**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676 (Ans)
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.

--FALSE

Given *μ* = 38 and Sd = 6 then according to above question z- value for employees at age 44 is 1. In standardized table 1 covers almost 84.37% data

So, people above 44 years are just 15.87% of data. It is approximately equal to 64 .

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

--TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS: - Here, both X1 and X2 follows normal distribution and X1,X2~ *N*(μ, σ2) are identically independent normal random variables.

2X1 = 2 *N*(μ, σ2) = N(2μ, 2σ2)

X1 + X2 = N(μ, σ2) + N(μ, σ2)

= N(2μ, 2σ2)

There is no significance difference between 2X1 and X1+X2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

ANS:

Given:  p(a<x<b) = 0.99 , mean =100,standardDeviation = 20

To Find:

Identify symmetric values for the standard normal distribution such that the area enclosed is .99

From the above details,we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

                                         Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

                         Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576)+100= 151.5

Two values symmetric about mean for the given standard normal distribution are [48.5,151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

ANS:

a) Rs 603.68

b) Rs 476.33

c) first division

**Step-by-step explanation:**

Total profit = profit 1 + profit 2 = P

hence P ~ N(12,74)

A ) Specifying a Rupee range ( centered on the mean ) that contains 95% probability for annual profit of the company

$13.41 = Rs 603.68

B) specifying the 5th percentile of profit

p = $10.59 ≈ Rs 476.33

C) The division that has a larger probability of making a loss in a given year is the first division