

Vivekanand Education Society's Institute of Technology, Chembur, Mumbai,
Department Of Artificial Intelligence and Data Science,
Year:2023-24 (ODD Sem)
MID TERM TEST

Class : D16AD	Division: -
Semester: VII	Subject: Game Theory for Data Science
Date: 8 th Sept, 2023	Time: 10:30 AM - 11:30 AM

Q.1)	(Attempt any five of the following)	Marks (20)
a)	How does game theory apply to the competition between two leading smartphone manufacturers, Apple, and Samsung, and what are the essential components of this strategic game?	2M
Ans	<p>Certainly, here is a different perspective on how game theory applies to the competition between Apple and Samsung in the smartphone market, along with the essential components of this strategic game:</p> <ol style="list-style-type: none"> Players: <ul style="list-style-type: none"> - Apple Inc. - Samsung Electronics Strategies: <p>Each player has a range of strategies at their disposal. These strategies can include product development, pricing, marketing, distribution, and innovation. For example, Apple may decide to focus on product differentiation and premium pricing, while Samsung may opt for a wider range of products and competitive pricing.</p> Payoffs: <p>Payoffs represent the outcomes or rewards that each player receives based on the combination of strategies they choose and the responses of their competitor. Payoffs can be measured in terms of market share, sales revenue, profit margins, brand loyalty, and overall market dominance.</p> Information and Uncertainty: <p>Information plays a crucial role in this strategic game. Both Apple and Samsung need to consider each other's past actions, market trends, customer preferences, and potential future moves when making their strategic decisions. Lack of information or misinformation can lead to suboptimal choices.</p> Game Dynamics: <p>The smartphone industry is characterized by rapid technological advancements, shorter product life cycles, and changing consumer demands. This dynamism requires constant adaptation and innovation from both players.</p> Nash Equilibrium: <p>In this game, the goal for each player is to reach a Nash equilibrium, where neither player can improve their payoff by unilaterally changing their strategy. It represents a stable point where both companies have found the best response to each other's actions.</p> 	
b)	What role does the ad quality score play in online advertising auctions, and how do advertisers strategically manipulate factors contributing to this score to gain a competitive advantage in the bidding process?	2M
Ans.	<p>The ad quality score plays a crucial role in online advertising auctions, particularly in the context of platforms like Google Ads. It helps determine the positioning and cost of an advertisement in the ad placement auction. Advertisers can strategically manipulate factors contributing to this score to gain a competitive advantage in the bidding process.</p> <p>Ad Quality Score Role:</p>	

	<ul style="list-style-type: none"> - Ad Quality Score is a metric used by online advertising platforms to assess the relevance and quality of an ad. It considers factors such as click-through rate (CTR), ad relevance, and landing page experience. - Advertisers with higher Ad Quality Scores may receive better ad placements and pay lower costs per click (CPC) compared to advertisers with lower scores. - It encourages advertisers to create high-quality, relevant ads that are more likely to provide value to users. <p>Strategic Manipulation:</p> <ul style="list-style-type: none"> - Advertisers strategically manipulate Ad Quality Score by optimizing their ad campaigns. - They improve CTR by crafting compelling ad copy and using relevant keywords. - Advertisers ensure that ad content aligns with user search queries to enhance ad relevance. - Landing pages are optimized for user experience and relevance, leading to a positive landing page experience score. - By consistently optimizing these factors, advertisers can achieve higher Ad Quality Scores, which can lead to improved ad positioning and lower advertising costs, providing a competitive advantage. <p>Advertisers aim to strike a balance between bid amount and Ad Quality Score to achieve the best return on investment in online advertising auctions.</p>	
c)	In a business context, can you elucidate how payoffs, as defined in game theory, were instrumental in shaping the outcome of a competitive bidding process among suppliers, and what specific factors influenced the determination of these payoffs?	2M
Ans.	<p>In a competitive bidding process among suppliers within a business context, payoffs, as defined in game theory, play a crucial role in shaping the outcome. Payoffs represent the rewards or gains that each supplier receives based on their strategic choices and the choices of their competitors. Here are four key factors that influence the determination of these payoffs:</p> <ol style="list-style-type: none"> 1. Bid Amount: The primary factor influencing payoffs is the bid amount submitted by each supplier. Higher bids typically lead to larger costs for the buyer, while lower bids may result in reduced profit margins for the supplier. The payoff for a supplier depends on whether their bid is successful or not, which in turn depends on the comparison of their bid amount with those of other competitors. 2. Market Demand: Payoffs are influenced by the underlying market demand for the product or service being procured. If the market demand is high, suppliers may have higher payoffs as they can secure more contracts and potentially charge higher prices. Conversely, in a low-demand market, payoffs may be lower due to reduced opportunities for winning contracts and competitive pricing pressures. 3. Competitive Strategies: The strategic choices made by suppliers also impact payoffs. Suppliers can adopt various strategies, such as aggressive bidding to win more contracts, or conservative bidding to maintain profit margins. Payoffs depend on how well a supplier's strategy aligns with the strategies of their competitors. For instance, a supplier employing a competitive bidding strategy may secure more contracts but with thinner profit margins. 4. Supplier Reputation and Quality: Payoffs can be affected by a supplier's reputation and the quality of their products or services. Suppliers with a strong track record of delivering high-quality goods or services may be able to command higher prices and win contracts with greater frequency. A positive reputation can lead to higher payoffs by increasing the likelihood of being selected by the buyer. <p>Therefore, payoffs in a competitive bidding process among suppliers are determined by bid amounts, market demand, competitive strategies, and supplier reputation and quality. These factors interact to shape the outcomes of the bidding process, influencing which suppliers win contracts and how much they gain from participating in the competition. Game theory provides a valuable framework for analyzing and understanding these dynamics in a business context.</p>	
d)	Can you describe a real-world scenario where athletes participated in a strictly competitive event, like the Olympics, and how did their preparation and performance reflect the nature of competitive games?	2M
Ans.	Certainly, here is another real-world scenario involving athletes participating in the Olympics, emphasizing how their preparation and performance reflect the nature of competitive games: Scenario: Olympic Gymnastics	

	<p>In the Olympic gymnastics event, particularly the uneven bars routine, athletes demonstrate the essence of competitive games through their meticulous preparation and high-pressure performances.</p> <ol style="list-style-type: none"> 1. Specialized Training: Gymnasts invest countless hours in specialized training, focusing on mastering their routines with precision and grace. They must excel in acrobatics, balance, and artistic flair. This level of dedication highlights the competitive nature of the event, as athletes aim to outperform their rivals through years of rigorous training. 2. Technical Mastery: The uneven bars routine demands an intricate series of movements, including flips, twists, and intricate handwork. Athletes must execute these maneuvers flawlessly to impress judges and outscore their competitors. The competitive aspect is evident as gymnasts strive for technical perfection in the face of intense competition. 3. Scoring and Judging: Competitive games often involve subjective judgment, and gymnastics is no exception. Judges evaluate each routine based on specific criteria, including execution, artistry, and difficulty. This competitive element adds an extra layer of pressure as athletes strive to earn the highest scores through impeccable performances, sometimes with minimal room for error. 4. High-Stakes Performances: Athletes in the uneven bars event understand that they have only one shot to showcase their skills on the Olympic stage. The competitive nature of the event is magnified by the high stakes and global audience. The pressure to perform flawlessly and outshine competitors is intense, highlighting the essence of competitive games where athletes must rise to the occasion. <p>In this scenario, the Olympic gymnastics uneven bars event exemplifies the competitive nature of sports. Athletes' specialized training, technical mastery, the influence of subjective judgment, and high-pressure performances all reflect the essence of competitive games at the highest level of international competition.</p>	
e)	<p>In the world of video games, could you discuss a scenario where players had dominant strategies in multiplayer games, and how did these strategies lead to a dominant strategy equilibrium or a Nash Equilibrium in the gameplay experience?</p>	2M
Ans.	<p>In the world of video games, a classic scenario where players often exhibit dominant strategies leading to a dominant strategy equilibrium or a Nash Equilibrium can be found in a simplified example from the game "Rock, Paper, Scissors."</p> <p>Scenario: Consider a multiplayer video game adaptation of "Rock, Paper, Scissors" where two players simultaneously choose one of three actions: "Rock," "Paper," or "Scissors." Each action has its own outcome when compared to the other player's choice: Rock beats Scissors, Scissors beats Paper, and Paper beats Rock.</p> <p>In this scenario:</p> <ul style="list-style-type: none"> - Player 1 chooses an action (Rock, Paper, or Scissors). - Player 2 also chooses an action (Rock, Paper, or Scissors). <p>Now, let us examine the dominant strategies and the resulting Nash Equilibrium:</p> <p>Dominant Strategies:</p> <ul style="list-style-type: none"> - If Player 1 chooses "Rock," Player 2's dominant strategy is to choose "Paper" since Paper beats Rock. - If Player 1 chooses "Paper," Player 2's dominant strategy is to choose "Scissors" since Scissors beats Paper. - If Player 1 chooses "Scissors," Player 2's dominant strategy is to choose "Rock" since Rock beats Scissors. <p>Similarly, for Player 1:</p> <ul style="list-style-type: none"> - If Player 2 chooses "Rock," Player 1's dominant strategy is to choose "Paper." - If Player 2 chooses "Paper," Player 1's dominant strategy is to choose "Scissors." - If Player 2 chooses "Scissors," Player 1's dominant strategy is to choose "Rock." <p>Nash Equilibrium:</p> <p>In this simplified scenario, we observe a Nash Equilibrium when both players employ their dominant strategies:</p> <ul style="list-style-type: none"> - Player 1 chooses "Paper." - Player 2 chooses "Scissors." <p>In this equilibrium, neither player has an incentive to change their strategy. If Player 1 deviates from choosing "Paper," they will lose to Player 2's "Scissors." Likewise, if Player 2 deviates from choosing "Scissors," they will lose to Player 1's "Paper." Thus, both players are "locked in" to their dominant strategies, resulting in a Nash Equilibrium.</p> <p>This equilibrium means that, in this simplified version of the game, the gameplay experience can become somewhat predictable because both players are essentially following a dominant</p>	

		strategy, and the outcome depends on the initial choices. Such dominant strategy equilibriums can simplify gameplay dynamics but may lead to repetitive or less engaging experiences. Game designers often introduce elements of uncertainty or complexity to encourage more varied and strategic gameplay.	
f)		What are some common examples or contexts where games with imperfect information are commonly encountered, and how do these scenarios illustrate the differences between imperfect and perfect information games in practical terms?	2M
Ans.		<p>Common examples or contexts where games with imperfect information are commonly encountered include:</p> <ol style="list-style-type: none"> 1. Card Games: Games like poker involve imperfect information because players don't know the exact cards held by their opponents. Each player has private information, and their decisions are influenced by the partial information available. 2. Auctions: Auctions, especially sealed-bid auctions, often involve imperfect information. Bidders typically have limited information about the valuations and strategies of other participants, which impacts their bidding decisions. 3. Business Negotiations: Negotiations in business, such as mergers and acquisitions or contract negotiations, often occur with imperfect information. Each party has limited knowledge about the other's financial situation, preferences, or negotiation tactics. 4. Military Strategies: In warfare, commanders often operate with incomplete information about the enemy's troop movements, capabilities, and intentions. Decisions must be made based on estimates and intelligence. <p>Practical Differences:</p> <ul style="list-style-type: none"> - Imperfect Information: In these scenarios, decision-makers lack complete information about the situation or the strategies of others. This leads to uncertainty and requires players to make decisions based on assumptions, predictions, and risk assessments. Strategies often involve bluffs, deception, and cautious decision-making to mitigate uncertainty. - Perfect Information: In contrast, perfect information games (like chess) involve complete knowledge of the game state. Players can see all the moves made and the entire game board. Decisions are based on logical analysis and can be planned several moves ahead without uncertainty about the opponent's actions. <p>Hence, imperfect information games introduce an element of unpredictability and strategic complexity, as players must make choices without knowing the full context or the strategies of their opponents. This often leads to more dynamic and strategic gameplay compared to perfect information games, where outcomes are entirely determined by the decisions and skill of the players without hidden information.</p>	
Q.2)	a)	What are some common real-world situations or domains where cooperative games are often encountered, and how does cooperation among participants lead to the pursuit of collective objectives or benefits?	5M
Ans		<p>Cooperative games are often encountered in various real-world situations and domains where participants work together to pursue collective objectives or benefits. Here are some common examples:</p> <ol style="list-style-type: none"> 1. Business Partnerships: Companies may form strategic partnerships or alliances to share resources, technology, or market access. Cooperation among partners can lead to mutual growth, increased market share, and cost efficiencies. 2. Research Collaborations: In academia and industry, researchers from different institutions or organizations often collaborate on projects. This cooperation allows them to pool their expertise, resources, and data to achieve scientific breakthroughs or develop innovative solutions. 3. Sports Teams: Team sports, such as soccer, basketball, and football, require cooperation among players to achieve victory. Effective teamwork, coordination, and communication are essential to pursuing the collective objective of winning the game. 4. Global Climate Agreements: International efforts to address climate change, such as the Paris Agreement, involve cooperation among countries. Nations work together to reduce greenhouse gas emissions and mitigate the effects of climate change for the benefit of the global environment. 5. Nonprofit Organizations: Charitable organizations and NGOs often collaborate to address complex social issues, such as poverty, healthcare, and education. Cooperation allows them to combine resources and expertise to make a more significant impact. 	

	<p>6. Military Alliances: Countries form military alliances, like NATO, to enhance their collective defense capabilities. Cooperation among member states helps deter potential threats and promotes stability.</p> <p>7. Supply Chain Management: In supply chain networks, manufacturers, suppliers, and distributors cooperate to streamline operations, reduce costs, and meet customer demands efficiently. Cooperation in the supply chain leads to improved product availability and customer satisfaction.</p> <p>8. Community Projects: Local communities and volunteers may cooperate to undertake projects like building parks, organizing festivals, or cleaning up neighborhoods. Cooperation fosters a sense of community and improves the quality of life for residents.</p> <p>Cooperation among participants in these scenarios leads to the pursuit of collective objectives or benefits through several mechanisms:</p> <ol style="list-style-type: none"> 1. Resource Sharing: Participants combine their resources, whether it's financial capital, knowledge, technology, or labor, to achieve goals that would be challenging or impossible to attain individually. 2. Specialization: Cooperation allows participants to specialize in their areas of expertise, optimizing their contributions to the collective effort and increasing overall efficiency. 3. Risk Sharing: Participants can share risks and uncertainties associated with the pursuit of objectives. This shared responsibility helps mitigate individual losses and enhances the likelihood of success. 4. Economies of Scale: Cooperation often leads to economies of scale, reducing per-unit costs and increasing the efficiency of production or service delivery. 5. Synergy: Collaboration often creates synergistic effects, where the collective output is greater than the sum of individual contributions. This results in enhanced outcomes and benefits. <p>In these cooperative contexts, participants recognize that working together can lead to outcomes that are superior or more sustainable than what they could achieve independently, making cooperation a vital strategy for pursuing collective objectives and reaping mutual benefits.</p>										
	OR										
b)	In terms of Nash equilibria, can you find and interpret the Nash equilibrium in the Matching Pennies game, and contrast it with the Nash equilibrium in the Prisoner's Dilemma, highlighting how each reflects rational decision-making by the players?	5M									
Ans	<p>Let us analyze the Nash equilibrium in both the Matching Pennies game and the Prisoner's Dilemma and contrast how each reflects rational decision-making by the players.</p> <p>Matching Pennies Game:</p> <p>In the Matching Pennies game, there are two players, Player A and Player B. Each player can choose to show either a "heads" or a "tails." The payoffs are as follows:</p> <ul style="list-style-type: none"> - If both players choose the same outcome (either both "heads" or both "tails"), Player A wins and receives a payoff of +1, while Player B loses and receives a payoff of -1. - If the players choose different outcomes, Player A loses and receives a payoff of -1, while Player B wins and receives a payoff of +1. <p>The Nash equilibrium in this game occurs when both players randomly choose between "heads" and "tails" with equal probability. This means that Player A has a 50% chance of choosing "heads" and a 50% chance of choosing "tails," and Player B does the same.</p> <p>Interpretation of Nash Equilibrium in Matching Pennies:</p> <p>In this Nash equilibrium, neither player has an incentive to unilaterally change their strategy. If Player A decides to always choose "heads," Player B can exploit this by always choosing "tails" to win consistently, and vice versa. Therefore, both players choose their strategies randomly to make it impossible for the opponent to predict their choices. This reflects rational decision-making as each player is minimizing their potential losses and maximizing their chances of winning over the long run.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td><td>Player 2 (Heads)</td><td>Player 2 (Tails)</td></tr> <tr> <td>Player 1 (Heads)</td><td>+1</td><td>-1</td></tr> <tr> <td>Player 1 (Tails)</td><td>-1</td><td>+1</td></tr> </table>		Player 2 (Heads)	Player 2 (Tails)	Player 1 (Heads)	+1	-1	Player 1 (Tails)	-1	+1	
	Player 2 (Heads)	Player 2 (Tails)									
Player 1 (Heads)	+1	-1									
Player 1 (Tails)	-1	+1									

Prisoner's Dilemma:

In the Prisoner's Dilemma, there are two players who are facing criminal charges, and they have the choice to either cooperate (remain silent) or betray (confess). The payoffs are as follows:

- If both players cooperate, they both receive a relatively low sentence, let us say 2 years each.
- If one player cooperates while the other betrays, the betrayer goes free (0 years) while the cooperator receives a severe sentence, say 5 years.
- If both players betray, they both receive a moderately long sentence, say 4 years each.

The Nash equilibrium in the Prisoner's Dilemma occurs when both players choose to betray (confess).

		Accused 2	
		Cooperate (remain silent)	Confess
Accused 1	Cooperate (remain silent)	(1, 1)	(5, 0)
	Confess	(0, 5)	(3, 3)

Interpretation of Nash Equilibrium in Prisoner's Dilemma:

In the Nash equilibrium of betraying, each player makes the rational decision to minimize their potential sentence, assuming the other player will also betray. However, this results in a suboptimal outcome for both players because if they both cooperated, they would have received a lower combined sentence (4 years instead of 8 years). The dilemma arises because each player's rational self-interest leads them to betray, even though cooperation would lead to a better overall outcome. This illustrates the tension between individual rationality and collective rationality in the context of the Prisoner's Dilemma.

Q.3)

a)

In the Matching Pennies game, how do players utilize mixed strategies to achieve a Nash equilibrium, and what is the significance of these mixed strategies in the context of the game's payoffs and outcomes?

5M

In the Matching Pennies game, players utilize mixed strategies to achieve a Nash equilibrium. Mixed strategies involve players randomizing their choices with specific probabilities. Here's how players use mixed strategies and the significance of these strategies in the context of the game's payoffs and outcomes:

1. Mixed Strategies in Matching Pennies:

Ans

let's solve above eq.
Find best response for each player.

player 1
play H with probability (p) &
" T " " " (1-p)

player 2
play H with probability (q)
" T " " " (1-q)

Finding BR for P1:

$$U_1(H, \sigma_2) = (+1)q + (-1)(1-q) = 2q - 1$$
$$U_1(T, \sigma_2) = (-1)q + (+1)(1-q) = 1 - 2q$$

If $U_1(H, \sigma_2) > U_1(T, \sigma_2) \Rightarrow H \text{ is BR. } (p=1)$

If $U_1(H, \sigma_2) < U_1(T, \sigma_2) \Rightarrow T \text{ is BR. } (p=0)$

If $U_1(H, \sigma_2) = U_1(T, \sigma_2) \Rightarrow \text{Any } p \in [0, 1] \text{ is BR.}$

Not only H or T but all combinations of H, T are BR.

BR2

NE

$p = 1/2$
 $q = 1/2$
 $\sigma_1 = (1/2, 1/2)$
 $\sigma_2 = (1/2, 1/2)$
NE.

	<p>for player 2</p> <p>Def (H, T)</p> $U_2(\sigma_1, H) = (-1)p + (+1)(1-p) = 1-2p$ $U_2(\sigma_1, T) = (+1)p + (-1)(1-p) = 2p-1$ <p>if $U_2(\sigma_1, H) > U_2(\sigma_1, T) \Rightarrow H \text{ is BR}$ $1-2p > 2p-1 \Rightarrow \frac{1}{2} > p \quad (q=1)$</p> <p>if $U_2(\sigma_1, H) < U_2(\sigma_1, T) \Rightarrow T \text{ is BR}$ $p > \frac{1}{2} \quad (q=0)$</p> <p>if $U_2(\sigma_1, H) = U_2(\sigma_1, T) \Rightarrow \text{Any } q \in [0, 1] \text{ is BR}$ $p = \frac{1}{2}$</p> <p>cc. to Nash equilibrium</p> $\sigma_i \in BR_i(\sigma_{-i}) \quad \forall i$ $(\sigma_1, \sigma_2) \text{ is NE iff}$ $\sigma_1 \in BR_1(\sigma_2)$ $\sigma_2 \in BR_2(\sigma_1)$ <p>2. Significance of Mixed Strategies:</p> <ul style="list-style-type: none"> - Mixed strategies are significant in this game because they prevent predictable patterns of play. This unpredictability forces both players to be cautious and choose their strategies with equal probabilities. - Mixed strategies ensure that, on average, each player's expected payoff is zero in the long run. This balance reflects the nature of the Matching Pennies game, where one player's gain is the other's loss. - Without mixed strategies, the game would have no Nash equilibrium in pure strategies, as any deterministic choice by one player could be exploited by the other. <p>Hence, in the Matching Pennies game, mixed strategies introduce an element of randomness and unpredictability into the players' choices, preventing exploitation and leading to a Nash equilibrium where neither player can gain an advantage by changing their strategy. This equilibrium represents a fair and balanced outcome where each player's expected payoff is zero.</p>	
	OR	
b)	<p>Could you provide an example of a real-world situation or game that can be represented as an extensive game with perfect information, and explain how the sequential decision-making process and perfect information affect the strategies of the players?</p>	5M
Ans	<p>Let us consider the game of chess as an example of a real-world situation or game that can be represented as an extensive game with perfect information. In chess, the sequential decision-making process and perfect information play a significant role in shaping the strategies of the players.</p> <p>Chess as an Extensive Game with Perfect Information:</p>	

	<ul style="list-style-type: none"> - Chess is a two-player strategy board game with well-defined rules and complete information. Each player has perfect information about the current state of the game, which includes the positions of all pieces on the board. - It is sequential in nature, with players taking turns to make moves. The order of play is fixed, and players have full knowledge of the moves made by their opponent. <p>Sequential Decision-Making Process:</p> <ol style="list-style-type: none"> 1. Sequential Moves: Players make sequential moves in chess, with each player taking turns. This sequential decision-making process allows players to plan their strategies several moves ahead, considering their own actions and their opponent's possible responses. 2. Analysis and Prediction: Because of perfect information, players can analyze the entire game tree, considering all possible moves and counter-moves. This deep analysis allows them to predict potential outcomes and select moves that maximize their chances of winning. 3. Long-Term Planning: Chess players often engage in long-term planning, setting up their pieces to control the board, protect their king, and create threats. They consider various tactical and strategic motifs, such as openings, middle-game plans, and endgame techniques. 4. Response to Opponent's Moves: Perfect information enables players to respond optimally to their opponent's moves. They can recognize threats, calculate consequences, and choose the most advantageous responses. 5. Bluff and Deception: While chess is a game of perfect information, players can introduce elements of bluff and deception through their moves, trying to mislead their opponent about their true intentions. <p>Effect on Players' Strategies:</p> <ul style="list-style-type: none"> - Optimal Play: Perfect information in chess encourages players to aim for optimal play. They strive to make the best moves at each decision point, considering both immediate tactical advantages and long-term strategic goals. - Depth of Calculation: Chess players often engage in deep calculations, analyzing multiple move sequences and variations to assess the consequences of different choices. This depth of calculation is a hallmark of perfect information games. - Strategy Development: Players develop opening strategies, tactics, and endgame techniques based on their understanding of the game's perfect information. They also adapt their strategies as the game progresses and new information is revealed. - Importance of Positional Play: Because players have full information, the positional aspect of chess becomes crucial. They focus on piece placement, control of key squares, and king safety, which contribute to their overall strategy. <p>In summary, chess is a prime example of an extensive game with perfect information. The sequential decision-making process and perfect information empower players to develop deep, strategic, and calculated approaches to the game, with each move being a carefully considered step toward achieving a winning position.</p>	
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