

# COL 100 - Lec 12

## 1. Currying

- Eg:

$$\frac{\text{Definition}}{\text{add } x \downarrow y = x + y}$$

$$\text{fn: } \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N}$$

- Note partial evaluation possible
  - i.e. add 3 will evaluate to a  $\lambda$ unction that adds 3 to its argument
- $\text{add } 3 = \text{fn: } \mathbb{Z} \rightarrow \mathbb{Z}$   
 $y \rightarrow y + 3$
- Where is currying critical?
  - where you expect the output to be a function too!

Eg: function derivatives, ...

Thus

Higher Order function we saw in the last class

$$\rightarrow \text{HOSumProd} : \text{Op termFnc nextfnc iden } l \cup$$
$$= \begin{cases} \text{if } l > u \text{ then iden} \\ \text{else Op (termFnc } l) (\text{HOSumProd} \\ \quad \quad \quad \text{Op termFnc nextfnc} \\ \quad \quad \quad \text{idен (nextfnc } l) \cup) \end{cases}$$

Could also be specified as  
the following:

- \* \* Show fact Computation!
- \* \* Show sum Computation!

$$\rightarrow \text{HOSumProd} (\text{Op, termFnc, nextfnc, iden, } l, \cup)$$
$$= \begin{cases} \text{if } (l > u) \text{ then iden} \\ \text{else Op ( termfnc } l), \text{ HOsumProd} ( \dots \dots \dots ) \end{cases}$$

↳ in the non-curried  
form

what have we looked so far

- Procedures as arguments  
(or functions)

Let us look now at constructing procedures  
as  $\lambda$  expressions.  
(lambda)

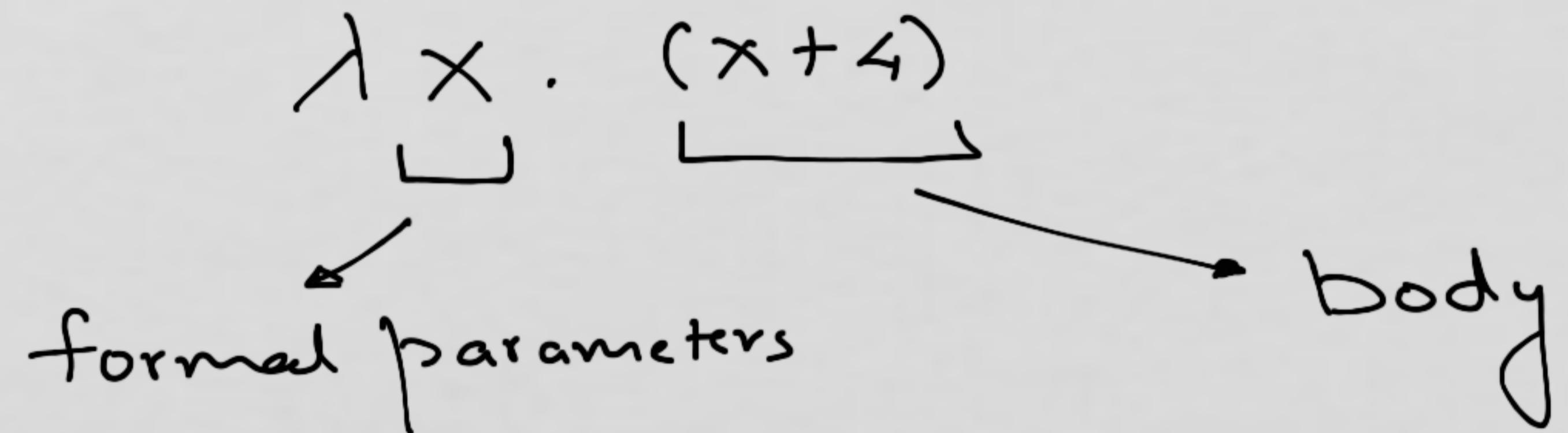
Motivation:

In the example to compute  
sum-series-pi we needed  
• pi-term to be  $(\frac{1}{x} \cdot \frac{1}{x+2})$   
pi-next to be  $(x+4)$

It is awkward to define trivial procedures  
pi-term & pi-next just so we can define our higher  
order function.

A more convenient way is to use lambda Expressions

Mathematically,



SML

; fn x => (x+4)

The resulting procedure is the same as what one would have obtained using keyword fun with a procedure name, except that lambda procedures are no-name (or nameless) procedures

# Procedures as general methods

## Elaborate Example

- finding roots of equations by the half-interval method

$$f(x) = 0 \quad [f \text{ is continuous}]$$

idea is if we are given pts  $a, b$

$$\text{s.t. } f(a) < 0 < f(b)$$

then  $f$  must have one root (or zero)

bet<sup>n</sup>  $a \& b$ .

take

$$x = \frac{a+b}{2}$$

- if  $f(x) > 0$  then the root is  
bet<sup>n</sup>  $a \& x$
- if  $f(x) < 0$  then the root is  
bet<sup>n</sup>  $x \& b$
- Continuing this way we reduce the  
interval by half at each step until  
some tolerance level is reached  
(let us call that T)

- Then, the process stops in  $O(\log(L/T))$  steps.  
[ $L$  is the length of original interval &  $T$  is the tolerance]
- function (or procedure) to implement our strategy

```
fun search f neg-pt pos-pt =  
let  
  val midpt = average neg-pt pos-pt  
in  
  if (close-enough? neg-pt pos-pt)  
  then midpt  
else  
  let  
    val test-value = f midpt  
    in  
      if test-value > 0.0 then  
        search f neg-pt midpt  
      else if test-value < 0.0 then  
        search f midpt pos-pt  
      else midpt
```

fun close-enough? x y  
= (abs(x-y)<0.001)

```
fun half-interval f a b =  
  let  
    val a-v = f a  
    val b-v = f b  
    in  
      if a-v < 0.0 andalso b-v ≥ 0.0 then  
        search f a b  
      else if a-v ≥ 0.0 andalso b-v < 0.0 then  
        search f b a  
      else raise NotOppositeSigns
```

half-interval-method sin 2.0 4.0

for  $x^3 - 2x - 3 = 0$ , bet<sup>n</sup> 182

half-interval-method (fn x  $\Rightarrow$   $x^3 - 2x - 3$ ) 1.0 2.0