Detection of condensate vortex states

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We discuss a detection scheme that reveals the existence of vortex states in a cylindrically symmetric condensate trap. It relies on the measurement of the second-order correlation function of the Schrödinger field and yields directly the topological charge of the vortex state, as well as direct evidence of the existence of persistent currents. [S1050-2947(98)07107-8]

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INTRODUCTION

The rapid progress in the experimental generation and manipulation of Bose-Einstein condensates in low-density trapped alkali-metal vapors [1–5] opens up the way to the detailed study of the thermodynamic and dynamical properties of weakly interacting quantum-degenerate gases. A topic of much current interest is the superfluidity of these samples. Superfluidity is related to the rotational properties and in particular to the existence of vortex states in quantum gases [6].

While no vortex states have been launched in low-density atomic condensates so far, their existence has been studied numerically [7–9] and their stability analyzed [10]. Optical methods to launch vortex states were recently proposed in Refs. [11,12] and Jackson *et al.* [13] have produced numerical solutions of vortex formation in a weakly interacting condensate by piercing it and subsequently slicing it with a blue-detuned laser at a velocity exceeding the critical velocity.

In view of these developments, it is therefore timely and important to discuss ways to detect vortices in trapped atomic condensates. One possibility suggested in Ref. [11] is based on measuring the spatial absorption profile of the sample. This method is sensitive to the rotation of the condensate via the rotational Doppler shift [14], but does not give a direct measure of its topological charge. Rather, the quantization of the vortex motion must be inferred from other condensate parameters such as the number of particles and system size.

In contrast, the scheme that we propose and analyze in this paper produces a spatial interference pattern that results directly from the nonzero atomic angular momentum of the vortex and gives a direct measurement of its topological charge. In addition, a slight variation of that method allows a direct demonstration of the existence of persistent currents in the sample. The method is based on the ionization detection scheme discussed in [15], whereby one measures normally ordered correlation functions of the Schrödinger field operator $\hat{\Psi}(\mathbf{r},t)$. Specifically, we show that the two-point second-order correlation functions provide direct information on the persistent currents and the topological charge of the vortex state.

THEORY

For the sake of concreteness, consider first the general physical setting where a vortex in a quantum state $|v\rangle$ is present along with a nonvortex state in a different quantum state $|g\rangle$ which we refer to as the ground state. We further assume that both states are cylindrically symmetric about a common axis, for example, in a magnetic trap or a dipole trap using a Gaussian beam, and that the system is effectively two dimensional as a result of strong confinement in the z direction. Since $\langle v|g\rangle = 0$, this system can conveniently be described in terms of the spinor $\boldsymbol{\phi}(\mathbf{r},t) = \mathrm{col}(\phi_g(\mathbf{r},t),\phi_v(\mathbf{r},t))$, where $\mathbf{r}=(\rho,\varphi)$ is the two-dimensional position vector. We write the vortex and ground states in the general form

$$\begin{aligned}\langle \mathbf{r}|v\rangle &= \phi_v(\mathbf{r},t) = \beta(t)e^{i\theta(\mathbf{r})}\psi_v(\rho),\\ \langle \mathbf{r}|g\rangle &= \phi_g(\mathbf{r},t) = \alpha(t)\psi_g(\rho),\end{aligned} \tag{1}$$

where $\alpha(t)$ and $\beta(t)$ account for any time variation, $\psi_v(\rho)$ and $\psi_g(\rho)$ are the spatial profiles of the vortex and ground states, respectively, and $\theta(\mathbf{r}) = m\varphi$, m being the topological charge of the vortex. Here we keep the phase $\theta(\mathbf{r})$ general, thus allowing us to point out the specific characteristics of the vortex state that distinguish it from a nonvortex state. The quantum state described by Eqs. (1) is broad enough to cover recent proposals to generate vortex states using Raman transitions between hyperfine levels of ground-state alkali-metal atoms in condensate traps [11,12], but is more generally applicable than to that specific geometry.

In the presence of both the vortex and nonvortex components the system is described by the two-component Schrödinger field operator $\hat{\mathbf{\Psi}}(\mathbf{r}) = (\hat{\Psi}_g(\mathbf{r}),\hat{\Psi}_v(\mathbf{r}))$ satisfying the boson commutation relation $[\hat{\Psi}_\ell(\mathbf{r}),\Psi^\dagger_{\ell'}(\mathbf{r}')] = \delta_{\ell\ell'}\delta(\mathbf{r}-\mathbf{r}')$, $\ell=\{v,g\}$. Assuming then a system composed of N atoms, the quantum state of the system becomes

$$|\Phi(t)\rangle = \frac{1}{\sqrt{N!}}\left[\int d^3r[\phi_g(\mathbf{r},t)\hat{\Psi}^\dagger_g(\mathbf{r}) + \phi_v(\mathbf{r},t)\hat{\Psi}^\dagger_v(\mathbf{r})]\right]^N|0\rangle \tag{2}$$

or, in terms of the mode creation and annihilation operators

$$a^\dagger_\ell = \int d^3r\ \psi_\ell(\mathbf{r})\hat{\Psi}^\dagger_\ell(\mathbf{r}),\quad a_\ell = \int d^3r\ \psi^\star_\ell(\mathbf{r})\hat{\Psi}_\ell(\mathbf{r}), \tag{3}$$

we obtain

$$|\Phi(t)\rangle=\frac{1}{\sqrt{N!}}\sum_k C_N^k[\alpha(t)]^k[\beta(t)]^{N-k}(a_g^\dagger)^k(a_v^\dagger)^{N-k}|0\rangle. \tag{4}$$

Hence the sample is generally in an entangled superposition of the ground and vortex states, a result of the fact that the total number of particles is conserved, but the individual particle numbers in the two states need not be. This is similar to the situation of split condensates discussed in Refs. [16,17]. We remark that the quantum state (4) does not display Bose broken symmetry, that is, we do not assume that the vortex and ground states have an *a priori* relative phase. We return to this important point later on.

Due to the assumed cylindrical symmetry of the system the existence of a vortex state cannot be demonstrated by off-resonance imaging, which measures correlation functions of the sample density $\hat{\rho}(\mathbf{r},t)\equiv\hat{\mathbf{\Psi}}^\dagger(\mathbf{r},t)\hat{\mathbf{\Psi}}(\mathbf{r},t)$. Since this is simply the sum of the condensate and vortex density profiles and the vortex density profile is cylindrically symmetric, the density profile does not reveal the phase singularity associated with the vortex, but only the density hole at the center of the vortex. What is needed instead is a measurement scheme that involves correlation functions of $\hat{\mathbf{\Psi}}(\mathbf{r},t)$ itself. As discussed in Ref. [15], these functions can be extracted in an ionization scheme whereby one or more tightly focused lasers are used to selectively ionize atoms in small regions of the condensate plus vortex system. The measurement proceeds then by detecting the ionized atoms, which play the role of a detector field.

We consider specifically a two-point detection scheme in which two ionizing laser beams are focused at locations $\mathbf{r}_1=(\rho_1,\varphi_1)$ and $\mathbf{r}_2=(\rho_2,\varphi_2)$. For that geometry and assuming that the lasers are focused onto spots small compared to the dimensions of the condensate, the ionization scheme measures the probability w_2 of jointly ionizing an atom at $\mathbf{r}_1$ and the other at $\mathbf{r}_2$ as a function of normally ordered Schrödinger field correlation functions whose explicit form is

$$\begin{aligned}w_2(t,\Delta t)\simeq{}&\eta(\mathbf{r}_1,\mathbf{r}_2)\,\eta(\mathbf{r}_2,\mathbf{r}_1)\int_t^{t+\Delta t}dt_1\int_t^{t+\Delta t}dt_2\langle\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_1,t_1)\\&\times\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_2,t_2)\hat{\mathbf{\Psi}}(\mathbf{r}_2,t_1)\hat{\mathbf{\Psi}}(\mathbf{r}_1,t_2)\rangle\\&+\eta(\mathbf{r}_1)\,\eta(\mathbf{r}_2)\int_t^{t+\Delta t}dt_1\int_t^{t+\Delta t}dt_2\langle\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_1,t_1)\\&\times\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_2,t_2)\hat{\mathbf{\Psi}}(\mathbf{r}_2,t_2)\hat{\mathbf{\Psi}}(\mathbf{r}_1,t_1)\rangle\\&+\eta_x(\mathbf{r}_1,\mathbf{r}_2)\int_t^{t+\Delta t}dt\langle\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_1,t)\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_2,t)\\&\times\hat{\mathbf{\Psi}}(\mathbf{r}_2,t)\hat{\mathbf{\Psi}}(\mathbf{r}_1,t)\rangle.\end{aligned} \tag{5}$$

Here $\eta(\mathbf{r})$ is the detector self-efficiency, $\eta(\mathbf{r}_1,\mathbf{r}_2)$ its cross efficiency, and $\eta_x(\mathbf{r}_1,\mathbf{r}_2)$ is its ‘‘exchange efficiency,’’ as discussed in Ref. [15], which gives the explicit form of these coefficients. Physically, the first term in w_2 is an exchange contribution resulting from the interference of the detector fields at positions $\mathbf{r}_1$ and $\mathbf{r}_2$, while the second term is the direct contribution familiar from photodetection theory. The last term finds its origin in the fact that the ionization scheme does not distinguish from which ionized atom the detected electrons originate. Optical photodetection theory normally considers the second term only.

Equation (5) can be considerably simplified for measurement intervals Δt small compared to the characteristic evolution time of the condensate. In this case we can set $t_1=t_2=t$ and we have simply

$$\begin{aligned}w_2(t)={}&G^{(2)}(\mathbf{r}_1,\mathbf{r}_2;t)\{[\eta(\mathbf{r}_1)\,\eta(\mathbf{r}_2)+\eta(\mathbf{r}_1,\mathbf{r}_2)\,\eta(\mathbf{r}_2,\mathbf{r}_1)]\\&\times(\Delta t)^2+\eta_x(\mathbf{r}_1,\mathbf{r}_2)\Delta t\},\end{aligned} \tag{6}$$

where

$$\begin{aligned}&G^{(2)}(\mathbf{r}_1,\mathbf{r}_2;t)\\&\quad\equiv\langle\Phi(0)|\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_1,t)\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_2,t)\hat{\mathbf{\Psi}}(\mathbf{r}_2,t)\hat{\mathbf{\Psi}}(\mathbf{r}_1,t)|\Phi(0)\rangle\\&\quad=\langle\Phi(t)|\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_1)\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}_2)\hat{\mathbf{\Psi}}(\mathbf{r}_2)\hat{\mathbf{\Psi}}(\mathbf{r}_1)|\Phi(t)\rangle.\end{aligned} \tag{7}$$

As follows from the definitions of the detector self, cross, and exchange efficiencies $\eta(\mathbf{r})$, $\eta(\mathbf{r}_1,\mathbf{r}_2)$, and $\eta_x(\mathbf{r}_1,\mathbf{r}_2)$ given in Ref. [15], the term in curly brackets in Eq. (6) does not vary azimuthally, which is important for the following considerations.

Specializing in anticipation of our subsequent discussion to the case $\rho_1=\rho_2=\rho$ and with Eq. (4), Eq. (7) gives readily

$$\begin{aligned}G^{(2)}(\mathbf{r}_1,\mathbf{r}_2;t)={}&N(N-1)(|\alpha(t)\phi_g(\rho)|^4+|\beta(t)\phi_v(\rho)|^4\\&+2|\psi_g(\rho)|^2|\psi_v(\rho)|^2|\alpha(t)|^2\\&\times|\beta(t)|^2\{1+\cos[\theta(\mathbf{r}_2)-\theta(\mathbf{r}_1)]\}).\end{aligned} \tag{8}$$

The key feature for the present discussion is the spatial dependence contained in the explicit phase difference $[\theta(\mathbf{r}_2)-\theta(\mathbf{r}_1)]$. This azimuthally varying interference term arises from the fact that the ionization scheme does not distinguish whether the detected electrons originated from the ground state or vortex state, thus yielding a quantum interference between the two alternatives where the electron at detector 1 originated from the ground state and the electron at detector 2 originated from the vortex and vice versa. Provided measurements are performed at a fixed time t, this dependence allows us to determine the existence of vortex motion, in a particularly simple way, as we show below.

PERSISTENT CURRENTS

The hydrodynamic formulation of superfluidity [18] introduces the velocity $\mathbf{v}_s=(\hbar/M)\nabla\theta(\mathbf{r})$, where M is the atomic mass and $\theta(\mathbf{r})$ is the phase of the superfluid component. In our case the vortex phase measured relative to the ground-state phase, which was tacitly taken equal to zero. Vortex states are characterized by the fact that the circulation of $\mathbf{v}_s$ is quantized,

$$\oint\mathbf{v}_s\cdot d\boldsymbol{l}=2\pi n(\hbar/M), \tag{9}$$

where n is an integer. In order to detect the circulation of the vortex, it is sufficient to determine its tangential velocity component v_φ. Hence the detectors can remain on a circle centered on the axis of rotation of the vortex and we have

$$\nabla\theta = \frac{1}{\rho}\frac{\partial\theta(\mathbf{r})}{\partial\varphi}\hat{\mathbf{e}}_\varphi, \tag{10}$$

where $\hat{\mathbf{e}}_\varphi$ is the unit vector tangential to the radial direction. For small distances $|\mathbf{r}_2-\mathbf{r}_1|$, the relative phase appearing in the last term in the correlation function (8) becomes

$$\theta(\mathbf{r}_2)-\theta(\mathbf{r}_1)\simeq(\nabla\theta)\cdot\hat{\mathbf{e}}_\varphi\rho d\varphi=(M/\hbar)v_\varphi(\varphi)\rho d\varphi. \tag{11}$$

For a general phase variation $\theta(\mathbf{r})$ the local velocity, and hence the current of atoms, will vary azimuthally. However, for a vortex of topological charge m, we have $v_\varphi = m\hbar/M\rho$ independent of the azimuthal position of the pair of closely spaced detectors and $n=m$ in Eq. (9). Hence, moving the pair of detectors along a circle while keeping their distance $\rho d\varphi$ fixed allows one to determine the presence of persistent currents $v_\varphi(\varphi)=\text{const}$. The detection of this persistent current is a key characteristic of a vortex state as it demonstrates that the state being detected is characterized by a single topological charge.

From the value of the persistent current one could infer the magnitude of m if all other parameters were known. Next we describe a second measurement that yields the topological charge more directly.

TOPOLOGICAL CHARGE

In addition, it is also possible to carry out a different class of measurements where detector 1 is held at a fixed position relative to the vortex core, while detector 2 is moved on a circle relative to detector 1. In that case, $G^{(2)}$ will exhibit oscillations as the relative azimuthal angle $(\varphi_1-\varphi_2)$ between the two detectors is varied, the phase difference being

$$\theta(\mathbf{r}_1)-\theta(\mathbf{r}_2)=m(\varphi_1-\varphi_2). \tag{12}$$

Thus the second-order correlation function shows interference fringes as the relative azimuthal angle of the detectors is varied and the topological charge of the vortex may be extracted as the number of (complete) bright fringes in the interference pattern as detector 2 is moved through a full circle. Note that in contrast to the preceding scheme, which requires knowledge of the system parameters, e.g. the atomic mass, to determine v_φ absolutely, the present method reveals the topological charge of the vortex m from a global property of the interference pattern, the number of bright fringes.

SUMMARY AND DISCUSSION

To summarize, we have discussed a measurement scheme that permits us to fully characterize vortex states in low-density atomic condensates, yielding both direct evidence of persistent currents and a parameter-free determination of the topological charge. While our specific example involves a vortex in an internal quantum state $|v\rangle$ different from that of the ground-state component, with a fixed total number of atoms, this restriction can readily be lifted. For instance, if it is known that the vortex component has exactly N_v atoms and the ground-state component N_g atoms, the system is described by the state

$$|\Phi(t)\rangle=|N_v,N_g\rangle=\frac{\alpha(t)^{N_g}\beta(t)^{N_v}}{\sqrt{N_v!N_g!}}(a_v^\dagger)^{N_v}(a_g^\dagger)^{N_g}|0\rangle \tag{13}$$

instead of the entangled state (2). In this case, one finds

$$\begin{aligned}G^{(2)}(\mathbf{r}_1,\mathbf{r}_2;t)=&N_v(N_v-1)|\beta(t)\phi_v(\rho)|^4\\&+N_g(N_g-1)|\alpha(t)\phi_g(\rho)|^4\\&+2N_vN_g|\beta(t)\phi_v|^2|\alpha(t)\phi_g|^2\\&\times\{1+\cos[\theta(\mathbf{r}_1)-\theta(\mathbf{r}_2)]\},\end{aligned} \tag{14}$$

identical in form to Eq. (8) for the case with differing internal quantum states, so the previous analysis and conclusions apply to this case also.

Finally, there is the issue of whether the vortex could be detected using a simpler scheme involving only the direct measurement of the density, e.g., via off-resonance imaging. Our previous discussion clearly shows that this is not possible if the vortex and ground-state components are in different electronic states. However, for the case that both components are in the same electronic state the topological charge may be inferred from the density interference pattern. To see this assume that there is an established phase relationship between a spatially homogeneous ground state and the vortex with azimuthal variation $\exp(im\varphi)$. The relative phase between the two components may arise from the assumption of Bose broken symmetry applicable to large condensates approaching the thermodynamic limit or from the measurement process itself as in Ref. [19]. In either case the density exhibits an interference pattern between the ground state and the vortex of the form $1+\eta\cos(m\varphi+\phi)$ for a fixed radius removed from the vortex core, where ϕ is a fixed but random phase and η the modulation depth. Thus the topological charge may be deduced from the interference between the ground state and the vortex if they have the same electronic state. In comparison, the virtues of the measurement scheme proposed here based on second-order correlation functions are that it does not require any phase relationship between the ground and vortex states, even when the electronic states are the same, and it does not require that the electronic states of the two components are the same.

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