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Evolution of phase singularities of vortex beams propagating in atmospheric turbulence

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Optical vortex beams propagating through atmospheric turbulence are studied by numerical modeling, and the phase singularities of the vortices existing in the turbulence-distorted beams are calculated. It is found that the algebraic sum of topological charges (TCs) of all the phase singularities existing in test aperture is approximately equal to the TC of the input vortex beam. This property provides us a possible approach for determining the TC of the vortex beam propagating through the atmospheric turbulence, which could have potential application in optical communication using optical vortices.

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1. INTRODUCTION

In recent years, much work has been done to investigate the properties of optical beams with optical vortices. The study of optical vortex beams and related phenomena have become an important subfield of optics in its own right, now known as singular optics [1]. A vortex beam possesses an intensity null in its core and an azimuthal phase dependence $\exp(im\varphi)$, where m is an integer number and refers to the topological charge (TC) of the beam, which is related to the orbital angular momentum (OAM) of photos. The OAM may help vortex beams propagate through optical turbulence with less distortion than conventional beams [2,3], and the TC of optical vortices can be transmitted without loss over significant distances in the atmosphere [4,5]. Because of these unique characters, optical vortex beams have been investigated for various applications such as optical micromanipulations [6,7], quantum optics information processing [8–11], and free-space communications [12]. Owing to the popular applicability of optical vortex beams, characterizing the OAM state or the TC of a vortex beam is important in exploiting their applications.

Over the past years, many methods have been proposed to determine the TC value of an optical vortex [13–23]. However, most of them are under the conditions of no optical turbulence or short propagation distance.

In remote communication systems, however, the optical vortex beams have to propagate through the turbulent atmosphere. Ascertaining the TC of an optical vortex beam propagating through the turbulence becomes a task of great significance [5,24]. In Ref. [5], Gbur and Tyson studied the TC properties of vortex beams propagating through turbulent atmosphere, in which the TC value is determined by a contour integration of the phase gradient along the perimeter of the detector aperture. In this paper, we simulated the propagation of a vortex beam through the turbulent atmosphere, and studied the phase singularities (PSs) (or named branch points) of the beam under different propagation parameters based on the method for locating the PSs proposed by Fried and Vaughn [25,26]. We calculated the algebraic sum of the TCs of all the phase singularities existing in variable detector apertures; hereafter, the expression about such an algebraic sum will be simplified as algebraic sum of the phase singularities and abbreviated as AS-PS for simplicity. It is demonstrated that the TC of the input vortex beam could be found out by calculating the AS-PS of the turbulence-distorted beam in detector aperture. In addition, the influences of the TC of the input vortex beam, the turbulence strength and the detector aperture on the AS-PS are also investigated.

2. THEORETICAL ANALYSIS

In general, the complex amplitude of a beam with phase singularities can be expressed as [5]

$$U(\mathbf{r}) = A(\mathbf{r}) \exp[i\varphi(\mathbf{r})], \quad (1)$$

in which the phase $\varphi(\mathbf{r})$ is well defined everywhere, except at the point where the amplitude $A(\mathbf{r})$ drops to zero. At such points, the phase could be ambiguous, or singular. The singularity of such a point can be characterized by a path integral along a closed loop surrounding it [25,26],

$$m = \frac{1}{2\pi} \oint_c \nabla\varphi(\mathbf{r}) \cdot \mathrm{d}l, \tag{2}$$

where C represents the contour of integration, $d\mathbf{l}$ is an infinitesimal vector path element, and m is the TC of the singularity. The TC could take any positive or negative integer for a given phase singularity [27,28]. In stochastic optical fields, however, only singularities or vortices with the TC of $m = \pm 1$ are stable [29–31]. Because the PSs are usually created and annihilated only in pairs of opposing charges in the turbulence-distorted optical field, we estimate that it could be possible to find out the TC of the input vortex beam by calculating the AS-PS of a turbulence-distorted beam at the receiver plane. For ascertaining the TC of the input vortex beam in the turbulence-distorted optical field, a basic task could be to obtain the AS-PS based on the phase distribution. In this paper, the summation of the phase gradients around a closed loop is adopted to identify PSs.

Suppose the complex amplitude of the turbulence-distorted beam at the detect plane is $U(\mathbf{r}_{p,q})$, in which $\mathbf{r}_{p,q}$ is the position vector of the sampling point (p, q) (p, q are the sequence number of the points in x and y direction, respectively). Its phase can be determined by [26]

$$\varphi(\mathbf{r}_{p,q}) = \tan^{-1}\left(\frac{\mathrm{Im}\{U(\mathbf{r}_{p,q})\}}{\mathrm{Re}\{U(\mathbf{r}_{p,q})\}}\right), \tag{3}$$

where $\mathrm{Re}\{U(\mathbf{r}_{p,q})\}$ and $\mathrm{Im}\{U(\mathbf{r}_{p,q})\}$ represent the real and the imaginary parts of the complex amplitude, respectively. The spatial gradient of the phase distribution can be defined as [26]

$$\mathbf{g}(\mathbf{r}_{p,q}) = \nabla\varphi(\mathbf{r}_{p,q}) \approx \frac{\{\varphi(\mathbf{r}_{p+1,q}) - \varphi(\mathbf{r}_{p,q})\}_{pv}}{d}\mathbf{l}_x + \frac{\{\varphi(\mathbf{r}_{p,q+1}) - \varphi(\mathbf{r}_{p,v})\}_{pv}}{d}\mathbf{l}_y, \tag{4}$$

where the notation $\{\ldots\}_{pv}$ denotes taking a principal value, d is the sampling interval, and $\mathbf{l}_x$ and $\mathbf{l}_y$ are unit vectors parallel to the x- and y-axes, respectively. Equation (4) can also be expressed as the function of the complex amplitude by substituting Eq. (3) into Eq. (4), that is,

$$\mathbf{g}(\mathbf{r}_{p,q}) = d^{-1}\left[\tan^{-1}\left(\frac{\mathrm{Im}\{U[(p+1)d, qd]U^*(pd, qd)\}}{\mathrm{Re}\{U[(p+1)d, qd]U^*(pd, qd)\}}\right)\right]\mathbf{l}_x + d^{-1}\left[\tan^{-1}\left(\frac{\mathrm{Im}\{U[pd, (q+1)d]U^*(pd, qd)\}}{\mathrm{Re}\{U[pd, (q+1)d]U^*(pd, qd)\}}\right)\right]\mathbf{l}_y. \tag{5}$$

According to the method proposed by Fried and Vaughn [25,26], we can judge whether there is a PS by calculating the following contour integral over a closed loop surrounding:

$$\oint_C \mathbf{g}(\mathbf{r}) \cdot \mathrm{d}\mathbf{r} = \begin{cases} \pm 2\pi, & \text{if a phase singularity is enclosesd} \\ 0, & \text{if no phase singularity is enclosesd} \end{cases}, \tag{6}$$

where C denotes the closed loop contour and $d\mathbf{r}$ is a vector parallel to the tangent of the contour C. Using the discrete form of the phase gradient described above, the discrete form of Eq. (6) can be written as

$$S_{p,q} = \mathbf{g}(\mathbf{r}_{p,q}) \cdot \mathbf{l}_x d + \mathbf{g}(\mathbf{r}_{p+1,q}) \cdot \mathbf{l}_y d - \mathbf{g}(\mathbf{r}_{p,q+1}) \cdot \mathbf{l}_x d - \mathbf{g}(\mathbf{r}_{p,q}) \cdot \mathbf{l}_y d. \tag{7}$$

Here, the integral contour is set to be each unit square of the computational array whose four corners are at $\mathbf{r}_{p,q}$, $\mathbf{r}_{p+1,q}$, $\mathbf{r}_{p+1,q+1}$, and $\mathbf{r}_{p,q+1}$. If $S_{p,q}$ in Eq. (7) equals $+2\pi$, it means that a positive PS (with a TC of $+1$) exists in the contour, while a negative PS (with a TC of -1) will appear in the contour if $S_{p,q}$ is equal to -2π. If $S_{p,q}$ equals zero, it means that no PS is enclosed in the contour. Thus, all the positive and negative PSs in the distorted optical field can be obtained and the algebraic sum of their TCs (AS-PS) can be calculated by the following formula:

$$S_{\mathrm{AS}} = \frac{1}{2\pi}\sum_{p,q}[\mathbf{g}(\mathbf{r}_{p,q}) \cdot \mathbf{l}_x d + \mathbf{g}(\mathbf{r}_{p+1,q}) \cdot \mathbf{l}_y d - \mathbf{g}(\mathbf{r}_{p,q+1}) \cdot \mathbf{l}_x d - \mathbf{g}(\mathbf{r}_{p,q}) \cdot \mathbf{l}_y d]. \tag{8}$$

As analyzed above, the TC of the input optical vortex beam could be determined by the AS-PS at the test plane.

3. SIMULATION AND DISCUSSION

To numerically simulate the vortex beam propagating through the turbulent atmosphere, we adopt the multiple phase screens method [32] and the modified von Karman spectrum model to describe the atmosphere turbulence [33]:

$$\Phi_n(\kappa) = 0.033 C_n^2 \frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2 + \kappa_0^2)^{11/6}}, \tag{9}$$

where $k_m = 5.92/l_0$ and $k_0 = 2\pi/L_0$. $l_0 = 0.025$ m is the inner scale of turbulence and $L_0 = 100$ m is the outer scale of turbulence. C_n^2 is refractive index structure constant that represents the atmospheric turbulence strength.

Suppose that the input vortex beam is a Laguerre–Gaussian (LG) beam with the following complex amplitude:

$$U_{nm}(r, \varphi) = C_{nm}\left(\frac{\sqrt{2}r}{\omega_0}\right)^{|m|} L_n^{(m)}\left(\frac{2r^2}{\omega_0^2}\right)\exp(im\varphi - r^2/\omega_0^2), \tag{10}$$

where (r, φ) are the polar coordinates in the input plane, C_{nm} is a normalization constant, ω_0 is the width of the beam, $L_n^{(m)}$ are the Laguerre polynomials, the integer n represents the beam's radial order, and m represents the beam's azimuthal order. The TC of such a beam is m. Without loss of generality, an unimportant amplitude constant factor has been omitted in this paper.

In our simulation, the propagation of LG beams through the turbulent atmosphere is based on the numerical solution of the parabolic equation along the z-axis. The size of all the phase screens is set to 512 × 512 pixels with 0.002 m sample spacing. Fifty equidistant phase screens are settled along the propagating path. A circular aperture with variable aperture diameter is placed at the receiver plan to simulate a detector aperture. Only the PSs within the aperture are taken into account in calculating the AS-PSs.

Figures 1(a) and 1(b) give an example of the intensity and phase distributions of an LG_{04} beam with parameters of $\omega_0 = 0.02$ m, $n = 0$, $m = 4$, and the wavelength

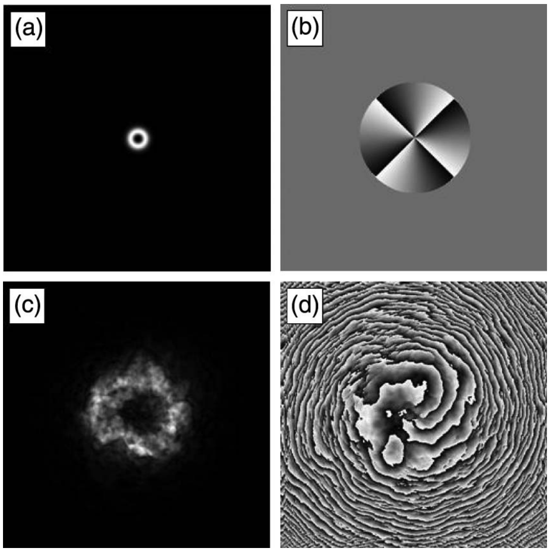

Fig. 1. (a) and (b) Intensity and phase distributions of an LG_{04} beam on the input plane $z = 0$. (c) and (d) Intensity and phase distributions of an LG_{04} beam on the receiver plane $z = 10$ km. Here the wavelength is taken to be $\lambda = 1.55$ μm, the turbulence strength is taken to be $C_n^2 = 10^{-15}$ $m^{-2/3}$, and the width of the beam is taken as $\omega_0 = 0.02$ m.

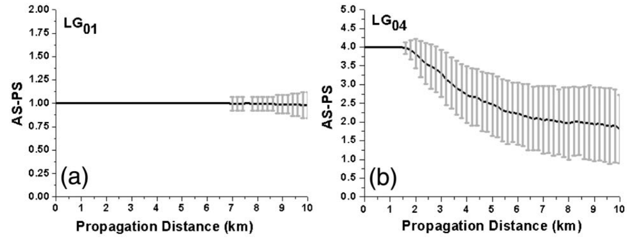

Fig. 2. Simulation of the average AS-PS for (a) LG_{01} and (b) LG_{04} beams propagating in atmospheric turbulence, $C_n^2 = 10^{-15}$ $m^{-2/3}$. Here, $\omega_0 = 0.02$ m, $\lambda = 1.55$ μm, and the diameter of a circular detector is 0.04 m.

$\lambda = 1.55$ μm on the input plane, while Figs. 1(c) and 1(d) show the simulated intensity and phase distribution after the beam propagates a distance of 10 km through the turbulent atmosphere with the turbulence strength $C_n^2 = 10^{-15}$ $m^{-2/3}$.

Because of the stochastic nature of the turbulence, the average parameters of beam realizations often are of interest, instead of the parameters of an individual realization in practice. In this paper, the average values are obtained by 200 propagation events for all the cases. The average AS-PS, $\overline{S_{AS}}$, and the standard deviation of the AS-PS, ΔS, are respectively, defined as

$$\overline{S_{AS}} = \frac{1}{N}\sum_{i=1}^{N} S_{AS}^{(i)}, \tag{11}$$

and

$$\Delta S = \left(\frac{1}{N}\sum_{i=1}^{N} (S_{AS}^{(i)})^2 - (\overline{S_{AS}})^2\right)^{1/2}, \tag{12}$$

where $S_{AS}^{(i)}$ is the AS-PS for the ith realization, and N is the repetitions of the realizations used to calculate the average.

Figures 2(a) and 2(b) give the relation curves of the average AS-PS and the propagation distance when the input beam is set to be LG_{01} and LG_{04}, respectively. It can be seen that the AS-PS is approximately equal to the TC of the input beams, with very little variance, if the propagation distance is smaller than about 7 km for the LG_{01} mode and 1.5 km for the LG_{04} mode under the same propagating conditions. With further propagation, the average AS-PS for LG_{04} mode will gradually decrease. This decline is perhaps caused by the finite size of the detector aperture and the spreading of the beam. As demonstrated in Ref. [5], the PSs will "wander" away from their original position when the beam propagates and spreads through the atmospheric turbulence. If the PSs wander outside the detector aperture, they will not be measured by the detector. Furthermore, there will be some PS pairs created by the turbulence near the border, in which one member of a pair occasionally drifts outside the detector region, resulting in an increase of the standard deviation.

It is known that, when the turbulence is strong, the wave field propagating in it will be severely distorted and more PSs will be created. Figures 3(a)–3(c) illustrate the average AS-PS as a function of the propagation distance when the turbulence strength is set to be 10^{-16} $m^{-2/3}$, 10^{-15} $m^{-2/3}$, and 10^{-14} $m^{-2/3}$, respectively. In the case of weak and moderate turbulence ($C_n^2 \sim 10^{-16}$–10^{-15} $m^{-2/3}$) [5], as shown in Figs. 3(a) and 3(b), the average AS-PS remains close to the original TC value of the input LG beam with little variance, even when the propagation distance exceeds 10 km, while in the case of strong turbulence ($C_n^2 \sim 10^{-14}$ $m^{-2/3}$), as shown in Fig. 3(c), the average AS-PS begins to decrease and the standard deviation relative to the original TC increases dramatically at roughly 1.5 km. The main reason may be that, under the weak turbulence strength condition, there exist fewer PS pairs produced by the turbulence and, thus, less influence on the detected AS-PS. While under the condition of strong turbulence, the wandering behavior of PSs becomes complicated and some of the PSs coming from the LG beam may wander from the detector aperture. In addition, longer propagation distance and/or stronger turbulence strength will result in more members of PS pairs being produced by turbulence drifting outside the detector region and, thus, result in an increase of the standard deviation.

From Ref. [5], we know that the standard deviation of the tested TC can be reduced by increasing the size of the detector aperture. Figures 4(a)–4(c) illustrate the average AS-PS as a function of the propagation distance when the diameter of the test region is set to be 0.04 m, 0.12 m, and 0.20 m, respectively. From Fig. 4, it can be seen that the average AS-PS is actually related to the size of the test region. Obviously, the distance with the average AS-PS consistent with the TC of the input beam will be extended with the increase of the test region. It is worth noting that bigger test region is not always better. Figure 5 further shows the relation curves of the relative error between the average AS-PS and the TC value of the input beam versus different test diameters and different propagation distance. From Fig. 5, we know that, to determine

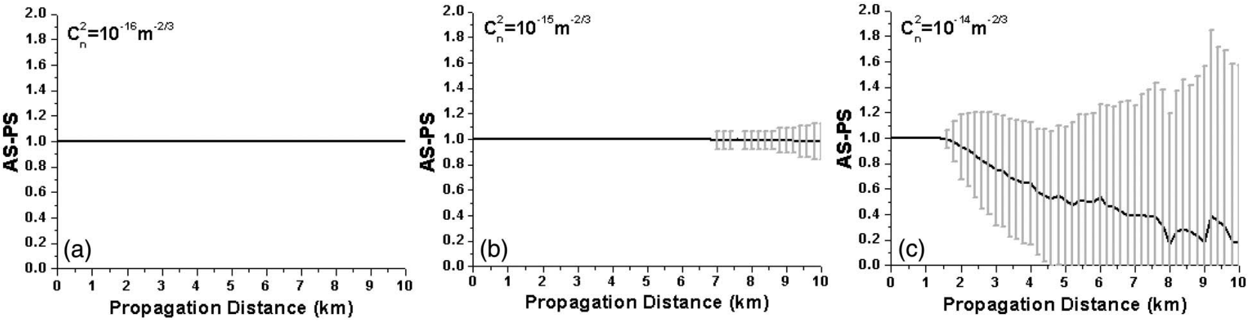

Fig. 3. Simulation of the average AS-PS for an LG_{01} beam for various turbulence strengths. (a) $C_n^2 = 10^{-16}$ m$^{-2/3}$, (b) $C_n^2 = 10^{-15}$ m$^{-2/3}$, and (c) $C_n^2 = 10^{-14}$ m$^{-2/3}$. All other parameters are as in Fig. 2.

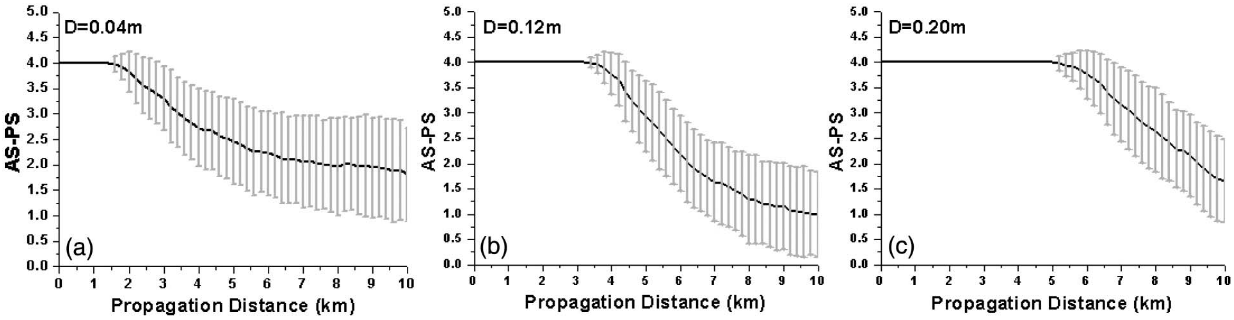

Fig. 4. Simulation of the average AS-PS for an LG_{04} beam for various diameters of the circular detector. (a) $D = 0.04$ m, (b) $D = 0.12$ m, and (c) $D = 0.20$ m. All other parameters are as in Fig. 2.

the TC of the input beam based on testing the average AS-PS, an appropriate diameter of the testing region should be adopted. If the test aperture is much larger than the size of the vortex beam, more PSs produced by optical turbulence will be included and the relative error will be increased. If too small a test aperture is used under long propagation distance, the size of the broadened beam will be larger than the aperture, and some of the PSs from the input vortex beam will not be detected, which will also lead to the increase of the relative error.

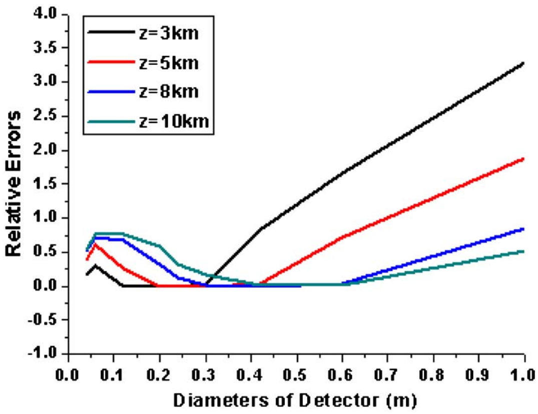

Fig. 5. Relative errors between the average AS-PS and the TC value of the input beam LG_{04} for different diameters of detector and different propagation distance. All other parameters are as in Fig. 2.

There seems to be an optimal test diameter for different propagation conditions. We found that, if the aperture size is approximately equal to the main spot size of the beam in the test plane, the relative error will trend to a minimum. Because the spreading of the vortex beam induced by atmospheric turbulence is less than the nonvortex one [34,35], the free-space beam spreading induced by diffraction is dominant. Thus, the radius of the main spot size of an LG_{0m} beam can be characterized by the radius of its intensity ring with the maximum intensity value in free-space propagation, which can be expressed as [35,36]

$$r(z) = \omega(z)\left(\frac{m}{2}\right)^{1/2} = \omega_0\left(\frac{m}{2}\right)^{1/2}\left[1+\left(\frac{\lambda z}{\pi\omega_0^2}\right)^2\right]^{1/2}. \quad \textbf{(13)}$$

Thus, we can estimate the optimal diameter according to Eq. (13) under weak turbulence strength conditions. For example, if $\omega_0 = 0.02$ m, $m = 4$, and $\lambda = 1.55$ μm, the optimal diameter should be about 0.22 m for a propagation distance of $z_1 = 3$ km and about 0.35 m for $z_2 = 5$ km, which are consistent with the simulated results as shown in Fig. 5.

Further simulations are also undertaken to analyze the influence of source size ω_0 on the average AS-PS. The simulation results are shown in Fig. 6. It can be seen that, under the conditions of our simulations, too small or too large a source size (such as $\omega_0 = 0.02$ m in Fig. 6(a)and $\omega_0 = 0.08$ m in Fig. 6(c)) will result in a rapid increase of the deviation with an increase of the propagation distance. A moderate source size may be a good choice for the purpose of a low increase of the

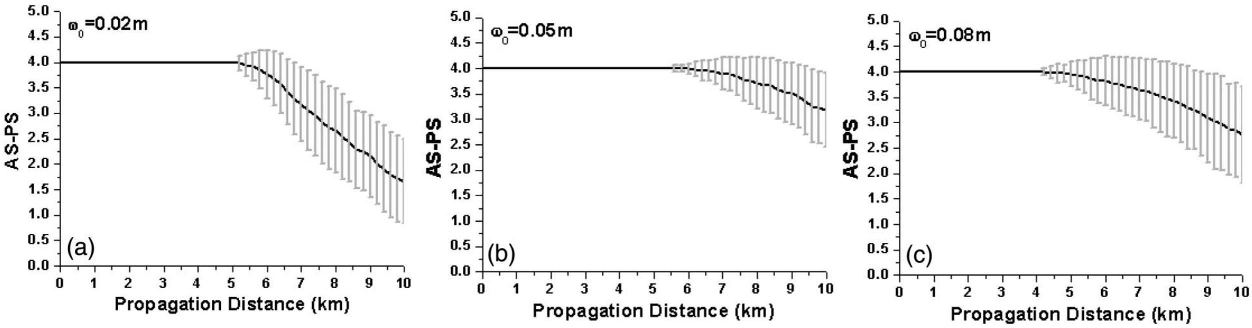

Fig. 6. Simulation of the average AS-PS for LG_{04} beams, for various source sizes. (a) $\omega_0 = 0.02$ m, (b) $\omega_0 = 0.05$ m, (c) $\omega_0 = 0.08$ m. Here, $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$, $\lambda = 1.55$ μm, and the diameter of a circular detector $D = 0.20$ m.

tested deviation with propagation. This result is in accordance with the theoretical prediction in Ref. [4], in which it is predicted that the minimum of OAM fluctuations will be achieved if the value of the diffraction Fresnel parameter is near one.

4. CONCLUSIONS

Our simulations demonstrated that the algebraic sum of the phase singularities of the turbulence-distorted beams is approximately equal to the TC of the input vortex beams within quite a long propagation distance. We also analyzed the influence of the detector aperture on the fluctuation of the calculated AS-PSs and found that, if the aperture size is approximately equal to the main spot size of the distorted beam on the test plane, the fluctuation or relative error will trend to a minimum. In comparison with the Gbur's method [5], in which the TC of the vortex beam propagating through the turbulent atmosphere was determined by directly evaluating Eq. (2) around the perimeter of the detect aperture, the AS-PS method described above could eliminate the influence of the local turbulence more effectively and thus diminish the fluctuation of the determined values under similar turbulence strength conditions. These simulations could be helpful to explore further the experimental methods for determining TC values of the distorted vortex beams based on the phase gradient information that can be retrieved in experiments using Shack–Hartmann wavefront measuring techniques [23].

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