Mathematics Talent Reward Programme

Model Solutions for Class IX

Multiple Choice Questions

[*Each question has only one correct option. You will be awarded 4 marks for the correct answer, 1 mark if the question is not attempted and 0 marks for wrong answer.*]

1. **(C)**	2. **(B)**	3. **(D)**	4. **(B)**	5. **(B)**
6. **(C)**	7. **(D)**	8. **(B)**	9. **(A)**	10. **(D)**
11. **(B)**	12. **(C)**	13. **(C)**	14. **(B)**	15. **(C)**

Short Answer Type Questions

[*Each question carries a total of 15 marks. Credit will be given to partially correct answers*]

1. Let velocity of the column be $u\ cm/s$ and velocity of the rebel ant be $v\ cm/s$. So, the relative velocity of the rebel ant with respect to the column will be $(v - u)\ cm/s$ when going forward and $v + u\ cm/s$ when coming back.

   $\therefore$ Total time taken $= \frac{15}{v-u} + \frac{15}{v+u}\ s$. Also distance travelled by the column is $\left(\frac{15}{v-u} + \frac{15}{v+u}\right) \cdot u$ which is given to be 15 cm.

$$\begin{aligned}
&\therefore \left(\frac{15}{v-u} + \frac{15}{v+u}\right) \cdot u = 15 \\
&\implies \frac{1}{\frac{v}{u} - 1} + \frac{1}{\frac{v}{u} + 1} = 1 \\
&\implies 2\left(\frac{v}{u}\right) = \left(\frac{v}{u}\right)^2 - 1 \\
&\implies \left(\frac{v}{u}\right)^2 - 2\left(\frac{v}{u}\right) - 1 = 0 \\
&\implies \left(\left(\frac{v}{u}\right) - 1\right)^2 = 2 \\
&\implies \left(\frac{v}{u}\right) - 1 = \pm\sqrt{2} \\
&\implies \left(\frac{v}{u}\right) = 1 + \sqrt{2}\ \left[\because 1 - \sqrt{2} < 0 \text{ and } \left(\frac{v}{u}\right) > 0\right]
\end{aligned}$$

   So, the distance travelled by the rebel ant is

$$\left(\frac{15}{v-u} + \frac{15}{v+u}\right) \cdot v = \left(\left(\frac{15}{v-u} + \frac{15}{v+u}\right) \cdot u\right) \cdot \frac{v}{u} = 15 \cdot \frac{v}{u} = 15(1 + \sqrt{2})\ cm.$$

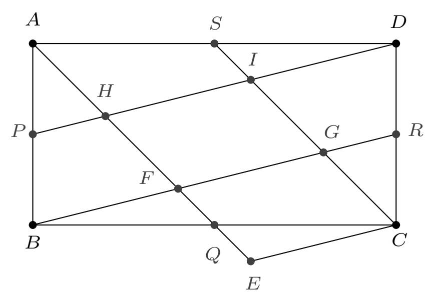
2. If we partition the numbers 1,2,...,13 into 2 groups then one of them must contain the number 13 and the other group will not. Then product pf the elements in the group containing 13 will be divisible by 13 whereas the product of the elements in he other group will not be divisible by 13. Hence a partition such that the product of elements in both groups is the same is not possible.

3. Consider a 3×3 chessboard and the label the squares as shown below.

1	4	7
2	5	8
3	6	9

Now consider the cyclic path of the knight's move $1 \to 8 \to 3 \to 4 \to 9 \to 2 \to 7 \to 6 \to 1$ So, in a 3×3 board the knight can move from any square to any square except the middlemost. Call this 3×3 board without the middlemost square a 3×3 ring. Now you can cover a 8×8 chessboard with 3×3 overlapping rings and so you can move from any square in a ring to any other square in that ring as well as the squares in rings with which it overlaps. Thus you can traverse the whole chessboard with a knight.

4. Draw a line through C parallel to BR. Let it intersect extended AQ at E. Let BR intersect AQ at F and SC at G and DP intersect them at H and I respectively.



Now consider ΔBFQ and ΔCEQ.

$$\begin{aligned} BQ &= QC \\ \angle BQF &= \angle CQE \ \text{[Vertically Opposite Angle]} \\ \angle QBF &= \angle QCE \ [\because BF \parallel EC\,] \\ \therefore \Delta BFQ &\cong \Delta CEQ \end{aligned}$$

Now observe that in ΔABQ and ΔCDS

$$\begin{aligned} AB &= CD \\ BQ &= DS \\ \angle ABQ &= \angle CDS \\ \therefore \Delta ABQ &\cong \Delta CDS \\ \therefore \angle AQB &= \angle DSC = \angle SCQ \ [\because BC \parallel AD\,] \\ \implies AE &\parallel CS \end{aligned}$$

Similarly, we can show that $BR \parallel DP$.
Now we get that, $DP \parallel BR \parallel CE$ so, we can conclude that

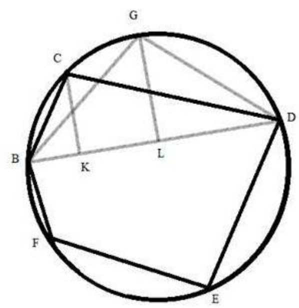
$$\frac{CG}{GI} = \frac{CR}{RD} = 1 \implies CG = GI.$$

Hence parallelograms $ECGF$ and $FGIH$ has same height $[\because AE \parallel CS\,]$ and same base length $[\because CG = GI\,]$. So, they have same area. Hence,

$$\text{Area of } \Delta CDI = \text{Area of } \Delta DAH = \text{Area of } \Delta ABF$$

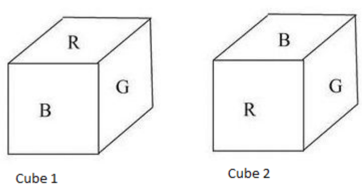
So finally we get Area of $FGIH = \frac{\text{Area of } ABCD}{5} = \frac{\Delta}{5}$.

5. Let us consider a pentagon $BCDEF$ inscribed in the circle which is irregular. Then $BCDEF$ has at least a pair of consecutive sides whose lengths are different. Let these sides be BC and CD (as shown in the figure). Now join BD and let G be the midpoint of the circular arc BCD. Perpendiculars CK and GL are dropped upon BD. It can be easily seen that $GL > CK$ and hence area of ΔBGD is greater than that of ΔBCD [as they both have same base BD]. Now area of the pentagon $BGDEF$ = area of the quadrilateral $BDEF$ + area of triangle BGD. Also area of the pentagon $BCDEF$ = area of the quadrilateral $BDEF$ + area of triangle BCD. Hence area of pentagon $BGDEF$ (which is also inscribed in the circle) is greater than the area of the pentagon $BCDEF$. So we conclude that $BCDEF$ cannot be a pentagon which has the maximum area among all the inscribed pentagons.

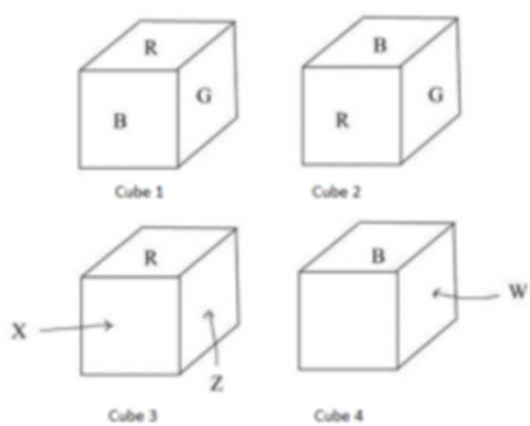


But note that if the pentagon was regular then the points C and G would have coincided, in other words, we could not have drawn another pentagon inscribed in the circle but with greater area. Thus for the area to be maximum the pentagon has to be regular.

6. (a) First we see that due to the colouring scheme each small cube has two blue faces, two red faces and two green faces. Let us draw a clearer picture of the adjacent cubes coloured red and blue.



We see that the touching faces cannot be blue because then blue would be used in four of the faces. Similarly it cannot be red and hence must be green. Then the top face of cube 1 must be red and that of cube 2 must be blue. Now we will look at the cube just below cube 1 and cube 2. Let us call them cube 3 and cube 4 respectively.

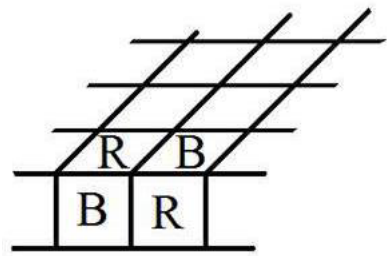


As the bottom face of cube 1 is also red the top face of cube 3 must be red. (As touching faces are of the same colour). Similarly the top face of cube 4 is blue.

Now the face marked X is either blue or green. If it is green then face marked Z is blue and the face of cube 4 touching Z must also be blue. But the top face of cube 4 is blue which makes blue appear 4 times in cube 4. So, X must be blue and Z is green. Hence W is also green which leaves the front face of cube 4 red in colour.

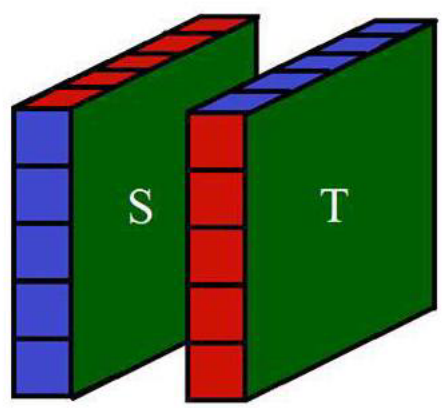
In the same process the cube lying below cube 3 will have blue colour in its front face and so on for the all the cubes lying below. By a similar argument the cubes lying above cube 1 will have blue colour in their front faces. So, the entire column will be blue in colour. Similarly the entire column containing the face with red colour will be red.

(b) Now consider the cubes lying at the top of the blue column and red column. It can easily be seen that their top faces are red and blue respectively as the touching face will be green as shown before.



Now rotate the large cube so that the face lying at the top previously now faces you. Then by the argument of part (a) the entire column containing the square with the red face must be coloured red. As P is a part of the strip Y, the colour of Y is red.

(c) Assume that none of the faces of the larger cube has squares of the same colour. Then there exists a face where there are adjacent squares of different colour. Let us assume that they are red and blue. By the argument in part (a) and (b) of the problem we get a figure as shown below.



Then the surfaces marked S and T will be green. As all touching faces and opposite faces are of the same colour the face of the large cube lying parallel to S and T will be green. Similarly if we would have started with the adjacent squares being red and green then we would have gotten a blue coloured face. In the remaining case of them being blue and green we would get a red face. So in any case we get a face with all 25 squares in it of the same colour.