Topics: Permutation tests, logistic regression   Due on November 16, 2019

Name of student:

Roll number:

1. Permutation test of independence   [8]

Perform a permutation test of independence on Table 1. Compare with the standard χ^2 and likelihood ratio tests.

| | Handedness | | | |
Gender	Right-handed	Left-handed	Ambidextrous	Total by Gender
Male	14	3	3	20
Female	13	1	2	16
Other	6	1	1	8
Total by Handedness	33	5	6	44

Table 1: A fictitious dataset.

2. Exact logistic regression   [12]

In a standard binomial logistic regression set-up

$$\text{logit}(\mathbb{P}(Y_i = 1)) = \beta_0 + \beta^\top X_i, \qquad i = 1, \ldots, n,$$

write down sufficient statitsics T_j for the parameters β_j, $j = 0, \ldots, p$. Show that the distribution of T_p conditional on $T_0, \ldots, T_{p-1}$ depends only on β_p. Thus, using this conditional distribution, one can estimate and perform inference on β_p. Does a conditional MLE always exist?

Write down the conditional distribution under $H_0 : \beta_p = 0$. Describe how you would do an exact test of this hypothesis.

Use the **R** packages `logistiX` and `elrm` to perform exact logistic regression on the data in Table 2 with "White-collar job" as response, and "Gender" and "College education" as explanatory variables. Also, perform

Gender	College education	White-collar job	Number of cases
M	No	1	8
F	No	1	6
M	Yes	7	10
F	Yes	6	6

Table 2: Another fictitious dataset.

a standard logistic regression on the same data and compare the results.