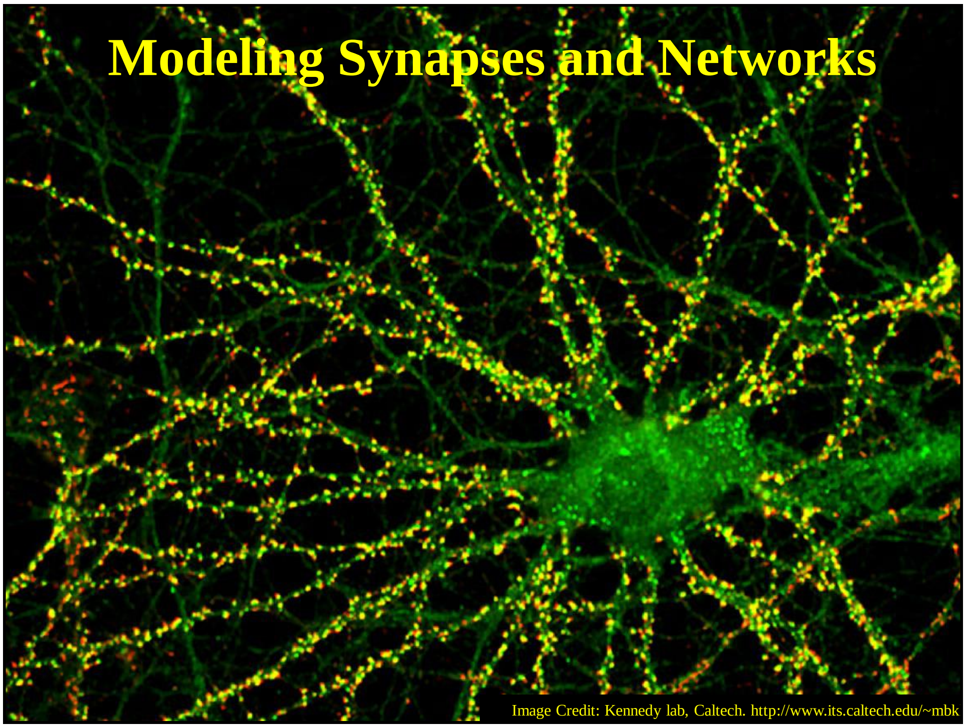
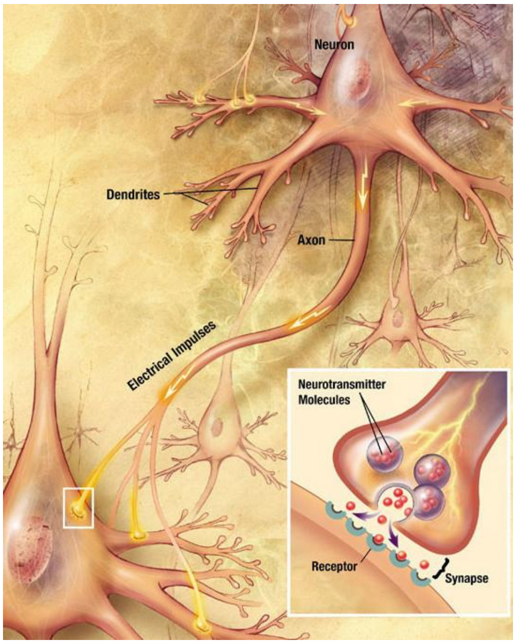


Highlights of our journey thus far…

- **Neuroscience Review**
  - Neurons, synapses, and brain regions
- **Neural Encoding**
  - What makes a neuron fire? (STA, covariance analysis)
  - Poisson model of spiking
- **Neural Decoding and Information Theory**
  - Stimulus discrimination and signal detection
  - Population decoding and Bayesian estimation
  - Information and neural coding principles
- **Single Neuron Models**
  - RC circuit model of membrane
  - Hodgkin-Huxley and compartmental models
  - Integrate-and-fire and simplified neuron models



How do neurons connect to form networks?

They use synapses!

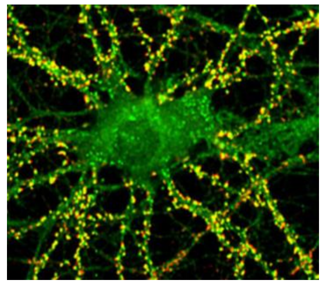
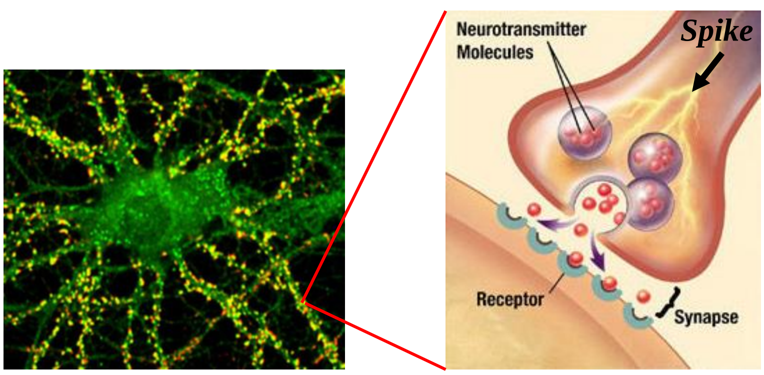


Image Source: Wikimedia Commons

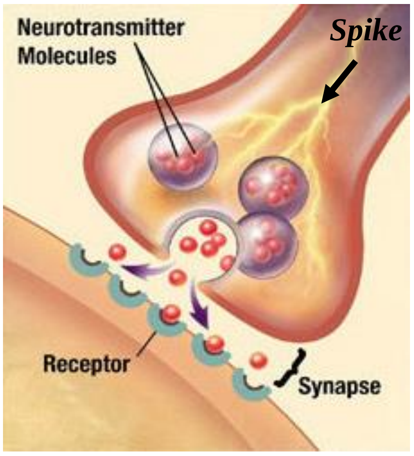
What do synapses do?



Increase or decrease postsynaptic membrane potential

Image Source: Wikimedia Commons

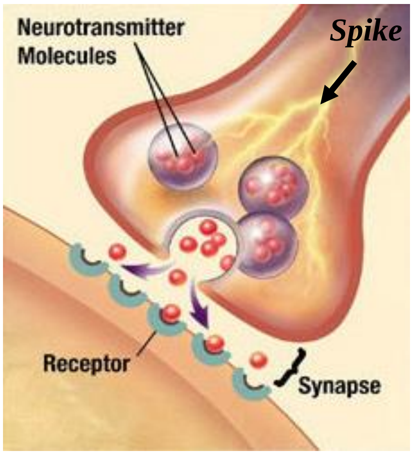
An Excitatory Synapse



Input spike →
Neurotransmitter release
(e.g., Glutamate) →
Binds to receptors →
Ion channels open →
positive ions (e.g. Na+)
enter cell →
Depolarization
(increases local
membrane potential)

Image Source: Wikimedia Commons

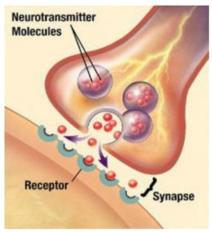
An Inhibitory Synapse

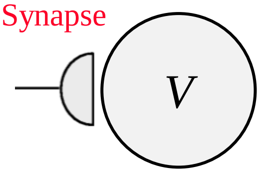


Input spike →
Neurotransmitter
release (e.g., GABA)
→ Binds to receptors
→ Ion channels open
→ positive ions (e.g.,
K+) leave cell →
Hyperpolarization
(decreases local
membrane potential)

Image Source: Wikimedia Commons

We want a *computational* model of the effects of a synapse on the membrane potential V

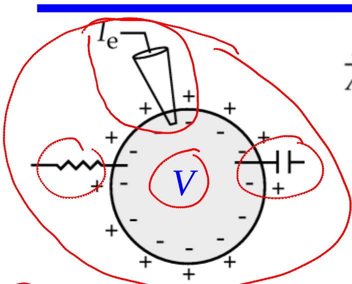


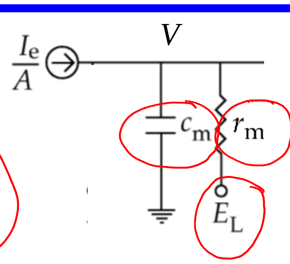


How do we do this?

Flashback

RC Circuit Model of the Membrane





$c_m \approx 10\ \mathrm{nF/mm^2}$

$r_m \approx 1\ \mathrm{M\Omega\ mm^2}$

$C_m = c_m A$

$R_m = r_m / A$

$E_L = -70\ mV$

$q = C_m V$

$\frac{dq}{dt} = i = C_m \frac{dV}{dt}$

$$c_m \frac{dV}{dt} = -\frac{(V - E_L)}{r_m} + \frac{I_e}{A}$$

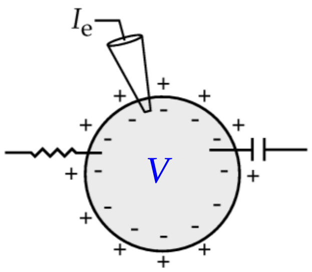
or equivalently:

$\tau_m = r_m c_m = R_m C_m$ is the membrane time constant

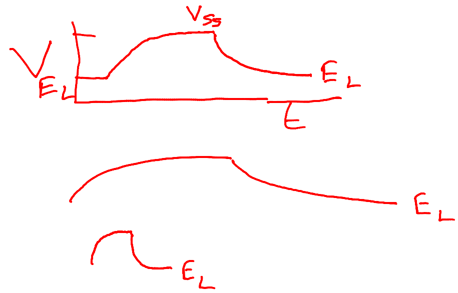
$$\boxed{\tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m}$$

Image Source: Dayan & Abbott textbook

What is this equation really saying?



$$\tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m \rightarrow 0$$

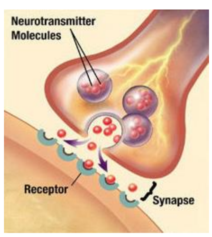


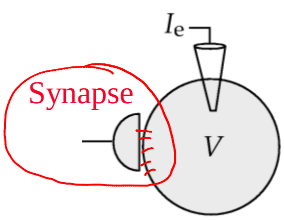
$0 = -(V - E_L) + I_e R_m$

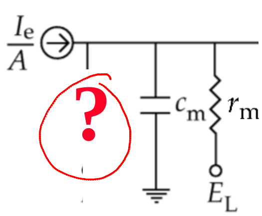
$V_{ss} = E_L + I_e R_m$

Image Source: Dayan & Abbott textbook

How do we model the effects of a synapse on the membrane potential V?

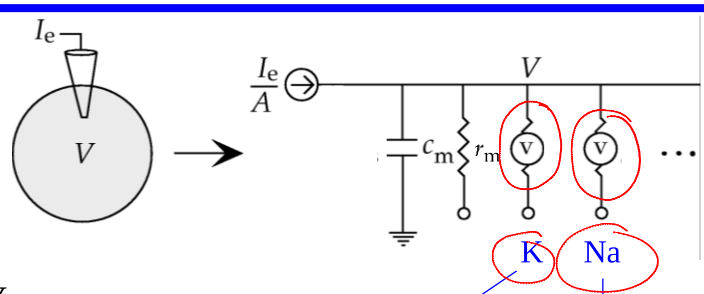






Hint!

Hodgkin-Huxley Model



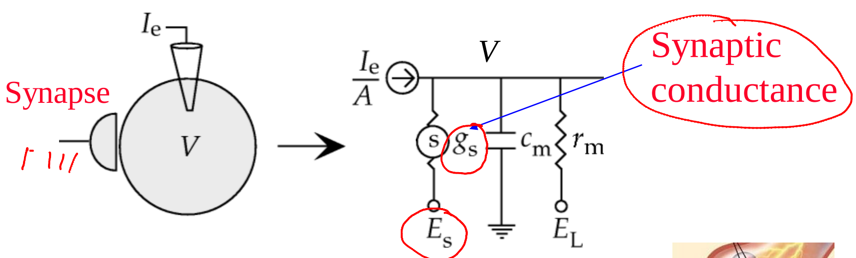
$$\tau_m \frac{dV}{dt} = -i_m r_m + I_e R_m$$

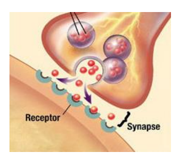
$$i_m = (1/r_m)(V - E_L) + g_{K,\max} n^4 (V - E_K) + g_{Na,\max} m^3 h (V - E_{Na})$$

E_L = -54 mV, E_K = -77 mV, E_{Na} = +50 mV

Image Source: Dayan & Abbott textbook

Modeling Synaptic Inputs





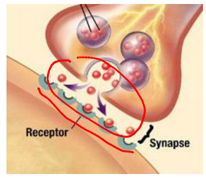
$$\tau_m \frac{dV}{dt} = -((V - E_L) + g_s (V - E_s) r_m) + I_e R_m$$

$$g_s = g_{s,\max} P_{rel} P_s$$

← Probability of postsynaptic channel opening (= fraction of channels opened)

Probability of transmitter release given an input spike

Basic Synapse Model



$g_s = g_{s,\max} P_{rel} P_s$

- Assume $P_{rel} = 1$
- Model the effect of a single spike input on P_s ← fraction of channels opened
- Kinetic Model of postsynaptic channels:

$$\textbf{Closed} \underset{\beta_s}{\overset{\alpha_s}{\rightleftharpoons}} \textbf{Open}$$

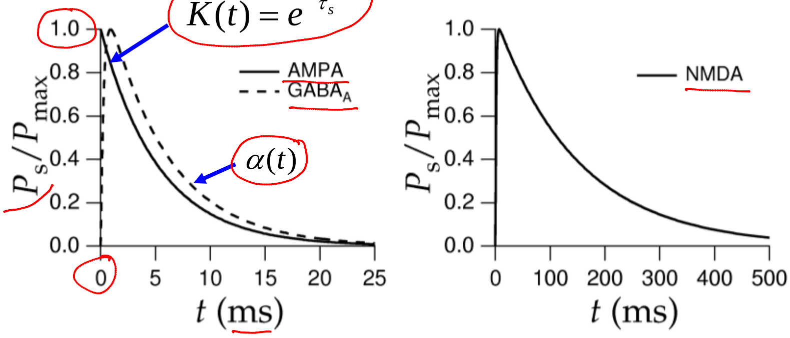
$$\frac{dP_s}{dt} = \alpha_s(1 - P_s) - \beta_s P_s$$

Opening rate (α_s)
Fraction of channels closed ($1 - P_s$)
Closing rate (β_s)
Fraction of channels open (P_s)

What does P_s look like over time given a spike?

$K(t) = e^{-\frac{t}{\tau_s}}$

$\alpha(t)$



Exponential function gives reasonable fit for some synapses
Others can be fit using "Alpha" function:

$$\alpha(t) = \frac{t}{\tau_{peak}} \cdot e^{(1 - \frac{t}{\tau_{peak}})}$$

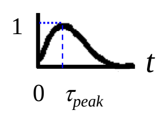
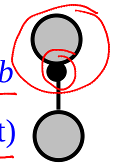


Image Source: Dayan & Abbott textbook

Linear Filter Model of a Synapse

Synapse b

Input Spike Train $\rho_b(t)$



$\rho_b(t) = \Sigma_i\, \delta(t\text{-}t_i)$ (t_i are the input spike times, δ = delta function)

Filter for synapse b = $K(t)$

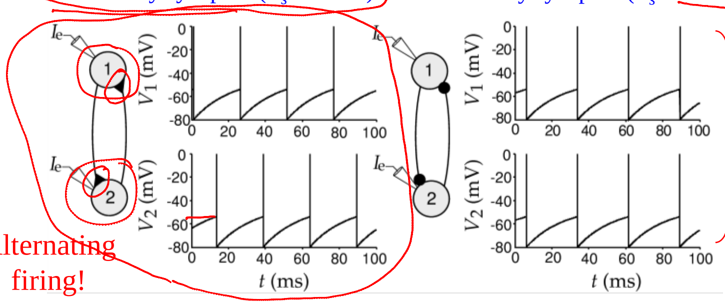
Synaptic conductance at b:

$$g_b(t) = g_{b,\max} \sum_{t_i<t} K(t-t_i)$$

$$= g_{b,\max} \int_{-\infty}^{t} K(t-\tau)\rho_b(\tau)d\tau$$

Example: Network of Integrate-and-Fire Neurons

Excitatory synapses (E_s = 0 mV) Inhibitory synapses (E_s = -80 mV)



Synchrony!

Alternating firing!

Each neuron: $\tau_m \frac{dV}{dt} = -((V-E_L) - g_s(t)(V-E_s)r_m) + I_e R_m$

Synapses : Alpha function

$E_L = -70\text{ mV}$ $V_{thresh} = -54\text{ mV}$

$\tau_m = 20\text{ ms}$ $\tau_{peak} = 10\text{ ms}$ $I_e R_m = 25\text{ mV}$

Image Source: Dayan & Abbott textbook