259 HW1
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Assignment 1

1. What happens to the turning angle if all the radii (R1, R2, R3) are the same? Does your simulation agree with your intuition?

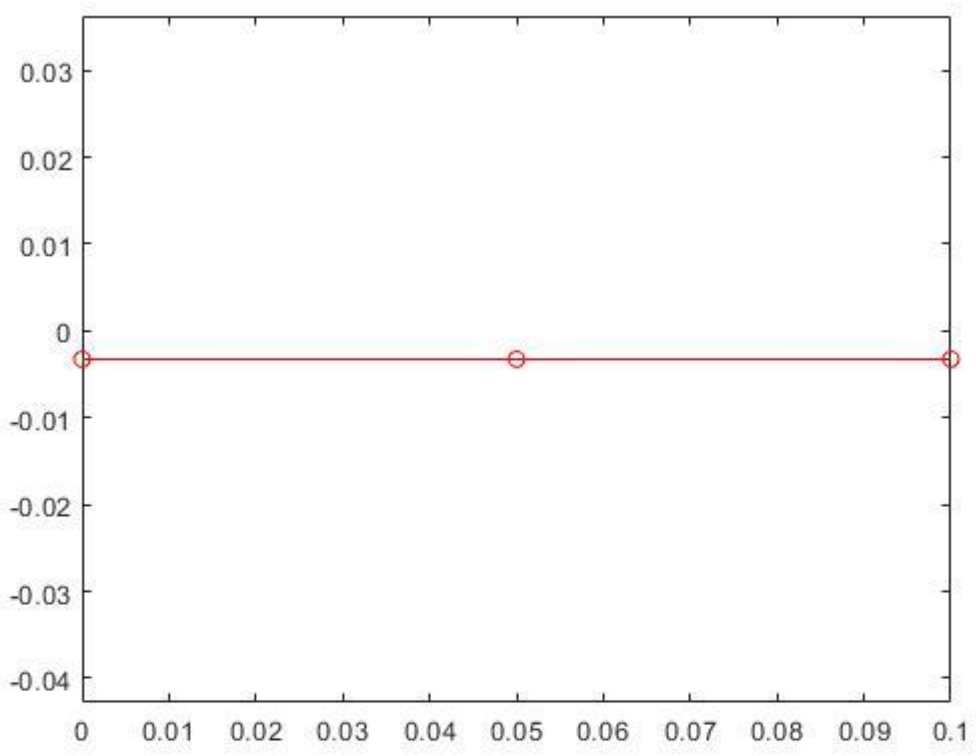

Figure 1. Simulation of 3 nodes with the same radii

Above is the simulated output for 3 spheres of the same radii. The turn angle between the 3 nodes would be 0. The simulated nodes would fall flat at the same rate. Since the radii are the same, with the same material density, the mass of the spherical bodies would be equal, thus there would be no turning angle.

2. Try changing the time step size (Δt), particularly for your explicit simulation, and use the observation to elaborate the benefits and drawbacks of the explicit and implicit approach.

When changing the step size to a larger value, the explicit simulation output becomes incorrect and deviates from the theoretical value by a lot. It can only produce the correct outputs when the step size is sufficiently small, say anything smaller than or equal to 1e-5 s. The explicit method, therefore, to be able to produce accurate outputs, requires a significant amount of computing power. This way of simulation is very inefficient. In comparison, the Newton Raphson approach is faster and more power efficient when it comes to simulation and predicting the output during a time marching scheme. The algorithm finds the next closest solution when a previous solution is found. With error bound, it is possible to maintain a relatively good accuracy while reducing computing times. However, it can be possible for the algorithm to not converge if unlucky.

Assignment 2

1. Include two plots showing the vertical position and velocity of the middle node with time. What is the terminal velocity?

The final velocity is -0.0058m/s

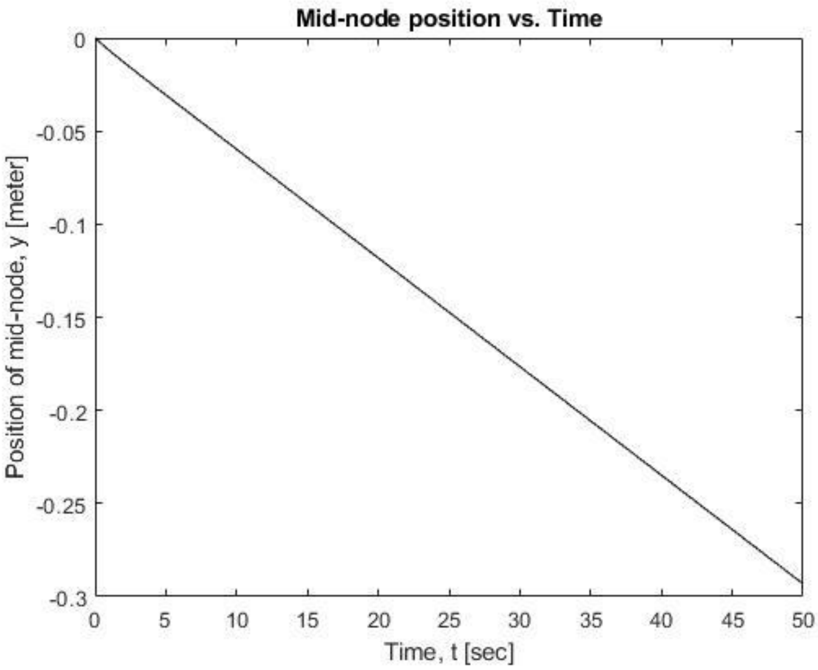

Figure 2.1.1 Mid-node position vs. Time

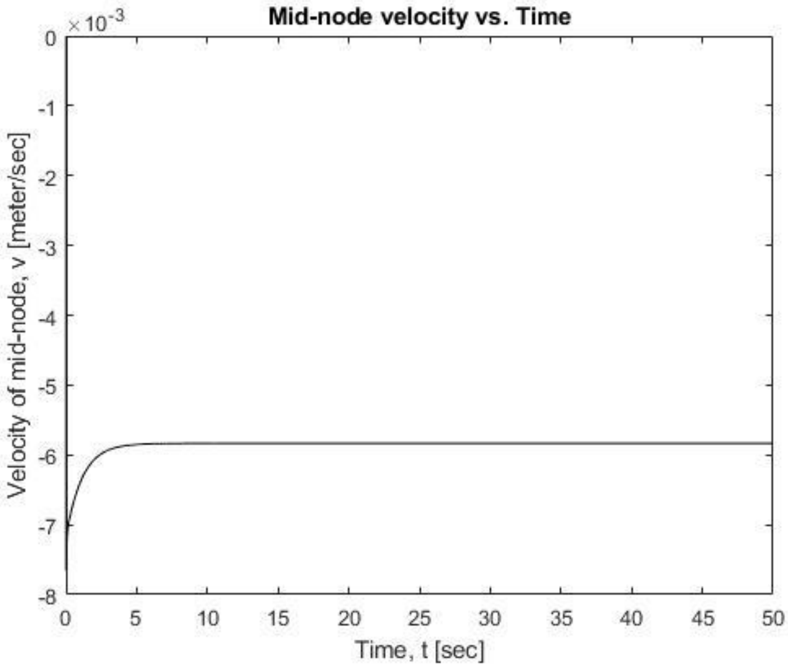

Figure 2.1.2 Mid-node velocity vs. Time

2. Include the final deformed shape of the beam.

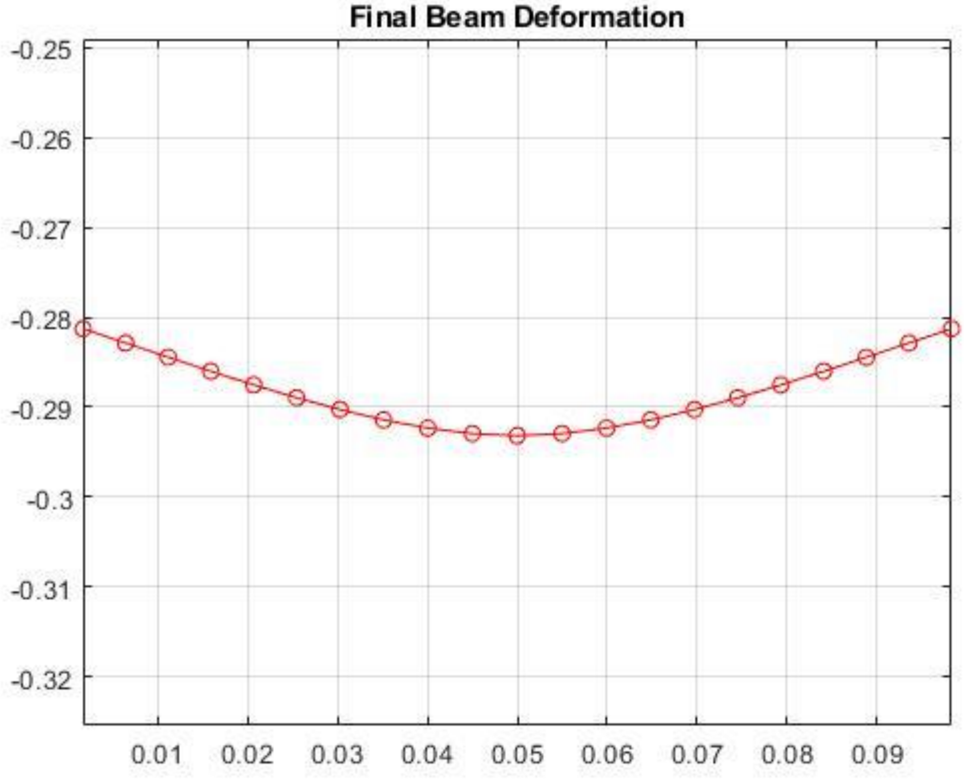

Figure 2.2 Final shape of deformed beam

3. Discuss the significance of spatial discretization (i.e. the number of nodes, N) and temporal discretization (i.e. time step size, Δt). Any simulation should be sufficiently discretized such that the quantifiable metrics, e.g. terminal velocity, do not vary much if N is increased and Δt is decreased. Include plots of terminal velocity vs. the number of nodes and terminal velocity vs. the time step size.

   With a greater number of nodes, the simulation can approach the actual experimental setup with a closer resemblance, thus producing more accurate results. With a smaller step size, more computing power is required, but the continuity of the simulation is preserved. As the number of nodes increase, the simulation output corresponding to each node becomes very close. If the number of nodes and step size is sufficiently discretized, then the terminal velocity matrix does not vary much across different settings. Below are the plots for terminal velocity vs. the number of nodes and terminal velocity vs. the time step size. It is evident that although the number of nodes vary, the timestep vary, the final velocity output does not vary by much

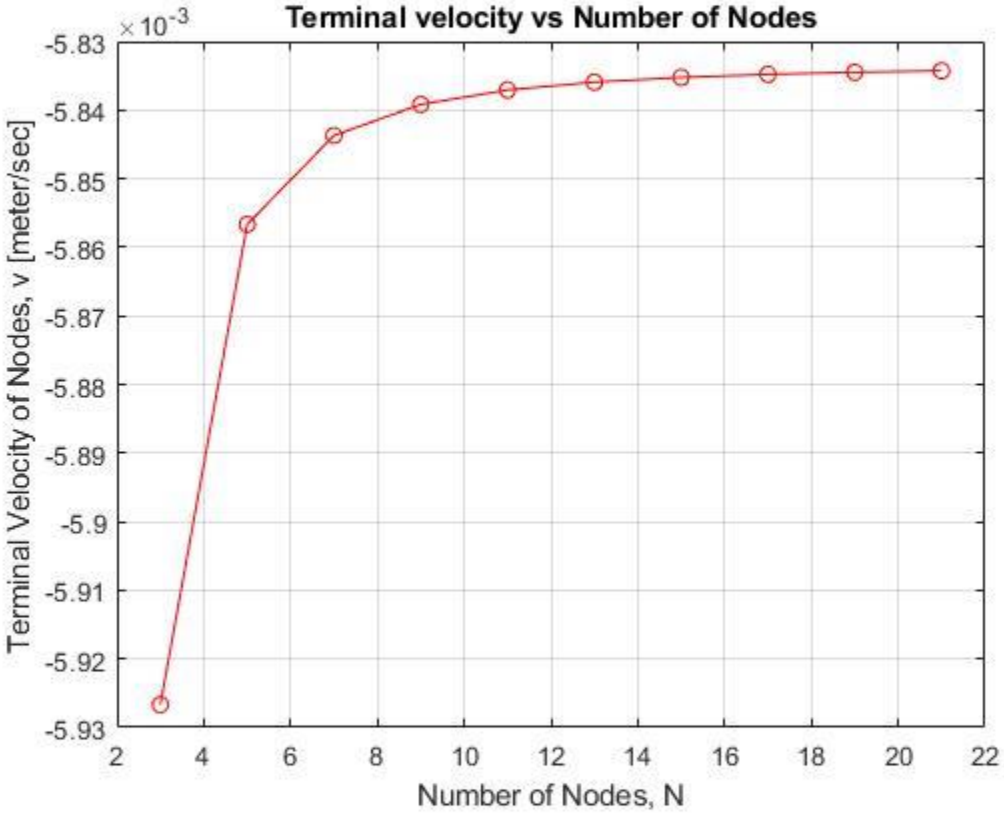

Figure 2.3.1 Terminal velocity vs. the number of nodes

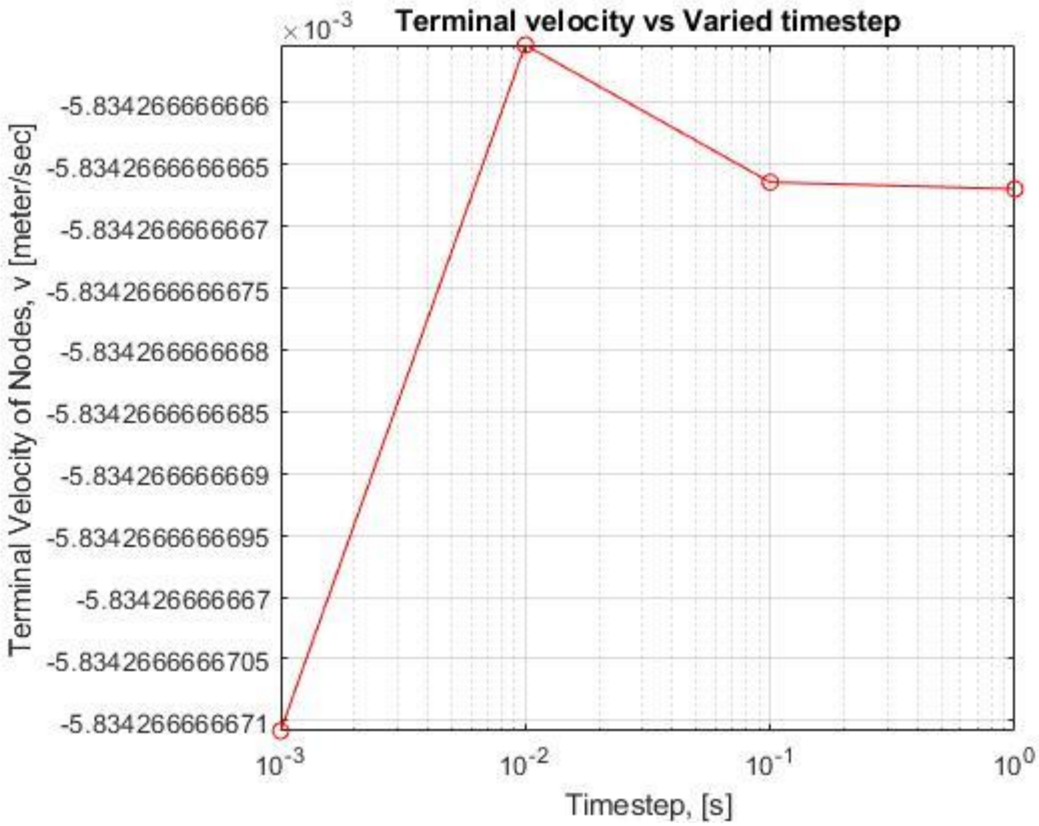

Figure 2.3.2 Terminal velocity vs. the time step size

Assignment 3

1. Plot the maximum vertical displacement, ymax, of the beam as a function of time. Depending on your coordinate system, ymax may be negative. Does ymax eventually reach a steady value? Examine the accuracy of your simulation against the theoretical prediction from Euler beam theory.

   y_max from Euler beam theory is 0.0380 m>>

   The maximum vertical displacement vs time is plotted below. The ymax eventually reaches a steady value of -0.0371093m while the output of the Euler beam theory is -0.0380m. There is a small difference of 2.34% from the theoretical output. This indicates good accuracy produced from the simulation.

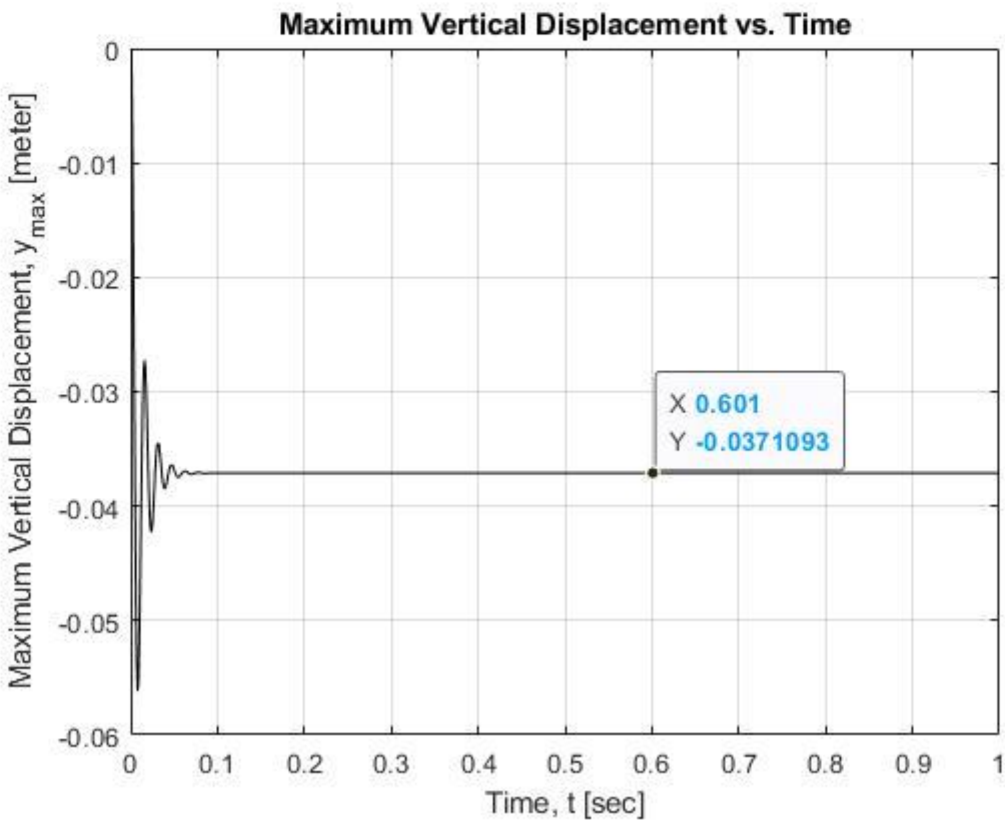

Figure 3.1 Maximum vertical displacement vs. Time for P = 2,000N

2. What is the benefit of your simulation over the predictions from beam theory? To address this, consider a higher load P = 20000 such that the beam undergoes large deformation. Compare the simulated result against the prediction from beam theory in Eq. 4.21. Euler beam theory is only valid for small deformation whereas your simulation, if done correctly, should be able to handle large deformation. Optionally, you can make a plot of P vs. ymax using data from both simulation and beam theory and quantify the value of P where the two solutions begin to diverge.

   y_max from Euler beam theory is 0.3804 m>>

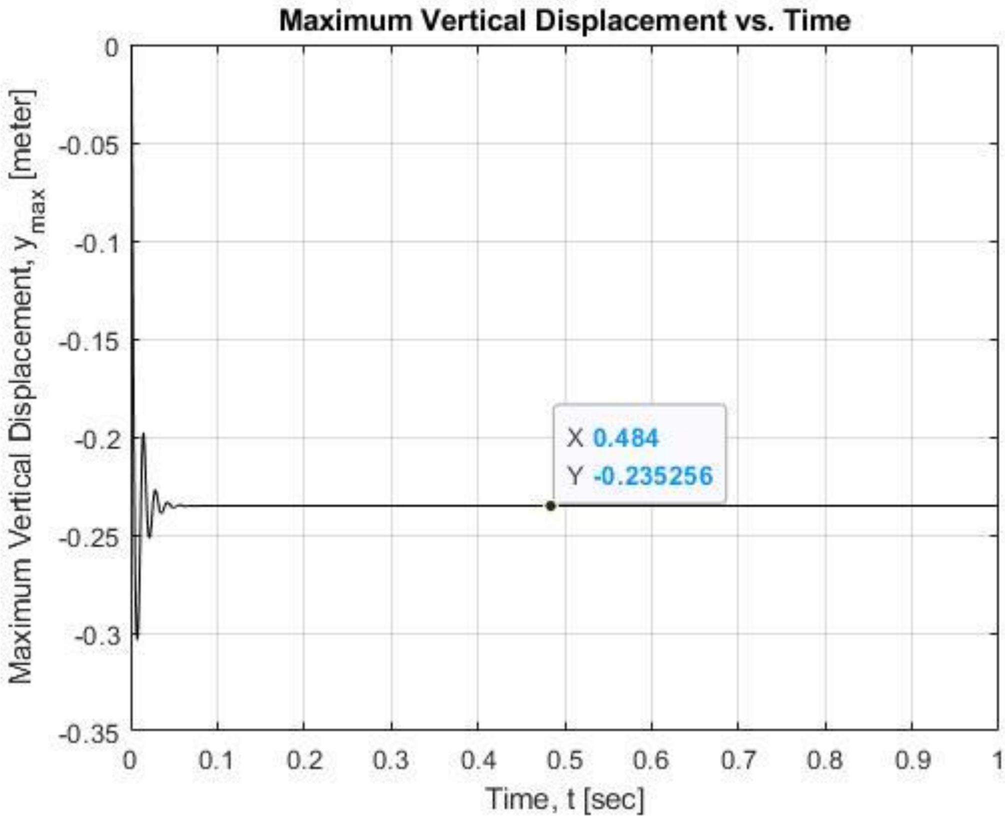

Figure 3.2 Maximum vertical displacement vs. Time for P = 20,000N

The Euler beam theory is only valid for small deformations like when the load is set to be 2,000N. However, if set to 20,000N, the deformation would become a large deformation and can no longer be fit to the Euler beam theory. When simulating P = 20,000N, the max vertical displacement becomes -0.235256m while the output from the Euler equation is -0.3804m. With a deviation of more than 60% from the simulated outputs. We are confident that the simulated results are closer to the actual output of the beam deformation since the calculations are based on a varying but converging time-march. The limitation of the Euler beam theory becomes clear.