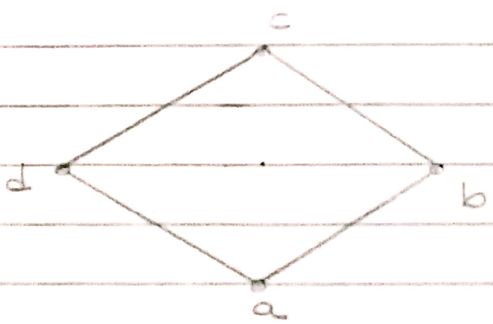
Assignment 2

(1) Explain the term poset

→ Partially ordered relation :- A relation R on a set A is called partial order if R is reflexive, anti symmetric and transitive poset
The set A together with the partial order R is called a partially ordered set or simply a poset It is denoted by (A, R)

(2) Explain the term Lattice
→ A Lattice is a poset $(L, \leq)$ in which every subset $\{a, b\}$ consisting of two elements has a least upper bound and a greatest lower bound we denote LUB $(\{a, b\})$ by $a \vee b$, and call it the Join of a and b. Similarly - we denote GLB $(\{a, b\})$ by $a \wedge b$ and call it the meet of a and b

Example on lattices.



(3) Let $A = \{a, b, c\}$, Draw Hasse Diagram for $\{P(A), \subseteq\}$

Set containment $\subseteq$ is always a partial order
Since for any subset B of A, $B \subseteq B$ i.e
$\subseteq$ is reflexive

If $B \subseteq C$ & $C \subseteq B$, $B = C$. so $\subseteq$ is anti-symmetry
If $B \subseteq C$, $C \subseteq D$ then $B \subseteq D$. so $\subseteq$ is transitive

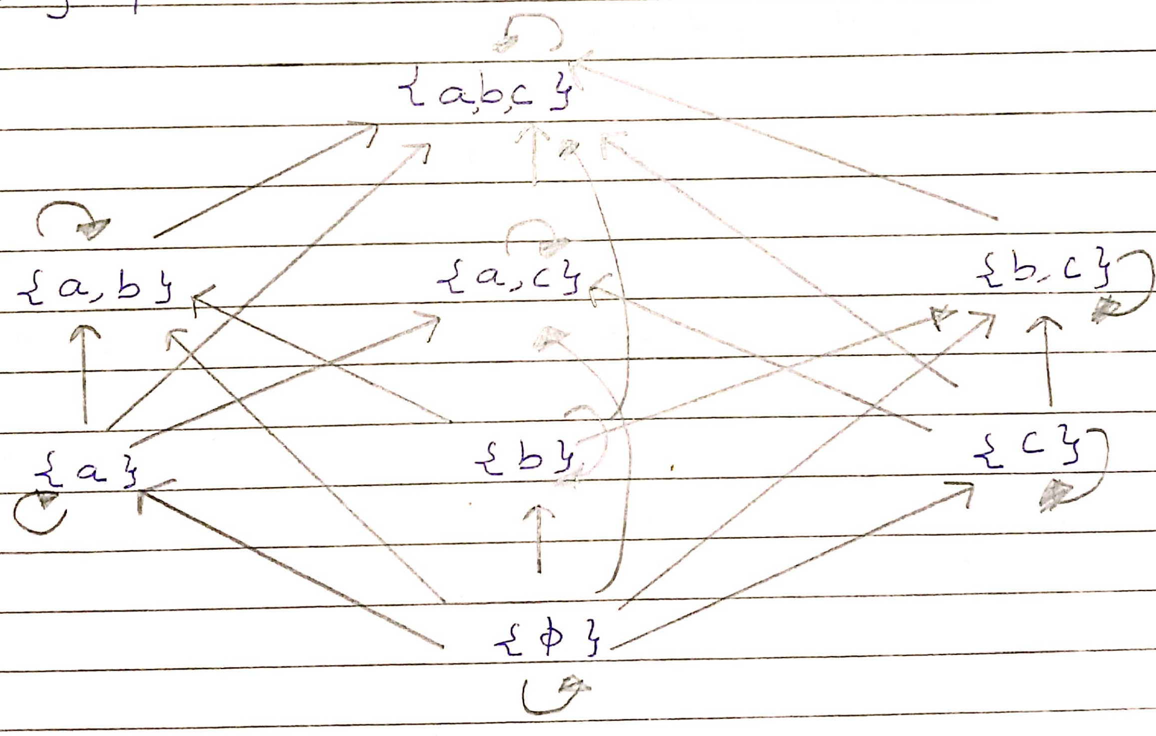
Partial ordered relation of set containment on set P(A) is as follows

$R = \{\{\{\phi\}, \{\phi\}\}, \{\{\phi\}, \{a\}\}, \{\{\phi\}, \{b\}\}, \{\{\phi\}, \{c\}\},$
$\{\{\phi\}, (a, b)\}, \{(\phi), (a, c)\}, \{(\phi), (b, c)\}, \{(\phi), (a, b, c)\},$
$\{(a), (a, b)\}, \{(a), (a, c)\}, \{(b), (a, b)\}, \{(b), (b, c)\},$
$\{(c), (a, c)\}, \{(c), (b, c)\}, \{(a, b), (a, b, c)\},$
$\{(a, c), (a, b, c)\}, \{(b, c), (a, b, c)\}, \{(a, b, c)$
$(a, b, c)\}$

$\therefore$ matrix of the above relation is as follows.

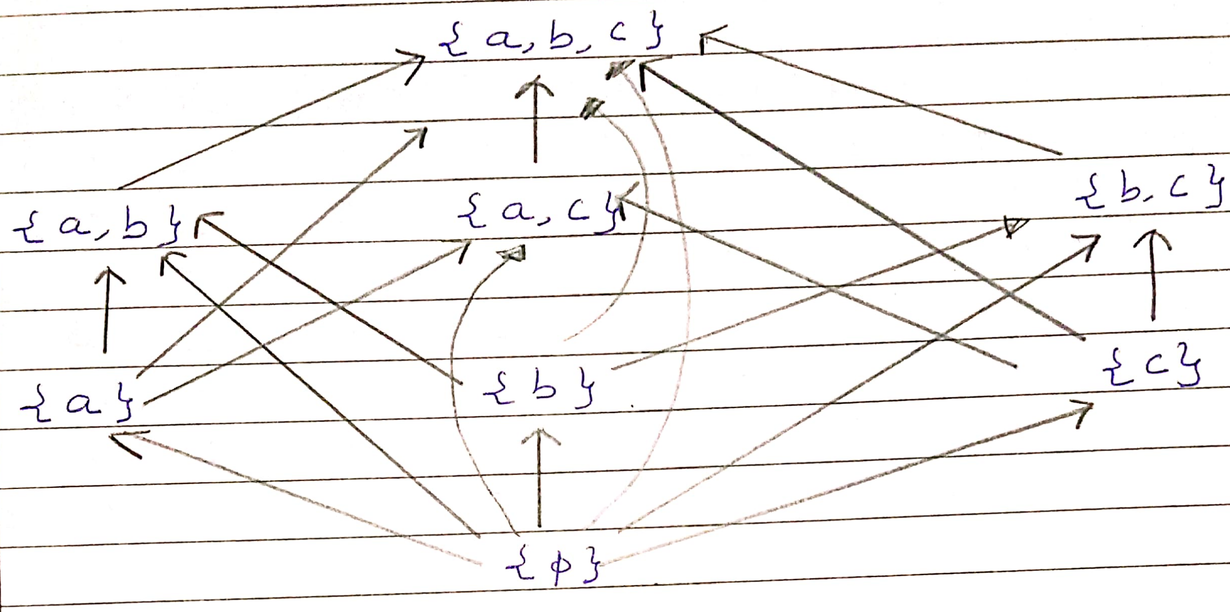
$$M_R = \begin{array}{c|cccccccc} & \phi & (a) & (b) & (c) & (a,b) & (a,c) & (b,c) & (a,b,c) \\ \hline \phi & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ (a) & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ (b) & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ (c) & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ (a,b) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ (a,c) & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ (b,c) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}$$

Digraph of the above matrix is

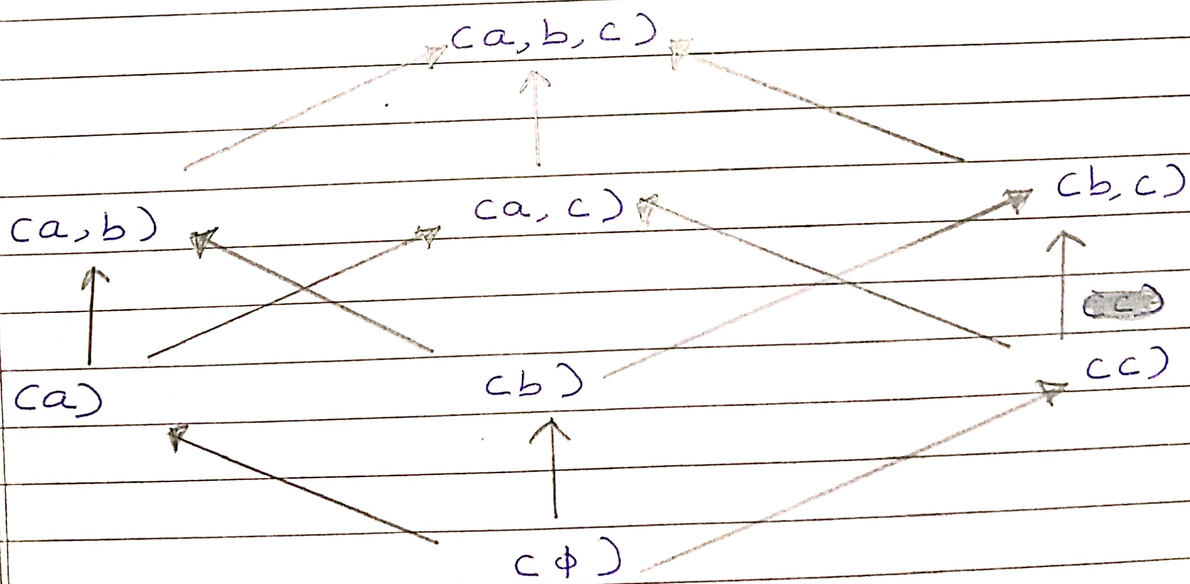


To convert thi digraph into hasse Digraph

(1) Remove cycles



(2) Remove transitive edges



Define Distributive lattice along with one appropriate example.

A lattice L is called distributive if for any elements a, b and c in L we have the following distributive properties,

(1) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

(2) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

If L is not distributive, we say that L is non-distributive

example

For a set S, the lattice P(S) is distributive, since union and intersection (the join and meet) each satisfy the distributive property.

$a \wedge (d \vee c) = a \wedge d = a$

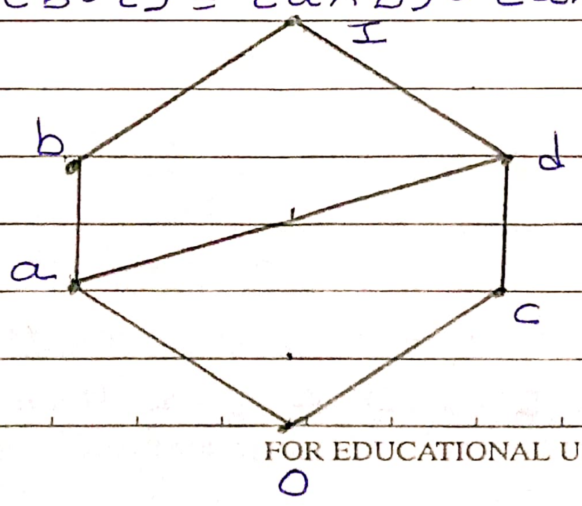
$(a \wedge d) \vee (a \wedge c) = a \vee 0 = a$

$a \wedge (d \vee c) = (a \wedge d) \vee (a \wedge c)$

$a \wedge (b \vee c) = a \wedge I = a$

$\therefore (a \wedge b) \vee (a \wedge c) = a \vee 0 = a$

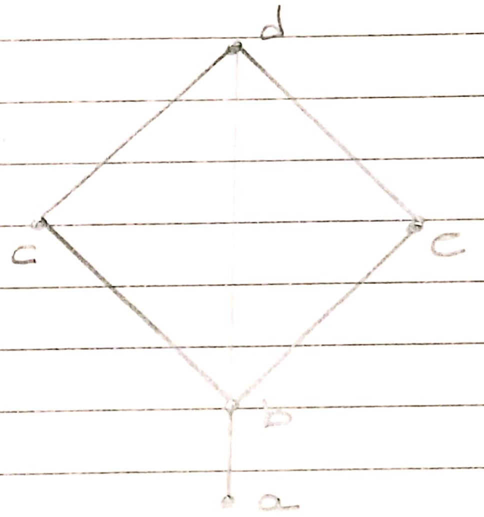
$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$



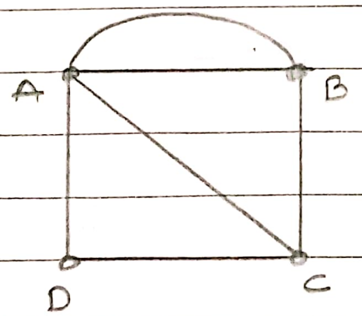
This lattice is distributive.

Define Hamiltonian path and Euler circuit.

A hamiltonian path is a path that contains each vertex exactly once. A Hamiltonian circuit is a circuit that contains each vertex exactly once except for the first vertex which is also the last



The path a, b, c, d, e is a hamiltonian path because it contains each vertex exactly once.
There is no hamiltonian circuit for this path.



There is an hamitonian circuit for this graph
The path A, B, C, B, A is a Hamiltonian Circuit.

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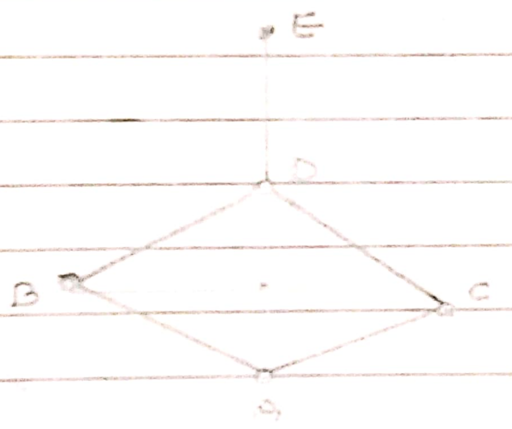
Define Euler path and Euler circuit?

A path in a graph G is called an Euler path if it includes every edge exactly once. An euler circuit is an euler Path that is a circuit.

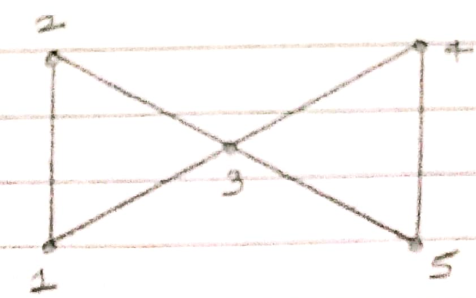
Examples -

(i) An Euler path is :- E, D, B, A, C, D

There is no euler circuit



(2) One Euler circuit in the graph of below is :-
5, 3, 2, 1, 3, 4, 5 Euler path is not possible for this graph.



Two questions arise naturally at this point. It is possible to determine whether an Euler path or Euler circuit exists without actually finding it? If there must be an Euler path or Euler circuit, is there an efficient way to find one?.

Q-7 How many vertices are necessary to construct a graph with exactly 6 edge in which each vertex is of degree 2.

→ Suppose there are n vertices in the graph with 6 edges also given the degree of each vertex is 2.

Therefore by handshaking lemma,

$$\sum_{i=1}^{n} d(v_i) = 2e$$

$$= 2 \times 6$$

$$d(v_1) + d(v_2) + \dots + d(v_n) = 2 \times 6$$

$$\underbrace{2+2+2+\dots+2}_{n \text{ times}} = 12$$

$$2n = 12$$

$$\boxed{n = 6}$$

Hence 6 nodes are required to construct a graph with 6 edges in which each node is of degree 2.

Q-8) Define Isomorphic graph with example

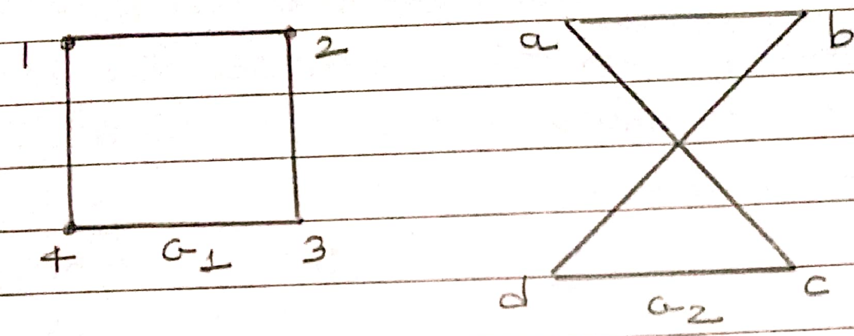
A. In geometry, two figures are thought of as equivalent if they have identical behaviour in terms of geometric properties. Likewise two graphs are thought of as equivalent or isomorphic if they have identical behaviours in terms of graph properties. The problem of graph isomorphism arises in many fields, such as chemistry, switching theory, information retrieval etc. Now we define Isomorphism of graph as follows.

Two graphs $G_1(V_1, E_1)$ and $G'(V', E')$ are said to be isomorphic to each other if there is a one-one correspondence between their vertices and between their edges such that the incidence relationship is preserved

In particular, adjacency between vertices is preserved. more generally two graph G_1 & G_2 are isomorphic if we can find the bijections.
$V_1 \to V_2$ and $E_1 \to E_2$ such that if $e \in E_1$, given by $e = (a, b)$ where a & b are vertices in V_1 then $f_E(e) = f_V(a), f_V(b)$ where $f_V(a)$ & $f_V(b)$ are vertices in V_2

Isomorphic graphs ae denoted by $G_1 = G_2$

G_1 is isomorphic to G_2.



The one-one correspondence between the vertices are

$1 \to a \quad 2 \to b$

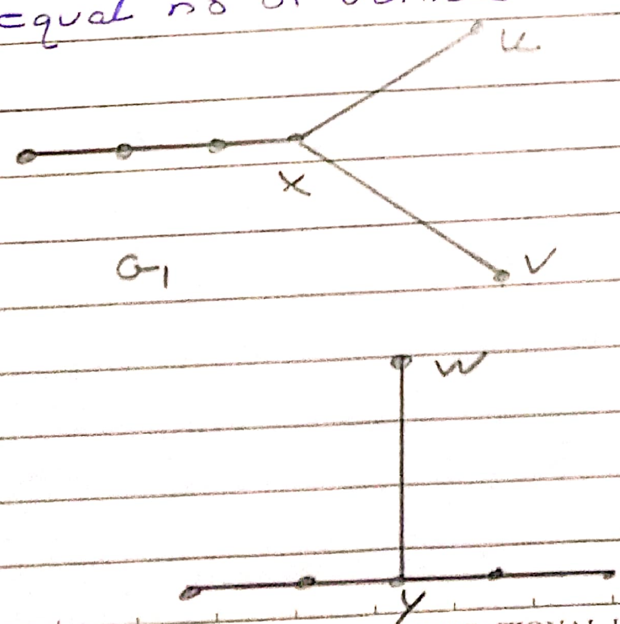
$3 \to d \quad 4 \to c$

It is immediately apparent by defination of isomorphism that two isomorphic graphs must have

(1) The same no of vertices

(2) The same no of edges

(3) An equal no of vertices with a given degree.



G_2

$G_1 \cong G_2$

Q-9) Find how many integers between 1 & 60 are not divisible by 2 nor by 3 and nor by 5 ?

→ Let A_1, A_2 & A_3 be the set of integers between 1 & 60 divisible by 2, 3 & 5

$$\therefore |A_1| = \left|\frac{60}{2}\right| = 30$$

$$|A_2| = \left|\frac{60}{3}\right| = 20$$

$$\therefore |A_3| = \left|\frac{60}{5}\right| = 12$$

and $|A_1 \cap A_2| = \left|\frac{60}{2\times 3}\right| = 10$

$$\therefore |A_1 \cap A_3| = \left|\frac{60}{2\times 5}\right| = 6$$

$$\therefore |A_2 \cap A_3| = \left|\frac{60}{3\times 5}\right| = 4$$

and $|A_1 \cap A_2 \cap A_3| = \left|\frac{60}{2\times 3\times 5}\right| = 2$

Number of integers between 1 & 60 which are divisible by 2, 3 or 5 are

$= |A_1 \cup A_2 \cup A_3|$

$= |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3|$

$- |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$

$= 30 + 20 + 12 - 10 - 6 - 4 + 2$

$= 44$

The no of integers between 1 & 60 are not divisible by 2, 3 or 5 $= 60 - 44 = 16$

Q-10) Determine the no of edges in graph with 6 nodes, 2 of degree 4 & 4 of degree 2. Draw two such graphs.

→ Suppose the graph with 6 vertices has a numbers of edges. Therefore, by handshaking lemma

$$\sum_{i=1}^{6} d(v_i) = 2e$$

$$\Rightarrow d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5) + d(v_6) = 2e$$

Now given 2 vertices are of degree 4 & 4 vertices are of degree 2

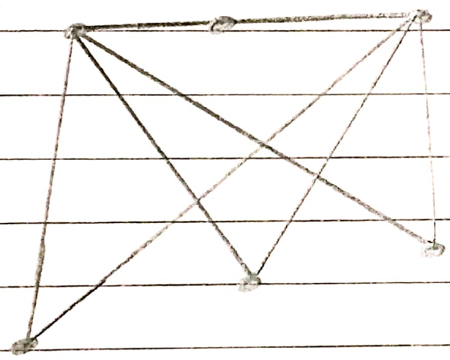
$(4+4) + (2+2+2+2) = 2e$

$\therefore 16 = 2e$

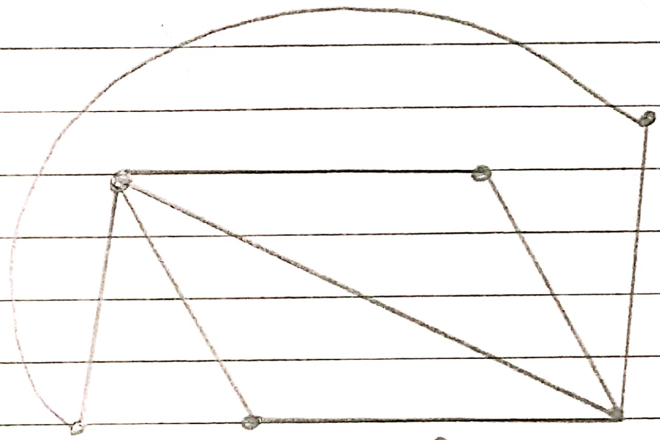
$e = 8$

The no of edges in a graph with 6 vertices with given conditions is 8

Two graphs are shown in below



(a)



(b)