Mathematics Talent Reward Programme

Question Paper for Class IX

18th January, 2015

Total Marks: 150

Allotted Time: 2:00 p.m. to 4:30 p.m.

Multiple Choice Questions

[**You should answer these questions in the first page according to the order given in the question**. *Each question has only one correct option. You will be awarded 4 marks for the correct answer, 1 mark if the question is not attempted and 0 marks for wrong answer.*]

1. In a ceremony all the guests present shook hands with each other. There were 55 handshakes observed. How many guests were there?

   (A) 9,　　　　　　　**(B)** 10,　　　　　　　**(C)** 11,　　　　　　　**(D)** 12.

2. Let x be a two-digit prime number such that if its digits are interchanged, we get a new prime number y. If the difference between x and y is 18, then what is the value of $5xy$?

   (A) 2005,　　　　　　**(B)** 2015,　　　　　　**(C)** 2025,　　　　　　**(D)** 2035.

3. How many times during a day do the hour hand and the minute hand of a clock make a right angle?

   (A) 23,　　　　　　　**(B)** 24,　　　　　　　**(C)** 25,　　　　　　**(D)** None of these.

4. Two trains start from A and B and travel towards each other at 50 km/h and 60 km/h respectively. At the time of meeting the second train has travelled 120 km more than the first one. Find the initial distance between them in kilometers.

   (A) 1280,　　　　　　**(B)** 1320,　　　　　　**(C)** 1300,　　　　　　**(D)** 1380.

5. What is the last digit of 7^{2015}?

   (A) 1,　　　　　　　**(B)** 3,　　　　　　　**(C)** 7,　　　　　　　**(D)** 9.

6. What is the area of the region in the figure below?
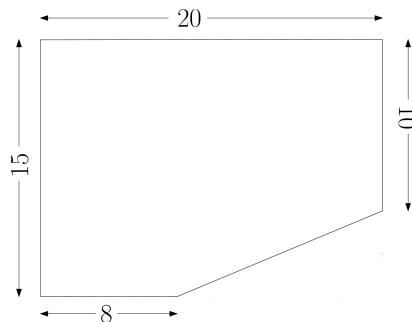
   (A) 250,　　　　　　**(B)** 270,　　　　　　**(C)** 300,　　　　　　**(D)** 320.

7. What is the length of the segment AB in the figure below?
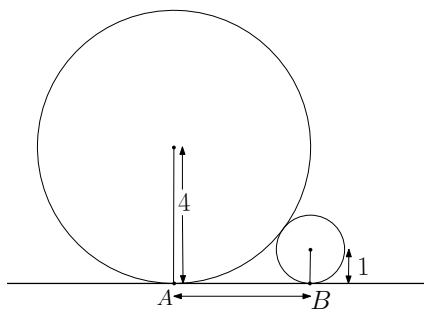
   (A) 4,　　　　　　　**(B)** 5,　　　　　　　**(C)** 3,　　　　　　　**(D)** $3\sqrt{3}$.

8. If the figure below, the square has sides of length 2 units. Two arcs of a circle are drawn with centers at A and C respectively and radius 2 units. What is the area of the shaded region?
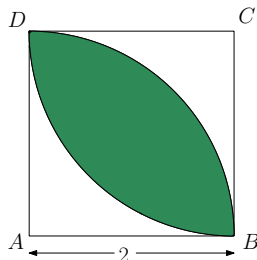
(A) 2π,   (B) $2\pi - 3$,   (C) $2(\pi - 2)$,   (D) $2(\pi - 3)$.

9. A can contains a mixture of two liquids A and B in the ratio 7:5. When 9 litres of the mixture are drawn and replaced by the same amount of liquid B, the ratio of A and B becomes 7:9. How many litres of liquid A was contained in the can initially?

(A) 18,   (B) 19,   (C) 20,   (D) None of these.

10. From a square with sides of length 2m, corners are cut away so as to form a regular octagon. What is the area of the octagon in sq.m?

(A) $2\sqrt{3}$,   (B) $\frac{4}{\sqrt{3}}$,   (C) $4(\sqrt{2} - 1)$,   (D) None of these.

11. Solve for x and y: $\sqrt{9x^2 - 30x + 74} + \sqrt{4y^2 + 28y + 74} = 12$

(A) $x = \frac{5}{3}, y = \frac{7}{2}$,   (B) $x = \frac{5}{3}, y = -\frac{7}{2}$,   (C) $x = \frac{7}{2}, y = \frac{5}{3}$,   (D) $x = \frac{7}{2}, y = -\frac{5}{3}$.

12. Define a sequence by $a_0 = a_1 = 1$ and $a_n = a_{n-1}a_{n-2} + 1$ for $n > 1$. Then

(A) a_{2015} is odd and a_{2016} is odd,   (B) a_{2015} is odd and a_{2016} is even,
(C) a_{2015} is even and a_{2016} is odd,   (D) a_{2015} is even and a_{2016} is even.

13. Consider the point (4,0) on the x, y-plane. Now if axes are rotated 45° anticlockwise then what are the new co-ordinates of this point in the new system?

(A) $(2\sqrt{2}, 2\sqrt{2})$,   (B) $(2\sqrt{2}, 0)$,   (C) $(-2\sqrt{2}, -2\sqrt{2})$,   (D) $(4\sqrt{2}, 2\sqrt{2})$.

14. Find the area of the region in the co-ordinate plane which satisfies all the following inequalities.

$$x + y \leq 1,$$
$$x - y \leq 1,$$
$$y - x \leq 1,$$
$$x + y \geq -1.$$

(A) $\sqrt{2}$,   (B) 2,   (C) $2\sqrt{2}$,   (D) 4.

15. Consider a hollow paper cone with slant height 4 cm. It is cut along the slant surface and unfolded to make a sector. This sector subtends an angle of 60° at the center. What is the surface area of the cone in sq.cm?

(A) $\frac{5\pi}{3}$,   (B) 2π,   (C) $\frac{8\pi}{3}$,   (D) 3π.

Short Answer Type Questions

[Each question carries a total of 15 marks. Credit will be given to partially correct answers]

1. A 15 cm long column of ants starts crawling. A rebel ant at the end of the column steps out and starts marching forward at a higher speed than the column. On reaching the front of the column, it immediately turns around and marches back at the same speed. When he reaches the end of the column he finds that the column of the remaining ants has moved exactly 15 cm. What distance did the rebel ant travel?

2. Consider the numbers 1,2,3,4,...,13. Can these numbers be partitioned into two groups such that the products of the elements are same in both groups?

3. In triangle $\triangle BAC$ with $\angle ABC = 30°$. D is the midpoint of BC. We join A and D and $\angle CDA = 45°$. Find $\angle BAC$.

C
45° D
30° B
A
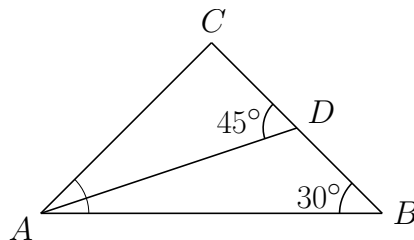
4. Let P, Q, R, S be the midpoints of the sides AB, BC, CD and DA respectively of a rectangle $ABCD$. If the area of the rectangle is Δ, then calculate the area bounded by the straight lines AQ, BR, CS and DP in terms of Δ.

5. A pentagon is inscribed inside a fixed circle. Show that for the area of the pentagon to be maximum, it must be a regular one.

6. There are 125 unit cubes each of whose faces are coloured with blue, green and red such that each colour is used at least once and opposite faces have the same colour. Now, a $5 \times 5 \times 5$ cube is constructed using these small cubes such that the touching faces are of the same colour.

   i) Suppose two adjacent squares on one face of that whole cube are of different colours (say blue and red as in the figure below). Then show that the entire column containing the red square will be coloured red. Also the same should hold for the column containing the blue square. [7]

  ii) Show that the squares in the strip marked Y should also be of the same colour. Also find this colour. [3]

 iii) Hence or otherwise show that there exists a face of the large cube in which all the squares are of the same colour. [5]

Y

B R
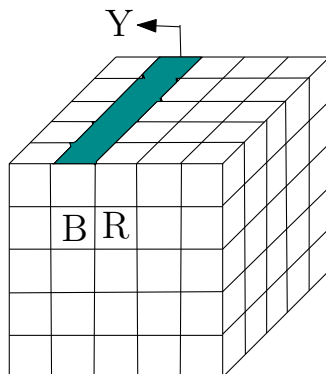
Use of calculators is not allowed. You may use a ruler and a compass for construction.
~ Best of Luck ~