

Mathematics Talent Reward Programme

Question Paper for Class IX

14th January, 2018

Total Marks: 100

Allotted Time: 02:00 p.m. to 04:30 p.m.

Multiple Choice Questions

[*You should answer these questions in the first page according to the order given in the question paper. Each question has only one correct option. You will be awarded 4 marks for the correct answer, 1 mark if the question is not attempted and 0 marks for wrong answer.*]

1. The number of solutions of the equation $\frac{5}{2-x} + \frac{x-5}{x+2} + \frac{3x+8}{x^2-4} = 0$ is
(A) 2, (B) 1, (C) 0, (D) None of these.
2. How many triangles ABC are there such that $\angle A = 60^\circ$, $AB = 3$ cm and $BC = 2$ cm ?
(A) 1, (B) 2, (C) 6, (D) None of these.
3. Let $3|a^2 + a + 1$, where a is an integer. Which of the following is false ?
(A) $3|a^2 - 1$, (B) 3 does not divide $a + 1$, (C) $a^3 + 2$ is not divisible by 3, (D) None of these.
4. Which of the following is true ?
(A) Last digit of 18^{19} is prime, (B) Last digit of 17^7 is 7, (C) Last digit of 17^9 is not 7, (D) None of these.
5. How many ordered pairs (a, b) of rational numbers are there such that $\frac{a}{b} + \frac{b}{a} = 3$?
(A) 1, (B) 2, (C) Infinitely many, (D) None of these.
6. Let ABC be an isosceles triangle with $AB = BC$ and $\angle B = 20^\circ$. Then which of the following is true ?
(A) $AB > 3AC$, (B) $2AC < AB < 3AC$, (C) $AC < AB < 2AC$, (D) None of these.
7. What is the product of the real roots of the equation $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$?
(A) 40, (B) 20, (C) $\sqrt{30}$, (D) None of these.

Short Answer Type Questions

[*Each question carries a total of 12 marks. Credits will be given to partially correct answers*]

1. Let $ABCD$ be a parallelogram. Let E be the midpoint of CD . Let O be the intersecting point of the lines AE and BD . Find $AO : OE$.
2. How many ordered triplets of integers (a, b, c) are there such that $a^3bc = 24$?
3. There are 5 pairs of balls which are kept in 5 different boxes (i.e. each box has 2 identical balls). In one of the boxes both the balls weigh $9g$ each. In the remaining 4 boxes all balls weigh $8g$ each. You have a weighing machine with 2 pans. If you put some balls on the left pan and some on the right, its reading will show you the (value of the weight on the left pan - value of weight on the right pan). Find the minimum number of times the weighing machine needs to be used in order to identify the balls which are heavier.
4. How many natural numbers are there such that the sum of the digits of the number equals the product of the digits of the number in its decimal representation? (Note that the sum and product of a single digit number is the number itself.)
5. Prove that there exist infinitely many perfect squares starting with the digits "2018".
6. Prove that there do not exist 2 natural numbers n_1, n_2 such that 9^{n_1} and 9^{n_2} are palindrome numbers and the difference in the number of digits of 9^{n_1} and 9^{n_2} is 2017. (Note: A palindromic number is a number that remains the same when its digits are reversed, for example 313, 121, 2332 are palindrome numbers.)

Use of calculators is not allowed. You may use a ruler and a compass for construction.

~ Best of Luck ~