DEMOGRAPHY

ASSIGNMENT 4: LIFE TABLES, FERTILITY RATES

Indian Statistical Institute, Kolkata

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SUBHRAJYOTY ROY (BS - 1613)

PALLAB DAS (BS - 1614)

UDIT SURYA SAHA (BS - 1619)

SUPRAVAT SARKAR (BS - 1648)

**Q1. The following table below gives the q-type age-specific death rates for male population in a developing country. Use it to calculate the life expectancies at birth for males in that country, assuming . Assume that no man survives until his 100-th birthday, i.e. .**

**Answer:**

From the q-type age-specific death rates we can figure out the p-type age-specific survival rates as, .

Since, the radix is given to be , hence we can figure out for each of the age groups.

Then we can obtain the number of person years lived between the ages and , using the formula;

Since the country is given to be a developing country, the mortality rates are high. In such a case, we use and , obtained from Coale-Demeny family of West Model life tables, and for higher age groups with age difference of 5 years, is taken as .

After this, we can compute the life expectancies at birth using the formula;

The calculation using the above mentioned formulas is summarized in the following table;

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Age,** | **Width,** |  |  |  |  |
| 0 | 1 | 0.1402 | 0.8598 | 10000 | 9060.66 |
| 1 | 4 | 0.0714 | 0.9286 | 8598 | 32766.13 |
| 5 | 5 | 0.0207 | 0.9793 | 7984 | 39505 |
| 10 | 5 | 0.015 | 0.985 | 7818 | 38795 |
| 15 | 5 | 0.022 | 0.978 | 7700 | 38075 |
| 20 | 5 | 0.0313 | 0.9687 | 7530 | 37060 |
| 25 | 5 | 0.0343 | 0.9657 | 7294 | 35842.5 |
| 30 | 5 | 0.0393 | 0.9607 | 7043 | 34522.5 |
| 35 | 5 | 0.0466 | 0.9534 | 6766 | 33040 |
| 40 | 5 | 0.0578 | 0.9422 | 6450 | 31317.5 |
| 45 | 5 | 0.0715 | 0.9285 | 6077 | 29297.5 |
| 50 | 5 | 0.0947 | 0.9053 | 5642 | 26872.5 |
| 55 | 5 | 0.125 | 0.875 | 5107 | 23937.5 |
| 60 | 5 | 0.1751 | 0.8249 | 4468 | 20382.5 |
| 65 | 5 | 0.2412 | 0.7588 | 3685 | 16202.5 |
| 70 | 5 | 0.3362 | 0.6638 | 2796 | 11627.5 |
| 75 | 5 | 0.4549 | 0.5451 | 1855 | 7165 |
| 80 | 5 | 0.6126 | 0.3874 | 1011 | 3505 |
| **Age,** | **Width,** |  |  |  |  |
| 85 | 5 | 0.7707 | 0.2293 | 391 | 1200 |
| 90 | 5 | 0.8984 | 0.1016 | 89 | 245 |
| 95 | 5 | 1 | 0 | 9 | 22.5 |
| **Total** | - | - | - | - | 470441.8 |

Therefore, the life expectancy at birth is given by; years, i.e. about 47 years and 15 days.

**Q2. Show that the crude birth rate in a stationary population corresponding to a life table is equal to , where is the life expectation at birth.**

**Answer:**

We know that;

where each of the age groups considered are of length equal to 1 year. Now, observe that, is the number of person years lived by the population between age to age .

Since, we are considering that the population is stationary, i.e.

1. The age-specific death rates are constant over time. So, is also constant over time.
2. The flow of births is constant over time.
3. Net migration is zero at all ages.

Therefore, the person year lived by the population between age to age is uniformly spread over the age interval. Consider a single person aged years, then as the population is stationary, so the person-year lived by that single person can be broken up into several parts, each representing the person-year lived by that person in each group, possibly in different time.

Therefore,

Now, observe that, because of stationary population, number of person year lived during the age in past times is constant over time, hence is equal to . Hence, we obtain that,

Also, note that, , the radix is the population in the age group to , i.e. is the population of new live birth cohorts. As the flow of the birth is constant in the stationary population, the number of live births during the year is equal to the number of birth cohorts i.e. .

Therefore,

**Q3. In a life table, is equal to , and**

**Show that, .**

**Answer:**

Observe that, is the probability that a person survives till age 1 given that he/she is born, i.e. of age 0, hence clearly,

Therefore, the given formula for can be rewritten as , where lies between 0 and 1.

Now, observe that, is the number of person years lived by the cohort between ages to . Clearly, should be the infinitesimal sums of number of surviving cohorts at every instant between age and . This reduces to the formula;

Using the above formula for and the given expression for , we can calculate;

Therefore, i.e. the number of person years lived by the cohort after age , can be expressed as;

Now, from the given information that , and , we obtain;

This proves the result.

**Q4. The table below gives estimates of the life expectation at various ages for females in Nicaragua, 1990-95 and the United States, 1989. Use them to estimate , , and for these two countries.**

**Answer:**

We know that, the life expectation at age , i.e. . Therefore, we can rewrite it as;

Therefore, we can consider the successive difference of the above to obtain;

However, as show in the answer to question 1, we can write as a weighted sum of the ’s where the weights depend only on . Therefore,

Since, , we obtain the final formula as;

To obtain the values of , we again use the Coale-Demeny family of West model life tables, and for female population, we have and and 2.5 for the rest of the age groups. The necessary calculations are summarized in the following table;

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Age,** | **Nicaragua, 1990-95** | | **United States, 1989** | |
|  |  |  |  |
| 0 | 67.7 | 0.046709 | 78.6 | 0.008866 |
| 1 | 70 | 0.024169 | 78.3 | 0.001298 |
| 5 | 67.7 | 0.00761 | 74.4 | 0.001389 |
| 10 | 63.2 | 0.004918 | 69.5 | 0.00149 |
| 15 | 58.5 | - | 64.6 | - |

**Q5. If the crude birth rate in a country remains constant over a number of years, but the general fertility rate increases steadily, what does this tell you about the country’s population?**

**Answer:**

We know that,

And

Now, if the Crude Birth Rate (CBR) remains constant over a number of years, that means the number of live births as a proportion of the whole population is not changing over the years. However, as General Fertility Rate (GFR) steadily increases, it indicates that the number of live births as proportion of the women population of reproductive age group is increasing. This indicates that the denominator is the GFR formula is decreasing over the years as proportion of the whole population, i.e. the number of women in reproductive age group as proportion of the whole population.

The above claim is also indicated by the following formula which can be obtained by taking the ratio of CBR to GFR, i.e.

As the numerator is held constant over time, but the denominator is increasing, this indicates the ratio being declined over time, which shows a steady decrement of the female share in the age distribution.

**Q6. The data below relate to fertility in a country in 1976 and 1993.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Age Group** | **1976** | | **1993** | |
| **No. of births (‘000)** | **Mid-year female population (‘000)** | **No. of births (‘000)** | **Mid-year female population (‘000)** |
| 15-19 | 57.9 | 1809 | 45.1 | 1455 |
| 20-24 | 182.2 | 1672 | 152.0 | 1831 |
| 25-29 | 220.7 | 1855 | 236.0 | 2070 |
| 30-34 | 90.8 | 1593 | 171.1 | 1967 |
| 35-39 | 26.1 | 1374 | 58.8 | 1729 |
| 40-44 | 6.5 | 1300 | 10.5 | 1750 |

1. **Calculate the general fertility rate for 1976 and 1993.**
2. **Calculate age specific fertility rate for these two years.**
3. **Using the 1976 population as standard calculate the standardized fertility rate for 1993.**

**Answer:**

To calculate the General Fertility Rate (GFR), we use the following formula;

The total number of live births during the year 1976 is = (57.9 + 182.2 + 220.7 + 90.8 + 26.1 + 6.5) = 584.2 thousands. And, the mid-year female population in reproductive period during the year 1976 is = (1809 + 1672 + 1855 + 1593 + 1374 + 1300) = 9603 thousands.

Therefore, children per thousand females.

The total number of live births during the year 1993 is = (45.1 + 152.0 + 236.0 + 171.1 + 58.8 + 10.5) = 673.5 thousands. And, the mid-year female population in reproductive period during the year 1993 is = (1455 + 1831 + 2070 + 1967 + 1729 + 1750) = 10802 thousands.

Therefore, children per thousand females.

**Hence, General Fertility Rate for the year 1976 is 60.83 children per thousand females and for the year 1993 is 62.35 children per thousand females.**

To calculate the age-specific fertility rates for different years, we use the following formula;

The necessary results for Age-specific fertility rates are shown in the following table;

|  |  |  |
| --- | --- | --- |
| **Age group** | **Age-specific Fertility Rate in the year 1976**  **(per ‘000 females)** | **Age-specific Fertility Rate in the year 1993**  **(per ‘000 females)** |
| 15-19 | 32.00663 | 30.99656 |
| 20-24 | 108.9713 | 83.01475 |
| 25-29 | 118.9757 | 114.0097 |
| 30-34 | 56.99937 | 86.98526 |
| 35-39 | 18.99563 | 34.0081 |
| 40-44 | 5 | 6 |

If we use the year 1976 as the standard, then we can calculate the expected number of live births to females in each age group, from the fertility distribution in different age groups in the year 1993, and using the age distribution of the female population in 1976 as the standard. We use the following formula for each age group to calculate the expected number of live births;

The necessary calculation is shown in the table below;

|  |  |  |  |
| --- | --- | --- | --- |
| **Age Group** |  | **Mid-year female population (‘000)** | **Expected number of live births (‘000)** |
| 15-19 | 30.99656 | 1809 | 56.07 |
| 20-24 | 83.01475 | 1672 | 138.8 |
| 25-29 | 114.0097 | 1855 | 211.49 |
| 30-34 | 86.98526 | 1593 | 138.57 |
| 35-39 | 34.0081 | 1374 | 46.73 |
| 40-44 | 6 | 1300 | 7.8 |
| **Total** | - | 9603 | 599.46 |

Therefore, the standardized fertility rate for the year 1993 is given as; children per thousand females.

Clearly, this also indicates that the fertility has been increased in 1993 compared to 1976 in the country.

**Q7. The table below gives the parity progression ratios for a number of recent birth cohorts in a country. Assuming that no woman in any of these birth cohorts had a fifth child, calculate;**

1. **The proportion of women in each birth cohort who had exactly 0, 1, 2, 3 and 4 children.**
2. **The total fertility rate for women in each birth cohort.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Calendar years of birth** | **Parity Progression Ratio (PPR)** | | | |
| **0-1** | **1-2** | **2-3** | **3-4** |
| 1931-33 | 0.861 | 0.804 | 0.555 | 0.518 |
| 1934-36 | 0.885 | 0.828 | 0.555 | 0.489 |
| 1937-39 | 0.886 | 0.847 | 0.543 | 0.455 |
| 1940-42 | 0.890 | 0.857 | 0.516 | 0.416 |
| 1943-45 | 0.892 | 0.854 | 0.458 | 0.378 |
| 1946-48 | 0.885 | 0.849 | 0.418 | 0.333 |

**Answer:**

Suppose, is the total female population in the reproductive period, and to be the number of females in the reproductive age group with parity . Hence,

Also, let us denote the parity progression ratio from parity to by . Clearly, is the probability that a women will have another child, i.e. will increase in parity given that she already is at parity .

Therefore, assuming no women has parity 5 or more,

Hence,

Hence, observe that;

which is the proportion of female population having parity exactly equal to .

Therefore, we use the following formula to calculate the proportions of female having parity ,

The answers are given in the following tabular form.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Calendar years of birth** | **Proportion of Women having exactly a specified parity** | | | | |
| **0** | **1** | **2** | **3** | **4** |
| 1931-33 | 0.139 | 0.168756 | 0.308049 | 0.185182 | 0.199013 |
| 1934-36 | 0.115 | 0.15222 | 0.326087 | 0.20782 | 0.198873 |
| 1937-39 | 0.114 | 0.135558 | 0.342952 | 0.222082 | 0.185408 |
| 1940-42 | 0.11 | 0.12727 | 0.369161 | 0.229844 | 0.163725 |
| 1943-45 | 0.108 | 0.130232 | 0.412878 | 0.217009 | 0.13188 |
| 1946-48 | 0.115 | 0.133635 | 0.437294 | 0.209485 | 0.104585 |

Now, to obtain the Total Fertility Rate (TFR) for women in each birth cohort, we compute the Completed Fertility Rate (CFR) for each birth cohort which is known to be equivalent of Total Fertility Rate (TFR). The formula for computing CFR is as follows;

since, for .

The answers are summarized in the following table;

|  |  |
| --- | --- |
| **Calendar years of birth** | **Completed Fertility Rate (CFR) or Total Fertility Rate (TFR) for birth cohorts (‘000)** |
| 1931-33 | 2.136453 |
| 1934-36 | 2.223346 |
| 1937-39 | 2.22934 |
| 1940-42 | 2.210023 |
| 1943-45 | 2.134538 |
| 1946-48 | 2.055021 |

THANK YOU