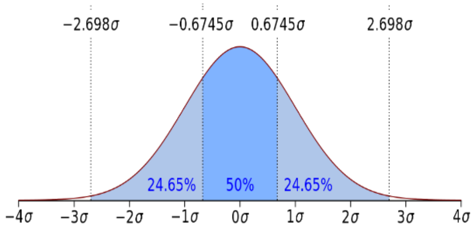
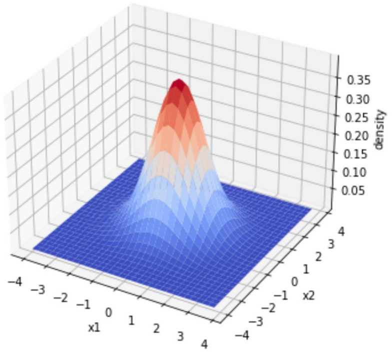
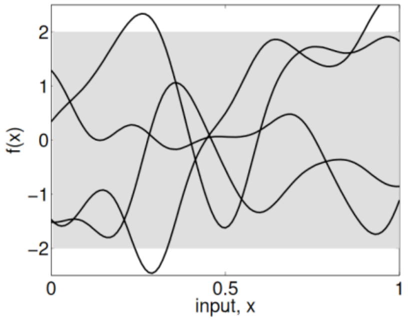
Train In Data

Gaussian Process

Gaussian Process

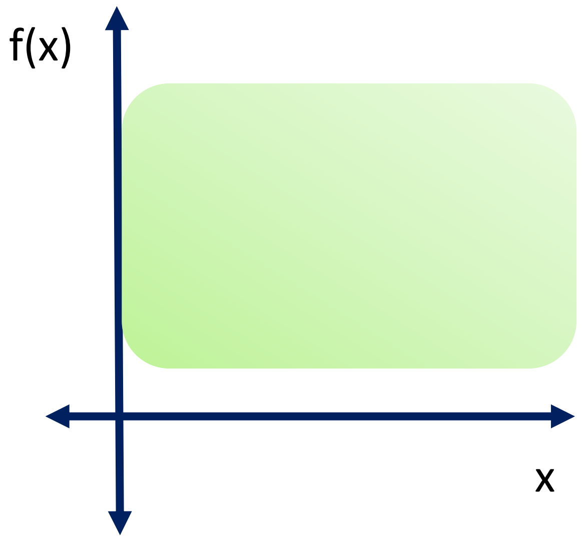






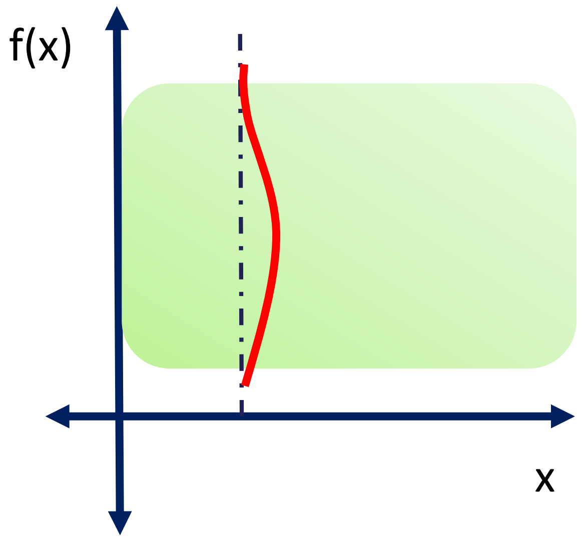
- Gaussian Distribution ➔ probability distribution of scalars
- Multivariate Gaussian ➔ probability distribution of vectors
- Gaussian Process ➔ Probability distribution of functions
  - Hyperparameter response function

Gaussian Process



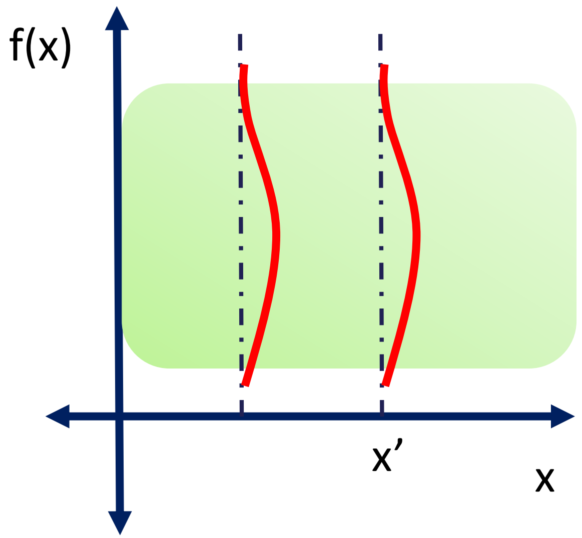
- Gaussian Process = GP = f(x)
- GP = Gaussian Distribution over functions
- X are functions
- Think of a function, X, as a very long vector, each entry in the vector specifying the function value f(x).

Gaussian Process



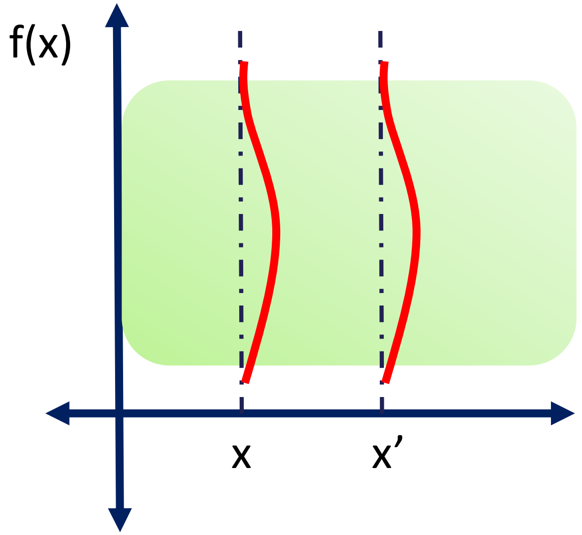
- Gaussian Process = GP = f(x)
- GP = Gaussian Distribution over functions
- X are functions
- Think of a function, X, as a very long vector, each entry in the vector specifying the function value f(x).

Gaussian Process



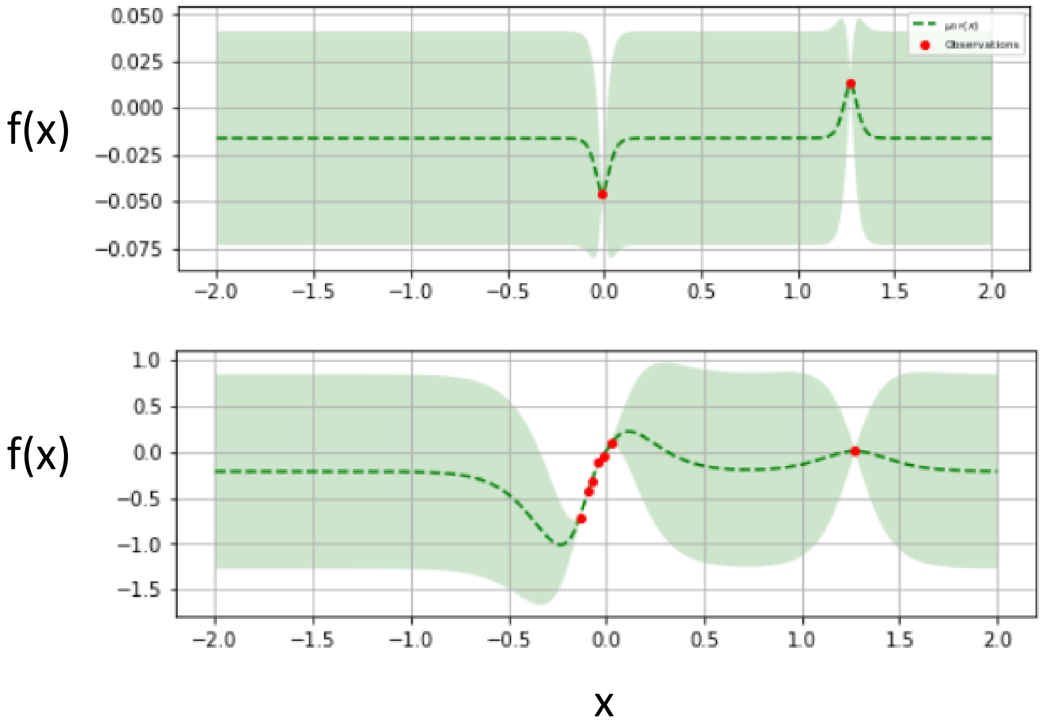
- GP = Gaussian Distribution over functions, f(x)
- x are functions
- Think of a function, x, as a very long vector, each entry in the vector specifying the function value f(x).
- The distribution of a Gaussian Process is the joint distribution of all those (infinitely many) random variables / functions.

Gaussian Process



- GP = Gaussian Distribution over functions
- $f(x) \sim GP(m(x), K(x, x'))$
- $m(x) = E[f(x)]$
- $K(x, x') = E[\,(f(x) - m(x)) \times (f(x') - m(x'))]$
- Computationally ➔ infinitely dimensional

Gaussian Process



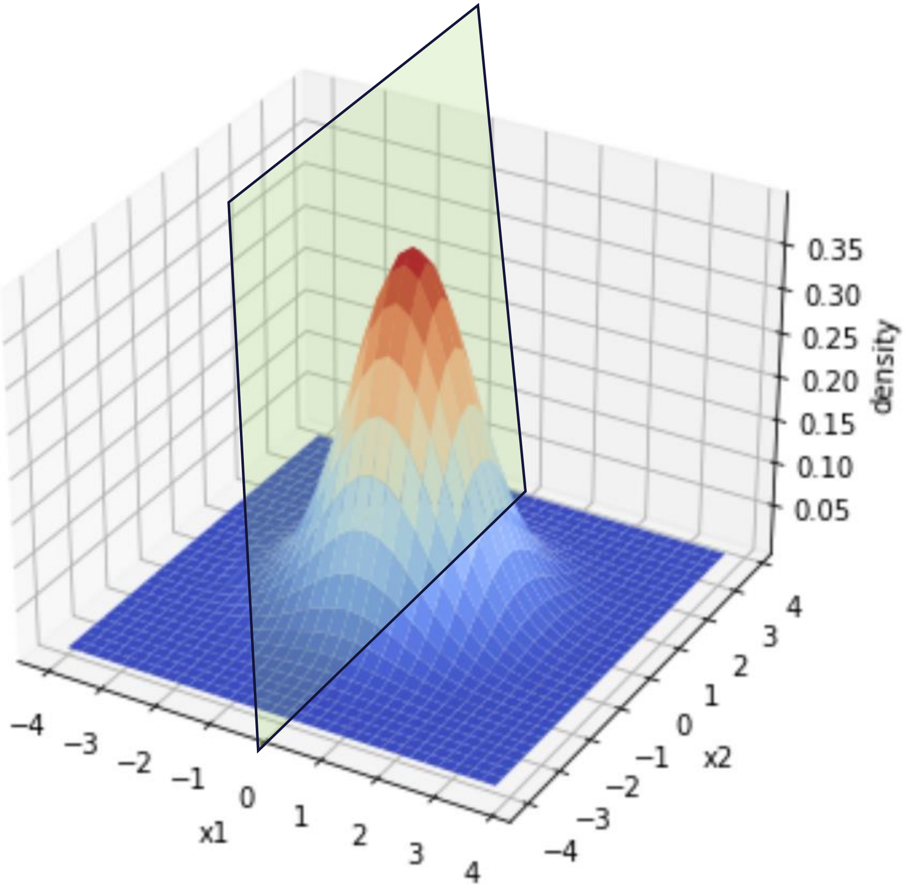
If we evaluate f(x) for a finite number of x, we can predict f(x) for all other values with an estimate plus an uncertainty.

GP ➔ prior

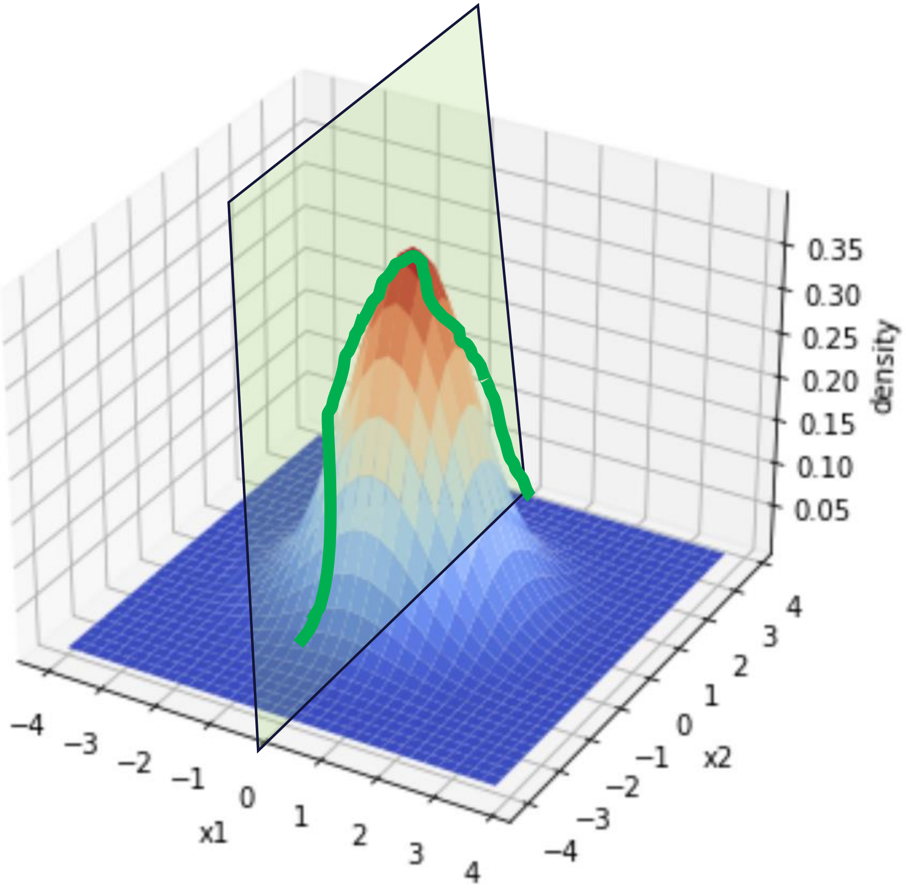
f(x) evaluations ➔ Data

Update f(x) ➔ the posterior

Joint Probability



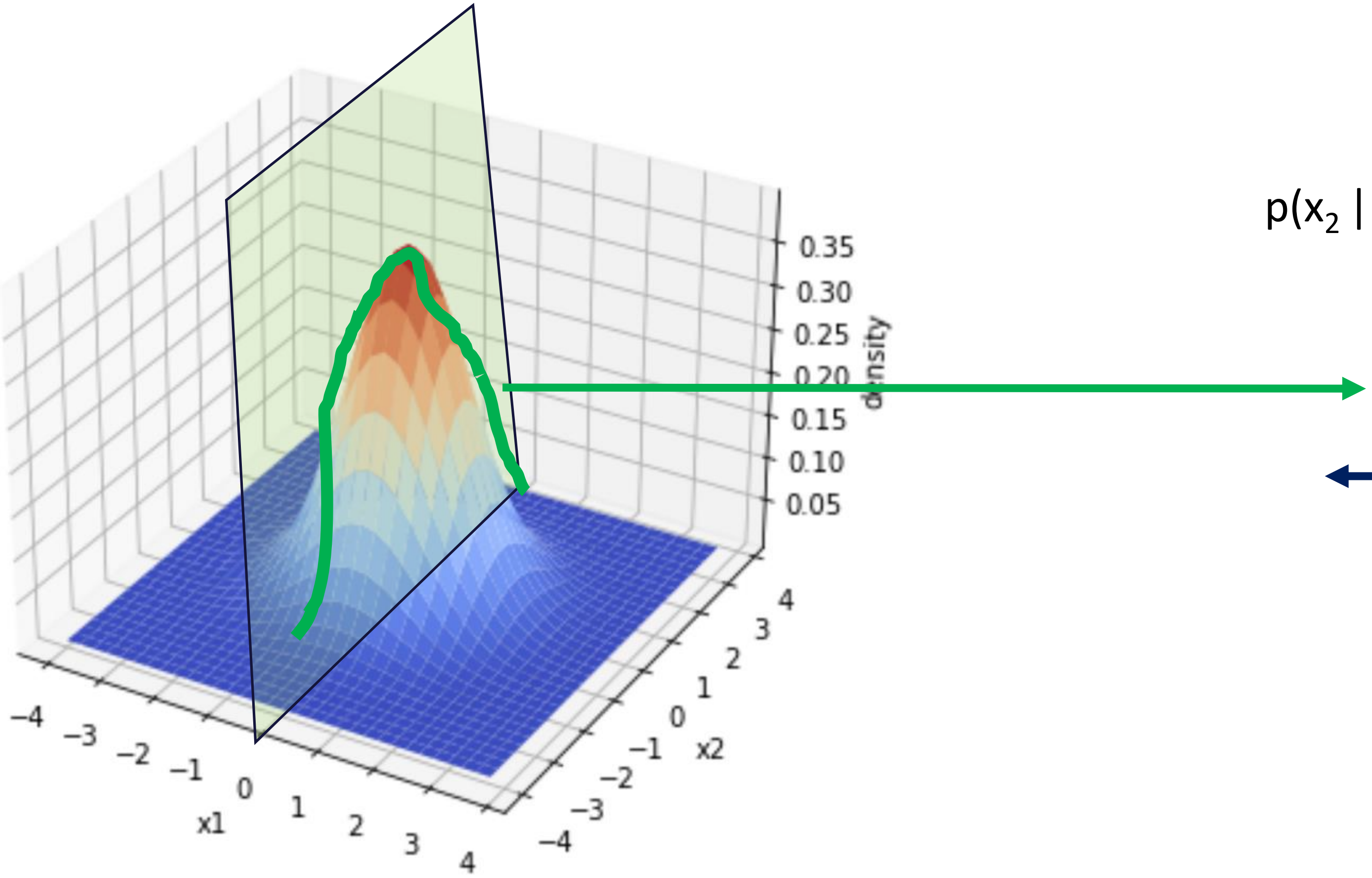
Joint Probability

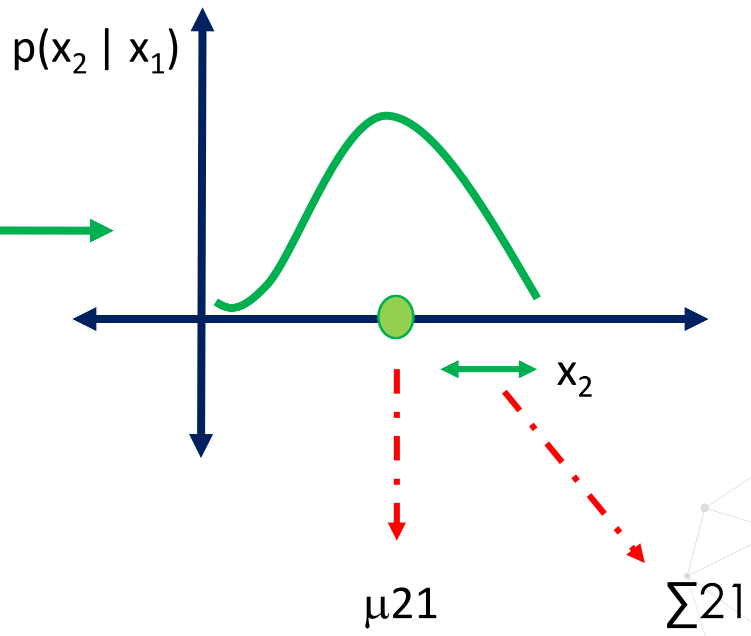


$$P(x2\ |x1 = x1)$$

Joint Probability

$$P(x2\ |x1 = x1)$$





Multivariate Gaussian Theorem

If $X = \begin{bmatrix} x1 \\ x2 \end{bmatrix} \sim N(\mu = \begin{bmatrix} m1 \\ m2 \end{bmatrix}, \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix})$

The marginal:

- P(x1) ~N(μ1, ∑11))
- P(x2) ~N(μ2, ∑22)

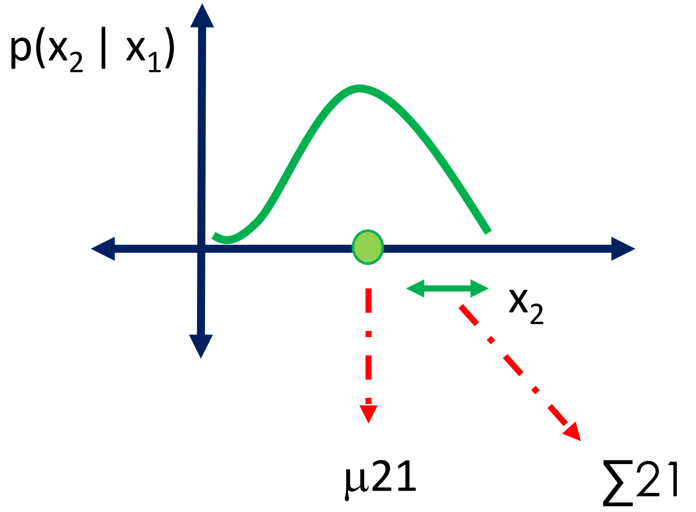
The posterior:

$$p(\mathbf{x}_1|\mathbf{x}_2) = \mathcal{N}(\mathbf{x}_1|\boldsymbol{\mu}_{1|2}, \boldsymbol{\Sigma}_{1|2})$$

$$\boldsymbol{\mu}_{1|2} = \boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\mathbf{x}_2 - \boldsymbol{\mu}_2)$$

$$\boldsymbol{\Sigma}_{1|2} = \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$$

Multivariate Gaussian Theorem



If $X = \begin{bmatrix} x1 \\ x2 \end{bmatrix} \sim N(\mu = \begin{bmatrix} m1 \\ m2 \end{bmatrix}, \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix})$

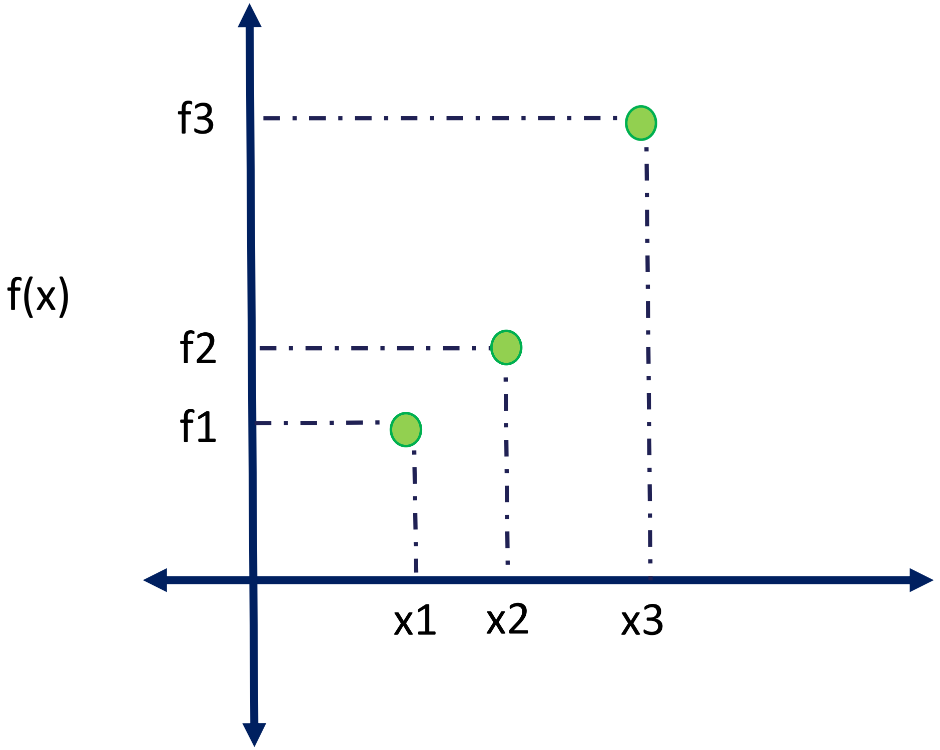
The marginal:

- P(x1) ~N(μ1, ∑11))
- P(x2) ~N(μ2, ∑22)

The posterior:

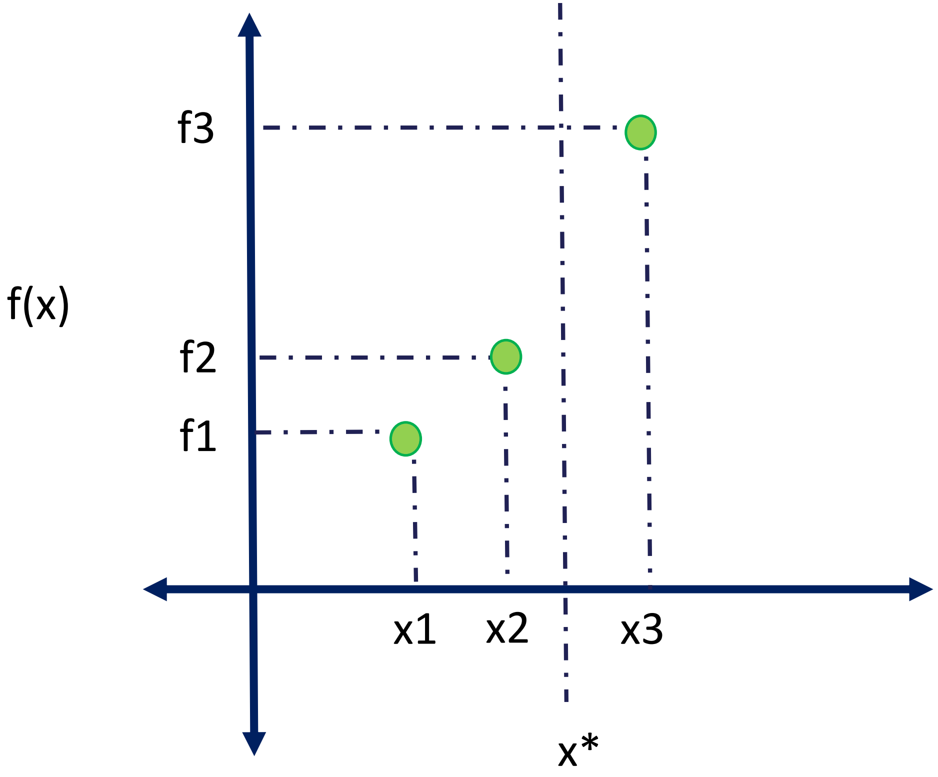
$$p(\mathbf{x}_1|\mathbf{x}_2) = \mathcal{N}(\mathbf{x}_1|\boldsymbol{\mu}_{1|2}, \boldsymbol{\Sigma}_{1|2})$$
$$\boldsymbol{\mu}_{1|2} = \boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\mathbf{x}_2 - \boldsymbol{\mu}_2)$$
$$\boldsymbol{\Sigma}_{1|2} = \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$$

Gaussian Regression



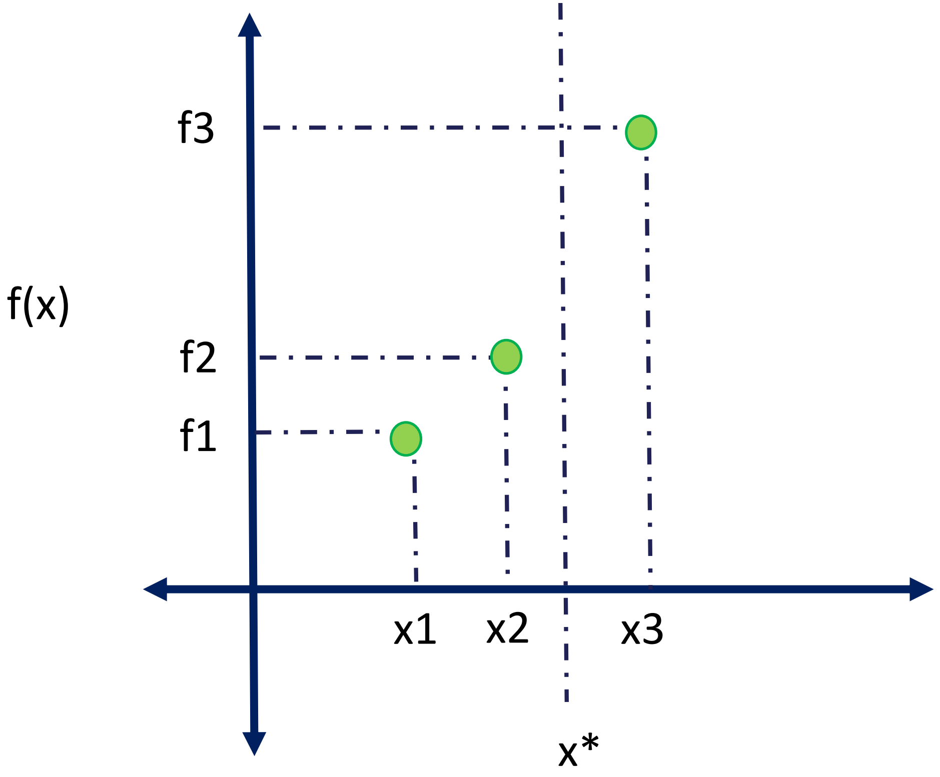
$$\begin{bmatrix} f1 \\ f2 \\ f3 \end{bmatrix} \sim \mathrm{N}(\mu = \begin{bmatrix} m1 \\ m2 \\ m3 \end{bmatrix}, \Sigma = \begin{bmatrix} K11 & K12 & K13 \\ K21 & K22 & K23 \\ K31 & K32 & K33 \end{bmatrix})$$

Gaussian Regression



$$\begin{bmatrix} f1 \\ f2 \\ f3 \end{bmatrix} \sim N(\mu = \begin{bmatrix} m1 \\ m2 \\ m3 \end{bmatrix}, \Sigma = \begin{bmatrix} K11 & K12 & K13 \\ K21 & K22 & K23 \\ K31 & K32 & K33 \end{bmatrix})$$

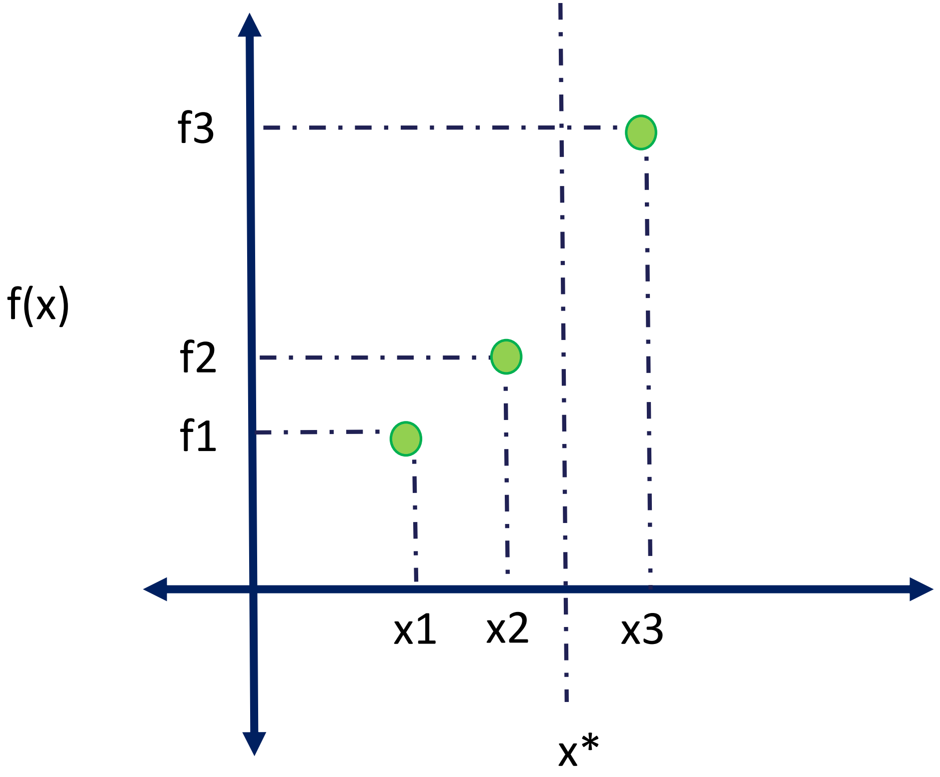
Gaussian Regression



$$\begin{bmatrix} f1 \\ f2 \\ f3 \end{bmatrix} \sim N(\mu = \begin{bmatrix} m1 \\ m2 \\ m3 \end{bmatrix}, \Sigma = \begin{bmatrix} K11 & K12 & K13 \\ K21 & K22 & K23 \\ K31 & K32 & K33 \end{bmatrix})$$

$$\begin{bmatrix} f1 \\ f2 \\ f3 \\ f* \end{bmatrix} \sim N(\mu = \begin{bmatrix} m1 \\ m2 \\ m3 \\ m* \end{bmatrix}, \Sigma = \begin{bmatrix} K11 & K12 & K13 & K1* \\ K21 & K22 & K23 & K2* \\ K31 & K32 & K33 & K3* \\ K^*1 & K^*2 & K^*3 & K** \end{bmatrix})$$

Gaussian Regression



$$\begin{bmatrix} f1 \\ f2 \\ f3 \end{bmatrix} \sim N(\mu = \begin{bmatrix} m1 \\ m2 \\ m3 \end{bmatrix}, \Sigma = \begin{bmatrix} K11 & K12 & K13 \\ K21 & K22 & K23 \\ K31 & K32 & K33 \end{bmatrix})$$

$$\begin{bmatrix} f \\ f* \end{bmatrix} \sim N(\mu = \begin{bmatrix} m \\ m* \end{bmatrix}, \Sigma = \begin{bmatrix} K & K* \\ K* & K** \end{bmatrix}$$

$$m* = \mu(x*) + K_* K^{-1}(f - \mu(f))$$

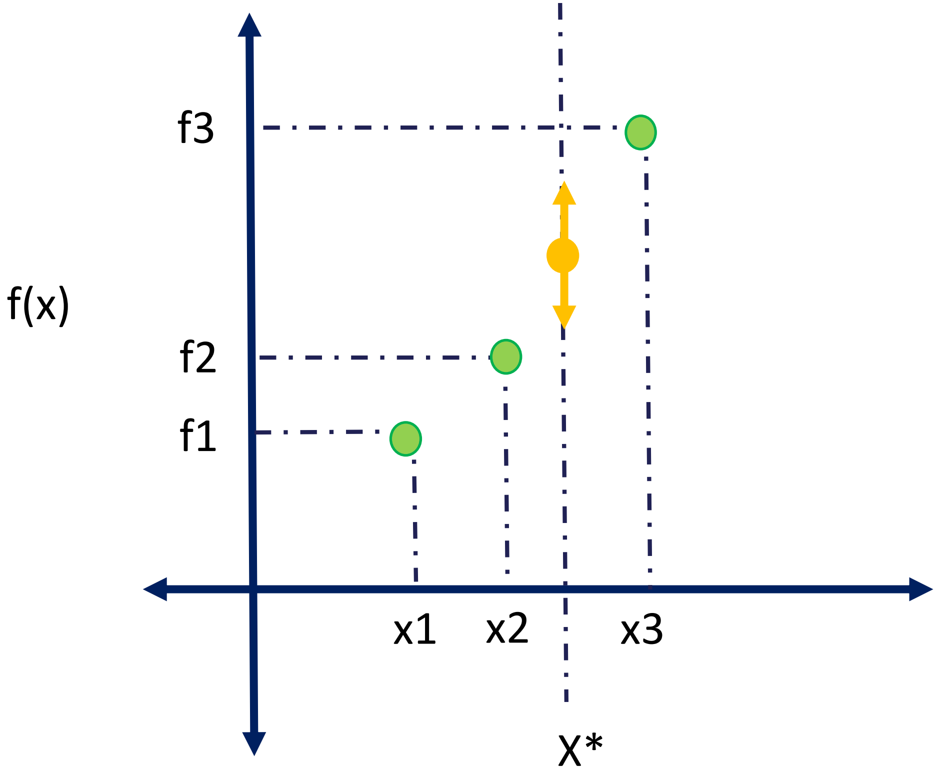
$$\Sigma = K_{**} - K_* K^{-1} K_*$$

$$p(\mathbf{x}_1|\mathbf{x}_2) = \mathcal{N}(\mathbf{x}_1|\boldsymbol{\mu}_{1|2}, \boldsymbol{\Sigma}_{1|2})$$

$$\boldsymbol{\mu}_{1|2} = \boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\mathbf{x}_2 - \boldsymbol{\mu}_2)$$

$$\boldsymbol{\Sigma}_{1|2} = \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$$

Gaussian Regression



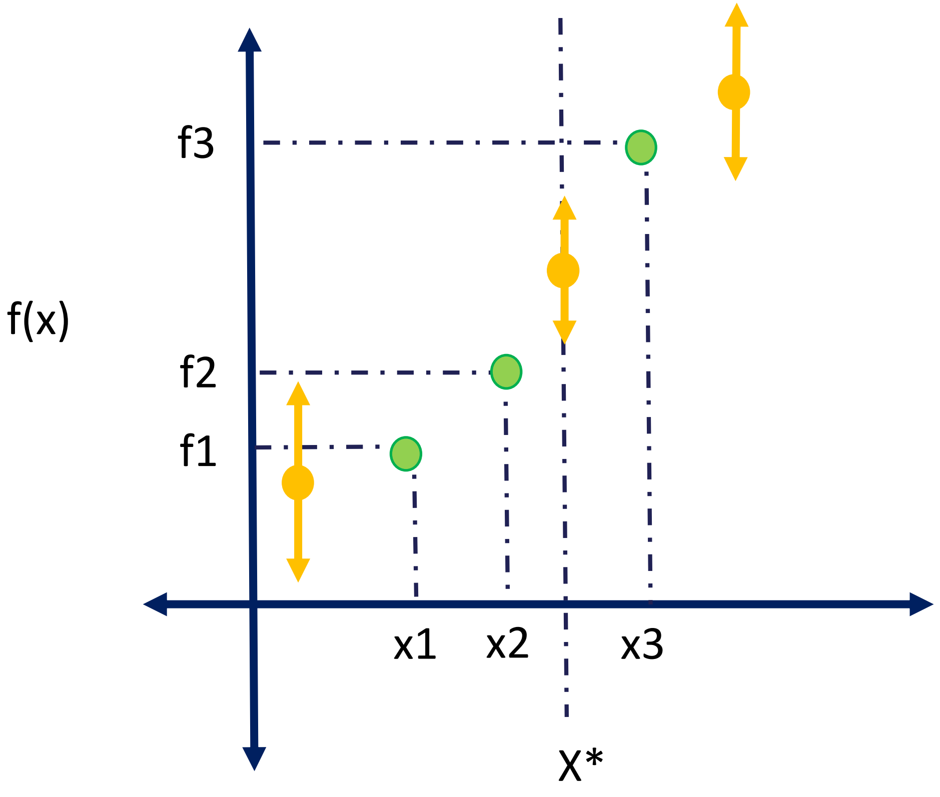
$$\begin{bmatrix} f1 \\ f2 \\ f3 \end{bmatrix} \sim \mathrm{N}(\mu = \begin{bmatrix} m1 \\ m2 \\ m3 \end{bmatrix}, \Sigma = \begin{bmatrix} K11 & K12 & K13 \\ K21 & K22 & K23 \\ K31 & K32 & K33 \end{bmatrix})$$

$$\begin{bmatrix} f \\ f* \end{bmatrix} \sim \mathrm{N}(\mu = \begin{bmatrix} m \\ m* \end{bmatrix}, \Sigma = \begin{bmatrix} K & K* \\ K* & K** \end{bmatrix}$$

$$m* = \mu(x*) + K_* K^{-1}(f - \mu(f))$$

$$\Sigma = K_{**} - K_* K^{-1} K_*$$

Gaussian Regression



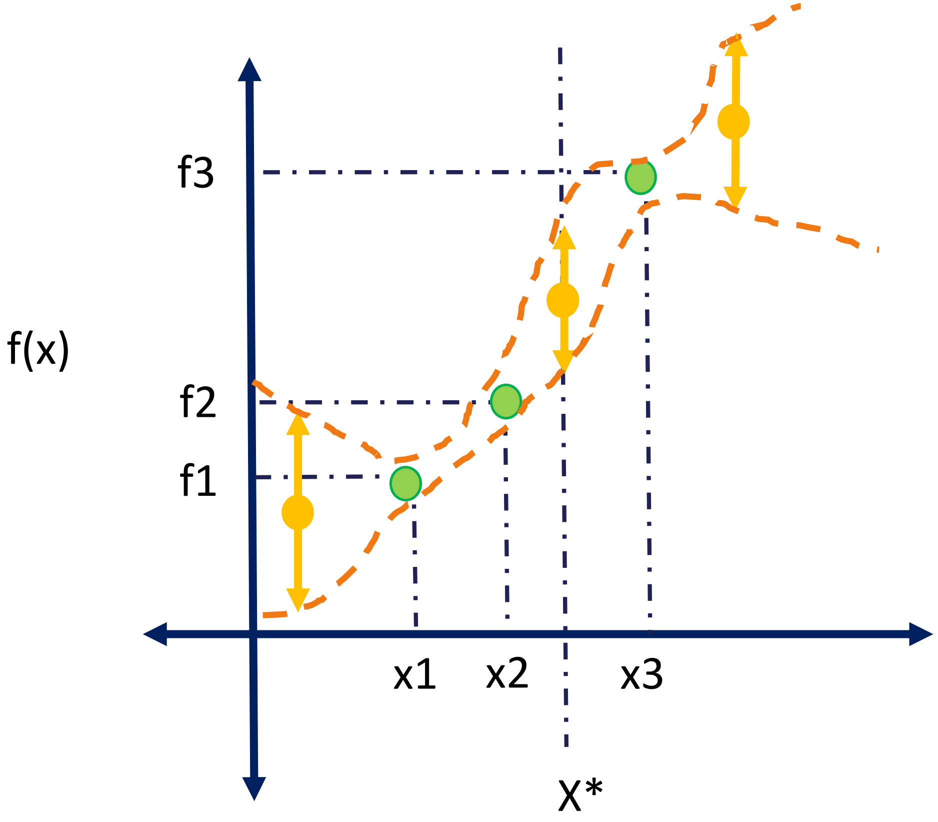
$$\begin{bmatrix} f1 \\ f2 \\ f3 \end{bmatrix} \sim N(\mu = \begin{bmatrix} m1 \\ m2 \\ m3 \end{bmatrix}, \Sigma = \begin{bmatrix} K11 & K12 & K13 \\ K21 & K22 & K23 \\ K31 & K32 & K33 \end{bmatrix})$$

$$\begin{bmatrix} f \\ f * \end{bmatrix} \sim N(\mu = \begin{bmatrix} m \\ m * \end{bmatrix}, \Sigma = \begin{bmatrix} K & K * \\ K * & K ** \end{bmatrix}$$

$$m^* = \mu(x^*) + K_* K^{-1}(f - \mu(f))$$

$$\Sigma = K_{**} - K_* K^{-1} K_*$$

Gaussian Regression



$$\begin{bmatrix} f1 \\ f2 \\ f3 \end{bmatrix} \sim \mathrm{N}(\mu = \begin{bmatrix} m1 \\ m2 \\ m3 \end{bmatrix}, \Sigma = \begin{bmatrix} K11 & K12 & K13 \\ K21 & K22 & K23 \\ K31 & K32 & K33 \end{bmatrix})$$

$$\begin{bmatrix} f \\ f* \end{bmatrix} \sim \mathrm{N}(\mu = \begin{bmatrix} m \\ m* \end{bmatrix}, \Sigma = \begin{bmatrix} K & K* \\ K* & K** \end{bmatrix}$$

$$m^* = \mu(x^*) + K_* K^{-1}(f - \mu(f))$$

$$\Sigma = K_{**} - K_* K^{-1} K_*$$

THANK YOU

www.trainindata.com