Bayes’ Rule

Bayes' Rule

Bayes' Rule, or Bayes' Theorem, is the mathematical relationship between the prior allocation of credibility (probability) and the posterior reallocation, conditioned on the new evidence.

Conditional Probability

- $P(A \mid B) = P(A, B) \mathbin{/} P(B)$
- $P(A \mid B) \times P(B) = P(A, B)$
- $P(B \mid A) = P(A, B) \mathbin{/} P(A)$
- $P(B \mid A) \times P(A) = P(A, B)$

Conditional Probability

- $P(A \mid B) = \mathbf{P(A, B)} / P(B)$
- $P(A \mid B) \times P(B) = P(A, B)$
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- $P(A \mid B) \times P(B) = P(A, B)$
- $P(B \mid A) = P(B, A) / P(A)$
- $P(B \mid A) \times P(A) = \mathbf{P(A, B)}$
- $P(A \mid B) = \dfrac{P(B \mid A) \times P(A)}{P(B)}$

Bayes' Rule

$$P(A\,|B) = \frac{P(B\,|A)\ \times P(A)}{P(B)}$$

- A and B are events, like breed and dysplasia.
- P(A | B) is the **posterior** (conditional) probability of A taking place given the new evidence B.
- P(B | A) is also a conditional probability, of B taking place given A.
- P(A) and P(B) are the marginal probability of A and B taking place independently.

Bayes' Rule

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Bayes' rule gets us from the prior p(A), to the posterior (conditional) distribution P(A | B), when focusing on a specific value of B.

Bayes' Rule in action: Fraud

A = fraud
B = ML decision

$$P(A\,|B) = \frac{P(B\,|A)\ \times P(A)}{P(B)}$$

- 1 in 1000 applications are fraudulent ➔ P(fraud)= 0.001
- ML model correctly identifies 99% of fraudsters ➔ P(positive | fraud) = 0.99
- ML model incorrectly flags 5% of non-fraudsters ➔ P(positive | non-fraud) = 0.05
- If application is flagged by ML model, what is the likelihood that it is fraudulent?

Bayes' Rule in action: Fraud

A = fraud
B = ML decision

$$P(fraud \mid ML = 1) = \frac{P(ML \mid fraud) \times P(fraud)}{P(ML)}$$

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Conditional Probability

- $P(A \mid B) = P(A, B) / P(B)$
- $P(A \mid B) \times P(B) =$ **$P(A, B)$**
- $P(B \mid A) = P(A, B) / P(A)$
- $P(B \mid A) \times P(A) =$ **$P(A, B)$**

Conditional Probability

- $P(A \mid B) = P(A, B) / P(B)$
- $P(A \mid B) \times P(B) = \mathbf{P(A, B)}$
- $P(B) = \sum \mathbf{P(A, B)}$

- $P(B \mid A) = P(A, B) / P(A)$
- $P(B \mid A) \times P(A) = \mathbf{P(A, B)}$

Conditional Probability

- $P(A \mid B) = P(A, B) \,/\, P(B)$
- $P(A \mid B) \times P(B) = \mathbf{P(A, B)}$
- $P(B \mid A) = P(A, B) \,/\, P(A)$
- $P(B \mid A) \times P(A) = \mathbf{P(A, B)}$

- $P(B) = \sum \mathbf{P(A, B)}$
- $P(B) \mathbf{=} \sum P(A \mid B) \times P(B) = \sum \mathbf{P(B \mid A) * P(A)}$

Bayes' Rule – Example: Fraud

A = fraud
B = ML decision

$$P(fraud \mid ML = 1) = \frac{0.99 \times 0.001}{\sum P(B \mid A*) * P(A*)}$$

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- If application is flagged by ML model, what is the likelihood that it is fraudulent?

Bayes' Rule – Example: Fraud

A = fraud
B = ML decision

$$P(fraud \mid ML = 1) = \frac{0.99 \times 0.001}{\mathbf{P(ML \mid fraud)} \times \mathbf{P(fraud) + P(ML|no\text{–}fraud)} \times \mathbf{P(no\text{–}fraud)}}$$

- 1 in 1000 applications are fraudulent ➔ P(fraud)= 0.001
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- If application is flagged by ML model, what is the likelihood that it is fraudulent?

Bayes' Rule – Example: Fraud

A = fraud
B = ML decision

$$P(fraud \mid ML = 1) = \frac{0.99 \times 0.001}{0.99 \times 0.001 + 0.05 \times 0.999}$$

- 1 in 1000 applications are fraudulent ➔ P(fraud)= 0.001
- ML model correctly identifies 99% of fraudsters ➔ P(positive | fraud) = 0.99
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- If application is flagged by ML model, what is the likelihood that it is fraudulent?

Bayes' Rule – Example: Fraud

A = fraud
B = ML decision

$$P(fraud \mid ML = 1) = 0.019$$

- 1 in 1000 applications are fraudulent ➔ P(fraud)= 0.001
- ML model correctly identifies 99% of fraudsters ➔ P(positive | fraud) = 0.99
- ML model incorrectly flags 5% of non-fraudsters ➔ P(positive | non-fraud) = 0.05
- **If application is flagged by ML model, what is the likelihood that it is fraudulent?**

Bayes' Rule

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)} = \frac{P(B|A) \times P(A)}{\sum P(B \mid A) * P(A)}$$

Bayes' rule gets us from the prior p(A), to the posterior (conditional) distribution P(A | B), when focusing on a specific value of B.

Bayes' Rule value

- Key application of Bayes' Rule when A is data and B is parameters.

- A model specifies P(data | parameters) and the prior P(parameters)

We use **Bayes' Rule** to convert that into what we are interested in:

- How strongly should we believe in the parameters given the data ➔ the posterior P(parameters | data)

THANK YOU

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