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Important Concepts

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Abstract

There are various concepts in probability theory and linear algebra that we keep on forgetting and they keep haunting us throughout our lifetime. I attempt to list and explain a few of them, which I myself need very much.

1 Eigen vectors and Eigen Values

2 Eigen Value decomposition

3 Singular value decomposition

4 Gram Schmidt Orthogonalization

5 Positive Definite Matrix

A symmetric (Hermitian) $n \times n$ real (complex) matrix M is said to be Positive Definite if $z^T M z > 0$ for every non-zero vector z. In case of positive semidefinite, $z^T M z \geq 0$.

5.1 Properties of Positive definite matrix

Let M be an $n \times n$ symmetric matrix.

- It has positive eigen values. Vice-versa, if a matrix has all it's eigen-values real and positive, the matrix is positive definite.
- For any real invertible matrix A, $A^T A$ **is always positive definite**
- Positive definite matrix has **positive determinant**. This means, that PD matrix is always **nonsingular**

6 Covariance Matrix

Let $\mathbf{X} = \{X_1, X_2 \cdots X_n\}$ be a random vector. Then,

$$Cov(\mathbf{X}) = [\Sigma]_{n \times n} \tag{1}$$

where $\Sigma_{ij} = cov(X_i, X_j) = E((X_i - \mu_i)(X_j - \mu_j))$
Also, Let $\mathbf{Y} = \{Y_1, Y_2 \cdots Y_m\}$ be another random vector. Then,

$$Cov(\mathbf{X, \ Y}) = E((\mathbf{X} - \mu_{\mathbf{X}})^T(\mathbf{Y} - \mu_{\mathbf{Y}})) = [\Sigma]_{n \times m} \tag{2}$$

6.1 Covariance

Covariance means how two things(random variables X_1 and X_2) vary with respect to each other. If on increase of X_1, X_2 increases and on decrease of X_1, X_2 decreases, we say covariance is positive. If on increase of X_1, X_2 decreases and vice versa, we say that covariance is negative. Covariance is 0 if no such relation exists. Understanding the magnitude of covariance is difficult.
Mathematically, let X_1 and X_2 are two real valued random variables. Then Cov(X_1, X_2) $= E[X_1 - \mu_1) \times (X_2 - \mu_2)]$

6.2 Properties of Covariance Matrices

- Positive semidefinite
- Symmetric
- If $(m = n)$

7 Random Variable

Random variable is a function that maps outcomes of an event to mathematically convenient form (real numbers). For example: Let our event be "*A coin is tossed 30 times*" and let random variable be **Number of heads occured while tossing 30 times**

7.1 Random Vector

Random vector are used when you want to view multiple events simultaneously. For example: You want to observe tossing of coin (X_1) and rolling of dice (X_2) simultaneously. Then, you define random vector $\mathbf{X} = \{X_1, X_2\}$. Here $X_1 : -1, 1$ and $X_2 : 1, 2, 3, 4, 5, 6$. In more machine learning pov, you have random vector as your feature set where each feature is a random variable.

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