full list

$$S^4 \times S^1$$

$$S^3 \times T^2$$

$$S^3 \times S^2$$

$$S^2 \times S^2 \times S^1$$

$$S^2 \times T^3$$

Topologies in seven dimensions

#### Topologies in seven dimensions

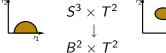
2/2

2009-05-01

- 1. again not full list
- 2. Ball, ring, torus.
- 3. hollow exist?.



$$S^4 \times S^1$$
 $\downarrow$ 
 $B^3 \times S^1$ 



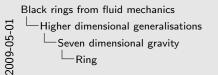
$$S^{3} \times S^{2} \qquad S^{2} \times S^{2} \times S^{1} \qquad S^{2} \times T^{3} \qquad S^{2$$

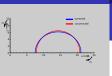
For ring, 
$$B^3 imes S^1$$
, take  $\epsilon = \frac{R_{B^3}}{R_{c^1}}$  small.

For 'torus', 
$$B^2 \times T^2$$
, take  $\epsilon = \frac{R_{B^2}}{R_{T^2}}$  small.

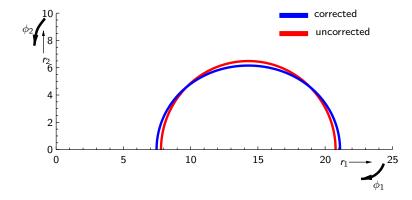
Expand in 
$$\epsilon$$
. At  $\mathcal{O}(\epsilon^0)$  – just a tube.

Similar to black-fold construction of Emparan et al.

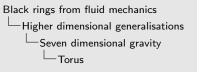








Higher dimensional generalisations Seven dimensional gravity

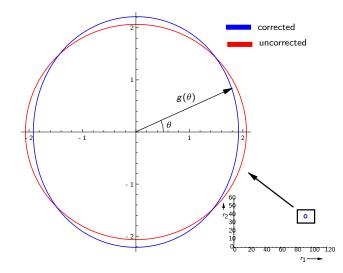




whole new topology

2009-05-01

#### Torus



Higher dimensional generalisations Seven dimensional gravity

Black rings from fluid mechanics

Summary

Summary

Summary

We can get height to some problems in classical gravity from floid mechanics in AdS/CFT.

In the dimensions—— qualitative agreement with flat space gravity.

In six dimensions—proposal for phase diagram.

Its waves dimensions—— ma topology.

Feature: manufacial volutions for D = T, phase diagram.

Grappy-Laffamma vs. Pictase-Rayleigh.

[Existential et al.]

#### Summary

We can get insight to some problems in classical gravity from fluid mechanics in AdS/CFT.

Summary

In five dimensions – qualitative agreement with flat space gravity.

In six dimensions – proposal for phase diagram.

In seven dimensions - new topology.

Future: numerical solutions for D = 7, phase diagram.

Gregory-Laflamme vs. Plateau-Rayleigh.

[Caldarelli et al.]

