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A simple method to create a vortex in Bose–Einstein condensate of alkali atoms

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Abstract

Bose–Einstein condensation in alkali atoms has materialized quite an interesting system, namely a condensate with a spin degree of freedom. In analogy with the A-phase of the superfluid ^{3}He, numerous textures with nonvanishing vorticity have been proposed. In the present paper, interesting properties of such spin textures are analyzed. We propose a remarkably simple method to create a vortex state of a BEC in alkali atoms.

Keywords: Bose–Einstein condensation; Texture; Vortex

The discovery of Bose–Einstein condensation (BEC) in alkali atoms has led to several exciting fields to study. One of these developments is a BEC with a spin degree of freedom [1,2]. When the spin exchange interaction is ferromagnetic or the confining magnetic field is strong, the BEC behaves somewhat similarly to superfluid ^{3}He-A, where topological objects called textures are known to exist. In the present paper, we investigate vortices in a BEC, which are analogous to the vortices and disgyrations in ^{3}He-A. We first explain the order parameter of a BEC with a spin degree of freedom. Then the cross disgyration, expected to exist in the Ioffe–Pritchard (IP) trap, is considered. Finally, we propose a simple method to create an ordinary vortex line in a BEC.

Suppose an alkali atom has a hyperfine-spin $|F|=1$. Then the order parameter has three components $\Psi_{\pm 1}$ and Ψ_0, which represent the amplitudes with $F_z=\pm 1,0$, respectively. The basis vectors in this representation are $\{|\pm\rangle, |0\rangle\}$. We introduce another set of basis vectors $|x\rangle$, $|y\rangle$ and $|z\rangle$, which are defined by $F_x|x\rangle = F_y\,|y\rangle = F_z|z\rangle = 0$. These vectors are related with the previous vectors as $|\pm 1\rangle = \mp(1/\sqrt{2})(|x\rangle \pm \mathrm{i}|y\rangle)$ and $|0\rangle = |z\rangle$. When the z-axis is taken parallel to the uniform magnetic field, the order parameter of the weak field seeking state takes the form $\Psi_{-1}=\psi$ and $\Psi_0=\Psi_1=0$, which is also written as $\Psi_x = \mathrm{i}\Psi_y = \psi/\sqrt{2}$. Let us denote this state in a vectorial form as $\boldsymbol{\Psi} = (\psi/\sqrt{2})(\hat{\boldsymbol{x}} - \mathrm{i}\hat{\boldsymbol{y}})$, where the common factor has been absorbed in the amplitude ψ. Suppose the magnetic field points to the direction $\hat{\boldsymbol{B}} = (\sin\beta\,\cos\alpha, \sin\beta\sin\alpha, \cos\beta)$. Then the weak field seeking state takes the form $\boldsymbol{\Psi} = (\psi/\sqrt{2})\mathrm{e}^{\mathrm{i}\gamma}(\hat{\boldsymbol{m}} - \mathrm{i}\hat{\boldsymbol{n}})$, where $\hat{\boldsymbol{m}} = (\cos\beta\,\cos\alpha, \cos\beta\sin\alpha, -\sin\beta)$ and $\hat{\boldsymbol{n}} = (-\sin\alpha, \cos\alpha, 0)$. The unit vector $\hat{\boldsymbol{l}} = -\hat{\boldsymbol{m}}\times\hat{\boldsymbol{n}}$ is the direction of the spin polarization. The same amplitudes in the basis $\{|0\rangle, |\pm\rangle\}$ are [3]

$$\Psi_1 = (\psi/2)(1-\cos\beta)\mathrm{e}^{-\mathrm{i}\alpha+\mathrm{i}\gamma},$$

$$\Psi_0 = -(\psi/\sqrt{2})\sin\beta\mathrm{e}^{\mathrm{i}\gamma},$$

$$\Psi_{-1} = (\psi/2)(1+\cos\beta)\mathrm{e}^{\mathrm{i}\alpha+\mathrm{i}\gamma}. \qquad (1)$$

The choice $\alpha = -\phi$, $\beta = \pi/2$ yields the cross disgyration shown in Fig. 1, where ϕ is the azimuthal angle. This texture has a nonvanishing vorticity n when $\gamma = n\phi$. It is expected that the cross disgyration is realized in the IP trap with $B_z = 0$.

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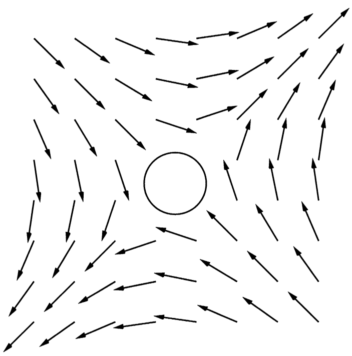

Fig. 1. The $\hat{l}$-vector field of a cross disgyration. The circle shows the laser beam, where no atoms exist.

Suppose a strong magnetic field B_z is applied to a BEC, along with the quadrupole field. Atoms are assumed to be in the weak field seeking state so that they are confined in the trap. The field B_z is so strong compared to the quadrupole field that the order parameter is virtually $\boldsymbol{\Psi} = (\psi/\sqrt{2})(\hat{\boldsymbol{x}} - \mathrm{i}\hat{\boldsymbol{y}})$. Clearly the vorticity of this texture vanishes. This configuration is derived from Eq. (1) by putting $\beta = 0$ and $\gamma = -\alpha = \phi$. Then B_z is adiabatically decreased, so that $\hat{\boldsymbol{l}}$ is always antiparallel to $\boldsymbol{B}$, until B_z vanishes. The adiabatic condition is required for atoms to remain in the weak field seeking state. Then the cross disgyration appears in the presence of the quadrupole field. In due process, the angle β increases from 0 to $\pi/2$ and γ and $-\alpha$ are identified with ϕ. Here, the trap must be plugged by laser beams along the axis, the top and the bottom of the trap so that the atoms do not escape from the trap. In the final step, the external field B_z is gradually increased in the opposite $(-z)$ direction. Then the $\hat{\boldsymbol{l}}$-vector points up so that $\beta = \pi$. Substituting these angles into (1), one obtains

$$\Psi_{-1} = \Psi_0 = 0, \quad \Psi_1 = \psi \mathrm{e}^{2\mathrm{i}\phi}. \tag{2}$$

This is nothing but the order parameter of a vortex with the winding number 2. The amplitude $\psi(r)$ is determined by solving the Gross–Pitaevskii equation with appropriate boundary conditions. The supercurrent of the texture has the ϕ-component

$$j_{\mathrm{s}\phi} = m|\psi|^2 v_{\mathrm{s}\phi}, \quad v_{\mathrm{s}\phi} = (\hbar/mr)(1 - \cos\beta), \tag{3}$$

where m is the atomic mass and v_s is the superfluid velocity. If the Na mass is substituted into Eq. (3), we obtain $v_{\mathrm{s}\phi} \simeq 0.5/r$ cm/s for $\beta = \pi$, where r is measured in units of μm.

In summary, we have proposed a simple method to create a vortex in a BEC of alkali atoms. A strong magnetic field is applied along the axis of the IP trap field, which is adiabatically varied toward a negative large value. Starting with a uniform order parameter field, we are eventually left with a vortex of the winding number 2. The initial state has no circulation while the final state does. This is because the external magnetic field transfers torque to the BEC while the spin vector is turned upside down. When the hyperfine-spin is F in general, we will end up with a vortex with the winding number $2F$ since Ψ_{-F} and Ψ_F have phases $F(\alpha + \gamma)$ and $F(-\alpha + \gamma)$, respectively.

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