ASSIGNMENT 2: SQC & OR

Formulation of Trial Control Limits

SUBHRAJYOTY ROY
Roll – BS1613
B.Stat. Year III
Indian Statistical Institute

Question

Data were obtained from a process that machines cylinder bores in an engine block. The inside diameter of the cylinder bore was measured following the boring operation. Measurements were made to 1/10,000 of an inch. Sample of size $n = 5$ were taken to obtain some data to initiate $\bar{X}$ and R control charts for the process. The samples were taken roughly every half-hour. The sample measurements were all taken on the same cylinder (position-wise) in the block. The results of the first 35 samples are shown in Table 1. The actual measurements are of the form 3.5205, 3.5202, 3.5204, and 3.5209 inches, and so on. The table provides only the last three digits in the measurement. Determine the trial control limits for the process.

Solution

From the given table containing the sampled data, we obtain $\bar{X}$ and R for each of the subgroups (i.e. for each of the shifts).

Table 1: Cylinder Boring Process Data

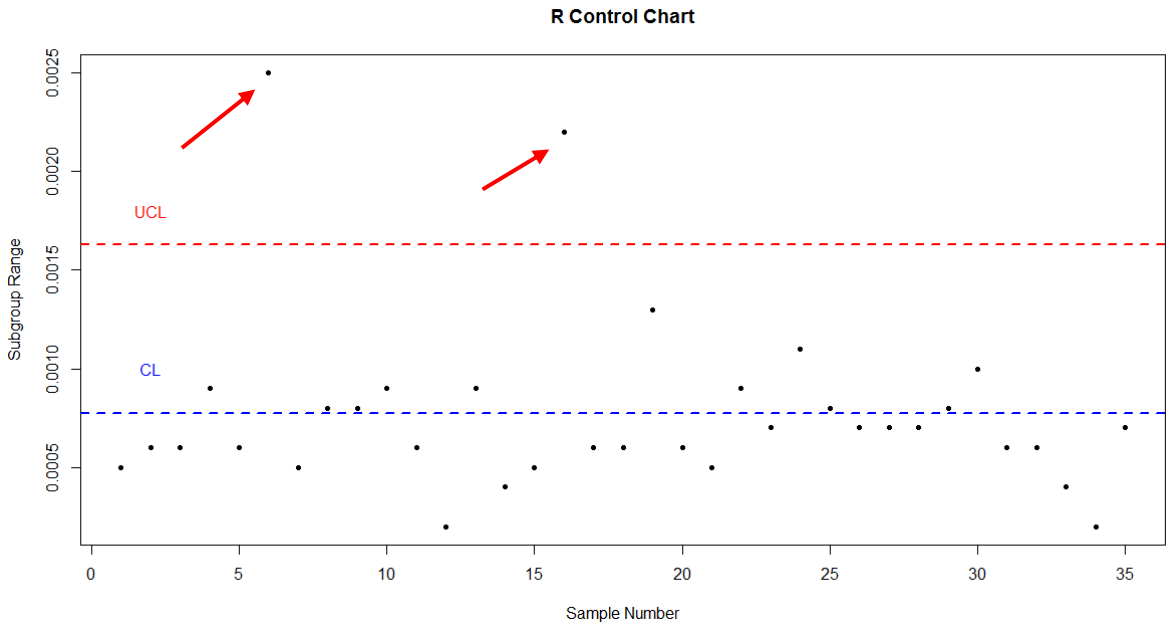
Time	Sample	1	2	3	4	5	Remarks	X(bar)	Range
8:00 AM	1	205	202	204	207	205	Startup of boring station	3.52046	0.0005
8:30 AM	2	202	196	201	198	202		3.51998	0.0006
9:00 AM	3	201	202	199	197	196		3.5199	0.0006
9:30 AM	4	205	203	196	201	197		3.52004	0.0009
10:00 AM	5	199	196	201	200	195		3.51982	0.0006
10:30 AM	6	203	198	192	217	196	Regular operator absent.	3.52012	0.0025
11:00 AM	7	202	202	198	203	202		3.52014	0.0005
11:30 AM	8	197	196	196	200	204		3.51986	0.0008
12:00 PM	9	199	200	204	196	202		3.52002	0.0008
12:30 PM	10	202	196	204	195	197		3.51988	0.0009
1:00 PM	11	205	204	202	208	205	Startup of boring station	3.52048	0.0006
1:30 PM	12	200	201	199	200	201		3.52002	0.0002
2:00 PM	13	205	196	201	197	198		3.51994	0.0009
2:30 PM	14	202	199	200	198	200		3.51998	0.0004
3:00 PM	15	200	200	201	205	201		3.52014	0.0005
3:30 PM	16	201	187	209	202	200	Regular operator absent.	3.51998	0.0022
4:00 PM	17	202	202	204	198	203		3.52018	0.0006
4:30 PM	18	201	198	204	201	201		3.5201	0.0006
5:00 PM	19	207	206	194	197	201		3.5201	0.0013
5:30 PM	20	200	204	198	199	199		3.52	0.0006
6:00 PM	21	203	200	204	199	200		3.52012	0.0005
6:30 PM	22	196	203	197	201	194		3.51982	0.0009
7:00 PM	23	197	199	203	200	196		3.5199	0.0007
7:30 PM	24	201	197	196	199	207		3.52	0.0011
8:00 PM	25	204	196	201	199	197		3.51994	0.0008
8:30 PM	26	206	206	199	200	203		3.52028	0.0007
9:00 PM	27	204	203	199	199	197		3.52004	0.0007
9:30 PM	28	199	201	201	194	200		3.5199	0.0007
10:00 PM	29	201	196	197	204	200		3.51996	0.0008
10:30 PM	30	203	206	201	196	201		3.52014	0.0010
11:00 PM	31	203	197	199	197	201		3.51994	0.0006
11:30 PM	32	197	194	199	200	199		3.51978	0.0006
12:00 AM	33	200	201	200	197	200		3.51996	0.0004
12:30 AM	34	199	199	201	201	201		3.52002	0.0002
1:00 AM	35	200	204	197	197	199		3.51994	0.0007
Average								3.52002514	0.000771

From the computation showed above, we obtain;

$\bar{\bar{X}} = 3.52002514$ and $\bar{R} = 0.000771$. Also note that, we have the subgroup sample size to be equal to 5. Hence, we obtain the first trial control limits for R-chart as follows:

$UCL_R = D_{4,5}\bar{R} = 2.114 \times 0.000771 = 0.001629894$
$LCL_R = D_{3,5}\bar{R} = 0 \times 0.000771 = 0$
And $CL_R = \bar{R} = 0.000771$.

Note that, the upper limit for the subgroup range is 0.00163 units approximately, and there are two subgroups for which the subgroup ranges exceed this control limit by a substantial amount.



Note that, specific to these subgroups, we indeed have an assignable cause of variation (i.e. absence of regular operator). Therefore, we need to remove these subgroups from consideration and recalculate the control limits. By removing those subgroups where the sample range exceeds the control limits, we obtain;

$\bar{\bar{X}} = 3.520024$ and $\bar{R} = 0.0006757$

Therefore, based on this, the new control limits for R-chart would be;

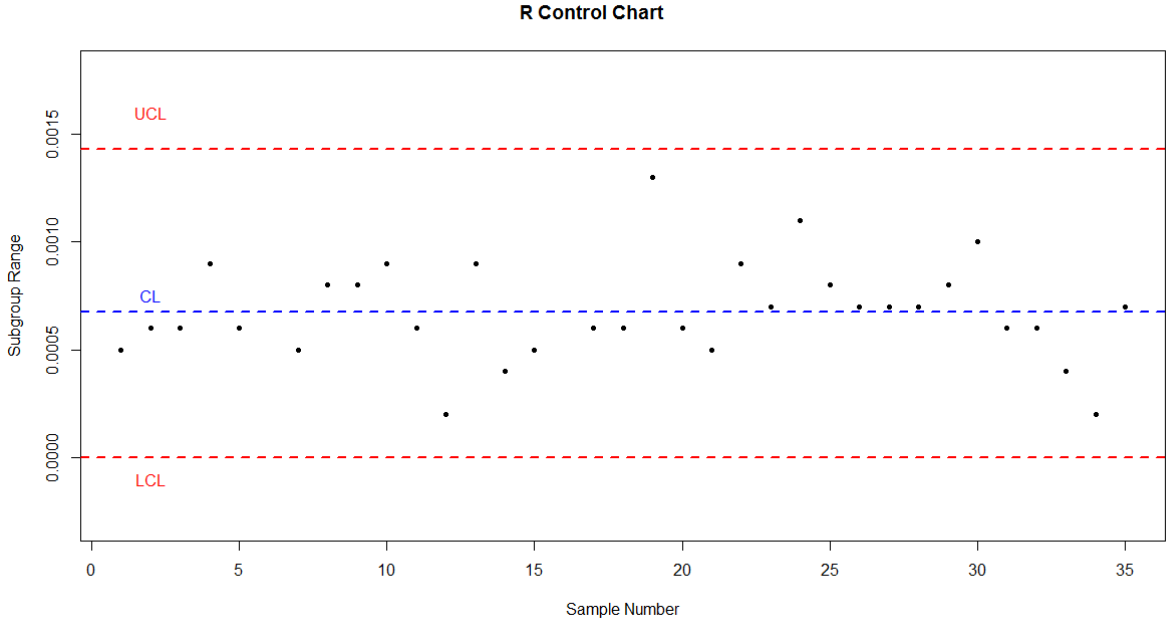
$UCL_R = D_{4,5}\bar{R} = 2.114 \times 0.0006757 = 0.001428552$
$LCL_R = D_{3,5}\bar{R} = 0 \times 0.0006757 = 0$
And $CL_R = \bar{R} = 0.0006757$.

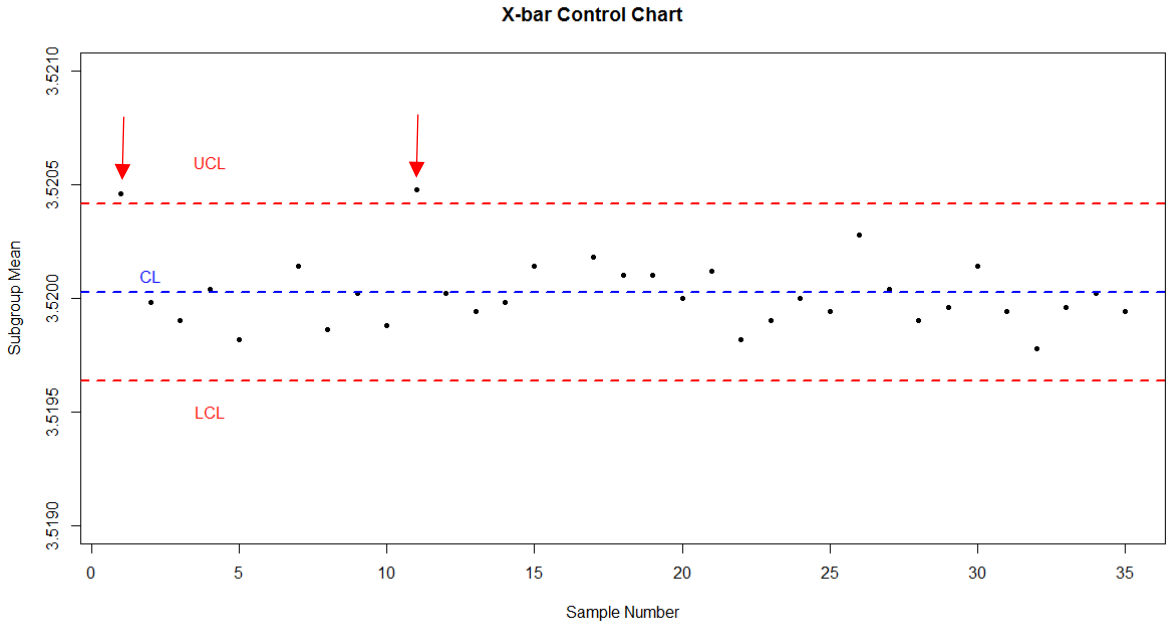
Now note that, none of the subgroups has ranges outside the new control limits.

Similarly, we obtain the control limits for $\bar{X}$-chart as follows;

$UCL_{\bar{X}} = \bar{\bar{X}} + A_{2,5}\bar{R} = 3.520024 + 0.577 \times 0.0006757 = 3.520414$
$CL_{\bar{X}} = \bar{\bar{X}} = 3.520024$
And $LCL_{\bar{X}} = \bar{\bar{X}} - A_{2,5}\bar{R} = 3.520024 - 0.577 \times 0.0006757 = 3.519634$

However, we have again two subgroups to be outside the control limits for $\bar{X}$ chart, but not by a "large" amount. But, we have an assignable cause of variation i.e. startup of boring station as possible reason. Therefore, we need to recalculate the control limits again, now removing those two subgroups.





Removing those two subgroups, and recomputing the all sample mean and ranges, we obtain;

$\bar{\bar{X}} = 3.519995$ and $\bar{R} = 0.000683871$

Similar as before, we finally obtain the control limits for R-chart as;

$\boldsymbol{UCL_R = D_{4,5}\bar{R} = 2.114 \times 0.000683871 = 0.0014457032}$

$\boldsymbol{LCL_R = D_{3,5}\bar{R} = 0 \times 0.000683871 = 0 \text{ and } CL_R = \bar{R} = 0.000683871.}$

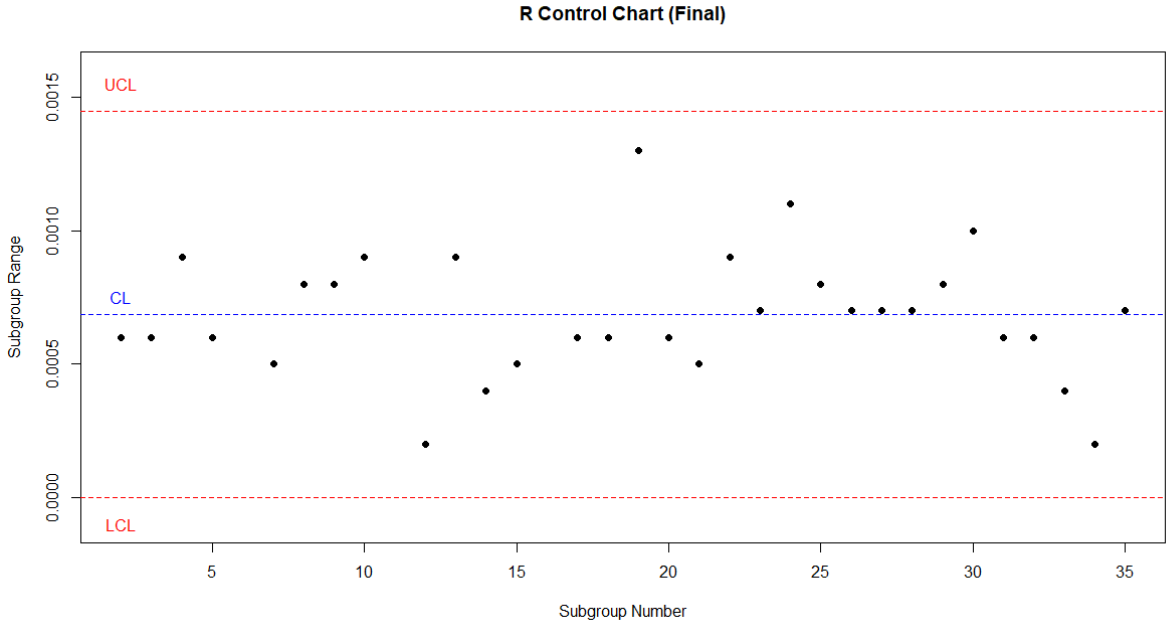
Similarly, we obtain the control limits for $\bar{X}$-chart as follows;

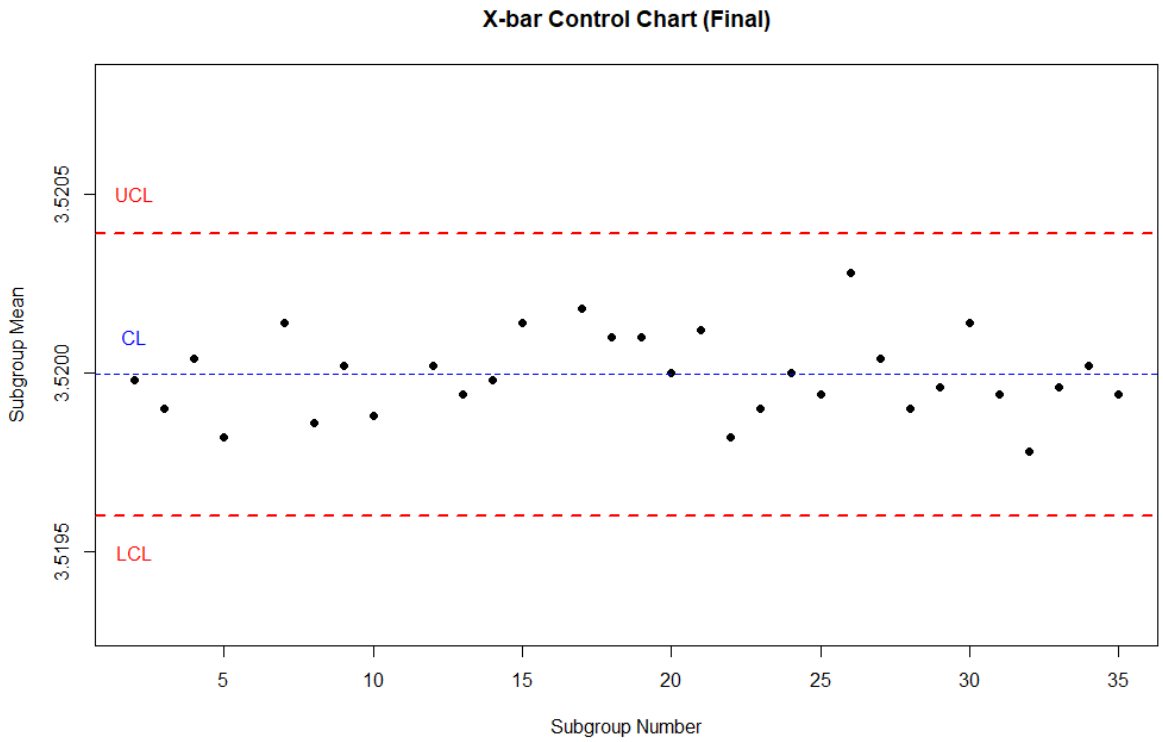
$\boldsymbol{UCL_{\bar{X}} = \bar{\bar{X}} + A_{2,5}\bar{R} = 3.519995 + 0.577 \times 0.000683871 = 3.520389}$

$\boldsymbol{CL_{\bar{X}} = \bar{\bar{X}} = 3.519995 \text{ and } LCL_{\bar{X}} = \bar{\bar{X}} - A_{2,5}\bar{R} = 3.519995 - 0.577 \times 0.0006838 = 3.5196}$

We note that, none of subgroup's mean or range now violates the control limits. Hence, we can adopt this for current use as the trial control limits.

The final charts are given in the following figures.





! THANK YOU !