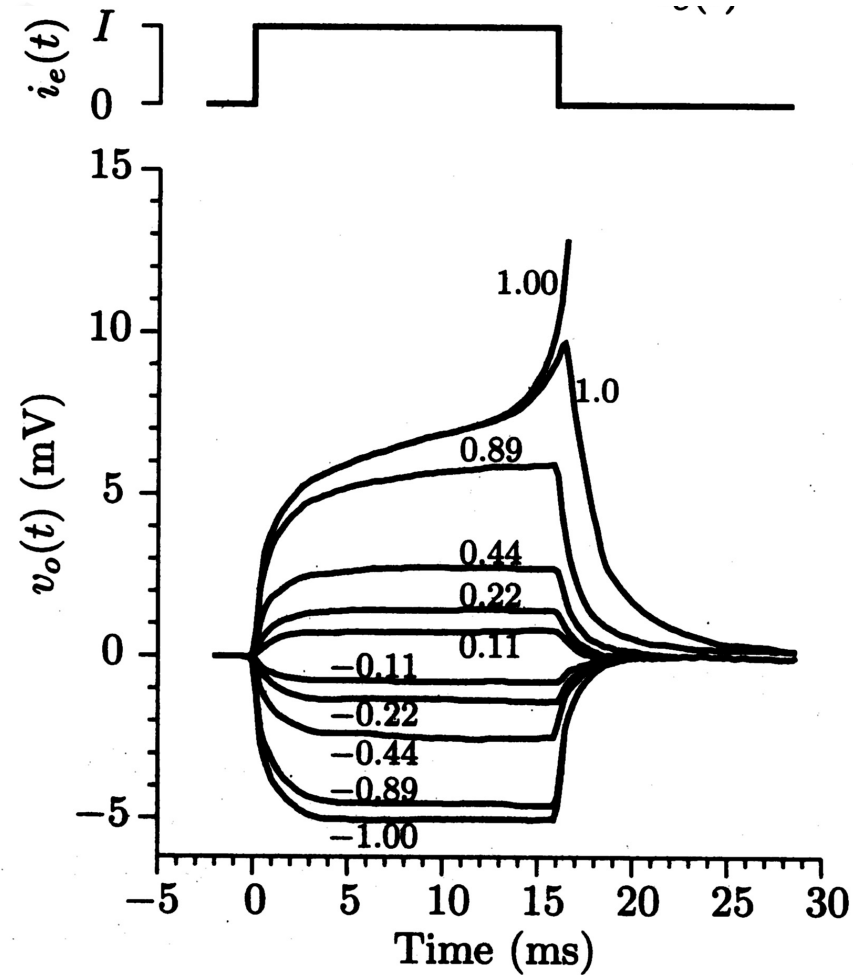
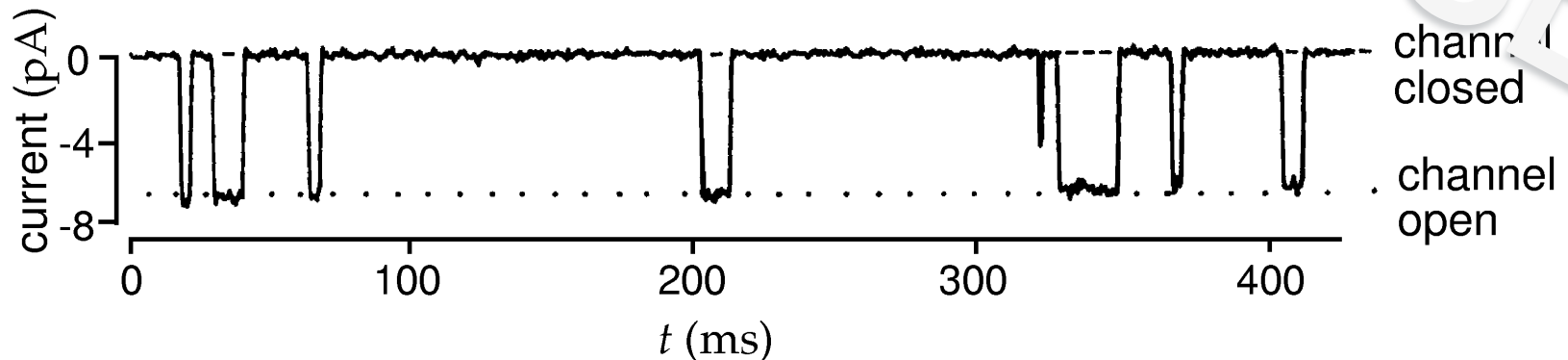
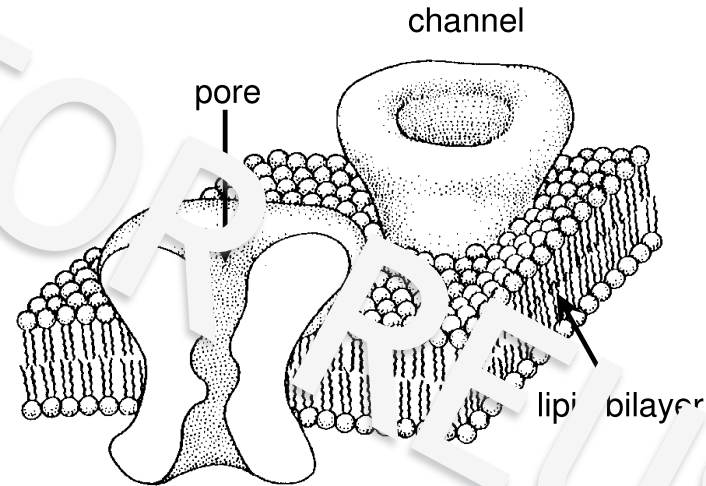


But what makes a neuron *compute*?



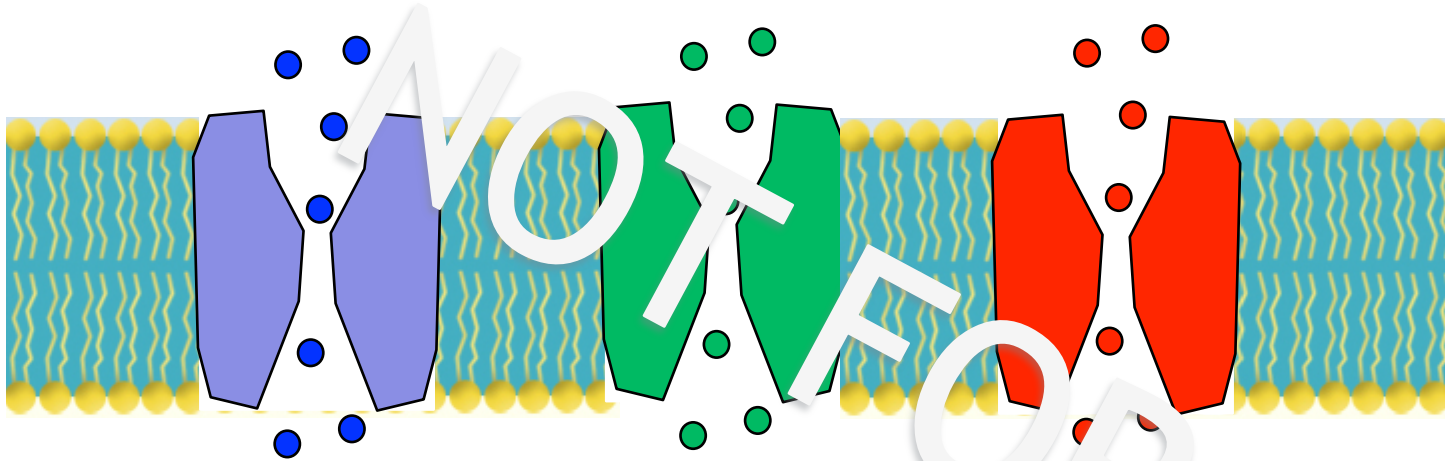
Excitability arises from ion channel nonlinearity

- voltage dependent
- transmitter dependent (synaptic)
- Ca dependent
- mechanosensitive

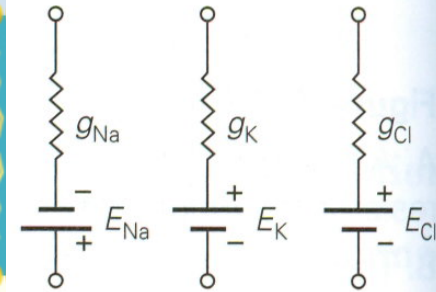


Best pics are all from Dayan and Abbott, *Theoretical Neuroscience*

Parallel paths for different ions to cross membrane

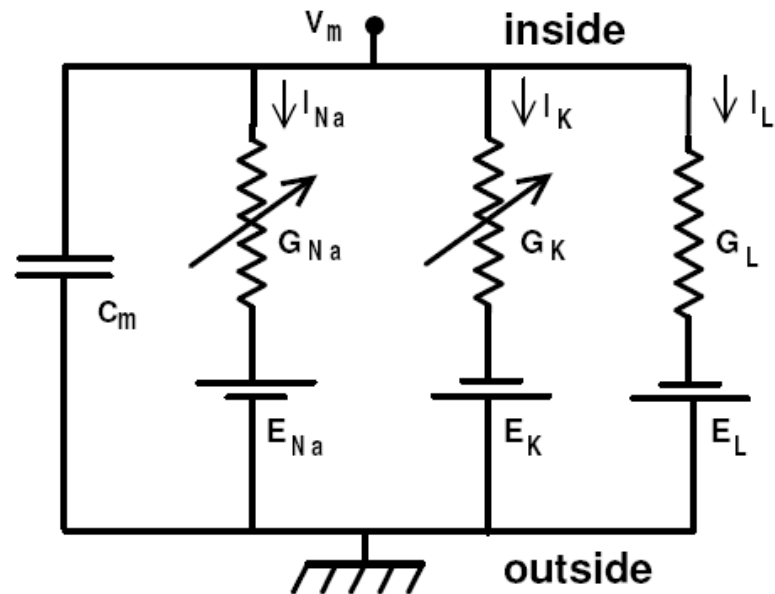


$$I_i = g_i (V - E_i)$$



New equivalent circuit:

Variable conductance

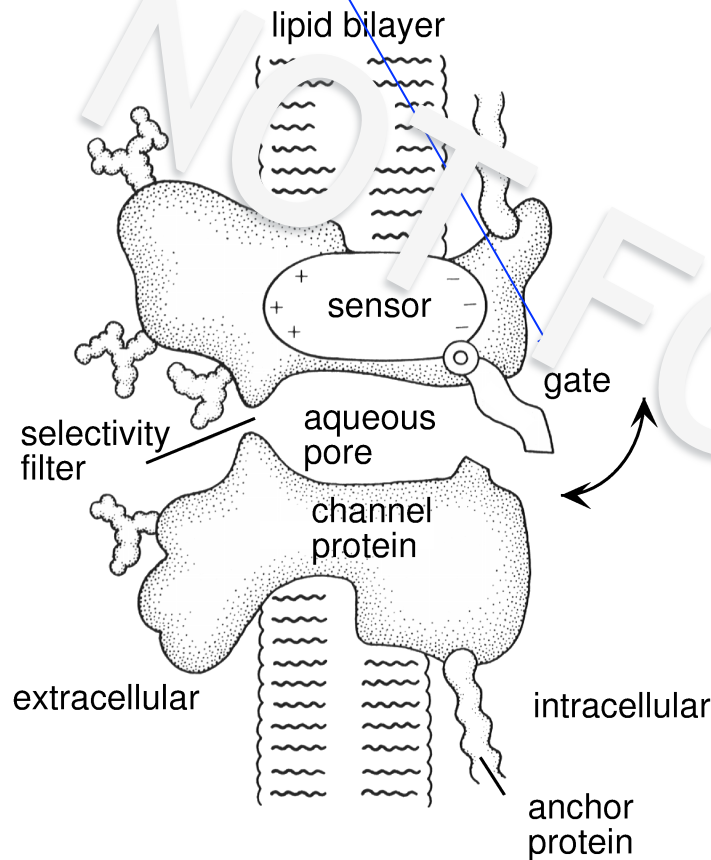


The ion channel is a cool molecular machine

K channel: open probability increases when depolarized

Gating depends on subunit state

$$P_K \sim n^4$$

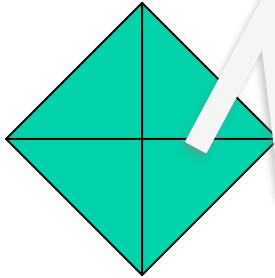


Persistent conductance

The ion channel is a cool molecular machine

n describes a subunit

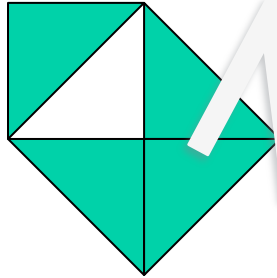
n	is open probability
$1 - n$	is closed probability



This picture is not from Dayan and Abbott, for instance

The ion channel is a cool molecular machine

n describes a subunit



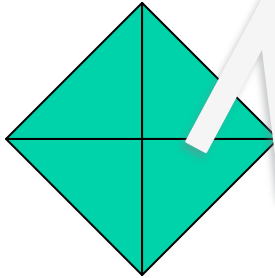
n is open probability
 $1 - n$ is closed probability

This picture is not from Dayan and Abbott, for instance

The ion channel is a cool molecular machine

n describes a subunit

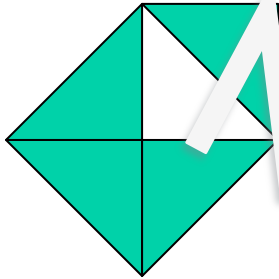
n	is open probability
$1 - n$	is closed probability



The ion channel is a cool molecular machine

n describes a subunit

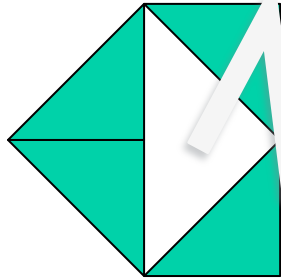
n	is open probability
$1 - n$	is closed probability



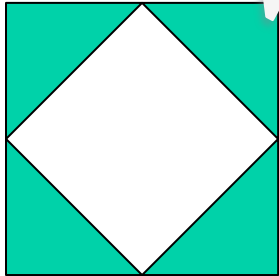
The ion channel is a cool molecular machine

n describes a subunit

n	is open probability
$1 - n$	is closed probability



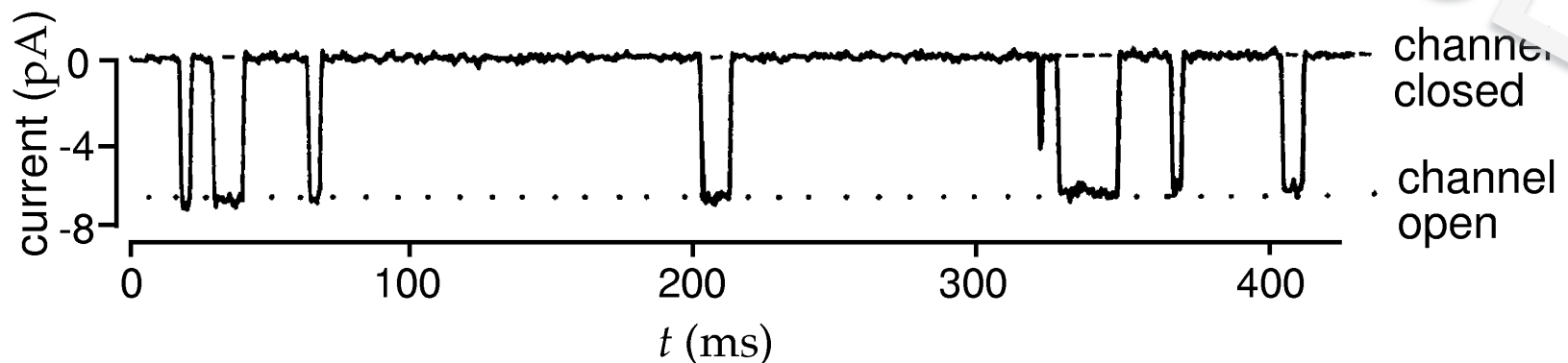
The ion channel is a cool molecular machine



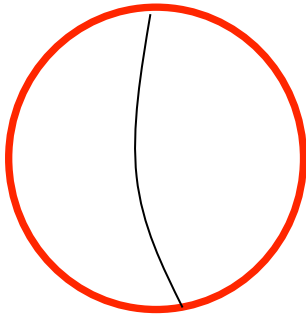
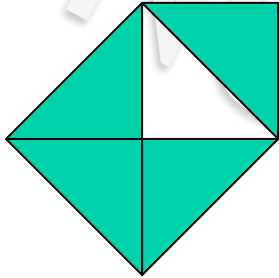
n describes a subunit

n is open probability
 $1 - n$ is closed probability

$$P_K \sim n^4$$



The ion channel is a cool molecular machine



n describes a subunit

n is open probability
 $1 - n$ is closed probability

Transitions between states
occur at voltage dependent
rates

$\alpha_n(V)$ $C \rightarrow O$

$\beta_n(V)$ $O \rightarrow C$

$$\frac{dn}{dt} = \alpha_n(V)(1 - n) - \beta_n(V)n$$

Dynamics of activation: persistent conductance

$$\frac{dn}{dt} = \alpha_n(V)(1 - n) - \beta_n(V)n$$

We can rewrite:

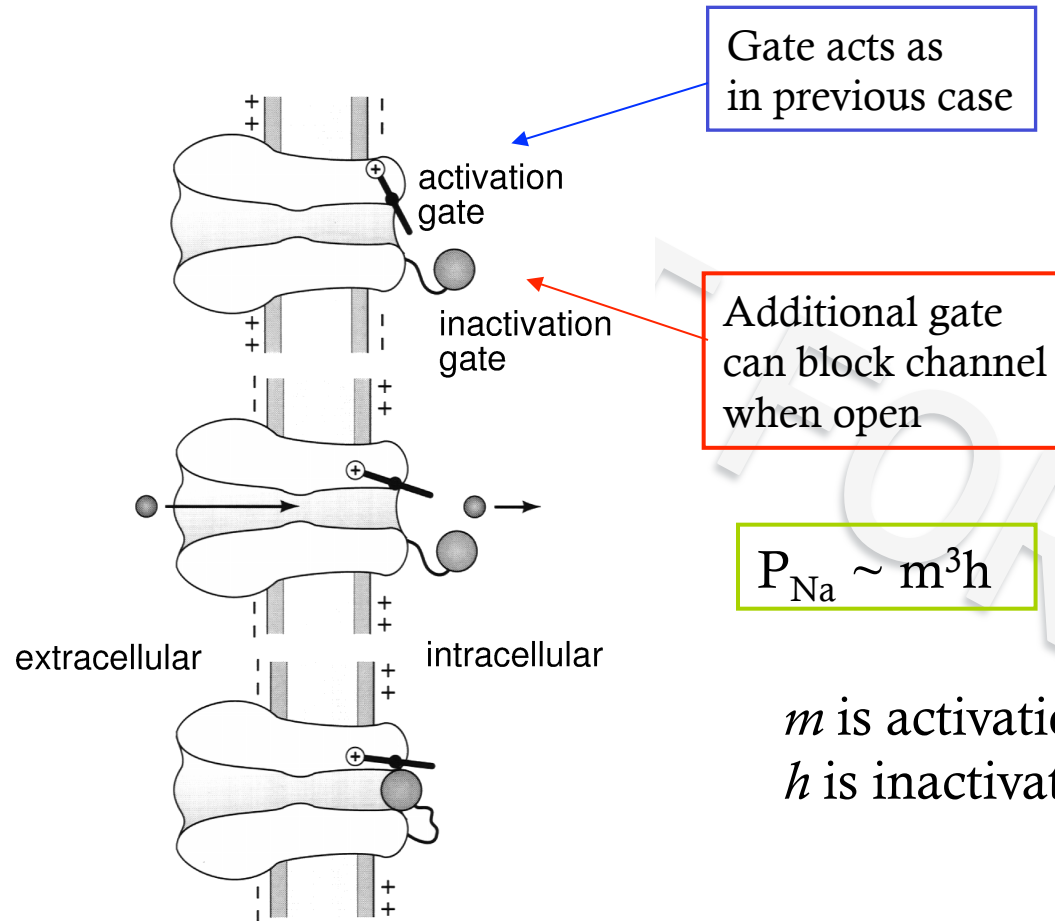
$$\tau_n(V) \frac{dn}{dt} = n_\infty(V) - n$$

where

$$\tau_n(V) = \frac{1}{\alpha_n(V) + \beta_n(V)}$$

$$n_\infty(V) = \frac{\alpha_n(V)}{\alpha_n(V) + \beta_n(V)}$$

Transient conductances



m is activation variable
 h is inactivation variable

m and h have opposite voltage dependences:
depolarization increases m , activation
hyperpolarization increases h , deinactivation

Dynamics of activation and inactivation

$$\begin{aligned}\frac{dn}{dt} &= \alpha_n(V)(1 - n) - \beta_n(V)n \\ \frac{dm}{dt} &= \alpha_m(V)(1 - m) - \beta_m(V)m \\ \frac{dh}{dt} &= \alpha_h(V)(1 - h) - \beta_h(V)h\end{aligned}$$

So will get equivalent forms as for n ...

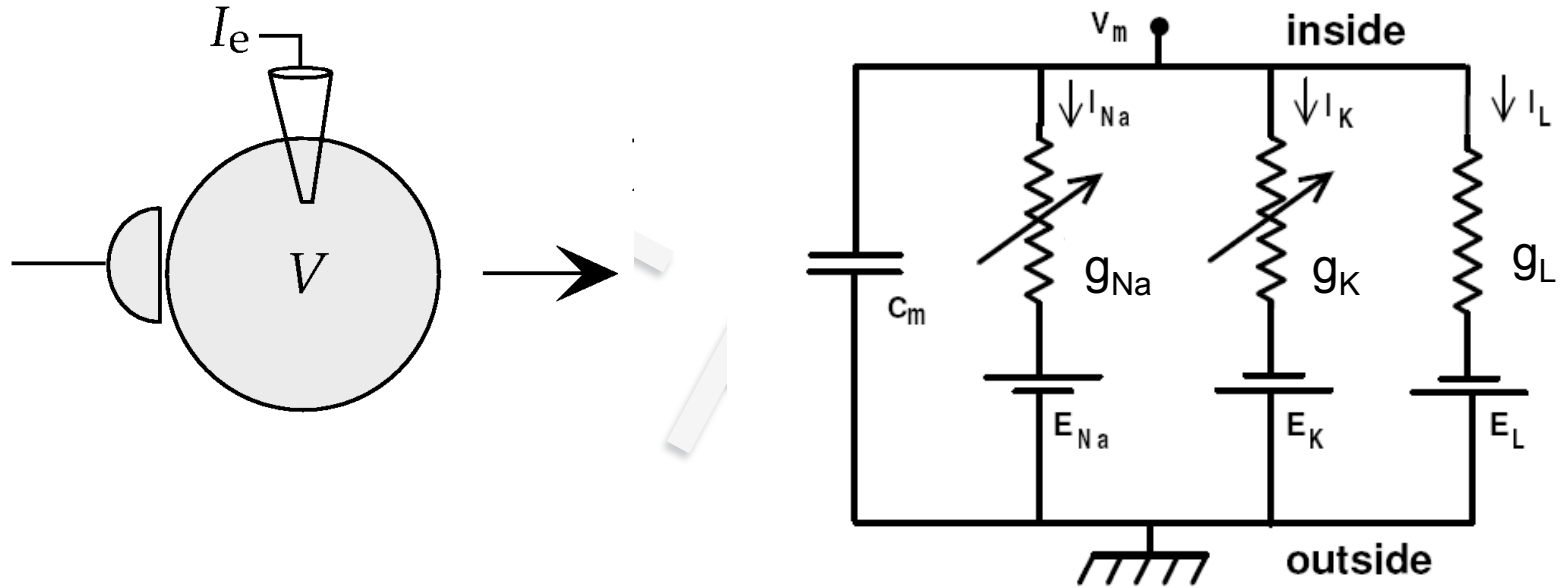
$$\tau_n(V) \frac{dn}{dt} = n_\infty(V) - n$$

V-dependent
conductances

$$g_K(V) = \bar{g}_K n^4$$

$$g_{Na}(V) = \bar{g}_{Na} m^3 h$$

Putting it together



Ohm's law: $V = IR$ and Kirchhoff's law

$$C_m \frac{dV}{dt} = - \sum_i g_i (V - E_i) + I_e$$

Capacitive
current

Ionic currents

Externally
applied current

Hodgkin and Huxley's Nobel equation



$$C_m \frac{dV}{dt} = - \sum_i g_i (V - E_i) + I_e$$

$$-C_m \frac{dV}{dt} = g_L(V - E_L) + \bar{g}_K n^4 (V - E_K) + \bar{g}_{Na} m^3 h (V - E_{Na}) - I_e$$

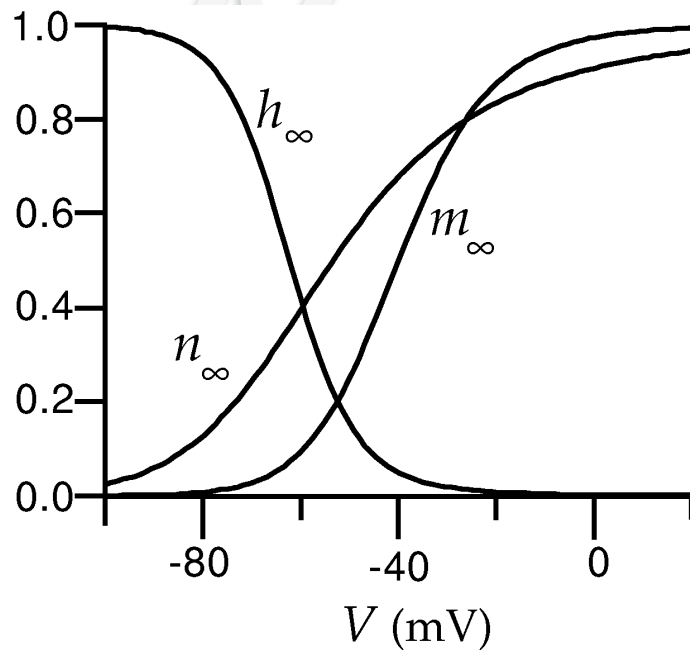
$$\frac{dn}{dt} = \alpha_n(V)(1 - n) - \beta_n(V)n$$

$$\frac{dm}{dt} = \alpha_m(V)(1 - m) - \beta_m(V)m$$

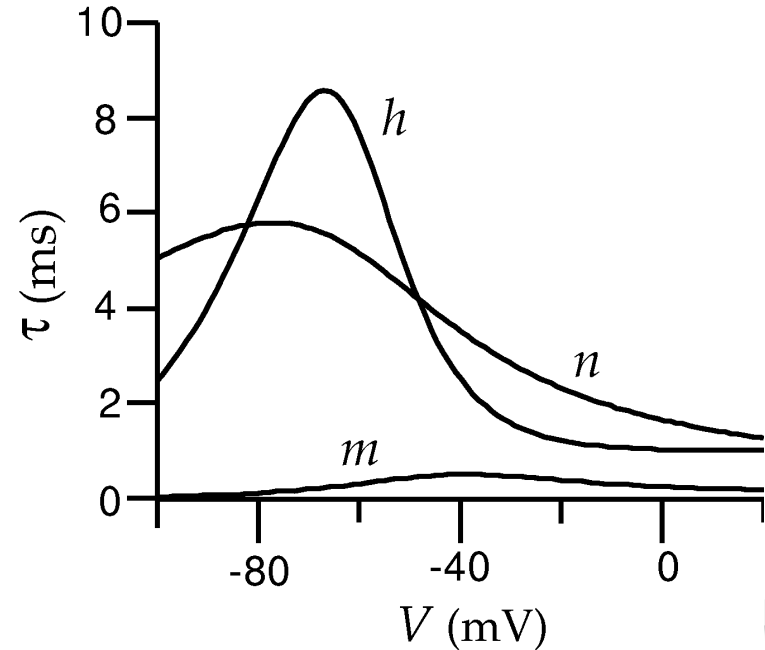
$$\frac{dh}{dt} = \alpha_h(V)(1 - h) - \beta_h(V)h$$

Dynamics of activation and inactivation

$$\tau_n(V) \frac{dn}{dt} = n_{\infty}(V) - n$$



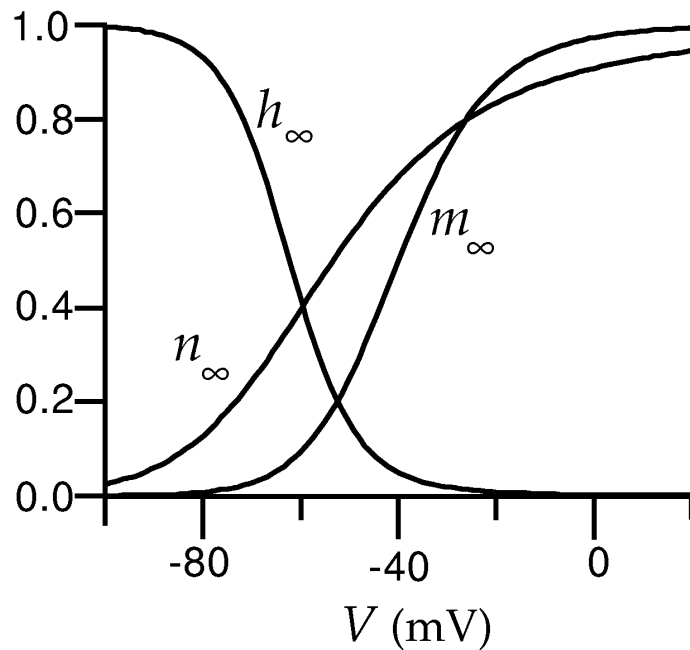
↑
This is where each
variable is going



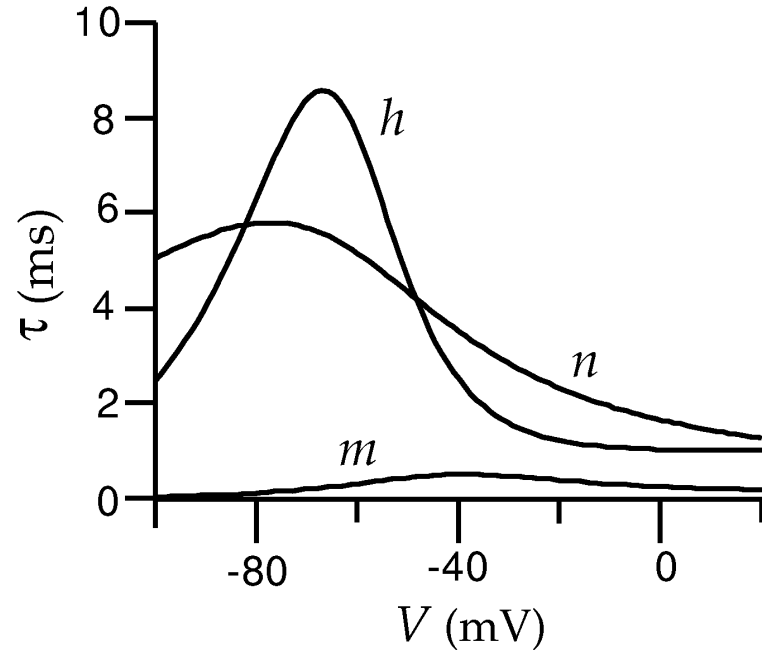
↑
This is how fast it
gets there

Dynamics of activation and inactivation

$$\tau_n(V) \frac{dn}{dt} = n_{\infty}(V) - n$$



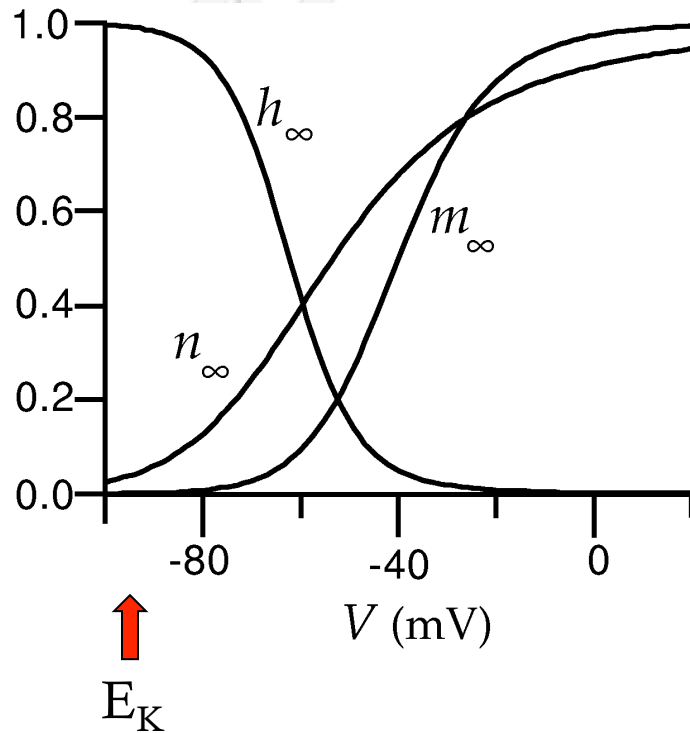
This is where each
variable is going



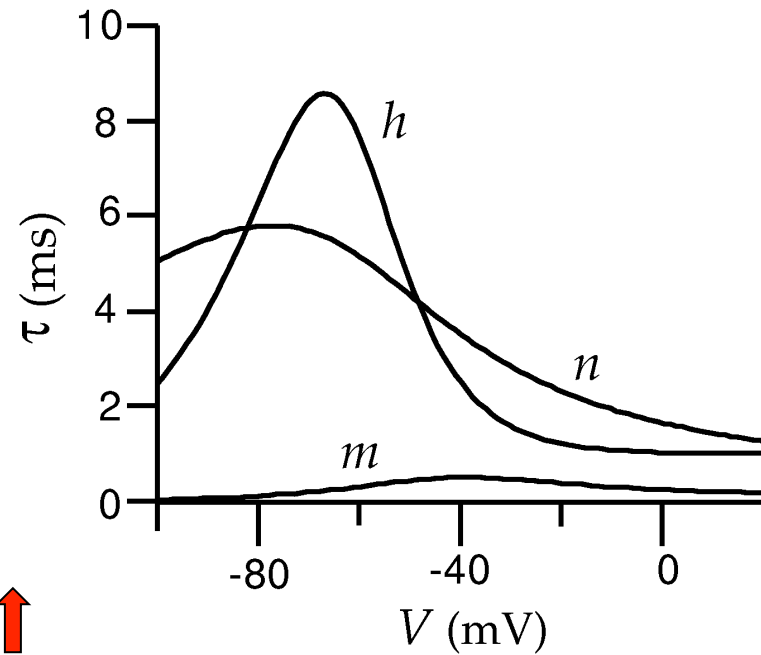
This is how fast it
gets there

Anatomy of a spike

Steady state



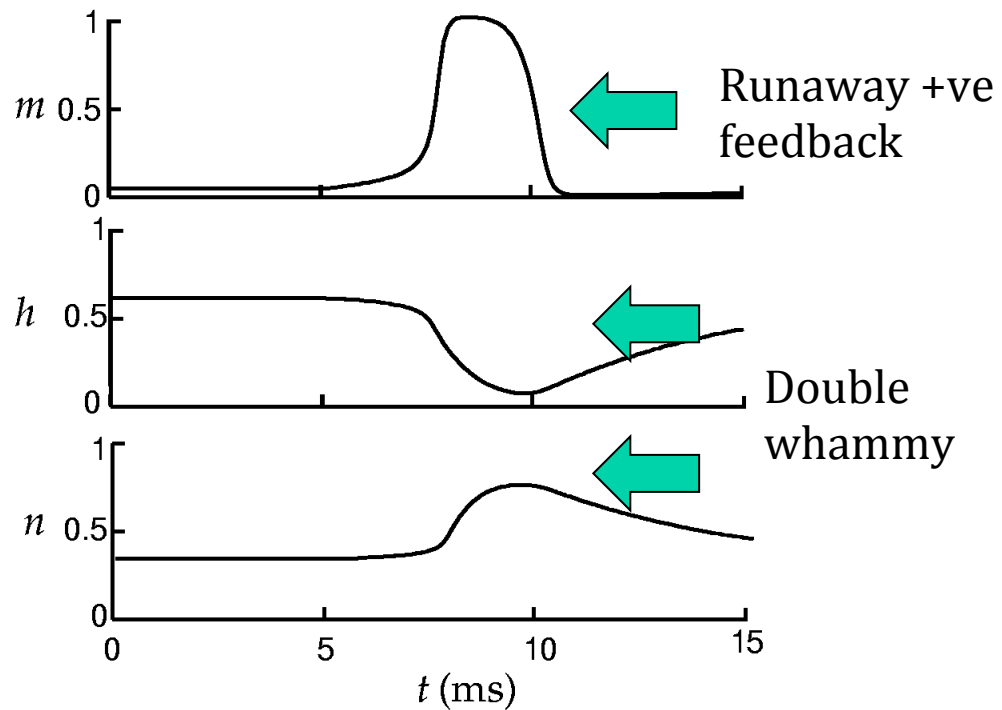
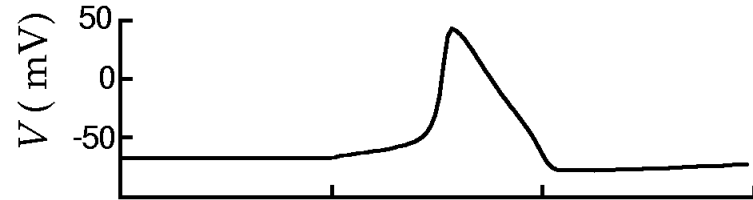
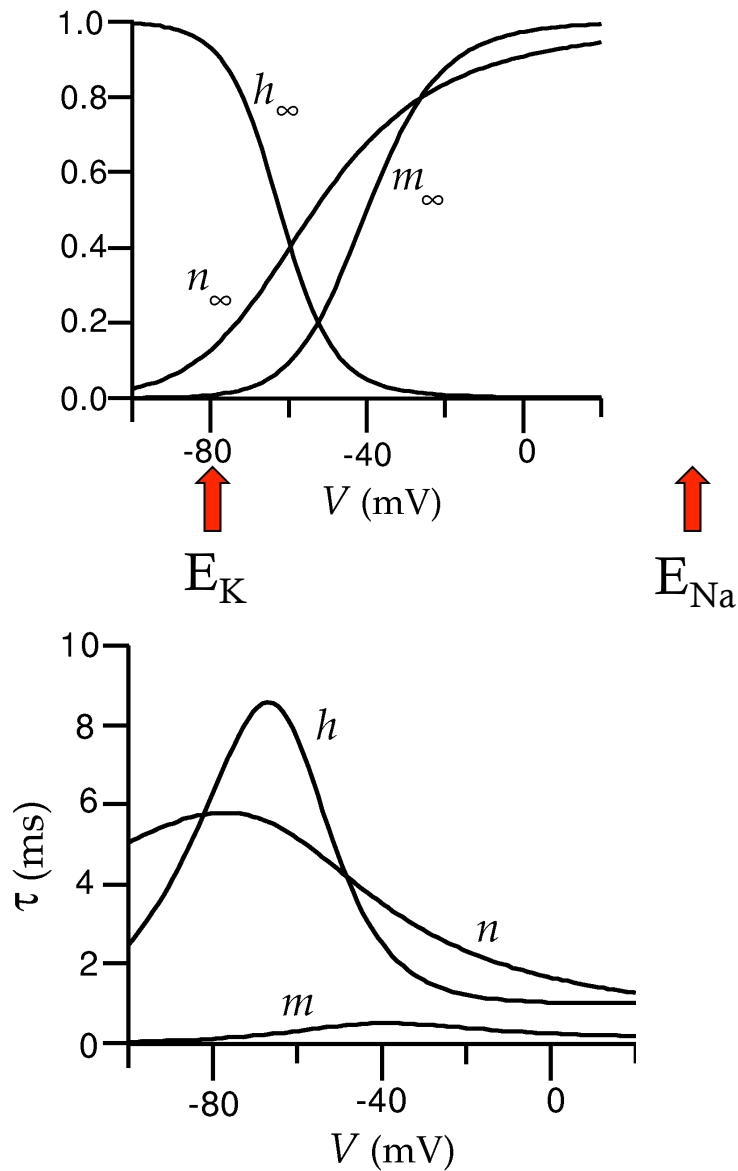
Time constant



$$g_K(V) = \bar{g}_K n^4$$

$$g_{Na}(V) = \bar{g}_{Na} m^3 h$$

Anatomy of a spike



Where to from here?

The diagram features a central red-outlined oval at the top labeled 'Hodgkin-Huxley'. Two large green arrows point downwards from this central oval to two separate red-outlined ovals below it. The left oval is titled 'Biophysical realism' and lists 'Ion channel physics', 'Additional channels', and 'Geometry'. The right oval is titled 'Simplified models' and lists 'Fundamental dynamics' and 'Analytical tractability'. A faint, large watermark 'CR' is visible in the background.

Hodgkin-Huxley

Biophysical realism
Ion channel physics
Additional channels
Geometry

Simplified models
Fundamental dynamics
Analytical tractability