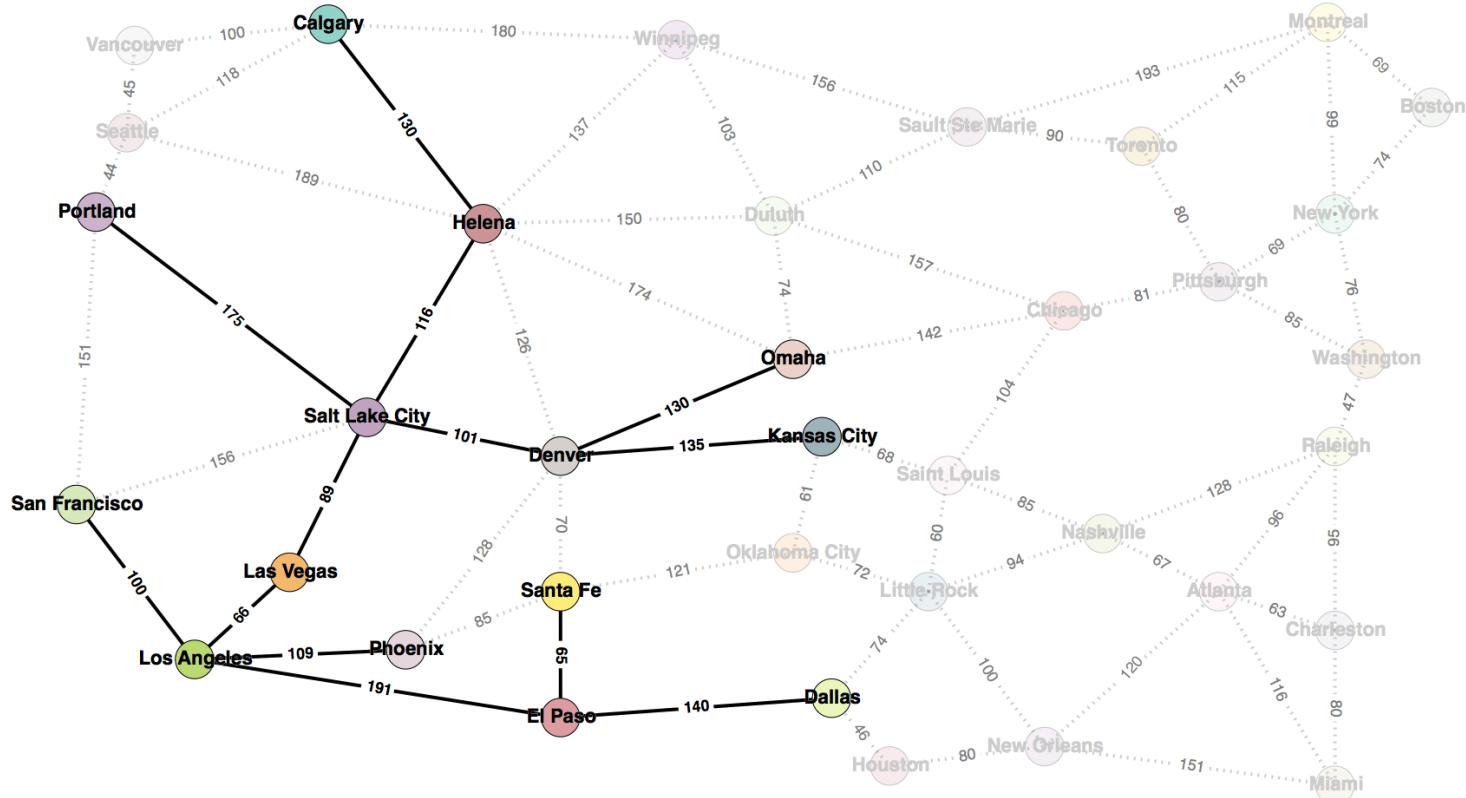


Artificial Intelligence

Search Agents

Uninformed search



Uninformed search

Use no domain knowledge!

Strategies:

1. Breadth-first search (BFS): Expand shallowest node

Uninformed search

Use no domain knowledge!

Strategies:

1. Breadth-first search (BFS): Expand shallowest node
2. Depth-first search (DFS): Expand deepest node

Uninformed search

Use no domain knowledge!

Strategies:

1. Breadth-first search (BFS): Expand shallowest node
2. Depth-first search (DFS): Expand deepest node
3. Depth-limited search (DLS): Depth first with depth limit

Uninformed search

Use no domain knowledge!

Strategies:

1. Breadth-first search (BFS): Expand shallowest node
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3. Depth-limited search (DLS): Depth first with depth limit
4. Iterative-deepening search (IDS): DLS with increasing limit

Uninformed search

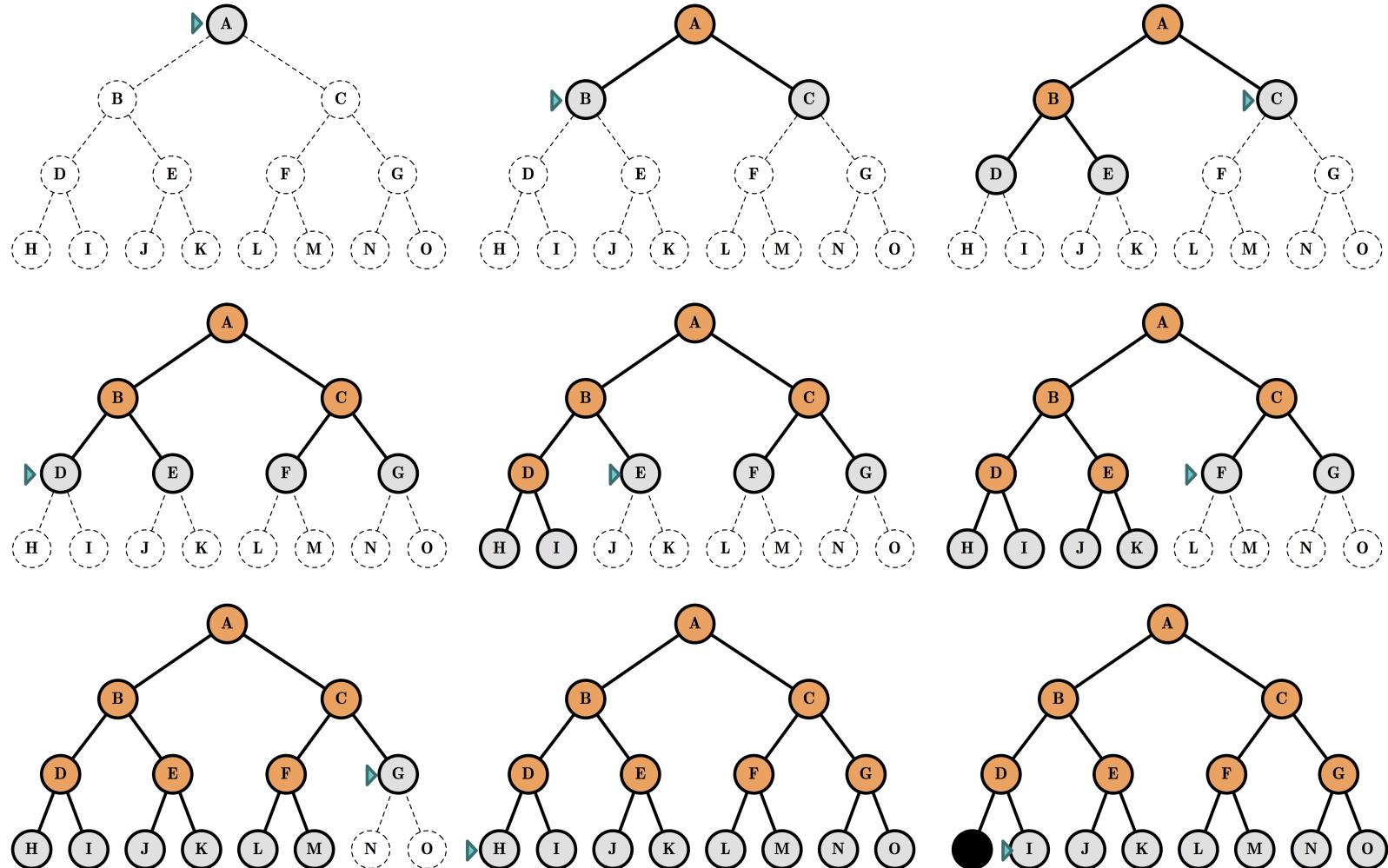
Use no domain knowledge!

Strategies:

1. Breadth-first search (BFS): Expand shallowest node
2. Depth-first search (DFS): Expand deepest node
3. Depth-limited search (DLS): Depth first with depth limit
4. Iterative-deepening search (IDS): DLS with increasing limit
5. Uniform-cost search (UCS): Expand least cost node

Breadth-first search (BFS)

BFS: Expand shallowest first.



BFS search

```
function BREADTH-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE :

    frontier = Queue.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.dequeue()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.enqueue(neighbor)

    return FAILURE
```

BFS Criteria

BFS criteria?

BFS

- **Complete** Yes (if b is finite)
- **Time** $1 + b + b^2 + b^3 + \dots + b^d = O(b^d)$
- **Space** $O(b^d)$
Note: If the *goal test* is applied at expansion rather than generation then $O(b^{d+1})$
- **Optimal** Yes (if cost = 1 per step).
- **implementation:** fringe: FIFO (Queue)

Question: If time and space complexities are exponential, why use BFS?

BFS

How bad is BFS?

BFS

How bad is BFS?

| Depth | Nodes | Time | Memory |
|-------|-----------|------------------|----------------|
| 2 | 110 | .11 milliseconds | 107 kilobytes |
| 4 | 11,110 | 11 milliseconds | 10.6 megabytes |
| 6 | 10^6 | 1.1 seconds | 1 gigabyte |
| 8 | 10^8 | 2 minutes | 103 gigabytes |
| 10 | 10^{10} | 3 hours | 10 terabytes |
| 12 | 10^{12} | 13 days | 1 petabyte |
| 14 | 10^{14} | 3.5 years | 99 petabytes |
| 16 | 10^{16} | 350 years | 10 exabytes |

Time and Memory requirements for breadth-first search for a branching factor $b=10$; 1 million nodes per second; 1,000 bytes per node.

BFS

How bad is BFS?

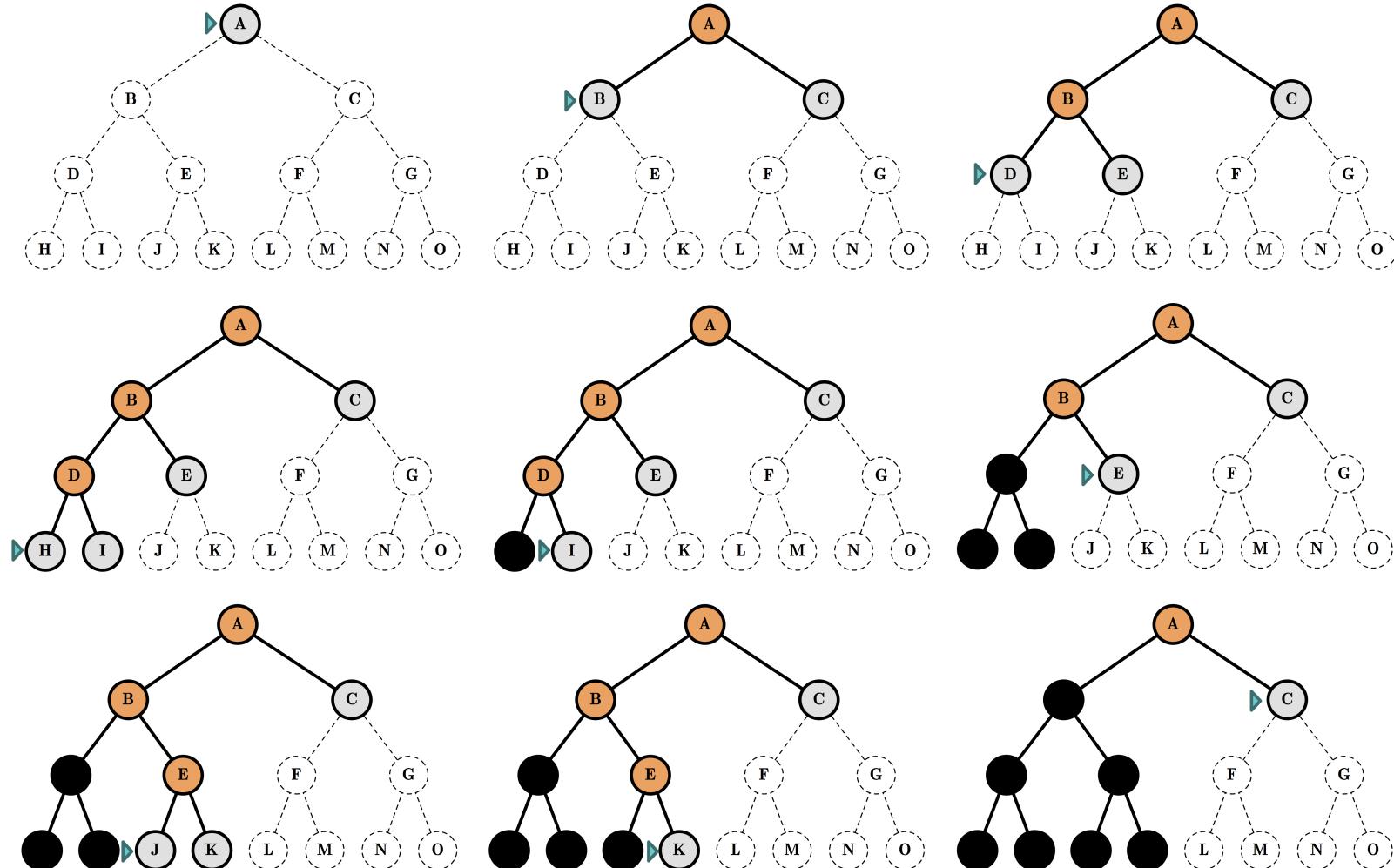
| Depth | Nodes | Time | Memory |
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Time and Memory requirements for breadth-first search for a branching factor $b=10$; 1 million nodes per second; 1,000 bytes per node.

Memory requirement + exponential time complexity are the biggest handicaps of BFS!

DFS

DFS: Expand deepest first.



DFS search

```
function DEPTH-FIRST-SEARCH(initialState, goalTest)
```

returns SUCCESS or FAILURE :

```
frontier = Stack.new(initialState)  
explored = Set.new()
```

```
while not frontier.isEmpty():  
    state = frontier.pop()  
    explored.add(state)
```

```
    if goalTest(state):  
        return SUCCESS(state)
```

```
    for neighbor in state.neighbors():  
        if neighbor not in frontier ∪ explored:  
            frontier.push(neighbor)
```

```
return FAILURE
```

DFS

DFS criteria?

DFS

- **Complete** No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path.
⇒ complete in finite spaces
- **Time** $O(b^m)$: $1 + b + b^2 + b^3 + \dots + b^m = O(b^m)$
bad if m is much larger than d
but if solutions are dense, may be much faster than BFS.
- **Space** $O(bm)$ linear space complexity! (needs to store only a single path from the root to a leaf node, **along with the remaining unexpanded sibling nodes for each node on the path, hence the m factor.**)
- **Optimal** No
- **Implementation:** fringe: LIFO (Stack)

DFS

How bad is DFS?

Recall for BFS...

| Depth | Nodes | Time | Memory |
|-------|-----------|------------------|----------------|
| 2 | 110 | .11 milliseconds | 107 kilobytes |
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Depth =16.

We go down from 10 exabytes in BFS to ... in DFS?

DFS

How bad is DFS?

Recall for BFS...

| Depth | Nodes | Time | Memory |
|-------|-----------|------------------|----------------|
| 2 | 110 | .11 milliseconds | 107 kilobytes |
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Depth =16.

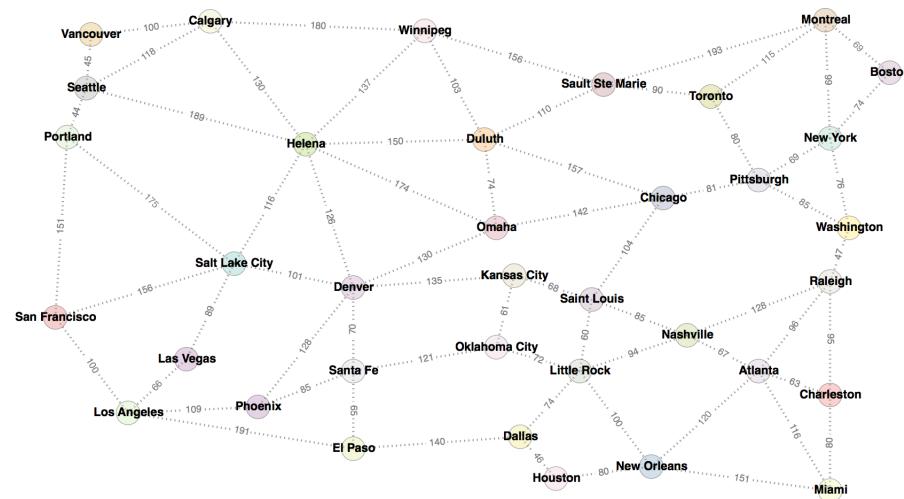
We go down from 10 exabytes in BFS to **156** kilobytes in DFS!

Depth-limited search

- DFS with depth limit l (nodes at level l has no successors).
- Select some limit L in depth to explore with DFS
- Iterative deepening: increasing the limit l

Depth-limited search

- If we know some knowledge about the problem, maybe we don't need to go to a full depth.



Idea: any city can be reached from another city in at most L steps with $L < 36$.

Iterative Deepening

- Combines the benefits of BFS and DFS.
- Idea: Iteratively increase the search limit until the depth of the shallowest solution d is reached.
- Applies DLS with increasing limits.
- The algorithm will stop if a solution is found or if DLS returns a failure (no solution).
- Because most of the nodes are on the bottom of the search tree, it not a big waste to iteratively re-generate the top
- Let's take an example with a depth limit between 0 and 3.

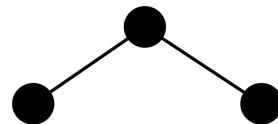
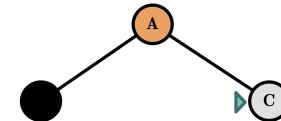
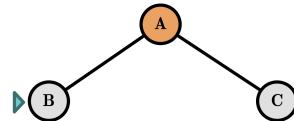
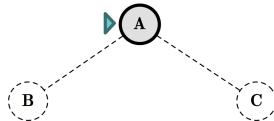
Iterative Deepening

Limit = 0



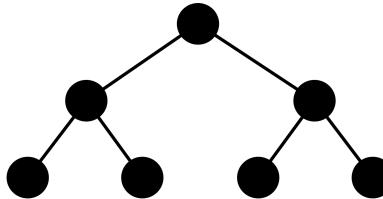
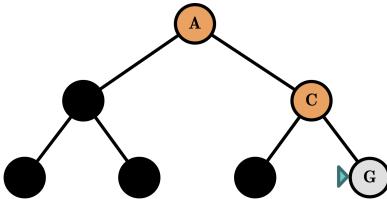
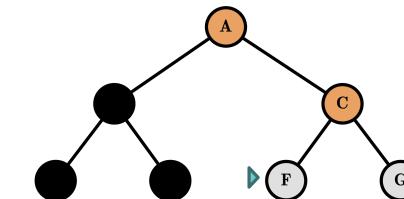
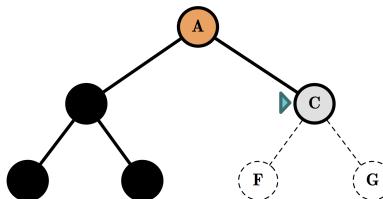
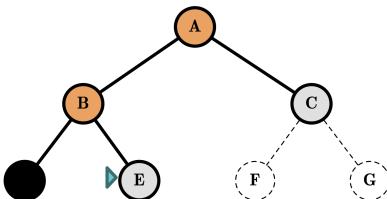
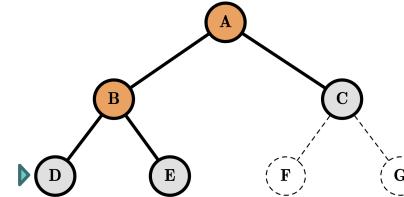
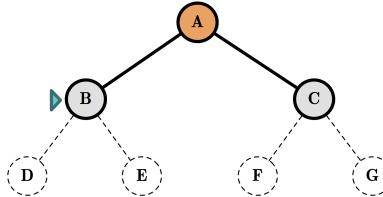
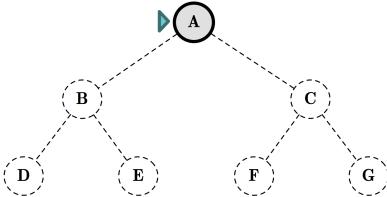
Iterative Deepening

Limit = 1



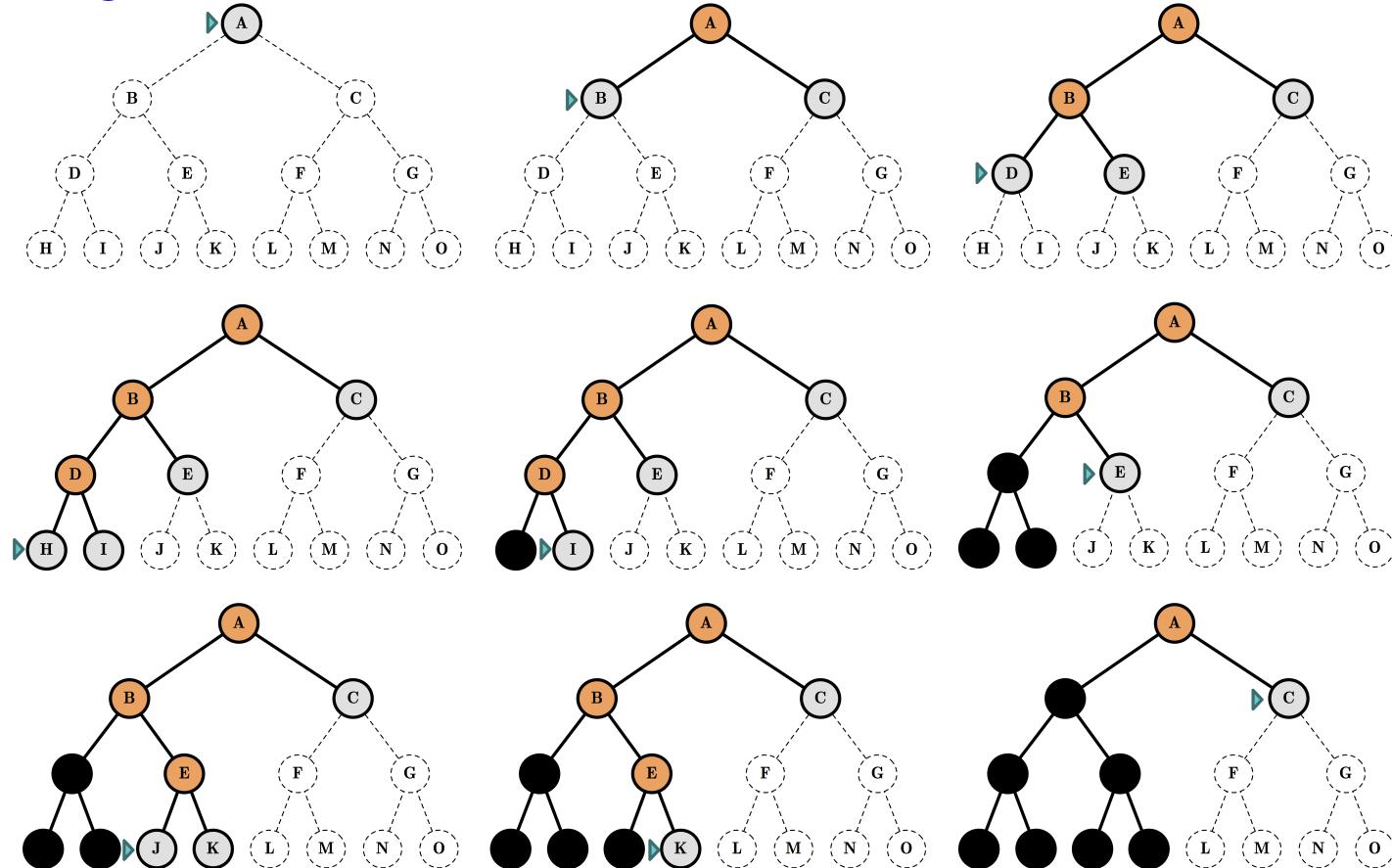
Iterative Deepening

Limit = 2

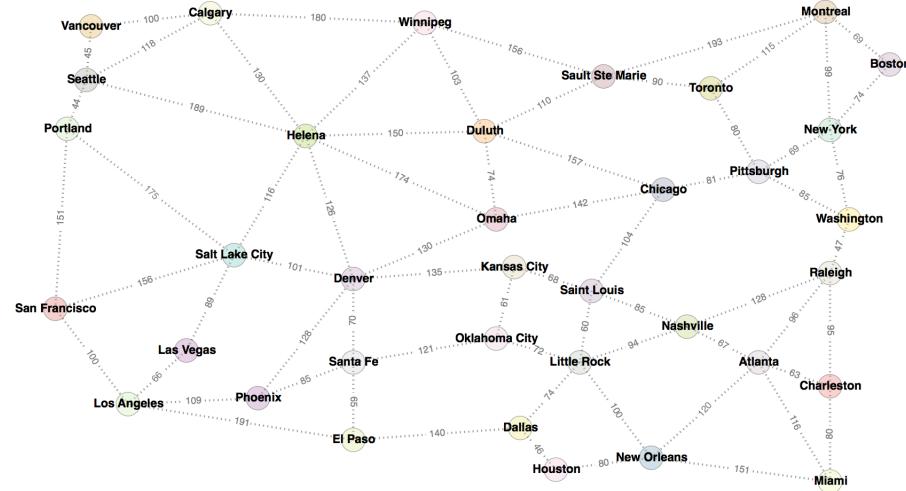


Iterative Deepening

Limit = 3

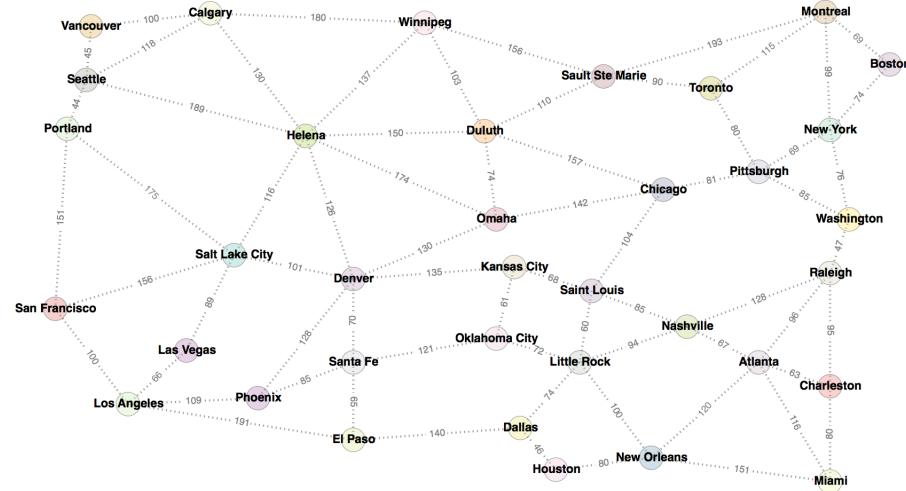


Uniform-cost search



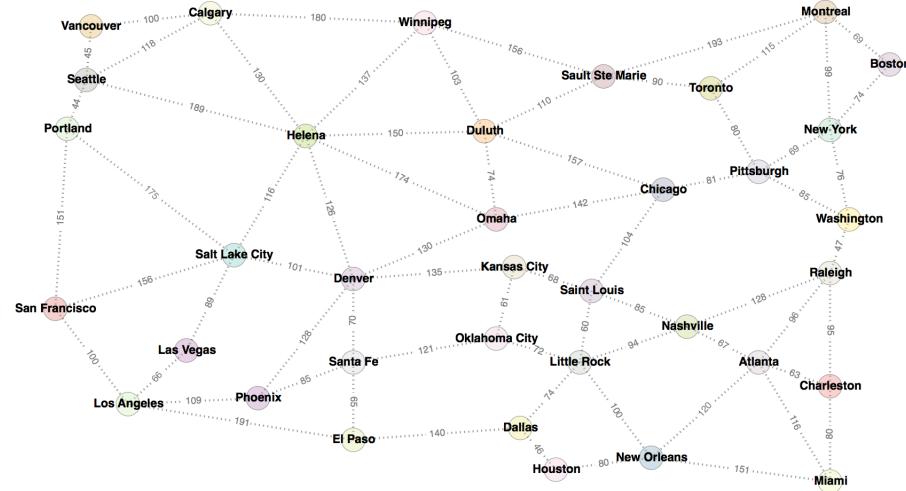
- The arcs in the search graph may have weights (different cost attached). How to leverage this information?

Uniform-cost search



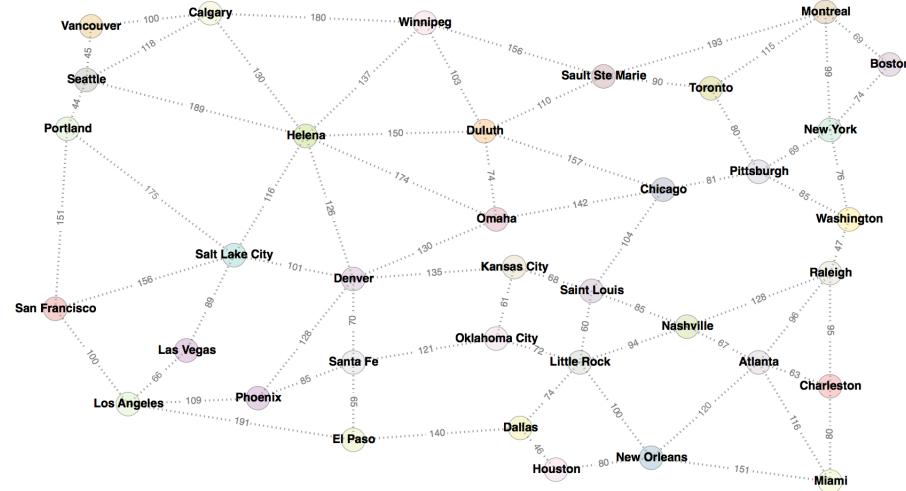
- The arcs in the search graph may have weights (different cost attached). How to leverage this information?
- BFS will find the shortest path which may be costly.
- We want the **cheapest** not shallowest solution.

Uniform-cost search



- The arcs in the search graph may have weights (different cost attached). How to leverage this information?
- BFS will find the shortest path which may be costly.
- We want the **cheapest** not shallowest solution.
- Modify BFS: Prioritize by cost not depth → **Expand node n with the lowest path cost $g(n)$**

Uniform-cost search



- The arcs in the search graph may have weights (different cost attached). How to leverage this information?
- BFS will find the shortest path which may be costly.
- We want the **cheapest** not shallowest solution.
- Modify BFS: Prioritize by cost not depth → **Expand node n with the lowest path cost $g(n)$**
- Explores increasing costs.

UCS algorithm

```
function UNIFORM-COST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE : /* Cost  $f(n) = g(n)$  */

    frontier = Heap.new(initialState)
    explored = Set.new()

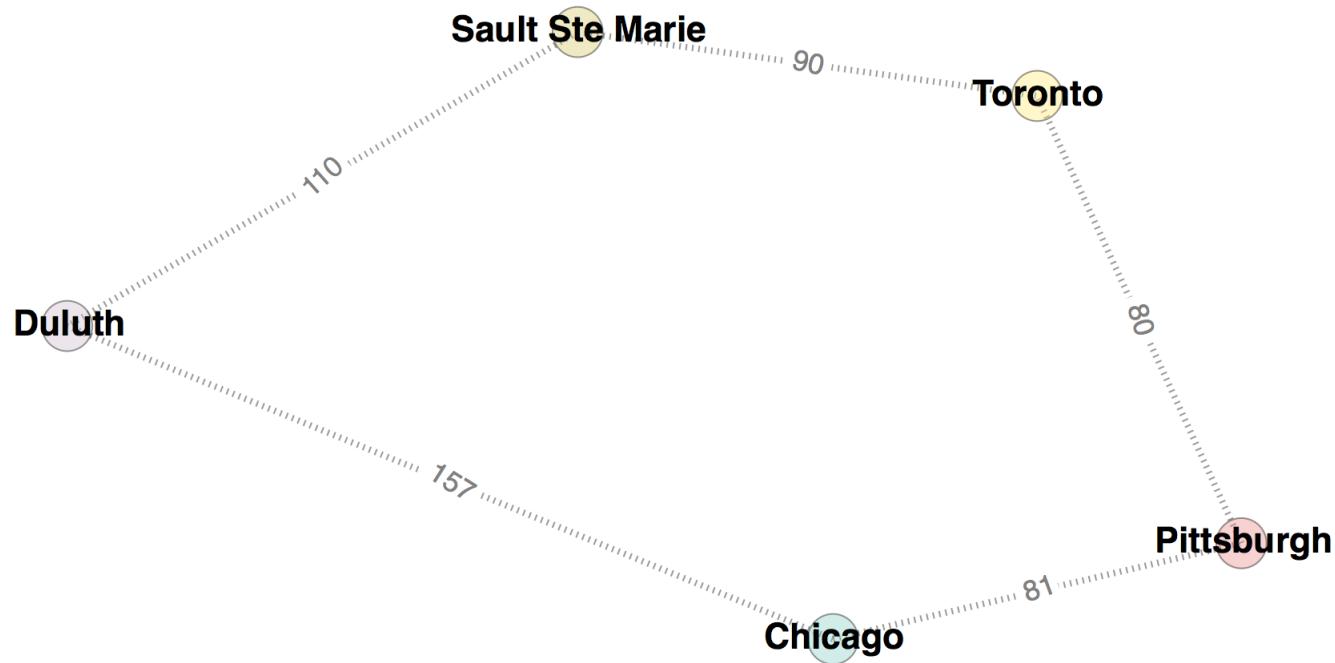
    while not frontier.isEmpty():
        state = frontier.deleteMin()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.insert(neighbor)
            else if neighbor in frontier:
                frontier.decreaseKey(neighbor)

    return FAILURE
```

Uniform-cost search



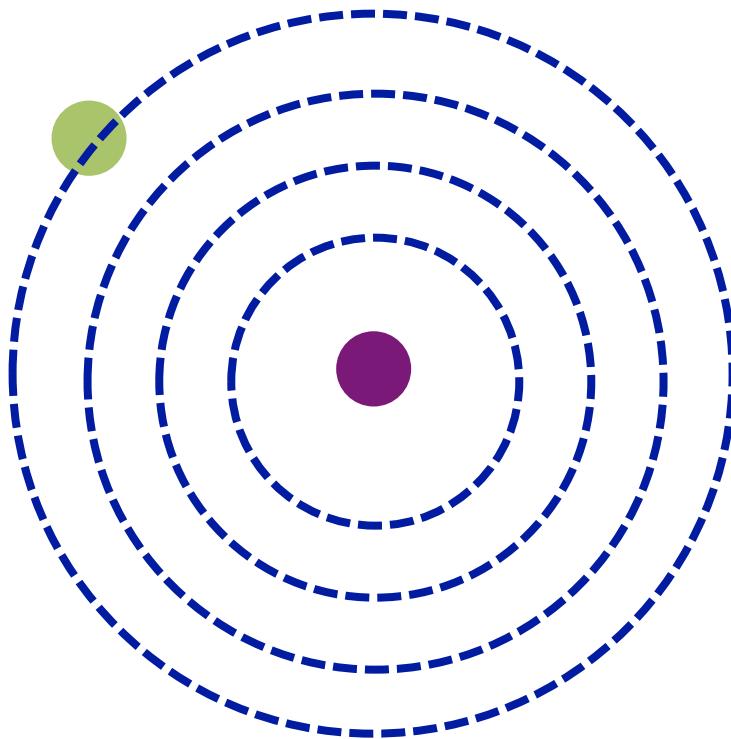
Go from Chicago to Sault Ste Marie. Using BFS, we would find Chicago-Duluth-Sault Ste Marie. However, using UCS, we would find Chicago-Pittsburgh-Toronto-Sault Ste Marie, which is actually the shortest path!

Uniform-cost search

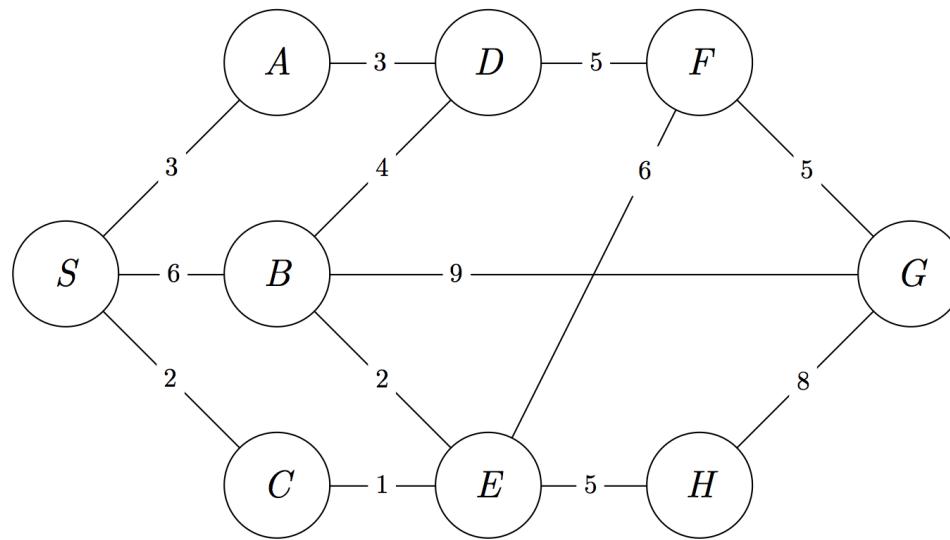
- **Complete** Yes, if solution has a finite cost.
- **Time**
 - Suppose C^* : cost of the optimal solution
 - Every action costs at least ϵ (bound on the cost)
 - The effective depth is roughly C^*/ϵ (how deep the *cheapest* solution could be).
 - $O(b^{C^*/\epsilon})$
- **Space** # of nodes with $g \leq$ cost of optimal solution, $O(b^{C^*/\epsilon})$
- **Optimal** Yes
- **Implementation**: fringe = queue ordered by path cost $g(n)$, lowest first = Heap!

Uniform-cost search

While complete and optimal, UCS explores the space in every direction because no information is provided about the goal!

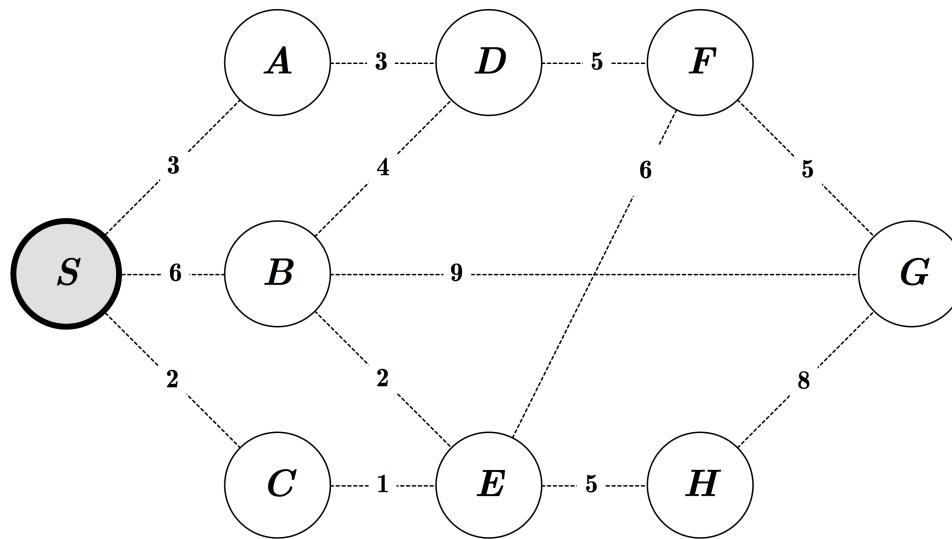


Exercise

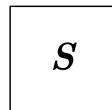


Question: What is the **order of visits of the nodes** and the **path** returned by BFS, DFS and UCS?

Exercise: BFS

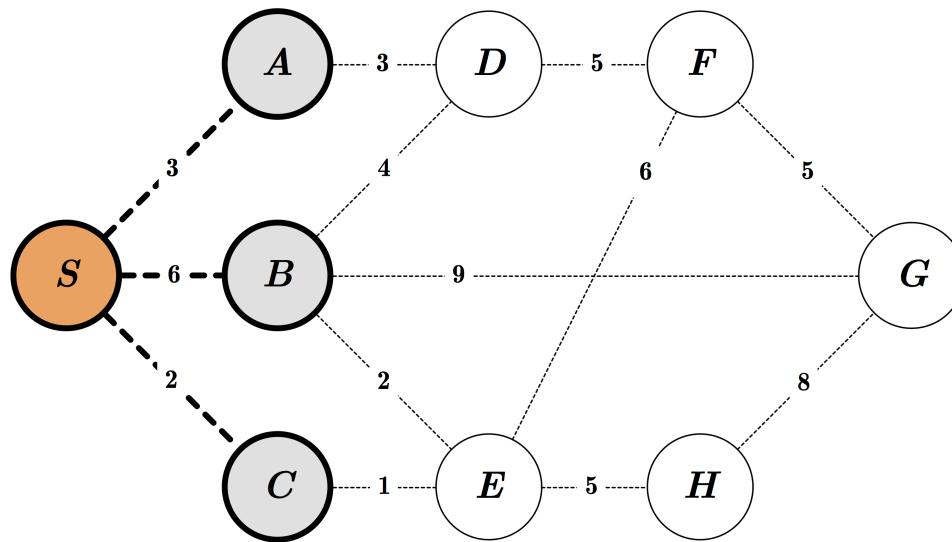


Queue:



Order of Visit:

Exercise: BFS



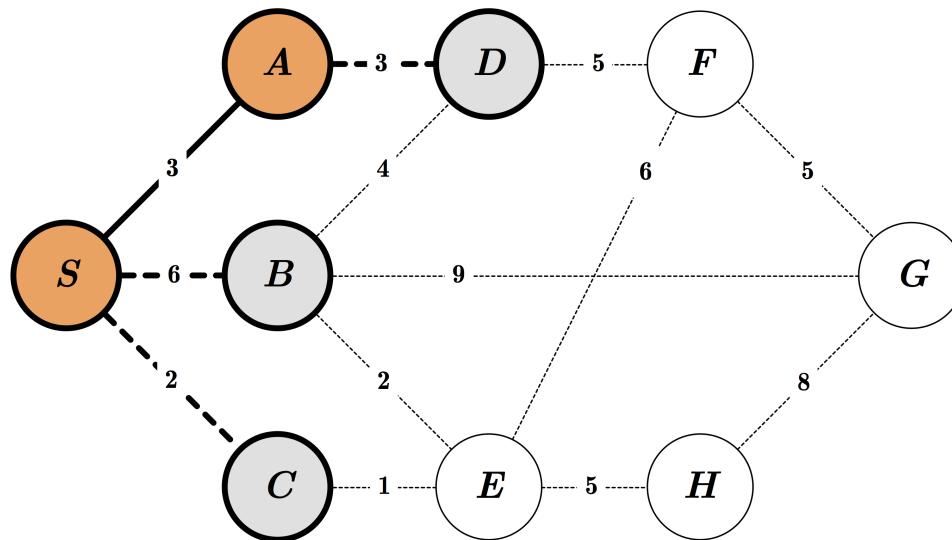
Queue:

| | | | |
|----------|---|---|---|
| S | A | B | C |
|----------|---|---|---|

Order of Visit:

S

Exercise: BFS



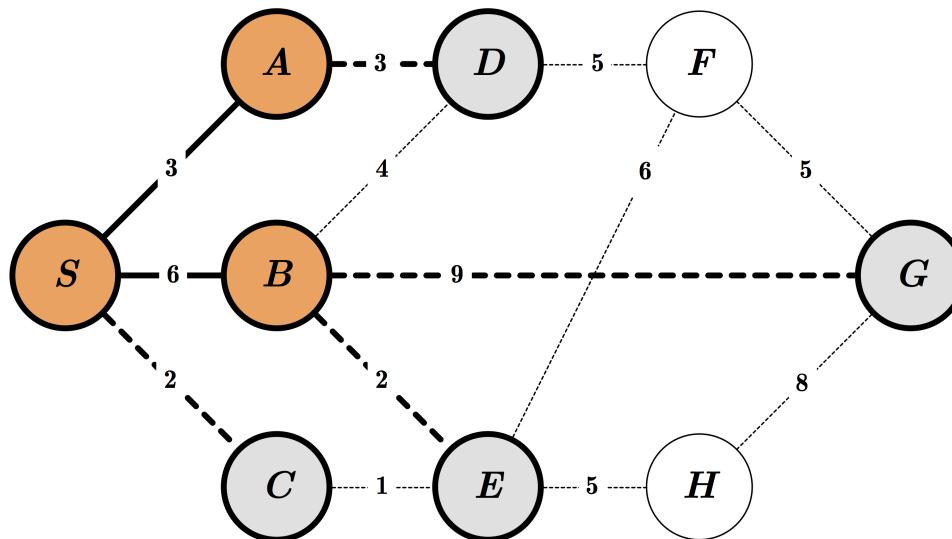
Queue:

| | | | | |
|----------|----------|----------|----------|----------|
| <i>S</i> | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> |
|----------|----------|----------|----------|----------|

Order of Visit:

S A

Exercise: BFS



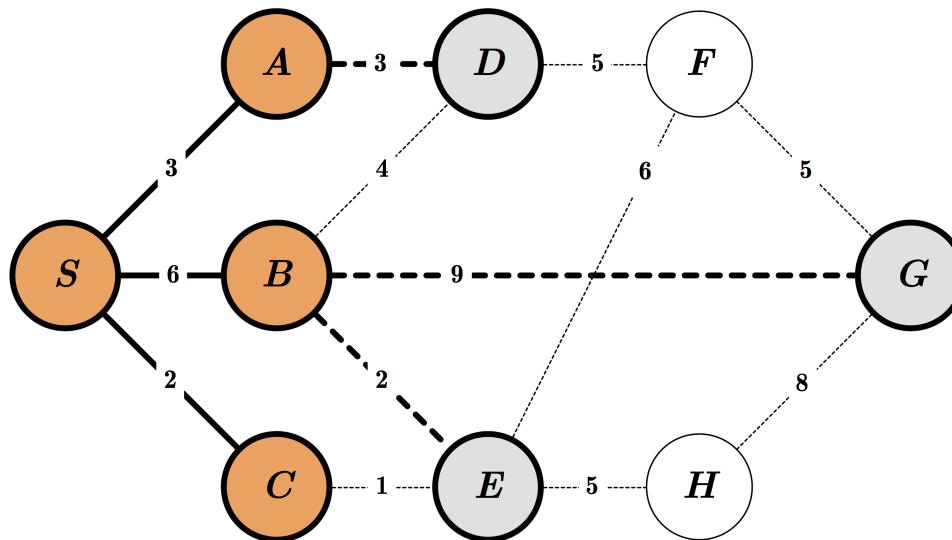
Queue:

| | | | | | | |
|---|---|---|---|---|---|---|
| S | A | B | C | D | E | G |
|---|---|---|---|---|---|---|

Order of Visit:

S A B

Exercise: BFS



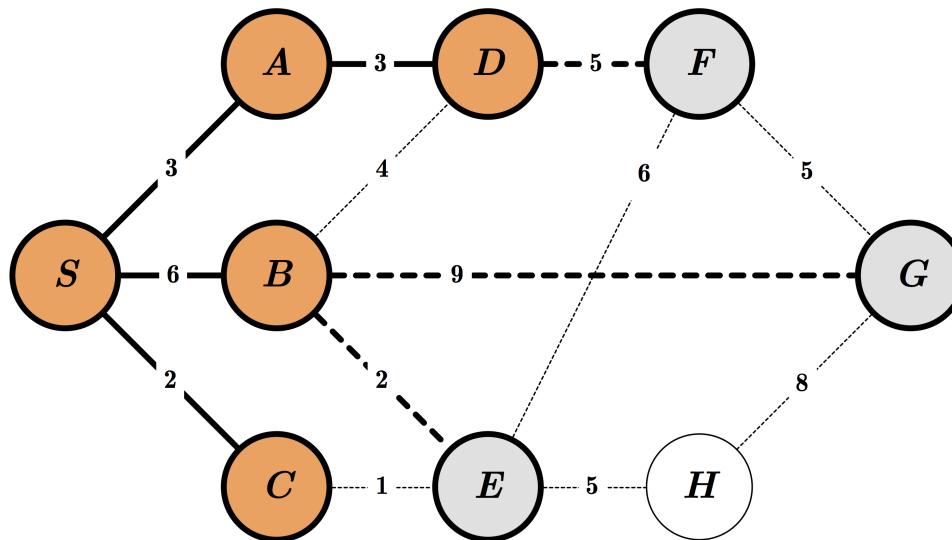
Queue:

| | | | | | | |
|---|---|---|---|---|---|---|
| S | A | B | C | D | E | G |
|---|---|---|---|---|---|---|

Order of Visit:

S A B C

Exercise: BFS



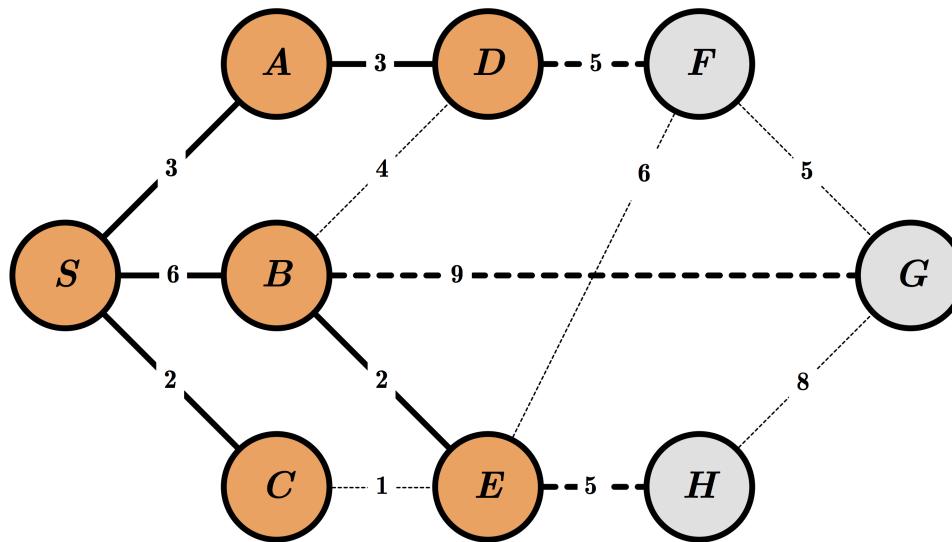
Queue:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| S | A | B | C | D | E | G | F |
|---|---|---|---|---|---|---|---|

Order of Visit:

S A B C D

Exercise: BFS



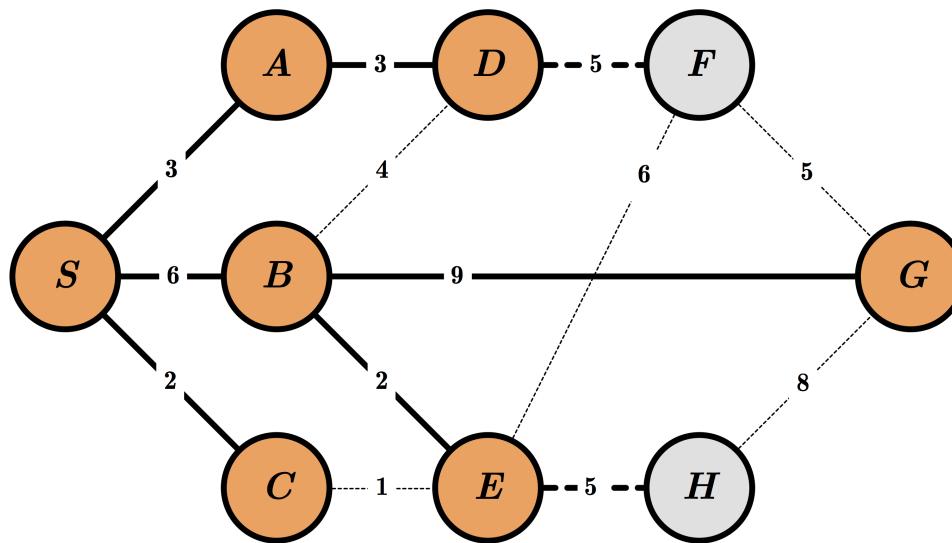
Queue:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| S | A | B | C | D | E | G | F | H |
|---|---|---|---|---|---|---|---|---|

Order of Visit:

S A B C D E

Exercise: BFS



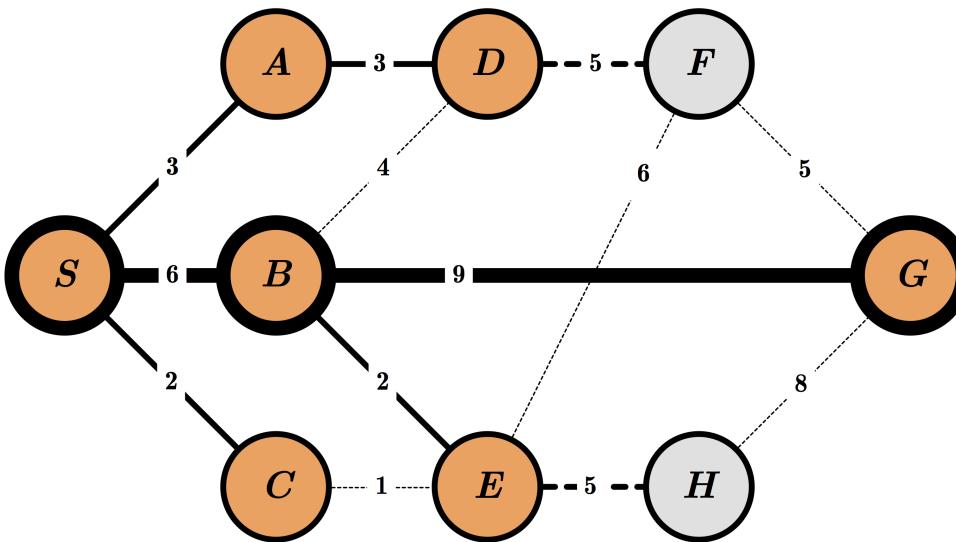
Queue:

| | | | | | | | | |
|---|---|---|---|---|---|----------|---|---|
| S | A | B | C | D | E | G | F | H |
|---|---|---|---|---|---|----------|---|---|

Order of Visit:

S A B C D E G

Exercise: BFS



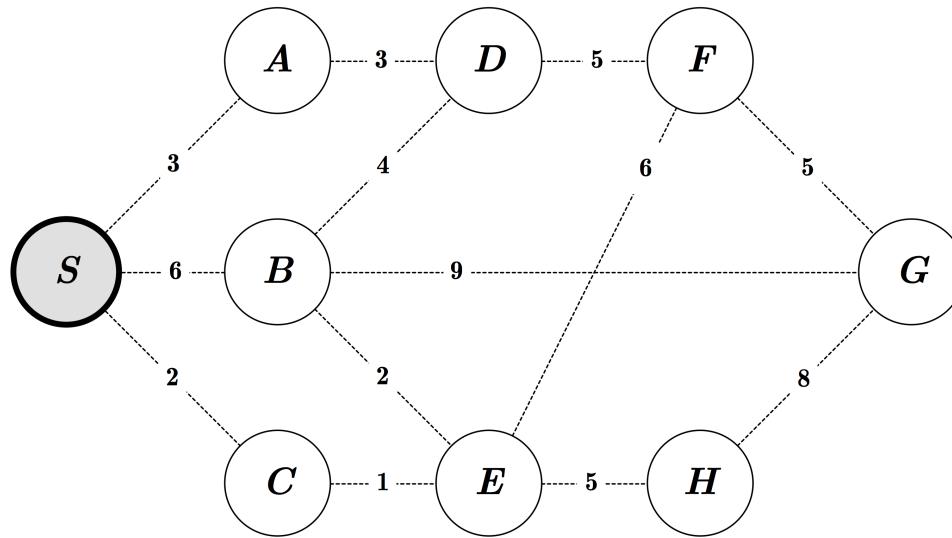
Queue:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| S | A | B | C | D | E | G | F | H |
|---|---|---|---|---|---|---|---|---|

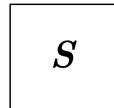
Order of Visit:

S A B C D E G

Exercise: DFS

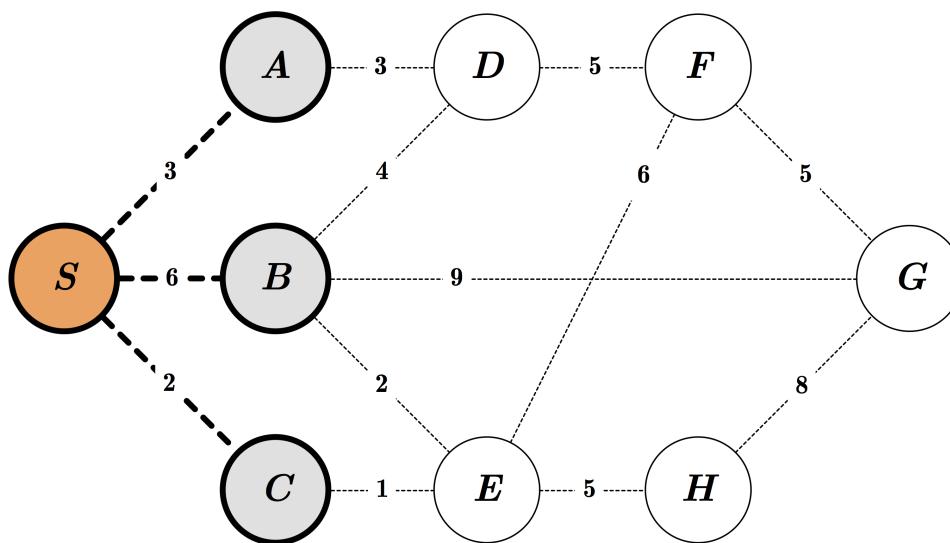


Stack:



Order of Visit:

Exercise: DFS



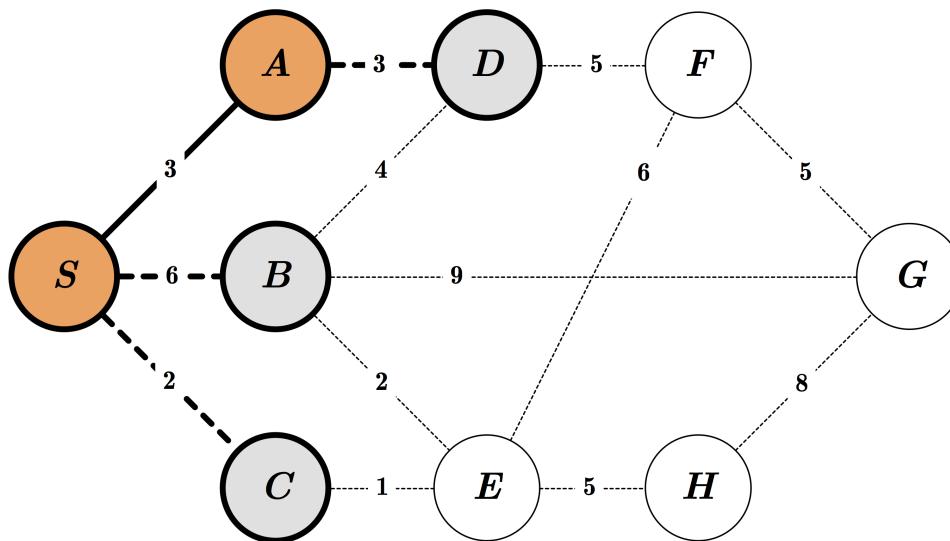
Stack:

| | | | |
|----------|---|---|---|
| S | C | B | A |
|----------|---|---|---|

Order of Visit:

S

Exercise: DFS



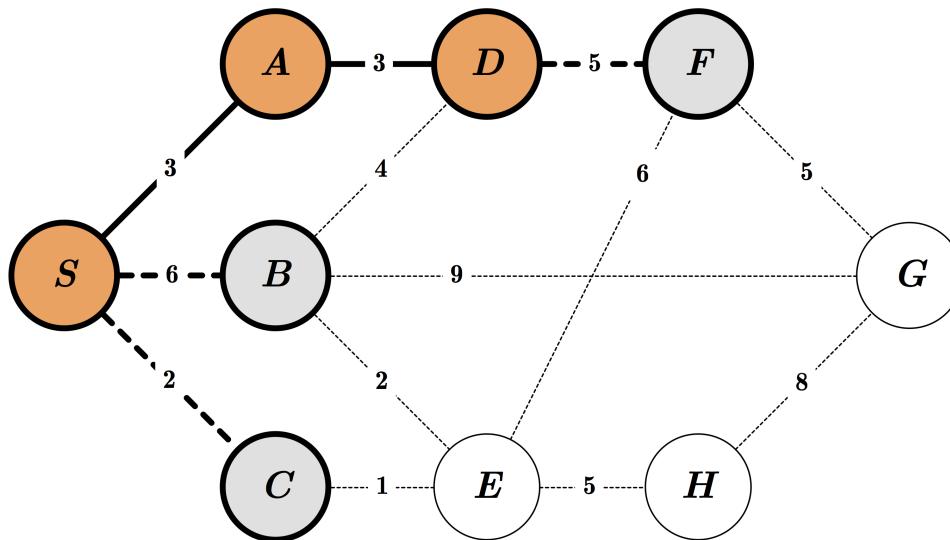
Stack:

| | | | | |
|----------|----------|----------|----------|----------|
| <i>S</i> | <i>C</i> | <i>B</i> | <i>A</i> | <i>D</i> |
|----------|----------|----------|----------|----------|

Order of Visit:

S A

Exercise: DFS



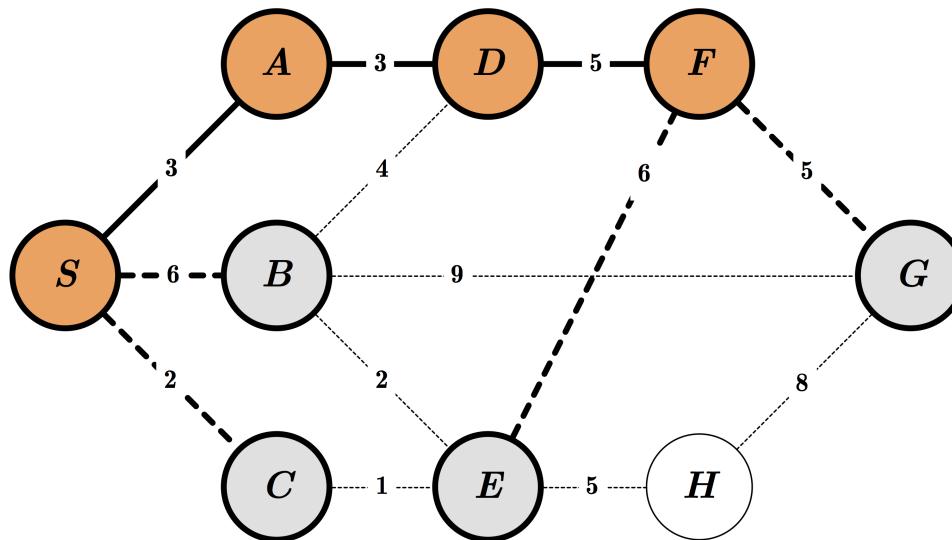
Stack:

| | | | | | |
|---|---|---|---|---|---|
| S | C | B | A | D | F |
|---|---|---|---|---|---|

Order of Visit:

S A D

Exercise: DFS



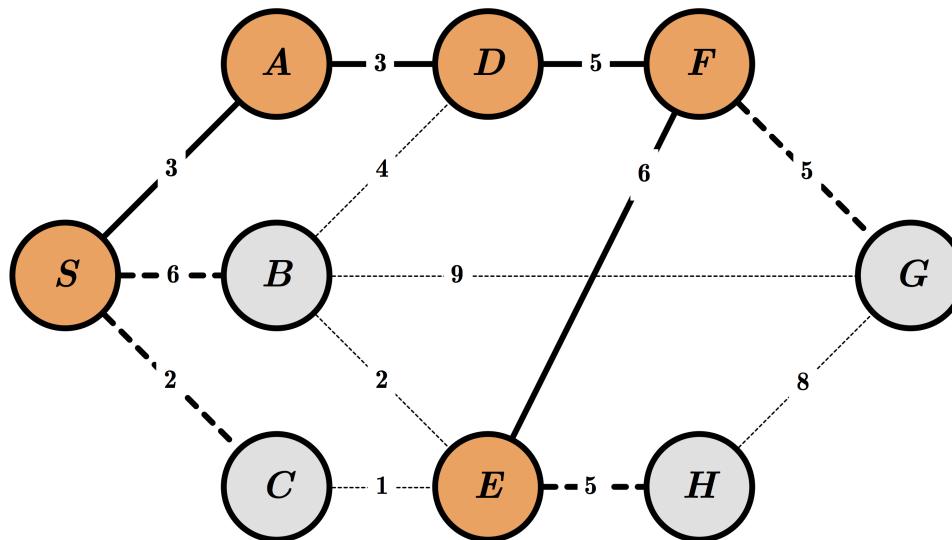
Stack:

| | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|
| <i>S</i> | <i>C</i> | <i>B</i> | <i>A</i> | <i>D</i> | <i>F</i> | <i>G</i> | <i>E</i> |
|----------|----------|----------|----------|----------|----------|----------|----------|

Order of Visit:

S A D F

Exercise: DFS



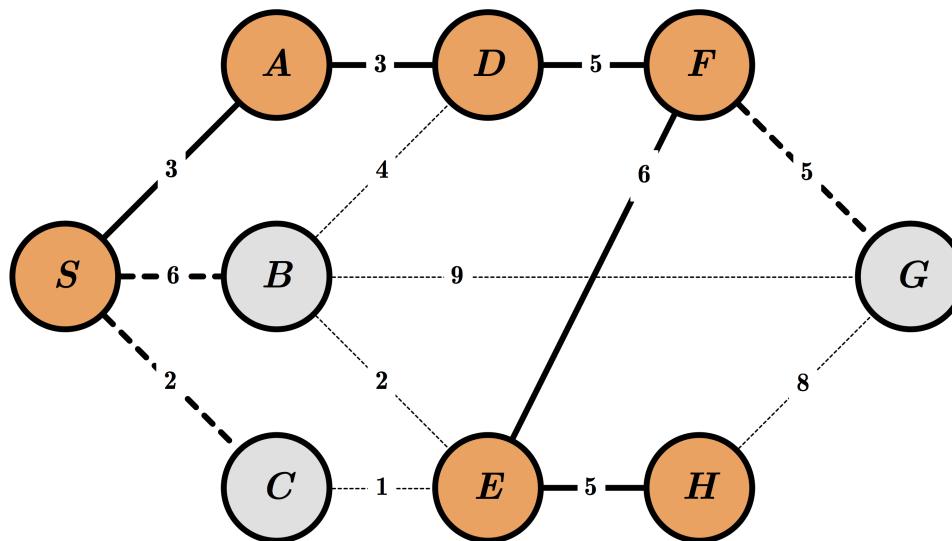
Stack:

| | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| <i>S</i> | <i>C</i> | <i>B</i> | <i>A</i> | <i>D</i> | <i>F</i> | <i>G</i> | <i>E</i> | <i>H</i> |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|

Order of Visit:

S A D F E

Exercise: DFS



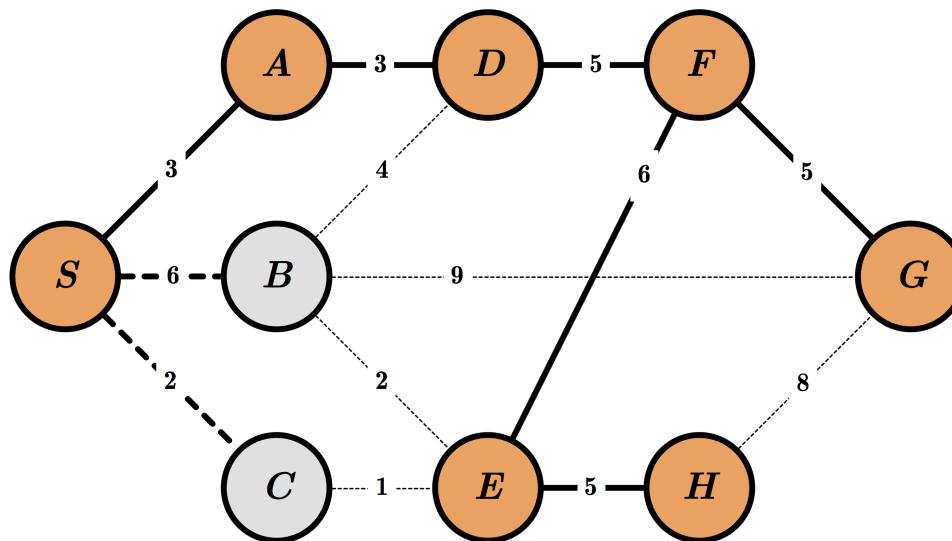
Stack:

| | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| <i>S</i> | <i>C</i> | <i>B</i> | <i>A</i> | <i>D</i> | <i>F</i> | <i>G</i> | <i>E</i> | <i>H</i> |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|

Order of Visit:

S A D F E H

Exercise: DFS



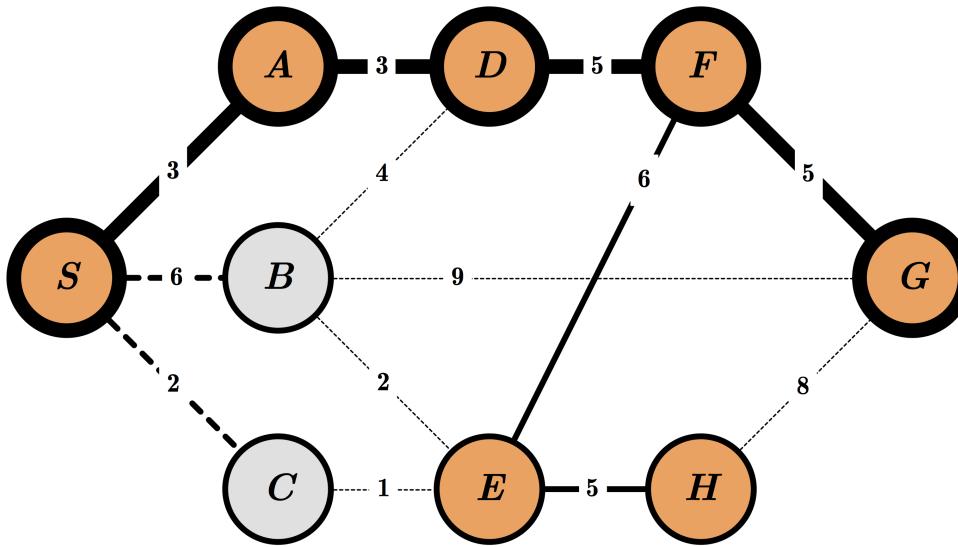
Stack:

| | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| <i>S</i> | <i>C</i> | <i>B</i> | <i>A</i> | <i>D</i> | <i>F</i> | <i>G</i> | <i>E</i> | <i>H</i> |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|

Order of Visit:

S A D F E H G

Exercise: DFS



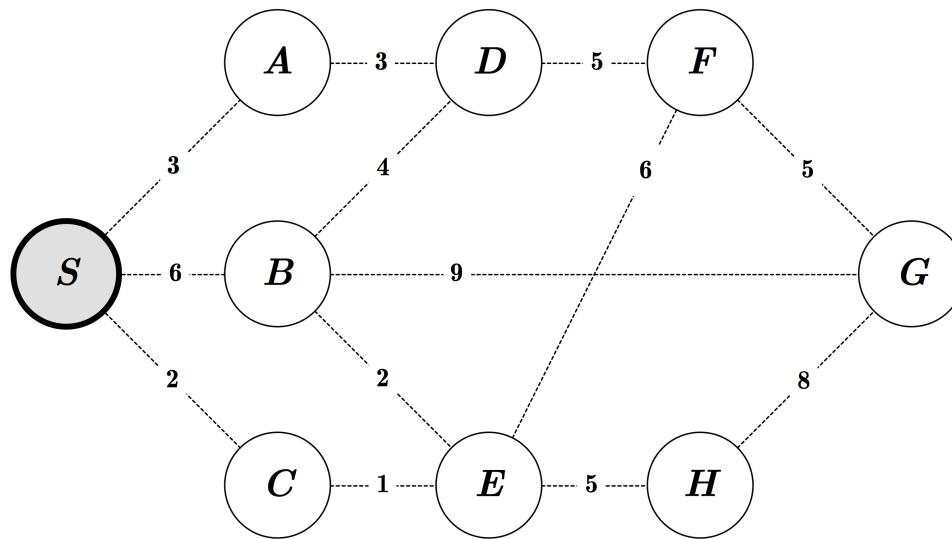
Stack:

| | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| <i>S</i> | <i>C</i> | <i>B</i> | <i>A</i> | <i>D</i> | <i>F</i> | <i>G</i> | <i>E</i> | <i>H</i> |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|

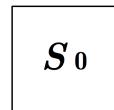
Order of Visit:

S A D F E H G

Exercise: UCS

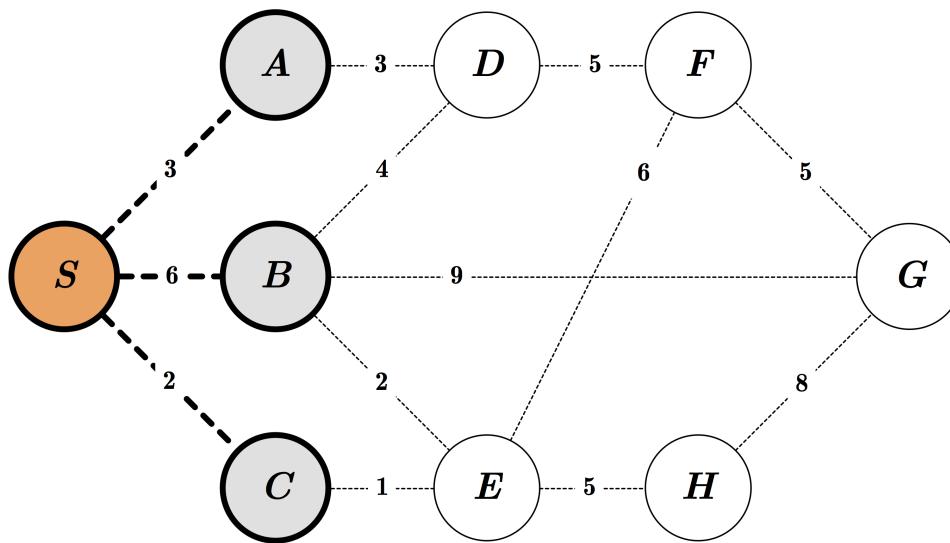


Priority Queue:



Order of Visit:

Exercise: UCS



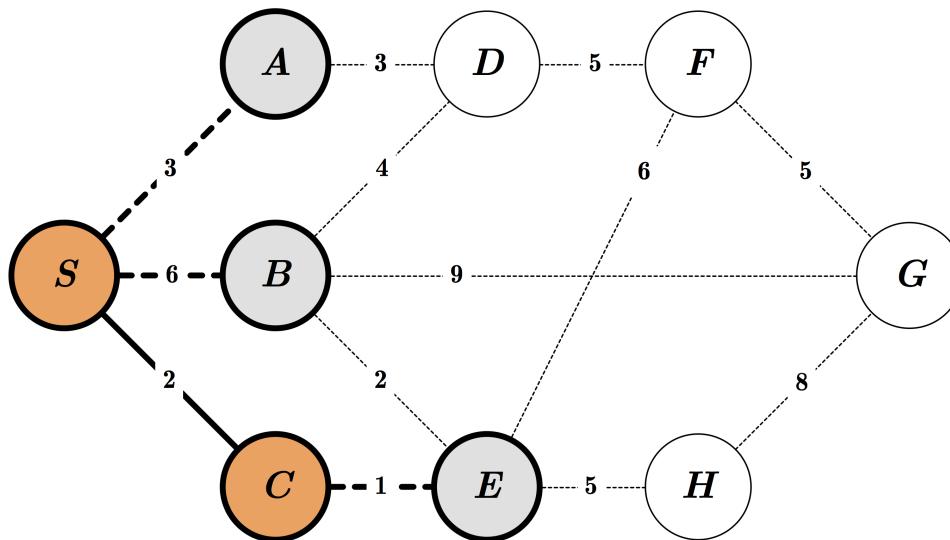
Priority Queue:

| | | | |
|---------|---------|---------|---------|
| $S \ 0$ | $C \ 2$ | $A \ 3$ | $B \ 6$ |
|---------|---------|---------|---------|

Order of Visit:

S

Exercise: UCS



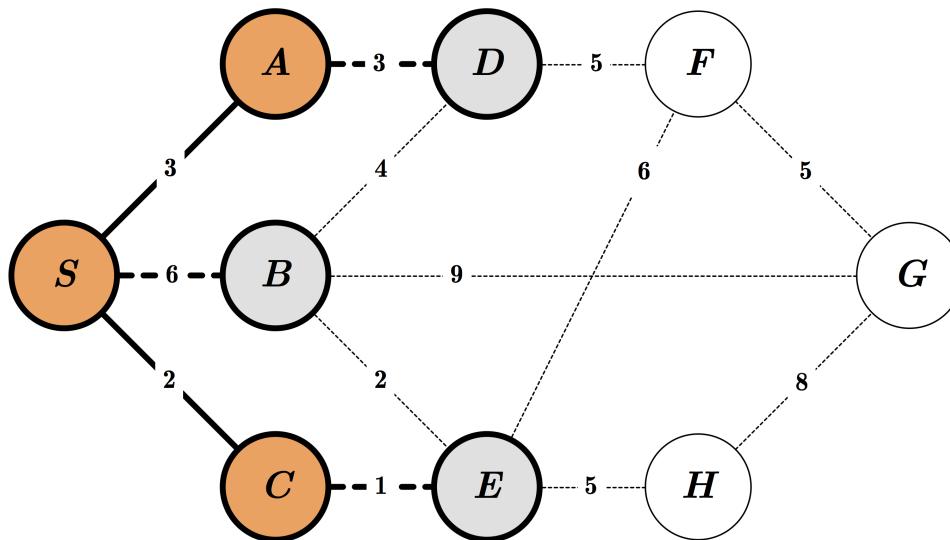
Priority Queue:

| | | | | |
|-------|-------|-------|-------|-------|
| S_0 | C_2 | A_3 | E_3 | B_6 |
|-------|-------|-------|-------|-------|

Order of Visit:

$S \quad C$

Exercise: UCS



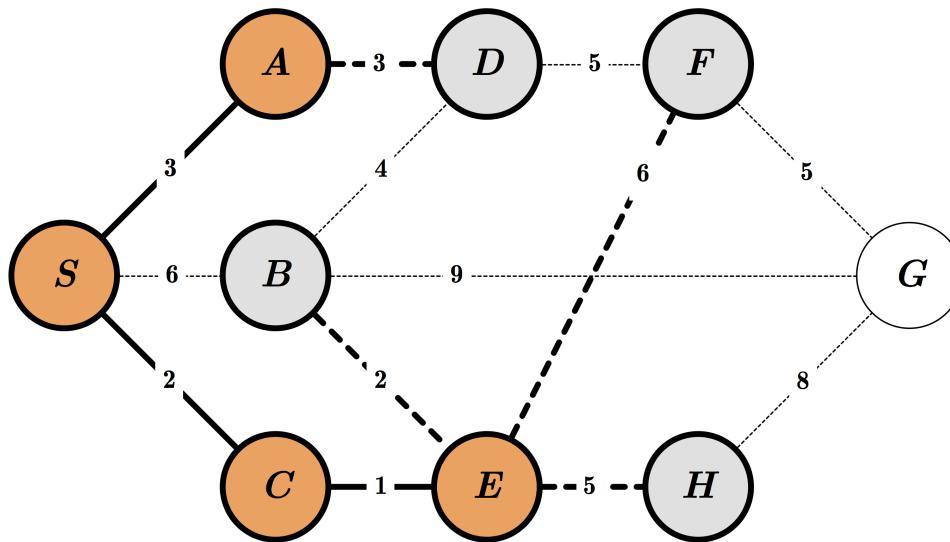
Priority Queue:

| | | | | | |
|-------|-------|-------|-------|-------|-------|
| S_0 | C_2 | A_3 | E_3 | B_6 | D_6 |
|-------|-------|-------|-------|-------|-------|

Order of Visit:

$S \quad C \quad A$

Exercise: UCS



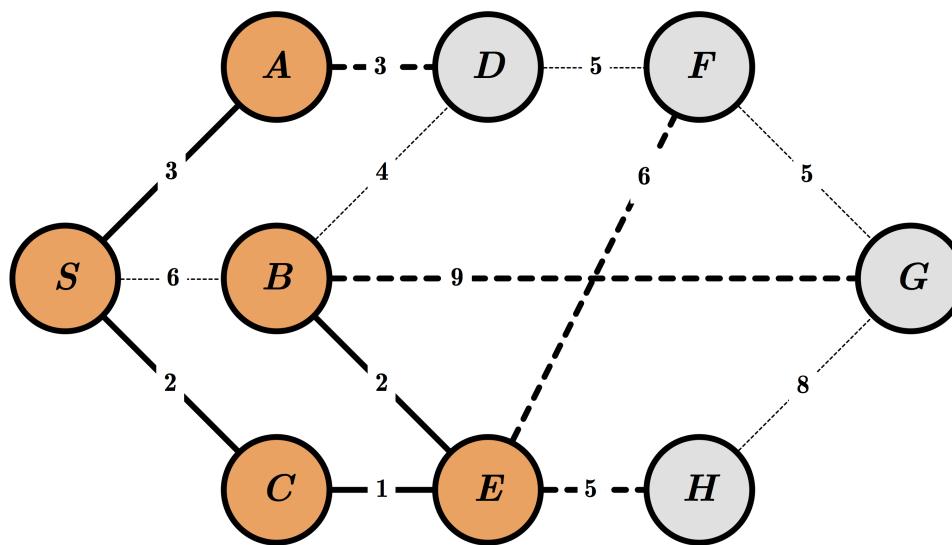
Priority Queue:

| | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|
| S_0 | C_2 | A_3 | E_3 | B_5 | D_6 | H_8 | F_9 |
|-------|-------|-------|-------|-------|-------|-------|-------|

Order of Visit:

$S \quad C \quad A \quad E$

Exercise: UCS



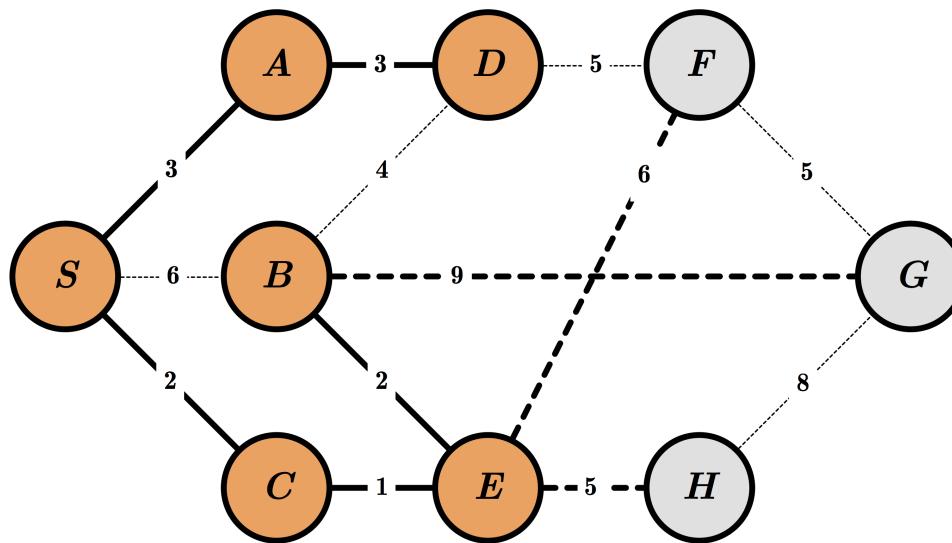
Priority Queue:

| | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| S_0 | C_2 | A_3 | E_3 | B_5 | D_6 | H_8 | F_9 | G_{14} |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|

Order of Visit:

$S \quad C \quad A \quad E \quad B$

Exercise: UCS



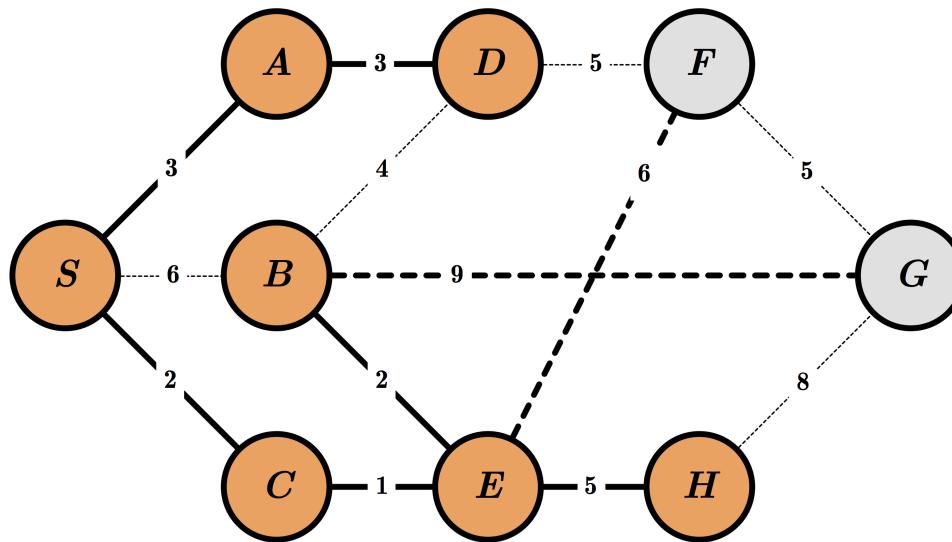
Priority Queue:

| | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| S_0 | C_2 | A_3 | E_3 | B_5 | D_6 | H_8 | F_9 | G_{14} |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|

Order of Visit:

$S \quad C \quad A \quad E \quad B \quad D$

Exercise: UCS



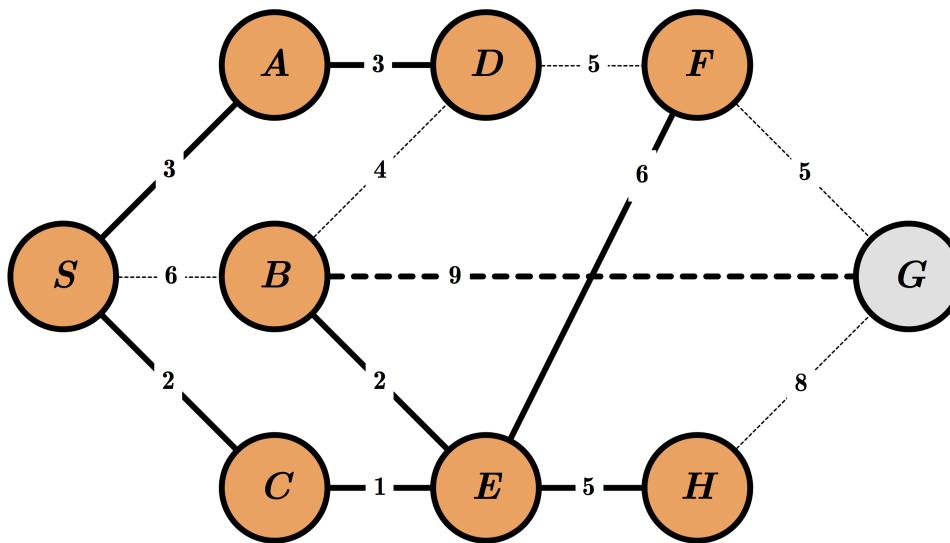
Priority Queue:

| | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| S_0 | C_2 | A_3 | E_3 | B_5 | D_6 | H_8 | F_9 | G_{14} |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|

Order of Visit:

$S \quad C \quad A \quad E \quad B \quad D \quad H$

Exercise: UCS



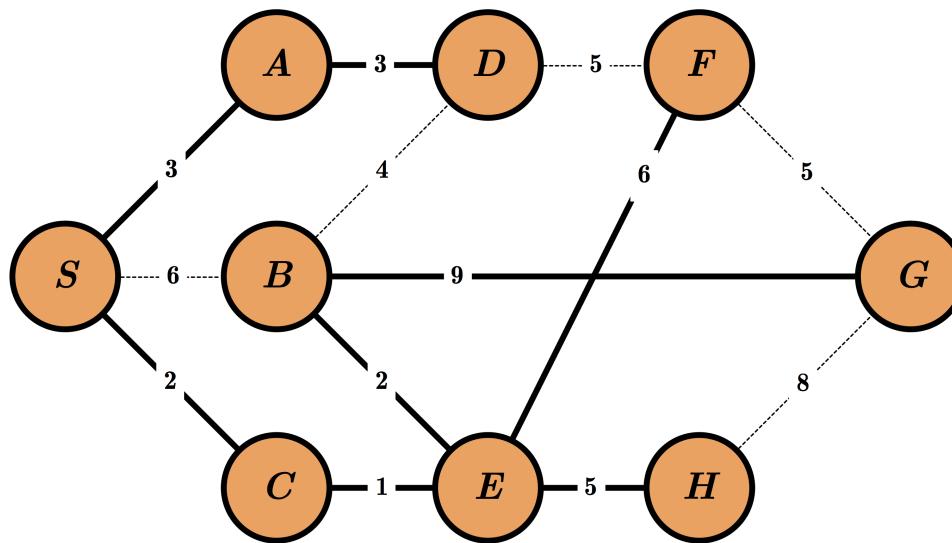
Priority Queue:

| | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| S_0 | C_2 | A_3 | E_3 | B_5 | D_6 | H_8 | F_9 | G_{14} |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|

Order of Visit:

$S \quad C \quad A \quad E \quad B \quad D \quad H \quad F$

Exercise: UCS



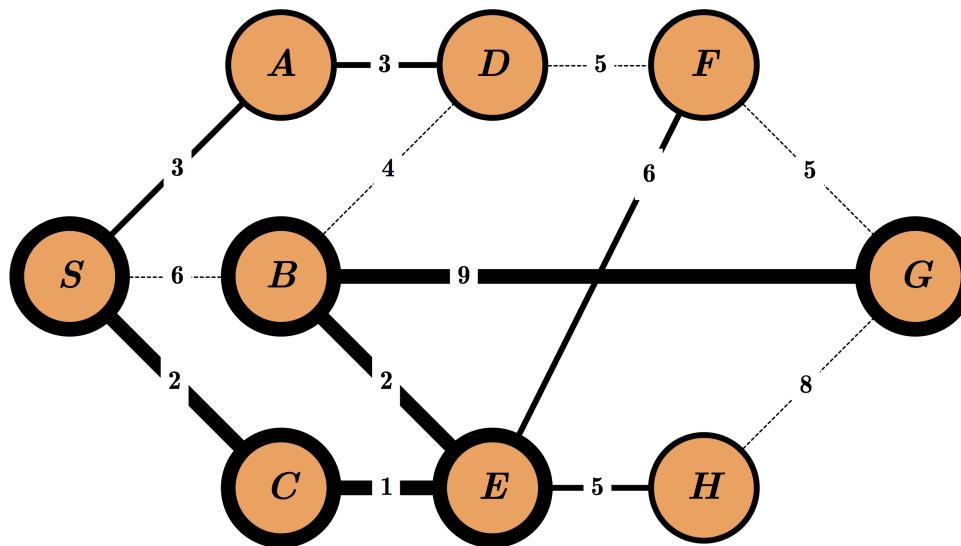
Priority Queue:

| | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| S_0 | C_2 | A_3 | E_3 | B_5 | D_6 | H_8 | F_9 | G_{14} |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|

Order of Visit:

$S \quad C \quad A \quad E \quad B \quad D \quad H \quad F \quad G$

Exercise: UCS



Priority Queue:

| | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| S_0 | C_2 | A_3 | E_3 | B_5 | D_6 | H_8 | F_9 | G_{14} |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|

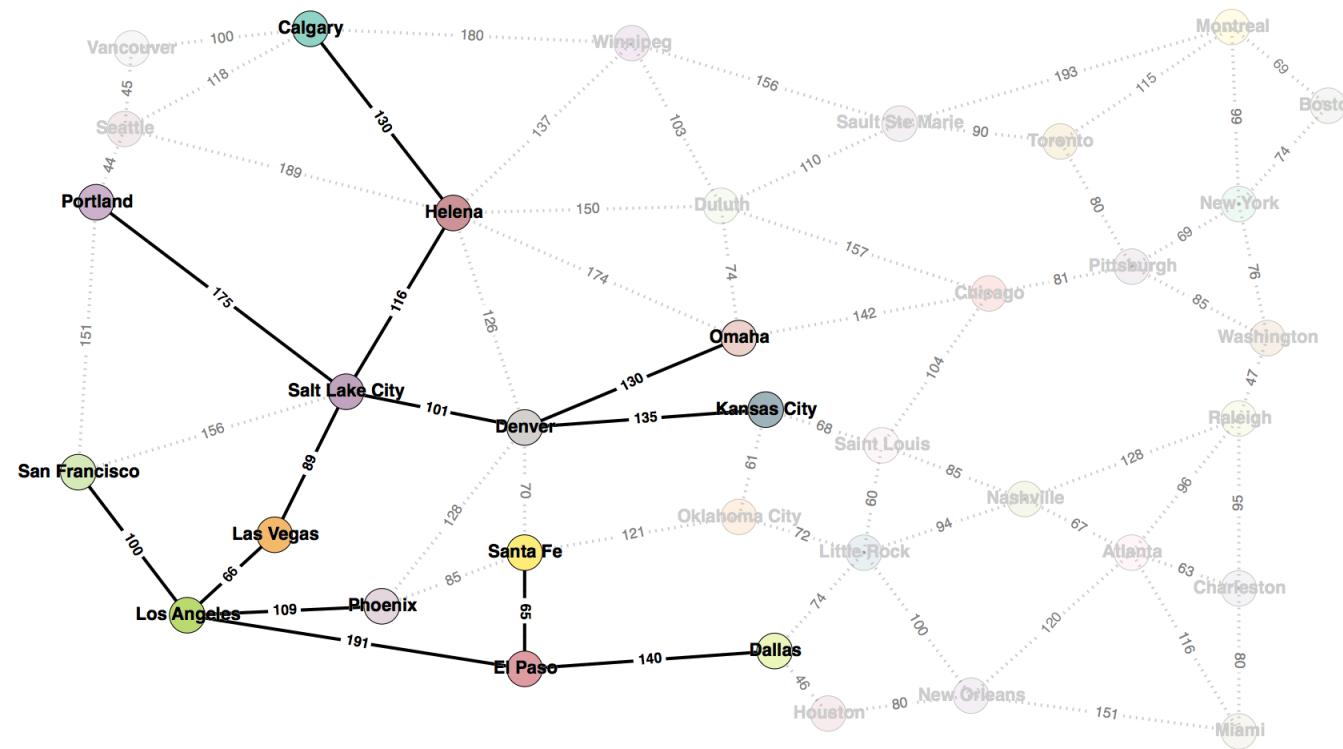
Order of Visit:

$S \quad C \quad A \quad E \quad B \quad D \quad H \quad F \quad G$

Examples using the map

Start: Las Vegas

Goal: Calgary



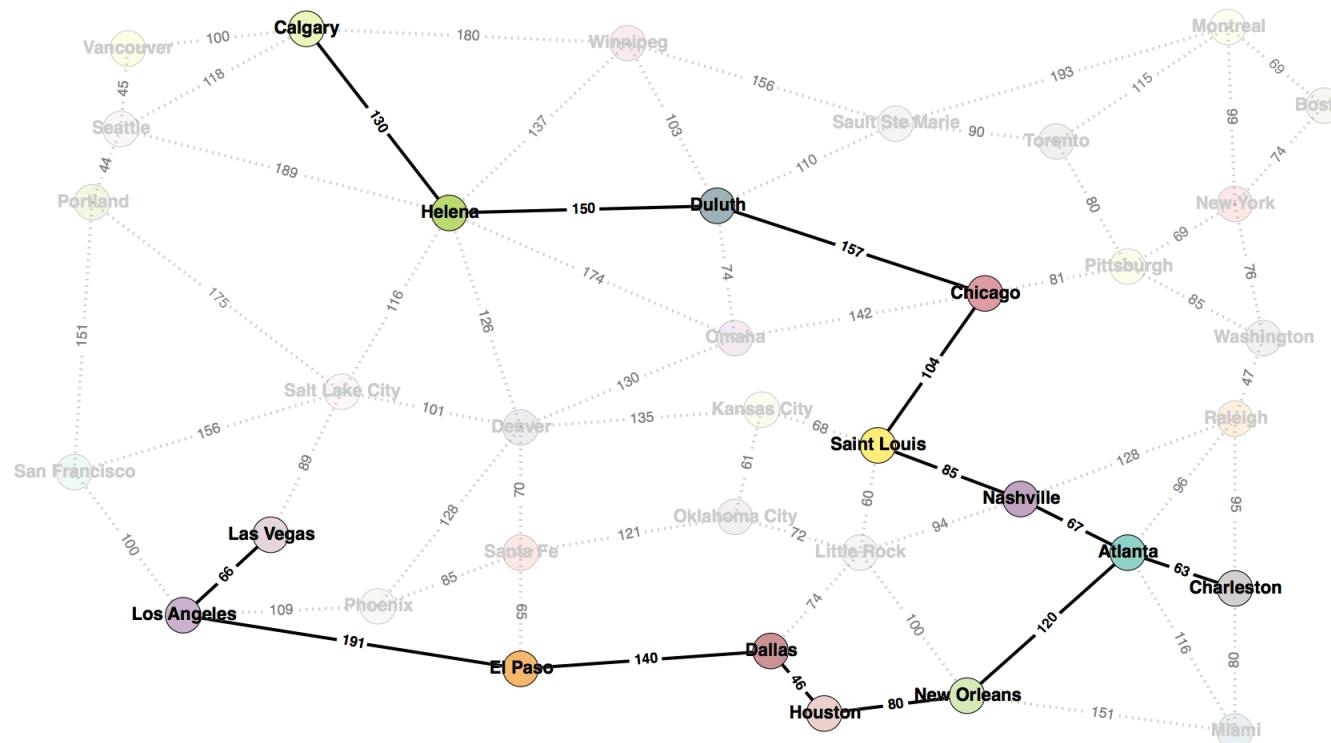
BFS

Order of Visit: Las Vegas, Los Angeles, Salt Lake City, El Paso, Phoenix, San Francisco, Helena, Portland, Dallas, Santa Fe, Kansas City, Omaha, Calgary.

Examples using the map

Start: Las Vegas

Goal: Calgary



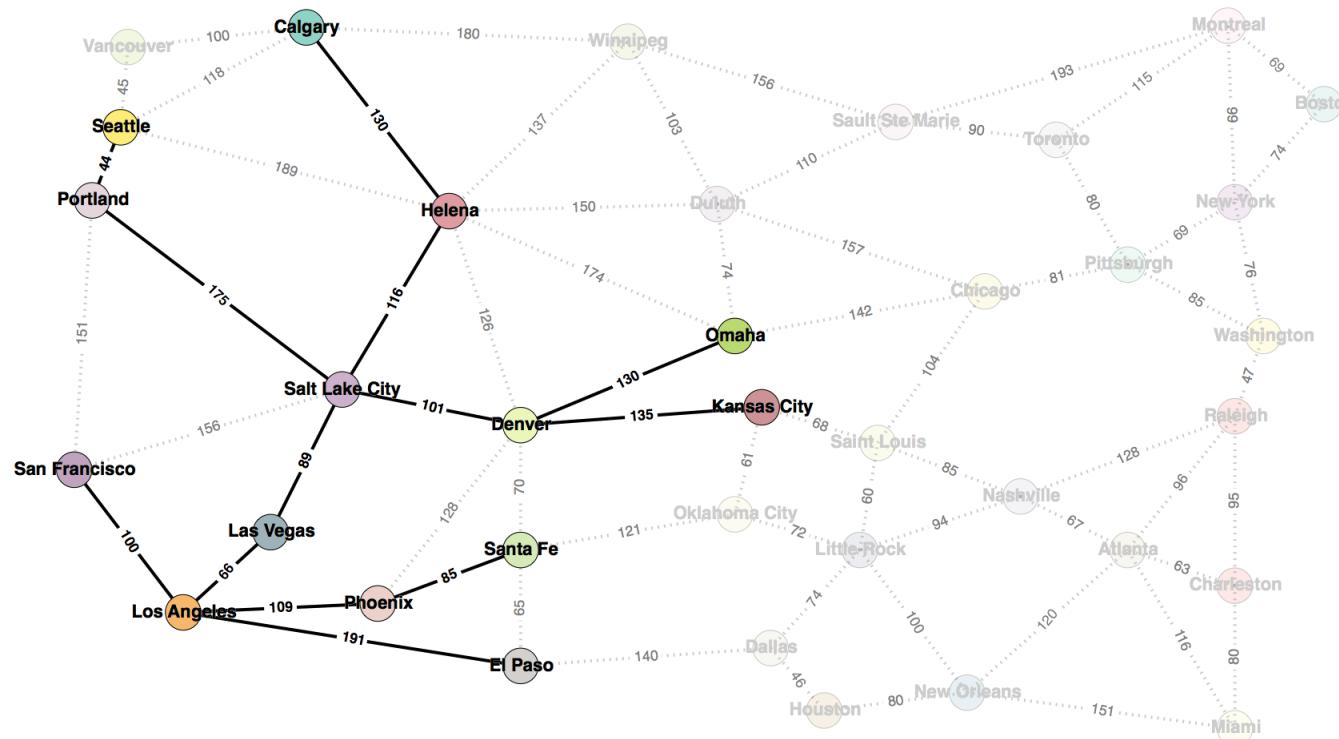
DFS

Order of Visit: Las Vegas, Los Angeles, El Paso, Dallas, Houston, New Orleans, Atlanta, Charleston, Nashville, Saint Louis, Chicago, Duluth, Helena, Calgary.

Examples using the map

Start: Las Vegas

Goal: Calgary



UCS

Order of Visit: Las Vegas, Los Angeles, Salt Lake City, San Francisco, Phoenix, Denver, Helena, El Paso, Santa Fe, Portland, Seattle, Omaha, Kansas City, Calgary.

Credit

- Artificial Intelligence, A Modern Approach. Stuart Russell and Peter Norvig. Third Edition. Pearson Education.

<http://aima.cs.berkeley.edu/>