Assignment2-Questions (Ref: Kernelization Chapter from 8-Authors book)

1. In the Min-Ones-2-SAT problem, we are given a 2-CNF formula φ and an integer k, and the objective is to decide whether there exists a satisfying assignment for φ with at most k variables set to true. Show that Min-Ones-2-SAT admits a polynomial kernel.

2. In the Connected Vertex Cover problem, we are given an undirected graph G and a positive integer k, and the objective is to decide whether there exists a vertex cover C of G such that $|C| \leq k$ and G[C] is connected.
a. Show that the problem admits a kernel with at most $2^k + O(k^2)$ vertices.
b. Show that if the input graph G does not contain a cycle of length 4 as a subgraph, then the problem admits a kernel with at most $O(k^2)$ vertices.

3. A split graph is a graph in which the vertices can be partitioned into a clique and an independent set. In the Vertex Disjoint Paths problem, we are given an undirected graph G and k pairs of vertices (s_i, t_i), $i \in \{1, . . . , k\}$, and the objective is to decide whether there exists paths P_i joining s_i to t_i such that these paths are pairwise vertex disjoint. Show that Vertex Disjoint Paths admits a polynomial kernel on split graphs (when parameterized by k).

4. In the Cluster Vertex Deletion problem, we are given as input a graph G and a positive integer k, and the objective is to check whether there exists a set $S \subseteq V(G)$ of size at most k such that $G - S$ is a cluster graph. Show a kernel for Cluster Vertex Deletion with $O(k^3)$ vertices.

5. In the Split Vertex Deletion problem, we are given an undirected graph G and a positive integer k and the objective is to test whether there exists a set $S \subseteq V(G)$ of size at most k such that $G - S$ is a split graph.
a) Show that a graph is split if and only if it has no induced subgraph isomorphic to one of the following three graphs: a cycle on four or five vertices, or a pair of disjoint edges.
b) Give a kernel with $O(k^5)$ vertices for Split Vertex Deletion.

(Ref: Iterative Compression Chapter from 8-Authors book)

6. An undirected graph G is called perfect if for every induced subgraph H of G, the size of the largest clique in H is the same as the chromatic number of H. We consider the Odd Cycle Transversal problem, restricted to perfect graphs. Obtain a $2^k n^{O(1)}$-time algorithm based on iterative compression.

7. Obtain an algorithm for 3-Hitting Set running in time $2.4656^k n^{O(1)}$ using iterative compression. Generalize this algorithm to obtain an algorithm for d-Hitting Set running in time $((d-1)+0.4656)^k n^{O(1)}$.

8. A graph G is a split graph if V (G) can be partitioned into sets C and I, such that C is a clique and I is an independent set. In the Split Vertex Deletion problem, given a graph G and an integer k, the task is to check if one can delete at most k vertices from G to obtain a split graph. Obtain a $2^k n^{O(1)}$-time algorithm for this problem using iterative compression.

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