# **Deep Integrated Explanations**

### **ACM Reference Format:**

## 1 GRADIENT ROLLOUT IMPLEMENTATION

The Gradient Rollout (**GR**) technique is a modified version of the Attention Rollout (**AR**) [1] method, which differentiates itself by including a Hadamard product between each attention map and its gradients in the computation, rather than relying solely on the attention map. The GR method can be expressed mathematically as follows:

$$A_b' = I + E_h(A_b \circ G_b), \tag{1}$$

$$GR = A_1' \cdot A_2' \cdot \cdot \cdot A_B'. \tag{2}$$

where  $A_b$  is a 3D tensor consisting of the 2D attention maps produced by each attention head in the transformer block b,  $G_b$  is the gradients w.r.t.  $A_b$ . I is the identity matrix, B is the number of transformer blocks in the model,  $E_h$  is the mean reduction operation (taken across the attention heads dimension), and  $\circ$  and  $\cdot$  are the Hadamard product and matrix multiplication operators, respectively.

## 2 COMPUTATIONAL COMPLEXITY OF DIX

The computational complexity of DIX is dominated by the order of the iterated integral to be computed. Practically, one will employ the discrete version from Eq. ?? which involves nested sums (each with n summands). Each summand requires the application of  $\mathbf{q}^l$ , whose computational complexity depends on the specific implementation. For example, in Sec. ??,  $\mathbf{q}^l$  combines both activation and attention maps, respectively, and their gradients, hence the computation involves both forward and backward passes. Therefore, assuming the computational complexity of  $\mathbf{q}^l$  is O(Q), the overall computational complexity of Eq. ?? is  $O(n^\beta Q)$ , with  $\beta = \sum_{l=0}^l b_l$ .

#### REFERENCES

Samira Abnar and Willem Zuidema. 2020. Quantifying Attention Flow in Transformers. arXiv preprint arXiv:2005.00928 (2020).

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