Forced 2D Turbulence: Experimental Evidence of Simultaneous Inverse Energy and Forward Enstrophy Cascades

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We have performed measurements on forced and decaying 2D turbulence in flowing soap films. Unlike previous experiments, turbulence was excited by grids (arrays of cylinders) lining the walls of the flow channel. The resulting turbulent steady state agrees with the ideas of an inverse energy cascade accompanied by a forward enstrophy cascade. Energy spectra agree, respectively, with $k^{-5/3}$ and k^{-3} power laws. Downstream from the forcing section of the channel, the turbulence decays freely, and the energy spectrum scales only as k^{-3}. Our observations unite the results of several theories, and a number of simulations and experiments, into a single self-consistent set of measurements. [S0031-9007(98)07093-8]

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Much of the physics of 3D turbulence is attributed to a strongly nonlinear term in the Navier-Stokes equation. The so called vortex stretching term is largely responsible for our surprising lack of understanding of turbulent flow. Going from three dimensions to two removes vortex stretching from the Navier-Stokes equation, yet enough nonlinearity remains to produce turbulent behavior. Evidence of quasi-2D turbulence in nature is ample, with the Jovian atmosphere and weather patterns on earth as examples. Despite its apparent simplicity compared with 3D, the 2D turbulence problem has thus far eluded rigorous theoretical interpretation. Various theories and computer simulations do not offer a consistent picture with respect to simple observable quantities such as the power spectrum of turbulent velocity fluctuations, $E(k)$. Experiments on 2D turbulence have been few and have not yet been able to resolve the disputes between theories and simulations. In this Letter we present the first (to our knowledge) experimental evidence which supports some of the most important theoretical results in 2D turbulence, namely, an inverse energy cascade (Kraichnan [1]) accompanied by a forward enstrophy (rms vorticity) cascade (Kraichnan [1], Batchelor [2]). Both steady state forced turbulence and decaying turbulence were studied in a single experiment, showing a smooth transition between the two cases.

If energy is uniformly injected into the flow at a particular length scale (ℓ_{inj}), Kraichnan [1] proposed that $E(k)$ will show two distinct power law regions. For $k < k_{\mathrm{inj}} \equiv 2\pi/\ell_{\mathrm{inj}}$, $E(k) \sim \epsilon^{2/3}k^{-5/3}$, where ϵ is the energy transfer rate. For $k > k_{\mathrm{inj}}$, $E(k) \sim \eta^{2/3}k^{-3}$, where η is the enstrophy transfer rate. This last regime was also suggested by Batchelor [2] for freely decaying 2D turbulence. There are other theoretical predictions for the energy spectrum of 2D turbulence. Saffman [3] has suggested that vorticity shocks can develop for the zero viscosity situation, leading to $E(k) \sim k^{-4}$.

Computer simulations generally support, but not unambiguously, the energy-enstrophy cascade picture. The $E(k) \sim \eta^{2/3}k^{-5/3}$ inverse energy cascade has been observed in many simulations (e.g., by Frisch and Sulem [4] and by Smith and Yakhot [5]) but was, for instance, not seen by Borue [6]. These simulations generally failed to reproduce the forward enstrophy cascade. Far less agreement exists between simulations which aim to capture this part of the spectrum. For a review, see Frisch [7]. Simulated spectra in the enstrophy cascade range generally abide $E(k) \sim k^{-3}$ (e.g., Staquet *et al.* [8] for a 2D mixing layer or Chasnov [9] for freely decaying 2D turbulence), but often decay more steeply, depending on simulation methods and initial conditions.

2D turbulence experiments are far less common than simulations, but in the last few years much has come to light. Magnetohydrodynamic (MHD) experiments by Sommértia [10] showed the first evidence for the inverse energy cascade. This has recently been improved upon by Paret and Tabeling [11]. Neither experiment was able to observe the enstrophy cascade. Experiments with flowing soap films have observed the enstrophy cascade (e.g., Kellay *et al.* [12] or Martin *et al.* [13]), as is expected for freely decaying 2D turbulence [2]. Since 2D turbulence in soap films has, up to now, been excited only at one point in the flow channel, experiments have lacked the steady state forcing environment (such as the MHD experiments) necessary to verify an inverse energy cascade. Gharib and Derango [14] did, however, report the simultaneous observation of energy and enstrophy cascades for decaying turbulence in soap films. This result goes against current theoretical understanding.

The experiments reported here finally provide strong evidence for the simultaneous existence of energy and enstrophy cascades in a single experiment and accurately measure the power law behavior of the energy spectra. Our measurements were performed with a soap film sheet flowing between two vertical wires. The workings of this technique have been described in detail by Rutgers [15,16], Goldburg [17], and Martin [13].

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In this work we have taken a new approach for generating steady state forced turbulence in flowing soap films. Instead of placing a horizontal comb across the mean flow [12–14,18], we have placed vertical combs along the channel walls. The teeth of the vertical comb perpetually generate small vortices, which are then quickly swept into the center of the channel by larger vortices. A forced, steady turbulent state ensues.

A photograph of a single vertical comb, consisting of needlelike staggered teeth (0.3 mm in diameter, 3 mm separation), in the flow is shown in Fig. 1a. The comb is placed in the center of the channel to demonstrate the turbulent structure which develops around it. The vertical stripes are interference fringes between the front and back surfaces of the film, indicating that its thickness changes 0.2 μm between consecutive bright fringes. The total film thickness is estimated at 5–6 μm. Near the comb, thickness variations occur over such short distances ($\ll$1 mm) that the photographic film can no longer distinguish individual fringes. For distances larger than 1 mm it is therefore reasonable to consider the film effectively smooth. The photograph was exposed for $1/8000$ s.

The turbulent envelope shows small vortices shedding from the teeth at the comb's leading edge, which merge with similar vortices downstream. Large semicircular vortices grow on opposite sides of the comb. They are spaced rather regularly, implying interactions between vortices on both sides of the comb. The structures are reminiscent of those observed in mixing layers [19]. Turbulent eddies are punctuated by laminar fluid, which is occasionally swept completely back into the comb.

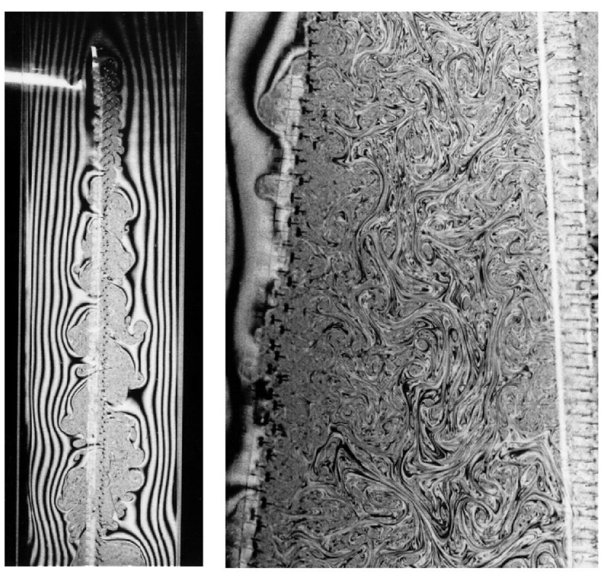

FIG. 1. Photographs of vertically flowing turbulent films. Interference of monochromatic light between the front and back surfaces of the film lends contrast to the images. (a) A single vertical comb at the center of an 8 cm wide channel. (b) Two slightly slanted combs, approximately 5 cm apart at the bottom of the image.

A single vertical comb alone is not a good device for generating isotropic homogeneous forced turbulence, but a pair of combs on either side of the flow channel does an acceptable job. Figure 1b shows a configuration with two long vertical combs arranged in an inverted "V." Transport of small vortices to the center of the channel is enhanced by a net fluid flux through the combs. The interior of the flow shows a mixture of large and small eddies, reasonably homogeneous over the entire length of the center of the channel. This photograph is strikingly different from those previously published with a single comb across the top of the channel [12–14,20]. In those photographs one observes a downstream coarsening of the turbulent vortices, typical of decaying 2D turbulence.

Quantitative measurements were taken with a two component laser Doppler velocimeter [(LDA) manufactured by TSI Inc.]. The flow was seeded with 0.22 μm diameter $\mathrm{TiO_2}$ spheres. Typical data rates were between 5 and 10 kHz. For a mean film speed of about 3 m/s, and making the frozen turbulence assumption, this translates to a measurement spacing of 0.3–0.6 mm. The actual measurement volume is less than 0.1 mm in diameter. Data were collected in the form of time series of about 200–300 s in duration. During this time, about 1 km of soap film has passed the measurement volume, and 2×10^6 data points are collected. Reynolds numbers for the largest vortices are estimated as greater than 10^3. We respectively denote the streamwise and cross stream directions as y and x.

We rely on the frozen turbulence assumption to convert the data from a time series to a measure of the spatial variation of the velocity. In our case, $v_i(y) = v_i(\langle v_y \rangle t)$. This is generally accepted as reasonable if the magnitude of the turbulent intensity (TI, defined as $\langle \Delta v_i^2 \rangle^{1/2} / \langle v_i \rangle$) is less than about 10%. In our flowing films the TI typically ranges from 10%–25% but can be as high as 55% very close to the turbulence generating grid.

The power spectra of $v_x(y)$ and $v_y(y)$ were taken by performing fast Fourier transforms (FFTs) on segments of 2048 consecutive data points, giving about 1000 averages for each time trace. Since LDV measurements arrive randomly in time we binned the data evenly in time, using a sample-hold technique, to perform the FFTs. The moduli of the FFTs, $|\hat{v}_x(k_y)|^2$ and $|\hat{v}_y(k_y)|^2$ are proportional to $E(k)$ if the turbulence is isotropic and homogeneous [21]. Since $|\hat{v}_x(k_y)|^2$ and $|\hat{v}_y(k_y)|^2$ are nearly identical at high wave numbers, we take the turbulence as isotropic. The probability distribution functions for v_x and v_y also have nearly identical widths. We deduce homogeneity from spectra measured at different points between the combs. In a region about $1/5$ the channel width, along the channel center, the spectra do not deviate significantly from one another.

Figure 2 shows a series of v_x power spectra measured between two vertical combs which form a parallel, rough walled, channel. From right to left, the curves follow the evolution of the spectrum downstream, from the forced region into the unforced region. The rightmost curve was taken at position A in the diagram, and successive curves to the left were taken increasingly downstream, at points B, C, D, and E. The exact position and geometry of the measurements are described in Fig. 2. Turbulent intensities were as follows: $\mathrm{TI}_A = 27\%$, $\mathrm{TI}_B = 22\%$, $\mathrm{TI}_C = 18\%$, $\mathrm{TI}_D = 11\%$, and $\mathrm{TI}_E = 6\%$. The spectrum at the bottom was taken for a laminar flow with the combs removed and serves as a noise floor for the measurements. The flattening of all the spectra at high frequency is thereby assumed to be noise and will be ignored in this discussion.

Spectrum A has three distinguishable features: (1) a region which scales as k^{-3} at higher wave numbers, (2) a region scaling as $k^{-5/3}$ at intermediate wave numbers, and (3) a pronounced peak at a wave number corresponding to about twice the distance between the combs. The power law regions of spectrum A fit nicely in the 2D energy and enstrophy cascade picture proposed by Kraichnan [1]. Figure 3 shows spectrum A, multiplied by $k^{5/3}$ and again, multiplied by k^3. Plateaus in the data correspond to the scaling regions of the energy spectra. We suggest that the peak at small k is due to vortices of alternating sign, which have grown to the width of the channel. The photo of the single vertical comb (Fig. 1a) shows the downstream growth of large vortical structures which would not surprisingly grow to crowd a channel made of two such combs. The length λ, which we take as twice the channel width, agrees well with the observed value of $2\pi/k_{\max} \sim 7$ cm. We defined $k_{\max}$ as the wave number where $E(k)$ reaches a maximum.

Spectra are identical to A further upstream in the channel, indicating the existence of a turbulent steady state. Beyond the exit of the channel, however, the nature of the spectra changes. The absence of channel walls allows the largest vortices to grow even larger, and we see a corresponding shift of $k_{\max}$ to larger scales. In the absence of forcing, the total energy of the velocity fluctuations (the area under the spectrum) steadily decreases. Energy is largely being dissipated by the air surrounding the film, as was shown by Martin *et al.* [13]. The most interesting change between the forced and unforced turbulence is the disappearance of the $-5/3$ power law portion of the spectrum. In a single experiment these observations relate the ideas of Batchelor on decaying 2D turbulence [2], and those of Kraichnan on forced 2D turbulence. Figure 3 accentuates the range of k over which $E(k) \sim k^{-3}$.

Finally we turn to identifying the magnitude of the energy injection scale, k_{inj}, for the forced steady state turbulence. The only obvious injection length scale is that of the small vortices shedding from the teeth at the channel walls. The dashed line in Fig. 4 is a spectrum measured in the weakly turbulent wake of an isolated tooth. This measurement implies $\ell_{\mathrm{inj}} \approx 0.2$ cm.

In the turbulent spectra k_{inj} corresponds to the "knee" where $E(k)$ goes from $k^{-5/3}$ to k^{-3}. For Fig. 2 this yields $\ell_{\mathrm{inj}} \sim 0.7$ cm, which is almost 4 times larger than the expected result from the single comb tooth. For a wider channel, as in Fig. 4, the discrepancy grows. The "knee" in the energy spectrum, however, approaches the single tooth injection scale as measurements are made closer to

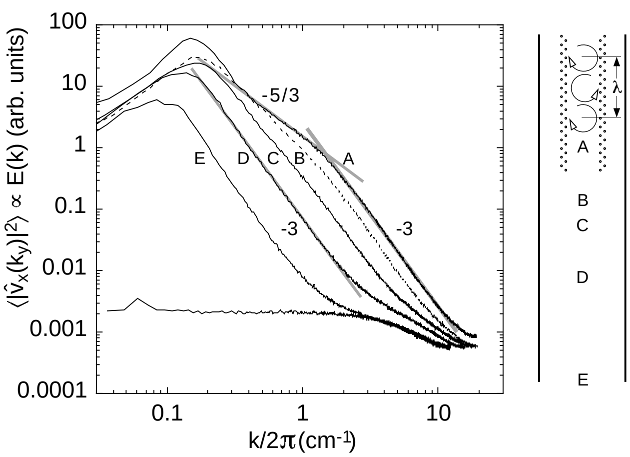

FIG. 2. Turbulent energy spectra derived from $v_x(t)$. From right to left, spectra were measured at positions $A-E$. The wires bounding the channel were 8 cm apart, the two vertical combs 2.5 cm apart. Downstream distances relative to the ends of the combs are $A = +5$ cm, $B = -5$ cm, $C = -10$ cm, $D = -20$ cm, and $E = -40$ cm. The bottom curve is a noise background measured for a laminar flow with the combs removed. Gray lines are not fits, but guides to the eye.

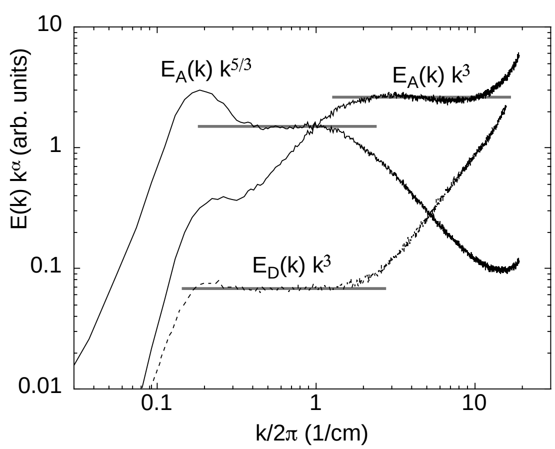

FIG. 3. The power law range of the spectra from Fig. 2 is illustrated by multiplying out the theoretically predicted behavior ($\alpha = 3$ or $5/3$). Solid curves are for forced turbulence at point A, which shows both energy and enstrophy cascades. The dashed curve is for decaying turbulence measured at position D.

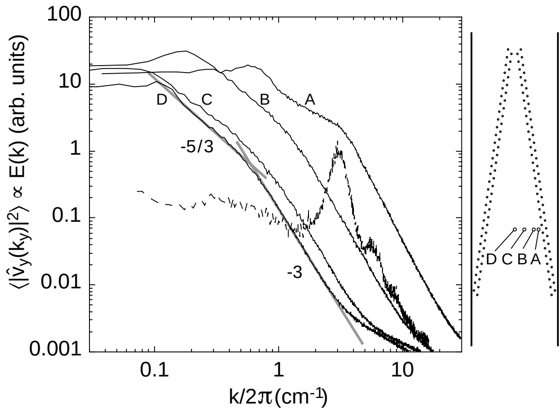

FIG. 4. Turbulent energy spectra derived from $v_y(t)$. From right to left, spectra were measured at positions $A-D$. The two vertical combs (30 cm long) gradually spread from 2 cm apart at the top to 7 cm apart at the bottom. Cross stream distances relative to the right comb are $A = 0$ cm, $B = 0.5$ cm, $C = 1.5$ cm, and $D = 2.5$ cm (which is the channel center). Turbulence intensities are $\mathrm{TI}_A = 56\%$, $\mathrm{TI}_B = 35\%$, $\mathrm{TI}_C = 15\%$, and $\mathrm{TI}_D = 11\%$. The dashed curve was measured 2.0 cm downstream from a single comb tooth. Gray lines are not fits, but guides to the eye.

the channel wall. This trend suggests that the small eddies near the channel walls coarsen into larger ones, during the time that they are advected to the center of the channel. A wider channel will therefore give an effectively larger value of ℓ_{inj} as deduced from the knee in the energy spectra.

Driving 2D turbulence from the channel walls has proven more effective at uncovering relevant physics than driving turbulence with a single upstream comb across the flow. The turbulence is reasonably isotropic (compare v_x spectra in Fig. 2 with v_y spectra in Fig. 4) and reasonably homogeneous in the center of the channel (spectrum C approaches spectrum D in Fig. 4). The technique will be made a more powerful test bed for 2D turbulence research by reducing drag on the film by evacuating the surrounding air [13,15]. MHD forcing may also be explored. But even at this point, the measurements give credence to the Kraichnan theory of forced 2D turbulence and show a continuous evolution toward Batchelor's ideas on decaying 2D turbulence. Our measurements are in agreement with various computer simulations for forced and decaying turbulence. Previous experiments have identified the enstrophy cascade in turbulent soap films and the energy cascade in MHD driven stratified fluids. Our experiments have now shown that both cascades can indeed exist simultaneously.

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