CS 769 Project Presentation Comparison of Multi-Armed Bandits Policies

Presented by -

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Introduction to Online Convex Optimization

K: convex decision set, and **F**: set of convex functions.

A repeated game of T rounds between a player and an adversary, at each round $\mathbf{t} \in \mathbf{T}$, then the OCO framework,

- player chooses a point $\mathbf{x_t} \in \mathbf{K}$
- adversary independently chooses a loss function $\mathbf{f_t} \in \mathbf{F}$
- player suffers a loss $\mathbf{f_t}\,(\mathbf{x_t})$ and receives a feedback $\mathbf{F_t}$

The regret of the setting can be defined as:

$$\boxed{Regret_T = \sum_{t=1}^{T} f_t(x_t) - \min_{x^* \in K} \sum_{t=1}^{T} f_t(x^*)}$$

Objective: To minimize the expected regret

Online Gradient Descent

Algorithm 1: Online Gradient Descent

Input: Convex set $K, T, x_1 \in K$, step sizes $\{\eta_t\}$

for $t = 1$ *to* T **do**

  Play x_t and observe loss $f_t(x_t)$

  Update and Project:

$$y_{t+1} = x_t = \eta_t \nabla f_t(x_t)$$

$$x_{t+1} = \prod_K (y_{t+1})$$

end

The Exploration-Exploitation Dilemma

- **Exploration** - improve knowledge for long-term benefit
- **Exploitation** - exploit knowledge for short-term benefit

Multi Armed Bandits (MAB)

Algorithm 20 Simple MAB algorithm

Input: OCO algorithm $\mathcal{A}$, parameter δ.
for $t = 1$ to T **do**
  Let b_t be a Bernoulli random variable that equals 1 with probability δ.
  if $b_t = 1$ **then**
    Choose $i_t \in \{1, 2, ..., n\}$ uniformly at random and play i_t.

    Let
$$\hat{\ell}_t(i) = \begin{cases} \frac{n}{\delta} \cdot \ell_t(i_t), & i = i_t \\ 0, & \text{otherwise} \end{cases}.$$
    Let $\hat{f}_t(\mathbf{x}) = \hat{\ell}_t^\top \mathbf{x}$ and update $\mathbf{x}_{t+1} = \mathcal{A}(\hat{f}_1, ..., \hat{f}_t)$.
  else
    Choose $i_t \sim \mathbf{x}_t$ and play i_t.
    Update $\hat{f}_t = 0, \hat{\ell}_t = \mathbf{0}$, $\mathbf{x}_{t+1} = \mathbf{x}_t$.
  end if
end for

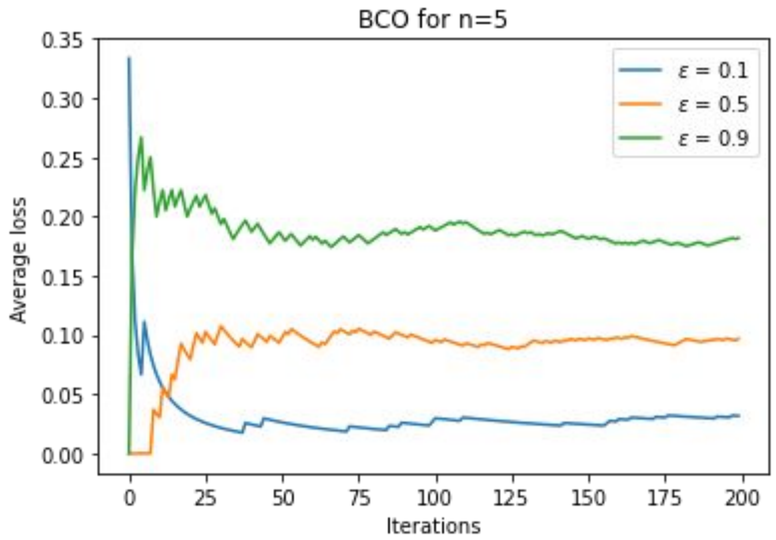
- In this setting the player only knows the loss of the action chosen and not the entire loss gradient.

Other Strategies for MAB

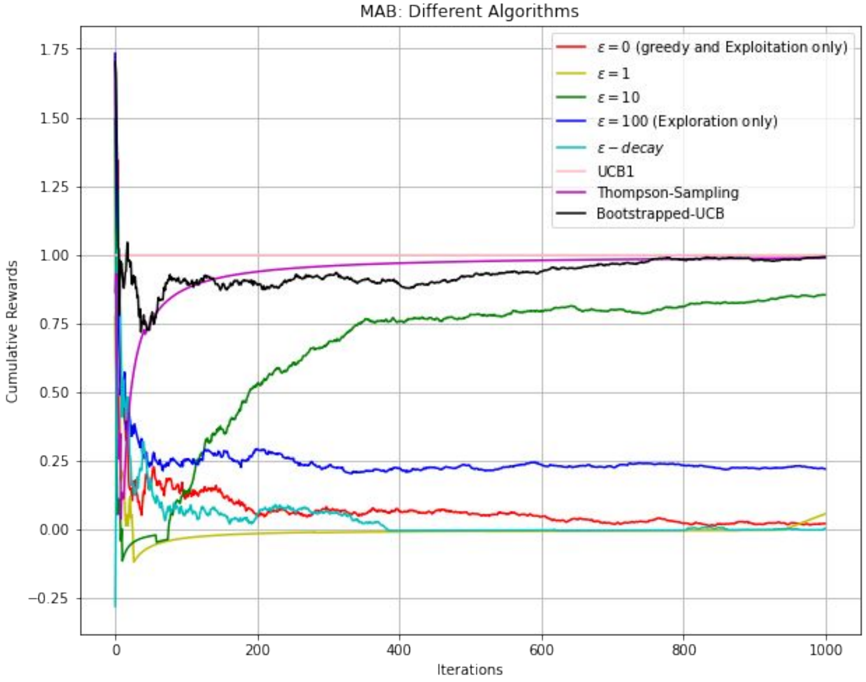
- Epsilon Decay Algorithm
  - Keep reducing the value of epsilon $\frac{1}{(1+\frac{t}{no.of\,arms})}$.
- UCB Algorithm
  - Reduce the uncertainty associated with an action
  - Done by sampling more number of times
- Thompson Sampling
  - Sample the next action using a Beta distribution of the number of times a reward was achieved and the number of times a loss was achieved by choosing that action.

Results

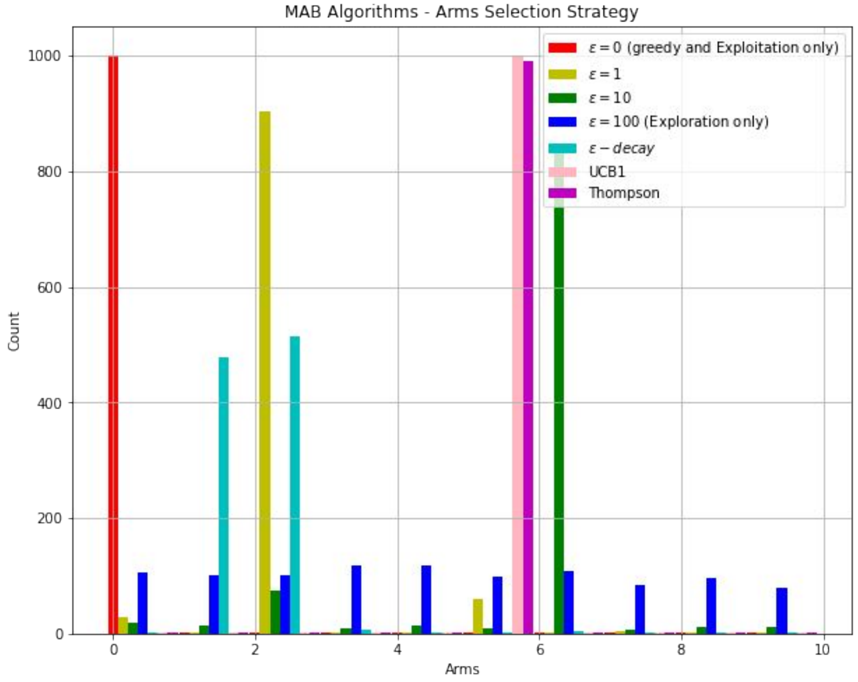
- Higher exploration rate leads to higher average loss over time
- This is due to the fact that the algorithm explores even when it is almost sure of the best action
- The other strategies and their convergence is given in the following slides.



Results



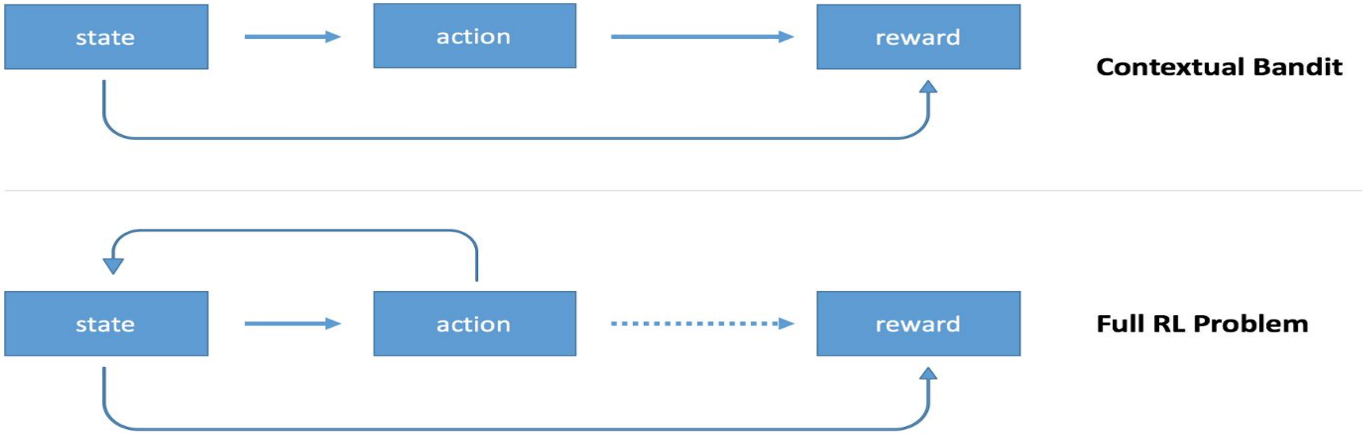
Convergence rates of different algorithms



Histogram of actions chosen by the algorithms

Contextual MAB

- It is an extension of multi-armed bandits.
- The algorithm observes a context, makes a decision, choosing one action from a number of alternative actions, and observes an outcome of that decision.



Bibtex Dataset:

- It is a Multilabel text classification for automated tag suggestion:
  - containing tags that people have assigned to different papers
  - the goal being to learn to suggest tags based on features from the papers
- dataset link

	Obs.	Feats.	Labels
BibTeX	7,395	1,836	159

Epsilon-Greedy for Contextual Bandits:

Inputs probability $p \in (0,1]$, decay rate $d \in (0,1]$, oracles $\hat{f}_{1:k}$

for each succesive round t with context $\mathbf{x}^t$ **do**

  With probability $(1-p)$:

    Select action $a = \text{argmax}_k \hat{f}_k(\mathbf{x}^t)$

  Otherwise:

    Select action a uniformly at random from 1 to k

  Update $p := p \times d$

  Obtain reward r_a^t, Add observation $\{\mathbf{x}^t, r_a^t\}$ to the history for arm a

  Update oracle $\hat{f}_a$ with its new history

Implementation:

- Firstly,the bibtex dataset is transformed into binary classification data for each label corresponding to each training instance.
- **MAB-FIRST:**

Inputs const. a, b, threshold m, contextual bandit policy π_k, covariates $\mathbf{x}$

Output score for arm $\hat{r}_k$

if $|\{r \in \mathcal{R}_k \mid r = 0\}| < m$ or $|\{r \in \mathcal{R}_k \mid r = 1\}| < m$ **then**

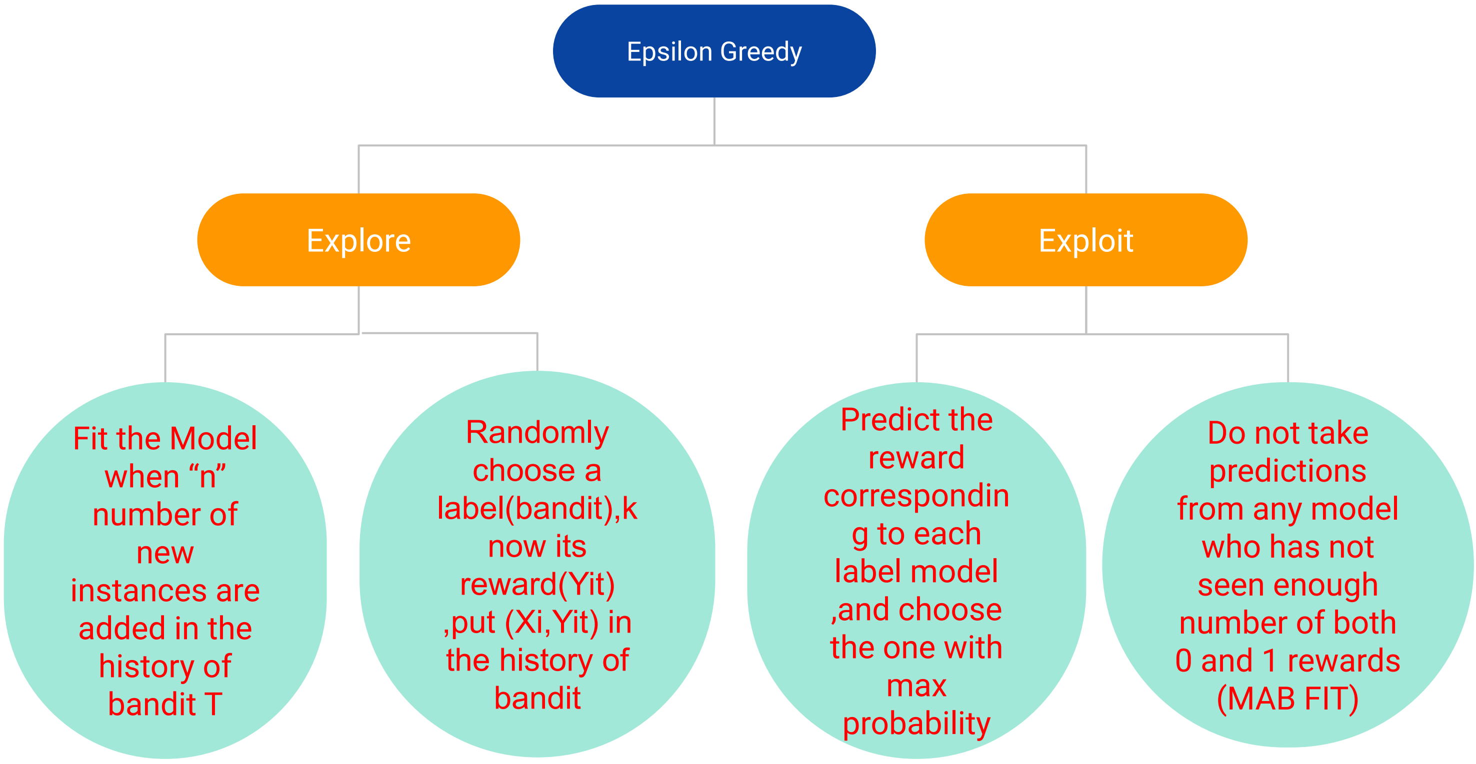
    Sample $\hat{r}_k \sim \text{Beta}(a + |\{r \in \mathcal{R}_k | r = 1\}|,\ b + |\{r \in \mathcal{R}_k | r = 0\}|)$

else

    Set $\hat{r}_k = \pi_k(x)$

return $\hat{r}_k$

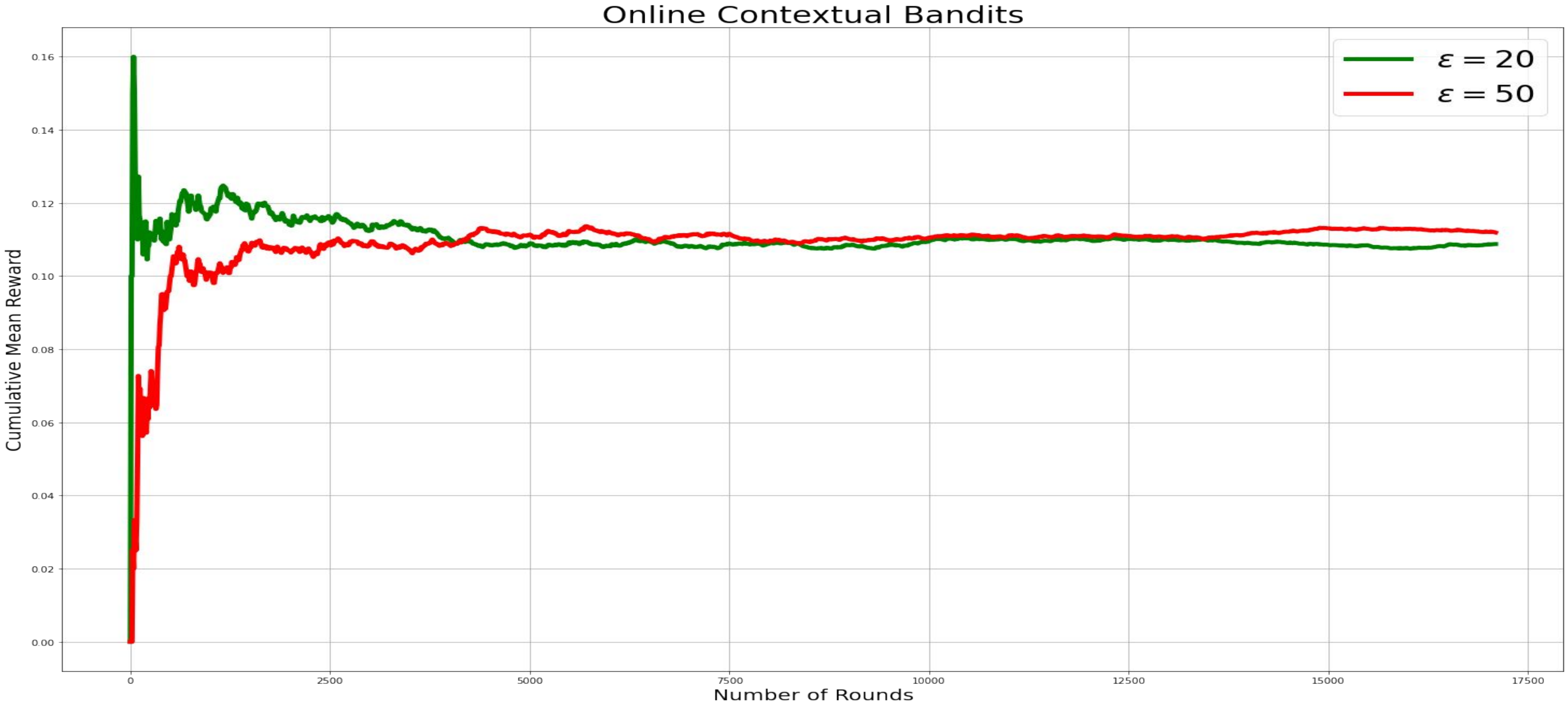
WorkFlow:


Epsilon Greedy
Explore
Exploit
Fit the Model when “n” number of new instances are added in the history of bandit T
Randomly choose a label(bandit),k now its reward(Yit) ,put (Xi,Yit) in the history of bandit
Predict the reward correspondin g to each label model ,and choose the one with max probability
Do not take predictions from any model who has not seen enough number of both 0 and 1 rewards (MAB FIT)

Parameters being Hypertuned:

- a and b values for beta distribution (a=3,b=7)
- Subset of bandits (labels) on which models are being tuned due to sparsity of data(opt subset length =10)
- Refit the corresponding bandit depending on the number of new training instances being added to its history, a tradeoff against computational power utilized (opt. value=5)

Results



Online Contextual Bandits
$\varepsilon = 20$
$\varepsilon = 50$
Cumulative Mean Reward
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
2500
5000
7500
10000
12500
15000
17500
Number of Rounds

Critical Analysis

- In the context of MAB, a higher probability of exploration leads to more regret as the algorithm is forced to explore bad choices.
- **ε-greedy family**: The algorithms are not at all concerned towards obtaining the true mean of an arm for exploitation
- **UCB, Thompson**: The algorithms tries to reduce the uncertainty about the true reward of an arm
- **UCB**:
    - UCB eliminates the randomness of the ε-greedy family algorithms
    - The algorithm gives the benefit of doubt to a less explored arm at a certain time t, and explores it to identify the true distribution/reward
- **Thompson**:
    - Tries to select the arm which has previously given more number of high rewards.
    - It tries to understand the true distribution of the rewards associated with the arms.

Thank you