

Black rings from fluid mechanics

Subhaneil Lahiri

based on arXiv:0705.3404 [hep-th] with Shiraz Minwalla
and arXiv:0903.4734 [hep-th] with Jyotirmoy Bhattacharya

May 1, 2009

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General relativity makes sense in any $\#$ of dimensions.

D is a parameter.

We vary parameters to understand the theory better: c.f. coupling constants, gauge groups, ...

Sometimes we need extra dimensions: string theory, large extra dimensions scenarios, ...

Is gravity the same in $D > 4$?

Higher dimensional gravity

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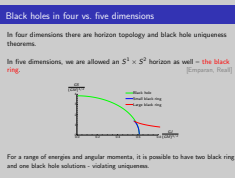
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Black rings from fluid mechanics

Motivation

Higher dimensional gravity

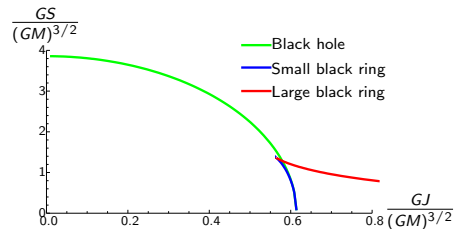
Black holes in four vs. five dimensions



Black holes in four vs. five dimensions

In four dimensions there are horizon topology and black hole uniqueness theorems.

In five dimensions, we are allowed an $S^1 \times S^2$ horizon as well – the black ring. [Empanan, Reall]



For a range of energies and angular momenta, it is possible to have two black ring and one black hole solutions - violating uniqueness.

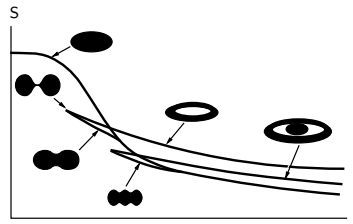
For $D \geq 6$: no exact solutions (except Myers-Perry). Approximate solutions for $R_{S^1} \gg R_{S^3}$.



[Emparan et al.]

Higher dimensions

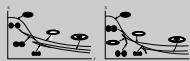
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1/3 One possible phase diag. \exists another

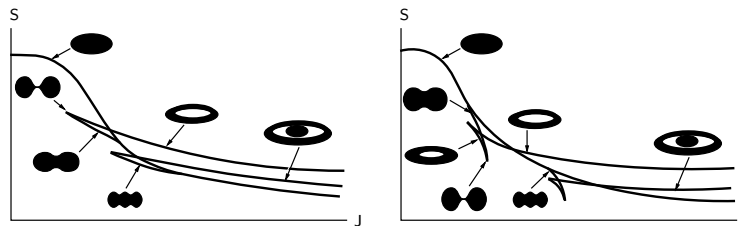
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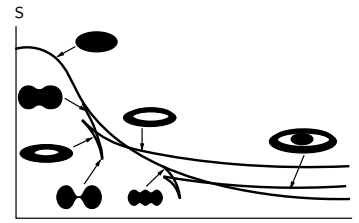
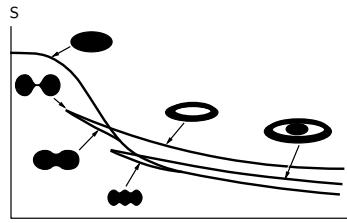


Other topologies?

[Empanan et al.]
[Galloway, Schön]

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Other topologies?

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The AdS/CFT correspondence

Gravitational theory \leftrightarrow Non-gravitational theory

Low curvature Strong coupling

Deconfinement Black holes

1. We'll look at deformed version: confining

Gravitational theory \Leftrightarrow Non-gravitational theory

Low curvature

Strong coupling

Deconfinement

Black holes

Black holes and fluid mechanics

1/2

1. Compute eos, trans, difficult in CFT.
2. Once we know them, strong coupling doesn't matter

At long wavelengths, deconfined plasma described by fluid mechanics.

Only input: equation of state, transport coefficients. Also works at strong coupling.

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Equation of state: black hole thermodynamics, static case.
Transport coefficients: small fluctuations, e.g. $\eta/s = 1/4\pi$. [Son,Starinets]

\Rightarrow universal features of black holes at long wavelengths.

Black holes and fluid mechanics

2/2

1. Compute eos, trans, difficult in CFT.
2. Once we know them, strong coupling doesn't matter
3. Take data from static. Predict features of non-static

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- 1 Motivation
- 2 Plasmaball setup
- 3 Relativistic fluid mechanics
- 4 Three dimensional configurations
- 5 Higher dimensional generalisations
- 6 Summary

Outline

- 1 Motivation
- 2 Plasmaball setup
- 3 Relativistic fluid mechanics
- 4 Three dimensional configurations
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Plasmaballs are a bubbles of deconfined phase, surrounded by confined phase, held together by surface tension.

Focus on theories that come from compactifying conformal theories on a Scherk-Schwarz circle.

Leads to confining theory.

Plasmaballs in confining theories

1. Why 1st order
2. with gravity dual
3. $I_{\text{mfp}} \sim R_\theta$. Reduce - $D = d + 2$. $n - 1$ angular momenta.

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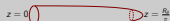
Leads to confining theory.

At low temperatures, gravity dual: **AdS soliton**:

$$ds^2 = \frac{R_{\text{AdS}}^2}{z^2} \left(-dt^2 + F_{R_\theta}(z) d\theta^2 + d\vec{x}^2 + \frac{1}{F_{R_\theta}(z)} dz^2 \right),$$

where $F_a(u) = 1 - \left(\frac{\pi u}{a}\right)^4$ and $R_{\text{AdS}}^2 = \sqrt{\lambda} \alpha'$.

[Witten]

Small z : Poincaré AdS_5 with one compact direction.At $z = R_\theta/\pi$, the θ circle contracts: space stops.

Confined phase

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Horizon at $z = \frac{\beta}{\pi}$. Temperature: $\mathcal{T} = 1/\beta$.

1/2

1. Double Wick rotation

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$$\mathcal{P} = \frac{\alpha}{\mathcal{T}_c} (\mathcal{T}^4 - \mathcal{T}_c^4).$$

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2/2

1. Conformal.
2. Different reference.
3. Reduction.

1. Vertical Hawking - AdS
2. Horizontal Hawking - flat
3. between flat and AdS. $C < 0$.
4. Only $\sigma(\mathcal{T}_c)$. Ignore temp dep

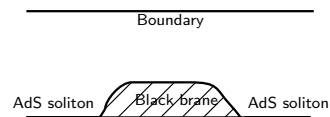
On the bulk side, deep interior looks like black brane. Far from the plasmaball, it looks like the AdS soliton. There is a domain wall that interpolates between the two.



In the limit of infinitely large radius, a numerical domain wall solution has been found. The surface tension and thickness can be computed from this solution. [Aharony, Minwalla, Wiseman]

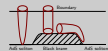
Plasmaball solutions

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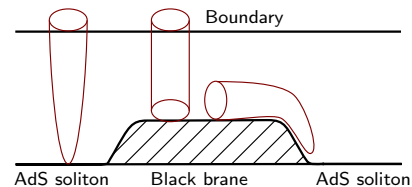
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Horizon topology: fibre circle over the plasmaball, contracting at surfaces.

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Black rings from fluid mechanics

└ Relativistic fluid mechanics

└ Fluid mechanics

Fluid mechanics

The equations of motion are $\nabla_\mu T^{\mu\nu} = 0$. The dynamical input is in specifying $T^{\mu\nu}$.For long wavelengths, we need only go up to one derivative terms:
 $T^{\mu\nu} = T_{\text{perfect}}^{\mu\nu} + T_{\text{dissipative}}^{\mu\nu}$ Coefficients depend on \mathcal{T} . Determined from static black brane.This approximation breaks down at surfaces – but at scales \gg surface thickness we can replace these regions with a δ -function localised surface tension.

Relativistic fluid mechanics

Fluid mechanics

Benefit of fluid pic. Know deriv exp in gravity. Surf, puzzle solved centuries ago

1. equilibrium - $\dot{\gamma}$ dissipative = 0
2. rest frame temp

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Three dimensional configurations

1/4

1. $\theta, \eta = 0$ - rotation + boost.
2. press grad - temp grad.

Rigid rotation: $(u^t, u^r, u^\phi) = \gamma(1, 0, \Omega)$, where $\gamma = \frac{1}{\sqrt{1-v^2}}$.
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Surfaces: $\mathcal{P} = \pm \frac{\sigma}{r}$. Relates (\mathcal{T}/γ) to Ω and position of surface.

Three dimensional configurations

4/4

1. $\theta, \eta = 0$ - rotation + boost.
2. press grad - temp grad.
3. extra term in q
4. inner / outer.
5. 2 param.

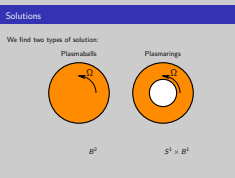
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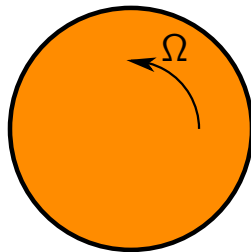
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Solutions

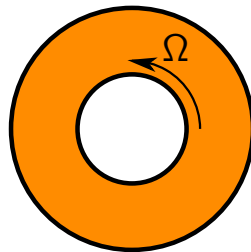
We find two types of solution:

Plasmaballs



$$B^2$$

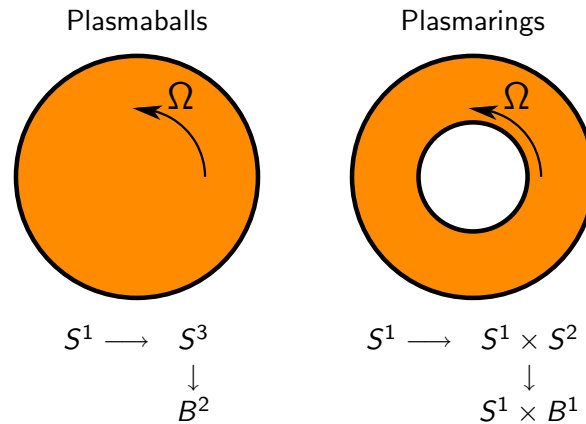
Plasmarings



$$S^1 \times B^1$$

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- Black rings from fluid mechanics
 - Three dimensional configurations
 - Thermodynamics
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We compute the thermodynamic properties of the whole solution with

$$E = \int d^2x (T^{tt}),$$

$$L = \int d^2x (r^2 T^{t\phi}),$$

$$S = \int d^2x (\gamma s).$$

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1/2

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$$T = \frac{\mathcal{T}}{\gamma}, \quad \Omega \text{ as before.}$$

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Then we compute an overall temperature and angular velocity via

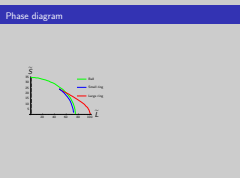
$$dE = T dS + \Omega dL,$$

we find

$$T = \frac{\mathcal{T}}{\gamma}, \quad \Omega \text{ as before.}$$

2/2

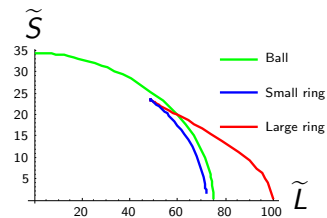
1. include KE. Different to \mathcal{E}
2. Different to \mathcal{T}
3. Identify with Hawking temp. const on horizon.
4. redshift glueballs

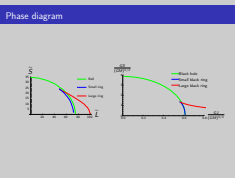


Phase diagram

1/2

1. approx break down at ext
2. c.f. flat

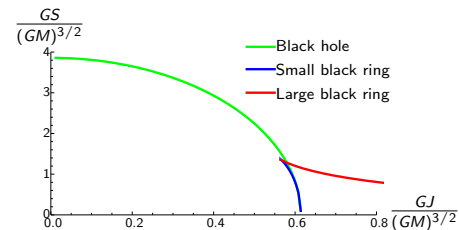
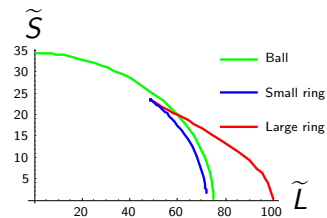




Phase diagram

2/2

1. approx break down at ext
2. c.f. flat
3. upper bound on L
4. Precise and rigorous for SSAdS. Qualitative for flat.
5. Left, easy. Right - hard.



Topologies in six dimensions

$$S^4$$

$$S^3 \times S^1$$

$$S^2 \times S^2$$

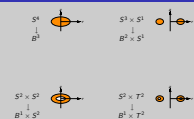
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1/3

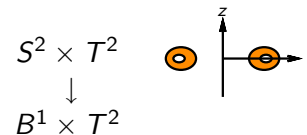
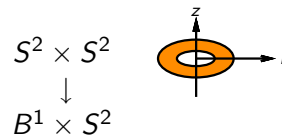
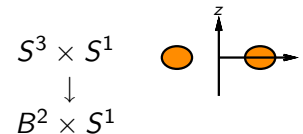
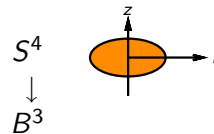
1. not full list. just ones we can imagine as fluids

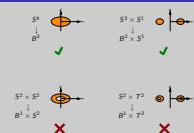


Topologies in six dimensions

2/3

1. not full list. just ones we can imagine as fluids
2. cylindrical coords. Suppress ϕ

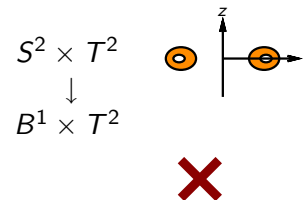
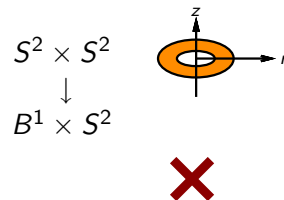
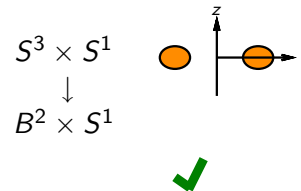
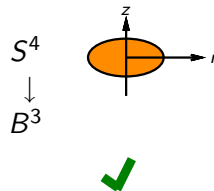




Topologies in six dimensions

3/3

1. not full list. just ones we can imagine as fluids
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- Black rings from fluid mechanics
 - Higher dimensional generalisations
 - Six dimensional gravity
 - Solving equations of motion

Again: rigid rotation $(u^t, u^r, u^\phi, u^z) = \gamma(1, 0, \Omega, 0)$.

Again: $\frac{\mathcal{I}}{\gamma} = T = \text{constant}$.

Now: surface satisfies $\mathcal{P} = \sigma K_\mu^\mu$.

Solving equations of motion

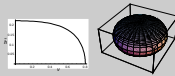
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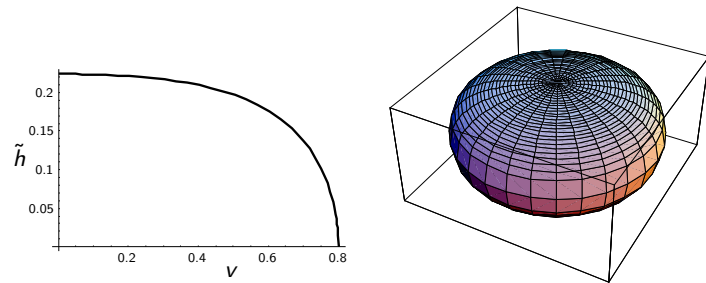
- Black rings from fluid mechanics
 - Higher dimensional generalisations
 - Solutions in six dimensions
 - Ordinary balls

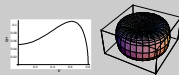
Ordinary balls



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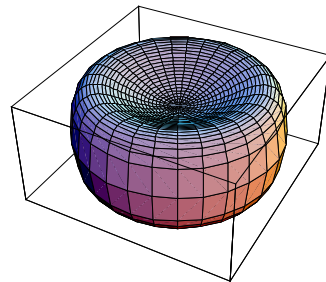
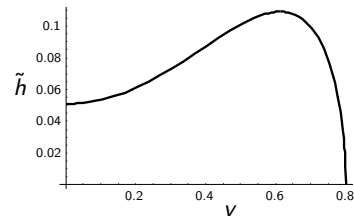
Increase Ω , $\mathcal{P}(0) \rightarrow 0$



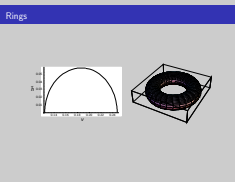


Pinched balls

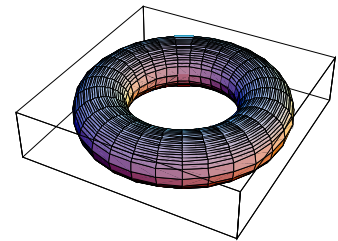
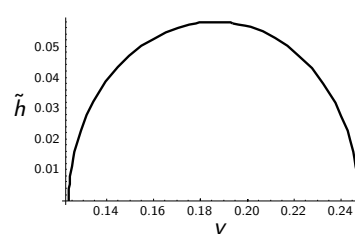
1. Emparan, Myers, wavy BH.
2. Increase Ω , $h(0) \rightarrow 0$

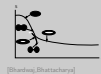


- Black rings from fluid mechanics
 - Higher dimensional generalisations
 - Solutions in six dimensions
 - Rings

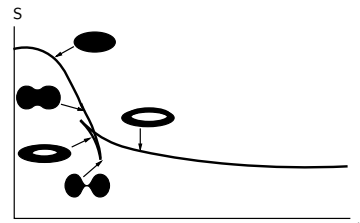


Rings





Phase diagram

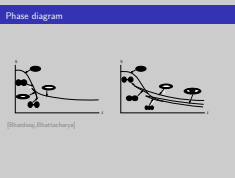


[Bhardwaj, Bhattacharya]

1/3

1. compare with proposals

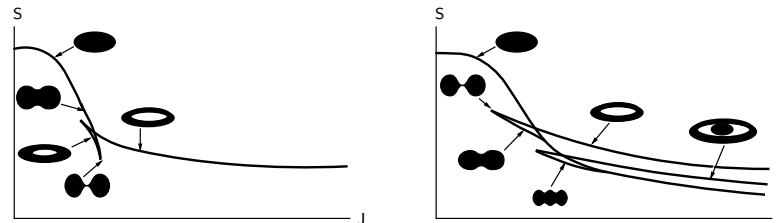
- Black rings from fluid mechanics
 - Higher dimensional generalisations
 - Solutions in six dimensions
 - Phase diagram



Phase diagram

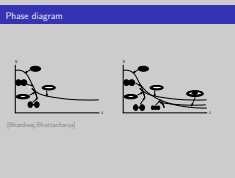
2/3

1. compare with proposals
2. not good



[Bhardwaj, Bhattacharya]

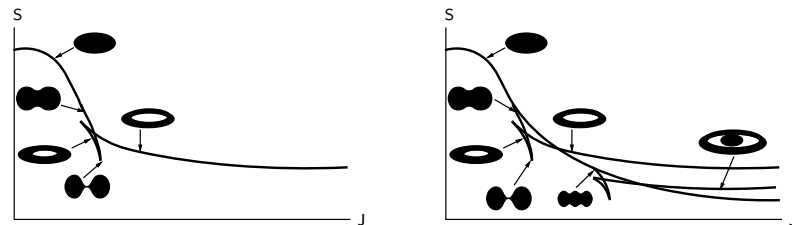
- Black rings from fluid mechanics
 - Higher dimensional generalisations
 - Solutions in six dimensions
 - Phase diagram



Phase diagram

3/3

1. compare with proposals
2. not good
3. good. Myers-Perry continues past wavy BH



[Bhardwaj, Bhattacharya]

$$S^5 \qquad S^4 \times S^1 \qquad S^3 \times T^2$$

$$S^3 \times S^2 \qquad S^2 \times S^2 \times S^1 \qquad S^2 \times T^3$$

Topologies in seven dimensions

1/2

1. again not full list

$$S^5$$

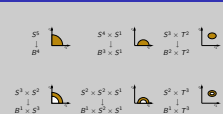
$$S^4 \times S^1$$

$$S^3 \times T^2$$

$$S^3 \times S^2$$

$$S^2 \times S^2 \times S^1$$

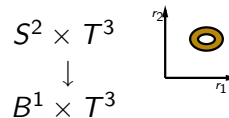
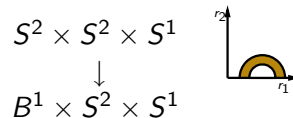
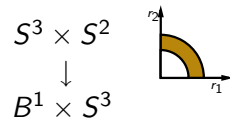
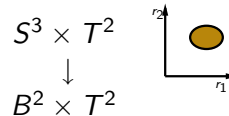
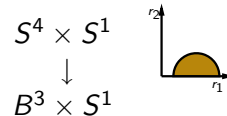
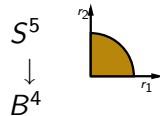
$$S^2 \times T^3$$



Topologies in seven dimensions

2/2

1. again not full list
2. Ball, ring, torus.
3. hollow exist?.



- Black rings from fluid mechanics
 - Higher dimensional generalisations
 - Seven dimensional gravity
 - Approximate solutions

Approximate solutions

For ring, $B^3 \times S^1$, take $\epsilon = \frac{R_{B^3}}{R_{S^1}}$ small.

For 'torus', $B^2 \times T^2$, take $\epsilon = \frac{R_{B^2}}{R_{T^2}}$ small.

Expand in ϵ . At $\mathcal{O}(\epsilon^0)$ – just a tube.

Similar to black-fold construction of Emparan et al.

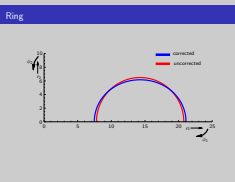
Approximate solutions

For ring, $B^3 \times S^1$, take $\epsilon = \frac{R_{B^3}}{R_{S^1}}$ small.

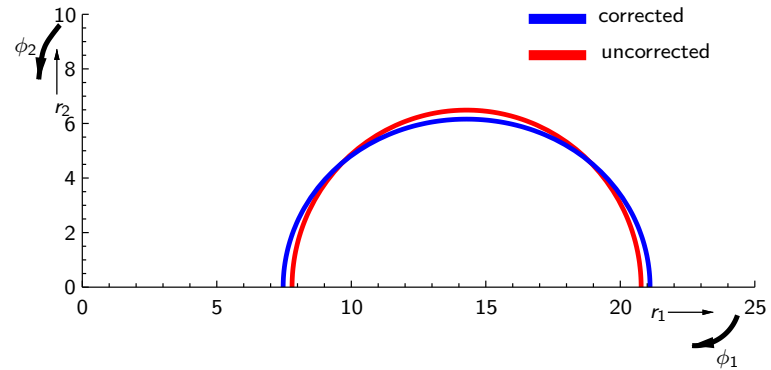
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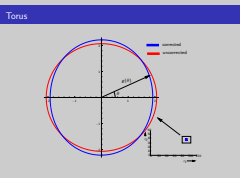
Similar to black-fold construction of Emparan et al.



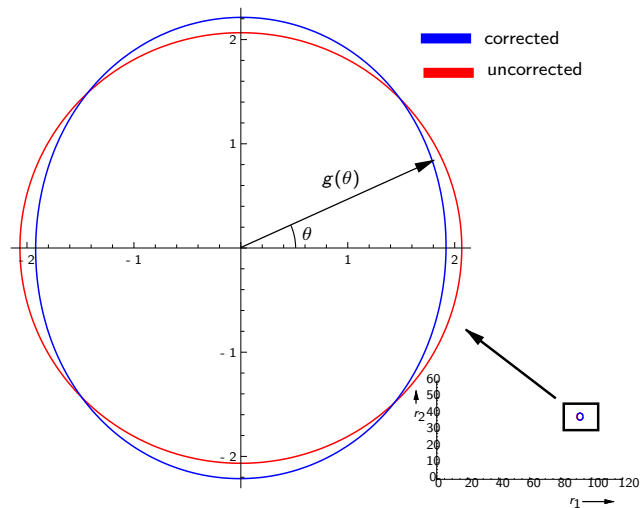
Ring



whole new topology



Torus



We can get insight to some problems in classical gravity from fluid mechanics in AdS/CFT.

In five dimensions – qualitative agreement with flat space gravity.

In six dimensions – proposal for phase diagram.

In seven dimensions – new topology.

Future: numerical solutions for $D = 7$, phase diagram.

Gregory-Laflamme vs. Plateau-Rayleigh.

[Caldarelli et al.]

Summary

We can get insight to some problems in classical gravity from fluid mechanics in AdS/CFT.

In five dimensions – qualitative agreement with flat space gravity.

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[Caldarelli et al.]