Homework I

Financial Econometrics

1) *Let X_t and Y_t be mutually uncorrelated covariance stationary processes, i.e, cov(X_s, Y_t) = 0 for all s, t. Show that $Z_t = X_t + Y_t$ is stationary with autocovariance function equal to the sum of the autocovariance function of X_t and Y_t.*

Since X_t and Y_t are mutually uncorrelated covariance stationary processes, X_t and Y_t are known to be independent.
Accordingly,

$$var(Z_t) = var(X_t + Y_t)$$
$$var(Z_t) = var(X_t) + var(Y_t) < \infty$$

$$E(Z_t) = E(X_t + Y_t) = \mu_x + \mu_y$$

$$\gamma_{x+y}(k) = cov(X_{t+k} + Y_{t+k}, X_t + Y_t)$$
$$= cov(X_{t+k}, X_t) + cov(X_{t+k}, Y_t) + cov(Y_{t+k}, X_t) + cov(Y_{t+k}, Y_t)$$
since X_t and Y_t are independent,
$$= cov(X_{t+k}, X_t) + cov(Y_{t+k}, Y_t)$$
$$\gamma_{x+y}(k) = \gamma_x(k) + \gamma_y(k)$$

Since the mean and autocovariance functions are independent of t, $Z_t = X_t + Y_t$ is stationary.

2) *Let Z_t be weakly stationary with mean zero. Consider*

$$X_t = Z_t + aZ_{t-1}, \quad Y_t = Z_t + bZ_{t-1}$$

where $|a|<1$ and $b = 1/a$.

(a) *Express the autocovariance functions of X_t and Y_t in terms of the autocovariance function of Z_t.*

Autocovariance function of X_t:

$$\gamma_x(k) = cov(X_{t+k}, X_t)$$
$$= cov(Z_{t+k} + aZ_{t+k-1}, Z_t + aZ_{t-1})$$

$$= cov(Z_{t+k}, Z_t) + cov(Z_{t+k}, aZ_{t-1}) + cov(aZ_{t+k-1}, Z_t) + cov(\,aZ_{t+k-1}, aZ_{t-1})$$
$$= cov(Z_{t+k}, Z_t) + a \times cov(Z_{t+k}, Z_{t-1}) + a \times cov(Z_{t+k-1}, Z_t)$$
$$+ a^2 \times cov(\,Z_{t+k-1}, Z_{t-1})$$
$$= \gamma(k) + a\gamma(k+1) + a\gamma(k-1) + a^2\gamma(k)$$
$$\gamma_x(k) = (1 + a^2)\gamma(k) + \, a\gamma(k+1) + a\gamma(k-1)$$

Autocovariance function of Y_t:

$$\gamma_y(k) = cov(Y_{t+k}, Y_t\,)$$
$$= cov(Z_{t+k} + \, bZ_{t+k-1}, Z_t + \, bZ_{t-1})$$
$$= cov(Z_{t+k}, Z_t) + cov(Z_{t+k}, bZ_{t-1}) + cov(bZ_{t+k-1}, Z_t) + cov(\,bZ_{t+k-1}, bZ_{t-1})$$
$$= cov(Z_{t+k}, Z_t) + b \times cov(Z_{t+k}, Z_{t-1}) + b \times cov(Z_{t+k-1}, Z_t)$$
$$+ b^2 \times cov(\,Z_{t+k-1}, Z_{t-1})$$
$$= \gamma(k) + b\gamma(k+1) + b\gamma(k-1) + b^2\gamma(k)$$
$$\gamma_y(k) = (1 + b^2)\gamma(k) + \, b\gamma(k+1) + b\gamma(k-1)$$

(b) Show that X_t and Y_t have the same autocorrelation functions.

The autocorrelation function formula for X_t is

$$\rho_x = \frac{\gamma_x(k)}{\gamma_x(0)}$$

Calculating for $\gamma_x(k)$ and $\gamma_x(0)$, we get

$$\gamma_x(k) = (1 + a^2)\gamma(k) + \, a\gamma(k+1) + a\gamma(k-1)$$
$$\gamma_x(0) = (1 + a^2)\gamma(0) + \, a\gamma(1) + a\gamma(-1)$$
$$\gamma(1) = \gamma(-1) \text{ from symmetry}$$
$$\gamma_x(0) = (1 + a^2)\gamma(0) + \, a\gamma(1) + a\gamma(1)$$
$$\gamma_x(0) = (1 + a^2)\gamma(0) + \, 2a\gamma(1)$$

Autocorrelation function of X_t is

$$\rho_x = \frac{\gamma_x(k)}{\gamma_x(0)}$$
$$\rho_x = \frac{(1 + a^2)\gamma(k) + \, a\gamma(k+1) + a\gamma(k-1)}{(1 + a^2)\gamma(0) + \, 2a\gamma(1)}$$

The autocorrelation function formula for Y_t is

$$\rho_y = \frac{\gamma_y(k)}{\gamma_y(0)}$$

Calculating for $\gamma_y(k)$ and $\gamma_y(0)$, we get

$$\gamma_y(k) = (1 + b^2)\gamma(k) + b\gamma(k + 1) + b\gamma(k - 1)$$

$$\gamma_y(0) = (1 + b^2)\gamma(0) + b\gamma(1) + b\gamma(-1)$$

Likewise, $\gamma(1) = \gamma(-1)$ from symmetry

$$\gamma_y(0) = (1 + b^2)\gamma(0) + b\gamma(1) + b\gamma(1)$$

$$\gamma_y(0) = (1 + b^2)\gamma(0) + 2b\gamma(1)$$

Autocorrelation function of Y_t is

$$\rho_y = \frac{\gamma_y(k)}{\gamma_y(0)}$$

$$\rho_y = \frac{(1 + b^2)\gamma(k) + b\gamma(k + 1) + b\gamma(k - 1)}{(1 + b^2)\gamma(0) + 2b\gamma(1)}$$

Given $b = 1/a$,

$$\rho_y = \frac{\left(1 + \frac{1}{a^2}\right)\gamma(k) + \frac{1}{a}\gamma(k + 1) + \frac{1}{a}\gamma(k - 1)}{\left(1 + \frac{1}{a^2}\right)\gamma(0) + 2\frac{1}{a}\gamma(1)}$$

$$\rho_y = \frac{a^2\gamma(k) + \gamma(k) + a\gamma(k + 1) + a\gamma(k - 1)}{a^2} \times \frac{a^2}{a^2\gamma(0) + \gamma(0) + 2a\gamma(1)}$$

$$\rho_y = \frac{(1 + a^2)\gamma(k) + a\gamma(k + 1) + a\gamma(k - 1)}{(1 + a^2)\gamma(0) + 2a\gamma(1)}$$

Therefore, X_t and Y_t have the same autocorrelation functions.

3) *Consider an AR(2) process*

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \varepsilon_t,\ \varepsilon_t \sim WN(0, \sigma^2)$$

(a) *Show that the AR(2) process becomes weakly stationary if*

$$\alpha_1 + \alpha_2 < 1, \quad -\alpha_1 + \alpha_2 < 1, \quad -1 < \alpha_2 < 1$$

Rewriting $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \varepsilon_t$, we get

$$(1 - \alpha_1 L - \alpha_2 L^2) X_t = \varepsilon_t$$

$$\frac{1}{\alpha_1} \text{ and } \frac{1}{\alpha_2}\text{: solutions of } \alpha(x) = 0$$

$$(1 - \alpha_1 L)(1 - \alpha_2 L) X_t = \varepsilon_t$$

$$\rightarrow X_t = \frac{1}{(1 - \alpha_1 L)(1 - \alpha_2 L)} \varepsilon_t$$

$$X_t = \left(\frac{\alpha_1/(\alpha_1 - \alpha_2)}{1 - \alpha_1 L} + \frac{-\alpha_2/(\alpha_1 - \alpha_2)}{1 - \alpha_2 L} \right) \varepsilon_t$$

For X_t to be stationary, $|\alpha_1| < 1$ and $|\alpha_2| < 1$.

Therefore, it can be said that X_t is stationary if $\alpha_1 + \alpha_2 < 1$, $-\alpha_1 + \alpha_2 < 1$, and $-1 < \alpha_2 < 1$.

(b) Under stationarity, obtain unconditional mean and variance of X_t.

Unconditional mean:

$$E(X_t) = E(\alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \varepsilon_t)$$

$$E(X_t) = \frac{c}{1 - \alpha}$$

(c=0)

$$E(X_t) = 0$$

Unconditional variance:

To get the variance, compute

$$E\left({X_t}^2\right) = E(\alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \varepsilon_t)^2$$

$$\rightarrow \gamma(0) = {\alpha_1}^2 \gamma(0) + {\alpha_2}^2 \gamma(0) + 2\alpha_1 \alpha_2 \gamma(1) + {\sigma_\varepsilon}^2$$

$$E(X_t X_{t-k}) = E[(\alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \varepsilon_t) X_{t-k}]$$

Since X_t is stationary,

$$\gamma(k) = E(X_t X_{t-k})$$

$$\gamma(k) = \alpha_1 \gamma(k-1) + \alpha_2 \gamma(k-2)$$

As k=1,

$$\gamma(1) = \frac{\alpha_1 \gamma(0)}{1 - \alpha_2}$$

$$(\gamma(1) = \gamma(-1))$$

Substituting $\gamma(1)$ into $\gamma(0)$

$$Var(X_t) = \gamma(0)$$

$$Var(X_t) = \frac{(1-\alpha_2){\sigma_\varepsilon}^2}{(1+\alpha_2)(1-\alpha_1-\alpha_2)(1+\alpha_1-\alpha_2)}$$

(c) Let $\rho(k)$ be the autocorrelation function of X_t. Show that we have

$$\rho(k) - \alpha_1\rho(k-1) - \alpha_2\rho(k-2) = 0 \ \ for \ k \geq 2$$

The autocorrelation function is

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)}$$

$$\gamma(0) = cov(X_t, X_t)$$

$$\gamma(0) = cov(\alpha_1 X_{t-1} + \ \alpha_2 X_{t-2} + \ \varepsilon_t, X_t)$$

$$\frac{\gamma(0)}{\gamma(0)} = \frac{\alpha_1\gamma(1) + \alpha_2\gamma(2) + \sigma^2}{\gamma(0)}$$

$$\alpha_1\rho(1) + \ \alpha_2\rho(2) + \ \frac{\sigma^2}{\gamma(0)} = 1$$

$$\gamma(0) = \frac{\sigma^2}{1-\alpha_1\rho(1) - \alpha_2\rho(2)}$$

$$\gamma(1) = cov(\alpha_1 X_{t-1} + \ \alpha_2 X_{t-2} + \ \varepsilon_t, X_{t-1})$$

$$\frac{\gamma(1)}{\gamma(0)} = \frac{\alpha_1\,\gamma(0) + \alpha_2\gamma(1)}{\gamma(0)}$$

$$\rho(1) = \alpha_1 + \alpha_2\rho(1)$$

$$\rho(1) = \frac{\alpha_1}{1-\alpha_2}$$

$$\gamma(2) = cov(\alpha_1 X_{t-1} + \ \alpha_2 X_{t-2} + \ \varepsilon_t, X_{t-2})$$

$$\frac{\gamma(2)}{\gamma(0)} = \frac{\alpha_1\,\gamma(1) + \alpha_2\gamma(0)}{\gamma(0)}$$

$$\rho(2) = \alpha_1\rho(1) + \alpha_2$$

$$\rho(2) = \frac{{\alpha_1}^2}{1-\alpha_2} + \alpha_2$$

$$\gamma(3) = cov(\alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \varepsilon_t, X_{t-3})$$
$$\frac{\gamma(3)}{\gamma(0)} = \frac{\alpha_1\,\gamma(2) + \alpha_2\gamma(1)}{\gamma(0)}$$
$$\rho(2) = \alpha_1\rho(2) + \alpha_2\rho(1)$$

Therefore,

$$\rho(k) = \alpha_1\rho(k-1) - \alpha_2\rho(k-2)$$
$$\rightarrow\ \rho(k) - \alpha_1\rho(k-1) - \alpha_2\rho(k-2) = 0 \ \text{ for } \ k \geq 2.$$

(d) Let $\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_t$*. Obtain the asymptotic distribution of* $\sqrt{T}\bar{X}$*.*

By LLN and CLT,

$$\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_t \xrightarrow{p} E(X_t) = 0$$
$$\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_t \xrightarrow{d} N(0, \sigma^2)$$

Therefore,

$$\sqrt{T}\bar{X} \xrightarrow{d} N(0, \frac{(1-\alpha_2)\sigma^2}{T(1+\alpha_2)(1-\alpha_1-\alpha_2)(1+\alpha_1-\alpha_2)})$$

(e) Construct a 95% confidence interval of $E(X_t)$*. (Use Part a)*

$$\bar{X} \pm z_{1-\alpha/2}\sqrt{\frac{var(X_t)}{T}}$$
$$\bar{X} \pm 1.96\sqrt{\frac{var(X_t)}{T}}$$

Therefore,

$$\bar{X} \pm 1.96\sqrt{\frac{(1-\alpha_2)\sigma^2}{T(1+\alpha_2)(1-\alpha_1-\alpha_2)(1+\alpha_1-\alpha_2)}}$$

4) Let X_t be an MA(1) process

$$X_t = \varepsilon_t + \beta\varepsilon_{t-1},\ |\beta| < 1,\ \varepsilon_t \sim WN(0,\sigma^2)$$

(a) Let $\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_t$. Obtain the asymptotic distribution of $\sqrt{T}\bar{X}$.

By LLN and CLT,

$$\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_t \xrightarrow{p} E(X_t) = 0$$

$$\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_t \xrightarrow{d} N(0,\sigma^2)$$

Therefore,

$$\sqrt{T}\bar{X} \xrightarrow{d} N(0, \frac{(1+\beta^2)\sigma^2}{T})$$

(b) Construct a 95% confidence interval of $E(X_t)$. (Use Part a)

$$\bar{X} \pm z_{1-\alpha/2}\sqrt{\frac{var(X_t)}{T}}$$

$$\bar{X} \pm 1.96\sqrt{\frac{var(X_t)}{T}}$$

Therefore,

$$\bar{X} \pm 1.96\sqrt{\frac{(1+\beta^2)\sigma^2}{T}}$$

5) Let X_t be an ARMA(1,1) process

$$X_t = \phi X_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1},$$

Where $\varepsilon_t \sim iidN(0,\sigma^2)$, $|\phi| < 1$ and $|\theta| < 1$. Let $E(X_t) = \mu$ and $var(X_t) = {\sigma_x}^2$

(a) Obtain μ and ${\sigma_x}^2$.

$$E(X_t) = \mu$$

$$var(X_t) = var(\phi X_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1})$$
$$var(X_t) = \phi^2 var(X_{t-1}) + \sigma^2 + \theta^2\sigma^2 + 2\phi\theta E(X_{t-1}\varepsilon_{t-1})$$

$$E(X_t\varepsilon_t) = \phi E(X_{t-1}\varepsilon_t) + \theta E(\varepsilon_{t-1}\varepsilon_t) + E(\varepsilon_t{}^2)$$
$$E(X_t\varepsilon_t) = E(\varepsilon_t{}^2) = \sigma^2$$
$$\rightarrow E(X_{t-1}\varepsilon_t) = 0, E(\varepsilon_t) = 0, E(\varepsilon_{t-1}\varepsilon_t) = 0$$

Going back

$$var(X_t) = \phi^2 var(X_{t-1}) + \sigma^2 + \theta^2\sigma^2 + 2\phi\theta E(X_{t-1}\varepsilon_{t-1})$$
$$var(X_t) = \phi^2 var(X_{t-1}) + \sigma^2 + \theta^2\sigma^2 + 2\phi\theta\sigma^2$$

If X_t is weakly stationary, then $var(X_t) = var(X_{t-1})$.

$$var(X_t) = \phi^2 var(X_{t-1}) + \sigma^2 + \theta^2\sigma^2 + 2\phi\theta\sigma^2$$
$$(1-\phi^2)var(X_t) = \sigma^2 + \theta^2\sigma^2 + 2\phi\theta\sigma^2$$

Therefore,

$$var(X_t) = \frac{(1 + 2\phi\theta + \theta^2)\sigma^2}{1-\phi^2}$$

(b) Determine the coefficients $\{\pi_i\}$ *in*

$$X_t = \sum_{i=0}^{\infty} \pi_i \, \varepsilon_{t-i}$$

$$X_t = \phi X_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$$
$$X_t - \phi X_{t-1} = \varepsilon_t + \theta\varepsilon_{t-1}$$
$$\phi(B)X_t = \theta(B)\varepsilon_t$$

➔ Using the backshift operator,

$$\phi(B) = 1 - \phi B, \ \theta(B) = 1 + \theta B$$

$$X_t = \frac{\theta(B)}{\phi(B)}\varepsilon_t \qquad (*)$$

For $|\phi| < 1$, (weakly stationary)

$$\frac{\theta(B)}{\phi(B)} = (1 + \phi B + \phi^2 B^2 + \cdots)(1 + \theta B)$$
$$= 1 + \phi B + \phi^2 B^2 + \cdots + \theta B + \phi\theta B^2 + \phi^2\theta B^3 + \cdots$$
$$= 1 + (\phi + \theta)B + (\phi + \theta)\phi B^2 + (\phi + \theta)\phi^2 B^3 + \cdots$$
$$= \sum_{i=0}^{\infty} \varphi_i B^i$$
$$\rightarrow \varphi_0 = 1,\ \varphi_i = (\phi + \theta)\phi^{i-1}, i = 1,2, \ldots$$

Rewriting $(*)$

$$X_t = \varepsilon_t + (\phi + \theta)\sum_{i=0}^{\infty} \phi^{i-1}\varepsilon_{t-i}$$

(c) Show that the autocorrelation function of X_t is given by

$$\rho(1) = \frac{(1 + \phi\theta)(\phi + \theta)}{1 + \theta^2 + 2\phi\theta}$$

And for $k \geq 2$ we have $\rho(k) = \phi^{k-1}\rho(1)$.

$$\gamma(k) = cov(X_{t+k}, X_t)$$
$$= E(X_{t+k}X_t)$$
$$= E[(\phi X_{t+k-1} + \varepsilon_{t+k} + \theta\varepsilon_{t+k-1})X_t]$$
$$= E[\phi X_{t+k-1}X_t + \varepsilon_{t+k}X_t + \theta\varepsilon_{t+k-1}X_t]$$
$$= \phi E[X_{t+k-1}X_t] + E[\varepsilon_{t+k}X_t] + \theta E[\varepsilon_{t+k-1}X_t]$$

Considering a causal ARMA (1,1) process

$$X_t = \sum_{i=0}^{\infty} \varphi_i B^i$$
$$E(\varepsilon_{t+k}X_t) = E(\varepsilon_{t+k}\sum_{i=0}^{\infty} \varphi_i \varepsilon_{t-i})$$
$$= \sum_{i=0}^{\infty} \varphi_i\, E(\varepsilon_{t+k}\varepsilon_{t-i})$$
$$\begin{cases} \varphi_1\sigma^2 & for\ k = 0 \\ 0 & for\ k \geq 1 \end{cases}$$

$$E(\varepsilon_{t+k-1}X_t) = E(\varepsilon_{t+k-1}\sum_{i=0}^{\infty}\varphi_i\varepsilon_{t-i})$$

$$= \sum_{i=0}^{\infty}\varphi_i\, E(\varepsilon_{t+k-1}\varepsilon_{t-i})$$

$$\begin{cases} \varphi_1\sigma^2 & for\ k = 0 \\ \varphi_0\sigma^2 & for\ k = 1 \\ 0 & for\ k \geq 2 \end{cases}$$

$$\varphi_0 = 1$$

$$\varphi_1 = \phi + \theta$$

$$\gamma(k) = \phi E[X_{t+k-1}X_t] + E[\varepsilon_{t+k}X_t] + \theta E[\varepsilon_{t+k-1}X_t]$$

$$\begin{cases} \phi\gamma(1) + \sigma^2(1 + \phi\theta + \theta^2) & for\ k = 0 \\ \phi\gamma(0) + \sigma^2\theta & for\ k = 1 \\ \phi\gamma(k-1) & for\ k \geq 2 \end{cases}$$

$$\gamma(0) = \phi\gamma(1) + \sigma^2(1 + \phi\theta + \theta^2)$$

$$\gamma(1) = \phi\gamma(0) + \sigma^2\theta$$

$$\gamma(k) = \phi\gamma(k-1)$$

$$\gamma(k) = \phi\gamma(k-1)$$

$$\gamma(2) = \phi\gamma(2) = \phi^2\gamma(1)$$

$$\gamma(3) = \phi\gamma(3) = \phi^3\gamma(1)$$

$$\vdots$$

$$\gamma(k) = \phi^{k-1}\gamma(1)$$

$$\gamma(0) = \frac{\sigma^2(1 + 2\phi\theta + \theta^2)}{1 - \phi^2}$$

$$\gamma(1) = \frac{\sigma^2(1 + \phi\theta)(\phi + \theta)}{1 - \phi^2}$$

$$\gamma(k) = \frac{\sigma^2(1 + \phi\theta)(\phi + \theta)}{1 - \phi^2}\phi^{k-1}\ for\ k \geq 1$$

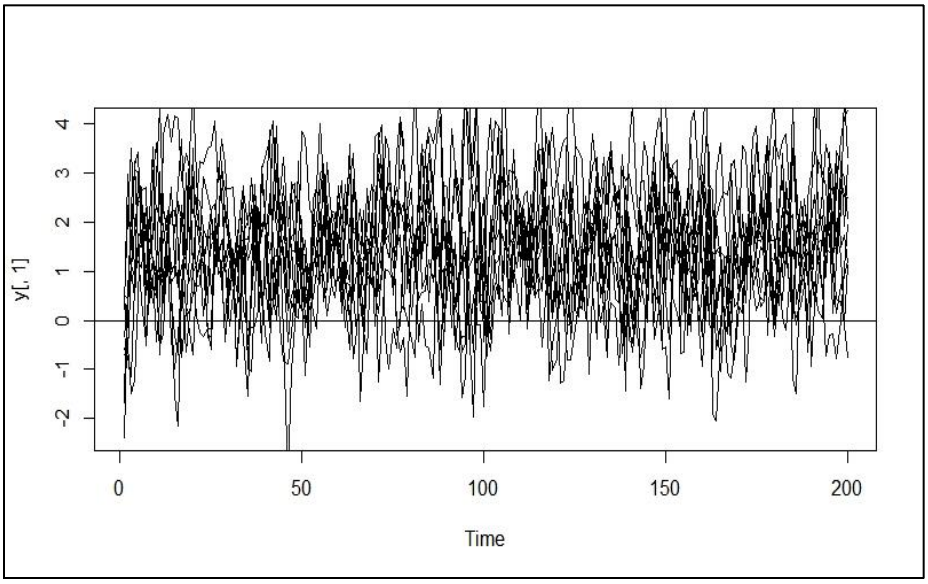
$$\frac{\gamma(k)}{\gamma(0)} = \frac{\frac{\sigma^2(1+\phi\theta)(\phi+\theta)}{1-\phi^2}\phi^{k-1}}{\frac{\sigma^2(1+2\phi\theta+\theta^2)}{1-\phi^2}}$$

Therefore,

$$\rho(1) = \frac{(1+\phi\theta)(\phi+\theta)}{1+\theta^2+2\phi\theta} \; for\; k \geq 2$$

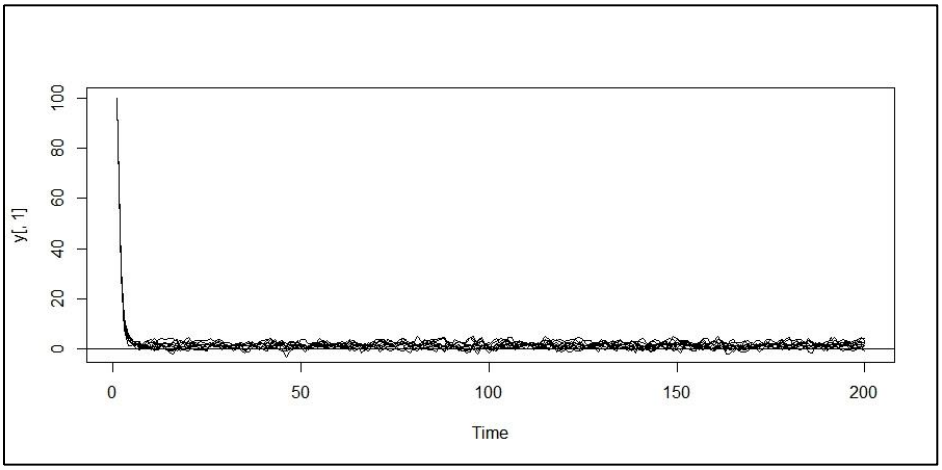
(d) Let $(\phi, \theta, \sigma^2) = (0.3, 0.3, 1)$. *Simulate the ARMA (1,1) model with T=200 and S=10 by drawing* $X_1 \sim N(0, {\sigma_x}^2)$. *Report a graph of S=10 simulated time series (x-axis: time (1-T), y-axis:* X_t*) in one plot.*

The parameters are ϕ, θ, σ^2. ϕ is the AR(1) coefficient and θ is the MA(1) coefficient. The sample size T is 200 and number of replications S is 10. There are two methods of simulation. The first method is invariant distribution which will be used in this problem. The second method will be used in part (e). The seed is set by 3565 as shown in the lecture. This is to attain the same results by fixing the random numbers. The matrix function to get a matrix with T (sample size) rows and S (number of replications) columns. The error matrix is also formed. Then, using the for loop, the y matrix will be filled by its appropriate invariant distribution. Consequently, plotting it will result to the graph below.



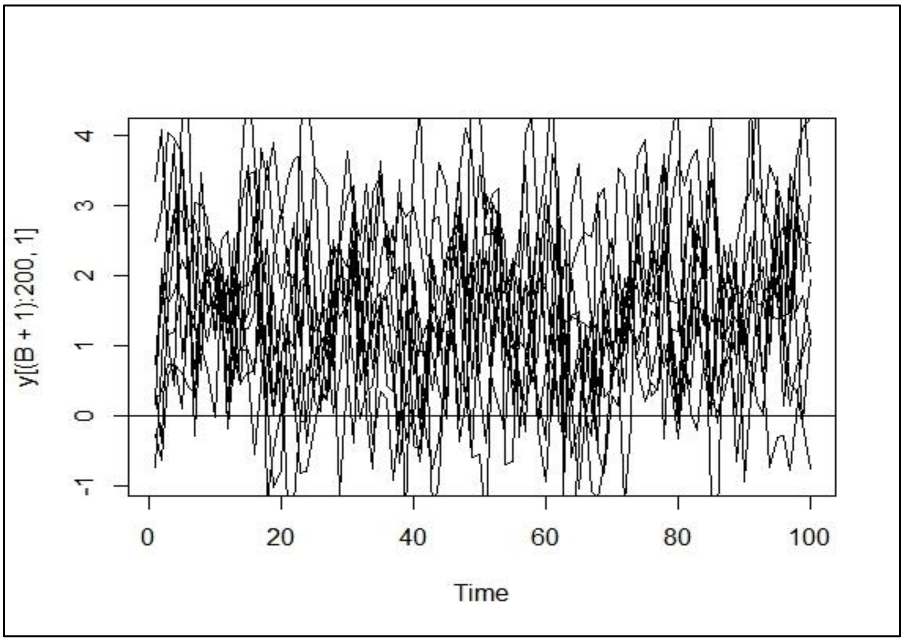
(e) *Simulate the same ARMA (1,1) model in part (d) with T=200 and S=10 by drawing* $X_1 = 10$. *Report a graph of S=10 simulated time series (x-axis: time (1-T), y-axis:* X_t*) in one plot. If you want to simulate a stationary process, how many samples would you recommend to throw away?*

Another method is by giving the initial point. The burn-in period B by 100. The size of the burn-in period depends on the nature of the process. The seed is set again to 3565, similar to part (d). The initial point X_1 is set to 10 as given. If plotted, the following result is obtained.



From the plot above, it can be seen that the first 10 samples are recommended to be thrown away to obtain a stationary process.

The graph below shows the period after the burn-in period.



6) *Consider the quarterly data from 1970/01/01 to 2017/12/31, and let* $Y_t = log(GPDI_t) - log(GPDI_{t-1})$.

(a) *Compute the Newey-West HAC estimator with a truncation parameter that is set according to the rule of thumb,* $L = 0.75T^{1/3}$ *(round this number to get an integer).*

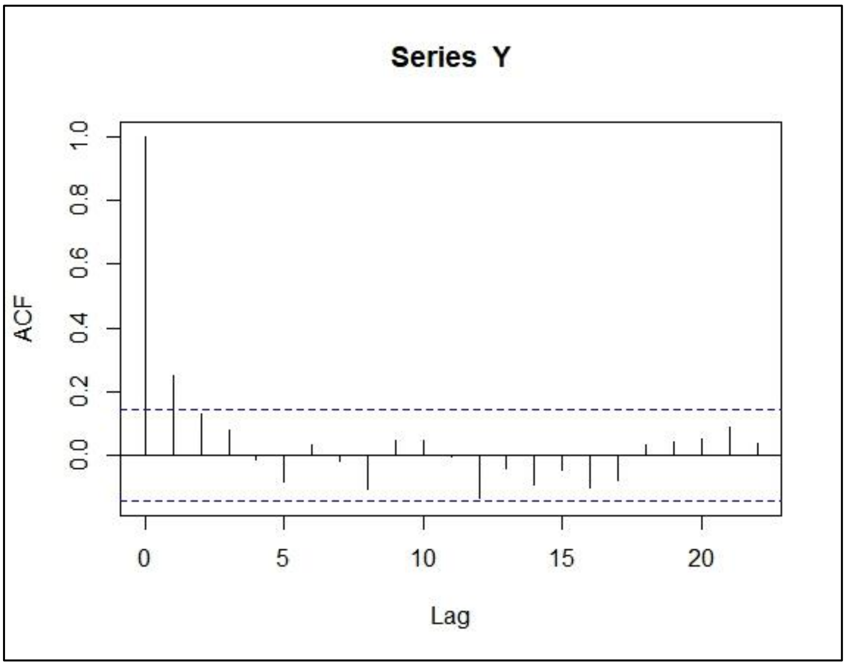
The Newey-West HAC estimator obtained by using the formula and result obtained is 1.8159.

(b) *Using the HAC estimator obtained above, compute the 95% confidence interval for* $E(Y_t)$.

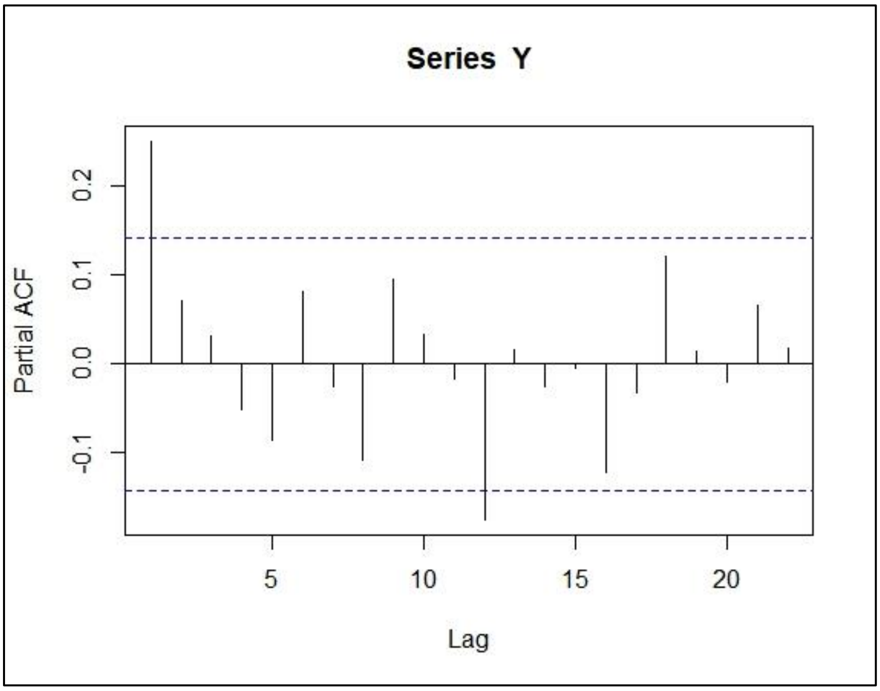
The 95% confidence interval for $E(Y_t)$ is (-0.1752341, 0.2069877).

(c) *For each time series, draw ACF and PACF.*

The ACF of the time series is obtained below.



The PACF of the time series is obtained below.



(d) Using the Ljung-Box test statistic, test

$$H_0: \rho(1) = \rho(2) = \rho(3) = 0$$

What are the value of the test statistic, p-value, and the test result at 5% significance level?

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        Box-Ljung test

data:  Y
X-squared = 18.301, df = 7, p-value = 0.01069
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The test statistic is 18.301 and the p-value is 0.01069.

Since the p-value is less than 0.05, we can reject the null hypothesis.