Assignment 4

Due date: 25/06/21

1. Given a string of n integers, find the longest increasing subsequence in $O(n \log n)$ times.

2. Given a set of n intervals (s_i, f_i), each with a value v_i , choose a subset S of non-overlapping intervals with $\sum_{i \in S} v_i$ maximized.

3. A subsequence is palindromic if it is the same whether read left to right or right to left. For instance, the sequence $A, C, G, T, G, T, C, A, A, A, A, T, C, G$ has many palindromic subsequences, including A, C, G, C, A and A, A, A, A (on the other hand, the subsequence A, C, T is not palindromic). Devise an algorithm that takes a sequence $x[1...n]$ and returns the longest palindromic subsequence.

4. Consider a directed graph in which the only negative edges are those that leave a vertex s; all other edges are positive. Can Dijkstra's algorithm, started at s, fail on such a graph? Prove your answer.

5. Let $G = (V, E)$ be a connected directed graph with non-negative edge weights,let s and t be vertices of G, and let H be a subgraph of G obtained by deleting some edges. Suppose we want to reinsert exactly one edge from G back into H, so that the shortest path from s to t in the resulting graph is as short as possible. Describe and analyze an algorithm that chooses the best edge to reinsert, in $O(|E| \log |V|)$ time.